Math 17 - Final Exam Details

The final exam will happen on Monday, December 10 @ 4:30pm. The exam is weighted as 22% of your final grade. A few things:

- **Please bring a calculator to the exam. You will not be permitted to use your phone as a calculator on the exam.**
- The exam is closed-book and closed-notes. You are not permitted to use technology other than your calculator.
- You must comply with the UVM Academic Code of Integrity. Cheating will not be tolerated. All work must be your own.
- You must show your work to receive full credit. Partial credit will be given if sufficient work is shown.

Format of Exam

The final exam will be of similar length as the previous exams and will be **semi-cumulative**. The official exam slot is 4:30-7:15pm, and I will allow students to be there the whole time if needed, but I do not anticipate this happening.

There will be ~ 4 questions (multi-part) on Graphs and Networks, and similarly there will be ~ 4 questions from previous chapters we have covered. See the study guide for specific chapters and concepts that you are responsible for knowing. You should find the study guide to be sufficient preparation for the exam.

Math 17 - Final Exam Study Guide

Chapter 5: Graphs and Networks

Term	Definition
Graph	A collection of objects and links between them
Vertex	The objects - usually labeled with letters A, B, C, D, etc. or numbers. Plural of vertex is vertices.
Edges	The links. An edge from vetex A to vertex B is denoted by AB.
Adjacent vertices	Two vertices are adjacent if an they are connected by an edge.
Loop	An edge from a vertex to itself, i.e. AA is a loop.
Degree (of a vertex)	The number of edges leading to the vertex. A loop counts as 2.
Even/Odd vertices	A vertex is even if its degree is even and odd if its degree is odd.
Clique	A set of vertices that are all connected to each other.
Simple graph	A graph with no loops (i.e., no AA) or multiple edges (i.e., no AB,AB)
Adjacent edges	Two edges are adjacent if they lead into the same vertex.
Path	A sequence of distinct edges from one vertex to another. We usually call paths by the sequence of vertices that are traveled through.
Circuit	A path which starts and ends at the same vertex.
Euler/Eulerian Path	A path that travels every edge (exactly once).
Euler/Eulerian Circuit	A circuit that travels every edge (exactly once).

Euler's Circuit Theorem:

- If a graph has all even vertices, then the graph must have an Euler Circuit.
- If a graph has any odd vertices, then the graph does not have an Euler Circuit.

Euler's Path Theorem:

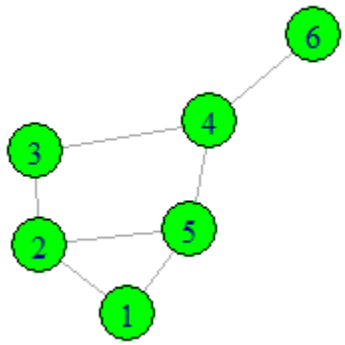
- If a graph has exactly two odd vertices, then the graph must have an Euler Path.
- If a graph has more than two odd vertices, then the graph does not have an Euler Path.

Other Definitions & Concepts:

- Adjacency Matrix and Edgelist representations of graphs
- Diameter: the maximum length of a shortest-path between two vertices. The diameter a measure of a network's connectedness/size.
- Trees: graphs with no circuits.
- Redundancy: $R = M - (N - 1)$ where M is the number of edges and N is the number of vertices. This tells us how many edges needed to be removed to reduce a graph to a tree. Thus, a tree has zero redundancy.
- Spanning Tree: a tree within a graph that spans (contains) all vertices.
- Weighted graph: a graph that has numeric weights associated with each edge.
- Minimum Spanning Tree (MST): a spanning tree in a graph that has the least total weight.
- Kruskal's Algorithm: repeatedly include the next cheapest (least weight) edge that does not create a circuit. Result is a MST.

Know the concepts and how to apply them! See examples in class and the following practice exercises.

1. Consider the following graph.



   (a) Represent the graph as an edgelist.
   (b) List a path of length 3 from vertex 1 to vertex 4.
   (c) List a circuit. Is this a tree?
   (d) List the degree of each vertex. Is there an Euler Circuit or Euler Path? If so, find one.
   (e) Calculate the diameter of the graph by listing out the lengths of the shortest path from each vertex X to each vertex Y.
   (f) Calculate the redundancy of the graph and find one possible spanning tree.

2. Consider the graph represented by the edgelist

$$\{AB, AC, BC, BD, BE, CD, CE, DE, DF, EF\}.$$

   (a) Draw the graph.
   (b) List the degree of each vertex. Is there an Euler Circuit or Euler Path? If so, find one.
   (c) Calculate the diameter of the graph – at this point you should be able to see what the longest shortest-path is without listing all of them.
   (d) Calculate the redundancy and find one possible spanning tree.
   (e) There are three unique cliques in the graph, list them.

3. Consider the graph represented by the adjacency matrix

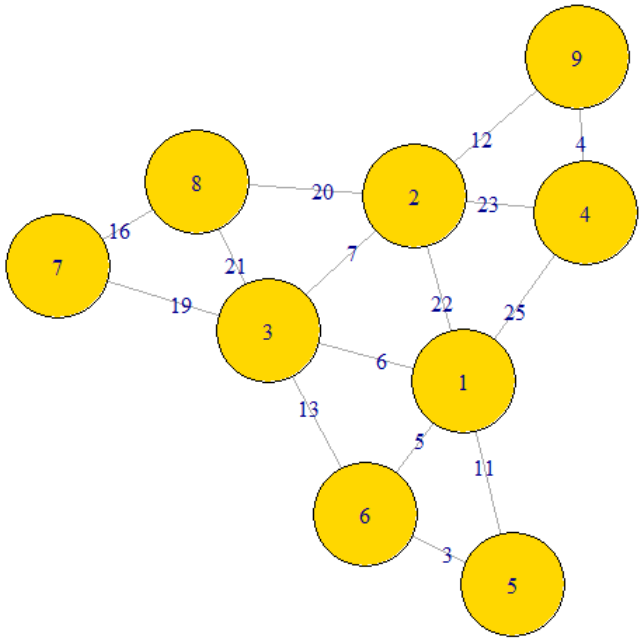
$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

(a) Draw the graph.

(b) *Fun fact: a nice property of adjacency matrices is that they allow for easy computation of graph metrics (degree, diameter, etc.). For example, the degree of a vertex is the sum of its corresponding row (or column).* Find the degree of each vertex.

(c) Is there an Euler Circuit or Euler Path? If so, find one.

4. Consider the graph



(a) Calculate the redundancy.

(b) Use Kruskal's algorithm to find the Minimum Spanning Tree.

Chapter 1: Voting Methods

Shown below is the preference schedule for an election with four candidates (A, B, C, D).

Number of voters	10	8	5	1
1st	C	A	B	B
2nd	B	B	A	A
3rd	A	D	D	C
4th	D	C	C	D

1. Use the **Borda Count method** to determine the full ranking of the election.
2. Use the **Pairwise Comparisons method** to determine the full ranking of the election.

Chapter 3: Fair Division

Suppose four items are to be divided among three people. Given the bids below, use the **Method of Sealed Bids** to determine the final settlement.

	Elder Wand	Firebolt	Invisibility Cloak	Time-Turner
Harry	600	300	200	100
Ron	200	450	150	100
Hermione	100	200	100	500

Chapter 10: Financial Mathematics

(General Compounding) Suppose $3000 is invested for 5 years with an APR of 3%. For each of the following compounding plans determine the final value.

Compounding plan	Final Value
annually	
semi-annually	
quarterly	
monthly	

(Simple Interest) With simple interest, what APR would be needed to end with the same final value as in monthly compounding?

Chapter 15: Statistics

Consider the dataset

$$6, 2, 5, 7, 5, 5, 1, 9, 6, 4$$

1. Compute the **five-number-summary**.
2. Create a **boxplot**.
3. Compute the **mean and standard deviation**.
4. Compute $\mu - .675\sigma$ and $\mu + .675\sigma$ using the mean and standard deviation from the last step. Compare with values you computed in part (1) to judge whether the data is approximately normal.