Quiz 6 Practice

1. Use implicit differentiation to find $\frac{dy}{dx}$.

$$3x^3 + y^5 = e^y$$

$$\frac{d}{dx}(3x^3) + \frac{d}{dx}\left((y(x))^5\right) = \frac{d}{dx}\left(e^{y(x)}\right)$$ chain rule

$$9x^2 + 5y^4 \cdot \frac{dy}{dx} = e^y \cdot \frac{dy}{dx}$$

$$\frac{dy}{dx}(5y^4 - e^y) = -9x^2$$

$$\boxed{\frac{dy}{dx} = \frac{-9x^2}{5y^4 - e^y}}$$

2. Use implicit differentiation to find y'.
Find the equation of the tangent line at $(2,1)$.

$$3xy - 2x - 2 = 0.$$

$$\frac{d}{dx}(3x \cdot y(x)) - \frac{d}{dx}(2x) - \frac{d}{dx}(2) = \frac{d}{dx}(0)$$

product rule

$$3y + y' \cdot 3x - 2 = 0$$

$$3xy' = 2 - 3y$$

$$\boxed{y' = \frac{2-3y}{3x}}$$

Point: $(2,1)$ slope: $y'|_{(2,1)} = \frac{2-3(1)}{3(2)} = \frac{-1}{6}$

equation: $\boxed{y - 1 = \frac{-1}{6}(x-2)}$

3. Helium is pumped into a spherical balloon at a constant rate of 4 cubic feet per second. How fast is the radius increasing after 1 minute. <u>Hint:</u> $V = \frac{4}{3}\pi r^3$

By implicit differentiation we get our related rates equation.

$$\frac{d}{dt} V(t) = \frac{d}{dt}\left(\frac{4}{3}\pi (r(t))^3\right)$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}\left((r(t))^3\right)$$

Chain rule

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3} r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Know $\frac{dV}{dt} = 4$ ft/sec. Want $\frac{dr}{dt}$.
Can find r. At 1 minute the volume is $\frac{4 \text{ cubic ft}}{\text{sec}} \cdot 60 \text{sec} = 240$ cubic ft.

$$\Longrightarrow 240 = \frac{4}{3}\pi r^3 \Longrightarrow r = \sqrt[3]{\frac{240}{\frac{4}{3}\pi}} = 3.86 \text{ ft}$$

Back to related rates equation...

$$4 = 4\pi (3.86)^2 \frac{dr}{dt}$$

$$\Longrightarrow \boxed{\frac{dr}{dt} = 0.02} \text{ ft/sec}$$