Solutions
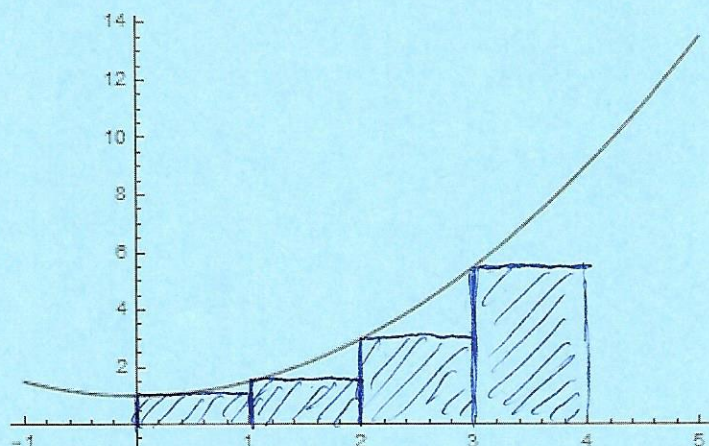
For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown.

For the function $f(x) = \frac{1}{2}x^2 + 1$ plotted below:

1. (5 pts) Approximate the area under the curve on the interval $[0, 4]$ using four left rectangles. Sketch the rectangles on the plot below.

$$\Delta x = \frac{b-a}{n}$$
$$= \frac{4-0}{4}$$
$$= 1$$

$$L_4 = 1 \cdot f(0) + 1 \cdot f(1) + 1 \cdot f(2) + 1 \cdot f(3)$$
$$= f(0) \quad + \quad f(1) \quad + \quad f(2) \quad + \quad f(3)$$
$$= \quad 1 \quad + \quad 1.5 \quad + \quad 3 \quad + \quad 5.5$$
$$= \boxed{11}$$

2. (5 pts) Compute the exact area under the curve on the interval $[0, 4]$ by evaluating

$$\int_0^4 \left(\frac{1}{2}x^2 + 1\right) dx.$$

$$= \frac{1}{6}x^3 + x \Big|_{x=0}^{x=4}$$
$$= \left[\frac{1}{6} \cdot 4^3 + 4\right] - \left[\frac{1}{6} \cdot 0^3 + 0\right]$$
$$= \frac{44}{3} \approx \boxed{14.67}$$

3. (Bonus: 1 pt) Compute the average value of f on $[0, 4]$.

$$= \frac{1}{4-0} \int_0^4 f(x)\,dx = \frac{1}{4} \cdot \frac{44}{3} = \frac{11}{3} \approx \boxed{3.67}$$