

This exam contains 7 pages and 7 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	12	
5	8	
6	8	
7	12	
Total:	100	

**HONORS PLEDGE** (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: \_\_\_\_\_

1. (20 points) Evaluate the following indefinite integrals

(a) (8 points)

$$\int (6x^8 - 5x^4 + 3)dx$$

(b) (6 points)

$$\int (8x^3 + 2)e^{x^4+x}dx$$

(c) (6 points)

$$\int \frac{6x-1}{3x^2-x}dx$$

2. (20 points)

(a) (6 points) Given that

$$\frac{d}{dx} \left( x \ln(x) - x \right) = \ln(x).$$

Find the particular antiderivative of

$$\ln(x)$$

which passes through the point  $(1, 0)$ .

(b) (6 points) Given that

$$\int_0^9 f(x) dx = 5; \quad \int_0^7 g(x) dx = 2; \quad \int_7^9 g(x) dx = 6$$

Compute  $\int_0^9 (2f(x) + g(x)) dx$ .

(c) (8 points) Compute the definite integral

$$\int_6^8 (7x + 3) dx$$

3. (20 points) When evaluating limits make sure to justify your answer. Write DNE if a limit does not exist.

(a) (6 points) Evaluate

$$\lim_{x \rightarrow 8} \frac{x - 7}{x - 8}$$

(b) (6 points) Evaluate

$$\lim_{x \rightarrow 0} \frac{x^2}{e^{3x} - 3x - 1}$$

(c) (8 points) Find all horizontal and vertical asymptotes of

$$f(x) = \frac{x^2 - 4}{5(x^2 - 6x + 8)}$$

4. (12 points) Compute the following derivatives

(a) (6 points)

$$\frac{d}{dx} (x^5 + \ln(1 + x^5))$$

(b) (6 points)

$$\frac{d}{dx} (e^x(x^2 + 2x + 1)^8)$$

5. (8 points) Find the equation of the tangent line to the function

$$f(x) = x^3 + x + 1$$

at the point  $(0, 1)$ .

6. (8 points) Find  $y'$  given that

$$xy - y^3 - e^x = 0$$

7. (12 points) For the function  $f(x) = x^3 - 9x^2 + 15x$

(a) (8 points) Find the intervals where  $f$  is increasing/decreasing

(b) (4 points) Find any local maxima or minima.

**Bonus (3 pts)** The 3rd-degree Taylor polynomial approximation for  $f(x)$  at  $x = 1$  is defined to be

$$p(x) = f(1) + f'(1)x + \frac{1}{2}f''(1)x^2 + \frac{1}{6}f'''(1)x^3$$

where  $f'(1)$ ,  $f''(1)$ , and  $f'''(1)$  are the 1st, 2nd, and 3rd derivatives of  $f$  evaluated at  $x = 1$ .

Compute the 3rd-degree Taylor polynomial at  $x = 1$  for  $f(x) = \ln(x)$ .