Math 17: Exam 2 Solutions Name:

There are 3 pages and 8 questions in total. There are 100 possible points and the point values for each page are 36, 30, and 34 respectively. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown.

1. Suppose you borrow \$3500 for a term of four years at simple interest and 3.4% APR. How much in total must you pay back on the loan?

$$F = 3500(1+.034\cdot 4) = \boxed{\$3976}$$

2. Find the APR of a bond that doubles its value in 15 years under simple interest.

$P = x$
$F = 2x$
$t = 15$

$$\frac{2x}{x} = \frac{x(1+r\cdot 15)}{x}$$
$$\Rightarrow 2 = 1 + 15r$$
$$\Rightarrow 15r = 1 \longrightarrow \boxed{r = \frac{1}{15} = 0.0\bar{6} = 6.\bar{6}\%}$$

3. Between 1990 and 1999 the average annual inflation rate was 3%. Find the salary in 1990 dollars that would be equivalent to a 36,000 salary in 1999.

$F = 36000$
$P = ???$
$t = 9$
$r = .03$

$$36000 = P(1+.03)^9$$
$$P = \frac{36000}{(1+.03)^9} = \boxed{\$27591}$$

4. Suppose you are investing in a CD. Which plan is better: (1) 8.2% APR compounded annually or (2) 8.0% compounded monthly? Use Annual Percentage Yield to justify your answer (that is, for both plans compute F assuming \$1 is invested for 1 year).

(1) $F = 1(1+.082)^1 = 1.082$

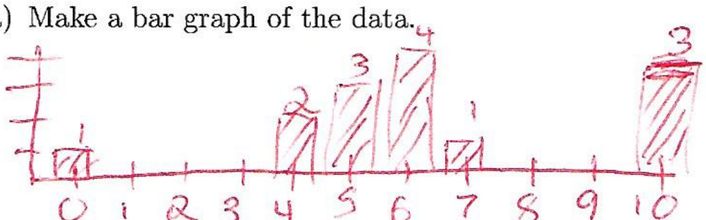
(2) $F = 1\left(1+\frac{.08}{12}\right)^{12} = 1.083$

Plan (2) is slightly better since $1.083 > 1.082$

5. Suppose we have the following dataset ($N = 14$).

$$0,\ 4,\ 4,\ 5,\ 5,\ 5,\ 6,\ 6,\ 6,\ 6,\ 7,\ 10,\ 10,\ 10.$$

(a) Make a bar graph of the data.



(b) Compute the mean and standard deviation of the data.

$\text{mean} = \frac{1}{14}(0+2(4)+3(5)+\ldots+3(10)) = \boxed{6}$

$sd^2 = \frac{1}{14}((0-6)^2+2(4-6)^2+3(5-6)^2+\ldots+3(10-6)^2) = 6.85$

$sd = \boxed{2.62}$

(c) Compute Q_1, Q_3, and the IQR.

$\boxed{Q_1}$ $L = \frac{25}{100} \cdot 14 = 3.5$ $\quad Q_1 = d_4 = \boxed{5}$

$\boxed{Q_3}$ $L = \frac{75}{100} \cdot 14 = 10.5$ $\quad Q_3 = d_{11} = \boxed{7}$

$IQR = 7-5 = \boxed{2}$

(d) Create a boxplot.

$\boxed{Med}$ $L = \frac{50}{100} \cdot 14 = 7$ $\quad Med = \frac{d_7+d_8}{2} = \boxed{6}$ $\quad min = 0,\ max = 10$

Boxplot

(e) Remove any datapoints that are less than $Q_1 - 1.5 \times IQR$ or greater than $Q_3 + 1.5 \times IQR$. What is the new dataset?

$Q_1 - 1.5 \cdot IQR = 5 - 1.5 \cdot 2 = 2$

$Q_3 + 1.5 \cdot IQR = 7 + 1.5 \cdot 2 = 10$

New dataset:
{4,4,5,5,5,6,6,6,6,7,10,10,1

~~(f) Recompute the mean and standard deviation of the new dataset.~~

6. Compute μ and σ for a normal distribution where $Q_3 = 900$ and upper point of inflection $P = 975$.

$Q_3 = 900 = \mu + .6750\sigma$

$P = 975 = \mu + \sigma$

$\ominus \quad -75 = -.3250\sigma$

$\sigma = 75/.325 = \boxed{230.8}$

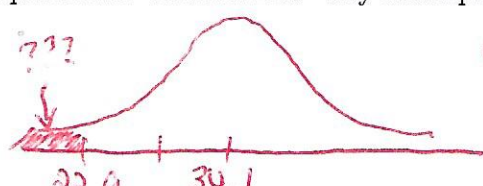
$\mu = P - \sigma$

$\mu = 975 - 230.8$

$\mu = \boxed{744.2}$

7. The forecast (high temperatures) here in VT from this Monday to next Monday is approximately normally distributed with $\mu = 34.1°$ and $\sigma = 5.6°$. The forecast in Los Angeles is approximately normally distributed with $\mu = 74.4°$ and $\sigma = 2.7°$.

   (a) What percentile would a 22.9° day correspond to in VT?



percentile $= 50 - \frac{95}{2}$

$= \boxed{2.5}$

   (b) What range of values are captured in the middle 95% of the LA forecast data.

-2σ to 2σ from mean

$74.4 - 2 \cdot 2.7 = 69°$

$74.4 + 2 \cdot 2.7 = 79.8°$

   (c) In the upcoming week, it more unlikely to see a 79.8° day in LA or a 21.2° day in VT?

$z_{LA} = \frac{79.8 - 74.4}{2.7} = 2$

$z_{VT} = \frac{21.2 - 34.1}{5.6} = -2.3$

VT more unlikely since z score is bigger in absolute value.

8. Suppose we have a dataset $\{x_1, x_2, \ldots, x_N\}$ of N datapoints, which is normal with mean μ and standard deviation σ. Let b be a number and consider the dataset $\{x_1 + b, x_2 + b, \ldots, x_N + b\}$. In other words, add b to every number in the dataset. Verify the mean is $\mu + b$ and the standard deviation is σ.

- Adding b to every value will shift the center of the data by b, so mean $= \mu + b$.

- Algebraically:

$$\frac{1}{N}((x_1+b)+(x_2+b)+\ldots+(x_N+b)) = \frac{1}{N}((x_1+\ldots+x_N)+Nb)$$
$$= \frac{1}{N}(x_1+\ldots+x_N) + b$$
$$= \mu + b$$

- Adding b to every # does not change the spread so sd $= \sigma$