Solutions

Math 19: Quiz 6 Name:

For full credit you must (NEATLY) show your work. Partial credit may be given for incorrect solutions if sufficient work is shown.

1. (5 pts) Use implicit differentiation to find y'.

$$x^2 + y^3 = 5y$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^3) = \frac{d}{dx}(5y)$$

$$2x + 3y^2 \cdot y' = 5y'$$

$$3y^2 y' - 5y' = -2x$$

$$y'(3y^2 - 5) = -2x$$

$$\boxed{y' = \frac{-2x}{3y^2 - 5}} = \frac{2x}{5 - 3y^2}$$

2. (5 pts) Helium is pumped into a spherical balloon at a constant rate of 8 cubic feet per second. How fast is the radius increasing after 1 minute.
Hint: The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$.

Related rates equation

$$\frac{d}{dt}V(t) = \frac{d}{dt}\left(\frac{4}{3}\pi (r(t))^3\right)$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \frac{d}{dt}\left((r(t))^3\right) \quad \text{chain rule}$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2 \cdot \frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Know $\frac{dV}{dt} = 8$ ft/sec. Want $\frac{dr}{dt}$.
Can find r. At 1 minute, the volume is $V = \frac{8 \text{ cubic ft}}{\text{sec}} \cdot 60 \text{ sec} = 480$ cubic ft.

$$\Rightarrow 480 = \frac{4}{3}\pi r^3 \Rightarrow r = \sqrt[3]{\frac{480}{\frac{4}{3}\pi}} \approx 4.86$$

Back to related rates equation...

$$8 = 4\pi (4.86)^2 \frac{dr}{dt}$$

$$\Rightarrow \boxed{\frac{dr}{dt} = 0.0269 \text{ ft/sec}}$$