Math 19: Quiz 2 Solutions Name:

For full credit you must (NEATLY) show your work. Partial credit may be given for incorrect solutions if sufficient work is shown.

1. (6 pts) Find the **vertical** asymptotes (if any) of the function

$$f(x) = \frac{x-1}{x^2+x-2}.$$

For a vertical asymptote we require the denominator to be zero

$$x^2+x-2=0 \implies (x+2)(x-1)=0 \implies x=-2, x=1$$

Thus $x=-2$ and $x=1$ are *candidates* for vertical asymptotes. We need to check that there is an infinite limit at each of these (i.e. $\frac{\text{some nonzero \#}}{0}$).

For $x=-2$ we have

$$\lim_{x\to -2} f(x) = \frac{-2-1}{(-2)^2-2-2} = \frac{-3}{0}.$$

Therefore $x=-2$ is a vertical asymptote.

For $x=1$ we have

$$\lim_{x\to 1} f(x) = \frac{1-1}{1^2+1-2} = \frac{0}{0}$$
$$= \lim_{x\to 1}\frac{\cancel{x-1}}{(x+2)\cancel{(x-1)}} = \lim_{x\to 1}\frac{1}{x+2} = \frac{1}{3}.$$

Therefore $x=1$ is not a vertical asymptote. $\boxed{\text{The only vertical asymptote is } x=-2.}$

2. (4 pts) Find the **horizontal** asymptotes (if any) of the function

$$f(x) = \frac{3x^4-x^2+1}{8x^6-10}.$$

To find a horizontal asymptote we need to take a limit at infinity.

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{3x^4}{8x^6} = \lim_{x\to\infty}\frac{3}{8x^2} = 0.$$

Therefore $\boxed{y=0 \text{ is a horizontal asymptote.}}$