Math 19: Quiz 2 Solutions   Name:

For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown.

For both questions, use $+\infty$ or $-\infty$ if appropriate. Justify your answers, i.e. by using "really big/small" terminology.

1. Determine the following limits. (4 pts)

$$\begin{aligned}
\lim_{x\to 1^-} \frac{2x+3}{x-1} &= \frac{2(1)+3}{1-1} && \text{[try to plug in]}\\
&= \frac{5}{0} && \text{[do more work]}\\
&= \frac{2(0.9999)+3}{0.9999-1}\\
&= \frac{5}{\text{small}-}\\
&= \text{big}-\\
&= \boxed{-\infty}.
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to\infty} \frac{2x+3}{x-1} &= \lim_{x\to\infty} \frac{2x}{x} && \text{[look at leading terms]}\\
&= \lim_{x\to\infty} 2\\
&= \boxed{2}.
\end{aligned}$$

2. Find the horizontal and vertical asymptotes of f. (6 pts)

$$f(x) = \frac{x+1}{(x-2)(x+1)}.$$

Vertical: A vertical asymptote occurs when the denominator is zero but the numerator is nonzero. When $x = 2$ the denominator is zero but the numerator is nonzero. However, $x = -1$ gives $f(x) = 0/0$ so $x = -1$ is not a vertical asymptote. $\boxed{\text{There is a vertical asymptote at } x = 2}$.

Horizontal: To find a horizontal asymptote we take the limit as $x \to \infty$. We have

$$\begin{aligned}
\lim_{x\to\infty} \frac{x+1}{(x-2)(x+1)} &= \lim_{x\to\infty} \frac{1}{x-2}\\
&= \lim_{x\to\infty} \frac{1}{x}\\
&= 0.
\end{aligned}$$

$\boxed{\text{There is a horizontal asymptote at } y = 0}$.