

Math 17: Exercise Set 3 – Posted 9/17

Recall a few facts.

- A fair division is **efficient** if there is no other fair division in which all players get higher valued shares. In other words, in an efficient fair division we cannot swap two players shares and make both of them happier – there **MUST** be a trade-off.
 - A fair division is **envy-free** if every player receives their most valued share.
 - There can only be zero or one envy-free fair division (unless multiple people value a share at exactly the same value, but this will not happen in our examples).
 - There is always at least one fair division that is efficient (in the case of there only being one possible fair division, we automatically call it efficient).
 - All envy-free fair divisions are efficient, but not all efficient fair divisions are envy-free. **Think of envy-free as being a “utopia” that doesn’t always exist, while efficient fair divisions are “doing the best we can”.**
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1. Suppose we are dividing three shares s_1, s_2, s_3 among three players A, B, C . The value system for each player is given in the table below. Note that, as in class, $33\frac{1}{3}\% = 0.3333\cdots = \frac{1}{3}$.

	s_1	s_2	s_3
A	38%	28%	34%
B	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
C	34%	40%	26%

- (a) Determine the fair shares for each player. Then come up with all possible fair divisions (hint: there are three of them).
 - (b) Which fair divisions are efficient?
 - (c) Are any fair divisions envy-free?
2. Suppose we are dividing four shares s_1, s_2, s_3, s_4 among four players A, B, C, D . The value system for each player is given in the table below.

	s_1	s_2	s_3	s_4
A	20%	32%	28%	20%
B	25%	25%	25%	25%
C	15%	15%	30%	40%
D	24%	24%	24%	28%

- (a) Determine the fair shares for each player. Then come up with all possible fair divisions (hint: there is only one).

- (b) Is the fair division efficient? (See bullet 4 in the list of facts above if unsure)
- (c) Is the fair division envy-free?
3. Suppose we are dividing four shares s_1, s_2, s_3, s_4 among four players A, B, C, D . The value system for each player is given **in terms of dollars** in the table below.

	s_1	s_2	s_3	s_4
A	\$3.00	\$5.00	\$5.00	\$3.00
B	\$4.50	\$3.50	\$4.50	\$5.50
C	\$4.25	\$4.50	\$3.50	\$3.75
D	\$5.50	\$4.00	\$4.50	\$6.00

- (a) Determine the fair shares for each player. Then come up with all possible fair divisions (hint: there are three of them).
- (b) Which fair divisions are efficient?
- (c) Are any fair divisions envy-free?
4. Suppose two roommates, Bob and Chris, are parting ways and need to divvy up some of the belongings in their apartment. They have joint ownership of an iPad, an Xbox One, and a PlayStation 4. Their value systems are given in the table below.

	iPad	Xbox	PS4
Bob	\$400	\$300	\$500
Chris	\$200	\$400	\$400

Use the **Method of Sealed Bids** to determine the allocation of items.