Solutions

Math 19 E
Spring 2019
Exam 1
February 15

Name: ____________________

This exam contains 5 pages and 6 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	18	
2	28	
3	12	
4	24	
5	12	
6	6	
Total:	100	

<u>HONORS PLEDGE</u> (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: ____

1. (18 points) Determine the following limits

(a) (6 points)

$$\lim_{x\to 8^-} \frac{x+3}{x-8} = \frac{8+3}{8-8} = \frac{11}{0}$$

$$= \frac{11}{7.99-8} = \frac{11}{\text{small} -} = \text{big} -$$

$$= \boxed{-\infty}$$

(b) (6 points)

$$\lim_{x\to 4^+} \frac{x^2-16}{x-4} = \frac{4^2-16}{4-4} = \frac{0}{0}$$

$$= \lim_{x\to 4^+} \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}}$$

$$= \lim_{x\to 4^+} x+4 = 4+4 = \boxed{8}$$

(c) (6 points)

$$\lim_{x\to\infty} \frac{x+9}{x^2+3x+2} = \lim_{x\to\infty} \frac{x}{x^2}$$

$$= \lim_{x\to\infty} \frac{1}{x}$$

$$= \boxed{0}$$

2. (28 points) For the function

$$f(x) = \frac{x^2 - 9}{x^2 + 1}$$

(a) (8 points) Find any vertical asymptotes of f.

$x^2 + 1 = 0 \Rightarrow x = \pm\sqrt{-1}$ no real solutions

$\Rightarrow$ no vertical asymptotes

(b) (8 points) Find any horizontal asymptotes of f.

$$\lim_{x \to \infty} \frac{x^2 - 9}{x^2 + 1} = \lim_{x \to \infty} \frac{x^2}{x^2} = 1$$

$\boxed{y = 1}$ is a horizontal asymptote

(c) (6 points) Find the partition numbers of f.

- $x^2 - 9 = 0 \Rightarrow x = \pm 3$
- $x^2 + 1 = 0$ has no solutions

Only partition #'s are $\boxed{x = \pm 3}$

(d) (6 points) Determine the sign chart for f.

$f(-4) = \frac{16-9}{16+1} = \frac{7}{17} > 0$

$+$ | $-$ | $+$

-3 3

$f(0) = \frac{0-9}{0+1} = -9 < 0$

$f(4) = \frac{16-9}{16+1} = \frac{7}{17} > 0$

3. (12 points) Consider the function

$$f(x) = x^2 + 7.$$

Use the limit definition of the derivative to compute $f'(x)$. No credit will be given for using shortcuts on this problem.

(a) (3 points)

$$f(x+h) = \cancel{(x+h)^2} \; (x+h)^2 + 7$$
$$= x^2 + 2xh + h^2 + 7$$

(b) (3 points)

$$f(x+h) - f(x) = x^2 + 2xh + h^2 + 7 - (x^2 + 7)$$
$$= \cancel{x^2} + 2xh + h^2 \cancel{+7} \cancel{-x^2} \cancel{-7}$$
$$= 2xh + h^2$$

(c) (3 points)

$$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2}{h} = 2x + h$$

(d) (3 points)

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
$$= \lim_{h \to 0} 2x + h$$
$$= 2x + 0$$
$$= 2x$$

4. (24 points) Compute the following quantities. You may use shortcuts.

(a) (8 points)

$$f'(x) \qquad \text{for} \qquad f(x) = x^5 - 2x^3 + 4x$$

$f'(x) = 5x^4 - 6x^2 + 4$

(b) (8 points)

$$\frac{d}{dx}f(x) \qquad \text{for} \qquad f(x) = \frac{1}{x^2} - 6x \quad = x^{-2} - 6x$$

$\frac{d}{dx} f(x) = -2x^{-3} - 6$

$= \frac{-2}{x^3} - 6$

(c) (8 points)

$$y' \qquad \text{for} \qquad y = 5\sqrt{x} + x^3 \quad = 5x^{1/2} + x^3$$

$y' = \frac{5}{2}x^{-1/2} + 3x^2$

$= \frac{5}{2\sqrt{x}} + 3x^2$

5. (12 points) Find the equation of the tangent line to $f(x) = x^5 - 2x^3 + 4x$ at $x = 1$. *Hint*: use your answer to part (a) of the previous page.

- Point: $x=1$, $y=f(1) = 1^5-2(1)^3+4(1) = 1-2+4=3$
- Slope: $f'(1) = 5(1)^4-6(1)^2+4 = 5-6+4 = 3$

Equation of tangent line: $y-3 = 3(x-1)$

$\Rightarrow \boxed{y = 3x}$

6. (6 points) Suppose $1000 is invested for 3 years with continuous compounding. At the end of the 3 years, the investment is worth $1500. Find r, the annual rate of compounding. *Hint*: the formula for continuous compounding is $A = Pe^{rt}$.

$1500 = 1000e^{3r}$

$1.5 = e^{3r}$

$\ln(1.5) = \ln(e^{3r})$

$\ln(1.5) = 3r\,\underbrace{\ln(e)}_{1}$

$\ln(1.5) = 3r$

$\boxed{r = \frac{\ln(1.5)}{3} \approx 0.135 = 13.5\%}$