

PRACTICE EXAM

This exam contains 8 pages and 8 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	15	
2	15	
3	15	
4	8	
5	15	
6	8	
7	8	
8	16	
Total:	100	

HONORS PLEDGE (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: _____

1. (15 points) Evaluate the following indefinite integrals

(a) (5 points)

$$\int (3x^5 - 2x + 3)dx$$

(b) (5 points)

$$\int (6x^2 + 10)e^{x^3+5x} dx$$

(c) (5 points)

$$\int \frac{2x}{x^2 - 8} dx$$

2. (15 points) More integration topics

(a) (5 points) Given that

$$\frac{d}{dx} \left(\frac{1}{1 + e^{-x}} \right) = \frac{e^{-x}}{(1 + e^{-x})^2}.$$

Find the particular antiderivative of

$$\frac{e^{-x}}{(1 + e^{-x})^2}$$

which passes through the point $(0, 0)$.

(b) (5 points) Estimate the area under $f(x) = x^2$ on $[1, 7]$ using $n = 3$ left rectangles.

(c) (5 points) Compute the definite integral

$$\int_3^8 (5x - 6) dx$$

3. (15 points) Limits – Evaluate the following limits. Make sure to briefly justify your answer. Write DNE if a limit does not exist.

(a) (5 points)

$$\lim_{x \rightarrow 1} \frac{x}{x-1}$$

(b) (5 points)

$$\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x^2 - 6x + 8}$$

(c) (5 points)

$$\lim_{x \rightarrow \infty} \frac{3x^8 - 6x^2}{-2x^6 + 3x^5 + 1}$$

4. (8 points) Consider the function

$$f(x) = x^2 - 3x + 9.$$

Use the limit definition of the derivative to compute $f'(x)$. No credit will be given for using shortcuts on this problem.

- (a) (2 points)

$$f(x + h) =$$

- (b) (2 points)

$$f(x + h) - f(x) =$$

- (c) (2 points)

$$\frac{f(x + h) - f(x)}{h} =$$

- (d) (2 points)

$$f'(x) =$$

5. (15 points) Compute the following derivatives. You may use shortcuts.

(a) (5 points)

$$\frac{d}{dx} \left(x^3 + 9 \ln(x) - \frac{2}{x^3} \right)$$

(b) (5 points)

$$\frac{d}{dx} \left((x^2 - 1)^4 (3x + 4) \right)$$

(c) (5 points)

$$\frac{d}{dx} \left(\frac{e^x + e^{-x}}{2} \right)$$

6. (8 points) Consider the function

$$f(x) = \frac{e^x}{e+x}$$

- (a) (4 points) Find the equation of the tangent line at $x = 1$.

- (b) (4 points) Find where the tangent line is horizontal.

7. (8 points) Find y' for the implicit curve defined by the equation

$$2y^2 + xy - 1 = 0.$$

8. (16 points) For the function $f(x) = -x^3 + 3x^2 + 9x - 9$

(a) (5 points) Find the intervals where f is increasing/decreasing

(b) (1 point) Find any local maxima or minima.

(c) (5 points) Find the intervals where f is concave up/concave down

(d) (1 point) Find any points of inflection.

(e) (4 points) Find the absolute maximum and absolute minimum of f on $[-4, 4]$.