Math 19 A&B
Fall 2019
Exam 1
October 3
Version A

Name: ____________________

Solutions

This exam contains 6 pages and 7 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

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1	24	
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4	18	
5	12	
6	6	
7	4	
Total:	100	

HONORS PLEDGE (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: ____________________

1. (24 points) Determine the following limits. If a limit does not exist, determine if it is $+\infty$ or $-\infty$.

   (a) (6 points)

$$\lim_{x\to 3} \frac{x^2+x-12}{x-3} = \frac{3^2+3-12}{3-3} = \frac{0}{0} \quad \text{factor}$$

$$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{x-3}} = \lim_{x\to 3} x+4 = 3+4 = \boxed{7}$$

   (b) (6 points)

$$\lim_{x\to 3^-} \frac{x+1}{x-3} = \frac{3+1}{3-3} = \frac{4}{0}$$

$$\approx \frac{4}{2.99-3} = \frac{4}{\text{small} -} = \text{big} - = \boxed{-\infty}$$

   (c) (6 points)

$$\lim_{x\to 6^-} \frac{x}{(x-6)^2} = \frac{6}{(6-6)^2} = \frac{6}{0}$$

$$\approx \frac{6}{(5.99-6)^2} = \frac{6}{\text{small} +} = \text{big} + = \boxed{+\infty}$$

   (d) (6 points)

$$\lim_{x\to\infty} \frac{5x^3+9x^2-3x}{4x^3-1} = \lim_{x\to\infty} \frac{5\cancel{x^3}}{4\cancel{x^3}} = \boxed{\frac{5}{4}}$$

(case where numerator exponent is = denominator exponent)

2. (24 points) For the function

$$f(x) = \frac{x-4}{x^2-3x-4} = \frac{x-4}{(x-4)(x+1)}$$

(a) (6 points) Find any vertical asymptotes of f.

$x^2-3x-4=0 \Rightarrow (x-4)(x+1)=0 \Rightarrow x=4,-1$ are "candidates"

Check:

$\lim_{x\to 4} f(x) = \frac{4-4}{4^2-3(4)-4} = \frac{0}{0} \ldots \Rightarrow$ no VA at $x=4$

$\lim_{x\to -1} f(x) = \frac{-1-4}{(-1)^2-3(-1)-4} = \frac{-5}{0} \Rightarrow \boxed{\text{VA at } x=-1}$

(b) (6 points) Find any horizontal asymptotes of f.

$\lim_{x\to\infty} \frac{x-4}{x^2-3x-4} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$

(this is the case where numerator exponent < denominator exponent)

$\Rightarrow \boxed{y=0 \text{ is HA}}$

(c) (6 points) Find the partition numbers of f.

① $f(x)=0$
$\Rightarrow x-4=0$
$\Rightarrow x=4$

② f is discontinuous at x
$\Rightarrow x^2-3x-4=0$
$\Rightarrow (x-4)(x+1)=0$
$\Rightarrow x=4, x=-1$

$\boxed{\text{Partition \#s are } x=4,\ x=-1}$

(d) (6 points) Make a sign chart for f.

$(-\infty,-1)$	-1	$(-1,4)$	4	$(4,\infty)$
−		+		+

$(-\infty,-1)$: Test $x=-2$

$f(-2) = \frac{-2-4}{(-2)^2-3(-2)-4} = \frac{-6}{6} = -1 < 0$

$(-1,4)$: Test $x=0$

$f(0) = \frac{0-4}{0^2-3\cdot 0-4} = \frac{-4}{-4} = 1 > 0$

$(4,\infty)$: Test $x=5$

$f(5) = \frac{5-4}{5^2-3\cdot 5-4} = \frac{1}{6} > 0$

3. (12 points) Consider the function

$$f(x) = 5x^2 - 9.$$

Use the limit definition of the derivative to compute $f'(x)$. No credit will be given for using shortcuts on this problem.

(a) (3 points)

$$f(x+h) = 5(x+h)^2 - 9$$
$$= 5(x^2+2xh+h^2) - 9$$
$$= 5x^2 + 10xh + 5h^2 - 9$$

(b) (3 points)

$$f(x+h) - f(x) = \cancel{5x^2} + 10xh + 5h^2 \cancel{-9} - (\cancel{5x^2} \cancel{-9})$$
$$= 10xh + 5h^2$$

(c) (3 points)

$$\frac{f(x+h) - f(x)}{h} = \frac{10xh + 5h^2}{h}$$
$$= 10x + 5h$$

(d) (3 points)

$$f'(x) = \lim_{h \to 0} 10x + 5h = 10x + 0 = \boxed{10x}$$

4. (18 points) Compute the following quantities. You may use shortcuts.

(a) (6 points)

$$f'(x) \quad \text{for} \quad f(x) = x^4 - 5x^3 + 3x + 2$$

$$\boxed{f'(x) = 4x^3 - 15x^2 + 3}$$

(b) (6 points)

$$\frac{d}{dx}f(x) \quad \text{for} \quad f(x) = \frac{1}{x^5} - \ln(x) = x^{-5} - \frac{1}{x}$$

$$\boxed{\frac{d}{dx}f(x) = -5x^{-6} - \frac{1}{x}}$$

(c) (6 points)

$$y' \quad \text{for} \quad y = 2e^x - \sqrt[4]{x} = 2e^x - x^{1/4}$$

$$\boxed{y' = 2e^x - \frac{1}{4}x^{-3/4}}$$

5. (12 points) For the function $f(x) = x^4 - \cancel{100}x^2$ $50x^2$ $f'(x) = 4x^3 - 100x$

(a) (6 points) Find the equation of the tangent line to f at $x = 1$.

point: $x_0 = 1$, $y_0 = f(1) = 1^4 - 50 \cdot 1^2 = -49$

slope: $f'(1) = 4(1)^3 - 100(1) = -96$

equation: $y - (-49) = -96(x-1)$

$\Rightarrow y + 49 = -96(x-1)$

$\Rightarrow \boxed{y = -96x + 47}$

any of these are acceptable answers

(b) (6 points) Find where the tangent line to f is horizontal.

$f'(x) = 0 \Rightarrow 4x^3 - 100x = 0$

$\Rightarrow 4x(x^2 - 25) = 0$

$\Rightarrow 4x(x+5)(x-5) = 0$

$\Rightarrow \boxed{x = 0, -5, 5}$

6. (6 points) Suppose \$2000 is invested with continuous compounding. At the end of 4 years, the investment is worth \$2500. Find r, the annual rate. *Hint*: the formula for continuous compounding is $F = Pe^{rt}$.

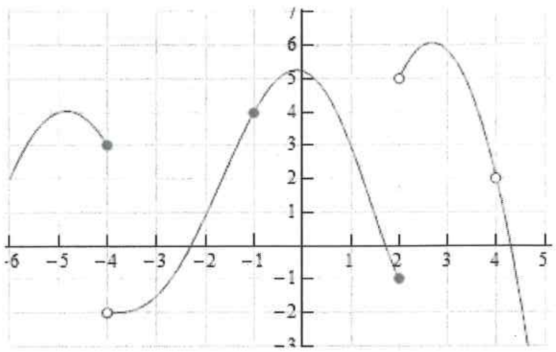
$2500 = 2000e^{r \cdot 4}$

$1.25 = e^{4r}$

$\ln(1.25) = 4r \cdot \cancel{\ln(e)}^{\,1}$

$\boxed{r = \frac{\ln(1.25)}{4} = 0.056}$

7. (4 points) Below is the graph of some function $f(x)$.



Criteria for continuity

$f(x)$ is continuous at $x = c$ if **all** of the following are true.

(f is discontinuous at $x = c$ if **one or more** of the following fail.)

(i) $f(c)$ exists

(ii) $\lim_{x \to c} f(x)$ exists

(iii) $\lim_{x \to c} f(x) = f(c)$

Where is f discontinuous? For each point of discontinuity, which of the three continuity criteria fails?

$x = -4$ (ii) + (iii) fail

$x = 2$ (ii) + (iii) fail

$x = 4$ (i) + (iii) fail

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   (a) (6 points)

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$$= \lim_{x\to 4}\frac{\cancel{(x-4)}(x+1)}{\cancel{x-4}} = \lim_{x\to 4} x+1 = 4+1 = \boxed{5}$$

   (b) (6 points)

$$\lim_{x\to\infty}\frac{7x^3+8x^2-4x}{5x^3-2} = \lim_{x\to\infty}\frac{7\cancel{x^3}}{5\cancel{x^3}} = \boxed{\frac{7}{5}}$$

(case where numerator exponent is = denominator exponent)

   (c) (6 points)

$$\lim_{x\to 6^-}\frac{x+5}{x-6} = \frac{6+5}{6-6} = \frac{11}{0}$$

$$\approx \frac{11}{5.99-6} = \frac{11}{\text{small} -} = \text{big} - = \boxed{-\infty}$$

   (d) (6 points)

$$\lim_{x\to 1^-}\frac{x}{(x-1)^2} = \frac{1}{(1-1)^2} = \frac{1}{0}$$

$$\approx \frac{1}{(0.99-1)^2} = \frac{1}{\text{small}+} = \text{big}+ = \boxed{+\infty}$$

2. (24 points) For the function

$$f(x) = \frac{x-3}{x^2+x-12} = \frac{x-3}{(x-3)(x+4)}$$

(a) (6 points) Find any vertical asymptotes of f.

$x^2+x-12=0 \Rightarrow (x-3)(x+4)=0 \Rightarrow x=3,-4$ are "candidates"

Check:

$\lim_{x\to 3} f(x) = \frac{3-3}{3^2+3-12} = \frac{0}{0} = \ldots \Rightarrow$ no VA at $x=3$

$\lim_{x\to -4} f(x) = \frac{-4-3}{(-4)^2-4-12} = \frac{-7}{0} \Rightarrow \boxed{\text{VA at } x=-4}$

(b) (6 points) Find any horizontal asymptotes of f.

$\lim_{x\to\infty} \frac{x-3}{x^2+x-12} = \lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$

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$\Rightarrow \boxed{y=0 \text{ is HA}}$

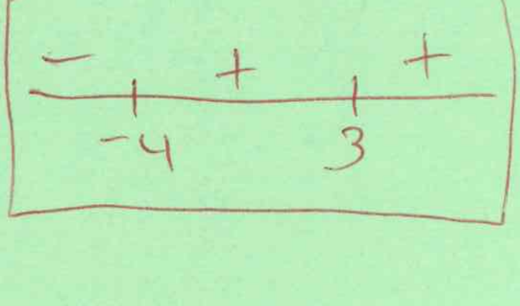
(c) (6 points) Find the partition numbers of f.

(1) $f(x)=0$
$\Rightarrow x-3=0$
$\Rightarrow x=3$

(2) f is discontinuous at x
$\Rightarrow x^2+x-12=0$
$\Rightarrow (x-3)(x+4)=0$
$\Rightarrow x=3,-4$

Partition #s are $x=3$, $x=-4$

(d) (6 points) Make a sign chart for f.



$(-\infty,-4)$: Test $x=-5$

$f(-5) = \frac{-5-3}{(-5)^2-5-12} = \frac{-8}{8} = -1 < 0$

$(-4,3)$: Test $x=0$

$f(0) = \frac{0-3}{0^2+0-12} = \frac{-3}{-12} = \frac{1}{4} > 0$

$(3,\infty)$: Test $x=4$

$f(4) = \frac{4-3}{4^2+4-12} = \frac{1}{8} > 0$

3. (12 points) Consider the function

$$f(x) = 4x^2 - 3.$$

Use the limit definition of the derivative to compute $f'(x)$. No credit will be given for using shortcuts on this problem.

(a) (3 points)

$$f(x+h) = 4(x+h)^2 - 3$$
$$= 4(x^2+2xh+h^2)-3$$
$$= 4x^2+8xh+4h^2-3$$

(b) (3 points)

$$f(x+h) - f(x) = \cancel{4x^2} + 8xh + 4h^2 \cancel{-3} - (\cancel{4x^2} \cancel{-3})$$
$$= 8xh + 4h^2$$

(c) (3 points)

$$\frac{f(x+h)-f(x)}{h} = \frac{8xh+4h^2}{h}$$
$$= 8x + 4h$$

(d) (3 points)

$$f'(x) = \lim_{h \to 0} 8x + 4h = 8x + 0 = \boxed{8x}$$

4. (18 points) Compute the following quantities. You may use shortcuts.

(a) (6 points)

$$f'(x) \quad \text{for} \quad f(x) = x^4 - 3x^3 + 2x + 1$$

$$\boxed{f'(x) = 4x^3 - 9x^2 + 2}$$

(b) (6 points)

$$\frac{d}{dx}f(x) \quad \text{for} \quad f(x) = \frac{1}{x^8} - \ln(x) = \cancel{x}^{-8} - \ln x$$

$$\boxed{\frac{d}{dx}f(x) = -8x^{-9} - \frac{1}{x}}$$

(c) (6 points)

$$y' \quad \text{for} \quad y = \sqrt[3]{x} - 6e^x = x^{1/3} - 6e^x$$

$$\boxed{y' = \frac{1}{3}x^{-2/3} - 6e^x}$$

5. (12 points) For the function $f(x) = x^4 - \cancel{64x^2}$ $32x^2$ $f'(x) = 4x^3 - 64x$

(a) (6 points) Find the equation of the tangent line to f at $x = 1$.

Point: $x_0 = 1$, $y_0 = f(1) = 1^4 - 32 \cdot 1^2 = -31$

slope: $f'(1) = 4 \cdot 1^3 - 64 \cdot 1 = -60$

equation: $y - (-31) = -60(x-1)$

$\Rightarrow y + 31 = -60(x-1)$

$\Rightarrow \boxed{y = -60x + 29}$

any of these are acceptable answers

(b) (6 points) Find where the tangent line to f is horizontal.

$f'(x) = 0 \Rightarrow 4x^3 - 64x = 0$

$\Rightarrow 4x(x^2 - 16) = 0$

$\Rightarrow 4x(x+4)(x-4) = 0$

$\Rightarrow \boxed{x = 0, -4, 4}$

6. (6 points) Suppose \$2500 is invested with continuous compounding. At the end of 6 years, the investment is worth \$3000. Find r, the annual rate. *Hint*: the formula for continuous compounding is $F = Pe^{rt}$.

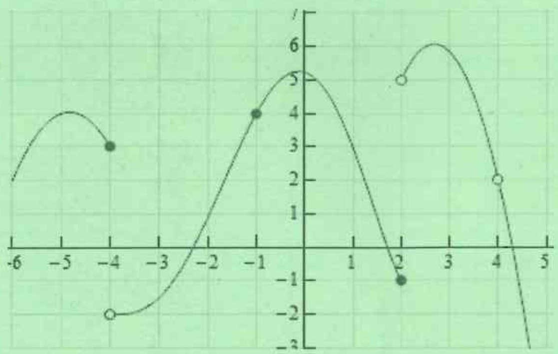
$3000 = 2000 e^{r \cdot 6}$

$1.2 = e^{6r}$

$\ln(1.2) = 6r \cdot \cancel{\ln(e)}^{\,1}$

$\boxed{r = \frac{\ln(1.2)}{6} = 0.030}$

7. (4 points) Below is the graph of some function $f(x)$.



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$x = -4$ (ii) + (iii) fail

$x = +2$ (ii) + (iii) fail

$x = 4$ (i) + (iii) fail