Math 17: Exercise Set 7 – Posted 11/5/15

## 17.1 & 17.2: Normal Distributions – Basic Properties

1. Compute $\mu$, $\sigma$, $Q_1$, and $Q_3$ for the following normal distributions.
   (a) $\mu = 80$ and upper point of inflection $P = 90$.
   (b) $\mu = 32$ and lower point of inflection $P' = 18$.
   (c) upper point of inflection $P = 400$ and lower point of inflection $P' = 220$.
   (d) $Q_1 = 950$ and $Q_3 = 1020$.
   (e) $Q_3 = 105$ and upper point of inflection $P = 120$.

2. Suppose we have a dataset $\{x_1, x_2, \ldots, x_N\}$ of $N$ datapoints, which is normal with mean $\mu$ and standard deviation $\sigma$.
   (a) Let $a$ be a number and consider the dataset $\{ax_1, ax_2, \ldots ax_N\}$. Verify the mean is $a\mu$ and the standard deviation is $a\sigma$.
   (b) Let $b$ be a number and consider the dataset $\{x_1 + b, x_2 + b, \ldots, x_N + b\}$. Verify the mean is $\mu + b$ and the standard deviation is $\sigma$.
   (c) Consider the dataset $\{ax_1 + b, ax_2 + b, \ldots, ax_N + b\}$. Combine the last two results to verify the mean is $a\mu + b$ and the standard deviation is $a\sigma$.
   (d) Suppose we have a dataset of temperatures in Fahrenheit that is normal with $\mu = 60°$ and standard deviation $\sigma = 10°$. The formula to go from Fahrenheit (F) to Celsius (C) is $\boxed{C = (5/9) * (F - 32°)}$ Find the mean and standard deviation of the dataset in Celsius. *Hint*: Use part (c) after identifying $a, b$.

3.
   (a) Explain why a distribution with $\mu = 195$, $Q_1 = 180$, and $Q_3 = 220$ cannot be a normal distribution.
   (b) Explain why a distribution with $\mu = 47$, $Q_1 = 35$, and $\sigma = 10$ cannot be a normal distribution.

4. Consider a normal distribution with $\mu = 110$ and $\sigma = 12$. Find the $z$-value of each of the following:
   (a) $x = 98$.
   (b) $x = 110$.
   (c) $x = 128$.

5. Consider a normal distribution with $\mu = 183.5$ and $\sigma = 31.2$. Find the data value corresponding to each of the following $z$-values.
   (a) $z = 0$.

(b) $z = 1.5$.

(c) $z = -2.2$.

6. In a normal distribution, what percent of data have $z$-values satisfying

   (a) $z \leq 2$.

   (b) $1 \leq z \leq 2$.

   (c) $-3 \leq z \leq 1$.

## 17.3 & 17.4: Normal Distributions – Applications

7. Packaged foods are not always the weight indicated on the package. Suppose the exact weight of a "12-ounce" bag of potato chips follows a normal distribution with $\mu = 12$ ounces and $\sigma = 0.5$ ounces.

   (a) If a bag is chosen at random, what is the chance that it weighs:

   - between 11 and 13 ounces.
   - between 12 and 12.5 ounces.
   - less than 11 ounces.

   (b) Suppose 1500 "12-ounce" bags are chosen at random. Estimate the number of bags that weigh less than 11 ounces.

8. *Clinical data.* Suppose the weight of six-month-old boys is normal with $\mu = 17.5$lbs and $\sigma = 1.0$lbs, and the weight of six-month-old girls is normal with $\mu = 16.1$lbs and $\sigma = 0.9$lbs.

   (a) If a six-month-old boy weighs 19.5lbs, what percentile is he in?

   (b) If a six-month-old boy weighs 15.5lbs, what percentile is he in?

   (c) If a six-month-old girl weighs 17lbs, what percentile is she in?

   (d) What is the range of weights for six-month-old girls with weights in the middle 68% of the data.

   (e) Find the interquartile range for the weight of six-month-old boys.

   (f) Is it more unlikely that a six-month-old boy weighs 18.5lbs or that a six-month-old girl weighs 13.4lbs.

<u>Some Useful Formulas:</u>

upper point of inflection: $P = \mu + \sigma$.
lower point of inflection: $P' = \mu - \sigma$.
third quartile: $Q_3 = \mu + 0.675\sigma$.
first quartile: $Q_1 = \mu - 0.675\sigma$.
$z$-value: $z = (x - \mu)/\sigma$.
$x$-value: $x = \sigma z + \mu$.
$68 - 95 - 99.7$ rule.