Quiz 2 Solutions

1 a) Gridlock occurs when $q > V$, where V is the total weight of the system. In this example, $\boxed{q > 40}$.

b) Anarchy occurs when $q \leq \frac{1}{2} V$ because there is a possibility for a split 50/50 vote. In this example, $\boxed{q \leq 20}$.

2. a) (Banzhaf)

winning coalitions	weight	critical players
$\{P_1, P_2\}$	9	P_1, P_2
$\{P_1, P_3\}$	8	P_1, P_3
~~$\{P_2, P_3\}$~~		
$\{P_1, P_2, P_3\}$	12	P_1

Critical counts: $b_1 = 3$, $b_2 = \cancel{1}$, $b_3 = 1$

$$T = 3 + 1 + 1 = 5$$

BPIs: $\boxed{B_1 = \frac{3}{5}, \quad B_2 = \frac{1}{5}, \quad B_3 = \frac{1}{5}}$

b) (Shapley-Shubik)

Sequential coalitions

$(P_1, \underline{P_2}, P_3)$
$(P_1, \underline{P_3}, P_2)$
$(P_2, \underline{P_1}, P_3)$
$(P_2, P_3, \underline{P_1})$
$(P_3, \underline{P_1}, P_2)$
$(P_3, P_2, \underline{P_1})$

Pivotal counts: $SS_1 = 4$, $SS_2 = 1$, $SS_3 = 1$

$$N! = 3! = 6$$

SSPIs: $\boxed{\sigma_1 = \frac{4}{6}, \quad \sigma_2 = \frac{1}{6}, \quad \sigma_3 = \frac{1}{6}}$