

Math 19 E
Spring 2019
Final Exam
May 6

Name: _____

PRACTICE EXAM

This exam contains 7 pages and 7 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	20	
4	12	
5	8	
6	8	
7	12	
Total:	100	

HONORS PLEDGE (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: _____

1. (20 points) Evaluate the following indefinite integrals

(a) (8 points)

$$\int (3x^5 - 2x + 3)dx$$

(b) (6 points)

$$\int (6x^2 + 10)e^{x^3+5x} dx$$

(c) (6 points)

$$\int \frac{2x}{x^2 - 8} dx$$

2. (20 points) More integration topics

(a) (6 points) Given that

$$\frac{d}{dx} \left(\frac{1}{1 + e^{-x}} \right) = \frac{e^{-x}}{(1 + e^{-x})^2}.$$

Find the particular antiderivative of

$$\frac{e^{-x}}{(1 + e^{-x})^2}$$

which passes through the point $(0, 0)$.

(b) (6 points) Given that

$$\int_0^{36} x dx = 648; \quad \int_0^9 \sqrt{x} dx = 18; \quad \int_9^{36} \sqrt{x} dx = 126$$

Compute $\int_0^{36} (2x + \sqrt{x}) dx$.

(c) (8 points) Compute the definite integral

$$\int_5^7 (3x + 2) dx$$

3. (20 points) Limits – When evaluating limits make sure to briefly justify your answer. Write DNE if a limit does not exist.

(a) (6 points) Evaluate

$$\lim_{x \rightarrow 1} \frac{x}{x-1}$$

(b) (6 points) Evaluate

$$\lim_{x \rightarrow 0^+} \frac{\ln(1+x)}{x}$$

(c) (8 points) Find all horizontal and vertical asymptotes of

$$f(x) = \frac{2(x^2 - 9)}{x^2 + 5x + 6}$$

4. (12 points) Compute the following derivatives

(a) (6 points)

$$\frac{d}{dx} (x^3 + 2 \ln(x) - 4)$$

(b) (6 points)

$$\frac{d}{dx} (xe^{2x})$$

5. (8 points) Find the equation of the tangent line to the function

$$f(x) = \frac{e^x}{x}$$

at the point $(1, e)$.

6. (8 points) Find y' given that

$$2y + xy - 1 = 0.$$

7. (12 points) For the function $f(x) = -x^3 + 12x + 2$

(a) (8 points) Find the intervals where f is increasing/decreasing

(b) (4 points) Find any local maxima or minima.

Bonus

1. Suppose f is a continuous function and $f(x) > 0$ for all x . Explain why the function

$$g(t) = \int_0^t f(x) dx$$

always increases as t increases.

2. Compute the 100th derivative of

$$f(x) = xe^x$$

3. The 3rd-degree Taylor polynomial for f at 0 is defined to be the function

$$p(x) = f(0) + f'(0)x + \frac{1}{2}f''(0)x^2 + \frac{1}{6}f'''(0)x^3$$

where $f'(0)$, $f''(0)$, and $f'''(0)$ are the 1st, 2nd, and 3rd derivatives of f evaluated at zero. This gives a polynomial function $p(x)$ which, near $x = 0$, has approximately the same graph as $f(x)$.

Compute the 3rd-degree Taylor polynomial for $f(x) = e^x$.