Solutions

Section 3.3 & 3.4: Product, Quotient, & Chain Rule

Find the derivative of each function

Product Rule

(a) $h(x) = (x^2 + 5)\underbrace{(3x - 14x)}_{= -11x}$

$h'(x) = 2x(-11x) + (-11)(x^2 + 5)$

(b) $h(x) = (3x^5 - 7x^2)e^x$

$h'(x) = (15x^4 - 14x)e^x + e^x(3x^5 - 7x^2)$

(c) $h(x) = (e^x + 1)\ln(x)$

$h'(x) = e^x \ln x + \frac{1}{x} \cdot (e^x + 1)$

Quotient Rule

(a) $h(x) = \dfrac{4x^2 - 1}{6x + 6}$

$h'(x) = \dfrac{8x(6x+6) - 6(4x^2 - 1)}{(6x+6)^2}$

(b) $h(x) = \dfrac{1 + e^x}{1 - e^x}$

$h'(x) = \dfrac{e^x(1 - e^x) - (-e^x)(1 + e^x)}{(1 - e^x)^2}$

(c) $h(x) = \dfrac{5\ln(x) + x^7}{x^2 - 1}$

$h'(x) = \dfrac{(5 \cdot \frac{1}{x} + 7x^6)(x^2 - 1) - 2x(5\ln x + x^7)}{(x^2 - 1)^2}$

Chain Rule

(a) $h(x) = (x^3 + 2x + 12)^8$

$h'(x) = 8(x^3+2x+12)^7 \cdot (3x^2+2)$

(b) $h(x) = \sqrt[4]{x^2 - 5x + 6}$ $= (x^2-5x+6)^{1/4}$

$h'(x) = \frac{1}{4}(x^2-5x+6)^{-3/4} \cdot (2x-5)$

(c) $h(x) = e^{-x^2}$

$h'(x) = e^{-x^2} \cdot (-2x)$

(d) $h(x) = \ln(x^3 - 3)$

$h'(x) = \frac{1}{x^3-3} \cdot 3x^2$

??? Rule(s)

(a) $h(x) = (x^4 + e^x)\ln(x)$ $\quad h'(x) = (4x^3+e^x)\ln x + \frac{1}{x} \cdot (x^4+e^x)$

(b) $h(x) = \dfrac{e^{18x}}{x}$ $\quad h'(x) = \dfrac{18e^{18x} \cdot x - e^{18x}}{x^2}$

(c) $h(x) = (5x + 5)^{55}$ $\quad h'(x) = 55(5x+5)^{54} \cdot 5$

(d) $h(x) = \dfrac{1}{\ln x} - \dfrac{1}{e^x}$ $= (\ln x)^{-1} - e^{-x} \Rightarrow \boxed{h'(x) = -(\ln x)^{-2} \cdot \frac{1}{x} - e^{-x} \cdot (-1)}$

(e) $h(x) = \ln\left(\dfrac{x^2}{e^x}\right)$ $= \ln(x^2) - \ln(e^x) = 2\ln x - x \Rightarrow \boxed{h'(x) = \frac{2}{x} - 1}$

Hint (d): Don't <u>have</u> to use quotient rule. Instead start by using $1/a = a^{-1}$.

Hint (e): Don't <u>have</u> to use quotient rule. Instead start by using $\ln(a/b) = \ln a - \ln b$.