Solutions

Math 19 E
Spring 2019
Exam 3
April 18
Version A

Name: ____________________

This exam contains 6 pages and 6 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	20	
2	20	
3	18	
4	20	
5	10	
6	12	
Total:	100	

<u>HONORS PLEDGE</u> (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: ____________________

1. (20 points) For the function $f(x) = x^3 - 9x^2 + 24x$

(a) (6 points) Calculate the first derivative of f.

$$f'(x) = 3x^2 - 18x + 24$$

(b) (6 points) Find the partition numbers of f'.

$$f'(x) = 0 \Rightarrow 3x^2 - 18x + 24 = 0$$
$$\Rightarrow 3(x^2 - 6x + 8) = 0$$
$$\Rightarrow x^2 - 6x + 8 = 0$$
$$\Rightarrow (x-2)(x-4) = 0$$
$$\Rightarrow \boxed{x = 2, \quad x = 4}$$

(c) (6 points) Find the intervals where f is increasing/decreasing.

$f'(0) = 24 > 0$
$f'(3) = -3 < 0$
$f'(5) = 9 > 0$

increasing on $(-\infty, 2) \cup (4, \infty)$
decreasing on $(2, 4)$

(d) (2 points) Find any local extrema. Make sure to specify whether each is a maximum or a minimum.

$x = 2$ is a local max
$x = 4$ is a local min

2. (20 points) For the function $f(x) = x^3 - 9x^2 + 24x$ from the previous page

(a) (6 points) Calculate the second derivative of f.

$f'(x) = 3x^2 - 18x + 24$

$\boxed{f''(x) = 6x - 18}$

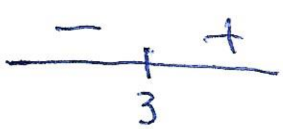
(b) (6 points) Find the partition numbers of f''.

$f''(x) = 0 \Rightarrow 6x - 18 = 0$

$\Rightarrow 6x = 18$

$\Rightarrow \boxed{x = 3}$

(c) (6 points) Find the intervals where f is concave up/down.



$f''(0) = -18 < 0$

$f''(4) = 6 > 0$

Concave up: $(3, \infty)$

Concave down: $(-\infty, 3)$

(d) (2 points) Find any points of inflection.

X=3 is a paint of inflection

3. (18 points) For the function $f(x) = x^3 + 3x^2 - 9x$

(a) (6 points) Calculate the first derivative.

$$f'(x) = 3x^2 + 6x - 9$$

(b) (6 points) Find the critical numbers of f.

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$
$$\Rightarrow 3(x^2 + 2x - 3) = 0$$
$$\Rightarrow x^2 + 2x - 3 = 0$$
$$\Rightarrow (x+3)(x-1) = 0$$
$$\Rightarrow \boxed{x = -3,\ x = 1}$$

(c) (6 points) Find the absolute maximum and minimum on the interval $[-4, 0]$.

<u>Note:</u> $x = 1$ is outside $[-4, 0]$

x	$f(x)$
-4	20
-3	27
0	0

$x = -3$ is ~~local~~ absolute max

$x = 0$ is ~~local~~ absolute min

4. (20 points) Evaluate the following limits. Use L'Hopital's rule if it applies – do NOT factor.

   (a) (10 points)

$$\lim_{x \to 7} \frac{x-7}{x^2+5x-84} = \frac{0}{0} \quad \text{apply LH rule}$$

$$= \lim_{x \to 7} \frac{1}{2x+5}$$

$$= \boxed{\frac{1}{19}}$$

   (b) (10 points)

$$\lim_{x \to 0} \frac{e^{3x}-1-3x}{x^2} = \frac{0}{0} \quad \text{apply LH rule}$$

$$= \lim_{x \to 0} \frac{3e^{3x}-3}{2x}$$

$$= \frac{0}{0} \quad \text{apply LH rule}$$

$$= \lim_{x \to 0} \frac{9e^{3x}}{2}$$

$$= \boxed{\frac{9}{2}}$$

5. (10 points) Evaluate the integral

$$\int (x^4 - 6x^2 + \sqrt{x} + 2)dx$$

$$= \int x^4 dx - 6\int x^2 dx + \int x^{1/2} dx + 2\int 1 dx$$

$$= \frac{x^5}{5} - 6\frac{x^3}{3} + \frac{x^{3/2}}{3/2} + 2x + C$$

$$= \boxed{\frac{x^5}{5} - 2x^3 + \frac{2}{3}x^{3/2} + 2x + C}$$

6. (12 points) Since 1950, U.S. consumption of renewable energy has been growing at a rate (in quadrillion Btu per year) given by

$$f'(t) = 0.002t + 0.03$$

where t is years after 1950.

(a) (8 points) Find $f(t)$ by taking the integral $\int f'(t)dt$.

$$f(t) = \int f'(t)dt = \int (0.002t + 0.03)dt$$

$$= 0.002\int t\,dt + 0.03\int 1 dt$$

$$= 0.002 \cdot \frac{t^2}{2} + 0.03t + C$$

$$= \boxed{0.001t^2 + 0.03t + C}$$

(b) (4 points) In 2016, U.S. consumption of renewable energy was 9.97 quadrillion Btu – that is, $\boldsymbol{f(66) = 9.97}$. Solve for C and use $f(t)$ to predict the U.S. consumption of renewable energy in 2030 ($t = 80$).

$$f(66) = 9.97 \implies 0.001(66)^2 + 0.03(66) + C = 9.97$$

$$\implies 6.336 + C = 9.97$$

$$\implies \boxed{C = 3.634}$$

Therefore

$$f(t) = 0.001t^2 + 0.03t + 3.634$$

The predicted U.S. consumption of renewable energy in 2030 is

$$\boxed{f(80) = 12.434}$$ (quadrillion Btu)

Solutions

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<u>**HONORS PLEDGE**</u> (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: ____

1. (20 points) For the function $f(x) = x^3 - 12x^2 + 45x$

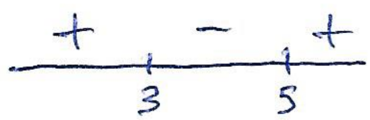
   (a) (6 points) Calculate the first derivative of f.

$$f'(x) = 3x^2 - 24x + 45$$

   (b) (6 points) Find the partition numbers of f'.

$$f'(x) = 0 \Rightarrow 3x^2 - 24x + 45 = 0$$
$$\Rightarrow 3(x^2 - 8x + 15) = 0$$
$$\Rightarrow x^2 - 8x + 15 = 0$$
$$\Rightarrow (x-3)(x-5) = 0$$
$$\Rightarrow \boxed{x = 3,\ x = 5}$$

   (c) (6 points) Find the intervals where f is increasing/decreasing.



$f'(0) = 45 > 0$
$f'(4) = -3 < 0$
$f'(6) = 9 > 0$

$\boxed{\text{increasing on } (-\infty, 3) \cup (5, \infty) \quad \text{decreasing on } (3, 5)}$

   (d) (2 points) Find any local extrema. Make sure to specify whether each is a maximum or a minimum.

$x = 3$ is a local max

$x = 5$ is a local min

2. (20 points) For the function $f(x) = x^3 - 12x^2 + 45x$ from the previous page

(a) (6 points) Calculate the second derivative of f.

$$f'(x) = 3x^2 - 24x + 45$$

$$\boxed{f''(x) = 6x - 24}$$

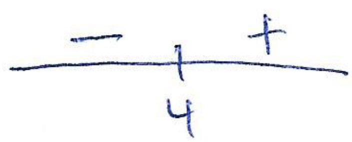
(b) (6 points) Find the partition numbers of f''.

$$f''(x) = 0 \Rightarrow 6x - 24 = 0$$

$$\Rightarrow 6x = 24$$

$$\Rightarrow \boxed{x = 4}$$

(c) (6 points) Find the intervals where f is concave up/down.



$$f''(0) = -24 < 0$$

$$f''(5) = 6 > 0$$

$$\boxed{\begin{array}{c} \text{concave up:} \\ (4, \infty) \\ \text{concave down:} \\ (-\infty, 4) \end{array}}$$

(d) (2 points) Find any points of inflection.

$x = 4$ is a point of inflection

3. (18 points) For the function $f(x) = x^3 - 9x^2 + 15x$

(a) (6 points) Calculate the first derivative of f.

$$f'(x) = 3x^2 - 18x + 15$$

(b) (6 points) Find the critical numbers of f.

$$f'(x) = 0 \implies 3x^2 - 18x + 15 = 0$$
$$\implies 3(x^2 - 6x + 5) = 0$$
$$\implies x^2 - 6x + 5 = 0$$
$$\implies (x-1)(x-5) = 0$$
$$\implies \boxed{x = 1,\ x = 5}$$

(c) (6 points) Find the absolute maximum and minimum on the interval $[0, 4]$.

Note: $x = 5$ is outside $[0, 4]$

x	$f(x)$
0	0
1	7
4	-20

$x = 1$ is absolute max

$x = 4$ is absolute min

4. (20 points) Evaluate the following limits. Use L'Hopital's rule if it applies – do NOT factor.

   (a) (10 points)

$$\lim_{x\to 7} \frac{x-7}{x^2+6x-91} = \frac{0}{0} \quad \text{apply LH rule}$$

$$= \lim_{x\to 7} \frac{1}{2x+6}$$

$$= \boxed{\frac{1}{20}}$$

   (b) (10 points)

$$\lim_{x\to 0} \frac{e^{2x}-1-2x}{x^2} = \frac{0}{0} \quad \text{apply LH rule}$$

$$= \lim_{x\to 0} \frac{2e^{2x}-2}{2x}$$

$$= \frac{0}{0} \quad \text{apply LH rule}$$

$$= \lim_{x\to 0} \frac{4e^{2x}}{2}$$

$$= \boxed{\frac{4}{2}} = 2$$

5. (10 points) Evaluate the integral

$$\int (x^4 + 9x^2 + \sqrt{x} - 3)dx$$

$$= \int x^4 dx + 9\int x^2 dx + \int x^{1/2} dx - 3\int 1 dx$$

$$= \frac{x^5}{5} + 9\frac{x^3}{3} + \frac{x^{3/2}}{3/2} - 3x + C$$

$$= \boxed{\frac{x^5}{5} + 3x^3 + \frac{2}{3}x^{3/2} - 3x + C}$$

6. (12 points) Since 1950, U.S. consumption of renewable energy has been growing at a rate (in quadrillion Btu per year) given by

$$f'(t) = 0.002t + 0.03$$

where t is years after 1950.

(a) (8 points) Find $f(t)$ by taking the integral $\int f'(t)dt$.

$$f(t) = \int f'(t)dt = \int (0.002t + 0.03)dt$$

$$= 0.002\int t\,dt + 0.03\int 1\,dt$$

$$= 0.002\frac{t^2}{2} + 0.03t + C$$

$$= \boxed{0.001t^2 + 0.03t + C}$$

(b) (4 points) In 2016, U.S. consumption of renewable energy was 9.97 quadrillion Btu – that is, $\boldsymbol{f(66) = 9.97}$. Solve for C and use $f(t)$ to predict the U.S. consumption of renewable energy in 2030 ($t = 80$).

$$f(66) = 9.97 \Rightarrow \underbrace{0.001(66)^2 + 0.03(66)}_{} + C = 9.97$$

$$\Rightarrow 6.336 + C = 9.97$$

$$\Rightarrow \boxed{C = 3.634}$$

Therefore

$$f(t) = 0.001t^2 + 0.03t + 3.634$$

The predicted U.S. consumption of renewable energy in 2030 is

$$\boxed{f(80) = 12.434} \text{ (quadrillion Btu)}$$