Math 19 A&B
Fall 2019
Exam 1
October 3

Name: _____

Solutions

PRACTICE EXAM

This exam contains 6 pages and 7 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	24	
2	24	
3	12	
4	18	
5	12	
6	6	
7	4	
Total:	100	

HONORS PLEDGE (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: _____

1. (24 points) Determine the following limits

(a) (6 points)

$$\lim_{x\to 8^-} \frac{x+3}{x-8} = \frac{8+3}{8-8} = \frac{11}{0}$$

$$\approx \frac{11}{7.99-8} = \frac{11}{\text{small}-} = \text{big}-$$

$$= \boxed{-\infty}$$

(b) (6 points)

$$\lim_{x\to 4^+} \frac{x^2-x-12}{x-4} = \frac{4^2-4-12}{4-4} = \frac{0}{0}$$

$$= \lim_{x\to 4^+} \frac{(x-4)(x+3)}{x-4} = \lim_{x\to 4^+} x+3 = 4+3$$

$$= \boxed{7}$$

(c) (6 points)

$$\lim_{x\to\infty} \frac{x+9}{x^2+3x+2} = \lim_{x\to\infty} \frac{x}{x^2}$$

(case where numerator exponent < denom)

$$= \lim_{x\to\infty} \frac{1}{x}$$

$$= \frac{1}{\text{big}+} = \boxed{0}$$

(d) (6 points)

$$\lim_{x\to 1^-} \frac{x}{(x-1)^2} = \frac{1}{(1-1)^2} = \frac{1}{0}$$

$$\approx \frac{1}{(0.99-1)^2}$$

$$= \frac{1}{(\text{small}-)^2}$$

$$= \frac{1}{\text{small}+}$$

$$= \text{big}+$$

$$= \boxed{+\infty}$$

2. (24 points) For the function

$$f(x) = \frac{x+1}{x^2-1}$$

(a) (6 points) Find any vertical asymptotes of f.

$x^2 - 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$ are "candidates"

(check) $\lim_{x\to 1} f(x) = \frac{1+1}{1^2-1} = \frac{2}{0} \Rightarrow$ $x=1$ is V-A

$\lim_{x\to -1} f(x) = \frac{-1+1}{(-1)^2-1} = \frac{0}{0} = \lim_{x\to -1} \frac{x+1}{(x+1)(x-1)} = \lim_{x\to -1} \frac{1}{x-1} = -\frac{1}{2}$

$\Rightarrow$ no V-A at $x=-1$

(b) (6 points) Find any horizontal asymptotes of f.

$\lim_{x\to\infty} \frac{x+1}{x^2-1} = \lim_{x\to\infty} \frac{x}{x^2}$

(case where exponent < denom) $= \lim_{x\to\infty} \frac{1}{x}$

$= \frac{1}{\text{big}+}$

$= 0 \Rightarrow$ $y=0$ is H-A

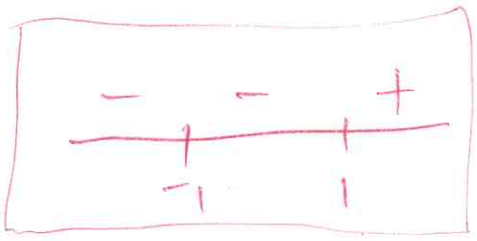
(c) (6 points) Find the partition numbers of f.

2 f is discont: $x^2 - 1 = 0 \Rightarrow x = \pm 1$

1 $f(x) = 0$: $x + 1 = 0 \Rightarrow x = -1$

$x = 1$ and $x = -1$

(d) (6 points) Make a sign chart for f.



$(-\infty, -1)$: Test $x = -2$

$f(-2) = \frac{-2+1}{(-2)^2-1} = \frac{-1}{3} < 0$

$(-1, 1)$: Test $x = 0$

$f(0) = \frac{0+1}{0^2-1} = -1 < 0$

$(1, \infty)$: Test $x = 2$

$f(2) = \frac{2+1}{2^2-1} = 1 > 0$

3. (12 points) Consider the function

$$f(x) = x^2 + 7.$$

Use the limit definition of the derivative to compute $f'(x)$. No credit will be given for using shortcuts on this problem.

(a) (3 points)

$f(x+h) =$ $(x+h)^2 + 7$

$= x^2 + 2xh + h^2 + 7$

(b) (3 points)

$f(x+h) - f(x) =$ $x^2 + 2xh + h^2 + 7 - (x^2 + 7)$

$= 2xh + h^2$

(c) (3 points)

$\dfrac{f(x+h) - f(x)}{h} =$ $\dfrac{2xh + h^2}{h}$

$= 2x + h$

(d) (3 points)

$f'(x) =$ $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim_{h \to 0} 2x + h = \boxed{2x}$

4. (18 points) Compute the following quantities. You may use shortcuts.

(a) (6 points)

$f'(x)$ for $f(x) = x^5 - 2x^3 + 4x$

$f'(x) = 5x^4 - 6x^2 + 4$

(b) (6 points)

$\frac{d}{dx} f(x)$ for $f(x) = \frac{6}{x^3} - e^x = 6x^{-3} - e^x$

$\frac{d}{dx} f(x) = 6 \cdot (-3) x^{-4} - e^x$

$= \boxed{-18x^{-4} - e^x}$

(c) (6 points)

y' for $y = \sqrt{x} - \ln(x) = x^{1/2} - \ln x$

$\boxed{y' = \frac{1}{2} x^{-1/2} - \frac{1}{x}}$

5. (12 points)

(a) (6 points) Find the equation of the tangent line to $f(x) = x^5 - 2x^3 + 4x$ at $x = 1$. *Hint*: use your answer to part (a) of the previous page. $f'(x) = 5x^4 - 6x^2 + 4$

Point: $x_0 = 1 \Rightarrow y_0 = f(1) = 1^5 - 2(1)^3 + 4(1) = 1 - 2 + 4 = 3$

Slope: $f'(1) = 5(1)^4 - 6(1)^2 + 4 = 5 - 6 + 4 = 3$

Eq'n: $\boxed{y - 3 = 3(x-1)} \iff y = 3x$

(b) (6 points) Find where the tangent line to $g(t) = t^4 - 18t^2$ is horizontal.

$g'(t) = 4t^3 - 36t$

$g'(t) = 0 \Rightarrow 4t^3 - 36t = 0$

$\Rightarrow 4t(t^2 - 9) = 0$

$\Rightarrow 4t(t+3)(t-3) = 0$

$\Rightarrow \boxed{t = 0, -3, 3}$

6. (6 points) Suppose \$1000 is invested with continuous compounding. At the end of 3 years, the investment is worth \$1500. Find r, the annual rate. *Hint*: the formula for continuous compounding is $F = Pe^{rt}$.

$F = 1500, P = 1000, t = 3, r = ??$

$1500 = 1000e^{3r}$

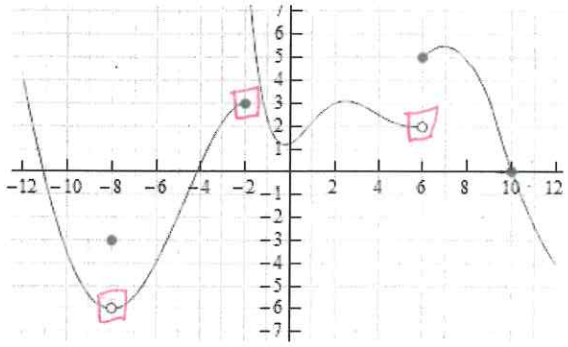
$1.5 = \frac{1500}{1000} = e^{3r}$

$\ln(1.5) = \ln(e^{3r})$

$\ln(1.5) = 3r \ln(e)$ ($\ln(e) \to 1$)

$\boxed{r = \frac{\ln(1.5)}{3} = 0.135}$

7. (4 points) Below is the graph of some function $f(x)$.



Criteria for continuity

(i) $f(c)$ exists

(ii) $\lim_{x\to c} f(x)$ exists

(iii) $\lim_{x\to c} f(x) = f(c)$

Where is f discontinuous? For each point of discontinuity, which of the three continuity criteria fails?

$x = -8$ (iii) fails

$x = -2$ (ii) + (iii) fail

$x = 6$ (ii) + (iii) fail