Math 17: Exam 1 Name:

There are 4 pages and 6 questions in total. There are 50 possible points and the point values for each page are 13, 12, 13, and 12 respectively. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown.

1. Shown below is the preference schedule for an election with four candidates (A, B, C, and D).

Number of voters	27	15	11	9	8	1
1st	C	A	B	D	B	B
2nd	D	B	D	A	A	A
3rd	B	D	A	B	C	D
4th	A	C	C	C	D	C

   (a) Determine the full ranking using the Plurality method. (3 pt)

   (b) Determine the full ranking using Plurality-with-Elimination. (8 pt)

   (c) What issue does the Plurality method have, but which the Plurality-with-Elimination method resolves? (2 pt)

2. Consider the weighted voting system $[7 : 5, 2, 1]$.

   (a) Which player(s), if any, have veto power? Justify your answer. (3 pt)

   (b) Determine the Banzhaf power distribution of this voting system. (7 pt)

Winning coalition	Weight of coalition	Critical players

$b_1 =$ _____, $b_2 =$ _____, $b_3 =$ _____

$T =$ _____

$\beta_1 =$ _____, $\beta_2 =$ _____, $\beta_3 =$ _____

3. What is the conceptual difference between the Banzhaf measurement of power and the Shapley-Shubik measurement of power? (2 pt)

4. Suppose we are dividing three shares s_1, s_2, s_3 among three players Alex, Benson, and Christine. The value system for each player is given in the table below.

	s_1	s_2	s_3
Alex	25%	40%	35%
Benson	28%	35%	37%
Christine	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$

(a) Determine the fair shares for each player. (2 pt)

(b) Determine all possible fair divisions. *Hint*: there are only two. (2 pt)

(c) Describe each possible fair division using the terms: efficient, inefficient, envy-free. Justify your answer. *Hint*: you may need to use more than one term. (2 pt)

5. Suppose we want to divide three items (a car, a laptop, and a guitar) among two siblings (Drake & Josh). From the bids shown below, use the Method of Sealed bids to determine the final settlement. (7 pt)

	Car	Laptop	Guitar
Drake	\$7000	\$500	\$500
Josh	\$5000	\$900	\$100

6. A university consists of five colleges: Arts and Sciences (A & S), Business, Education, Nursing, and Engineering. The university wants to hire 250 faculty in total. The amount of faculty that are hired for each college should be in proportion to the student enrollment for the college. Below is the student enrollment for each college.

College	A & S	Business	Education	Nursing	Engineering	Total
Enrollment	1,646	762	2,081	1,066	6,945	12,500
Standard quota						
Lower quota						
Residue						
Apportionment						

(a) Calculate the standard divisor and interpret it in the context of the problem. (2 + 2 pt)

(b) Find the apportionment of faculty using Hamilton's Method. (8 pt)
Show your work for this problem by adding to the table above.