Math 19 A&B **Name:** _____
Fall 2019
Exam 2
November 7

PRACTICE EXAM

This exam contains 8 pages and 6 questions. Total of points is 100. For full credit you must show your work. Partial credit may be given for incorrect solutions if sufficient work is shown. Messy/unorganized answers may be penalized, even if correct.

Grade Table (for teacher use only)

Question	Points	Score
1	30	
2	10	
3	15	
4	10	
5	25	
6	10	
Total:	100	

HONORS PLEDGE (sign after exam is completed): I have neither given nor received aid on this exam, nor have I observed a violation of the UVM Code of Academic Integrity.

Signature: _____

1. (30 points) Calculate the derivatives of the following functions.

   (a) (6 points)

$$h(x) = 3(2x^2 + 5x - 8)^4$$

   (b) (6 points)

$$h(x) = 10 \ln(1 - 7x)$$

   (c) (6 points)

$$h(x) = \frac{1 - e^x}{1 + e^x}$$

(d) (6 points)

$$h(x) = x^2 \ln(x)$$

(e) (6 points)

$$h(x) = 5xe^{2x}$$

2. (10 points)

(a) (5 points) Suppose $h(x) = \dfrac{f(x)}{g(x)}$ and suppose we know that $f(1) = 1$, $f'(1) = 4$, $g(1) = 2$, and $g'(1) = 7$. Calculate $h'(1)$.

(b) (5 points) Consider the function $T(x)$ defined as

$$T(x) = \ln\left(\frac{x^3 f(x)}{g(x)}\right).$$

Find $T'(x)$. Hint 1: use properties of logarithms to simplify (otherwise you have to do product + quotient rule = yikes!!!). Hint 2: your answer will involve $f'(x)$ and $g'(x)$ due to chain rule.

3. (15 points) For the implicit curve defined by the equation: $y^4 - 3y^3 + x^3 = 6$

   (a) (10 points) Use implicit differentiation to find $\dfrac{dy}{dx}$.

   (b) (5 points) Find the equation of the tangent line at the point $(2, 1)$.

4. (10 points) **One of these two problems (but not both) will appear on the exam!**

   - A 40-foot ladder is placed against a wall. If the top of the ladder is sliding down the wall at 3 feet per second, at what rate is the bottom of the ladder moving away from the wall when the bottom of the ladder is 22 feet away from the wall?

   - The radius of a spherical balloon is increasing at the rate of 6 centimeters per minute. How fast is the volume changing when the radius is 24 centimers?

     Note: the formula for the volume V of a sphere with radius r is given by

     $$V = \frac{4}{3}\pi r^3.$$

5. (25 points) Consider the function

$$f(x) = 3x^3 - 9x$$

(a) (8 points) Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

(b) (2 points) Find the local extrema and identify whether each is a local maximum or local minimum.

(c) (8 points) Find the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

(d) (2 points) Find the point(s) of inflections.

(e) (5 points) Find the absolute maximum and absolute minimum on the interval $[0, 3]$.

6. (10 points) Evaluate the limits

(a) (5 points)

$$\lim_{x\to\infty} \frac{x^3 - 1000}{\ln(x)}$$

(b) (5 points)

$$\lim_{x\to 0} \frac{x^2}{e^{3x} - 3x - 1}$$