

Homework 3 Solutions

1

	S_1	S_2	S_3
A	38%	28%	34%
B	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$	$33\frac{1}{3}\%$
C	34%	40%	26%

Fair Share threshold = $\frac{1}{3} = 33\frac{1}{3}\%$

Fair Shares

A: S_1, S_3
 B: S_1, S_2, S_3
 C: S_1, S_2

Fair Divisions

- (i) A: S_1 , B: S_3 , C: S_2
- (ii) A: S_3 , B: S_1 , C: S_2
- (iii) A: S_3 , B: S_2 , C: S_1

Value of FD

	A	B	C
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- (i) 38% $33\frac{1}{3}\%$ 40%
- (ii) 34% $33\frac{1}{3}\%$ 40%
- (iii) 34% $33\frac{1}{3}\%$ 34%

← envy-free and efficient
 ← not efficient
 ← not efficient

2

	S_1	S_2	S_3	S_4
A	20%	32%	28%	20%
B	25%	25%	25%	25%
C	15%	15%	30%	40%
D	24%	24%	24%	28%

Fair Share threshold
 = $\frac{1}{4} = 25\%$

Fair Shares

A: S_2, S_3
 B: S_1, S_2, S_3, S_4
 C: S_3, S_4
 D: S_4

Fair Div

A: S_2 , B: S_1 , C: S_3 , D: S_4

- Not envy-free since C doesn't get S_4
- Efficient because it's the only Fair Div (nothing to compare to)

3

	S ₁	S ₂	S ₃	S ₄
A	\$3	\$5	\$5	\$3
B	\$4.50	\$3.50	\$4.50	\$5.50
C	\$4.25	\$4.50	\$2.50	\$3.75
D	\$5.50	\$4	\$4.50	\$6

Total	Fair Share Value
\$16	\$4
\$18	\$4.50
\$16	\$4
\$20	\$5

Fair shares

- A: S₂, S₃
 B: S₁, S₃
 C: S₁, S₂
 D: S₁, S₄

Fair Divisions

- (i) A: S₂, B: S₃, C: S₁, D: S₄
 (ii) A: S₃, B: S₁, C: S₂, D: S₄
 (iii) A: S₃, B: S₄, C: S₂, D: S₁

Value of FD

	A	B	C	D	
(i)	\$5	\$4.50	\$4.25	\$6	← not efficient
(ii)	\$5	\$4.50	\$4.50	\$6	← efficient
(iii)	\$5	\$5.50	\$4.50	\$5.50	←

• None are envy-free since S₄ valued most by C and D.

4

	iPad	Xbox	PS4	Total	Fair Share Value
Bob	(400)	300	(500)	1200	600
Chris	200	(400)	400	1000	500

Pot
 Bob 300 in
 Chris 100 out
 Surplus = 200
 Surplus division
 = 200/2 = 100

Bob gets iPad, PS4, pays 300, gets 100

Chris gets Xbox, gets 100 + 100