

Homework 2 Solutions

1 a) $q \geq 31$ 30 total votes

b) $q \leq 15$ (in this case $q \leq 14$ because no config of votes can give 15/15 split but I will accept either answer)

c) $q = 30$ or $q = 29$

2 a) P_1, P_2

b) P_1, P_2, P_3, P_4

c) P_1

d) P_1, P_2, P_3

3 a)

winning coalitions	weight	critical players
$\{P_1, P_2\}$	12	P_1, P_2
$\{P_1, P_3\}$	10	P_1, P_3
$\{P_1, P_2, P_3\}$	15	P_1

crit counts ~~points~~: $b_1 = 3, b_2 = 1, b_3 = 1$

$$T = 3 + 1 + 1 = 5$$

Banzhaf power distribution: $\beta_1 = \frac{3}{5}, \beta_2 = \frac{1}{5}, \beta_3 = \frac{1}{5}$

b) Sequential coalitions

(P_1, P_2, P_3)

(P_1, P_3, P_2)

(P_2, P_1, P_3)

(P_2, P_3, P_1)

(P_3, P_1, P_2)

(P_3, P_2, P_1)

$$SS_1 = 4, SS_2 = 1, SS_3 = 1$$

$$\sigma_1 = \frac{4}{3!} = \frac{4}{6}, \sigma_2 = \frac{1}{6}, \sigma_3 = \frac{1}{6}$$

4, a)

winning coalitions	weight	Critical players
$\{P_1, P_2\}$	7	P_1, P_2
$\{P_1, P_3\}$	6	P_1, P_3
$\{P_1, P_2, P_3\}$	8	P_1

Critical counts $b_1 = 3, b_2 = 1, b_3 = 1, T = 5$
 Banzhaf Power distribution $\beta_1 = \frac{3}{5}, \beta_2 = \frac{1}{5}, \beta_3 = \frac{1}{5}$

b) Sequential coalitions

(P_1, P_2, P_3)

$$SS_1 = 4, SS_2 = 1, SS_3 = 1$$

(P_1, P_3, P_2)

$$\sigma_1 = \frac{4}{6}, \sigma_2 = \frac{1}{6}, \sigma_3 = \frac{1}{6}$$

(P_2, P_1, P_3)

(P_2, P_3, P_1)

(P_3, P_1, P_2)

(P_3, P_2, P_1)

5. Clearly $[8: 5, 3, 2]$ and $[2: 1, 1, 0]$ have the same number of players.

$[8: 5, 3, 2]$ has winning coalitions $\{P_1, P_2\}, \{P_1, P_3\}, \{P_1, P_2, P_3\}$

and $[2: 1, 1, 0]$ has same.

Thus they are equivalent.

*Notice 4a,b is the same power distribution as 3a,b.
 As I said in class, there aren't many possibilities for only 3 player systems (and it isn't very interesting) but doing the computation by hand for ≥ 4 players is tedious, so we will limit ourselves.