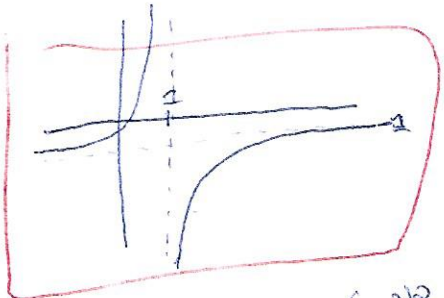
Written HW 1 Solutions

1. $\lim_{x \to 1^+} f(x) = -\infty$ $\lim_{x \to \infty} f(x) = -1$



2. a) $\lim_{x \to -2^-} \frac{x^2+4x+4}{x+2} = \frac{(-2)^2+4(-2)+4}{-2+2} = \frac{0}{0} \Rightarrow$ Factor

$= \lim_{x \to -2^-} \frac{(x+2)\cancel{(x+2)}}{\cancel{x+2}} = \lim_{x \to -2^-} x+2 = \boxed{0}$

b) $\lim_{x \to 3^-} \frac{3x+3}{x-3} = \frac{3(3)+3}{3-3} = \frac{12}{0} \Rightarrow$ more work

$= \frac{12}{2.999-3} = \frac{12}{\text{small} -} = \text{big} - = \boxed{-\infty}$

c) $\lim_{x \to 1^+} \frac{x^3+1}{x^2+1} = \frac{1+1}{1+1} = \boxed{1}$

d) $\lim_{x \to 4^+} \frac{2x^2}{x-4} = \frac{2(4)^2}{4-4} = \frac{32}{0} \Rightarrow$ more work

$= \frac{32}{4.0001-4} = \frac{32}{\text{small}+} = \text{big} + = \boxed{+\infty}$

e) $\lim_{x \to -3^-} \frac{x^2+3x+1}{(x+3)^3} = \frac{(-3)^2+3(-3)+1}{(-3+3)^3} = \frac{9-9+1}{0} = \frac{1}{0} \Rightarrow$ more work

$= \frac{1}{(2.999-3)^3} = \frac{1}{\text{small} -} = \text{big} - = \boxed{-\infty}$

3 a) $\lim_{x \to -\infty} \frac{x^2}{10x^3+55} = \lim_{x \to -\infty} \frac{x^2}{10x^3} = \lim_{x \to -\infty} \frac{1}{10x} = \boxed{0}$

3 b) $\lim_{x\to\infty} \frac{-3x^4+1}{x^2-10} = \lim_{x\to\infty} \frac{-3x^4}{x^2} = \lim_{x\to\infty} -3x^2$

$= \boxed{-\infty}$

c) $\lim_{x\to\infty} \frac{9x-4}{x-5} = \lim_{x\to\infty} \frac{9x}{x} = \lim_{x\to\infty} 9 = \boxed{9}$

4 a) $f(x) = \frac{x^3+x}{x^2-4}$

Vertical: $x^2-4=0 \Rightarrow (x+2)(x-2)=0 \Rightarrow x=\pm 2$

the numerator is nonzero at both $x=2$, $x=-2$
so there are ~~horiz~~ vertical asymptotes
at $\boxed{x=\pm 2}$

Horizontal:

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{x^3+x}{x^2-4} = \lim_{x\to\infty} \frac{x^3}{x^2} = \lim_{x\to\infty} x$

$= \infty$

and $\lim_{x\to-\infty} f(x) = -\infty$

There are no horizontal asymptotes

b) $f(x) = \frac{x^2}{x^2+9}$

Vertical: $x^2+9=0 \Rightarrow x^2=-9 \Rightarrow x=\pm\sqrt{-9}$
imaginary # so no vertical asymptotes

Horizontal: $\lim_{x\to\infty} \frac{x^2}{x^2+9} = \lim_{x\to\infty} \frac{x^2}{x^2} = \boxed{1}$

There is a horizontal asymptote at $\boxed{y=1}$

4c) $f(x)=\dfrac{x+3}{x^2+x-6}$

Vertical: $x^2+x-6=0 \Rightarrow (x+3)(x-2)=0 \Rightarrow x=2$ or $x=-3$

The numerator is zero when $x=-3$, but nonzero for $x=2$. Thus, there is a vertical asymptote at $\boxed{x=2}$

Horizontal: $\lim_{x\to\infty} \dfrac{x+3}{x^2+x-6} = \lim_{x\to\infty} \dfrac{x}{x^2} = \lim_{x\to\infty} \dfrac{1}{x} = 0$

There is a horizontal asymptote at $\boxed{y=0}$

$\boxed{2.3}$

1. • f is not continuous at $x=0$ because $f(0)$ is not defined

• f is not continuous at $x=1$ because $\lim_{x\to 0} f(x)$ DNE

2. a) x^4-x^2-x is a polynomial so is continuous for all x, i.e. on the interval $\boxed{(-\infty,\infty)}$

b) $(2x+5)^{1/3} = \sqrt[3]{2x+5}$ is an odd root function so is continuous for all x such that $2x+5$ is continuous. Thus f is continuous on $\boxed{(-\infty,\infty)}$

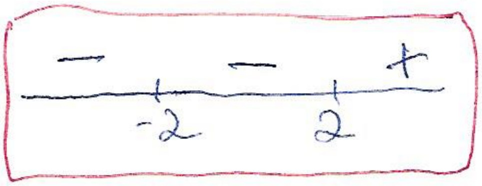
c) $\dfrac{x}{x^2+1}$ is a rational function so is continuous whenever $x^2+1 \neq 0$. Note x^2+1 is never zero (for real-numbers), thus f is continuous on $\boxed{(-\infty,\infty)}$

3 a) $f(x) = \dfrac{x^2-4}{x+2}$

Partition #s

- $f(x)=0 \Rightarrow x^2-4=0 \Rightarrow x=\pm 2$
- f discontinuous $\Rightarrow x+2=0 \Rightarrow x=-2$

$\left\{ \begin{array}{l} \text{Part \#s} \\ x=\pm 2 \end{array} \right.$

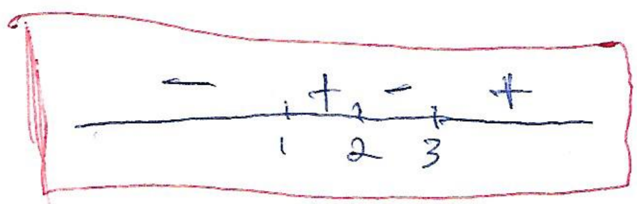


- $f(-3) = \dfrac{(-3)^2-4}{-3+2} = \dfrac{9-4}{-1} = -5 < 0$
- $f(0) = \dfrac{0-4}{0+2} = -2 < 0$
- $f(3) = \dfrac{3^2-4}{3+2} = \dfrac{5}{5} = 1 > 0$

b) $f(x) = \dfrac{(x-1)(x-2)(x-3)}{x^2+2}$

- $f(x)=0 \Rightarrow x=1, x=2, x=3$
- f discont $\Rightarrow$ ... nowhere! x^2+2 is never 0

Partition #s: $x=1,2,3$



- $f(0) = \dfrac{(0-1)(0-2)(0-3)}{0^2+2} = -3 < 0$
- $f(1.5) = 0.08 > 0$
- $f(2.5) = -0.04 < 0$
- $f(4) = \frac{1}{3} > 0$

4. a) $\dfrac{x^2-4}{x+2} < 0$ for x in $(-\infty, 2)$

b) $\dfrac{(x-1)(x-2)(x-3)}{x^2+2} > 0$ for x in $(1,2) \cup (3,\infty)$

$\boxed{2.4}$ 1. a) $f(x) = \sqrt{x+7}$

$$ARC = \frac{f(1)-f(0)}{1-0} = f(1)-f(0)$$
$$= \sqrt{8} - \sqrt{7}$$
$$= \boxed{0.183}$$

b) $f(x) = x^4 + 3x$

$$ARC = \frac{f(2)-f(1)}{2-1} = f(2)-f(1)$$
$$= [2^4 + 3(2)] - [1^4 + 3(1)]$$
$$= \boxed{18}$$

2 a) $f(x) = 2x^2 + x + 3$

$$f(x+h) = 2(x+h)^2 + (x+h) + 3$$
$$= 2(x^2 + 2xh + h^2) + x + h$$
$$= 2x^2 + 4xh + 2h^2 + x + h + 3$$
$$f(x+h) - f(x) = \cancel{2x^2} + 4xh + 2h^2 + \cancel{x} + \cancel{3} - (\cancel{2x^2} + \cancel{x} + \cancel{3}) + h$$
$$= 4xh + 2h^2 + h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{4xh + 2h^2 + h}{h} = 4x + 2h + 1$$

$$f'(x) = \lim_{h \to 0} 4x + 2h + 1 = \boxed{4x+1}$$

2b) $f(x) = 5 + 2\sqrt{x}$

$f(x+h) = 5 + 2\sqrt{x+h}$

$$f(x+h) - f(x) = \cancel{5} + 2\sqrt{x+h} - (\cancel{5} + 2\sqrt{x})$$

$$= 2(\sqrt{x+h} - \sqrt{x})$$

$$= 2(\sqrt{x+h} - \sqrt{x}) \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= 2\,\frac{\cancel{x} + h - \cancel{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \frac{2h}{\sqrt{x+h} + \sqrt{x}}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{2h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{2}{\sqrt{x+h} + \sqrt{x}}$$

$$f'(x) = \lim_{h \to 0} \frac{2}{\sqrt{x+h} + \sqrt{x}} = \frac{2}{\sqrt{x} + \sqrt{x}} = \frac{2}{2\sqrt{x}} = \boxed{\frac{1}{\sqrt{x}}}$$

c) $f(x) = \frac{2x}{x-1}$

$f(x+h) = \frac{2(x+h)}{x+h-1}$

$$f(x+h) - f(x) = \frac{2(x+h)}{x+h-1} - \frac{2x}{x-1}$$

$$= 2\left[\frac{x-1}{x-1} \cdot \frac{x+h}{x+h-1} - \frac{x}{x-1} \cdot \frac{x+h-1}{x+h-1}\right]$$

$$= 2\left[\frac{(x-1)(x+h) - x(x+h-1)}{(x-1)(x+h-1)}\right]$$

$$= 2\left[\frac{x^2 + xh - x - h - (x^2 + xh - x)}{(x-1)(x+h-1)}\right]$$

$$= 2\left[\frac{\cancel{x^2} + \cancel{xh} - \cancel{x} - h - \cancel{x^2} - \cancel{xh} + \cancel{x}}{(x-1)(x+h-1)}\right] = \frac{-2h}{(x-1)(x+h-1)}$$

$$\frac{f(x+h) - f(x)}{\cancel{h}} = \frac{-2\cancel{h}}{(x-1)(x+h-1)} \cdot \frac{1}{\cancel{h}} = -\frac{2}{(x-1)(x+h-1)}$$

$$f'(x) = \lim_{h \to 0} \frac{-2}{(x-1)(x+h-1)} = \frac{-2}{(x-1)(x+0-1)} = \boxed{\frac{-2}{(x-1)^2}}$$

3a) • Point on line: $x_0 = 10$ $\quad y_0 = f(10) = 5 + 2\sqrt{10}$

≈ 11.3

• Slope of line: $f'(10) = \frac{1}{\sqrt{10}} \approx 3.16$

Equation

$$y - 11.3 = 3.2(x-10) \Rightarrow \boxed{y = 3.2x - 20.7}$$

b) • Point on line: $x_0 = 10$, $y_0 = f(10) = \frac{2 \cdot 10}{10-1} = \frac{20}{9}$

• Slope of line: $f'(10) = \frac{-2}{(10-1)^2} = \frac{-2}{81}$

Equation:

$$y - \frac{20}{9} = \frac{-2}{81}(x-10) \Rightarrow \boxed{y = \frac{-2}{81}x + \frac{200}{81}}$$

or in decimal

$$y - 2.22 = -0.02(x-10) \Rightarrow y = -.02x + 2.47$$