ENGR 380 Gearbox Design Project - Group 6

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ENGR 380 Design of Machine Elements

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1.0. Abstract

This report outlines the design process of a single-stage reduction gearbox for the ENGR 380 course project. The goal was to design a gearbox that could take input from a motor, reduce the speed and increase the torque, and deliver the power to a propeller. The design process involved multiple iterations and redesigns to optimize the geometry and material of each part. Once the final design of the primary components was complete, we modeled the gearbox in SOLIDWORKS. The model included the design of a housing to align all components in place. This housing design was refined to minimize the amount of material used and produce a cost-effective product. Each component had safety factors and geometric requirements that needed to be met to ensure the gearbox would have a lifespan of at least 20,000 hours. Whenever possible, commonly available parts were sourced from online suppliers to reduce the total manufacturing time. Finally, full shop drawings of each part were made to simplify the manufacturing process. The SOLIDWORKS file and the Microsoft Excel spreadsheets have been included as part of the supplementary material.

2.0. Table of Contents

3.0. Nomenclature

3.1. Gear Nomenclature

Name	Symbol	Units	Name	Symbol	Units
Power	P	HP	Overload Factor	K_o	N/A
Angular Velocity	$n_{pinion/gear}$	rpm	Size Factor	$K_{s,\ pinion/gear}$	N/A
Diametral Pitch	P_d	in^{-1}	Thickness Factor	$K_{B,\ pinion/gear}$	N/A
Pressure Angle	f	degrees	Dynamic Factor	K_v	N/A
Number of teeth	$N_{pinion/gear}$	N/A	Hardness Ratio Factor	C_H	N/A
Gear Ratio	$m_{pinion/gear}$	N/A	Temperature Factor	K_T	N/A
Pitch Diameter	$D_{pinion/gear}$	in	Reliability Factor	K_R	N/A
Centre Distance	C	in	Load Cycles	$N_{Pinion/Gear}$	rotations
Pitch Line Speed	V	ft/min	Bending Stress Factor	$Y_{N,\ pinion/gear}$	N/A
Transmitted Load	W_t	lb	Pitting Stress Factor	$Z_{N,\ pinion/gear}$	N/A
Face Width	F	in	Bending Stress	$\sigma_{b,\ pinion/gear}$	psi
Elastic Coefficient	C_p	$(psi)^{1/2}$	Max Bending Stress	$\sigma_{b,\ pinion/gear\ Max}$	psi
Quality Number	Q_v	N/A	Contact Stress	$\sigma_{c,\ pinion/gear}$	psi
Lewis Form Factor	$Y_{pinion/gear}$	N/A	Max Contact Stress	$\sigma_{c,\ pinion/gear\ Max}$	psi
Reliability	R	%	Safety Factor	η	N/A
Geometry Factor	$J_{pinion/gear}$	N/A	Mesh Alignment Factor	C_{ma}	N/A
Pitting Factor	I	N/A	Load Distribution Factor	K_m	N/A

Name	Symbol	Units	Name	Symbol	Units
Proportion Factor	C_{pf}	N/A			

3.2. Shaft Nomenclature

Name	Symbol	Units
Torque	T	lbf-in
Power	P_{hp}	hp
Angular velocity	n	rpm
Gear radius	r_{gear}	in
Tangential force	$W_{tangential}$	lbf
Pressure angle	ϕ	degrees
Normal force	W_{Normal}	lbf
Reaction force at point 2 in the y direction	$R_{2,y}$	lbf
Distance between two bearings	b	in
Distance between the bearing and center of the gear	p	in
Reaction force at point 1 in the y direction	$R_{1,y}$	lbf
Reaction force at point 2 in the z direction	$R_{2,z}$	lbf
Reaction force at point 1 in the z direction	$R_{2,z}$	lbf
Bending moment in the y direction at point B	$M_{B,y}$	lbf-in
Bending moment in the z direction at point B	$M_{B,z}$	lbf-in
Ultimate tensile strength	S_{ut}	kpsi
Yield strength	S_{yt}	kpsi

3.3. Bearing Nomenclature

Name	**Symbol**	**Units**
Endurance strength factor	S_e'	NA
Marin Factor	k_a	NA
Marin Factor	a	NA
Marin Factor	b	NA
Marin Factor	k_b	NA
Marin Factor	k_c	NA
Marin Factor	k_d	NA
Marin Factor	k_e	NA
Neuber Constant	$sqrta$	
Geometric stress concentration factor	K_t	NA
Geometric stress concentration factor for shear	K_{ts}	NA
Notch Radius	r	in
Fatigue stress concentration factor	K_f	NA
Fatigue stress concentration factor for shear	K_{fs}	NA
Intermediate Calculation Value	A	lbf-in
Fatigue safety factor	*n*	NA
Endurance strength limit	S_e	NA
Safety factor against yielding on first cycle	n_y	NA
Mean stress	σ'_m	kpsi
Alternating stress	σ'_a	kpsi
Mean shear stress	τ'_m	kpsi

Name	Symbol	Units
Alternating stress	τ'_a	kpsi
Max stress	σ_m	kpsi
Distance from center	c	in
Polar moment of inertia	I	$lbm - in^2$

3.4. Key Nomenclature

Name	Symbol	Units
Power	P	HP
Angular Velocity	n	rpm
Torque	T	lb-in
Force	F	lb-f
Radius	R	in
Thickness	t_{key}	in
Length	l_{key}	in
Shear Strength	S_{yt}	psi
Shear Yield Strength	S_{sy}	psi
Shear Stress	τ	psi
Compressive Stress	σ	psi

4.0. Introduction

For the gearbox project, our group was tasked with designing a one-stage reduction gearbox. This gearbox needed to meet design requirements that include reducing an input speed of 1750 rpm to 500 rpm while delivering 25HP. This gearbox also needed to be designed with a total lifespan of approximately 20,000 hours. Input power is received from an AC electric motor, and the gearbox output shaft drives a propeller. Each shaft must extend 4 inches outside the gearbox so that the motor and propeller drive shafts can be connected to the gearbox.

This project focused on making design decisions that a mechanical engineer would make when designing a gearbox. This includes determining how to mount the gearbox, the design of the outer casing, shaft, gear pinion, gaskets, bearings and keys. For each of these it was important to not only consider form factor, but price, weight, and material selection as well. Overall the ideal design would be the easiest to manufacture and the cheapest to produce while still meeting all specifications.

To verify the design and determine the final specifications, both manual calculations and finite element analysis were performed. The results of this analysis include the creation of spreadsheets which show the iterative design process, and detailed drawings to allow for ease of machining. As part of the design process, it was important that we could iterate on individual part design. This required creating multiple spreadsheets and simple programs to perform the calculations.

This report will present sections on Synthesis and Decisions on General Layout, Design Procedure and Sample Calculations, Simulation Setup and Results, a Discussion and our final Conclusions.

5.0. Synthesis and Decision on General Layout

This section will outline the general design process for the gears, shaft, keys and bearings. The primary focus is on the iterative design process rather than calculating specific values. The exact calculations associated with each design will be discussed in Section 6.

5.1. Gear Design

5.1.1. Gear Summary. For the gears we decided to choose spur gears over helical gears to transmit power from the input shaft to the output shaft. This decision was made for a couple reasons. Firstly, spur gears do not produce axial forces so they only have to be accounted for in the radial direction. Secondly, the shafts are parallel and therefore we have no need for asymmetric force transmission, which is one of the reasons helical gears are chosen. Also, as this is a one-stage gearbox, the smoother operation associated with helical gears is not necessary. Furthermore, spur gears are generally of lower cost than helical gears, and cost was an optimization criteria for the project.

5.1.2. First Iteration. For the first iteration of the gears we started by choosing an 18-tooth pinion. This was chosen because it is the smallest number of teeth a pinion can have without causing interference in the gear teeth. Next, the gear ratio was used to find the number of gear teeth. A diametrical pitch of 4 was selected based on these two values, resulting in a centre distance that was slightly below the design constraint maximum of 11 inches. Then, after calculating all relevant factors and choosing the highest quality factor available for commercial quality gears (7) the bending and contact stress associated with this configuration was noted.

Comparing that value to the bending and contact stress number associated with gray cast iron class 40, it was found that the safety factors were above the minimum required. This iteration ended up being the most economical iteration because, despite being the largest iteration in terms of diameter, it was the only iteration that had bending and contact stresses low enough to allow the use of gray cast iron.

5.1.3. Second Iteration. For the second iteration, the number of teeth were increased by the minimum amount possible for integer values which resulted in a pinion tooth number of 20. The diametrical pitch was increased to 6, in order to satisfy the max center distance constraint. This changed the applicable range of face width values and thus the face width had to be lowered. The change in teeth number also affected the geometry factors and Lewis form factors. Another change, based on the diameters of the pinion and gear, was the pitch line speed and transmitted load. All these changes culminated in a much higher bending and contact stress that the pinion and gear had to withstand. To satisfy the minimum safety factor values the bending and contact stress number of the pinion and gear had to be raised drastically by choosing a material with a much higher Brinell hardness. The material chosen was case hardened steel. This configuration yielded higher safety factors than the first iteration, although it used a material that was more expensive.

5.1.4. Third Iteration. The third iteration of the pinion and gear was similar to the second with a few changes. The goal for the third iteration was to improve the safety factors of the pinion and gear without changing the material. This was accomplished by increasing the number of teeth on the pinion and gear. It was raised to the max number of teeth allowed in the project outline which is 84 gear teeth and 24 pinion teeth. This, similar to the second iteration, changed the diameters of the pinion and gear.

Changing the number of teeth also consequently changes the geometry factors and Lewis form factors. The face width for this iteration is the same as the second. In total, without changing the material, the net result of the third iteration was an increase of all safety factors.

5.1.5. Gear Conclusion. The two iterations worth considering are the first and third. The second iteration uses the same material as the third with inferior safety factor values and should therefore be disregarded. The decision between the first and third comes down to cost. The first iteration has a larger diameter which allows it to use a weaker and cheaper material. In comparison, the third iteration is smaller in diameter and requires a much harder and more expensive material. The first iteration was choose because it met the minimum safety factor requirements while also using the more economical material of gray cast iron class 40.

5.2. Shaft Design

5.2.1. Summary. Once gears had been decided the shaft parameters were calculated. A simple design of the shaft was created with parameters that were to be calculated. The key design features that needed to be optimized were the length of the shaft, the shaft material and methods for preventing side-to-side motion. One of the selected methods to prevent lateral motion was to add a shoulder to the shaft. Its specific size was identified as a key design variable that will be calculated in Section 6. The other key feature would be some way to prevent the gear from sliding axially while also preventing the gear face from rubbing against the housing. For this a c-clip and a shoulder on the gear were both considered. The c-clip did a better job at preventing friction but required a groove to be cut in the shaft that would concentrate stress.

This stress concentration would result in a thicker overall shaft that would ultimately raise the price of the gearbox. It was therefore decided to use a shoulder on the gear as it still reduces friction but does not require a groove. The final key design feature was a way for torque to be applied to or by the shaft. To accomplish this, two key ways were cut in each shaft. The design of these keyways will be outlined in Section 5.3.

5.2.2. Shaft Material. The first set of design choices that were made revolved around material selection. The optimal material would be low cost, fall within the design specifications and be easy to machine. For these reasons a shaft from machined 1006 was used as a base for calculations. This is low carbon steel and relatively economical. Upon performing the calculations, it was found that the relative weakness of the material required a shaft diameter that was greater than 2 inches. For this reason this possibility was discarded and a new material was chosen. This process was iterated on until 1018 was eventually chosen to be the ideal shaft material that minimized cost while meeting the minimum design requirements.

5.2.3. Shaft Dimensions. The second key parameter explored was the lengths of the shoulder on the shaft and gear. For initial considerations, both were assumed to have a length of 1 inch. These calculations were set as the baseline for which other results could be compared against. For comparison both shaft lengths were increased to 1.5 inches. This resulted in a longer total shaft and higher stress concentrations and was therefore discarded. The second iteration involved setting the lengths to 0.5 inches; this resulted in decreasing shaft diameters slightly but more significantly made the overall design more compact. 0.5 inches was therefore deemed to be the best choice for the lengths of these components. The calculation of optimal parameters for shaft diameter will be presented in the design procedure section.

5.3. Key Design

The design variables for the keys are the dimensions of the key (thickness and length) and the material. For this decision, key dimensions and materials that are available from the source listed in the outline were referenced (Misumi, 2023). This gave a range of thicknesses and lengths as well as two different materials to choose from.

For all keys use 1045 carbon steel from the manufacturer. The dimensions of the keys were chosen to have the smallest key possible that met the safety factor requirements from the outline, with the additional requirement that the input shaft key had the smallest safety factor and would therefore be the first part to fail in the case of overloading the gearbox. This is done to protect all consequent components from failure due to increased input power. The size of the key depends on the radius of the shaft that it acts on. In this way, the iterative process for the keys depends on the iterative process for the shafts. The decision on which iteration to use however, was not made based on key size. The key is a relatively small and low cost part in comparison to the shaft and gears and, as a consequence, was not the deciding factor and which iteration to choose.

5.4. Bearing Design

There are many varieties of bearings that are useful for many different applications. The main parts of a bearing design that need to be decided upon include the type of rollers, amount of rows, sealing type and other potential features. The main reason ball bearings were selected was due to the relatively small loads and minimal shock being applied to the bearings, which encourages the trade off of lower contact area for higher rotational speeds. Ball bearings were also selected due to ample radial space allowed by the casing design, which left plenty of room for the balls and accompanying raceways. A single-row design is viable for the bearings due to the simplistic nature of the application; more balls would incur further cost for little benefit when compared to the standardized single-row format. Essentially pure radial loads on the bearings led to a selection of simple deep-groove bearings instead of an alternative such as angular-contact bearings. The bearing design is standard, but also perfect for keeping high speeds while producing minimal noise and vibration. Contact sealed bearings were the best choice for completing the seal of the casing, which is desirable so as to reduce the entry of foreign material into the gearbox. Besides providing the gearbox with water protection, double-sided sealed bearings also come pre-lubricated, reducing maintenance. The other main requirements were a bore size matching the shaft diameter, as well as load ratings sufficient enough to provide a long life to the bearings. This grocery list of features provided a narrow scope to look through when browsing for bearings, ultimately 6206-2RS1 and 6208-2RS1 bearings were selected from the SKF catalog as they perfectly fit the description of the design.

6.0. Design Procedure and Sample Calculations

The design procedure and sample calculations will be presented in this section. For more detail on the calculations performed please see the Excel spreadsheet attached as supplementary materials.

6.1. Gear Calculations

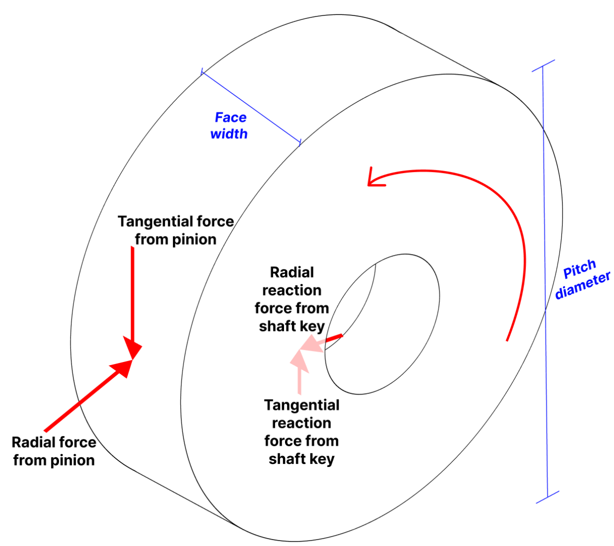

Figure 6.1.1. Free Body Diagram: Gear

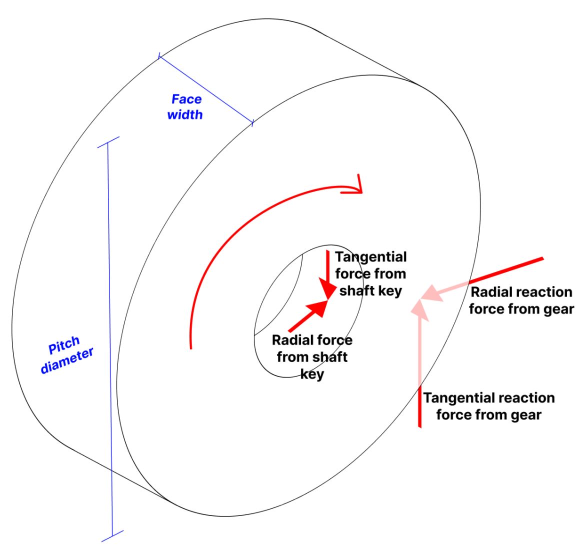

Figure 6.1.2. Free Body Diagram: Pinion

The pitch diameter, center distance, pitch line speed and transmitted load for the pinion/gear is all determined based on the number of teeth included on the pinion and gear and the diametral pitch. These two variables are to be chosen based on specifications described in the project outline while also satisfying safety factor calculations. The only other variables that are open to change is the material chosen for the gear and the face width of the gear teeth. These can also be used to obtain necessary safety factors.

Pitch Diameter

$$D_{pinion} = \frac{N_{pinion}}{P_d}$$

Centre Distance

$$C = \frac{N_{pinion} + N_{gear}}{2 \times P_d}$$

Pitch Line Speed

$$V = \frac{\pi \times P_d \times D_{pinion}}{12}$$

Transmitted Load

$$W_t = \frac{33{,}000 \times n_{pinion}}{V}$$

The following equations are all based on the pressure angle of the gears, which is given in the project outline, and the face width of the pinion/gear teeth which is to be chosen within constraints based on the diametral pitch. It should be noted that many factor coefficients referenced in the following equations are found in figures, not calculated directly.

Pitting Geometry Factor

$$I = \frac{cos(f) \times sin(f) \times m_G}{2 \times m_N \times (m_G + 1)} \qquad \textit{(eq. 14-23)}$$

Pinion Proportion Factor

$$C_{pf} = \frac{F}{10 \times D_p} - 0.0375 + (0.0125 \times F) \qquad \textit{(eq. 14-32)}$$

Mesh Alignment Factor

$$C_{ma} = A + BF + CF^2 \qquad \textit{(eq. 14-34)}$$

Load Distribution Factor

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad \textit{(eq. 14-30)}$$

Pinion Size Factor

$$K_{sP} = 1.192 \times (\frac{F \times \sqrt{Y}}{P_d})^{0.0535} \qquad \textit{(eq. (a) pg.765)}$$

Dynamic Factor

$$K_v = (\frac{A+\sqrt{V}}{A})^B \qquad \textit{(eq. 14-27)}$$

The following equations are based on the number of cycles that the gear is desired to function correctly within. This number is based on the rotations per minute of the pinion/gear and the number of hours of operation.

Bending Stress Cycle Factor

$$Y_{NP} = 1.3558 \times N_P^{-0.0178} \textit{(Figure 14-14)}$$

Pitting Stress Factor

$$Z_{NP} = 1.4488 \times N_P^{-0.023} \qquad \textit{(Figure 14-15)}$$

Following are the calculations made based on the values found above. The stresses are found using calculated and chosen values as well as the material properties associated with the chosen material.

Pinion Bending Stress

$$\sigma_{bP} = W_t K_o K_v K_{sp} K_m K_{BP} \times \left(\frac{P_d}{(F \times J_P)}\right)$$

Max Allowed Pinion Bending Stress

$$\sigma_{bP\ Max} = \sigma_{tP} \times \left(\frac{Y_{NP}}{(K_T \times K_R)}\right)$$

Safety Factor For Bending

$$\eta = \frac{\sigma_{bP\ Max}}{\sigma_{bP}}$$

Pinion Contact Stress

$$\sigma_{cP} = C_p \sqrt{\frac{W_t K_o K_v K_{sp} K_m}{D_P F I}}$$

Max Allowed Pinion Contact Stress

$$\sigma_{cP\ Max} = \sigma_{cP} \times \left(\frac{Z_{NP}}{(K_T \times K_R)}\right)$$

Safety Factor For Contact Stress

$$\eta = \frac{\sigma_{cP\ Max}}{\sigma_{cP}}$$

6.2. Shaft Calculation

Prior to designing the specific parameters for each shaft, a free body diagram was drawn. These FBD's are shown in Figure 6.2.1, and 6.2.1.

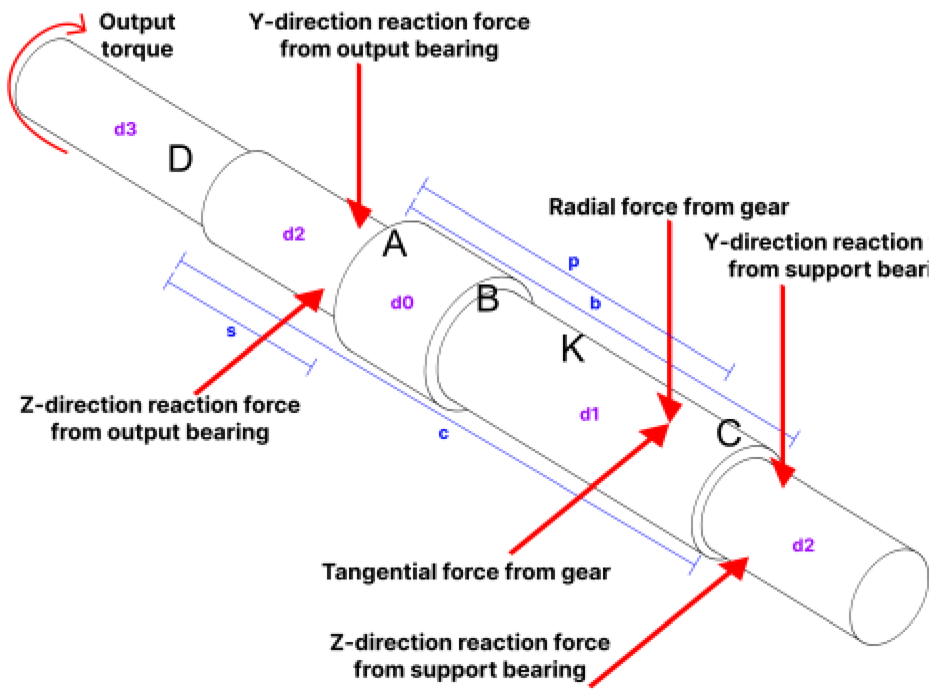

Figure 6.2.1. Free Body Diagram: Output Shaft

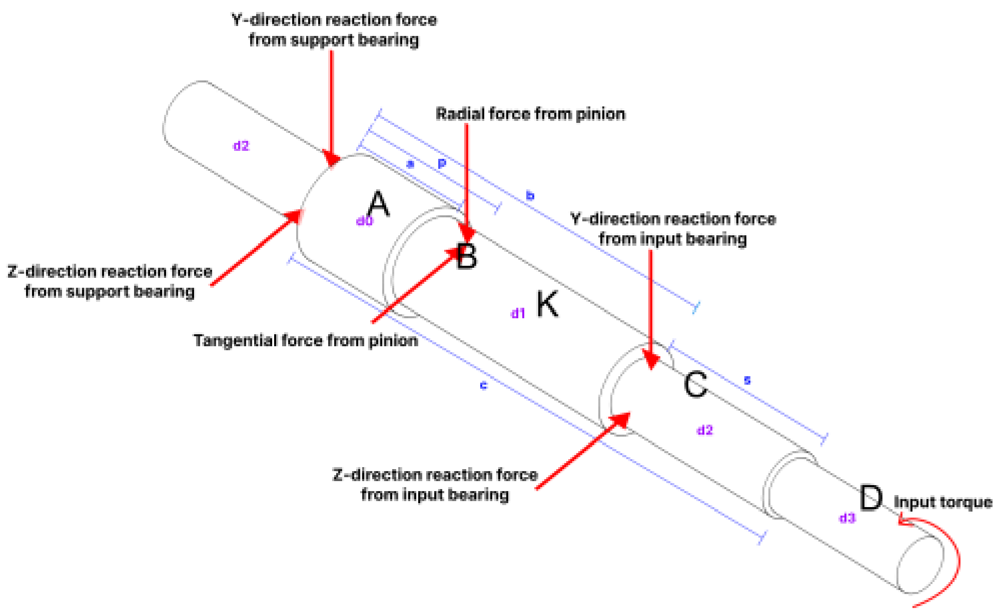

Figure 6.2.2. Free Body Diagram: Input Shaft

For force calculations, all axial forces were assumed to be zero. While this may not be entirely accurate, the axial forces should be negligible relative to forces in other directions. The torque was calculated using the motor horsepower and the gear speed.

$$T = \frac{P_{hp} \times 63025}{\text{n}}$$

The forces acting to create a bending moment were calculated by first using the gear size and shaft diameter.

$$W_{Tangential} = \frac{T}{r_{gear}}$$

This force calculation only applies to the force acting tangent to the shaft. To determine the force perpendicular to the shaft, we used the gear pressure angle. The trigonometric formula is shown below.

$$W_{Normal} = W_{Tangential} \times tan(\phi)$$

The reaction forces at each of the two supports then need to be calculated using a torque balance. By creating a pivot about point A in the shaft drawing, the force at point C was found using the following torque balance.

$$R_{2,y} \times p = W_{Normal} \times b$$

$$R_{2,y} = \frac{W_{Normal} \times b}{p}$$

The forces that are acting in the y direction will all sum to zero by Newton's second law. This can be used to calculate the last reaction force in the y direction under the assumption that only the reaction forces and normal force are acting on the gear.

$$R_{1,y} = -W_{Normal} - R_{2,y}$$

Similar calculations can be performed for forces in the z direction except in this case the tangential forces are used.

$$R_{2,z} \times p = W_{Tangential} \times b$$

$$R_{2,z} = \frac{W_{Tangential} \times b}{p}$$

$$R_{1,y} = -W_{Tangential} - R_{2,y}$$

For calculating bending moments, the forces begin acting at the bearings so the moment there must be zero. The bending moment at the point of application of the normal force is the integral of the force acting at the bearing over the distance from the bearing to the shaft. This results in bending moments shown below

$$M_{B,y} = R_{1,y} \times p$$

$$M_{B,z} = R_{1,z} \times p$$

For calculating stress concentration factor we must determine the material. For our iterations we chose to use 1018 steel. The strength factors were found in table A20.

$$S_{yt} = 54ksi$$

For strength calculations, the endurance strength limit can be found using the parameter S'_e. This parameter is found using the Equation 6-10 for $S_{ut} < 200kpsi$.

$$S'_e = 0.5S_{ut}$$

The marin factors were also found for calculating stress.

$$k_a = a \times S_{ut}^b$$

The values for a and b were found using Table 6-2 for a machined shaft.

$$a = 2$$

$$b = -0.217$$

$$k_c = k_d = k_e = 0$$

For calculating stress concentration factors we assumed a notch radius of 0.01 in. The Neuber equation was used to approximate stress concentration factors. The stress concentration factor was approximated using equation 6-35

$$\sqrt{a} = 0.246 - 3.08 \times 10^{-3} S_{ut} + 1.51 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3$$

The Neuber constant was then used in equation 6-34 to calculate the stress concentration factor assuming a noth radius of 0.01 inches and a value for K_t extracted from table 7-1.

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

A similar process was performed for the shear stress concentration factors except with a Neuber constant calculated by equation 6-36.

$$\sqrt{a} = 0.0.19 - 2.51 \times 10^{-3} S_{ut} + 1.35 \times 10^{-5} S_{ut}^2 - 2.67 \times 10^{-8} S_{ut}^3$$

The shear stress concentration factors were than found using equation an adaption of 6-24 shown below.

$$K_{fs} = 1 + \frac{K_{ts} - 1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

All calculations were repeated at locations K, C, and D. The individual shaft diameter was then calculated by solving equation 7-6 shown below. Due to the nature of the load, the

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$
$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

These values were then inserted into the DE-Goodman equations labeled equation 7-7 and 7-8 in the textbook, along with a safety factor of 2.5.

$$n = \frac{\pi d^3}{16}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)^{-1}$$
$$d = \left[\frac{16n}{\pi}\left(\frac{A}{S_e} + \frac{B}{S_{ut}}\right)\right]^{1/3}$$

The values of Se were found using equation 6-17

$$S_e = k_a k_b k_c k_d k_e S'_e$$

And kb was using equation 6-10

$$k_b = \begin{cases} (d/0.3)^{-0.107} = 0.879d^{-0.107} & 0.3 \leq d \leq 2 \text{ in} \\ 0.91d^{-0.157} & 2 < d \leq 10 \text{ in} \\ (d/7.62)^{-0.107} = 1.24d^{-0.107} & 7.62 \leq d \leq 51 \text{ mm} \\ 1.51d^{-0.157} & 51 < d \leq 254 \text{ mm} \end{cases}$$

Solving all of these equations simultaneously yields a minimum value for the shaft diameter at a given point based on fatigue. The first cycle yielding safety factor then had to be calculated using equations 6-43, 6-66, and 6-67, 3-28. The average bending moment is 0, and the shear stress is a constant value.

$$n_y = \frac{S_y}{\sigma'_m + \sigma'_a}$$

$$\sigma'_a = \{[(K_f)_{\text{bending}}(\sigma_{a0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{a0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{a0})_{\text{torsion}}]^2\}^{1/2}$$

$$\sigma'_m = \{[(K_f)_{\text{bending}}(\sigma_{m0})_{\text{bending}} + (K_f)_{\text{axial}}(\sigma_{m0})_{\text{axial}}]^2 + 3[(K_{fs})_{\text{torsion}}(\tau_{m0})_{\text{torsion}}]^2\}^{1/2}$$

$$\sigma_{m0} = \frac{Mc}{I}$$

$$\sigma_{a0} = 0$$

$$\tau_{a0} = My/I$$

$$\tau_{m0} = 0$$

If the prior value calculated at a given point creates a safety factor less than 2.5, the diameter is increased until such a point as the minimum safety factor is satisfied. This method will produce the minimum values for each section however these values are not the final shaft diameters. The final shaft diameter is chosen as a function of the available bearings. The diameter was therefore increased until the diameter at the point of the bearings equals an available bore diameter. It is then verified that all of the diameter shifts are equal to approximately 20% of the diameter at that point. One final part to determine was the starting diameter of the shaft. For this, a list of available 1045 rods was found for Varsteel (*COLD FINISH BARS & SHAFTING*). The initial rod diameter was chosen to be approximately equal to the shoulder diameter. Final specifications can be found in Sheets 13 and 14 of Appendix B.

6.3. Key Calculations

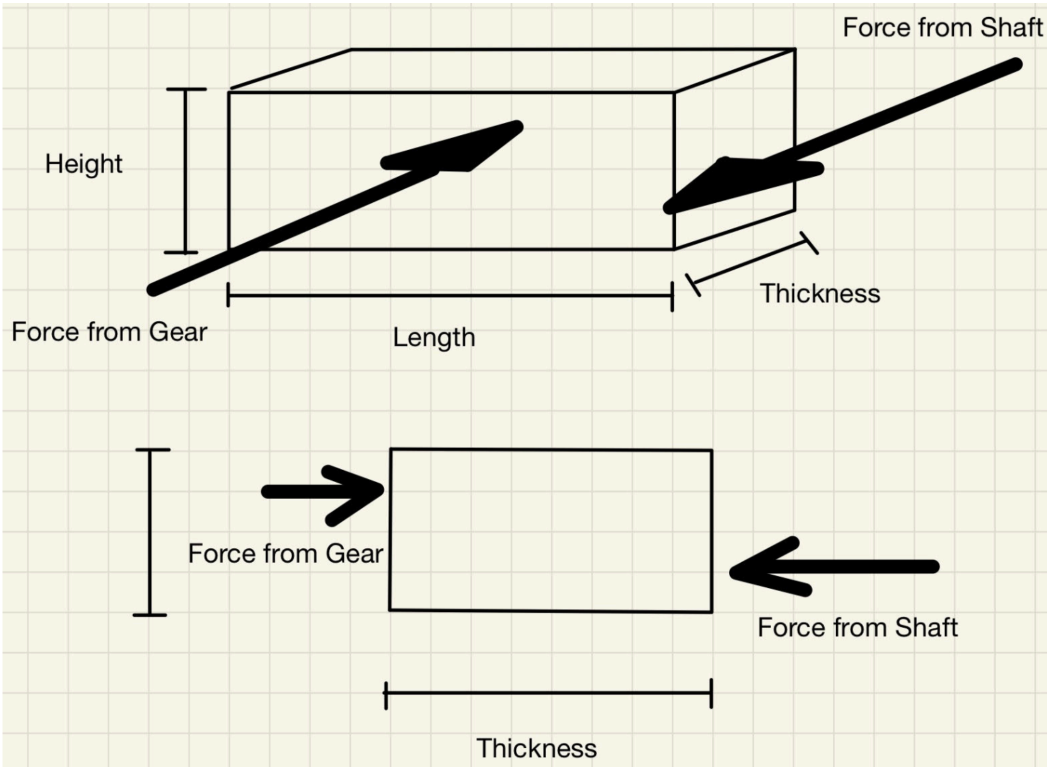

Figure 6.3.1. Free Body Diagram: Key

Following are the force equations for the key. It is based on the rotational speed and horsepower of the shaft that it acts on.

Torque

$$T = \frac{P_{HP} \times 63{,}025}{n}$$

Force On Key

$$F = \frac{T}{R_{shaft}}$$

The following equations calculate both the shear and compressive stresses that the key will need to withstand and the safety factors that the key will have, based on the material and dimensions chosen for the key.

Shear Stress

$$\tau = \frac{F}{t_{key} \times l_{key}}$$

Compressive Stress

$$\sigma = \frac{F}{(\frac{t_{key}}{2}) \times l_{key}}$$

Yield Safety Factor

$$\eta_Y = \frac{S_{sy}}{\tau}$$

Compressive Safety Factor

$$\eta_C = \frac{S_{yt}}{\sigma}$$

6.4. Bearings Calculations

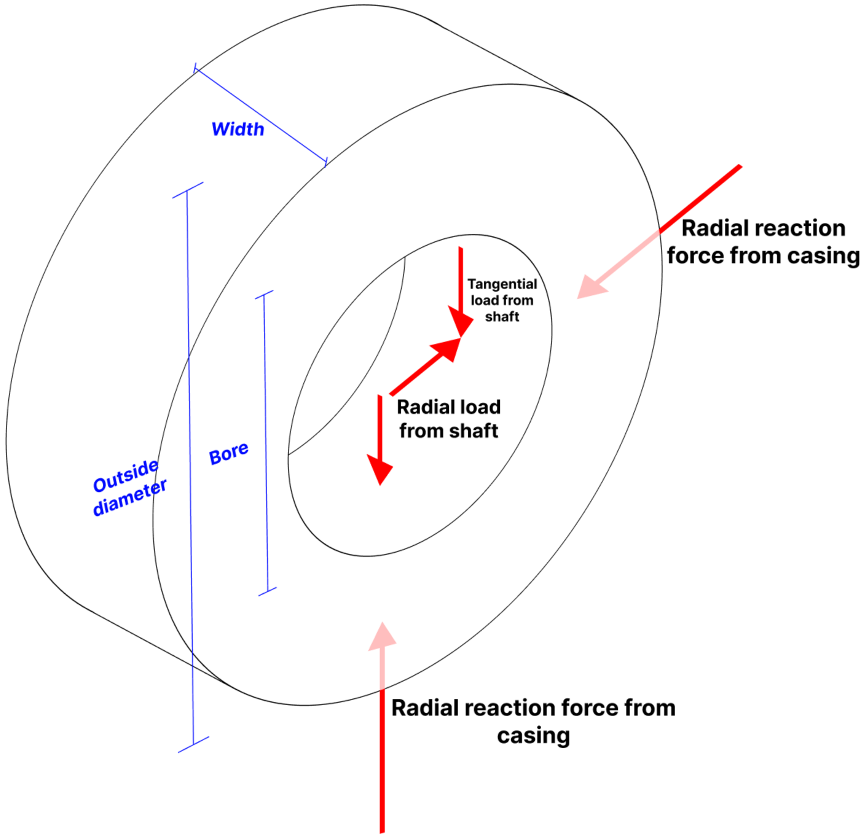

Figure 6.4.1. Free Body Diagram: Bearing

Given: L_{10} time, Reliability, ω_{input}, ω_{output}, shaft diameter, rotation factor (V) = 1

The given specifications can be used to calculate L_{10} values for the bearings on both the input and output shafts. Note that the angular velocity values must be converted to revs/hr for this calculation as the L_{10} time has units of hours. Using equation 11-2b.

$$L_{10} \text{ Pinion Bearings} = (\omega_{input})(60 \text{ m/hr})(L_{10} \text{ time})$$

$$L_{10} \text{ Gear Bearings} = (\omega_{output})(60 \text{ m/hr})(L_{10} \text{ time})$$

Assumed that $a_f = 1$ and from previous assumptions that the axial load on the bearings is negligible. Therefore the F_D can be found from the resultant of the forces on the bearing found from shaft and gear calculations. Given R_y and R_z from shaft and gear calculations.

$$F_D = \sqrt{R_y^2 + R_z^2} * V$$

Calculated shaft diameters at the bearing locations are utilized to find the bearing bore size. The shaft diameter is taken, then the value is rounded up to the nearest 5 mm interval to obtain the bore size (assuming diameter > 17 mm). The desired bearing design features single-row deep-groove ball bearings. Table 11-2 is referenced in order to obtain C_{10} and C_0 load ratings for the found bore sizes. Due to a desired reliability greater than 90%, Weibull parameters are implemented in the calculation. From table 11-6, manufacturer 1's Weibull parameters are selected for the calculation. A rearrangement of equation 11-10 gives a solution for L_5. a = 3 for ball bearings.

$$L_5 = (\frac{C_{10}}{a_f * F_D})^a * \theta(L_{10})(1 - R_D)^{(\frac{1}{b})}$$

The life of all bearings designed are extremely large, these bearings will be suitable for the gearbox design due to the high load rating and appropriate bore. A SKF catalog can be used to select real bearings that closely match the created design.

The pinion shaft bearings have a bore of 1.181 inches. The 6206-2RS1 or 6206-RZ bearings are nearly equal to the design, with identical bores and similar load ratings. The difference between the two options is a both sides contact versus non-contact seals. For the application the 6206-2RS1 would be recommended due to the contact seals greater water protection, even if at the expense of a very minor amount of torque loss.

The bearings on the gear shaft have a bore of 1.575 inches. 6208-2RS1 bearings have been selected for the gear shaft because of their identical bore size and appropriate load rating when compared to the produced design. These single-row deep-groove bearings also feature contact seals for water protection.

6.5. Housing Design

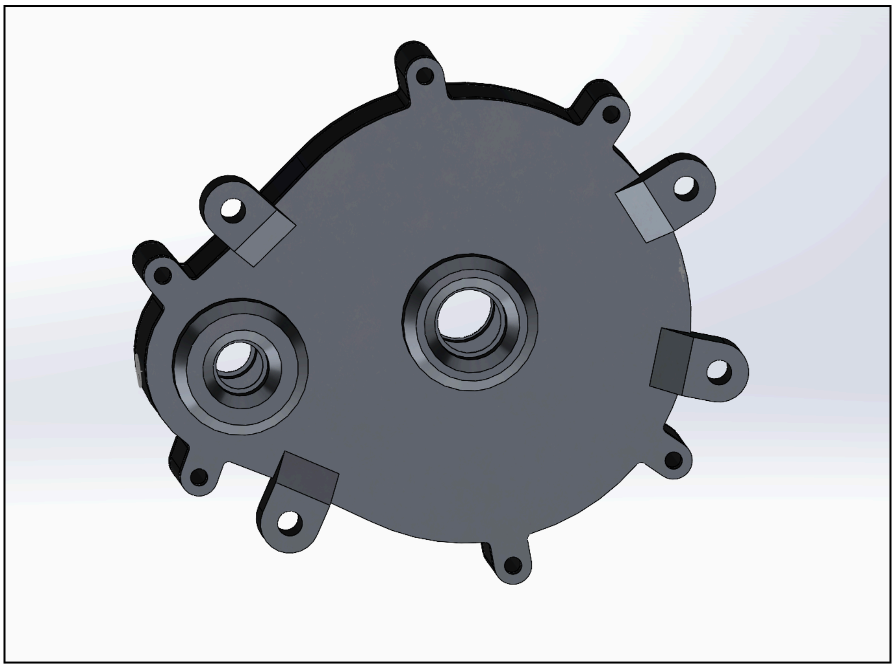

Figure 6.5.1. Initial Gearbox Housing

When designing the gearbox housing, it was important to make sure that it met the design requirements. These requirements include making sure that the components are reasonable to manufacture, and that the design is convenient to assemble. To accomplish this three main components are used. The first is the midframe of the gearbox, this part can be created through waterjet cutting . This process is used as it allows for ease of creation of one off parts with complex geometries, as waterjets can effectively cut up to 10-12 inches this component could be made without having to fit subparts together. The part has 6 bolt brackets that are used to attach the top and bottom of the housing to the extruded midframe. During manufacturing these could be created with the waterjet. One design choice that was made was to avoid any threading in the midframe or the top and bottom of the housing for holding the gearbox together. This was to reduce manufacturing complexity. The only threading operations made were for the lubricant fill and drain hole. To further reduce complexity the fill hole also doubles as a location for a dipstick. The alignment, mounting, and sealing of the housing is handled through the top and bottom plates, which are similar but have slight differences. The top plate can be formed through a simple waterjet operation to achieve the general form, and a CNC machining operation can be used to create a small groove around the perimeter of the top plate. This groove is used to insert a rubber gasket to seal the lubricant inside the housing. The bottom plate is very similar however it has an extra 4 extrusions used for mounting the gearbox housing. For extra strength and support extra metal cutouts are welded to the backplate which provides more rigidity and helps reduce vibration and forces from moments which could lead to torsion on the gearbox housing. In our first simulation of the gearbox we had deformation of less than 1mm anywhere on the housing.

7.0 Simulation Setup and Results

To ensure the housing design can withstand the gear set's applied forces, a static finite element analysis (FEA) simulation was conducted in Solidworks. An FEA can simulate the forces applied to a rigid body with a simulated material and visualize the distributions of the loaded mechanical properties that the force induces on the body using contour plots. For the gearbox housing, the FEA was set up to create plots for material displacement, static nodal stress, and factor of safety. To prepare the FEA, the model must be assigned a number of qualities, including the theoretical forces applied to the model, the model fixture points, solid body contact points if there are multiple bodies, fastener connections, and the material from which the model is made.

In the case of the gearbox housing, a bearing load was applied to the bearing mounts using the force values acquired from the Excel sheet used to calculate the optimal values for the gearbox. The model was then fixed in space by the modeled mounting pins on the gearbox. The fastener toolbox was then used to create bolts to keep the 3-piece housing assembly together. Regarding the gearbox housing material, 6061-T4 aluminum was selected for its high machinability, lightweight, and versatility. Solidworks was then asked to create a model mesh where the software takes the model and splits it into smaller polygons of a selected dimension. Finer meshes allow for more precise plots at the expense of computer memory and time compared to coarse meshes. After the mesh was created, the simulation was run and subsequently calculated the results for static nodal stress and factor of safety, as seen in Figure 7.1 and Figure 7.2, respectively. From the calculated plots, it can be observed that the case design is heavily over-engineered. The nodal stress plot shows that the stress distribution on the gearbox housing never comes close to the yield stress of 6061-T4 aluminum, with the peak stress only 2.639x106 Pa compared to the yield stress of 6061-T4 aluminum being 9.0x107 Pa. The factor of safety plot confirmed the previous observation with the minimum factor of safety across the gearbox housing equal to 34. The gearbox housing was also found to weigh an excessive 82.42 lbs, proving this design over engineering state.

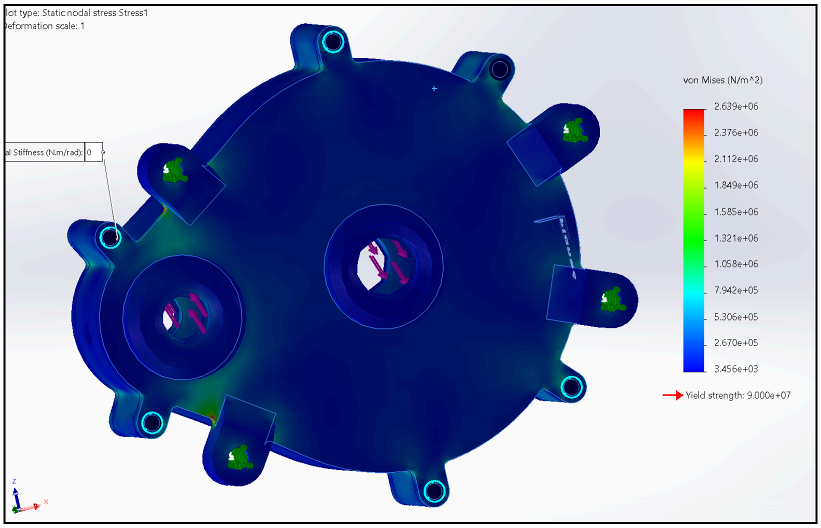

Figure 7.1. Static Nodal Stress Plot of First Housing.

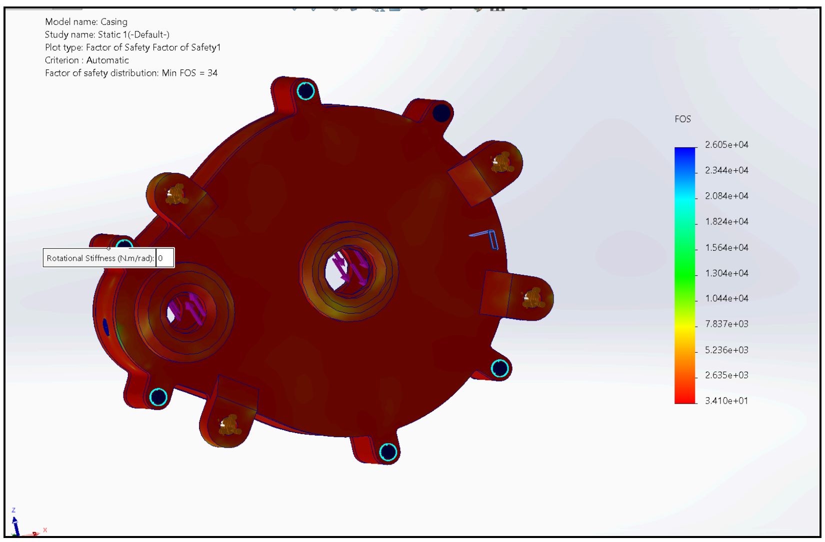

Figure 7.2. Factor of Safety Plot of First Housing.

After the first simulation, it was clear that the gearbox housing could be reduced in size immensely. Using the data from the previous simulations, the housing was lightened by removing material from places under low concentrations of stress. The following iteration is seen in Figure 7.3. weights 45.41 lbs. However, manufacturing of the housing has become more complex due to extra machining required to create the more intricate shapes. Nonetheless, a second FEA simulation was conducted on the new gearbox housing. Unfortunately, the results of the new simulation show that the second iteration of the housing was also over-engineered. Although the nodal stress concentration, particularly around the pinion bearing, was much more concentrated and dispersed than the first gearbox housing, it only peaked at approximately 3.00x106 Pa on the edges of the gearbox mounting points seen in Figure 7.3. The factor of safety plot confirms the over-engineering, with Figure 7.5 displaying a minimum factor of safety of 30. However, although the gearbox housing can be reduced in size further, the amount of extra machining will also increase. Due to the increased machining costs, time constraints, the not insignificant amount of weight reduction, and the consideration of material fatigue, the 2nd iteration of the gearbox housing was selected.

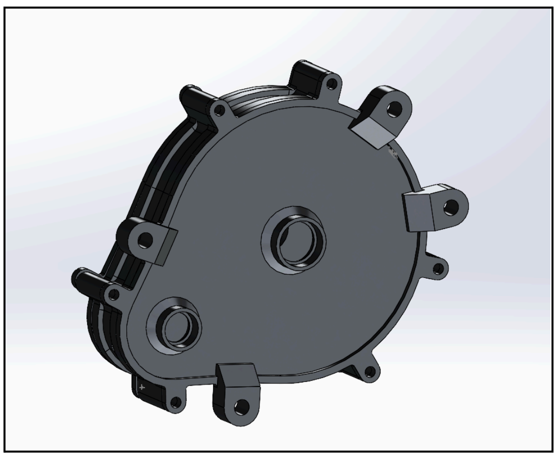

Figure 7.3. 2nd Gearbox Housing.

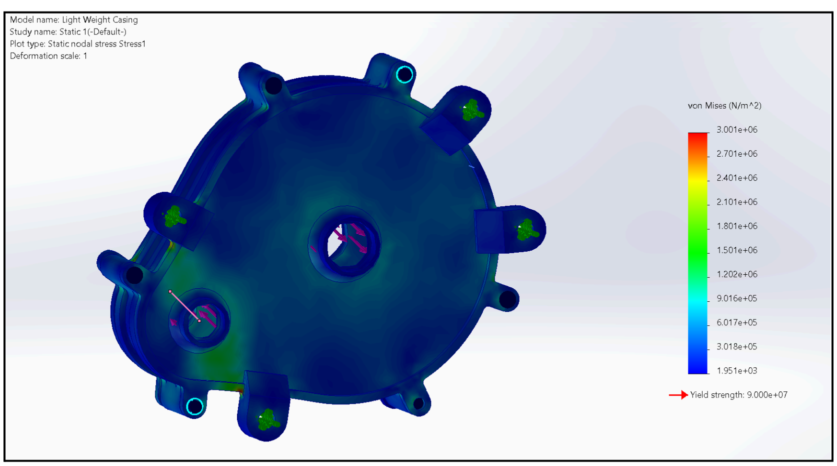

Figure 7.4. Static Nodal Stress Plot of 2nd Housing.

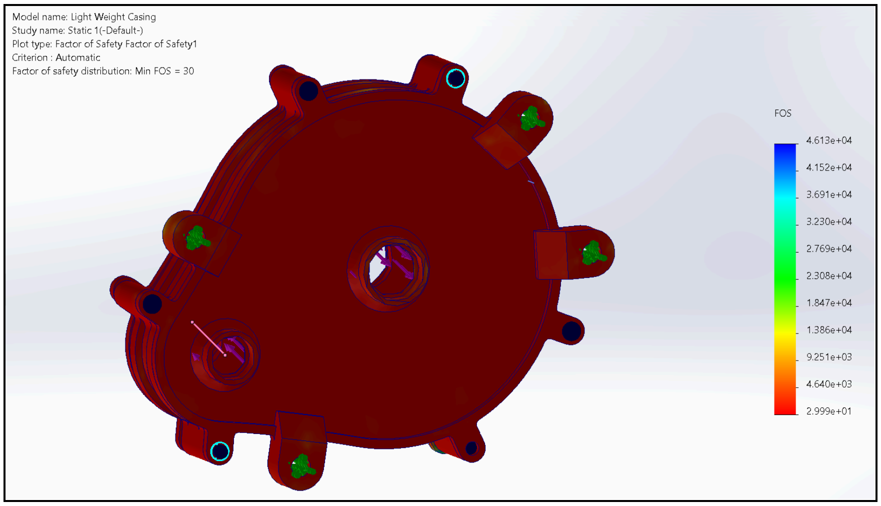

Figure 5. 7. Factor of Safety Plot of First Housing.

8.0 Discussion

This discussion will show that all requirements have been met on the gearbox design. Tables 8.1, 8.2, 8.3, 8.4 and 8.5 present each of the criteria, the completion status, related files, and any notes associated with that particular criteria.

Table 8.1. Design Requirements: Gears

Design Criteria	Value	Status	Related files	Notes
Pressure Angle	20°	Met	Design Drawings: Pinion/Gear Spreadsheets: Pinion/Gear	
Diametral Pitch	4,6,8,10,12	Met	Design Drawings: Pinion/Gear Spreadsheets: Pinion/Gear	We chose 4
Commercial Quality, Reliability	99%	Met	Spreadsheets: Pinion/Gear	

Design Criteria	Value	Status	Related files	Notes
Teeth Number	≤ 84	Met	Design Drawings: Pinion/Gear Spreadsheets: Pinion/Gear	Max is 63
Hours of Operation	20,000	Met	Spreadsheets: Pinion/Gear	
Power Delivered	25 HP	Met	Spreadsheets: Pinion/Gear	
Input Speed	1750 rpm	Met	Spreadsheets: Pinion/Gear	
Output Speed	500 rpm	Met	Spreadsheets: Pinion/Gear	
Gear Ratio	3.5	Met	Spreadsheets: Pinion/Gear	
Max Centre Distance	11 in	Met	Design Drawings: Pinion/Gear Spreadsheets: Pinion/Gear	
Safety Factor, Pinion Contact	> 1.2	Met	Spreadsheets: Pinion/Gear	
Safety Factor, Pinion Bending	> 1.2	Met	Spreadsheets: Pinion/Gear	
Safety Factor, Gear Contact	> 1.2	Met	Spreadsheets: Pinion/Gear	
Safety Factor, Gear Bending	> 1.5	Met	Spreadsheets: Pinion/Gear	

Table 8.2. Design Requirements: Shafts

Design Criteria	Value	Status	Related files	Notes
Design a pinion shaft,	0.5”	Met	Design Drawings:	

Design Criteria	**Value**	**Status**	**Related files**	**Notes**
provide positive axial location			Pinion Shaft, Spreadsheets: Shaft	
Design a gear pinion shaft, provide positive axial location	0.5”	Met	Design Drawings: Gear Shaft, Spreadsheets: Shaft	
The input shaft must be designed to extend beyond the housing to enable the motor shaft to be coupled to it. The output shaft must accommodate a coupling that mates with the drive shaft of the propeller. Use a design reliability of 0.999.		Met	Design Drawings: Gear Shaft, Design Drawings: Gear gears, Spreadsheets: Shaft	We assumed the mating could be done with a shaft and key. This assumption was confirmed to be valid by the professor.
Design shafts for infinite life using Goodman criterion. Fatigue safety factor $n_f \geq 2.5$	> 2.5 for all areas	Met	Design Drawings: Gear Shaft, Design Drawings: Gear gears, Spreadsheets: Shaft	
Design shafts with first-cycle yielding safety factor $n_y \geq 2.5$	2.5	Met	Design Drawings: Gear Shaft, Design Drawings: Gear gears, Spreadsheets: Shaft	
Diameter of the shafts should not exceed 2 inches	Max Diameter 2”	Met	Design Drawings: Gear Shaft, Design Drawings: Pinion Shaft, Spreadsheets: Shaft	
Total Shaft Length	<= 10”	Met	Design Drawings: Gear Shaft, Design Drawings: Pinion Shaft, Spreadsheets:	

Design Criteria	Value	Status	Related files	Notes
			Shaft	
Select shaft materials from: Varsteel or Metal Supermarkets	Varsteel	Met	Spreadsheets: Shaft	

Table 8.3. Design Requirements: Keys

Design Criteria	Value	Status	Related files	Notes
Pinion Key Safety Factor, Yielding	≥ 2	Met	Design Drawings: Pinion Key, Spreadsheets: Keys	
Gear Key Safety Factor, Yielding	≥ 2	Met	Design Drawings: Gear Key, Spreadsheets: Keys	
Input Key Safety Factor, Yielding	≥ 2	Met	Design Drawings: Input Key, Spreadsheets: Keys	The input key has the lowest safety factor and will therefore be the first part to fail in the event of an overload to the gearbox.
Output Key Safety Factor, Yielding	≥ 2	Met	Design Drawings: Output Key, Spreadsheets: Keys	
Select Keys From Suppliers (Material/Dimensions)	Misumi (1045 Carbon Steel)	Met	Spreadsheets: Keys	

Table 8.4. Design Requirements: Bearings

Design Criteria	Value	Status	Related files	Notes
Reliability	95%	Met	Spreadsheets: Bearings	
Design Life	20,000 Hours	Met	Spreadsheets: Bearings	

Table 8.5. Design Requirements: Housing

Design Criteria	Value	Status	Related files	Notes
The housing should have at least two mounting points such as lugs or feet to provide reaction torque, assuming that the shaft couplings do not provide any radial reaction	4 mounting points	Met	Design Drawings: Housing,	
It should have a fill hole, a drain hole and a means to check lubricant level;	Fill hole, drain hole, dipstick	Met	Design Drawings: Assembly	
It should have sufficient strength to support the enclosed moving parts and reactions from the mounting points at rated torque with factor of safety against yield no less than the factor of safety against ultimate shear of the input key.	Safety factor is above 2 FoS = 30	Met	See finite element analysis	
It should have a means to prevent leakage of lubricant	2 Gaskets	Met	Design Drawings: Assembly	

9.0 Conclusion

The final parts and assembly for this project were designed and implemented after multiple iterations of the design process. The final materials and dimensions for each part have a reason for being chosen, whether to meet a design constraint laid out in the outline, or to make that part work well with the other parts in the assembly. Some parts drove the constraints and others changed based on the constraints of the others. For example, the gears had safety factors that needed to be met so they had to be designed to be above those constraint values. This, in turn, changed the transmitted load W_t. This load goes directly through the keys, and into the shafts to transmit force. As a result, the design specs of the shaft and key, as well as many other parts, are directly correlated to the size and shape of the gears we chose. However, the shaft and keys also have design constraints of their own that need to be accounted for. In the end, the components that we chose all meet their design requirements while also optimizing for space, cost, and reliability. This optimization was performed using both calculations in Excel and using a finite element analysis. The result is a practical design that is relatively inexpensive.

References

"6206-2RS1." *SKF*, https://www.skf.com/ca/en/products/rolling-bearings/ball-bearings/deep-groove-ball-bearings/productid-6206-2RS1.

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"Aluminum 6061." Diversified Ulbrich, https://www.diversifiedulbrich.ca/products/6061-aluminum-products.

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"Prototypes." *Rushgears.com*, https://www.rushgears.com/services/prototypes.

"Oil Seals." *All Seals*, https://www.allsealsinc.com/oilsseals.html

"Machine Keys." Misumi. (2023). Retrieved April 11, 2023, from https://us.misumi-ec.com/vona2/mech_screw/M1808000000/

Appendix A: Component References

Component	Quantity	Source
Deep Groove Ball Bearing 6208-2RS1	2	(*6208-2RS1*)
Deep Groove Ball Bearing 6206-2RS1	2	(*6206-2RS1*)
63 and 18 tooth gears Gray Cast Iron Class 40	1 for each size	(*Prototypes*)
Square Keys 1045 carbon steel	2	(*COLD FINISH BARS & SHAFTING*)
Shaft	1 for each size	(*COLD FINISH BARS & SHAFTING*)
Lubricant Bolt	2	(Aluminum 6061)
Sealing Gaskets	2	(Oil Seals)
Housing Top Plate	1	(Aluminum 6061)
Housing Mounting Plate	1	(Aluminum 6061)
Housing Mid Frame	1	(Aluminum 6061)

Appendix B: Design Drawings

Drawing will be presented in the order shown in Table B-1.

#	Name	Quantity	Material
1	Case Bottom Light	1	Aluminum 6061
2	Case Walls	1	Aluminum 6061
3	Case Top Light	1	Aluminum 6061
4	Dipstick	1	Aluminum 6061
5	Gasket	2	Rubber
6	63 Tooth Gear from Rush Gears	1	Cast Iron Class 40
7	HF Bolt 0.75 12x6.6x2.5	6	Steel
8	HNutt 0.75-10-D-C	6	Steel
9	Gear Key	1	1045 Steel
10	Input Key	1	1045 Steel
11	Pinion Key	1	1045 Steel
12	18 Tooth Pinion from Rush Gears	1	Cast Iron Class 40
13	Output Shaft	1	1018 Steel
14	Input Shaft	1	1018 Steel
15	0.75” Washer	6	Steel
16	SKF 6206-2RS1 Ball	40	Steel
17	SKF 6206-2RS1 Assembly	2	Steel
18	SKF 6206-2RS1 Inner Ring	2	Steel
19	SKF 6206-2RS1 Outer Ring	2	Steel
20	SKF 6208-2RS1 Assembly	2	Steel
21	SKF 6208-2RS1 Inner Ring	2	Steel

22	SKF 6208-2RS1 Outer Ring	2	Steel
23	Bolt	1	Steel
24	Assembly	1	NA