Back pain: four outcomes

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1 Approach

For each outcome, the simplest model that has appropriate fit will be sought. Models have been classified in rough ordering from "simplest" below using A, B, C, or D with A representing the common/easily understood models

Outcome variable/*Model*	*Multiple linear regression* (A)	*Ordinal logistic regression* (B*)	*Logistic regression* (B)	*Poisson/negative binomial regression* (C)	*Beta regression* (D)
Binary	-	-	+	-	-
Ordinal	-	+	+ (if outcome made binary)	-	-
Values from 0 to 100	+ (outcome potentially transformed)	- (if outcome made ordinal but bad option)	+ (if outcome made binary)	+	+

*Probably "C" not "B" but is basically multiple logistic regressions performed with different dichotomisations of the order levels in the outcome

2 Set up

2.1 Packages

```r
suppressPackageStartupMessages(suppressWarnings({

  library("dplyr") # tidyverse
  library("tidyr")
  library("readr")
  library("forcats")
  library("ggplot2")

  library("GGally") # additional ggplot-type plotting

  library("compositions")
  library("zCompositions") # this one for lr_EM

  library("performance") # model checking
  library("mice")        # missing data functions
  library("car")         # Anova() for comparing models
  library("knitr")       # kable() for pretty printing
  library("foreach")     # powerful looping

  library("boot")        # bootstrap confidence intervals
  library("tictoc")      # check time between tic() and toc()

}))
```

2.2 Constants

```r
pred_comps <- c("Time_Sleep", "Time_Sedentary", "Time_LPA", "Time_MVPA")
(D <- length(pred_comps))
```

```
[1] 4
```

```r
pred_covs <- c("age", "sex", "bmi", "stress", "smoking", "education", "ses")
outcs <-
  c(
    "LBP_frequency_year", "LBP_intensity_year",
```

```r
    "LBP_intensity_month", "LBP_intensity_week"
  )

# default RHS of model formulas
# (rhs_formula <- paste(c(paste(pred_covs, collapse = " + "), "ilr"), collapse = " + "))
(rhs_formula <- paste(pred_covs, collapse = " + "))
```

```
[1] "age + sex + bmi + stress + smoking + education + ses"
```

```r
# this is the sequential binary partition matrix to be used for ilr creation
sbp1 <- matrix(
  c(
    1, 1, -1, -1,
    1, -1, 0, 0,
    0, 0, 1, -1
  ),
  ncol = 4, byrow = TRUE
)

# a way of creating ilr names automatically from SBP matrix
create_ilr_names <- function(sbp_matrix) {
  ilr_sbp_nms <- apply(sbp_matrix, 1, paste, collapse = "")
  ilr_sbp_nms <- gsub("-1", "-", ilr_sbp_nms)
  ilr_sbp_nms <- gsub("1", "+", ilr_sbp_nms)
  ilr_sbp_nms <- gsub("0", ".", ilr_sbp_nms)
  return(paste0("ilr(", ilr_sbp_nms, ")"))
}
create_ilr_names(sbp1)
```

```
[1] "ilr(++--)" "ilr(+-..)" "ilr(..+-)"
```

```r
do_closure <- function(x, clo_val = 1) {
  return(clo_val * x / sum(x))
}
calc_comp_mean <- function(x, clo_val = 1) {
  unclose_mean <- NULL
  if (is.null(dim(x))) {
    return(x)
  } else if (ncol(x) == 1) { # column matrix
```

```
    return(as.numeric(x))
  } else {
    unclose_mean <- apply(x, 2, function(x) exp(mean(log(x))))
  }

  return(do_closure(unclose_mean, clo_val = clo_val))

}
```

3 Data wrangling

3.1 Read data

```r
bpd_col_spec <-
  cols(
    Time_Sleep = col_double(),
    Time_Sedentary = col_double(),
    Time_LPA = col_double(),
    Time_MVPA = col_double(),
    age = col_double(),
    sex = col_character(),
    bmi = col_double(),
    stress = col_character(),
    smoking = col_character(),
    education = col_character(),
    ses = col_character(),
    LBP_sufferer = col_character(),
    LBP_frequency_year = col_character(),
    LBP_intensity_year = col_double(),
    LBP_intensity_month = col_double(),
    LBP_intensity_week = col_double()
  )


bpd <- read_csv("dat/bpd.csv", col_types = bpd_col_spec)
# head(bpd)

summary(bpd)
```

```
   Time_Sleep      Time_Sedentary       Time_LPA            Time_MVPA
 Min.   : 87.14   Min.   :  10.0   Min.   :   9.857   Min.   :  0.00
 1st Qu.:400.00   1st Qu.: 305.4   1st Qu.: 363.286   1st Qu.: 12.00
 Median :443.57   Median : 440.9   Median : 512.143   Median : 31.14
 Mean   :439.55   Mean   : 451.9   Mean   : 504.967   Mean   : 43.55
 3rd Qu.:480.71   3rd Qu.: 588.3   3rd Qu.: 642.286   3rd Qu.: 60.00
 Max.   :757.14   Max.   :1100.1   Max.   :1160.286   Max.   :514.00

      age             sex                  bmi            stress
 Min.   :18.00   Length:2333         Min.   :15.10   Length:2333
 1st Qu.:38.00   Class :character    1st Qu.:21.95   Class :character
```

```
 Median :49.00   Mode  :character   Median :24.25   Mode  :character
 Mean   :48.11                      Mean   :24.94
 3rd Qu.:58.00                      3rd Qu.:27.15
 Max.   :92.00                      Max.   :66.02

   smoking            education             ses            LBP_sufferer
 Length:2333        Length:2333        Length:2333        Length:2333
 Class :character   Class :character   Class :character   Class :character
 Mode  :character   Mode  :character   Mode  :character   Mode  :character




 LBP_frequency_year LBP_intensity_year LBP_intensity_month LBP_intensity_week
 Length:2333        Min.   :  0.0      Min.   :  0.00      Min.   :  0.00
 Class :character   1st Qu.: 19.0      1st Qu.:  9.00      1st Qu.:  0.00
 Mode  :character   Median : 30.0      Median : 25.00      Median : 10.00
                    Mean   : 33.8      Mean   : 30.78      Mean   : 20.36
                    3rd Qu.: 49.0      3rd Qu.: 50.00      3rd Qu.: 31.00
                    Max.   :100.0      Max.   :100.00      Max.   :100.00
                    NA's   :673        NA's   :673         NA's   :673
```

3.2 Tidy data

```r
# relevel categories in LBP_frequency_year
y_lab <- "LBP_frequency_year"
sort(unique(bpd[[y_lab]]))
```

```
[1] "0days"                                 "1-7days"
[3] "31-90days"                             "8-30days"
[5] "everyday"                              "more_than90days_but_not_everyday"
```

```r
bpd[[y_lab]] <-
  if_else(
    bpd[[y_lab]] == "more_than90days_but_not_everyday",
    "91+_not_evday",
    bpd[[y_lab]]
  )

# check
```

```r
table(bpd[[y_lab]], useNA = "ifany")
```

```
      0days        1-7days      31-90days       8-30days 91+_not_evday 
        673            760            146            451           203 
   everyday 
        100 
```

```r
bpd[[y_lab]] <- factor(bpd[[y_lab]])
levels(bpd[[y_lab]])
```

```
[1] "0days"         "1-7days"       "31-90days"     "8-30days"     
[5] "91+_not_evday" "everyday"     
```

```r
lvls_ord <- c(1, 2, 4, 3, 5, 6)
# right order?
# levels(bpd[[y_lab]])[lvls_ord]
bpd[[y_lab]] <- lvls_reorder(bpd[[y_lab]], lvls_ord)
### right order?
levels(bpd[[y_lab]])
```

```
[1] "0days"         "1-7days"       "8-30days"      "31-90days"    
[5] "91+_not_evday" "everyday"     
```

```r
with(bpd, table(LBP_frequency_year, LBP_sufferer, useNA = "ifany"))
```

```
                  LBP_sufferer
LBP_frequency_year  no yes
     0days         673   0
     1-7days         0 760
     8-30days        0 451
     31-90days       0 146
     91+_not_evday   0 203
     everyday        0 100
```

```r
### comment these lines for sensitivity analysis
bpd$age <-
```

```
  cut(
    bpd$age,
    breaks = c(17, 44, 64, 100),
    labels = c("1_younger", "2_middle", "3_older")
  )
table(bpd$age)
```

```
1_younger  2_middle   3_older
      896      1153       284
```

```
bpd$bmi <-
  cut(
    bpd$bmi,
    breaks = c(15, 18.5, 25, 70),
    right = FALSE,
    labels = c("1_underweight", "2_normal", "3_overweight")
  )
table(bpd$bmi)
```

```
1_underweight      2_normal  3_overweight
           44          1309           980
```

3.3 Impute missing values in compositions

This code is thanks to Kaja!

Missing data is assumed to be below detectable threshold and imputed.

```
# Do I have zero values in my composition? (yes in MVPA)
### See: summary(bpd)

# We need to make compositions before we do the lrEM method. The most straightforward way
comp1 <- bpd[, pred_comps]

# How much participants have zero MVPA? 159 participants (6.8% of the sample)
missingSummary(comp1)
```

```
                missingType
variable          NMV   BDL   MAR MNAR    SZ   Err
```

```
  Time_Sleep     2333    0    0    0    0    0
  Time_Sedentary 2333    0    0    0    0    0
  Time_LPA       2333    0    0    0    0    0
  Time_MVPA      2174  159    0    0    0    0
```

```r
sum(rowSums(is.na(comp1) | (comp1 < 0.1)), na.rm = FALSE)
```

```
[1] 159
```

```r
sum(which_0 <- as.logical(rowSums(is.na(comp1) | (comp1 < 0.1))))
```

```
[1] 159
```

```r
# these are 0 vals anywhere in composition (or NA)
bpd[which_0, pred_comps]
```

```
# A tibble: 159 x 4
   Time_Sleep Time_Sedentary Time_LPA Time_MVPA
        <dbl>          <dbl>    <dbl>     <dbl>
 1       475            148.     817.         0
 2       437.           826.     177.         0
 3       479.           522.     440.         0
 4       427.           429      584.         0
 5       429.           468.     544.         0
 6       424.           296.     720.         0
 7       506.           727.     207.         0
 8       456.           208      776.         0
 9       475            607.     358.         0
10       272.           783.     384.         0
# i 149 more rows
```

```r
# I have zeroes in MVPA - lrEM function will be applied

# ?lrEM
# what is the smallest time-use value above 0? [in minutes]
min(comp1[comp1 > 0])
```

```
[1] 0.1428571
```

```r
thresh_detect <- 10 / 1440
# thresh_detect <- 0.01

comp1.a <- comp1 / 1440 # Create % based composition
dl <- c(rep(thresh_detect, times = D)) # threshold limit for the replacement

comp1.zr <- lrEM(comp1.a, label = 0, dl = dl) # conduct the lrEM Zero Replacement
```

```
No. iterations to converge:  6
```

```r
comp1.zr <- as_tibble(comp1.zr * 1440)
# composition is larger than 1440 for those who have imputated MVPA
# (all behaviours will be proportionally downscaled to fit 1440 min when constructing the

# look at imputed values
comp1.zr[which_0, ]
```

```
# A tibble: 159 x 4
   Time_Sleep Time_Sedentary Time_LPA Time_MVPA
        <dbl>          <dbl>    <dbl>     <dbl>
 1        475            148.     817.      2.82
 2        437.           826.     177.      3.21
 3        479.           522.     440.      3.08
 4        427.           429      584.      2.99
 5        429.           468.     544.      3.01
 6        424.           296.     720.      2.91
 7        506.           727.     207.      3.21
 8        456.           208      776.      2.87
 9        475            607.     358.      3.12
10        272.           783.     384.      3.00
# i 149 more rows
```

```r
# build dataset that contain imputed values for our 24-h composition

# add new compositions to other noncompositional data
# remove 24-h data from the datase
# bpd <- subset(bpd, select = -c(id, Time_Sleep, Time_Sedentary, Time_LPA, Time_MVPA))
head(bpd[which_0, pred_comps])
```

```
# A tibble: 6 x 4
  Time_Sleep Time_Sedentary Time_LPA Time_MVPA
       <dbl>          <dbl>    <dbl>     <dbl>
1        475           148.     817.         0
2        437.          826.     177.         0
3        479.          522.     440.         0
4        427.          429      584.         0
5        429.          468.     544.         0
6        424.          296.     720.         0
```

```
bpd <- bpd[, !(colnames(bpd) %in% pred_comps)] # remove ori time-use cols
bpd <- bind_cols(comp1.zr, bpd) # add imputed 24-h data
head(bpd[which_0, pred_comps])
```

```
# A tibble: 6 x 4
  Time_Sleep Time_Sedentary Time_LPA Time_MVPA
       <dbl>          <dbl>    <dbl>     <dbl>
1        475           148.     817.      2.82
2        437.          826.     177.      3.21
3        479.          522.     440.      3.08
4        427.          429      584.      2.99
5        429.          468.     544.      3.01
6        424.          296.     720.      2.91
```

3.4 Compositions transformation to *ilr*s

The below function will allow us to automatically add ilrs to a dataset

```
add_ilrs_to_data <- function(dataset, comp_vars = pred_comps, sbp_matrix = sbp1) {

  # the time-use composition
  comp <- dataset[, comp_vars]
  comp <- acomp(comp) # designate it as a compositional variable

  # define sequential binary partition (SBP)
  psi1 <- gsi.buildilrBase(t(sbp_matrix)) #  The orthonormal matrix

  # find the mean composition
  (m <- mean(comp)) # comp has been designated as acomp, therefore R knows it's a composit
  # cat(
  #   "\nThis is the compositional mean [in mins] of the columns (",
```

```r
    #   paste(comp_vars, collapse = ", "),
    #   ")\n\n",
    #   sep = ""
    # )
    # print(clo(m, total = 1440)) # to look at the mean in minutes/day.
    # cat("\n\n")

    # create isometric log ratios (ilr.1) using the above SBP and orthonormal b asis V=psi1.
    ilrs_from_comp <- ilr(comp, V = psi1)
    colnames(ilrs_from_comp) <- create_ilr_names(sbp_matrix)
    # colnames(ilrs_from_comp) <- paste0("coord", 1:(length(comp_vars) - 1))
    dataset$ilr <- ilrs_from_comp

    return(dataset)

  }

  # use function: creates the ilr columns nested in the single column "ilr"
  bpd <- add_ilrs_to_data(bpd)
  # check
  bpd[, c("ilr", pred_comps)]
```

```
# A tibble: 2,333 x 5
   ilr[,"ilr(++--)"] [,"ilr(+-..)"] Time_Sleep Time_Sedentary Time_LPA Time_MVPA
               <dbl>          <dbl>      <dbl>          <dbl>    <dbl>     <dbl>
 1              1.04         0.355        435            263.     722.      19.7
 2              2.29        -0.401        410            723.     297.      10.3
 3              1.57        -0.336        426.           685.     285.      44.3
 4              1.10        -0.0811       486.           545.     318.      91.3
 5              0.801        0.220        484.           355.     536.      64.6
 6              1.71        -0.134        494.           598.     318.      30.4
 7              1.03        -0.0333       447.           469.     467.      57.6
 8              1.29         0.189        546.           418.     435.      40
 9              1.69        -0.243        452.           638      320.      30.4
10              1.36        -0.329        354.           565.     494.      26.7
# i 2,323 more rows
# i 1 more variable: ilr[3] <rmult>
```

```r
  # also create version of data without the nested ilrs
  bpd_clean <- as.data.frame(bpd)
  bpd_clean$ilr <- NULL # remove nested cols
```

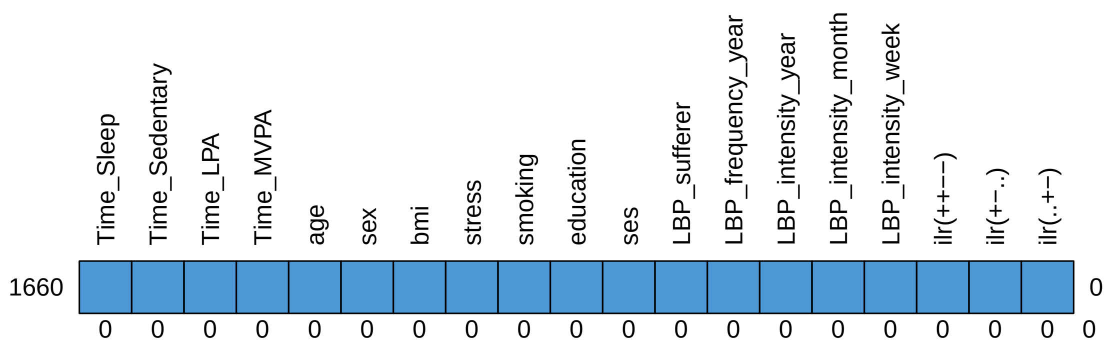
```r
bpd_clean <- cbind(bpd_clean, as.data.frame(bpd$ilr))
```

4 Exploratory analysis

4.1 Missing/`NA` value summaries

```r
### Missing data summary for LBP suffers
bpd_clean %>%
  dplyr::filter(LBP_sufferer == "yes") %>%
  md.pattern(., rotate.names = TRUE)
```

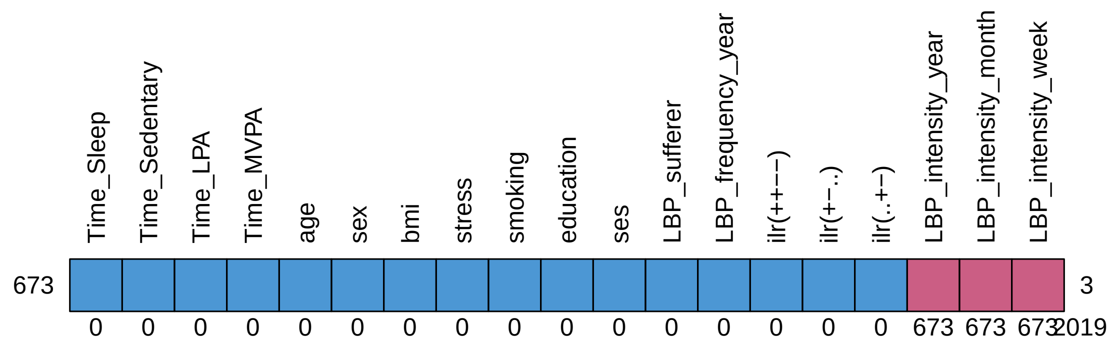
```
 /\     /\
{  `---'  }
{  O   O  }
==>  V <==  No need for mice. This data set is completely observed.
 \  \|/  /
  `-----'
```




```
     Time_Sleep Time_Sedentary Time_LPA Time_MVPA age sex bmi stress smoking
1660          1              1        1         1   1   1   1      1       1
              0              0        0         0   0   0   0      0       0
     education ses LBP_sufferer LBP_frequency_year LBP_intensity_year
1660         1   1            1                  1                  1
             0   0            0                  0                  0
```

```
     LBP_intensity_month LBP_intensity_week ilr(++--) ilr(+-..) ilr(..+-)
1660                   1                  1         1         1         1 0
                       0                  0         0         0         0 0
```

```r
### Missing data summary for _non_ LBP suffers
bpd_clean %>%
  dplyr::filter(LBP_sufferer == "no") %>%
  md.pattern(., rotate.names = TRUE)
```

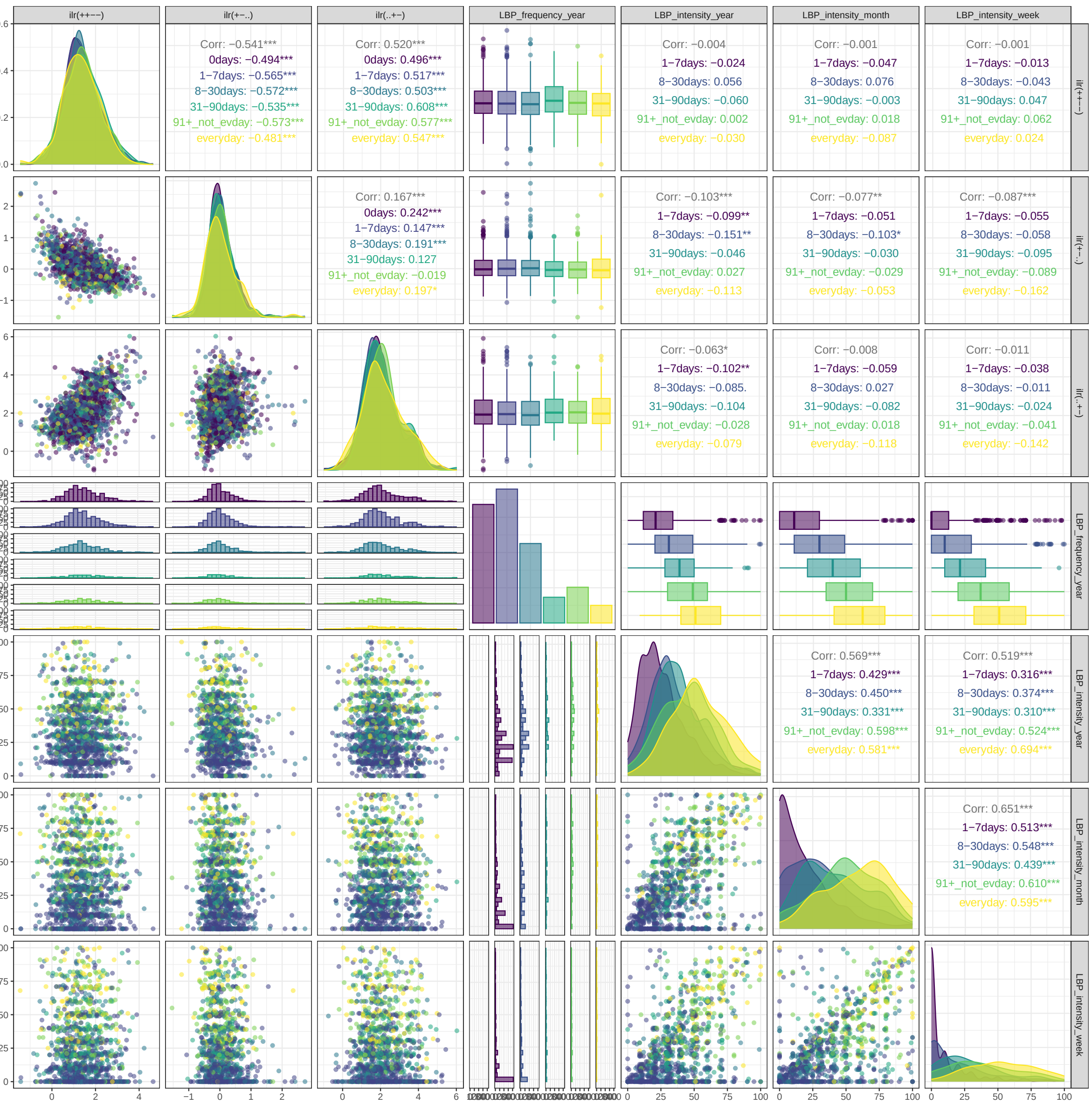



```
    Time_Sleep Time_Sedentary Time_LPA Time_MVPA age sex bmi stress smoking
673          1              1        1         1   1   1   1      1       1
             0              0        0         0   0   0   0      0       0
    education ses LBP_sufferer LBP_frequency_year ilr(++--) ilr(+-..) ilr(..+-)
673         1   1            1                  1         1         1         1
            0   0            0                  0         0         0         0
    LBP_intensity_year LBP_intensity_month LBP_intensity_week
673                  0                   0                  0    3
                   673                 673                673 2019
```

```r
### ===> data doesn't have mistiness for analysis
```

4.2 Pairwise plots between *ilr*s and outcome variables

```
### plot pairwise comparisons of time-use and outcomes
if (FALSE) { # takes 30 sec
  suppressWarnings({
    bpd_clean %>%
      dplyr::select(all_of(pred_comps), all_of(outcs)) %>%
      ggpairs(
        .,
        progress = FALSE,
        ggplot2::aes(
          colour = LBP_frequency_year,
          fill = LBP_frequency_year,
          alpha = 0.25
        )
      ) +
      theme_bw() +
      scale_colour_viridis_d()+
      scale_fill_viridis_d()
  })
}

### plot pairwise comparisons of _ilrs_ and outcomes
if (TRUE) { # takes 30 sec
  suppressWarnings({
    bpd_clean %>%
      dplyr::select(starts_with("ilr"), all_of(outcs)) %>%
      ggpairs(
        .,
        progress = FALSE,
        ggplot2::aes(
          colour = LBP_frequency_year,
          fill = LBP_frequency_year,
          alpha = 0.25
        )
      ) +
      theme_bw() +
      scale_colour_viridis_d()+
      scale_fill_viridis_d()
  })
}
```



ilr(++−−)
ilr(+−..)
ilr(..+−)
LBP_frequency_year
LBP_intensity_year
LBP_intensity_month
LBP_intensity_week
Corr: −0.541***
0days: −0.494***
1−7days: −0.565***
8−30days: −0.572***
31−90days: −0.535***
91+_not_evday: −0.573***
everyday: −0.481***
Corr: 0.520***
0days: 0.496***
1−7days: 0.517***
8−30days: 0.503***
31−90days: 0.608***
91+_not_evday: 0.577***
everyday: 0.547***
Corr: −0.004
1−7days: −0.024
8−30days: 0.056
31−90days: −0.060
91+_not_evday: 0.002
everyday: −0.030
Corr: −0.001
1−7days: −0.047
8−30days: 0.076
31−90days: −0.003
91+_not_evday: 0.018
everyday: −0.087
Corr: −0.001
1−7days: −0.013
8−30days: −0.043
31−90days: 0.047
91+_not_evday: 0.062
everyday: 0.024
Corr: 0.167***
0days: 0.242***
1−7days: 0.147***
8−30days: 0.191***
31−90days: 0.127
91+_not_evday: −0.019
everyday: 0.197*
Corr: −0.103***
1−7days: −0.099**
8−30days: −0.151**
31−90days: −0.046
91+_not_evday: 0.027
everyday: −0.113
Corr: −0.077**
1−7days: −0.051
8−30days: −0.103*
31−90days: −0.030
91+_not_evday: −0.029
everyday: −0.053
Corr: −0.087***
1−7days: −0.055
8−30days: −0.058
31−90days: −0.095
91+_not_evday: −0.089
everyday: −0.162
Corr: −0.063*
1−7days: −0.102**
8−30days: −0.085.
31−90days: −0.104
91+_not_evday: −0.028
everyday: −0.079
Corr: −0.008
1−7days: −0.059
8−30days: 0.027
31−90days: −0.082
91+_not_evday: 0.018
everyday: −0.118
Corr: −0.011
1−7days: −0.038
8−30days: −0.011
31−90days: −0.024
91+_not_evday: −0.041
everyday: −0.142
Corr: 0.569***
1−7days: 0.429***
8−30days: 0.450***
31−90days: 0.331***
91+_not_evday: 0.598***
everyday: 0.581***
Corr: 0.519***
1−7days: 0.316***
8−30days: 0.374***
31−90days: 0.310***
91+_not_evday: 0.524***
everyday: 0.694***
Corr: 0.651***
1−7days: 0.513***
8−30days: 0.548***
31−90days: 0.439***
91+_not_evday: 0.610***
everyday: 0.595***

5 Statistical analysis

5.1 Outcome 0: binary outcome of `Pain = "yes"`

5.1.1 Model fit

```
bpd <-
  bpd %>%
  mutate(lbp_occurr = as.integer(LBP_sufferer == "yes"))

(this_outcome <- "lbp_occurr")
```

```
[1] "lbp_occurr"
```

```
# (mod_form_null <-as.formula(paste0(this_outcome, " ~ ", rhs_formula)))
(mod_form_ilrs <-as.formula(paste0(this_outcome, " ~ ", rhs_formula, " + ilr")))
```

```
lbp_occurr ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```
table(bpd[, this_outcome], useNA = "ifany")
```

```
lbp_occurr
   0    1
 673 1660
```

```
# logistic regression model __with__ ilrs
bpd_occurr_ilrs <- glm(mod_form_ilrs, data = bpd, family = binomial())
summary(bpd_occurr_ilrs)
```

```
Call:
glm(formula = mod_form_ilrs, family = binomial(), data = bpd)

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         1.053768   0.384211   2.743 0.006094 **
age2_middle         0.340378   0.103168   3.299 0.000969 ***
```

```
age3_older          0.262018   0.159537   1.642 0.100513
sex2_male           0.062784   0.111862   0.561 0.574618
bmi2_normal         0.164453   0.327142   0.503 0.615178
bmi3_overweight     0.378117   0.333707   1.133 0.257180
stress2_stressed    0.407584   0.103641   3.933  8.4e-05 ***
smoking2_nonsmoker -0.105372   0.126368  -0.834 0.404365
education2_higher  -0.274614   0.112119  -2.449 0.014313 *
ses2_middle        -0.328406   0.177563  -1.850 0.064383 .
ses3_higher        -0.723348   0.215902  -3.350 0.000807 ***
ilrilr(++--)       -0.083947   0.103592  -0.810 0.417735
ilrilr(+-..)       -0.078951   0.177757  -0.444 0.656933
ilrilr(..+-)        0.005253   0.072374   0.073 0.942138
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2803.2  on 2332  degrees of freedom
Residual deviance: 2736.7  on 2319  degrees of freedom
AIC: 2764.7

Number of Fisher Scoring iterations: 4
```

```r
Anova(bpd_occurr_ilrs)
```

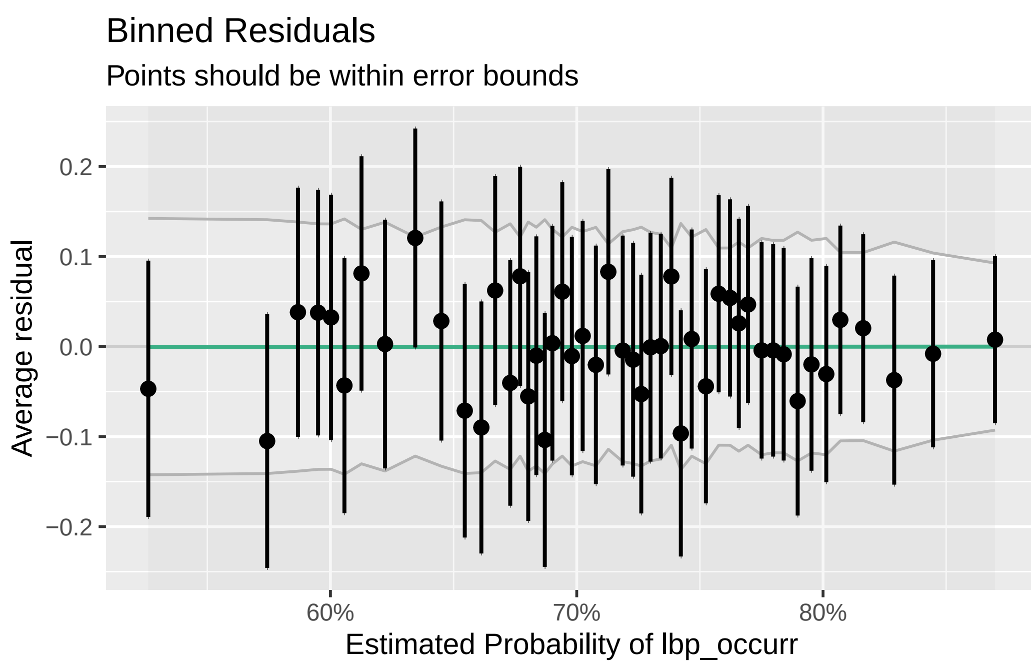
```
Analysis of Deviance Table (Type II tests)

Response: lbp_occurr
          LR Chisq Df Pr(>Chisq)
age        11.0155  2   0.004055 **
sex         0.3161  1   0.573932
bmi         5.0563  2   0.079808 .
stress     15.7925  1  7.068e-05 ***
smoking     0.7022  1   0.402035
education   6.1009  1   0.013511 *
ses        12.5248  2   0.001907 **
ilr         1.2749  3   0.735097
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5.1.2 Model diagnostics

```
### check binned residuals are acceptable
# From the help file:
# Binned residual plots are achieved by "dividing the data into categories
# (bins) based on their fitted values, and then plotting the average residual
# versus the average fitted value for each bin." (Gelman, Hill 2007: 97).
# If the model were true, one would expect about 95% of the residuals to
# fall inside the error bounds.
bin_res_overall <- binned_residuals(bpd_occurr_ilrs)
bin_res_overall
```

```
Ok: About 100% of the residuals are inside the error bounds.
```

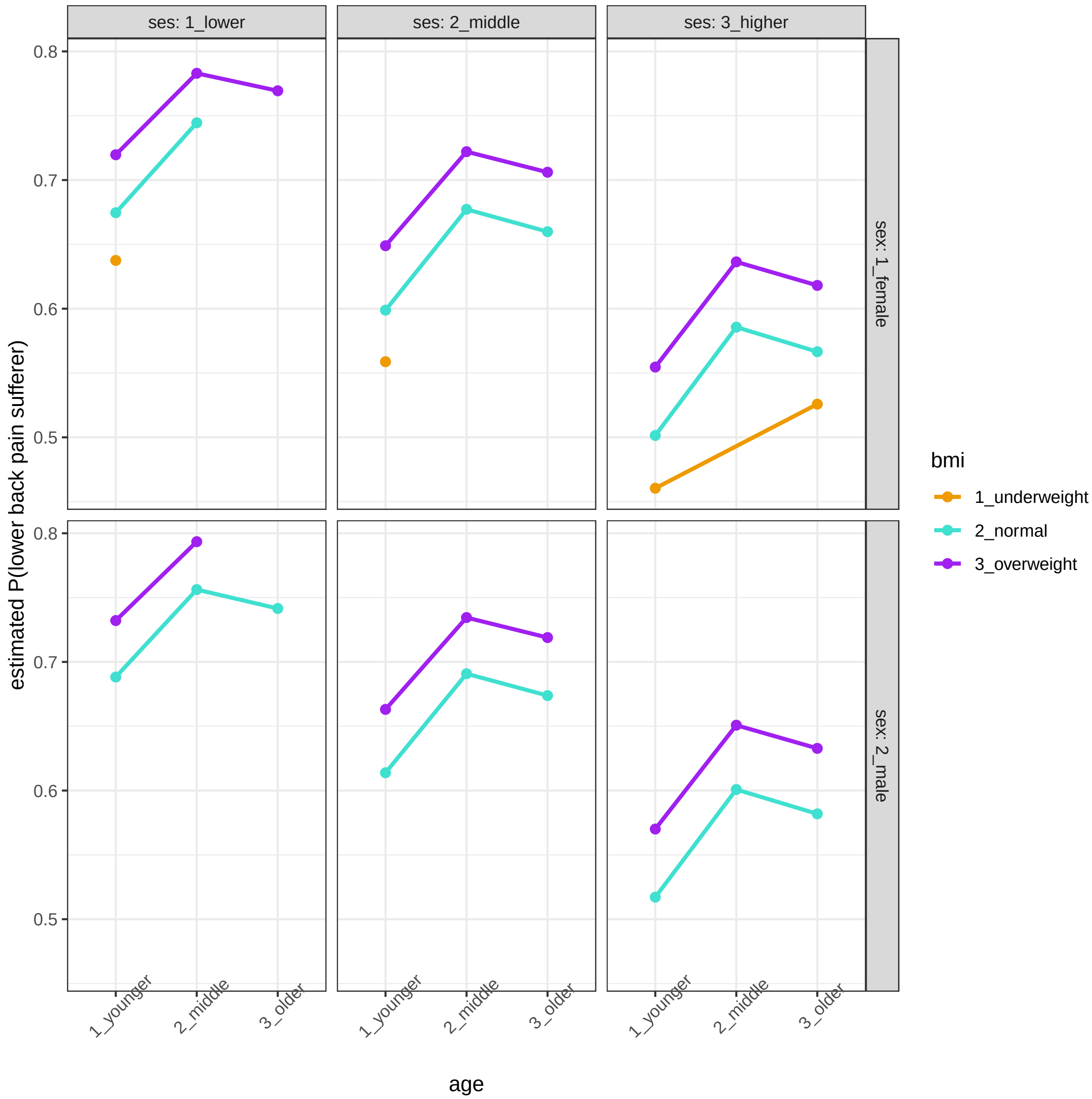
```
plot(bin_res_overall)
```



5.1.3 Model predictions

```r
# create dataset for predictions
newdata <-
  bpd %>%
  dplyr::select(all_of(pred_covs), ilr) %>%
  distinct(pick(all_of(pred_covs)), .keep_all = TRUE) %>%
  arrange(pick(all_of(pred_covs)))

mean_ilr <- mean(bpd$ilr)
dev_null <- foreach(i = 1:nrow(newdata)) %do% {
  newdata$ilr[i, ] <- mean_ilr
}

# make preds and then put in long format for ggplot
predictions_probs <-
  cbind(
    `P(LBP)` = predict(bpd_occurr_ilrs, newdata, type = "response"),
    newdata
  ) %>%
  dplyr::select(-ilr)

# predictions_probs


## model predictions for specific values
predictions_probs %>%
  dplyr::filter(
    # sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
    # ses == "2_middle"
  ) %>%
  ggplot(., aes(age, `P(LBP)`, group = bmi)) +
  geom_line(aes(colour = bmi), linewidth = 1) +
  geom_point(aes(colour = bmi), size = 2) +
  facet_grid(sex~ ses, labeller = label_both) +
  labs(x = "age", y = "estimated P(lower back pain sufferer)") +
  theme_bw() +
  scale_color_manual(values = c("orange2", "turquoise", "purple")) +
```

```r
  theme(axis.text.x = element_text(angle = 45))
```




```r
# create a RHS of regression equation dataset for time-reallocation
predict_basis <-
  bpd %>%
  dplyr::select(all_of(pred_covs), all_of(pred_comps)) %>%
```

```r
  dplyr::filter(
    age == "2_middle",
    sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
    ses == "2_middle",
    bmi == "2_normal"
  )

### continuous scenario
# predict_basis$age <- mean(predict_basis$age)

(predict_basis <-
  predict_basis %>%
  distinct(across(all_of(pred_covs)), .keep_all = TRUE) %>%
  as.data.frame())
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   546.4286
  Time_Sedentary Time_LPA Time_MVPA
1       418.2857 435.1429        40
```

```r
# compositional mean: geometric mean to closure
# (comp_mean <- mean(acomp(bpd[, pred_comps])))
(comp_mean <- calc_comp_mean(bpd[, pred_comps], clo_val = 1440))
```

```
    Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
     474.36588      439.73363      499.43836       26.46213
```

```r
predict_basis0 <- predict_basis
predict_basis0[, pred_comps] <- comp_mean

predict_basis0
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   474.3659
  Time_Sedentary Time_LPA Time_MVPA
1       439.7336 499.4384  26.46213
```

```r
# +15 minutes to Time_MVPA and -15 minutes from Time_Sedentary
comp_mean_changed <- comp_mean
comp_mean_changed["Time_MVPA"] <- comp_mean_changed["Time_MVPA"] + 15
comp_mean_changed["Time_Sedentary"] <- comp_mean_changed["Time_Sedentary"] - 15
# check
comp_mean_changed - comp_mean
```

```
   Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA 
            0            -15              0             15 
```

```r
predict_basis1 <- predict_basis
predict_basis1[, pred_comps] <- comp_mean_changed

pred_df <- rbind(predict_basis0, predict_basis1)
pred_df <- add_ilrs_to_data(pred_df, comp_vars = pred_comps, sbp_matrix = sbp1)
pred_df
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   474.3659
2 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   474.3659
  Time_Sedentary Time_LPA Time_MVPA      ilr.1      ilr.2      ilr.3
1       439.7336 499.4384  26.46213 1.37947472 0.05360560 2.07731687
2       424.7336 499.4384  41.46213 1.13758831 0.07814711 1.75977934
```

```r
predict(bpd_occurr_ilrs, pred_df, type = "link")
```

```
        1         2 
0.7410846 0.7577845 
```

```r
# ratio of odds ratios
exp(diff(predict(bpd_occurr_ilrs, pred_df, type = "link")))
```

```
      2 
1.01684 
```

```r
get_pred_diff <- function(mod, new_dat) {
  log_odds_pred <- predict(mod, new_dat, type = "link")
  odds_ratio_ratio <- exp(log_odds_pred[2] - log_odds_pred[1])
  return(odds_ratio_ratio)
}
(est_v1 <- get_pred_diff(bpd_occurr_ilrs, pred_df))
```

```
      2
1.01684
```

```r
fit_mod_boot <- function(data, i, pred_dat) {

  this_dat <- data[i, ]
  this_logis <- glm(mod_form_ilrs, data = this_dat, family = binomial())
  est <- get_pred_diff(this_logis, new_dat = pred_dat)
  return(est)

}

### CI method #1 (bootstrapping):
alpha <- 0.05
(ci_v1 <-
  c(
    est = est_v1,
    quantile(
      boot(bpd, fit_mod_boot, R = 100, pred_dat = pred_df)$t,
      c(alpha / 2, 1 - alpha / 2)
    )))
```

```
    est.2      2.5%     97.5%
1.0168402 0.9860732 1.0521782
```

```r
### alternative CI method #2 (Wald approximation - re-transformed):
pred_df[, "ilr"]
```

```
         [,1]       [,2]     [,3]
[1,] 1.379475 0.05360560 2.077317
[2,] 1.137588 0.07814711 1.759779
attr(,"class")
[1] "rmult"
```

```r
diff(pred_df[, "ilr"])
```

```
           [,1]       [,2]       [,3]
[1,] -0.2418864 0.02454151 -0.3175375
attr(,"class")
[1] "rmult"
```

```r
x_0_red <- matrix(as.numeric(diff(pred_df[, "ilr"])), nrow = 1)
x_0_red
```

```
           [,1]       [,2]       [,3]
[1,] -0.2418864 0.02454151 -0.3175375
```

```r
betas <- coef(bpd_occurr_ilrs)
nms_kp <- grepl("^ilr", names(betas))
betas_red <- as.matrix(betas[nms_kp])
Sigma <- stats::vcov(bpd_occurr_ilrs)
nms_kp <- grepl("^ilr", colnames(Sigma))
sigma_red <- Sigma[nms_kp, nms_kp]
sigma_red
```

```
             ilrilr(++--) ilrilr(+-..) ilrilr(..+-)
ilrilr(++--)  0.010731347   0.01369669 -0.005543191
ilrilr(+-..)  0.013696690   0.03159738 -0.008105390
ilrilr(..+-) -0.005543191  -0.00810539  0.005237994
```

```r
est_red <- x_0_red %*% betas_red
se_red <- sqrt(x_0_red %*% sigma_red %*% t(x_0_red))
z_star <- qnorm(0.975)
(ci_v2 <-
  exp(c(
    est = est_red,
    lo = est_red - z_star * se_red,
    hi = est_red + z_star * se_red
  )))
```

```
      est        lo        hi
1.0168402 0.9836173 1.0511852
```

```r
### alternative CI method #3 (delta method)
# (first order approximation, although still linear combin of param ests):
approx_ci <-
  deltaMethod(
    bpd_occurr_ilrs,
    "-0.2418864 * `ilrilr(++--)` + 0.02454151 * `ilrilr(+-..)` + -0.3175375 * `ilrilr(..+-
  )
(ci_v3 <-
  exp(c(
    est = approx_ci[["Estimate"]],
    lo = approx_ci[["2.5 %"]],
    hi = approx_ci[["97.5 %"]]
  )))
```

```
      est        lo        hi
1.0168402 0.9836173 1.0511852
```

```r
### compare CIs
kable(rbind(ci_v1, ci_v2, ci_v3))
```

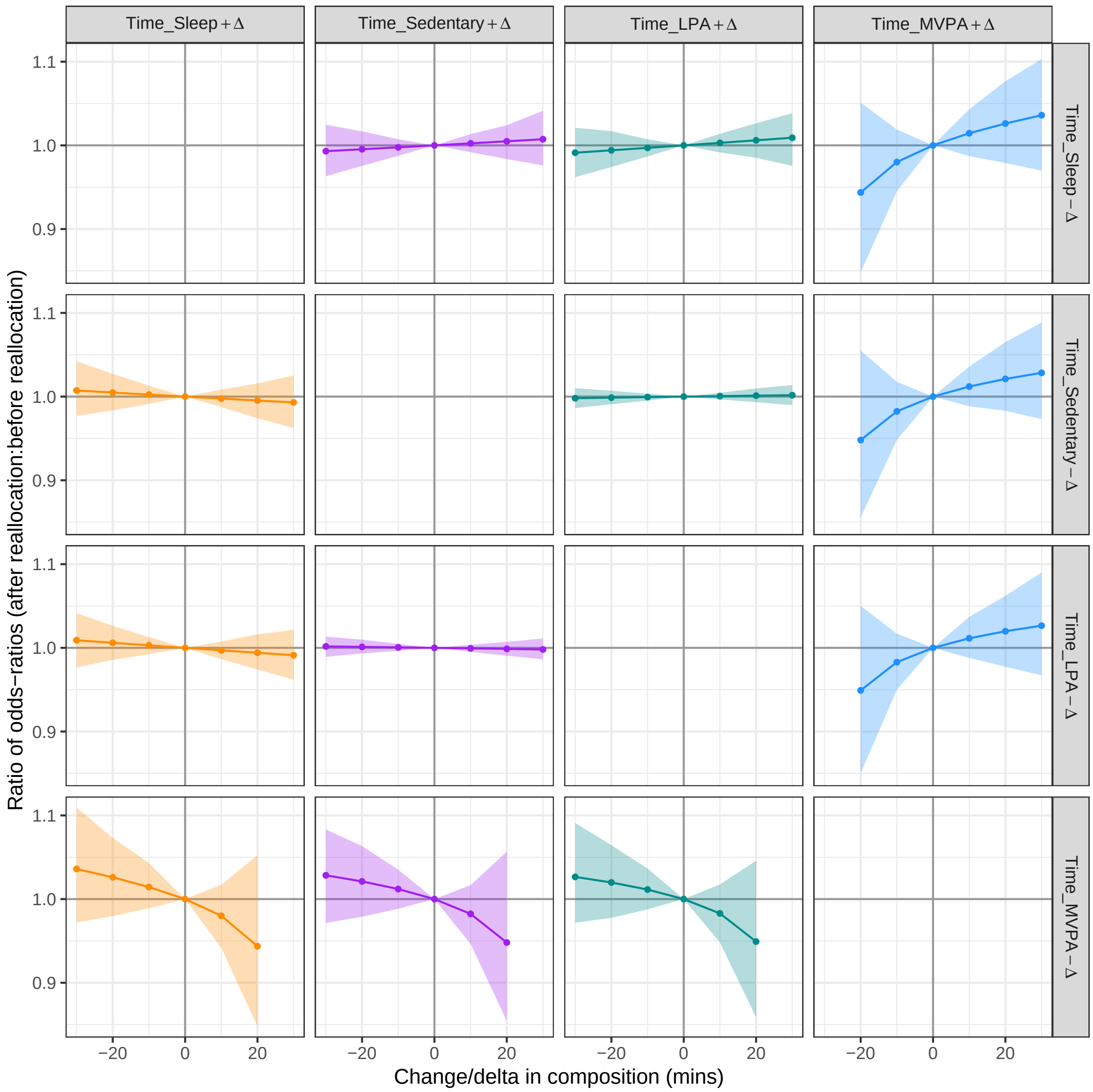
	est.2	2.5%	97.5%
ci_v1	1.01684	0.9860732	1.052178
ci_v2	1.01684	0.9836173	1.051185
ci_v3	1.01684	0.9836173	1.051185

```r
do_multi_realloc <- function(mod, basis_data, timeusenames, time_changes, sbp_matrix = sbp


  x0 <- basis_data

  plot_dat <-
    foreach(i = 1:length(timeusenames), .combine = bind_rows) %do% {
      print(paste("i: ", i))
      foreach(j = 1:length(timeusenames), .combine = bind_rows) %do% {
        print(paste("  j: ", j))
        foreach(d = 1:length(time_changes), .combine = bind_rows) %do% {
          print(paste("    d: ", d))

          timeuse_to <- timeusenames[i]
```

```
        timeuse_from <- timeusenames[j]
        change_time <- time_changes[d]

        proposed_change_1 <- x0[timeuse_to] + change_time
        proposed_change_2 <- x0[timeuse_from] - change_time

        if (timeuse_to == timeuse_from) {
          NULL # reallocation exceeds 0 or max time
        } else if ((proposed_change_1 < 0) | (proposed_change_1 > 1440)) {
          NULL # reallocation exceeds 0 or max time
        } else if ((proposed_change_2 < 0) | (proposed_change_2 > 1440)) {
          NULL # reallocation exceeds 0 or max time
        } else {

          x1 <- x0
          x1[timeuse_to] <- x1[timeuse_to] + change_time
          x1[timeuse_from] <- x1[timeuse_from] - change_time

          pred_df <- rbind(x0, x1)
          pred_df <- add_ilrs_to_data(pred_df, comp_vars = timeusenames, sbp_matrix = sb

          ratio_of_odds_ratios <- get_pred_diff(mod, pred_df)

          bootstrapped_ests <- boot(bpd, fit_mod_boot, R = 1000, pred_dat = pred_df)$t
          ci_est <- quantile(as.numeric(bootstrapped_ests), c(alpha / 2, 1 - alpha / 2))

          tibble(
            to = timeuse_to,
            from = timeuse_from,
            change_time = change_time,
            ratio_of_odds_ratios = ratio_of_odds_ratios,
            ci_lo = ci_est[1],
            ci_hi = ci_est[2]
          )
        }
      }
    }
  }

plot_dat$to <- factor(plot_dat$to, levels = timeusenames)
plot_dat$from <- factor(plot_dat$from, levels = timeusenames)
```

```r
  return(plot_dat)

}




# takes ~25 min (single core)

### Uncomment to generate bootstrapping

# tic()
# set.seed(1234)
# realloc_plot_data <-
#   do_multi_realloc(
#     bpd_occurr_ilrs,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )
# saveRDS(realloc_plot_data, file = "res/logistic_realloc_boot_res.rda")
# toc()


realloc_plot_data <- readRDS(file = "res/logistic_realloc_boot_res.rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
  geom_line(aes(x = change_time , y = ratio_of_odds_ratios, col = to)) +
  geom_point(aes(x = change_time , y = ratio_of_odds_ratios, col = to), size = 1) +
  facet_grid(from ~ to, labeller = label_parsed) +
  theme_bw() +
  scale_colour_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
  scale_fill_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
  labs(
```

```
  x = paste0("Change/delta in composition (mins)"),
  y = paste0("Ratio of odds-ratios (after reallocation:before reallocation)")
) +
theme(legend.position = "none")
```



```r
ggsave(
  filename = "fig/lbp_occur_logistic_odds.png",
  dpi = 600, # print quality
  width = 10,
  height = 10
)
```

5.2 Note for outcomes 1 to 2

The dataset for the remain outcomes will be limited to people who responded:

```
bpd_yes <- bpd %>% dplyr::filter(LBP_sufferer == "yes")
nrow(bpd)
```

```
[1] 2333
```

```
nrow(bpd_yes)
```

```
[1] 1660
```

```
bpd_clean_yes <- bpd_clean %>% dplyr::filter(LBP_sufferer == "yes")
```

5.3 Outcome 1: LBP_frequency_year

5.3.1 Model fit

```
(this_outcome <- outcs[1])
```

```
[1] "LBP_frequency_year"
```

```
# (mod_form_null <-as.formula(paste0(this_outcome, " ~ ", rhs_formula)))
(mod_form_ilrs <-as.formula(paste0(this_outcome, " ~ ", rhs_formula, " + ilr")))
```

```
LBP_frequency_year ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```
table(bpd_yes[, this_outcome], useNA = "ifany")
```

```
LBP_frequency_year
        0days       1-7days      8-30days     31-90days 91+_not_evday
            0           760           451           146           203
     everyday
          100
```

```
bpd_yes[[this_outcome]] <- fct_drop(bpd_yes[[this_outcome]])
table(bpd_yes[, this_outcome], useNA = "ifany")
```

```
LBP_frequency_year
      1-7days      8-30days     31-90days 91+_not_evday      everyday
          760           451           146           203           100
```

```
## model without ilrs
# bpd_ordinal_null <- polr(mod_form_null, data = bpd, Hess = TRUE, method = "logistic")
# summary(bpd_ordinal_null)


## model __with__ ilrs
bpd_ordinal_ilrs <- polr(mod_form_ilrs, data = bpd_yes, Hess = TRUE, method = "logistic")
```

```r
summary(bpd_ordinal_ilrs)
```

```
Call:
polr(formula = mod_form_ilrs, data = bpd_yes, Hess = TRUE, method = "logistic")

Coefficients:
                     Value Std. Error t value
age2_middle        0.33918    0.10482  3.2360
age3_older         0.84204    0.16404  5.1331
sex2_male         -0.28829    0.11085 -2.6006
bmi2_normal       -0.02595    0.34969 -0.0742
bmi3_overweight    0.03744    0.35410  0.1057
stress2_stressed   0.57164    0.09929  5.7575
smoking2_nonsmoker -0.17602   0.11788 -1.4932
education2_higher -0.11931    0.10479 -1.1386
ses2_middle       -0.34624    0.14759 -2.3460
ses3_higher       -0.49685    0.20599 -2.4120
ilrilr(++--)      -0.30211    0.10322 -2.9268
ilrilr(+-..)      -0.56557    0.17197 -3.2888
ilrilr(..+-)       0.19891    0.07352  2.7055

Intercepts:
                        Value   Std. Error t value
1-7days|8-30days        -0.3467  0.3935    -0.8812
8-30days|31-90days       0.8660  0.3939     2.1987
31-90days|91+_not_evday  1.3945  0.3953     3.5277
91+_not_evday|everyday   2.6782  0.4038     6.6327

Residual Deviance: 4392.209
AIC: 4426.209
```

```r
Anova(bpd_ordinal_ilrs)
```

```
Analysis of Deviance Table (Type II tests)

Response: LBP_frequency_year
          LR Chisq Df Pr(>Chisq)
age         27.596  2  1.018e-06 ***
sex          6.822  1   0.009005 **
bmi          0.429  2   0.806925
```

```
stress      33.255  1  8.084e-09 ***
smoking      2.218  1   0.136374
education    1.294  1   0.255262
ses          6.813  2   0.033158 *
ilr         11.658  3   0.008651 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# pr <- profile(bpd_ordinal_ilrs)
# confint(pr)
# plot(pr)
# pairs(pr)


est_ci_df <- cbind(est = coef(bpd_ordinal_ilrs), confint(bpd_ordinal_ilrs)) # profiled CIs
```

```
Waiting for profiling to be done...
```

```
kable(est_ci_df, digits = 3)      # these are the log-odds scale estimates (and CI)
```

	est	2.5 %	97.5 %
age2_middle	0.339	0.134	0.545
age3_older	0.842	0.520	1.164
sex2_male	-0.288	-0.506	-0.072
bmi2_normal	-0.026	-0.704	0.675
bmi3_overweight	0.037	-0.649	0.747
stress2_stressed	0.572	0.377	0.767
smoking2_nonsmoker	-0.176	-0.406	0.056
education2_higher	-0.119	-0.325	0.086
ses2_middle	-0.346	-0.635	-0.056
ses3_higher	-0.497	-0.902	-0.094
ilrilr(++−)	-0.302	-0.505	-0.100
ilrilr(+-..)	-0.566	-0.904	-0.229
ilrilr(..+-)	0.199	0.055	0.343

```
kable(exp(est_ci_df), digits = 3) # these are the odds ratios (and approx CIs)
```

	est	2.5 %	97.5 %
age2_middle	1.404	1.144	1.725
age3_older	2.321	1.683	3.202
sex2_male	0.750	0.603	0.931
bmi2_normal	0.974	0.495	1.965
bmi3_overweight	1.038	0.523	2.111
stress2_stressed	1.771	1.458	2.152
smoking2_nonsmoker	0.839	0.666	1.057
education2_higher	0.888	0.723	1.090
ses2_middle	0.707	0.530	0.945
ses3_higher	0.608	0.406	0.910
ilrilr(++−)	0.739	0.603	0.905
ilrilr(+-..)	0.568	0.405	0.795
ilrilr(..+-)	1.220	1.057	1.410

Ordinal logistic regression has fit the model:

$$\begin{aligned}
logit(\hat{P}(Y \leq \texttt{1-7days})) &= \hat{\beta}_{0,\texttt{1-7days|8-30days}} - \hat{\beta}_1 \times (age) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)} \\
logit(\hat{P}(Y \leq \texttt{8-30days})) &= \hat{\beta}_{0,\texttt{8-30days|31-90days}} - \hat{\beta}_1 \times (age) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)} \\
logit(\hat{P}(Y \leq \texttt{31-90days})) &= \hat{\beta}_{0,\texttt{31-90days|91+_not_evday}} - \hat{\beta}_1 \times (age) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)} \\
logit(\hat{P}(Y \leq \texttt{91+_not_evday})) &= \hat{\beta}_{0,\texttt{91+_not_evday|everyday}} - \hat{\beta}_1 (age == \texttt{2_middle}) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)}
\end{aligned}$$

5.3.2 Model diagnostics

```
# deviance test
g2 <- deviance(bpd_ordinal_ilrs)
df <- df.residual(bpd_ordinal_ilrs)
1 - pchisq(g2, df)
```

```
[1] 0
```

```
with(bpd_yes,
  table(
    LBP_frequency_year,
    as.numeric(LBP_frequency_year),
    useNA = "ifany"
```

```
  )
)
```

```
LBP_frequency_year   1   2   3   4   5
     1-7days       760   0   0   0   0
     8-30days        0 451   0   0   0
     31-90days       0   0 146   0   0
     91+_not_evday   0   0   0 203   0
     everyday        0   0   0   0 100
```

```
## checking parallel slopes assumptions can be done by fitting successive logistic regress
## while creating a binary outcome using different thresholds of the ordinal outcome
### (note the rhs/linear predictor is negative so coefs should be approx same
### as main model except negative)

# e.g. this is a single logistic regression
coef(glm(
  I(as.numeric(LBP_frequency_year) <= 1) ~
    age + sex + bmi + stress + smoking + education + ses + ilr,
  family = "binomial",
  data = bpd_yes
))
```

```
       (Intercept)        age2_middle         age3_older           sex2_male
       -0.60603839        -0.25411567        -0.65013781          0.24867275
        bmi2_normal     bmi3_overweight   stress2_stressed  smoking2_nonsmoker
        0.17851384         0.11126358        -0.50892555          0.22359339
 education2_higher         ses2_middle         ses3_higher       ilrilr(++--)
        0.08669237         0.33663193         0.46257283          0.28067682
      ilrilr(+-..)       ilrilr(..+-)
        0.44988574        -0.16448339
```

```
# this is running multiple logistic regressions
## we want to see the coefficients to be roughly the same (intercepts and negative coefs -

### note that the below shows there may be reason to include an age variable that has
### non-constant coefficient for each level of the outcome (or subgroup analyses for each
### we can see this because the age coefs increase/decrease monotonically

foreach(i = 1:(length(levels(bpd_yes$LBP_frequency_year)) - 1), .combine = cbind) %do% {
```

```
  log_coefs <-
    coef(glm(
      I(as.numeric(LBP_frequency_year) <= i) ~
         age + sex + bmi + stress + smoking + education + ses + ilr,
      family = "binomial",
      data = bpd_yes
    ))
  log_coefs <- as.data.frame(log_coefs)
  colnames(log_coefs) <- paste0("logit(P(Y<=", i, "))")
  log_coefs
} %>%
  kable(., digits = 2)
```

	logit(P(Y<=1))	logit(P(Y<=2))	logit(P(Y<=3))	logit(P(Y<=4))
(Intercept)	-0.61	1.15	1.99	3.51
age2_middle	-0.25	-0.43	-0.50	-0.80
age3_older	-0.65	-0.99	-1.09	-1.69
sex2_male	0.25	0.33	0.29	0.31
bmi2_normal	0.18	0.00	-0.39	0.01
bmi3_overweight	0.11	0.01	-0.50	-0.25
stress2_stressed	-0.51	-0.65	-0.59	-0.94
smoking2_nonsmoker	0.22	0.12	0.10	-0.19
education2_higher	0.09	0.12	0.14	0.49
ses2_middle	0.34	0.32	0.35	0.31
ses3_higher	0.46	0.51	0.74	0.52
ilrilr(++–)	0.28	0.29	0.37	0.38
ilrilr(+-..)	0.45	0.69	0.71	0.54
ilrilr(..+-)	-0.16	-0.25	-0.25	-0.25

5.3.3 Model predictions

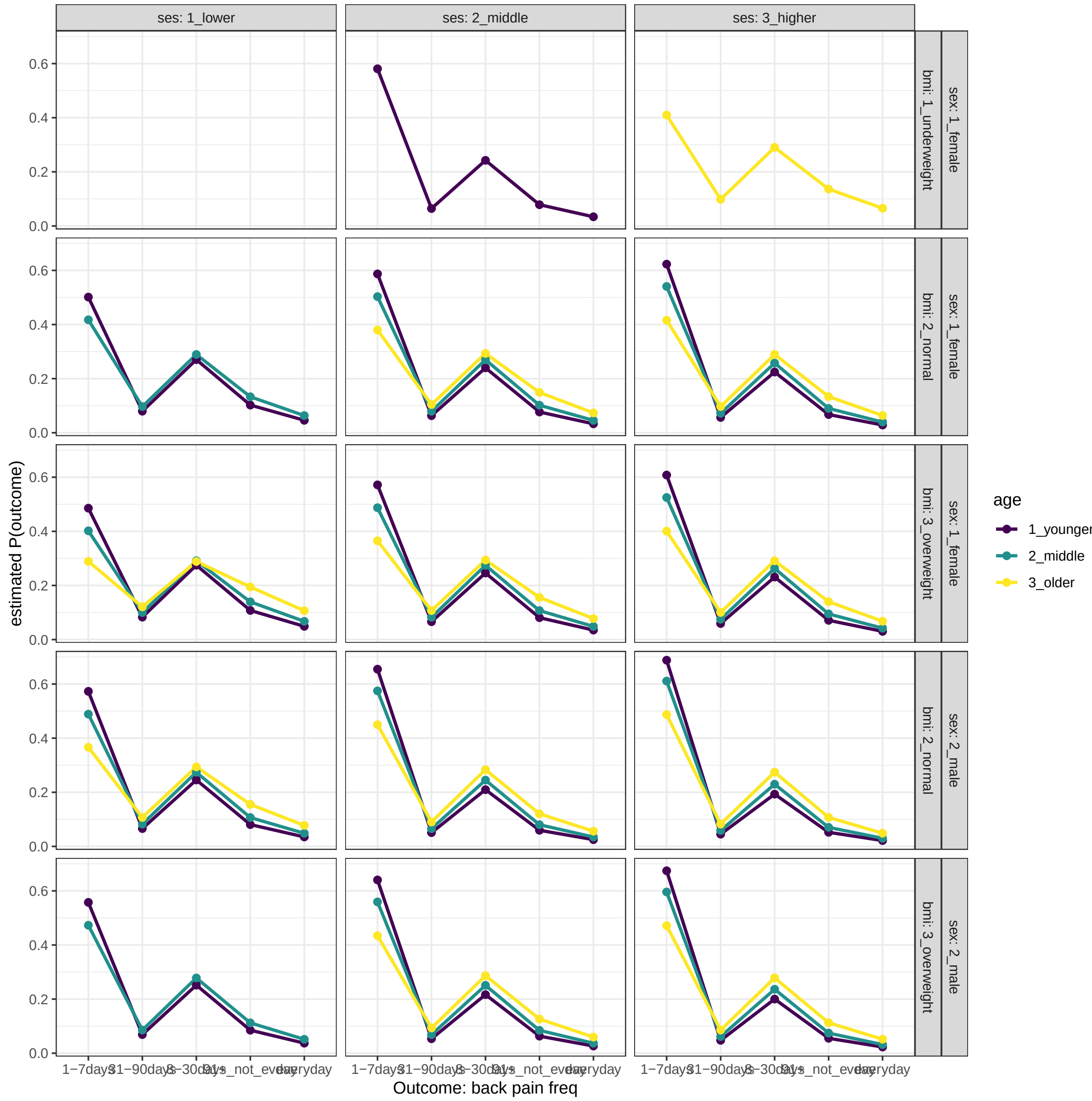
```
# create dataset for predictions
newdata <-
  bpd_yes %>%
  dplyr::select(all_of(pred_covs), ilr) %>%
  distinct(pick(all_of(pred_covs)), .keep_all = TRUE) %>%
  arrange(pick(all_of(pred_covs)))

mean_ilr <- mean(bpd_yes$ilr)
```

```r
dev_null <- foreach(i = 1:nrow(newdata)) %do% {
  newdata$ilr[i, ] <- mean_ilr
}

# make preds and then put in long format for ggplot
predictions_probs <-
  cbind(
    predict(bpd_ordinal_ilrs, newdata, type = "probs"),
    newdata
  ) %>%
  dplyr::select(-ilr) %>%
  pivot_longer(
    cols = -all_of(pred_covs),
    names_to = "outcome",
    values_to = "P(outc)"
  )

predictions_probs
```

```
# A tibble: 1,030 x 9
   age       sex      bmi       stress smoking education ses   outcome `P(outc)`
   <fct>     <chr>    <fct>     <chr>  <chr>   <chr>     <chr> <chr>       <dbl>
 1 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 1-7days    0.451
 2 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 8-30da~    0.283
 3 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 31-90d~    0.0899
 4 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 91+_no~    0.120
 5 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ everyd~    0.0558
 6 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 1-7days    0.537
 7 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 8-30da~    0.259
 8 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 31-90d~    0.0727
 9 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 91+_no~    0.0910
10 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ everyd~    0.0401
# i 1,020 more rows
```

```r
## model predictions for specific values
predictions_probs %>%
  dplyr::filter(
    # sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
```

```
  # ses == "2_middle"
) %>%
ggplot(., aes(outcome, `P(outc)`)) +
geom_line(aes(colour =  age, group =  age), linewidth = 1) +
geom_point(aes(colour =  age), size = 2) +
facet_grid(sex * bmi ~ ses, labeller = label_both) +
labs(x = "Outcome: back pain freq", y = "estimated P(outcome)") +
theme_bw() +
scale_colour_viridis_d()
```




```r
# create a RHS of regression equation dataset for time-reallocation
predict_basis <-
   bpd_yes %>%
   dplyr::select(all_of(pred_covs), all_of(pred_comps)) %>%
   dplyr::filter(
     age == "2_middle",
     sex == "1_female",
```

```r
      stress == "1_normal",
      smoking == "2_nonsmoker",
      education == "2_higher",
      ses == "2_middle",
      bmi == "2_normal"
    )

### continuous situation
# predict_basis$age <- mean(predict_basis$age)

(predict_basis <-
  predict_basis %>%
   distinct(across(all_of(pred_covs)), .keep_all = TRUE) %>%
   as.data.frame())
```

```
      age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   546.4286
  Time_Sedentary Time_LPA Time_MVPA
1       418.2857 435.1429        40
```

```r
# compositional mean: geometric mean to closure
# (comp_mean <- mean(acomp(bpd_yes[, pred_comps])))
(comp_mean <- calc_comp_mean(bpd_yes[, pred_comps], clo_val = 1440))
```

```
  Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
    472.8407       438.4062       502.1666        26.5865
```

```r
predict_basis0 <- predict_basis
predict_basis0[, pred_comps] <- comp_mean

predict_basis0
```

```
      age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
  Time_Sedentary Time_LPA Time_MVPA
1       438.4062 502.1666   26.5865
```

```r
# +15 minutes to Time_MVPA and -15 minutes from Time_Sedentary
comp_mean_changed <- comp_mean
comp_mean_changed["Time_MVPA"] <- comp_mean_changed["Time_MVPA"] + 15
comp_mean_changed["Time_Sedentary"] <- comp_mean_changed["Time_Sedentary"] - 15
# check
comp_mean_changed - comp_mean
```

```
  Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA 
           0            -15              0             15 
```

```r
predict_basis1 <- predict_basis
predict_basis1[, pred_comps] <- comp_mean_changed

pred_df <- rbind(predict_basis0, predict_basis1)
pred_df <- add_ilrs_to_data(pred_df, comp_vars = pred_comps, sbp_matrix = sbp1)
pred_df <- pred_df[, !(colnames(pred_df) %in% pred_comps)] # get rid of compositions
pred_df
```

```
       age      sex      bmi   stress     smoking education      ses      ilr.1
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle 1.37128443
2 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle 1.13019151
       ilr.2      ilr.3
1 0.05346627 2.07785318
2 0.07808340 1.76151343
```

```r
# model.matrix(formula(bpd_ordinal_ilrs), data = cbind(LBP_frequency_year = 0, pred_df))


df <- bpd_yes[, attr(formula(bpd_ordinal_ilrs), "term.labels")]
# this is a list of levels for each factor in the original df (after applying factor functi
xlevs <- lapply(df[,sapply(df, is.character), drop = F], function(j) {
  levels(factor(j))
})

# calling "xlev = " builds out a model.matrix with identical levels as the original df
mm_new <- model.matrix( ~ ., data = pred_df, xlev = xlevs)
colnames(mm_new)
```

```
 [1] "(Intercept)"        "age2_middle"        "age3_older"
```

```
 [4] "sex2_male"          "bmi2_normal"        "bmi3_overweight"
 [7] "stress2_stressed"   "smoking2_nonsmoker" "education2_higher"
[10] "ses2_middle"        "ses3_higher"        "ilr1"
[13] "ilr2"               "ilr3"
```

```
mm_new <- mm_new[, -1] # remove intercept


colnames(mm_new)[grepl("^ilr", colnames(mm_new))] <- paste0("ilr", create_ilr_names(sbp1))
# colnames(mm_new)
# don't need intercept  # c("(Intercept)" = 1, coef(bpd_ordinal_ilrs))
betas <- as.matrix(coef(bpd_ordinal_ilrs)) # should be col matrix
# rownames(betas)

if (!all(colnames(mm_new) == rownames(betas))) {
  stop("design and parameter est matrices non-conform")
}

# note as linear predictor is taken from the K intercepts the ratio of odds ratios is flip
# i.e. after:before of odds is calculated as exp(before_log_odds / after_log_odds)
preds <- mm_new %*% betas
exp(preds[1] - preds[2])
```

```
[1] 1.004017
```

```
# check manual calcs agree with model
mm_old <- model.matrix( ~ ., data = df, xlev = xlevs)
mm_old <- mm_old[, -1] # remove intercept
# colnames(mm_old)

# model and manual calcs agree?
# note that bpd_ordinal_ilrs$lp are the eta/linear predictor that is taken
# away from the xi_k intercept
all(abs(as.numeric(mm_old %*% betas) - bpd_ordinal_ilrs$lp) < 1e-9)
```

```
[1] TRUE
```

```
# bpd_ordinal_ilrs$lp # linear predictor
```

```r
get_pred_diff <- function(mod, new_dat) {
  betas_ <- as.matrix(coef(mod))
  if (!all(colnames(new_dat) == rownames(betas_))) {
    print(paste(paste(colnames(new_dat), collapse = "|"), "vs", paste(rownames(betas_), co
    stop("design and parameter est matrices non-conform")
  }
  log_odds_pred <- as.numeric(new_dat %*% betas_)
  # note reversal of order (see above)
  odds_ratio_ratio <- exp(log_odds_pred[1] - log_odds_pred[2])
  return(odds_ratio_ratio)
}
(est_v1 <- get_pred_diff(bpd_ordinal_ilrs, mm_new))
```

```
[1] 1.004017
```

```r
fit_mod_boot <- function(data, i, pred_dat) {

  this_dat <- data[i, ]
  this_ordinal <- polr(mod_form_ilrs, data = this_dat, Hess = TRUE, method = "logistic")

  df <- this_dat[, attr(formula(this_ordinal), "term.labels")]
  # this is a list of levels for each factor in the original df (after applying factor fun
  xlevs <- lapply(df[,sapply(df, is.character), drop = F], function(j) {
    levels(factor(j))
  })


  # calling "xlev = " builds out a model.matrix with identical levels as the original df
  mm_new <- model.matrix( ~ ., data = pred_dat, xlev = xlevs)
  mm_new <- mm_new[, -1] # remove intercept
  # make sure ilr colnames are legit/match coeffs
  colnames(mm_new)[grepl("^ilr", colnames(mm_new))] <- paste0("ilr", create_ilr_names(sbp1
  colnames(mm_new)

  est <- get_pred_diff(this_ordinal, new_dat = mm_new)
  return(est)

}

### CI method #1 (bootstrapping):
alpha <- 0.05
```

```r
(ci_v1 <-
    c(
      est = est_v1,
      quantile(
        boot(bpd_yes, fit_mod_boot, R = 100, pred_dat = pred_df)$t,
        c(alpha / 2, 1 - alpha / 2)
      )))
```

```
      est      2.5%     97.5% 
1.0040167 0.9732311 1.0473858 
```

```r
### alternative CI method #2 (Wald approximation - re-transformed):
pred_df[, "ilr"]
```

```
         [,1]       [,2]     [,3]
[1,] 1.371284 0.05346627 2.077853
[2,] 1.130192 0.07808340 1.761513
attr(,"class")
[1] "rmult"
```

```r
diff(pred_df[, "ilr"])
```

```
           [,1]       [,2]       [,3]
[1,] -0.2410929 0.02461713 -0.3163398
attr(,"class")
[1] "rmult"
```

```r
# x_0_red <- matrix(- as.numeric(diff(pred_df[, "ilr"])), nrow = 1)
x_0_red <- matrix(as.numeric(pred_df[1, "ilr"] - pred_df[2, "ilr"]), nrow = 1)
x_0_red
```

```
          [,1]        [,2]      [,3]
[1,] 0.2410929 -0.02461713 0.3163398
```

```r
betas <- coef(bpd_ordinal_ilrs)
nms_kp <- grepl("^ilr", names(betas))
```

```r
betas_red <- as.matrix(betas[nms_kp])
Sigma <- stats::vcov(bpd_ordinal_ilrs)
nms_kp <- grepl("^ilr", colnames(Sigma))
sigma_red <- Sigma[nms_kp, nms_kp]
sigma_red
```

```
             ilrilr(++--) ilrilr(+-..) ilrilr(..+-)
ilrilr(++--)   0.01065435   0.01320158  -0.00568300
ilrilr(+-..)   0.01320158   0.02957295  -0.00789973
ilrilr(..+-)  -0.00568300  -0.00789973   0.00540513
```

```r
est_red <- x_0_red %*% betas_red
se_red <- sqrt(x_0_red %*% sigma_red %*% t(x_0_red))
z_star <- qnorm(0.975)
(ci_v2 <-
    exp(c(
      est = est_red,
      lo = est_red - z_star * se_red,
      hi = est_red + z_star * se_red
    )))
```

```
      est        lo        hi
1.0040167 0.9717601 1.0373441
```

```r
### alternative CI method #3 (delta method)
# (first order approximation, although still linear combin of param ests):
as.numeric(x_0_red)
```

```
[1]  0.24109292 -0.02461713  0.31633975
```

```r
(g_form <- paste(
  paste(
    as.numeric(x_0_red),
    "*",
    c("`ilrilr(++--)`", "`ilrilr(+-..)`", "`ilrilr(..+-)`")
  ),
  collapse = " + "
))
```

```
[1] "0.241092921978823 * `ilrilr(++--)` + -0.0246171278023131 * `ilrilr(+-..)` + 0.316339752
```

```
approx_ci <-deltaMethod(bpd_ordinal_ilrs, g_form)
(ci_v3 <-
    exp(c(
      est = approx_ci[["Estimate"]],
      lo = approx_ci[["2.5 %"]],
      hi = approx_ci[["97.5 %"]]
    )))
```

```
      est        lo        hi
1.0040167 0.9717601 1.0373441
```

```
### compare CIs
kable(rbind(ci_v1, ci_v2, ci_v3))
```

	est	2.5%	97.5%
ci_v1	1.004017	0.9732311	1.047386
ci_v2	1.004017	0.9717601	1.037344
ci_v3	1.004017	0.9717601	1.037344

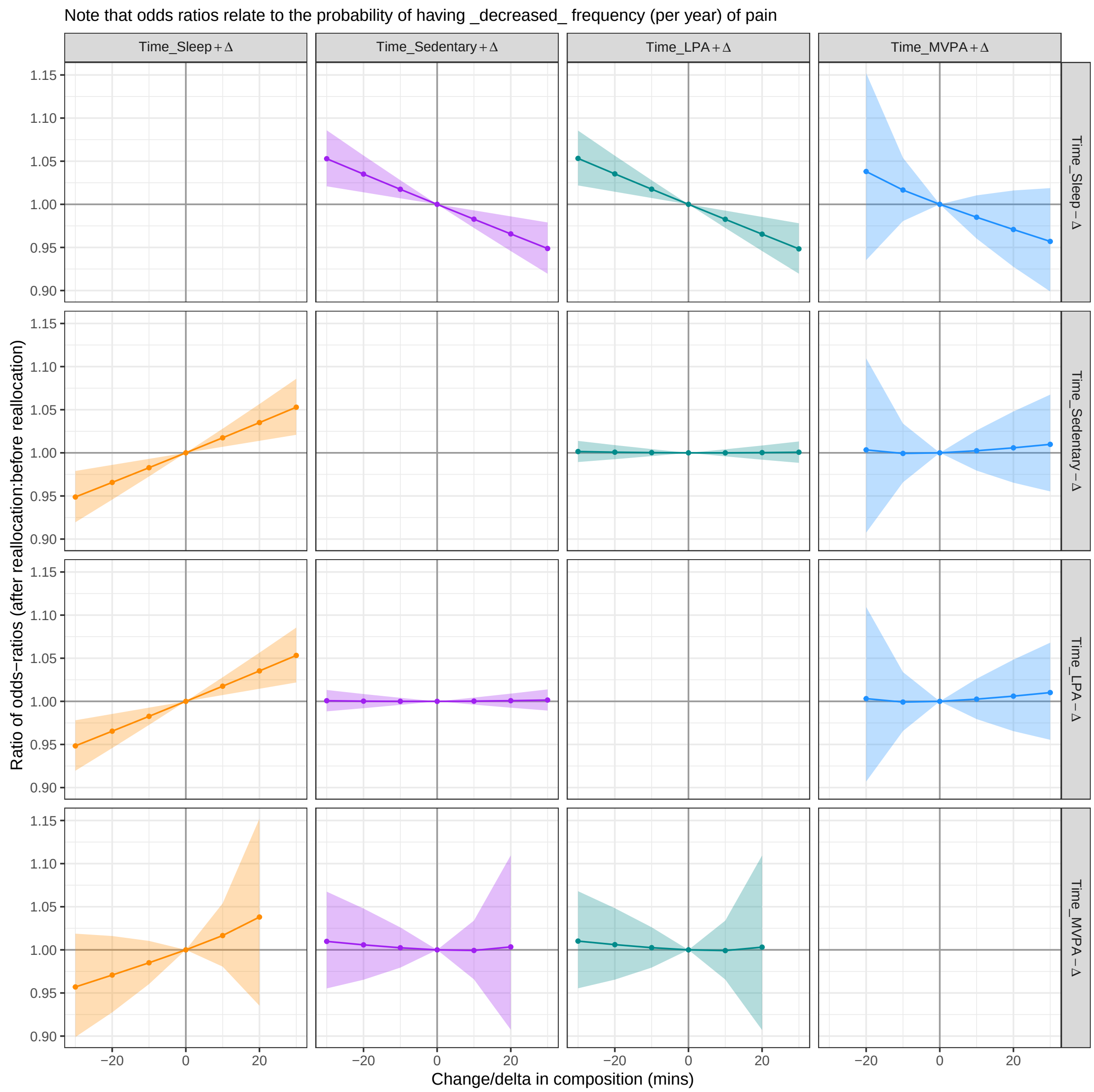
```
do_multi_realloc <- function(mod, basis_data, timeusenames, time_changes, sbp_matrix = sbp


  x0 <- basis_data

  plot_dat <-
    foreach(i = 1:length(timeusenames), .combine = bind_rows) %do% {
      print(paste("i: ", i))
      foreach(j = 1:length(timeusenames), .combine = bind_rows) %do% {
        print(paste("  j: ", j))
        foreach(d = 1:length(time_changes), .combine = bind_rows) %do% { # %dopar%
          print(paste("    d: ", d))

          timeuse_to <- timeusenames[i]
          timeuse_from <- timeusenames[j]
          change_time <- time_changes[d]

          proposed_change_1 <- x0[timeuse_to] + change_time
```

```
proposed_change_2 <- x0[timeuse_from] - change_time

if (timeuse_to == timeuse_from) {
  NULL # reallocation exceeds 0 or max time
} else if ((proposed_change_1 < 0) | (proposed_change_1 > 1440)) {
  NULL # reallocation exceeds 0 or max time
} else if ((proposed_change_2 < 0) | (proposed_change_2 > 1440)) {
  NULL # reallocation exceeds 0 or max time
} else {

  x1 <- x0
  x1[timeuse_to] <- x1[timeuse_to] + change_time
  x1[timeuse_from] <- x1[timeuse_from] - change_time

  pred_df <- rbind(x0, x1)
  pred_df <- add_ilrs_to_data(pred_df, comp_vars = timeusenames, sbp_matrix = sb

  ### alternative CI method #3 (delta method)
  # x_0_red <- -as.numeric(diff(pred_df[, "ilr"]))
  x_0_red <- as.numeric(pred_df[1, "ilr"] - pred_df[2, "ilr"])
  # (first order approximation, although still linear combin of param ests):
  (g_form <- paste(
    paste(
      x_0_red,
      "*",
      c("`ilrilr(++--)`", "`ilrilr(+-..)`", "`ilrilr(..+-)`")
    ),
    collapse = " + "
  ))
  approx_ci <-deltaMethod(bpd_ordinal_ilrs, g_form)
  this_ci <-
      exp(c(
        est = approx_ci[["Estimate"]],
        lo = approx_ci[["2.5 %"]],
        hi = approx_ci[["97.5 %"]]
      ))


  ### bootstrapping takes too long
  # pred_df <- pred_df[, !(colnames(pred_df) %in% timeusenames)] # get rid of co
  #
```

```
            # df <- bpd_yes[, attr(formula(bpd_ordinal_ilrs), "term.labels")]
            # # this is a list of levels for each factor in the original df (after applyin
            # xlevs <- lapply(df[,sapply(df, is.character), drop = F], function(j) {
            #   levels(factor(j))
            # })
            #
            # # calling "xlev = " builds out a model.matrix with identical levels as the o
            # mm_new <- model.matrix( ~ ., data = pred_df, xlev = xlevs)
            # mm_new <- mm_new[, -1] # remove intercept
            # # make sure ilr colnames are legit/match coeffs
            # colnames(mm_new)[grepl("^ilr", colnames(mm_new))] <- paste0("ilr", create_il
            #
            # ratio_of_odds_ratios <- get_pred_diff(mod, new_dat = mm_new)
            # bootstrapped_ests <- boot(bpd_yes, fit_mod_boot, R = 10, pred_dat = pred_df)
            # ci_est <- quantile(as.numeric(bootstrapped_ests), c(alpha / 2, 1 - alpha / 2

            tibble(
              to = timeuse_to,
              from = timeuse_from,
              change_time = change_time,
              ratio_of_odds_ratios = this_ci["est"],
              ci_lo = this_ci["lo"],
              ci_hi = this_ci["hi"]
            )
          }
        }
      }
    }

  plot_dat$to <- factor(plot_dat$to, levels = timeusenames)
  plot_dat$from <- factor(plot_dat$from, levels = timeusenames)

  return(plot_dat)

}


# takes ~ 3h (single core) for bootstrapping
# takes ~ 4sec (single core) for delta/wald method
```

```r
### Uncomment to generate bootstrapping

# tic()
# set.seed(1234)

  # # library("doParallel")
  # # no_cores <- detectCores() - 1 # Calculate the number of cores (leave one free)
  # # cl <- makeCluster(no_cores) # Create clusters
  # # registerDoParallel(cl) # and register

# realloc_plot_data <-
#   do_multi_realloc(
#     bpd_ordinal_ilrs,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )

  # # # close para comp
  # # stopCluster(cl)

# saveRDS(realloc_plot_data, file = "res/ordinal_realloc_wald_res.rda")
# toc()

# saveRDS(realloc_plot_data, file = "res/ordinal_realloc_boot_res.rda")
# realloc_plot_data <- readRDS(file = "res/ordinal_realloc_boot_res.rda")


realloc_plot_data <- readRDS(file = "res/ordinal_realloc_wald_res.rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
  geom_line(aes(x = change_time , y = ratio_of_odds_ratios, col = to)) +
  geom_point(aes(x = change_time , y = ratio_of_odds_ratios, col = to), size = 1) +
  facet_grid(from ~ to, labeller = label_parsed) +
```

```
theme_bw() +
scale_colour_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
scale_fill_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
labs(
  x = paste0("Change/delta in composition (mins)"),
  y = paste0("Ratio of odds-ratios (after reallocation:before reallocation)"),
  subtitle = "Note that odds ratios relate to the probability of having _decreased_ freq
) +
theme(legend.position = "none")
```




```
ggsave(
  filename = "fig/lbp_freq_ordinal_odds_v1.png",
  dpi = 600, # print quality
  width = 10,
  height = 10
)
```

```r
time_lvls <- gsub("Time_", "", pred_comps)


rep_char <- function(n, char = " ") paste(rep(char, n), collapse = "")
rep_char(3)
```

```
[1] "   "
```

```r
rep_char(0)
```

```
[1] ""
```

```r
rep_char <- Vectorize(rep_char, vectorize.args = "n")
rep_char(0:7)
```

```
[1] ""        " "       "  "      "   "     "    "    "     "   "      " 
[8] "       "
```

```r
pd2 <-
  realloc_plot_data %>%
  mutate(
    to = gsub("Time_", "", to),
    from = gsub("Time_", "", from),
    to = gsub("+Delta", "", to, fixed = TRUE),
    from = gsub("-Delta", "", from, fixed = TRUE),
    to_len = nchar(to),
    to_max = max(to_len),
    from_len = nchar(from),
    from_max = max(from_len),
    to_pad =  rep_char(pmax(0, from_max - to_len)),
    from_pad = rep_char(pmax(0, to_max - from_len)),
    to = factor(to, levels = time_lvls),
    from = factor(from, levels = time_lvls),
    to_num = as.numeric(to),
```

```r
    from_num = as.numeric(from)
  ) %>%
  dplyr::filter(to_num > from_num) %>%
  mutate(
    ratio_of_odds_ratios = 1 / ratio_of_odds_ratios,
    tmp = 1 / ci_lo,
    ci_lo = 1 / ci_hi,
    ci_hi = tmp,
    # from_to = paste0("      ", "+", from, rep_char(10), from_pad, "\u2194", to_pad, rep_c
    from_to = paste0("+", from, rep_char(13), from_pad, "", to_pad, rep_char(13), "+", to)
  ) %>%
  arrange(from, to)

unique(pd2$from_to)
```

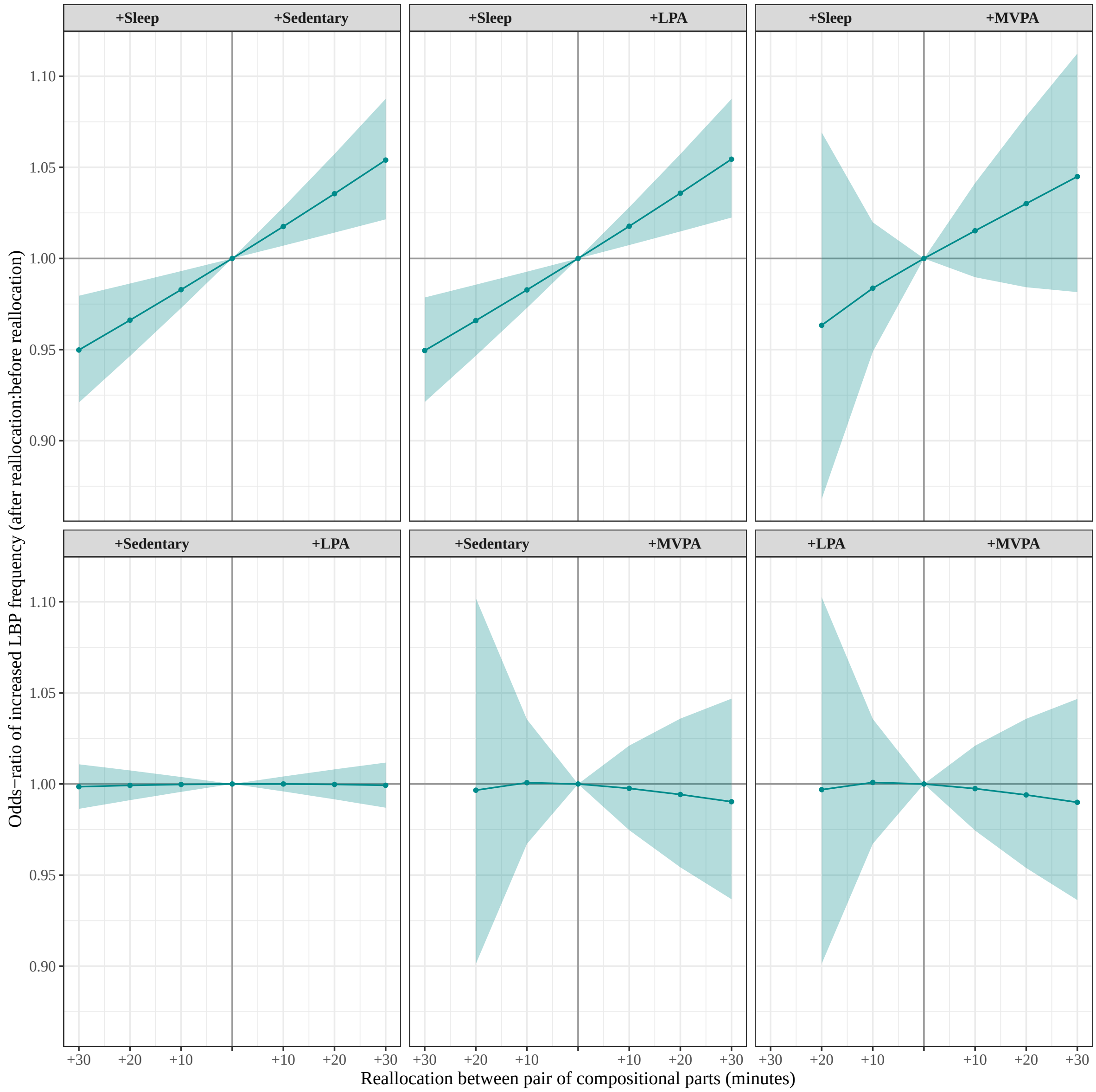
```
[1] "+Sleep                              +Sedentary"
[2] "+Sleep                                    +LPA"
[3] "+Sleep                                   +MVPA"
[4] "+Sedentary                                +LPA"
[5] "+Sedentary                               +MVPA"
[6] "+LPA                                     +MVPA"
```

```r
pd2$from_to <- factor(pd2$from_to, levels = unique(pd2$from_to))

this_breaks <- seq(-30, 30, 10)
this_labs <- sprintf("+%2.0f", abs(seq(-30, 30, 10)))
this_labs[this_labs == "+ 0"] <- ""
this_labs
```

```
[1] "+30" "+20" "+10" ""    "+10" "+20" "+30"
```

```r
ggplot(pd2) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3, co
  geom_line(aes(x = change_time , y = ratio_of_odds_ratios, col = to), col = "cyan4") +
  geom_point(aes(x = change_time , y = ratio_of_odds_ratios, col = to), size = 1, col = "c
  facet_wrap(~ from_to, labeller = label_bquote(.(from_to))) +
  theme_bw() +
```

```
scale_x_continuous(breaks = this_breaks, labels = this_labs) +
labs(
  x = paste0("Reallocation between pair of compositional parts (minutes)"),
  y = paste0("Odds-ratio of increased LBP frequency (after reallocation:before reallocat
  # subtitle = "Note that odds ratios relate to the probability of having _increased_ fr
) +
theme(
  legend.position = "none",
  text = element_text(family = "serif"),
  strip.text = element_text(size = 10, face = "bold"),
  axis.text =  element_text(size = 10),
  axis.title =  element_text(size = 12)
)
```




```
ggsave(filename = "fig/lbp_freq_ordinal_odds_v2.png", width = 14, height = 9, dpi = 600)
# ggsave(filename = "fig/lbp_freq_ordinal_odds.pdf", width = 10, height = 8)
```

```
# ---- outcome1_pred_not_use ----
```

```r
# logitP(Y k x)= _k-
# zeta_{1-7days|8-30days} = -0.2910
# eta = 0.3184 + -0.1786 + -0.1110 + -0.3463 + -0.3017
coef(bpd_ordinal_ilrs)
```

```
     age2_middle         age3_older           sex2_male          bmi2_normal
      0.33918326         0.84203588         -0.28828961          -0.02594625
 bmi3_overweight  stress2_stressed smoking2_nonsmoker  education2_higher
      0.03744251         0.57164263         -0.17602164          -0.11931436
     ses2_middle         ses3_higher         ilrilr(++--)         ilrilr(+-..)
     -0.34624303        -0.49685106         -0.30210820          -0.56557283
    ilrilr(..+-)
      0.19890657
```

```r
# summary(bpd_ordinal_ilrs)
bpd_ordinal_ilrs$zeta
```

```
     1-7days|8-30days      8-30days|31-90days 31-90days|91+_not_evday
           -0.3467297               0.8659799               1.3944591
91+_not_evday|everyday
            2.6782000
```

```r
# bpd_ordinal_ilrs$lp
# p_0 <- predict(bpd_ordinal_ilrs, pred_df, type = "prob")
# (lodr <- log(p_0 / (1-p_0)))
# # ratio of odds ratios
# exp(apply(lodr, 2, diff))
# # predicted class  argmin_k{abs(zeta_k - eta)}?
# predict(bpd_ordinal_ilrs, type = "class")[1:3]
# p_m <- matrix(rep(bpd_ordinal_ilrs$lp, 5), ncol = 5)
# co_m <- matrix(rep(c(bpd_ordinal_ilrs$zeta, 0), nrow(p_m)), ncol = 5, byrow = TRUE)
# apply(abs(p_m - co_m), 1, which.min)[1:3]
#
# table(
#   predict(bpd_ordinal_ilrs, type = "class"),
#   apply(abs(p_m - co_m), 1, which.min) # + 1) %% 5
# )
```

5.4 Outcome 2: LBP_intensity_year

5.4.1 Model fit

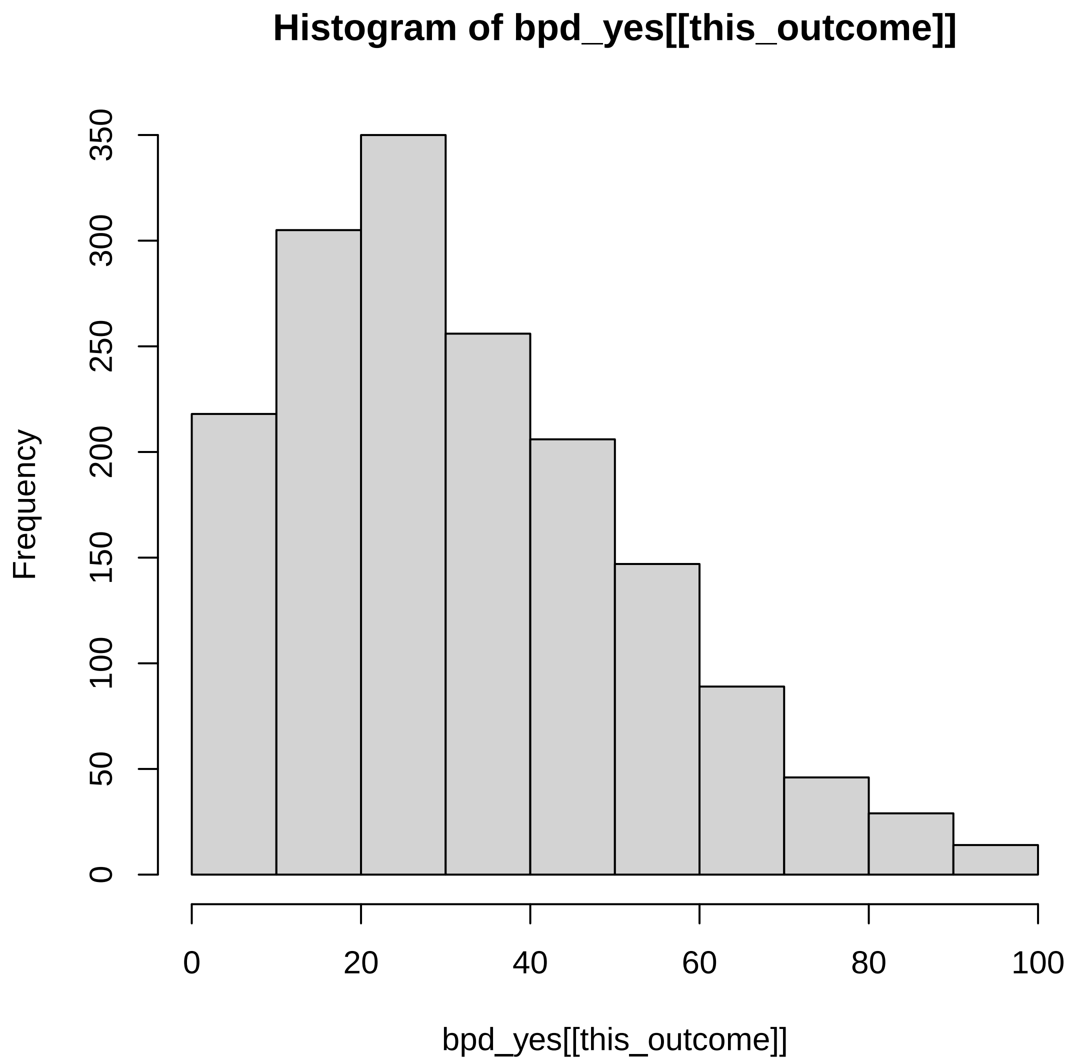
```r
(this_outcome <- outcs[2])
```

```
[1] "LBP_intensity_year"
```

```r
# (mod_form_null <-as.formula(paste0(this_outcome, " ~ ", rhs_formula)))
(mod_form_ilrs <- as.formula(paste0(this_outcome, " ~ ", rhs_formula, " + ilr")))
```

```
LBP_intensity_year ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```r
hist(bpd_yes[[this_outcome]])
```




```r
lbp_intensity_lm <- lm(mod_form_ilrs, data = bpd_yes)
summary(lbp_intensity_lm)
```

```
Call:
lm(formula = mod_form_ilrs, data = bpd_yes)

Residuals:
   Min      1Q Median      3Q     Max
```

```
-42.94 -14.78  -3.03  12.16  69.47

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)
(Intercept)         40.3475     4.3139   9.353  < 2e-16 ***
age2_middle          0.5727     1.1059   0.518 0.604605
age3_older           7.2422     1.7224   4.205 2.75e-05 ***
sex2_male           -3.5483     1.1587  -3.062 0.002231 **
bmi2_normal         -0.5308     3.8464  -0.138 0.890251
bmi3_overweight      2.4503     3.8931   0.629 0.529177
stress2_stressed     4.4659     1.0541   4.237 2.39e-05 ***
smoking2_nonsmoker   1.3548     1.2786   1.060 0.289471
education2_higher   -3.2982     1.1148  -2.959 0.003134 **
ses2_middle         -4.2698     1.6008  -2.667 0.007722 **
ses3_higher         -7.4351     2.1993  -3.381 0.000740 ***
ilrilr(++--)        -2.0302     1.0869  -1.868 0.061949 .
ilrilr(+-..)        -6.9931     1.7974  -3.891 0.000104 ***
ilrilr(..+-)        -0.4219     0.7671  -0.550 0.582430
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 19.81 on 1646 degrees of freedom
Multiple R-squared:  0.06251,   Adjusted R-squared:  0.0551
F-statistic: 8.442 on 13 and 1646 DF,  p-value: < 2.2e-16
```

```r
car::Anova(lbp_intensity_lm)
```

```
Warning in printHypothesis(L, rhs, names(b)): one or more coefficients in the hypothesis incl
     arithmetic operators in their names;
  the printed representation of the hypothesis will be omitted

Anova Table (Type II tests)

Response: LBP_intensity_year
          Sum Sq   Df F value    Pr(>F)
age         7709    2  9.8229 5.744e-05 ***
sex         3680    1  9.3784  0.002231 **
bmi         3279    2  4.1782  0.015489 *
stress      7043    1 17.9495 2.394e-05 ***
smoking      441    1  1.1228  0.289471
education   3435    1  8.7534  0.003134 **
```

```
ses          4605    2  5.8679  0.002888 **
ilr         11652    3  9.8986 1.810e-06 ***
Residuals 645859 1646
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
### THis is logodds transform of outcome, not a good fit
# # move extreme values off boundary
# bpd_yes$intensity <- bpd_yes$LBP_intensity_year
# bpd_yes$intensity[bpd_yes$LBP_intensity_year < 0.5] <- 0.5
# bpd_yes$intensity[bpd_yes$LBP_intensity_year > (100 - 0.5)] <- 100 - 0.5
# bpd_yes$logodds_intensity <- with(bpd_yes, log((intensity / 100) / (1 - intensity / 100)
# bpd_yes$intensity <- NULL
# (mod_form_logodds_ilrs <- as.formula(paste0("logodds_intensity ~ ", rhs_formula, " + ilr
# lbp_intensity_logodds_lm <- lm(mod_form_logodds_ilrs, data = bpd_yes)
# summary(lbp_intensity_logodds_lm)
# car::Anova(lbp_intensity_logodds_lm)
# check_model(lbp_intensity_logodds_lm)

lbp_intensity_pois <- glm(mod_form_ilrs, family = "poisson", data = bpd_yes)
summary(lbp_intensity_pois)
```

```

Call:
glm(formula = mod_form_ilrs, family = "poisson", data = bpd_yes)

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         3.693194   0.037276  99.077  < 2e-16 ***
age2_middle         0.016658   0.009724   1.713 0.086684 .
age3_older          0.205646   0.014486  14.197  < 2e-16 ***
sex2_male          -0.105547   0.010228 -10.319  < 2e-16 ***
bmi2_normal        -0.017978   0.033512  -0.536 0.591644
bmi3_overweight     0.069579   0.033867   2.054 0.039929 *
stress2_stressed    0.129912   0.009082  14.305  < 2e-16 ***
smoking2_nonsmoker  0.040861   0.011211   3.645 0.000268 ***
education2_higher  -0.095585   0.009529 -10.031  < 2e-16 ***
ses2_middle        -0.112664   0.013084  -8.611  < 2e-16 ***
ses3_higher        -0.209084   0.019007 -11.000  < 2e-16 ***
ilrilr(++--)       -0.057087   0.009171  -6.225 4.83e-10 ***
ilrilr(+-..)       -0.202889   0.015413 -13.164  < 2e-16 ***
ilrilr(..+-)       -0.014136   0.006519  -2.168 0.030126 *
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 21235  on 1659  degrees of freedom
Residual deviance: 19981  on 1646  degrees of freedom
AIC: 28412

Number of Fisher Scoring iterations: 5
```

```
# check the goodness of fit test not significant,
# p > 0.05: indicates model fit the data
# p < 0.05: indicates model DOES NOIT fit the data
with(
  lbp_intensity_pois,
  cbind(
    res.deviance = deviance,
    df = df.residual,
    p = pchisq(deviance, df.residual, lower.tail = FALSE)
  )
)
```

```
     res.deviance   df p
[1,]     19981.33 1646 0
```

```
lbp_intensity_nb <- glm.nb(mod_form_ilrs, data = bpd_yes)
summary(lbp_intensity_nb)
```

```
Call:
glm.nb(formula = mod_form_ilrs, data = bpd_yes, init.theta = 2.48631581,
    link = log)

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         3.679463   0.143204  25.694  < 2e-16 ***
age2_middle         0.016056   0.036738   0.437 0.662083
age3_older          0.200337   0.057057   3.511 0.000446 ***
sex2_male          -0.104393   0.038509  -2.711 0.006710 **
bmi2_normal        -0.003187   0.127766  -0.025 0.980099
```

```
bmi3_overweight     0.086268   0.129300   0.667 0.504650
stress2_stressed    0.133104   0.034966   3.807 0.000141 ***
smoking2_nonsmoker  0.042290   0.042475   0.996 0.319415
education2_higher  -0.099687   0.036963  -2.697 0.006998 **
ses2_middle        -0.105312   0.052931  -1.990 0.046636 *
ses3_higher        -0.207810   0.072984  -2.847 0.004409 **
ilrilr(++--)       -0.053990   0.036021  -1.499 0.133915
ilrilr(+-..)       -0.199774   0.059664  -3.348 0.000813 ***
ilrilr(..+-)       -0.019407   0.025432  -0.763 0.445391
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Negative Binomial(2.4863) family taken to be 1)

    Null deviance: 1983.2  on 1659  degrees of freedom
Residual deviance: 1897.9  on 1646  degrees of freedom
AIC: 14547

Number of Fisher Scoring iterations: 1

              Theta:  2.4863
          Std. Err.:  0.0942

 2 x log-likelihood:  -14517.0260
```

```r
car::Anova(lbp_intensity_nb)
```

```
Analysis of Deviance Table (Type II tests)

Response: LBP_intensity_year
          LR Chisq Df Pr(>Chisq)
age        14.0426  2  0.0008927 ***
sex         7.2582  1  0.0070576 **
bmi         6.8347  2  0.0327987 *
stress     14.4891  1  0.0001410 ***
smoking     0.9800  1  0.3221987
education   7.2378  1  0.0071383 **
ses         8.0144  2  0.0181840 *
ilr        23.4695  3  3.223e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
(est <- cbind(Estimate = coef(lbp_intensity_nb), confint(lbp_intensity_nb)))
```

```
Waiting for profiling to be done...
```

```
                      Estimate       2.5 %       97.5 %
(Intercept)        3.679462775  3.40299626  3.969011870
age2_middle        0.016055946 -0.05598109  0.087856035
age3_older         0.200337071  0.08850561  0.313532033
sex2_male         -0.104393094 -0.17957684 -0.028571870
bmi2_normal       -0.003187056 -0.26156507  0.239011069
bmi3_overweight    0.086268083 -0.17496158  0.331713392
stress2_stressed   0.133103716  0.06446472  0.201989018
smoking2_nonsmoker 0.042290260 -0.04175439  0.125095866
education2_higher -0.099687445 -0.17285762 -0.026981253
ses2_middle       -0.105311819 -0.21064524 -0.001965261
ses3_higher       -0.207809938 -0.35179957 -0.063592059
ilrilr(++--)      -0.053989622 -0.12392266  0.015888706
ilrilr(+-..)      -0.199773802 -0.31581509 -0.083907349
ilrilr(..+-)      -0.019407373 -0.06865999  0.029729611
```

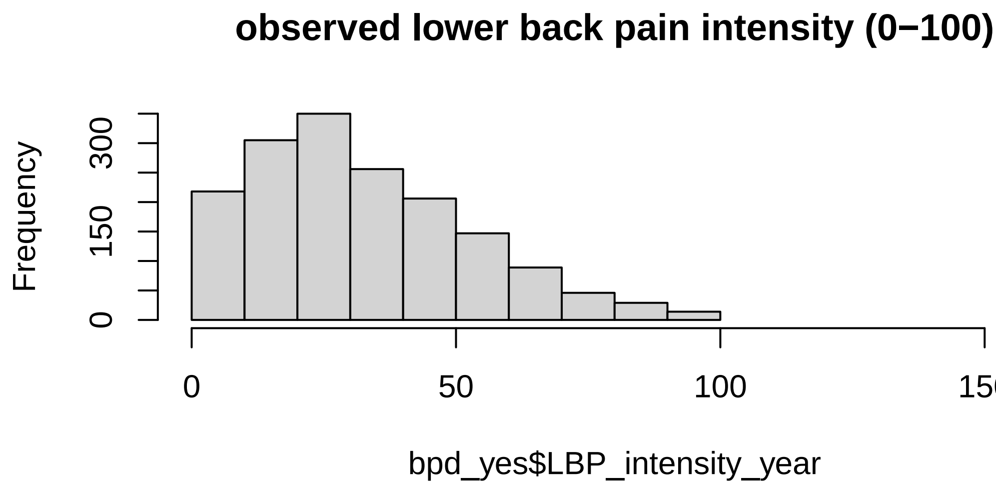
```r
exp(est)
```

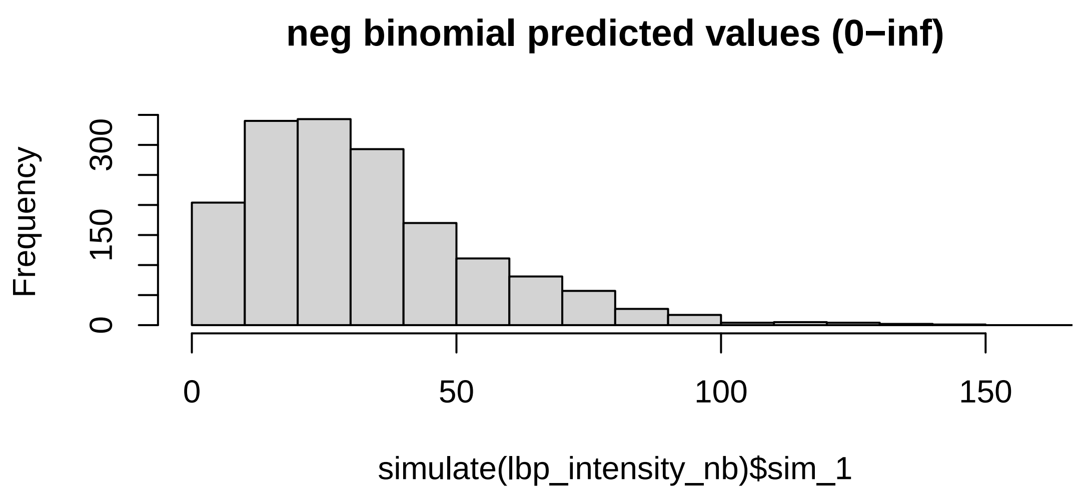
```
                    Estimate      2.5 %     97.5 %
(Intercept)       39.6251008 30.0540149 52.9322011
age2_middle        1.0161855  0.9455570  1.0918309
age3_older         1.2218145  1.0925404  1.3682493
sex2_male          0.9008711  0.8356237  0.9718324
bmi2_normal        0.9968180  0.7698458  1.2699926
bmi3_overweight    1.0900985  0.8394893  1.3933534
stress2_stressed   1.1423685  1.0665879  1.2238346
smoking2_nonsmoker 1.0431972  0.9591053  1.1332571
education2_higher  0.9051203  0.8412574  0.9733795
ses2_middle        0.9000438  0.8100614  0.9980367
ses3_higher        0.8123614  0.7034211  0.9383877
ilrilr(++--)       0.9474419  0.8834482  1.0160156
ilrilr(+-..)       0.8189160  0.7291943  0.9195164
ilrilr(..+-)       0.9807797  0.9336441  1.0301759
```

```r
# likelihood ratio test
# dispersion parameter check is it equal to zero (if not, NB mod preferred)
pchisq(
  2 * (logLik(lbp_intensity_nb) - logLik(lbp_intensity_pois)),
  df = 1,
  lower.tail = FALSE
)
```

```
'log Lik.' 0 (df=15)
```

```r
par(mfrow = c(2, 1))
hist(bpd_yes$LBP_intensity_year, xlim = c(0, 160), breaks = 10,
     main = "observed lower back pain intensity (0-100)")
hist(simulate(lbp_intensity_nb)$sim_1, xlim = c(0, 160), breaks = 20,
     main = "neg binomial predicted values (0-inf)")
```






```
par(mfrow = c(1, 1))


# Pain intensity could be categorised as (Boonstra et al., 2014):
# - no pain (0)
# - mild pain (1-38)
# - moderate pain (39-57)
# - severe pain (58-100)
```

```r
bpd_yes$intens_ord <-
  cut(
    bpd_yes[[this_outcome]],
    breaks = c(-1, 0, 38, 57, 101),
    labels = c(
      "no pain (0)", "mild pain (1-38)",
      "moderate pain (39-57)", "severe pain (58-100)"
    )
  )
class(bpd_yes$intens_ord)
```

```
[1] "factor"
```

```r
table(
  bpd_yes$intens_ord,
  cut(bpd_yes[[this_outcome]], breaks = c(-1, 0, 38, 57, 101)),
  useNA = "ifany"
)
```

```
                        (-1,0] (0,38] (38,57] (57,101]
  no pain (0)               45      0       0        0
  mild pain (1-38)           0    974       0        0
  moderate pain (39-57)      0      0     401        0
  severe pain (58-100)       0      0       0      240
```

```r
(mod_form_ord_ilrs <-as.formula(paste0("intens_ord ~ ", rhs_formula, " + ilr")))
```

```
intens_ord ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```r
## model __with__ ilrs
bpd_intens_ord_ilrs <- polr(mod_form_ord_ilrs, data = bpd_yes, Hess = TRUE, method = "logi
summary(bpd_intens_ord_ilrs)
```

```
Call:
polr(formula = mod_form_ord_ilrs, data = bpd_yes, Hess = TRUE,
    method = "logistic")
```

```
Coefficients:
                      Value Std. Error t value
age2_middle         0.05704    0.11183  0.5100
age3_older          0.60483    0.16942  3.5700
sex2_male          -0.41189    0.11909 -3.4585
bmi2_normal         0.14649    0.38765  0.3779
bmi3_overweight     0.47744    0.39207  1.2177
stress2_stressed    0.43625    0.10534  4.1416
smoking2_nonsmoker  0.15893    0.12884  1.2335
education2_higher  -0.29335    0.11024 -2.6612
ses2_middle        -0.43164    0.15546 -2.7766
ses3_higher        -0.79524    0.22275 -3.5701
ilrilr(++--)       -0.14674    0.10909 -1.3451
ilrilr(+-..)       -0.60042    0.18119 -3.3138
ilrilr(..+-)       -0.04818    0.07715 -0.6245

Intercepts:
                                           Value   Std. Error t value
no pain (0)|mild pain (1-38)               -4.0583  0.4556    -8.9078
mild pain (1-38)|moderate pain (39-57)      0.1189  0.4312     0.2757
moderate pain (39-57)|severe pain (58-100)  1.4918  0.4332     3.4438

Residual Deviance: 3330.714
AIC: 3362.714
```

```r
Anova(bpd_intens_ord_ilrs)
```

```
Analysis of Deviance Table (Type II tests)

Response: intens_ord
          LR Chisq Df Pr(>Chisq)
age        13.8442  2  0.0009858 ***
sex        12.1816  1  0.0004826 ***
bmi        10.7801  2  0.0045617 **
stress     17.1554  1  3.444e-05 ***
smoking     1.5339  1  0.2155292
education   7.0539  1  0.0079094 **
ses        13.2177  2  0.0013484 **
ilr        22.0152  3  6.476e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# profiled CIs
est_ci_df <- cbind(est = coef(bpd_intens_ord_ilrs), confint(bpd_intens_ord_ilrs))
```

```
Waiting for profiling to be done...
```

```r
kable(est_ci_df, digits = 3)      # these are the log-odds scale estimates (and CI)
```

	est	2.5 %	97.5 %
age2_middle	0.057	-0.162	0.277
age3_older	0.605	0.272	0.937
sex2_male	-0.412	-0.647	-0.180
bmi2_normal	0.146	-0.595	0.932
bmi3_overweight	0.477	-0.273	1.271
stress2_stressed	0.436	0.230	0.643
smoking2_nonsmoker	0.159	-0.092	0.413
education2_higher	-0.293	-0.509	-0.077
ses2_middle	-0.432	-0.736	-0.126
ses3_higher	-0.795	-1.234	-0.360
ilrilr(++–)	-0.147	-0.361	0.067
ilrilr(+-..)	-0.600	-0.957	-0.246
ilrilr(..+-)	-0.048	-0.199	0.103

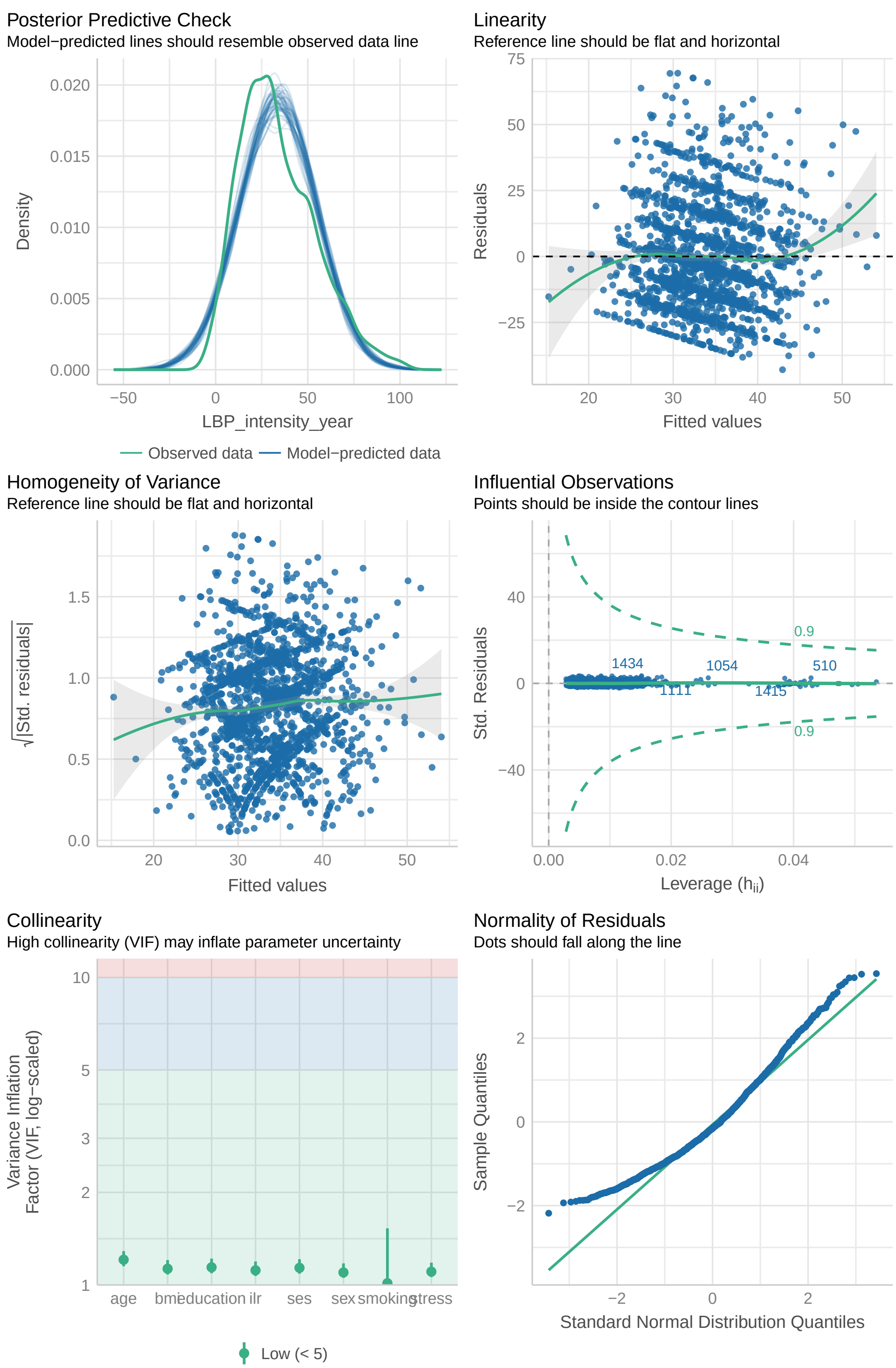
```r
kable(exp(est_ci_df), digits = 3) # these are the odds ratios (and approx CIs)
```

	est	2.5 %	97.5 %
age2_middle	1.059	0.851	1.319
age3_older	1.831	1.313	2.551
sex2_male	0.662	0.524	0.835
bmi2_normal	1.158	0.551	2.539
bmi3_overweight	1.612	0.761	3.564
stress2_stressed	1.547	1.258	1.902
smoking2_nonsmoker	1.172	0.912	1.512
education2_higher	0.746	0.601	0.926
ses2_middle	0.649	0.479	0.882
ses3_higher	0.451	0.291	0.697
ilrilr(++–)	0.864	0.697	1.069
ilrilr(+-..)	0.549	0.384	0.782

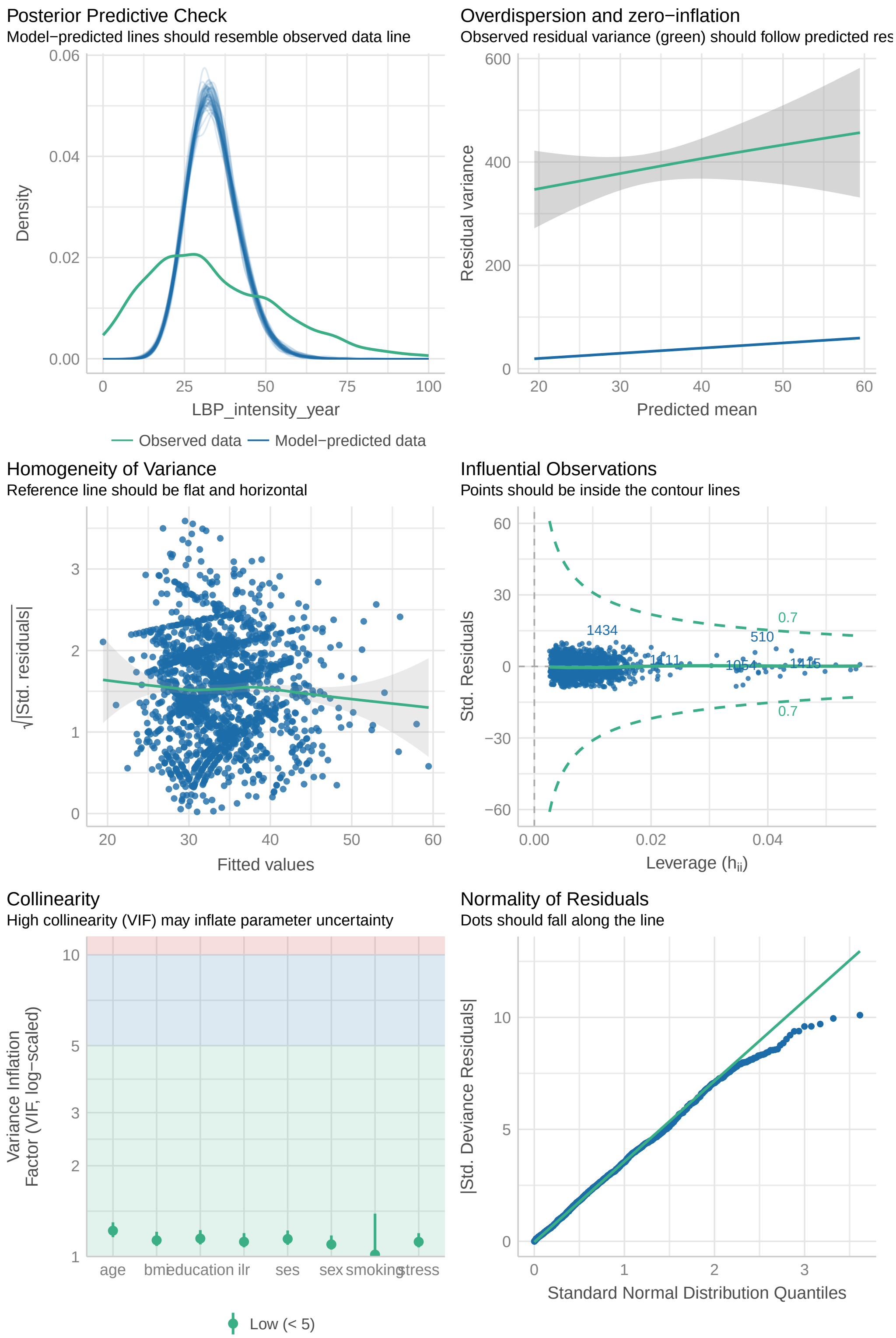
	est	2.5 %	97.5 %
ilrilr(..+-)	0.953	0.819	1.109

5.4.2 Model diagnostics

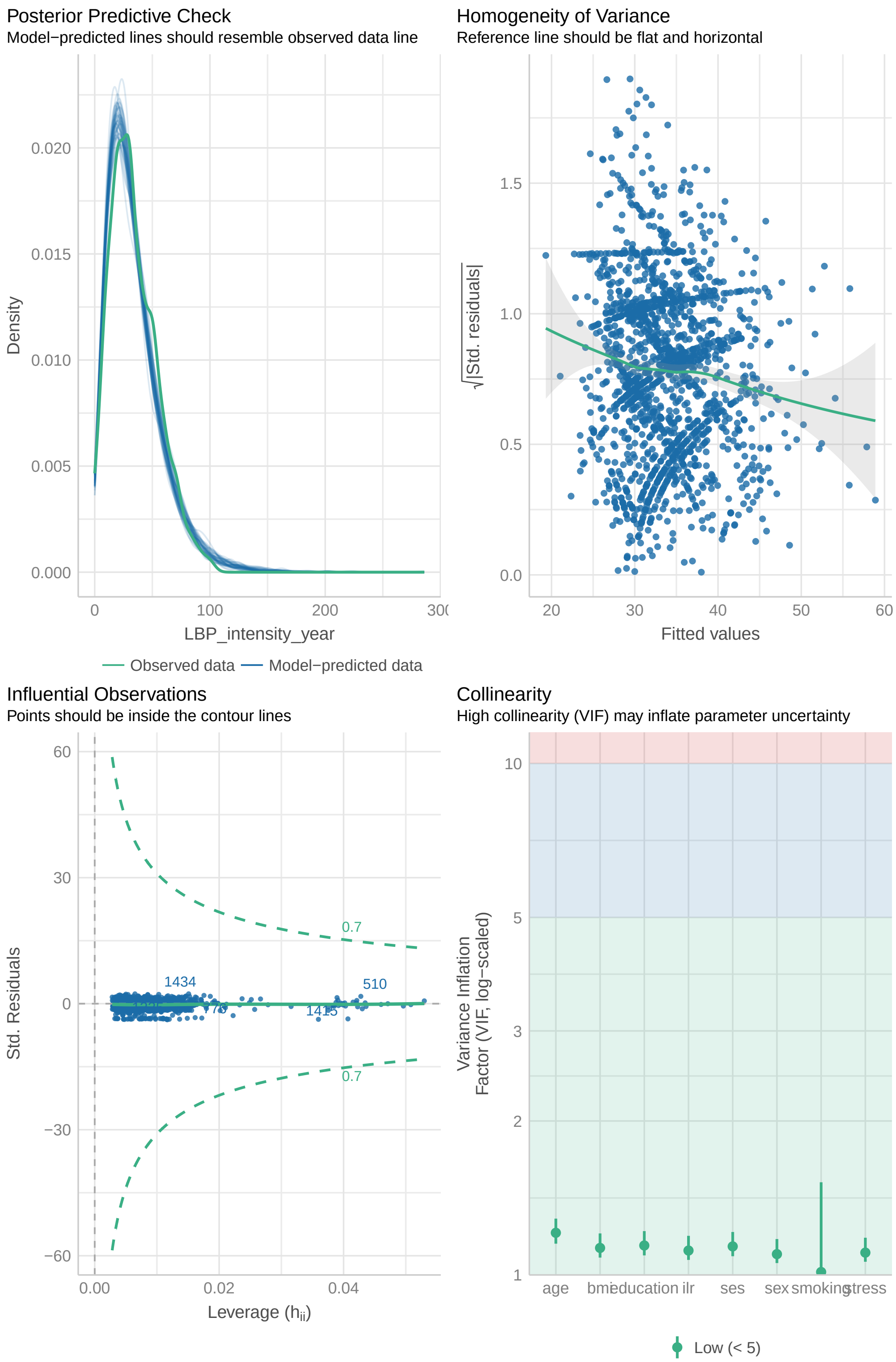
```
## plain linear model
check_model(lbp_intensity_lm) # acceptable?
```


Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
0.020
0.015
0.010
0.005
0.000
−50
0
50
100
LBP_intensity_year
Observed data
Model−predicted data
Linearity
Reference line should be flat and horizontal
Residuals
75
50
25
0
−25
20
30
40
50
Fitted values
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
1.5
1.0
0.5
0.0
20
30
40
50
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
40
0
−40
0.9
0.9
1434
1054
510
1111
1415
0.00
0.02
0.04
Leverage (h_{ii})
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation Factor (VIF, log−scaled)
10
5
3
2
1
age
bmi
education
ilr
ses
sex
smoking
stress
Low (< 5)
Normality of Residuals
Dots should fall along the line
Sample Quantiles
2
0
−2
−2
0
2
Standard Normal Distribution Quantiles

```
## Poisson regression (bad)
check_model(lbp_intensity_pois) # horrible
```

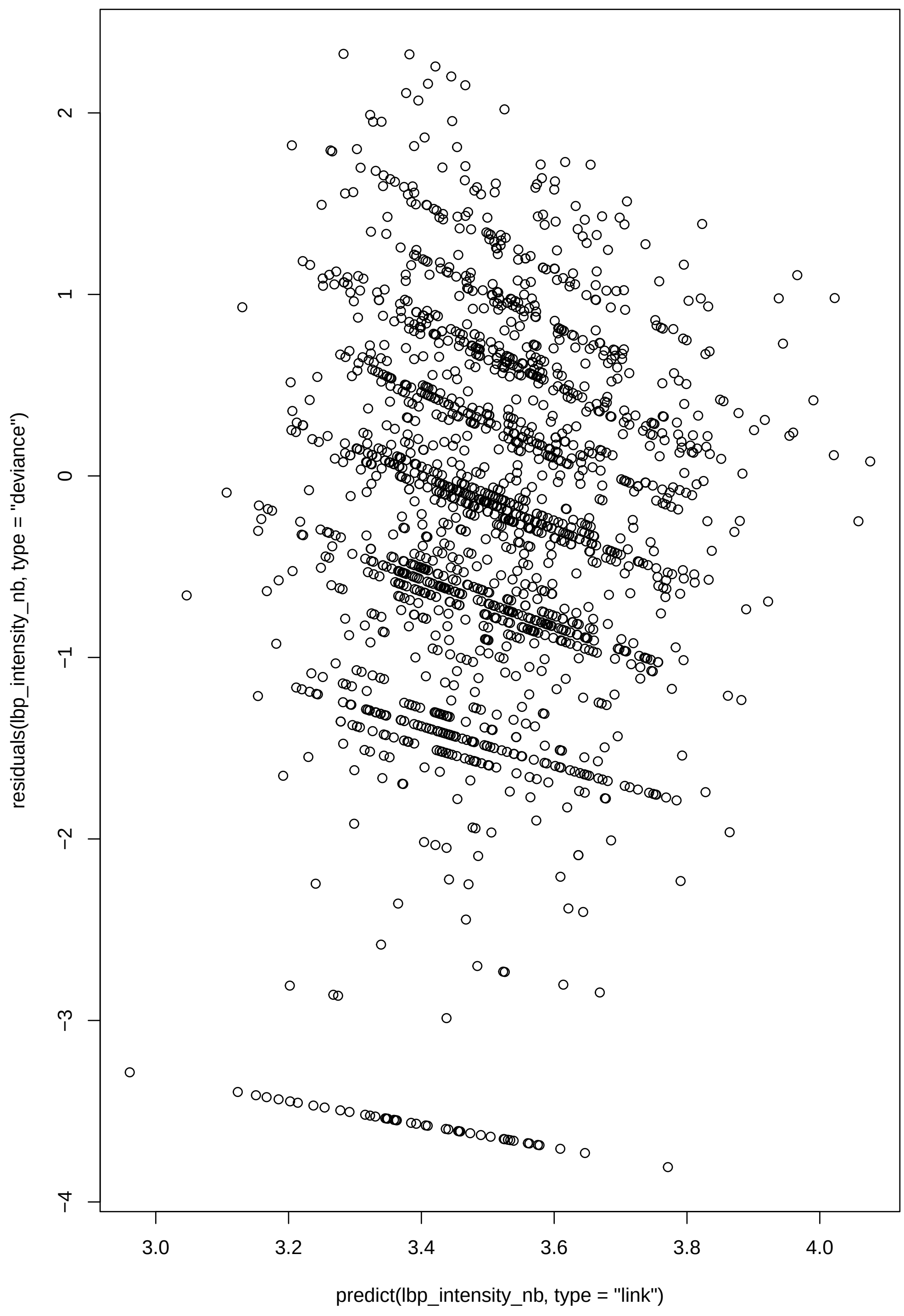

Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
LBP_intensity_year
Observed data
Model−predicted data
Overdispersion and zero−inflation
Observed residual variance (green) should follow predicted res
Residual variance
Predicted mean
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
Leverage (hii)
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation Factor (VIF, log−scaled)
age
bmi
education ilr
ses
sex
smoking
stress
Low (< 5)
Normality of Residuals
Dots should fall along the line
|Std. Deviance Residuals|
Standard Normal Distribution Quantiles

```r
## Negative Binomial regression
check_model(
  lbp_intensity_nb,
  check = c("pp_check", "homogeneity", "outliers", "vif")
)
```



Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
0.020
0.015
0.010
0.005
0.000
0
100
200
300
LBP_intensity_year
Observed data
Model−predicted data
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
1.5
1.0
0.5
0.0
20
30
40
50
60
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
60
30
0
−30
−60
0.7
0.7
1434
510
1415
0.00
0.02
0.04
Leverage (h_{ii})
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation Factor (VIF, log−scaled)
10
5
3
2
1
age
bmi
education
ilr
ses
sex
smoking
stress
Low (< 5)

```r
plot(
  predict(lbp_intensity_nb, type = "link"),
  residuals(lbp_intensity_nb, type = "deviance")
)
```


residuals(lbp_intensity_nb, type = "deviance")
predict(lbp_intensity_nb, type = "link")
2
1
0
−1
−2
−3
−4
3.0
3.2
3.4
3.6
3.8
4.0

```r
deviance(lbp_intensity_nb)/lbp_intensity_nb$df.residual
```

```
[1] 1.153043
```

```r
## Ordinal logistic regression (looks ok)
# this is running multiple logistic regressions
## we want to see the coefficients to be roughly the same EXCEPT for the
## (intercept) values
foreach(i = 2:length(levels(bpd_yes$intens_ord)), .combine = cbind) %do% {
  log_coefs <-
    coef(glm(
      I(as.numeric(intens_ord) >= i) ~
        age + sex + bmi + stress + smoking + education + ses + ilr,
      family = "binomial",
      data = bpd_yes
    ))
  log_coefs <- as.data.frame(log_coefs)
  colnames(log_coefs) <- paste0("logit(P(Y>=", i, "))")
  log_coefs
} %>%
  kable(., digits = 2)
```

	logit(P(Y>=2))	logit(P(Y>=3))	logit(P(Y>=4))
(Intercept)	5.80	-0.17	-1.95
age2_middle	-0.28	0.08	0.16
age3_older	0.49	0.62	0.65
sex2_male	-0.76	-0.38	-0.41
bmi2_normal	0.56	0.06	0.28
bmi3_overweight	0.73	0.38	0.68
stress2_stressed	0.52	0.46	0.40
smoking2_nonsmoker	0.34	0.09	0.37
education2_higher	-0.21	-0.36	-0.07
ses2_middle	-0.40	-0.34	-0.70
ses3_higher	-1.14	-0.68	-0.96
ilrilr(++–)	0.23	-0.17	-0.01
ilrilr(+-..)	-0.67	-0.57	-0.38
ilrilr(..+-)	-1.03	0.03	-0.07

5.4.3 Model predictions

```r
# create dataset for predictions
newdata <-
  bpd_yes %>%
  dplyr::select(all_of(pred_covs), ilr) %>%
  distinct(pick(all_of(pred_covs)), .keep_all = TRUE) %>%
  arrange(pick(all_of(pred_covs)))

(mean_ilr <- mean(bpd_yes$ilr))
```

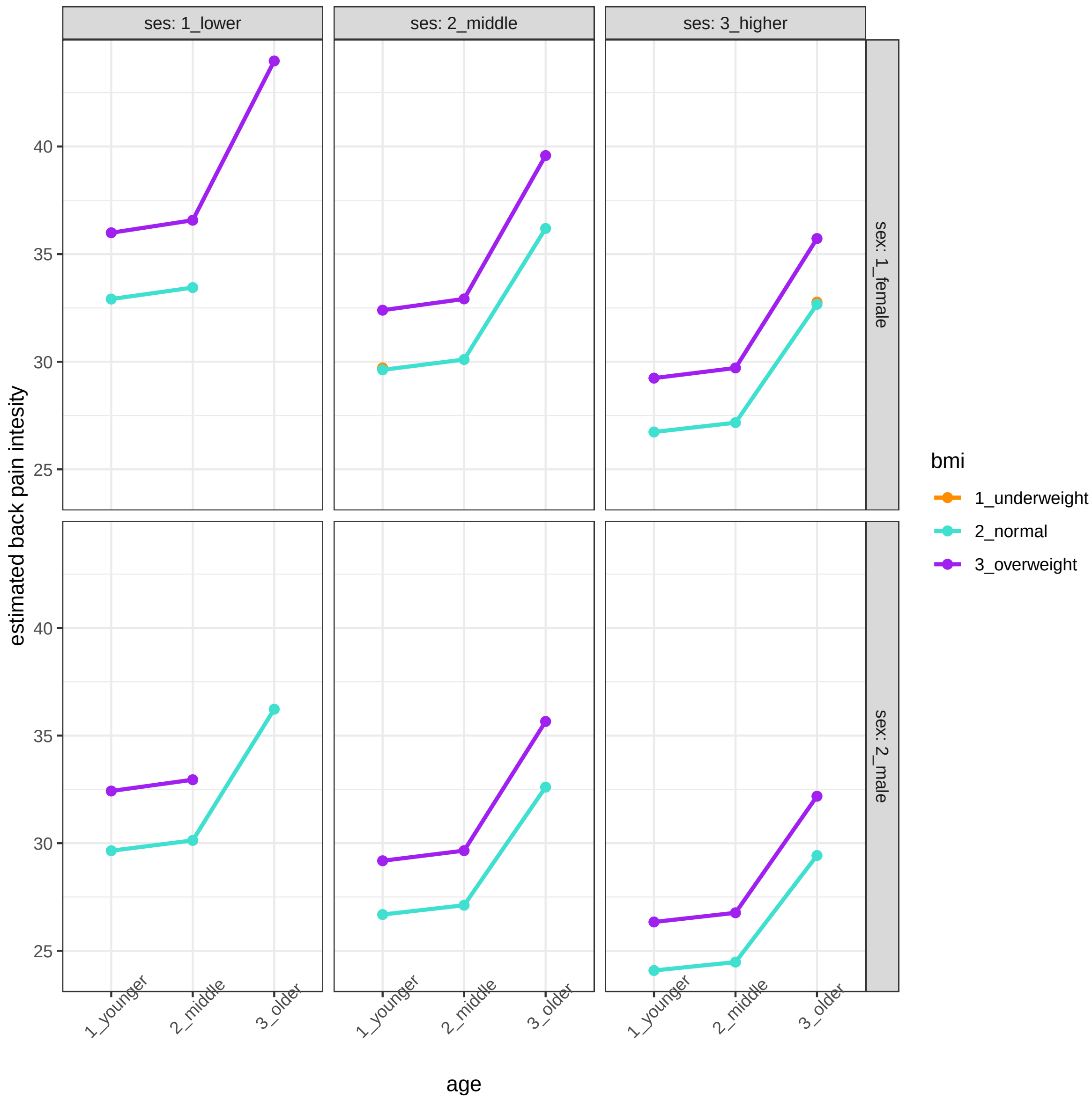
```
 ilr(++--)  ilr(+-..)  ilr(..+-)
1.37128443 0.05346627 2.07785318
attr(,"class")
[1] "rmult"
```

```r
dev_null <- foreach(i = 1:nrow(newdata)) %do% {
  newdata$ilr[i, ] <- mean_ilr
}

# make preds and then put in long format for ggplot
predictions_intens <-
  cbind(
    pain_intens = predict(lbp_intensity_nb, newdata, type = "response"),
    newdata
  ) %>%
  dplyr::select(-ilr)

head(predictions_intens)
```

```
  pain_intens       age       sex           bmi     stress     smoking education
1    31.64985 1_younger 1_female 1_underweight   1_normal    1_smoker  2_higher
2    28.48625 1_younger 1_female 1_underweight   1_normal    1_smoker  2_higher
3    36.47806 1_younger 1_female 1_underweight   1_normal 2_nonsmoker   1_lower
4    32.83186 1_younger 1_female 1_underweight   1_normal 2_nonsmoker   1_lower
5    29.71678 1_younger 1_female 1_underweight   1_normal 2_nonsmoker  2_higher
6    32.54180 1_younger 1_female 1_underweight 2_stressed    1_smoker  2_higher
       ses
1  1_lower
2 2_middle
```

```
3  1_lower
4 2_middle
5 2_middle
6 2_middle
```

```
# newdata2 <- cbind(newdata2, predict(lbp_nb, newdata2, type = "link", se.fit=TRUE))
# newdata2 <- within(newdata2, {
#   lbp_pred <- exp(fit)
#   LL <- exp(fit - 1.96 * se.fit)
#   UL <- exp(fit + 1.96 * se.fit)
# })



predictions_intens %>%
  dplyr::filter(
    # sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
    # ses == "2_middle"
  ) %>%
  ggplot(., aes(age, pain_intens, group = bmi)) +
  geom_line(aes(colour = bmi), linewidth = 1) +
  geom_point(aes(colour = bmi), size = 2) +
  facet_grid(sex~ ses, labeller = label_both) +
  labs(x = "age", y = "estimated back pain intesity") +
  theme_bw() +
  scale_color_manual(values = c("darkorange", "turquoise", "purple")) +
  theme(axis.text.x = element_text(angle = 45))
```


ses: 1_lower
ses: 2_middle
ses: 3_higher
sex: 1_female
sex: 2_male
estimated back pain intesity
40
35
30
25
1_younger
2_middle
3_older
age
bmi
1_underweight
2_normal
3_overweight

5.4.3.1 Absolute scale

```r
# create a RHS of regression equation dataset for time-reallocation
(predict_basis <-
    bpd_yes %>%
    dplyr::select(all_of(pred_covs), all_of(pred_comps)) %>%
    dplyr::filter(
      age == "2_middle",
      sex == "1_female",
      stress == "1_normal",
      smoking == "2_nonsmoker",
      education == "2_higher",
      ses == "2_middle",
      bmi == "2_normal"
    ) %>%
    distinct(across(all_of(pred_covs)), .keep_all = TRUE) %>%
    as.data.frame())
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   546.4286
  Time_Sedentary Time_LPA Time_MVPA
1       418.2857 435.1429        40
```

```r
# compositional mean: geometric mean to closure
# (comp_mean <- mean(acomp(bpd_yes[, pred_comps])))
(comp_mean <- calc_comp_mean(bpd_yes[, pred_comps], clo_val = 1440))
```

```
  Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
    472.8407       438.4062       502.1666        26.5865
```

```r
predict_basis0 <- predict_basis
predict_basis0[, pred_comps] <- comp_mean

predict_basis0
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
  Time_Sedentary Time_LPA Time_MVPA
1       438.4062 502.1666   26.5865
```

```r
# +15 minutes to Time_MVPA and -15 minutes from Time_Sedentary
comp_mean_changed <- comp_mean
comp_mean_changed["Time_MVPA"] <- comp_mean_changed["Time_MVPA"] + 15
comp_mean_changed["Time_Sedentary"] <- comp_mean_changed["Time_Sedentary"] - 15
# check
comp_mean_changed - comp_mean
```

```
  Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA 
           0            -15              0             15 
```

```r
predict_basis1 <- predict_basis
predict_basis1[, pred_comps] <- comp_mean_changed

pred_df <- rbind(predict_basis0, predict_basis1)
pred_df <- add_ilrs_to_data(pred_df, comp_vars = pred_comps, sbp_matrix = sbp1)
pred_df
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
2 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
  Time_Sedentary Time_LPA Time_MVPA      ilr.1      ilr.2      ilr.3
1       438.4062 502.1666   26.5865 1.37128443 0.05346627 2.07785318
2       423.4062 502.1666   41.5865 1.13019151 0.07808340 1.76151343
```

```r
predict(lbp_intensity_nb, pred_df, type = "link")
```

```
       1        2 
3.404581 3.418819 
```

```r
# exponentiate difference in the log back pain intensity (ratio of back pain preds)
exp(diff(predict(lbp_intensity_nb, pred_df, type = "link")))
```

```
      2 
1.01434 
```

```r
# abs difference in the mean back pain intensity
diff(predict(lbp_intensity_nb, pred_df, type = "response"))
```

```
        2
0.4316527
```

```r
(p_0 <- predict(lbp_intensity_nb, pred_df, type = "response"))
```

```
       1        2
30.10167 30.53332
```

```r
# % increase in pain intensity
(p_0[2]  - p_0[1]) / p_0[1]
```

```
         2
0.01433983
```

```r
# ratio version
get_pred_diff_rat <- function(mod, new_dat) {
  log_ratio_pred <- predict(mod, new_dat, type = "link")
  ratio_outc <- exp(log_ratio_pred[2] - log_ratio_pred[1])
  return(ratio_outc)
}
get_pred_diff_rat(lbp_intensity_nb, pred_df)
```

```
      2
1.01434
```

```r
# absolute difference version
get_pred_diff_abs <- function(mod, new_dat) {
  log_ratio_pred <- predict(mod, new_dat, type = "response")
  ratio_outc <- log_ratio_pred[2] - log_ratio_pred[1]
  return(ratio_outc)
}
get_pred_diff_abs(lbp_intensity_nb, pred_df)
```

```
        2
0.4316527
```

```r
# wrapper:
get_pred_diff <- function(mod, new_dat, type = "abs") {
  if (type == "abs") {
    return(get_pred_diff_abs(mod = mod, new_dat = new_dat))
  } else if (type == "rat") {
    return(get_pred_diff_rat(mod = mod, new_dat = new_dat))
  } else {
    stop("'type' must be 'abs' (absolute differnce) or 'rat' (ratio)")
  }
}
get_pred_diff(lbp_intensity_nb, pred_df, type = "abs")
```

```
        2
0.4316527
```

```r
get_pred_diff(lbp_intensity_nb, pred_df, type = "rat")
```

```
      2
1.01434
```

```r
fit_mod_boot <- function(data, i, pred_dat, type = "abs") {

  this_dat <- data[i, ]
  this_nbr <- glm.nb(mod_form_ilrs, data = this_dat)
  est <- get_pred_diff(this_nbr, new_dat = pred_dat, type = type)
  return(est)

}
alpha <- 0.05
quantile(boot(bpd_yes, fit_mod_boot, R = 10, pred_dat = pred_df)$t, c(alpha / 2, 1 - alpha
```

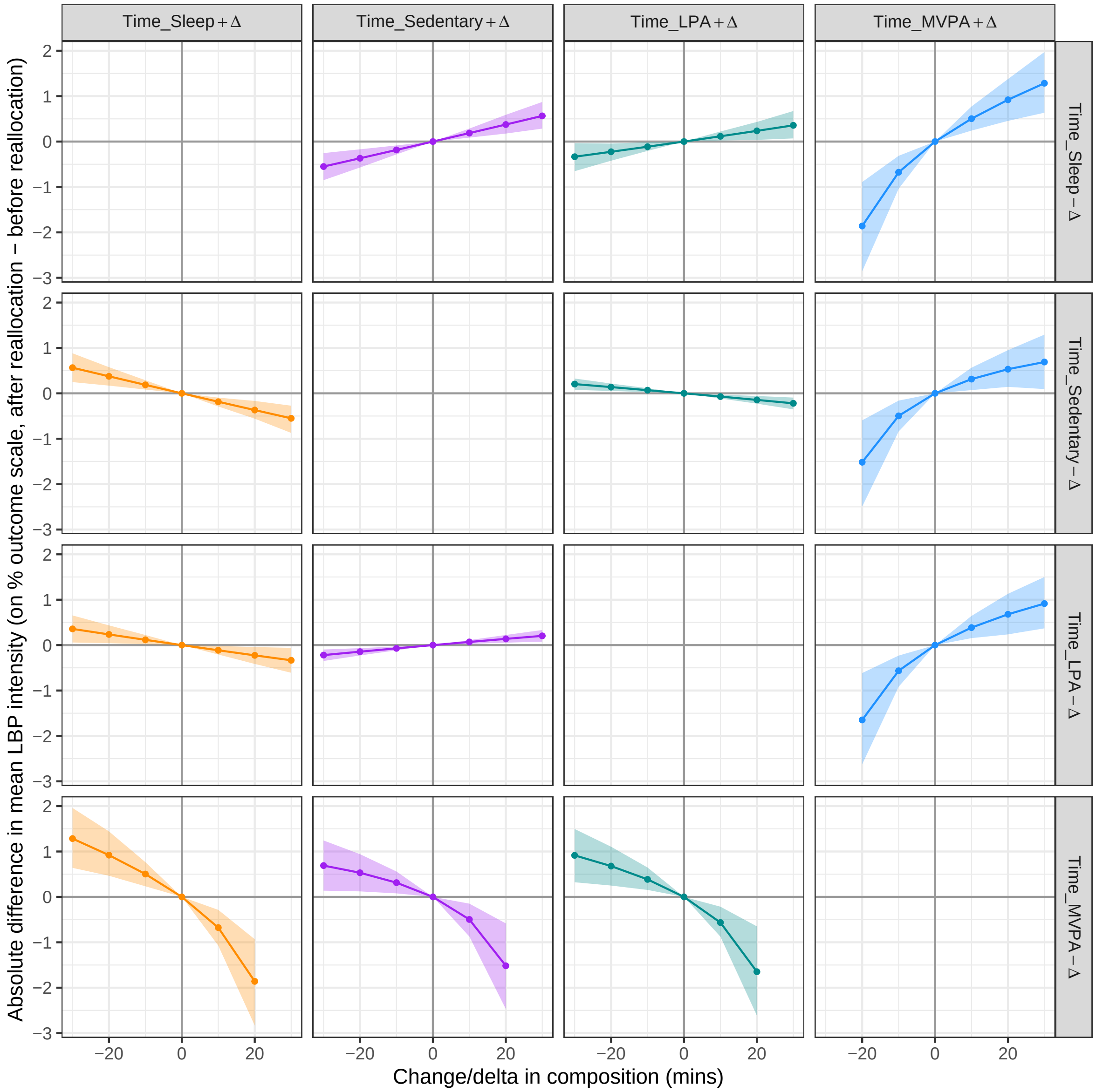
```
     2.5%     97.5%
0.1762889 0.6908688
```

```r
do_multi_realloc <- function(mod, basis_data, timeusenames, time_changes, sbp_matrix = sbp


  x0 <- basis_data
```

```
plot_dat <-
  foreach(i = 1:length(timeusenames), .combine = bind_rows) %do% {
    print(paste("i: ", i))
    foreach(j = 1:length(timeusenames), .combine = bind_rows) %do% {
      print(paste("  j: ", j))
      foreach(d = 1:length(time_changes), .combine = bind_rows) %do% {
        print(paste("    d: ", d))

        timeuse_to <- timeusenames[i]
        timeuse_from <- timeusenames[j]
        change_time <- time_changes[d]

        proposed_change_1 <- x0[timeuse_to] + change_time
        proposed_change_2 <- x0[timeuse_from] - change_time

        if (timeuse_to == timeuse_from) {
          NULL # reallocation exceeds 0 or max time
        } else if ((proposed_change_1 < 0) | (proposed_change_1 > 1440)) {
          NULL # reallocation exceeds 0 or max time
        } else if ((proposed_change_2 < 0) | (proposed_change_2 > 1440)) {
          NULL # reallocation exceeds 0 or max time
        } else {

          x1 <- x0
          x1[timeuse_to] <- x1[timeuse_to] + change_time
          x1[timeuse_from] <- x1[timeuse_from] - change_time

          pred_df <- rbind(x0, x1)
          pred_df <- add_ilrs_to_data(pred_df, comp_vars = timeusenames, sbp_matrix = sb

          outc_ratio <- get_pred_diff(mod, pred_df)

          bootstrapped_ests <- boot(bpd_yes, fit_mod_boot, R = 1000, pred_dat = pred_df)
          ci_est <- quantile(as.numeric(bootstrapped_ests), c(alpha / 2, 1 - alpha / 2))

          tibble(
            to = timeuse_to,
            from = timeuse_from,
            change_time = change_time,
            outc_ratio = outc_ratio,
            ci_lo = ci_est[1],
```

```
              ci_hi = ci_est[2]
            )
          }
        }
      }
    }

  plot_dat$to <- factor(plot_dat$to, levels = timeusenames)
  plot_dat$from <- factor(plot_dat$from, levels = timeusenames)

  return(plot_dat)

}

set.seed(1234)


# takes ~60 min (single core) for bootstrapped CIs (R = 1000)
# takes ~ 6 min (single core) for bootstrapped CIs (R = 100)

### Uncomment to generate bootstrapping
# tic()
# realloc_plot_data <-
#   do_multi_realloc(
#     lbp_intensity_nb,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )
# saveRDS(realloc_plot_data, file = "res/negbin_realloc_boot_res(abs).rda")
# toc()



realloc_plot_data <- readRDS(file = "res/negbin_realloc_boot_res(abs).rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
```

```r
geom_vline(xintercept = 0, col = "grey60") +
geom_hline(yintercept = 0, col = "grey60") +
geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
geom_line(aes(x = change_time , y = outc_ratio, col = to)) +
geom_point(aes(x = change_time , y = outc_ratio, col = to), size = 1) +
facet_grid(from ~ to, labeller = label_parsed) +
theme_bw() +
scale_colour_manual(values = c("darkorange","purple","cyan4", "dodgerblue")) +
scale_fill_manual(values = c("darkorange","purple","cyan4", "dodgerblue")) +
labs(
  x = paste0("Change/delta in composition (mins)"),
  y = paste0(
    "Absolute difference in mean LBP intensity (on % outcome scale, ",
    "after reallocation - before reallocation)")
) +
theme(legend.position = "none")
```




```r
ggsave(
  filename = "fig/lbp_intens_negbin_abs_v1.png",
  dpi = 600, # print quality
  width = 10,
  height = 10
)
```

```
pd2 <-
  realloc_plot_data %>%
  mutate(
    to = gsub("Time_", "", to),
    from = gsub("Time_", "", from),
    to = gsub("+Delta", "", to, fixed = TRUE),
    from = gsub("-Delta", "", from, fixed = TRUE),
    to_len = nchar(to),
    to_max = max(to_len),
    from_len = nchar(from),
    from_max = max(from_len),
    to_pad =  rep_char(pmax(0, from_max - to_len)),
    from_pad = rep_char(pmax(0, to_max - from_len)),
    to = factor(to, levels = time_lvls),
    from = factor(from, levels = time_lvls),
    to_num = as.numeric(to),
    from_num = as.numeric(from)
  ) %>%
  dplyr::filter(to_num > from_num) %>%
  mutate(
    # from_to = paste0("     ", "+", from, rep_char(10), from_pad, "\u2194", to_pad, rep_c
    from_to = paste0("+", from, rep_char(13), from_pad, "", to_pad, rep_char(13), "+", to)
  ) %>%
  arrange(from, to)

unique(pd2$from_to)
```

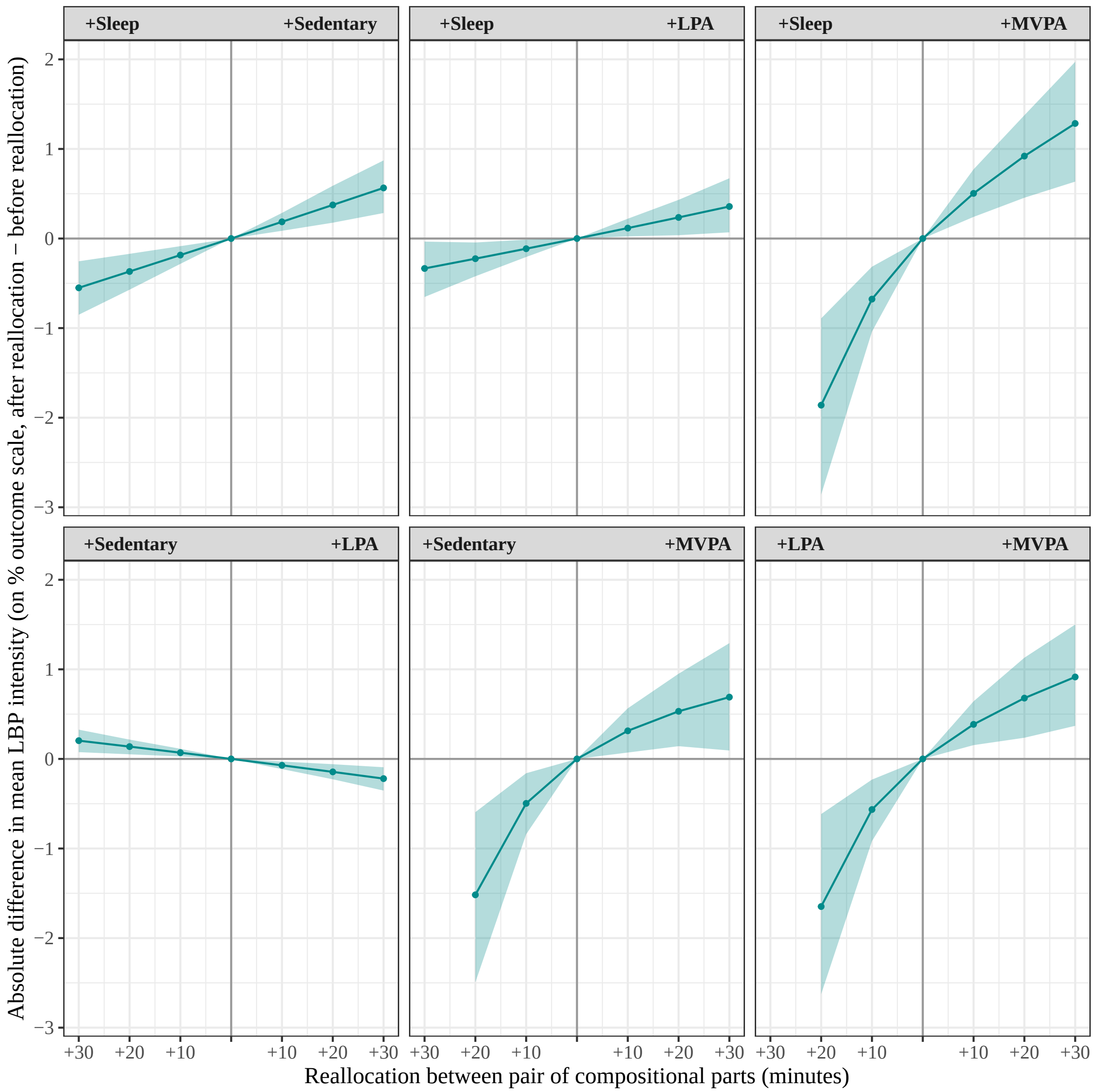
```
[1] "+Sleep                              +Sedentary"
[2] "+Sleep                                    +LPA"
[3] "+Sleep                                   +MVPA"
[4] "+Sedentary                                +LPA"
[5] "+Sedentary                               +MVPA"
[6] "+LPA                                     +MVPA"
```

```
pd2$from_to <- factor(pd2$from_to, levels = unique(pd2$from_to))

this_breaks <- seq(-30, 30, 10)
this_labs <- sprintf("+%2.0f", abs(seq(-30, 30, 10)))
this_labs[this_labs == "+ 0"] <- ""
```

```
this_labs
```

```
[1] "+30" "+20" "+10" ""    "+10" "+20" "+30"
```

```
ggplot(pd2) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 0, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3, co
  geom_line(aes(x = change_time , y = outc_ratio, col = to), col = "cyan4") +
  geom_point(aes(x = change_time , y = outc_ratio, col = to), size = 1, col = "cyan4") +
  facet_wrap(~ from_to, labeller = label_bquote(.(from_to))) +
  theme_bw() +
  scale_x_continuous(breaks = this_breaks, labels = this_labs) +
  labs(
    x = paste0("Reallocation between pair of compositional parts (minutes)"),
    y = paste0(
      "Absolute difference in mean LBP intensity (on % outcome scale, ",
      "after reallocation - before reallocation)"
    )
    # subtitle = "Note that odds ratios relate to the probability of having _increased_ fr
  ) +
  theme(
    legend.position = "none",
    text = element_text(family = "serif"),
    strip.text = element_text(size = 10, face = "bold"),
    axis.text =  element_text(size = 10),
    axis.title =  element_text(size = 12)
  )
```




```r
ggsave(filename = "fig/lbp_intens_negbin_abs_v2.png", width = 14, height = 9, dpi = 600)
```

5.4.3.2 Ratio scale

```r
# wrapper:
get_pred_diff <- function(mod, new_dat, type = "rat") {
  if (type == "abs") {
    return(get_pred_diff_abs(mod = mod, new_dat = new_dat))
  } else if (type == "rat") {
    return(get_pred_diff_rat(mod = mod, new_dat = new_dat))
  } else {
    stop("'type' must be 'abs' (absolute differnce) or 'rat' (ratio)")
  }
}
get_pred_diff(lbp_intensity_nb, pred_df, type = "abs")
```

```
        2
0.4316527
```

```r
get_pred_diff(lbp_intensity_nb, pred_df, type = "rat")
```

```
      2
1.01434
```

```r
get_pred_diff(lbp_intensity_nb, pred_df)
```

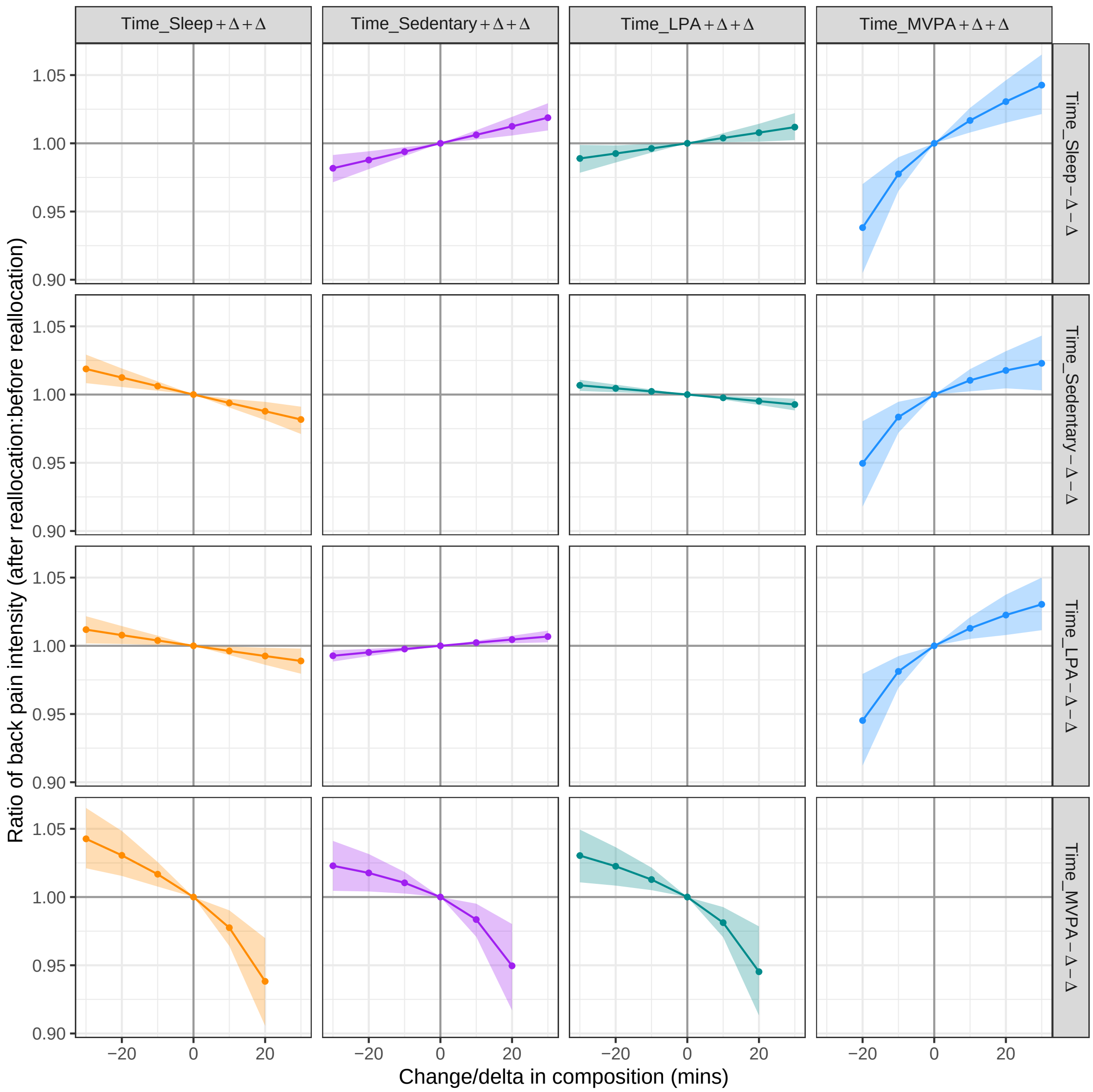
```
      2
1.01434
```

```r
fit_mod_boot <- function(data, i, pred_dat, type = "rat") {

  this_dat <- data[i, ]
  this_nbr <- glm.nb(mod_form_ilrs, data = this_dat)
  est <- get_pred_diff(this_nbr, new_dat = pred_dat, type = type)
  return(est)

}
alpha <- 0.05
quantile(boot(bpd_yes, fit_mod_boot, R = 10, pred_dat = pred_df)$t, c(alpha / 2, 1 - alpha
```

```
   2.5%    97.5% 
1.009664 1.026014 
```

```r
# takes ~60 min (single core) for bootstrapped CIs (R = 1000)
# takes ~ 6 min (single core) for bootstrapped CIs (R = 100)


### Uncomment to generate bootstrapping
# set.seed(1234)
# tic()
# realloc_plot_data <-
#   do_multi_realloc(
#     lbp_intensity_nb,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )
# saveRDS(realloc_plot_data, file = "res/negbin_realloc_boot_res(rat).rda")
# toc()





realloc_plot_data <- readRDS(file = "res/negbin_realloc_boot_res(rat).rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
  geom_line(aes(x = change_time , y = outc_ratio, col = to)) +
  geom_point(aes(x = change_time , y = outc_ratio, col = to), size = 1) +
  facet_grid(from ~ to, labeller = label_parsed) +
  theme_bw() +
  scale_colour_manual(values = c("darkorange","purple","cyan4", "dodgerblue")) +
  scale_fill_manual(values = c("darkorange","purple","cyan4", "dodgerblue")) +
  labs(
    x = paste0("Change/delta in composition (mins)"),
```

```
    y = paste0("Ratio of back pain intensity (after reallocation:before reallocation)")
  ) +
  theme(legend.position = "none")
```




```
ggsave(
  filename = "fig/lbp_intens_negbin_rat_v1.png",
```

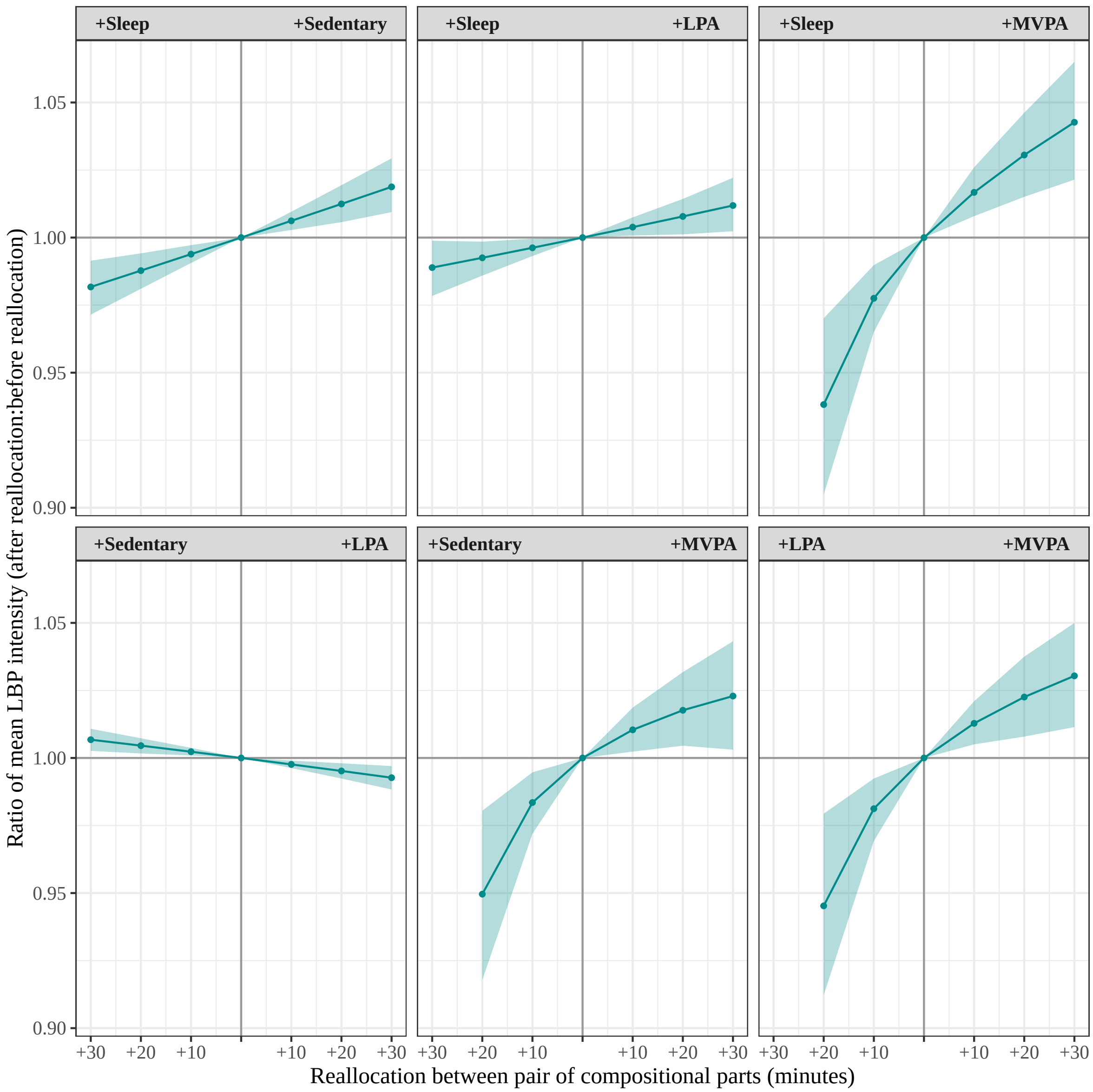
```r
    dpi = 600, # print quality
    width = 10,
    height = 10
  )



  pd2 <-
    realloc_plot_data %>%
    mutate(
      to = gsub("Time_", "", to),
      from = gsub("Time_", "", from),
      to = gsub("+Delta", "", to, fixed = TRUE),
      from = gsub("-Delta", "", from, fixed = TRUE),
      to_len = nchar(to),
      to_max = max(to_len),
      from_len = nchar(from),
      from_max = max(from_len),
      to_pad =  rep_char(pmax(0, from_max - to_len)),
      from_pad = rep_char(pmax(0, to_max - from_len)),
      to = factor(to, levels = time_lvls),
      from = factor(from, levels = time_lvls),
      to_num = as.numeric(to),
      from_num = as.numeric(from)
    ) %>%
    dplyr::filter(to_num > from_num) %>%
    mutate(
      # from_to = paste0("     ", "+", from, rep_char(10), from_pad, "\u2194", to_pad, rep_c
      from_to = paste0("+", from, rep_char(13), from_pad, "", to_pad, rep_char(13), "+", to)
    ) %>%
    arrange(from, to)

  unique(pd2$from_to)
```

```
[1] "+Sleep                              +Sedentary"
[2] "+Sleep                                    +LPA"
[3] "+Sleep                                   +MVPA"
[4] "+Sedentary                                +LPA"
[5] "+Sedentary                               +MVPA"
[6] "+LPA                                     +MVPA"
```

```r
pd2$from_to <- factor(pd2$from_to, levels = unique(pd2$from_to))

this_breaks <- seq(-30, 30, 10)
this_labs <- sprintf("+%2.0f", abs(seq(-30, 30, 10)))
this_labs[this_labs == "+ 0"] <- ""
this_labs
```

```
[1] "+30" "+20" "+10" ""    "+10" "+20" "+30"
```

```r
ggplot(pd2) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3, co
  geom_line(aes(x = change_time , y = outc_ratio, col = to), col = "cyan4") +
  geom_point(aes(x = change_time , y = outc_ratio, col = to), size = 1, col = "cyan4") +
  facet_wrap(~ from_to, labeller = label_bquote(.(from_to))) +
  theme_bw() +
  scale_x_continuous(breaks = this_breaks, labels = this_labs) +
  labs(
    x = paste0("Reallocation between pair of compositional parts (minutes)"),
    y = paste0("Ratio of mean LBP intensity (after reallocation:before reallocation)")
    # subtitle = "Note that odds ratios relate to the probability of having _increased_ fr
  ) +
  theme(
    legend.position = "none",
    text = element_text(family = "serif"),
    strip.text = element_text(size = 10, face = "bold"),
    axis.text =  element_text(size = 10),
    axis.title =  element_text(size = 12)
  )
```




```
ggsave(filename = "fig/lbp_intens_negbin_rat_v2.png", width = 14, height = 9, dpi = 600)
```

6 Session information

```
format(Sys.time(), '%d %b %Y')
```

```
[1] "26 Sep 2023"
```

```
sessionInfo()
```

```
R version 4.3.1 (2023-06-16 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19045)

Matrix products: default


locale:
[1] LC_COLLATE=English_Australia.utf8  LC_CTYPE=English_Australia.utf8
[3] LC_MONETARY=English_Australia.utf8 LC_NUMERIC=C
[5] LC_TIME=English_Australia.utf8

time zone: Australia/Darwin
tzcode source: internal

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] tictoc_1.2          boot_1.3-28.1       foreach_1.5.2
 [4] knitr_1.43          car_3.1-2           carData_3.0-5
 [7] mice_3.16.0         performance_0.10.4  zCompositions_1.4.0-1
[10] truncnorm_1.0-9     NADA_1.6-1.1        survival_3.5-5
[13] MASS_7.3-60         compositions_2.0-6  GGally_2.1.2
[16] ggplot2_3.4.2       forcats_1.0.0       readr_2.1.4
[19] tidyr_1.3.0         dplyr_1.1.2

loaded via a namespace (and not attached):
 [1] tidyselect_1.2.0   viridisLite_0.4.2 farver_2.1.1       fastmap_1.1.1
 [5] reshape_0.8.9      tensorA_0.36.2     bayestestR_0.13.1 digest_0.6.33
 [9] rpart_4.1.19       lifecycle_1.0.3    magrittr_2.0.3     compiler_4.3.1
[13] rlang_1.1.1        tools_4.3.1        utf8_1.2.3         yaml_2.3.7
```

```
[17] labeling_0.4.2      bit_4.0.5           plyr_1.8.8          RColorBrewer_1.1-3
[21] abind_1.4-5         withr_2.5.0         purrr_1.0.1         datawizard_0.8.0
[25] nnet_7.3-19         grid_4.3.1          fansi_1.0.4         jomo_2.7-6
[29] colorspace_2.1-0    scales_1.2.1        iterators_1.0.14    insight_0.19.3
[33] cli_3.6.1           rmarkdown_2.23      crayon_1.5.2        ragg_1.2.5
[37] generics_0.1.3      rstudioapi_0.15.0   robustbase_0.99-0   tzdb_0.4.0
[41] minqa_1.2.5         bayesm_3.1-5        splines_4.3.1       parallel_4.3.1
[45] vctrs_0.6.3         glmnet_4.1-8        Matrix_1.6-0        jsonlite_1.8.7
[49] patchwork_1.1.3     hms_1.1.3           ggrepel_0.9.3       bit64_4.0.5
[53] mitml_0.4-5         systemfonts_1.0.4   see_0.8.0           glue_1.6.2
[57] nloptr_2.0.3        DEoptimR_1.1-1      pan_1.9             codetools_0.2-19
[61] shape_1.4.6         gtable_0.3.3        lme4_1.1-34         munsell_0.5.0
[65] tibble_3.2.1        pillar_1.9.0        htmltools_0.5.5     R6_2.5.1
[69] textshaping_0.3.6   vroom_1.6.3         evaluate_0.21       lattice_0.21-8
[73] backports_1.4.1     broom_1.0.5         Rcpp_1.0.11         nlme_3.1-162
[77] mgcv_1.8-42         xfun_0.39           pkgconfig_2.0.3
```