Back pain: four outcomes

Ty and Kaja

Table of contents

1 Approach

For each outcome, the simplest model that has appropriate fit will be sought. Models have been classified in rough ordering from "simplest" below using A, B, C, or D with A representing the common/easily understood models

Outcome variable/*Model*	*Multiple linear regression* (A)	*Ordinal logistic regression* (B*)	*Logistic regression* (B)	*Poisson/negative binomial regression* (C)	*Beta regression* (D)
Binary	-	-	+	-	-
Ordinal	-	+	+ (if outcome made binary)	-	-
Values from 0 to 100	+ (outcome potentially transformed)	- (if outcome made ordinal but bad option)	+ (if outcome made binary)	+	+

*Probably "C" not "B" but is basically multiple logistic regressions performed with different dichotomisations of the order levels in the outcome

2 Set up

2.1 Packages

```r
suppressPackageStartupMessages(suppressWarnings({

  library("dplyr") # tidyverse
  library("tidyr")
  library("readr")
  library("forcats")
  library("ggplot2")

  library("GGally") # additional ggplot-type plotting

  library("compositions")
  library("zCompositions") # this one for lr_EM

  library("performance") # model checking
  library("mice")        # missing data functions
  library("car")         # Anova() for comparing models
  library("knitr")       # kable() for pretty printing
  library("foreach")     # powerful looping

  library("boot")        # bootstrap confidence intervals
  library("tictoc")      # check time between tic() and toc()

}))
```

2.2 Constants

```r
pred_comps <- c("Time_Sleep", "Time_Sedentary", "Time_LPA", "Time_MVPA")
(D <- length(pred_comps))
```

```
[1] 4
```

```r
pred_covs <- c("age", "sex", "bmi", "stress", "smoking", "education", "ses")
outcs <-
  c(
```

```r
    "LBP_frequency_year", "LBP_intensity_year",
    "LBP_intensity_month", "LBP_intensity_week"
  )

# default RHS of model formulas
# (rhs_formula <- paste(c(paste(pred_covs, collapse = " + "), "ilr"), collapse = " + "))
(rhs_formula <- paste(pred_covs, collapse = " + "))
```

```
[1] "age + sex + bmi + stress + smoking + education + ses"
```

```r
# this is the sequential binary partition matrix to be used for ilr creation
sbp1 <- matrix(
  c(
    1, 1, -1, -1,
    1, -1, 0, 0,
    0, 0, 1, -1
  ),
  ncol = 4, byrow = TRUE
)

# a way of creating ilr names automatically from SBP matrix
create_ilr_names <- function(sbp_matrix) {
  ilr_sbp_nms <- apply(sbp_matrix, 1, paste, collapse = "")
  ilr_sbp_nms <- gsub("-1", "-", ilr_sbp_nms)
  ilr_sbp_nms <- gsub("1", "+", ilr_sbp_nms)
  ilr_sbp_nms <- gsub("0", ".", ilr_sbp_nms)
  return(paste0("ilr(", ilr_sbp_nms, ")"))
}
create_ilr_names(sbp1)
```

```
[1] "ilr(++--)" "ilr(+-..)" "ilr(..+-)"
```

```r
do_closure <- function(x, clo_val = 1) {
  return(clo_val * x / sum(x))
}
calc_comp_mean <- function(x, clo_val = 1) {
  unclose_mean <- NULL
  if (is.null(dim(x))) {
    return(x)
```

```r
  } else if (ncol(x) == 1) { # column matrix
    return(as.numeric(x))
  } else {
    unclose_mean <- apply(x, 2, function(x) exp(mean(log(x))))
  }

  return(do_closure(unclose_mean, clo_val = clo_val))

}
```

3 Data wrangling

3.1 Read data

```
bpd_col_spec <-
  cols(
    Time_Sleep = col_double(),
    Time_Sedentary = col_double(),
    Time_LPA = col_double(),
    Time_MVPA = col_double(),
    age = col_double(),
    sex = col_character(),
    bmi = col_double(),
    stress = col_character(),
    smoking = col_character(),
    education = col_character(),
    ses = col_character(),
    LBP_sufferer = col_character(),
    LBP_frequency_year = col_character(),
    LBP_intensity_year = col_double(),
    LBP_intensity_month = col_double(),
    LBP_intensity_week = col_double()
  )


bpd <- read_csv("dat/bpd.csv", col_types = bpd_col_spec)
# head(bpd)

summary(bpd)
```

```
  Time_Sleep      Time_Sedentary      Time_LPA          Time_MVPA
 Min.   : 87.14   Min.   :  10.0   Min.   :   9.857   Min.   :  0.00
 1st Qu.:400.00   1st Qu.: 305.4   1st Qu.: 363.286   1st Qu.: 12.00
 Median :443.57   Median : 440.9   Median : 512.143   Median : 31.14
 Mean   :439.55   Mean   : 451.9   Mean   : 504.967   Mean   : 43.55
 3rd Qu.:480.71   3rd Qu.: 588.3   3rd Qu.: 642.286   3rd Qu.: 60.00
 Max.   :757.14   Max.   :1100.1   Max.   :1160.286   Max.   :514.00

      age            sex                 bmi            stress
 Min.   :18.00   Length:2333        Min.   :15.10   Length:2333
 1st Qu.:38.00   Class :character   1st Qu.:21.95   Class :character
```

```
 Median :49.00   Mode  :character   Median :24.25   Mode  :character
 Mean   :48.11                      Mean   :24.94
 3rd Qu.:58.00                      3rd Qu.:27.15
 Max.   :92.00                      Max.   :66.02

   smoking           education             ses            LBP_sufferer
 Length:2333        Length:2333        Length:2333        Length:2333
 Class :character   Class :character   Class :character   Class :character
 Mode  :character   Mode  :character   Mode  :character   Mode  :character




 LBP_frequency_year LBP_intensity_year LBP_intensity_month LBP_intensity_week
 Length:2333        Min.   :  0.0      Min.   :  0.00      Min.   :  0.00
 Class :character   1st Qu.: 19.0      1st Qu.:  9.00      1st Qu.:  0.00
 Mode  :character   Median : 30.0      Median : 25.00      Median : 10.00
                    Mean   : 33.8      Mean   : 30.78      Mean   : 20.36
                    3rd Qu.: 49.0      3rd Qu.: 50.00      3rd Qu.: 31.00
                    Max.   :100.0      Max.   :100.00      Max.   :100.00
                    NA's   :673        NA's   :673         NA's   :673
```

3.2 Tidy data

```r
# relevel categories in LBP_frequency_year
y_lab <- "LBP_frequency_year"
sort(unique(bpd[[y_lab]]))
```

```
[1] "0days"                              "1-7days"
[3] "31-90days"                          "8-30days"
[5] "everyday"                           "more_than90days_but_not_everyday"
```

```r
bpd[[y_lab]] <-
  if_else(
    bpd[[y_lab]] == "more_than90days_but_not_everyday",
    "91+_not_evday",
    bpd[[y_lab]]
  )

# check
```

```r
table(bpd[[y_lab]], useNA = "ifany")
```

```

       0days        1-7days       31-90days        8-30days 91+_not_evday 
         673            760             146             451           203 
    everyday 
         100 
```

```r
bpd[[y_lab]] <- factor(bpd[[y_lab]])
levels(bpd[[y_lab]])
```

```
[1] "0days"         "1-7days"       "31-90days"     "8-30days"     
[5] "91+_not_evday" "everyday"     
```

```r
lvls_ord <- c(1, 2, 4, 3, 5, 6)
# right order?
# levels(bpd[[y_lab]])[lvls_ord]
bpd[[y_lab]] <- lvls_reorder(bpd[[y_lab]], lvls_ord)
### right order?
levels(bpd[[y_lab]])
```

```
[1] "0days"         "1-7days"       "8-30days"      "31-90days"    
[5] "91+_not_evday" "everyday"     
```

```r
with(bpd, table(LBP_frequency_year, LBP_sufferer, useNA = "ifany"))
```

```
                  LBP_sufferer
LBP_frequency_year  no yes
     0days         673   0
     1-7days         0 760
     8-30days        0 451
     31-90days       0 146
     91+_not_evday   0 203
     everyday        0 100
```

```r
### comment these lines for sensitivity analysis
bpd$age <-
  cut(
    bpd$age,
    breaks = c(17, 44, 64, 100),
    labels = c("1_younger", "2_middle", "3_older")
  )
table(bpd$age)
```

```

1_younger  2_middle   3_older
      896      1153       284
```

```r
bpd$bmi <-
  cut(
    bpd$bmi,
    breaks = c(15, 18.5, 25, 70),
    right = FALSE,
    labels = c("1_underweight", "2_normal", "3_overweight")
  )
table(bpd$bmi)
```

```

1_underweight      2_normal  3_overweight
           44          1309           980
```

3.3 Impute missing values in compositions

This code is thanks to Kaja!

Missing data is assumed to be below detectable threshold and imputed.

```r
# Do I have zero values in my composition? (yes in MVPA)
### See: summary(bpd)
# bpd %>%
#   summarise(
#     Time_MVPA.min = min(Time_MVPA),
#     Time_MVPA.max = max(Time_MVPA),
#     Time_LPA.min = min(Time_LPA),
#     Time_LPA.max = max(Time_LPA),
#     Time_Sedentary.min = min(Time_Sedentary),
```

```r
#     Time_SB.max = max(Time_Sedentary),
#     Time_Sleep.min = min(Time_Sleep),
#     Time_Sleep.max = max(Time_Sleep),
#     n = n()
#   )

# We need to make compositions before we do the lrEM method. The most straightforward way
comp1 <- bpd[, pred_comps]

# How much participants have zero MVPA? 159 participants (6.8% of the sample)
missingSummary(comp1)
```

```
                missingType
variable          NMV  BDL  MAR MNAR   SZ  Err
  Time_Sleep     2333    0    0    0    0    0
  Time_Sedentary 2333    0    0    0    0    0
  Time_LPA       2333    0    0    0    0    0
  Time_MVPA      2174  159    0    0    0    0
```

```r
sum(rowSums(is.na(comp1) | (comp1 < 0.1)), na.rm = FALSE)
```

```
[1] 159
```

```r
sum(which_0 <- as.logical(rowSums(is.na(comp1) | (comp1 < 0.1))))
```

```
[1] 159
```

```r
# these are 0 vals anywhere in composition (or NA)
bpd[which_0, pred_comps]
```

```
# A tibble: 159 x 4
   Time_Sleep Time_Sedentary Time_LPA Time_MVPA
        <dbl>          <dbl>    <dbl>     <dbl>
 1       475            148.     817.         0
 2       437.           826.     177.         0
 3       479.           522.     440.         0
```

```
 4        427.           429      584.          0
 5        429.           468.     544.          0
 6        424.           296.     720.          0
 7        506.           727.     207.          0
 8        456.           208      776.          0
 9        475            607.     358.          0
10        272.           783.     384.          0
# i 149 more rows
```

```r
# I have zeroes in MVPA - lrEM function will be applied

# ?lrEM
# what is the smallest time-use value above 0? [in minutes]
min(comp1[comp1 > 0])
```

```
[1] 0.1428571
```

```r
thresh_detect <- 10 / 1440
# thresh_detect <- 0.01

comp1.a <- comp1 / 1440 # Create % based composition
dl <- c(rep(thresh_detect, times = D)) # threshold limit for the replacement

comp1.zr <- lrEM(comp1.a, label = 0, dl = dl) # conduct the lrEM Zero Replacement
```

```
No. iterations to converge:  6
```

```r
comp1.zr <- as_tibble(comp1.zr * 1440)
# composition is larger than 1440 for those who have imputated MVPA
# (all behaviours will be proportionally downscaled to fit 1440 min when constructing the

# look at imputed values
comp1.zr[which_0, ]
```

```
# A tibble: 159 x 4
   Time_Sleep Time_Sedentary Time_LPA Time_MVPA
        <dbl>          <dbl>    <dbl>     <dbl>
```

```
 1        475             148.     817.      2.82
 2        437.            826.     177.      3.21
 3        479.            522.     440.      3.08
 4        427.            429      584.      2.99
 5        429.            468.     544.      3.01
 6        424.            296.     720.      2.91
 7        506.            727.     207.      3.21
 8        456.            208      776.      2.87
 9        475             607.     358.      3.12
10        272.            783.     384.      3.00
# i 149 more rows
```

```r
# build dataset that contain imputed values for our 24-h composition

# add new compositions to other noncompositional data
# remove 24-h data from the datase
# bpd <- subset(bpd, select = -c(id, Time_Sleep, Time_Sedentary, Time_LPA, Time_MVPA))
head(bpd[which_0, pred_comps])
```

```
# A tibble: 6 x 4
  Time_Sleep Time_Sedentary Time_LPA Time_MVPA
       <dbl>          <dbl>    <dbl>     <dbl>
1        475           148.     817.         0
2        437.          826.     177.         0
3        479.          522.     440.         0
4        427.          429      584.         0
5        429.          468.     544.         0
6        424.          296.     720.         0
```

```r
bpd <- bpd[, !(colnames(bpd) %in% pred_comps)] # remove ori time-use cols
bpd <- bind_cols(comp1.zr, bpd) # add imputed 24-h data
head(bpd[which_0, pred_comps])
```

```
# A tibble: 6 x 4
  Time_Sleep Time_Sedentary Time_LPA Time_MVPA
       <dbl>          <dbl>    <dbl>     <dbl>
1        475           148.     817.      2.82
2        437.          826.     177.      3.21
3        479.          522.     440.      3.08
```

```
4       427.            429      584.      2.99
5       429.            468.     544.      3.01
6       424.            296.     720.      2.91
```

3.4 Compositions transformation to *ilr*s

The below function will allow us to automatically add ilrs to a dataset

```
add_ilrs_to_data <- function(dataset, comp_vars = pred_comps, sbp_matrix = sbp1) {

  # the time-use composition
  comp <- dataset[, comp_vars]
  comp <- acomp(comp) # designate it as a compositional variable

  # define sequential binary partition (SBP)
  psi1 <- gsi.buildilrBase(t(sbp_matrix)) #  The orthonormal matrix

  # find the mean composition
  (m <- mean(comp)) # comp has been designated as acomp, therefore R knows it's a composit
  # cat(
  #   "\nThis is the compositional mean [in mins] of the columns (",
  #   paste(comp_vars, collapse = ", "),
  #   ")\n\n",
  #   sep = ""
  # )
  # print(clo(m, total = 1440)) # to look at the mean in minutes/day.
  # cat("\n\n")

  # create isometric log ratios (ilr.1) using the above SBP and orthonormal b asis V=psi1.
  ilrs_from_comp <- ilr(comp, V = psi1)
  colnames(ilrs_from_comp) <- create_ilr_names(sbp_matrix)
  # colnames(ilrs_from_comp) <- paste0("coord", 1:(length(comp_vars) - 1))
  dataset$ilr <- ilrs_from_comp

  return(dataset)

}

# use function: creates the ilr columns nested in the single column "ilr"
bpd <- add_ilrs_to_data(bpd)
# check
```

```r
bpd[, c("ilr", pred_comps)]
```

```
# A tibble: 2,333 x 5
   ilr[,"ilr(++--)"] [,"ilr(+-..)"] Time_Sleep Time_Sedentary Time_LPA Time_MVPA
               <dbl>          <dbl>      <dbl>          <dbl>    <dbl>     <dbl>
 1             1.04          0.355        435            263.     722.      19.7
 2             2.29         -0.401        410            723.     297.      10.3
 3             1.57         -0.336        426.           685.     285.      44.3
 4             1.10         -0.0811       486.           545.     318.      91.3
 5             0.801         0.220        484.           355.     536.      64.6
 6             1.71         -0.134        494.           598.     318.      30.4
 7             1.03         -0.0333       447.           469.     467.      57.6
 8             1.29          0.189        546.           418.     435.      40
 9             1.69         -0.243        452.           638      320.      30.4
10             1.36         -0.329        354.           565.     494.      26.7
# i 2,323 more rows
# i 1 more variable: ilr[3] <rmult>
```

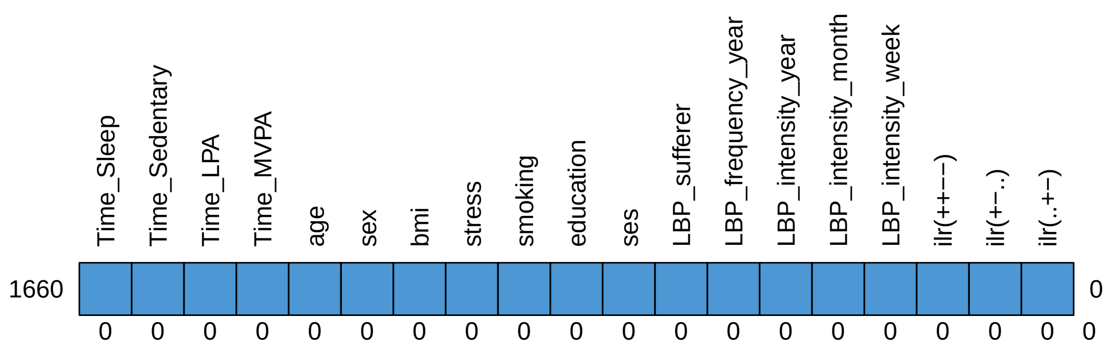
```r
# also create version of data without the nested ilrs
bpd_clean <- as.data.frame(bpd)
bpd_clean$ilr <- NULL # remove nested cols
bpd_clean <- cbind(bpd_clean, as.data.frame(bpd$ilr))
```

4 Exploratory analysis

4.1 Missing/NA value summaries

```r
### Missing data summary for LBP suffers
bpd_clean %>%
  dplyr::filter(LBP_sufferer == "yes") %>%
  md.pattern(., rotate.names = TRUE)
```

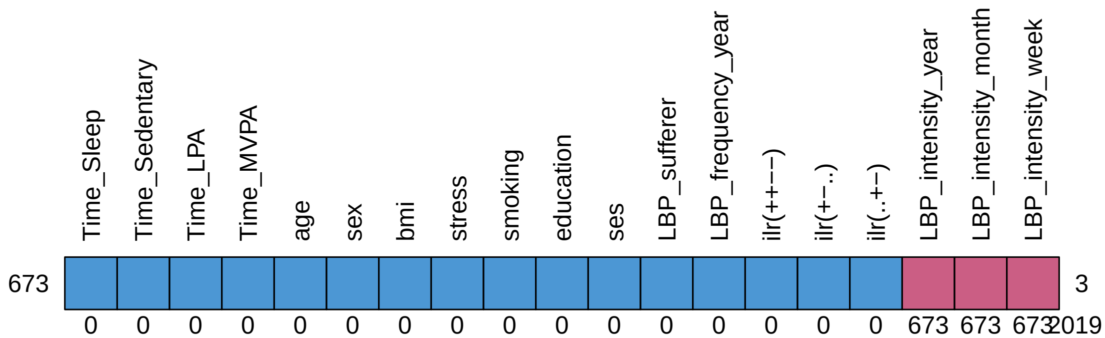
```
 /\     /\
{  `---'  }
{  O   O  }
==>  V <==  No need for mice. This data set is completely observed.
 \  \|/  /
  `-----'
```




```
     Time_Sleep Time_Sedentary Time_LPA Time_MVPA age sex bmi stress smoking
1660          1              1        1         1   1   1   1       1       1
              0              0        0         0   0   0   0       0       0
     education ses LBP_sufferer LBP_frequency_year LBP_intensity_year
1660         1   1            1                  1                  1
             0   0            0                  0                  0
```

```
     LBP_intensity_month LBP_intensity_week ilr(++--) ilr(+-..) ilr(..+-)
1660                   1                  1         1         1         1 0
                       0                  0         0         0         0 0
```

```
### Missing data summary for _non_ LBP suffers
bpd_clean %>%
  dplyr::filter(LBP_sufferer == "no") %>%
  md.pattern(., rotate.names = TRUE)
```

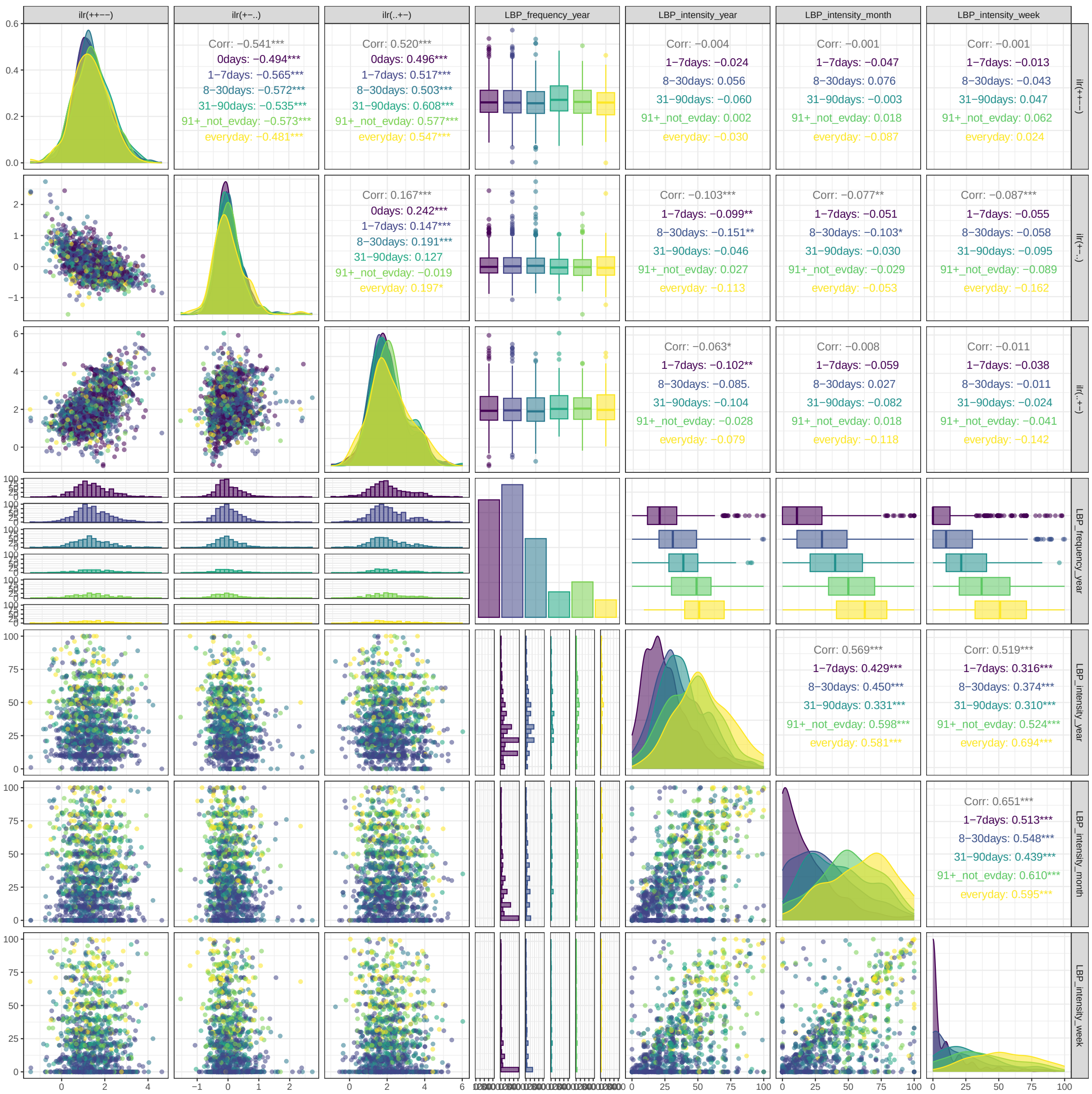



```
    Time_Sleep Time_Sedentary Time_LPA Time_MVPA age sex bmi stress smoking
673          1              1        1         1   1   1   1      1       1
             0              0        0         0   0   0   0      0       0
    education ses LBP_sufferer LBP_frequency_year ilr(++--) ilr(+-..) ilr(..+-)
673         1   1            1                  1         1         1         1
            0   0            0                  0         0         0         0
    LBP_intensity_year LBP_intensity_month LBP_intensity_week
673                  0                   0                  0    3
                   673                 673                673 2019
```

```
### ===> data doesn't have mistiness for analysis
```

4.2 Pairwise plots between *ilr*s and outcome variables

```
### plot pairwise comparisons of time-use and outcomes
if (FALSE) { # takes 30 sec
  suppressWarnings({
    bpd_clean %>%
      dplyr::select(all_of(pred_comps), all_of(outcs)) %>%
      ggpairs(
        .,
        progress = FALSE,
        ggplot2::aes(
          colour = LBP_frequency_year,
          fill = LBP_frequency_year,
          alpha = 0.25
        )
      ) +
      theme_bw() +
      scale_colour_viridis_d()+
      scale_fill_viridis_d()
  })
}

### plot pairwise comparisons of _ilrs_ and outcomes
if (TRUE) { # takes 30 sec
  suppressWarnings({
    bpd_clean %>%
      dplyr::select(starts_with("ilr"), all_of(outcs)) %>%
      ggpairs(
        .,
        progress = FALSE,
        ggplot2::aes(
          colour = LBP_frequency_year,
          fill = LBP_frequency_year,
          alpha = 0.25
        )
      ) +
      theme_bw() +
      scale_colour_viridis_d()+
      scale_fill_viridis_d()
  })
}
```



ilr(++−−)
ilr(+−..)
ilr(..+−)
LBP_frequency_year
LBP_intensity_year
LBP_intensity_month
LBP_intensity_week
Corr: −0.541***
0days: −0.494***
1−7days: −0.565***
8−30days: −0.572***
31−90days: −0.535***
91+_not_evday: −0.573***
everyday: −0.481***
Corr: 0.520***
0days: 0.496***
1−7days: 0.517***
8−30days: 0.503***
31−90days: 0.608***
91+_not_evday: 0.577***
everyday: 0.547***
Corr: −0.004
1−7days: −0.024
8−30days: 0.056
31−90days: −0.060
91+_not_evday: 0.002
everyday: −0.030
Corr: −0.001
1−7days: −0.047
8−30days: 0.076
31−90days: −0.003
91+_not_evday: 0.018
everyday: −0.087
Corr: −0.001
1−7days: −0.013
8−30days: −0.043
31−90days: 0.047
91+_not_evday: 0.062
everyday: 0.024
Corr: 0.167***
0days: 0.242***
1−7days: 0.147***
8−30days: 0.191***
31−90days: 0.127
91+_not_evday: −0.019
everyday: 0.197*
Corr: −0.103***
1−7days: −0.099**
8−30days: −0.151**
31−90days: −0.046
91+_not_evday: 0.027
everyday: −0.113
Corr: −0.077**
1−7days: −0.051
8−30days: −0.103*
31−90days: −0.030
91+_not_evday: −0.029
everyday: −0.053
Corr: −0.087***
1−7days: −0.055
8−30days: −0.058
31−90days: −0.095
91+_not_evday: −0.089
everyday: −0.162
Corr: −0.063*
1−7days: −0.102**
8−30days: −0.085.
31−90days: −0.104
91+_not_evday: −0.028
everyday: −0.079
Corr: −0.008
1−7days: −0.059
8−30days: 0.027
31−90days: −0.082
91+_not_evday: 0.018
everyday: −0.118
Corr: −0.011
1−7days: −0.038
8−30days: −0.011
31−90days: −0.024
91+_not_evday: −0.041
everyday: −0.142
Corr: 0.569***
1−7days: 0.429***
8−30days: 0.450***
31−90days: 0.331***
91+_not_evday: 0.598***
everyday: 0.581***
Corr: 0.519***
1−7days: 0.316***
8−30days: 0.374***
31−90days: 0.310***
91+_not_evday: 0.524***
everyday: 0.694***
Corr: 0.651***
1−7days: 0.513***
8−30days: 0.548***
31−90days: 0.439***
91+_not_evday: 0.610***
everyday: 0.595***

5 Statistical analysis

5.1 Outcome 0: binary outcome of Pain = "yes"

5.1.1 Model fit

```
bpd <-
  bpd %>%
  mutate(lbp_occurr = as.integer(LBP_sufferer == "yes"))

(this_outcome <- "lbp_occurr")
```

```
[1] "lbp_occurr"
```

```
# (mod_form_null <-as.formula(paste0(this_outcome, " ~ ", rhs_formula)))
(mod_form_ilrs <-as.formula(paste0(this_outcome, " ~ ", rhs_formula, " + ilr")))
```

```
lbp_occurr ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```
table(bpd[, this_outcome], useNA = "ifany")
```

```
lbp_occurr
   0    1
 673 1660
```

```
# logistic regression model __with__ ilrs
bpd_occurr_ilrs <- glm(mod_form_ilrs, data = bpd, family = binomial())
summary(bpd_occurr_ilrs)
```

```
Call:
glm(formula = mod_form_ilrs, family = binomial(), data = bpd)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-2.1078  -1.3516   0.7320   0.8578   1.1920
```

```
Coefficients:
                    Estimate Std. Error z value Pr(>|z|)    
(Intercept)         1.053768   0.384211   2.743 0.006094 ** 
age2_middle         0.340378   0.103168   3.299 0.000969 ***
age3_older          0.262018   0.159537   1.642 0.100513    
sex2_male           0.062784   0.111862   0.561 0.574618    
bmi2_normal         0.164453   0.327142   0.503 0.615178    
bmi3_overweight     0.378117   0.333707   1.133 0.257180    
stress2_stressed    0.407584   0.103641   3.933  8.4e-05 ***
smoking2_nonsmoker -0.105372   0.126368  -0.834 0.404365    
education2_higher  -0.274614   0.112119  -2.449 0.014313 *  
ses2_middle        -0.328406   0.177563  -1.850 0.064383 .  
ses3_higher        -0.723348   0.215902  -3.350 0.000807 ***
ilrilr(++--)       -0.083947   0.103592  -0.810 0.417735    
ilrilr(+-..)       -0.078951   0.177757  -0.444 0.656933    
ilrilr(..+-)        0.005253   0.072374   0.073 0.942138    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 2803.2  on 2332  degrees of freedom
Residual deviance: 2736.7  on 2319  degrees of freedom
AIC: 2764.7

Number of Fisher Scoring iterations: 4
```

```r
Anova(bpd_occurr_ilrs)
```

```
Analysis of Deviance Table (Type II tests)

Response: lbp_occurr
          LR Chisq Df Pr(>Chisq)    
age        11.0155  2   0.004055 ** 
sex         0.3161  1   0.573932    
bmi         5.0563  2   0.079808 .  
stress     15.7925  1  7.068e-05 ***
smoking     0.7022  1   0.402035    
education   6.1009  1   0.013511 *  
ses        12.5248  2   0.001907 ** 
```

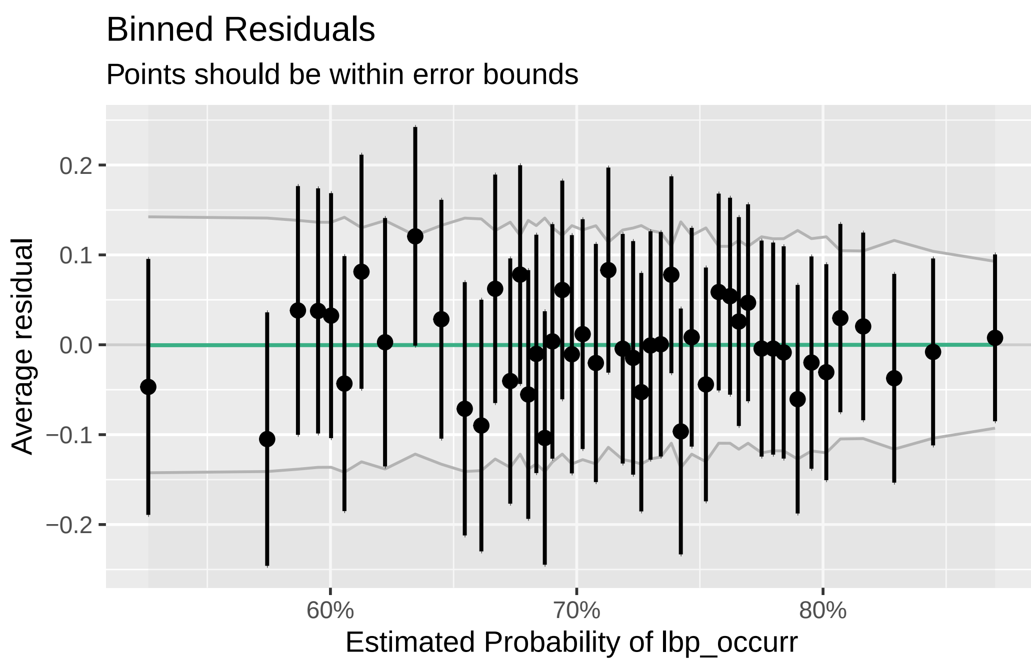
```
ilr          1.2749  3   0.735097
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

5.1.2 Model diagnostics

```
### check binned residuals are acceptable
# From the help file:
# Binned residual plots are achieved by "dividing the data into categories
# (bins) based on their fitted values, and then plotting the average residual
# versus the average fitted value for each bin." (Gelman, Hill 2007: 97).
# If the model were true, one would expect about 95% of the residuals to
# fall inside the error bounds.
bin_res_overall <- binned_residuals(bpd_occurr_ilrs)
bin_res_overall
```

```
Ok: About 100% of the residuals are inside the error bounds.
```

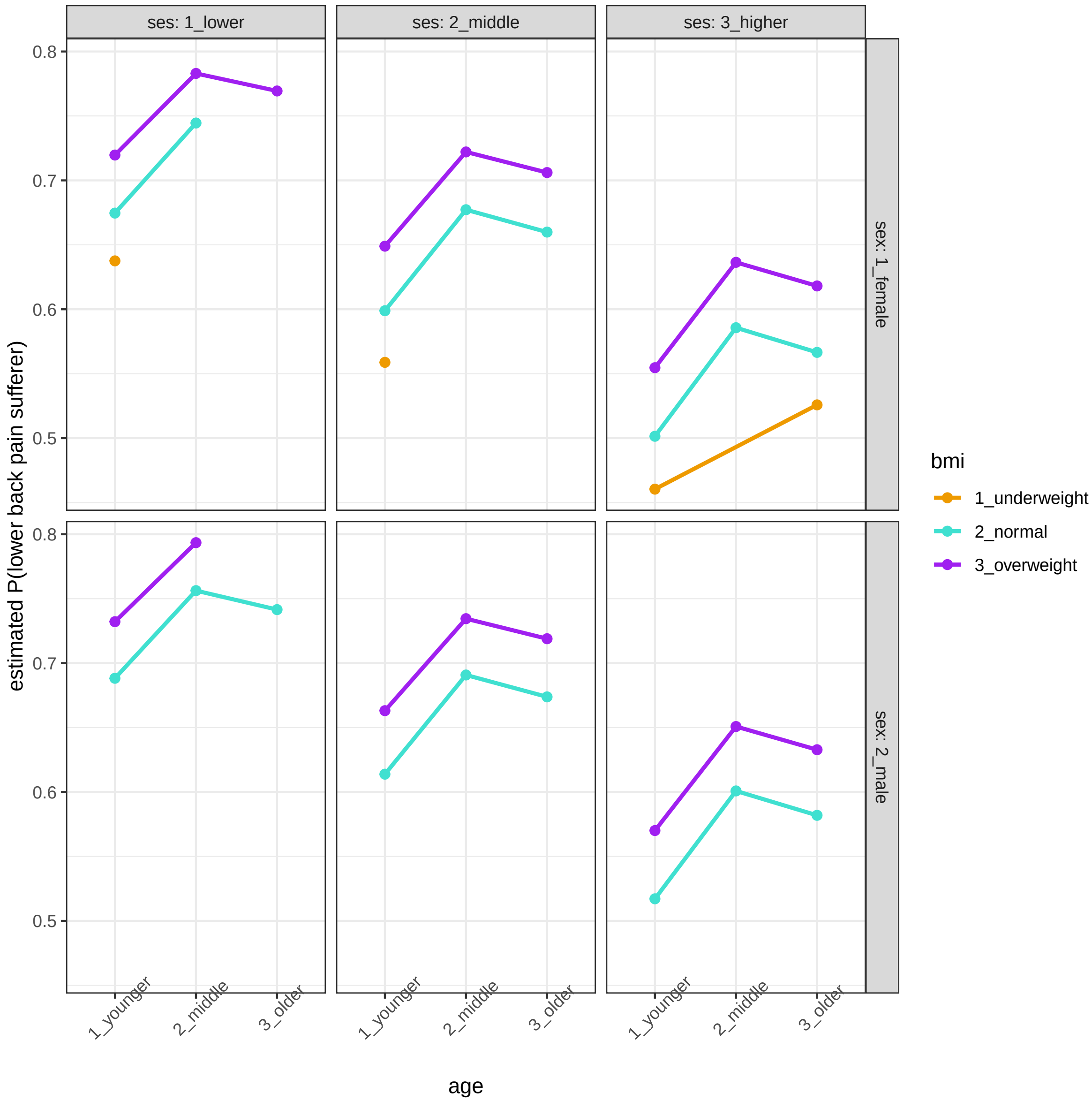
```
plot(bin_res_overall)
```



5.1.3 Model predictions

```r
# create dataset for predictions
newdata <-
  bpd %>%
  dplyr::select(all_of(pred_covs), ilr) %>%
  distinct(pick(all_of(pred_covs)), .keep_all = TRUE) %>%
  arrange(pick(all_of(pred_covs)))

mean_ilr <- mean(bpd$ilr)
dev_null <- foreach(i = 1:nrow(newdata)) %do% {
  newdata$ilr[i, ] <- mean_ilr
}

# make preds and then put in long format for ggplot
predictions_probs <-
  cbind(
    `P(LBP)` = predict(bpd_occurr_ilrs, newdata, type = "response"),
    newdata
  ) %>%
  dplyr::select(-ilr)

# predictions_probs


## model predictions for specific values
predictions_probs %>%
  dplyr::filter(
    # sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
    # ses == "2_middle"
  ) %>%
  ggplot(., aes(age, `P(LBP)`, group = bmi)) +
  geom_line(aes(colour = bmi), linewidth = 1) +
  geom_point(aes(colour = bmi), size = 2) +
  facet_grid(sex~ ses, labeller = label_both) +
  labs(x = "age", y = "estimated P(lower back pain sufferer)") +
  theme_bw() +
  scale_color_manual(values = c("orange2", "turquoise", "purple")) +
```

```r
  theme(axis.text.x = element_text(angle = 45))
```




```r
# create a RHS of regression equation dataset for time-reallocation
predict_basis <-
  bpd %>%
  dplyr::select(all_of(pred_covs), all_of(pred_comps)) %>%
```

```r
  dplyr::filter(
    age == "2_middle",
    sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
    ses == "2_middle",
    bmi == "2_normal"
  )

### continuous scenario
# predict_basis$age <- mean(predict_basis$age)

(predict_basis <-
  predict_basis %>%
  distinct(across(all_of(pred_covs)), .keep_all = TRUE) %>%
  as.data.frame())
```

```
      age       sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   546.4286
  Time_Sedentary Time_LPA Time_MVPA
1       418.2857 435.1429        40
```

```r
# compositional mean: geometric mean to closure
# (comp_mean <- mean(acomp(bpd[, pred_comps])))
(comp_mean <- calc_comp_mean(bpd[, pred_comps], clo_val = 1440))
```

```
    Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
     474.36588      439.73363      499.43836       26.46213
```

```r
predict_basis0 <- predict_basis
predict_basis0[, pred_comps] <- comp_mean

predict_basis0
```

```
      age       sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   474.3659
  Time_Sedentary Time_LPA Time_MVPA
1       439.7336 499.4384  26.46213
```

```r
# +15 minutes to Time_MVPA and -15 minutes from Time_Sedentary
comp_mean_changed <- comp_mean
comp_mean_changed["Time_MVPA"] <- comp_mean_changed["Time_MVPA"] + 15
comp_mean_changed["Time_Sedentary"] <- comp_mean_changed["Time_Sedentary"] - 15
# check
comp_mean_changed - comp_mean
```

```
   Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
            0            -15              0             15
```

```r
predict_basis1 <- predict_basis
predict_basis1[, pred_comps] <- comp_mean_changed

pred_df <- rbind(predict_basis0, predict_basis1)
pred_df <- add_ilrs_to_data(pred_df, comp_vars = pred_comps, sbp_matrix = sbp1)
pred_df
```

```
       age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   474.3659
2 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   474.3659
  Time_Sedentary Time_LPA Time_MVPA      ilr.1      ilr.2      ilr.3
1       439.7336 499.4384  26.46213 1.37947472 0.05360560 2.07731687
2       424.7336 499.4384  41.46213 1.13758831 0.07814711 1.75977934
```

```r
predict(bpd_occurr_ilrs, pred_df, type = "link")
```

```
        1         2
0.7410846 0.7577845
```

```r
# ratio of odds ratios
exp(diff(predict(bpd_occurr_ilrs, pred_df, type = "link")))
```

```
      2
1.01684
```

```r
get_pred_diff <- function(mod, new_dat) {
  log_odds_pred <- predict(mod, new_dat, type = "link")
  odds_ratio_ratio <- exp(log_odds_pred[2] - log_odds_pred[1])
  return(odds_ratio_ratio)
}
(est_v1 <- get_pred_diff(bpd_occurr_ilrs, pred_df))
```

```
      2
1.01684
```

```r
fit_mod_boot <- function(data, i, pred_dat) {

  this_dat <- data[i, ]
  this_logis <- glm(mod_form_ilrs, data = this_dat, family = binomial())
  est <- get_pred_diff(this_logis, new_dat = pred_dat)
  return(est)

}

### CI method #1 (bootstrapping):
alpha <- 0.05
(ci_v1 <-
  c(
    est = est_v1,
    quantile(
      boot(bpd, fit_mod_boot, R = 100, pred_dat = pred_df)$t,
      c(alpha / 2, 1 - alpha / 2)
    )))
```

```
    est.2      2.5%     97.5%
1.0168402 0.9873628 1.0511730
```

```r
### alternative CI method #2 (Wald approximation - re-transformed):
pred_df[, "ilr"]
```

```
         [,1]       [,2]     [,3]
[1,] 1.379475 0.05360560 2.077317
[2,] 1.137588 0.07814711 1.759779
```

```
attr(,"class")
[1] "rmult"
```

```
diff(pred_df[, "ilr"])
```

```
           [,1]       [,2]       [,3]
[1,] -0.2418864 0.02454151 -0.3175375
attr(,"class")
[1] "rmult"
```

```
x_0_red <- matrix(as.numeric(diff(pred_df[, "ilr"])), nrow = 1)
x_0_red
```

```
           [,1]       [,2]       [,3]
[1,] -0.2418864 0.02454151 -0.3175375
```

```
betas <- coef(bpd_occurr_ilrs)
nms_kp <- grepl("^ilr", names(betas))
betas_red <- as.matrix(betas[nms_kp])
Sigma <- stats::vcov(bpd_occurr_ilrs)
nms_kp <- grepl("^ilr", colnames(Sigma))
sigma_red <- Sigma[nms_kp, nms_kp]
sigma_red
```

```
             ilrilr(++--) ilrilr(+-..) ilrilr(..+-)
ilrilr(++--)  0.010731347   0.01369669 -0.005543191
ilrilr(+-..)  0.013696690   0.03159738 -0.008105390
ilrilr(..+-) -0.005543191  -0.00810539  0.005237994
```

```
est_red <- x_0_red %*% betas_red
se_red <- sqrt(x_0_red %*% sigma_red %*% t(x_0_red))
z_star <- qnorm(0.975)
(ci_v2 <-
  exp(c(
    est = est_red,
    lo = est_red - z_star * se_red,
```

```
    hi = est_red + z_star * se_red
  )))
```

```
      est        lo        hi 
1.0168402 0.9836173 1.0511852 
```

```
### alternative CI method #3 (delta method)
# (first order approximation, although still linear combin of param ests):
approx_ci <-
  deltaMethod(
    bpd_occurr_ilrs,
    "-0.2418864 * `ilrilr(++--)` + 0.02454151 * `ilrilr(+-..)` + -0.3175375 * `ilrilr(..+-
  )
(ci_v3 <-
  exp(c(
    est = approx_ci[["Estimate"]],
    lo = approx_ci[["2.5 %"]],
    hi = approx_ci[["97.5 %"]]
  )))
```

```
      est        lo        hi 
1.0168402 0.9836173 1.0511852 
```

```
### compare CIs
kable(rbind(ci_v1, ci_v2, ci_v3))
```

	est.2	2.5%	97.5%
ci_v1	1.01684	0.9873628	1.051173
ci_v2	1.01684	0.9836173	1.051185
ci_v3	1.01684	0.9836173	1.051185

```
do_multi_realloc <- function(mod, basis_data, timeusenames, time_changes, sbp_matrix = sbp


  x0 <- basis_data

  plot_dat <-
```

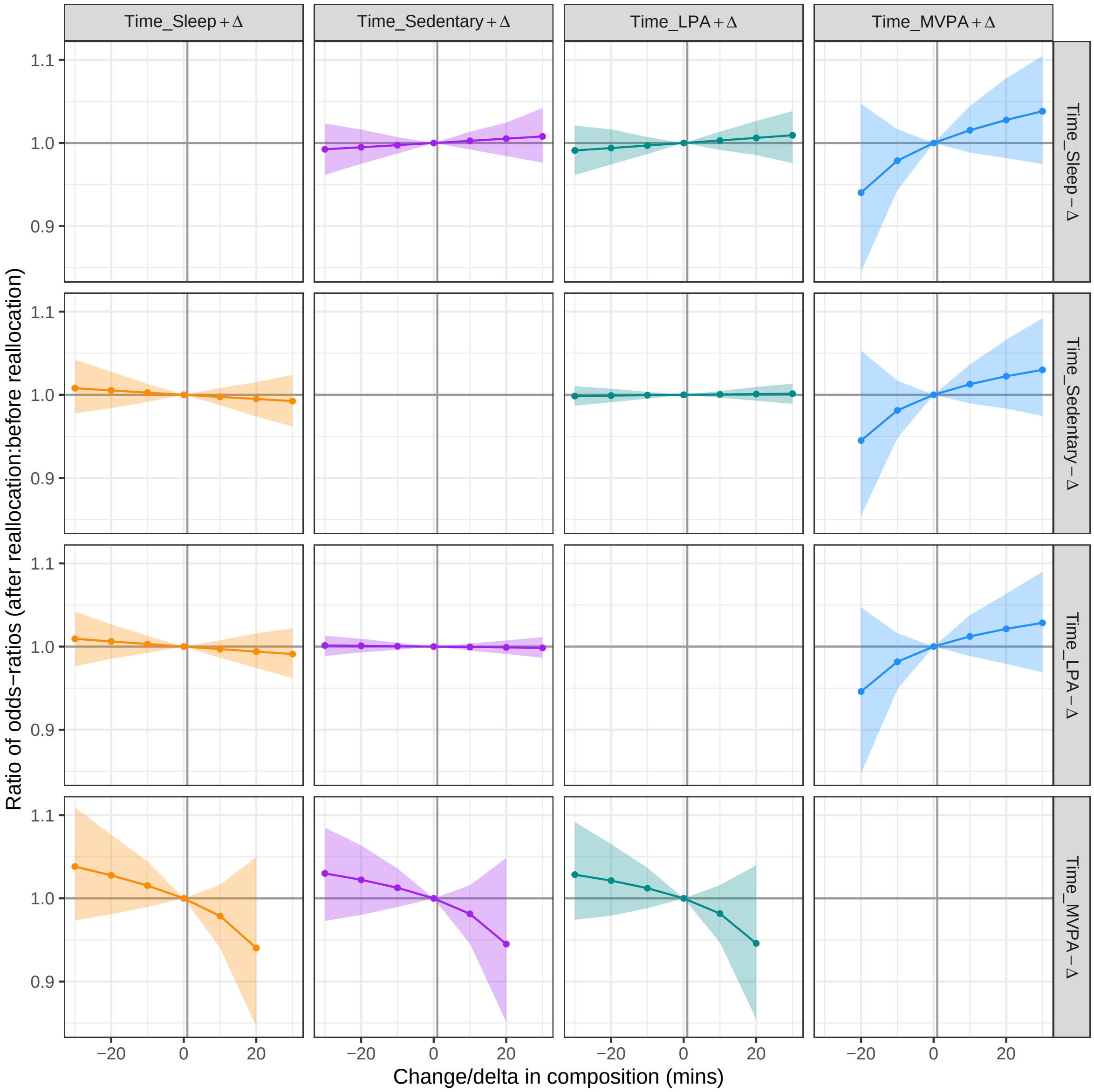
```
foreach(i = 1:length(timeusenames), .combine = bind_rows) %do% {
  print(paste("i: ", i))
  foreach(j = 1:length(timeusenames), .combine = bind_rows) %do% {
    print(paste("  j: ", j))
    foreach(d = 1:length(time_changes), .combine = bind_rows) %do% {
      print(paste("    d: ", d))

      timeuse_to <- timeusenames[i]
      timeuse_from <- timeusenames[j]
      change_time <- time_changes[d]

      proposed_change_1 <- x0[timeuse_to] + change_time
      proposed_change_2 <- x0[timeuse_from] - change_time

      if (timeuse_to == timeuse_from) {
        NULL # reallocation exceeds 0 or max time
      } else if ((proposed_change_1 < 0) | (proposed_change_1 > 1440)) {
        NULL # reallocation exceeds 0 or max time
      } else if ((proposed_change_2 < 0) | (proposed_change_2 > 1440)) {
        NULL # reallocation exceeds 0 or max time
      } else {

        x1 <- x0
        x1[timeuse_to] <- x1[timeuse_to] + change_time
        x1[timeuse_from] <- x1[timeuse_from] - change_time

        pred_df <- rbind(x0, x1)
        pred_df <- add_ilrs_to_data(pred_df, comp_vars = timeusenames, sbp_matrix = sb

        ratio_of_odds_ratios <- get_pred_diff(mod, pred_df)

        bootstrapped_ests <- boot(bpd, fit_mod_boot, R = 1000, pred_dat = pred_df)$t
        ci_est <- quantile(as.numeric(bootstrapped_ests), c(alpha / 2, 1 - alpha / 2))

        tibble(
          to = timeuse_to,
          from = timeuse_from,
          change_time = change_time,
          ratio_of_odds_ratios = ratio_of_odds_ratios,
          ci_lo = ci_est[1],
          ci_hi = ci_est[2]
```

```r
            )
          }
        }
      }
    }

  plot_dat$to <- factor(plot_dat$to, levels = timeusenames)
  plot_dat$from <- factor(plot_dat$from, levels = timeusenames)

  return(plot_dat)

}

set.seed(1234)

# takes ~25 min (single core)
tic()
# realloc_plot_data <-
#   do_multi_realloc(
#     bpd_occurr_ilrs,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )
toc()
```

```
0 sec elapsed
```

```r
# saveRDS(realloc_plot_data, file = "res/logistic_realloc_boot_res.rda")

realloc_plot_data <- readRDS(file = "res/logistic_realloc_boot_res.rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
  geom_vline(xintercept = 1, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
```

```r
geom_line(aes(x = change_time , y = ratio_of_odds_ratios, col = to)) +
geom_point(aes(x = change_time , y = ratio_of_odds_ratios, col = to), size = 1) +
facet_grid(from ~ to, labeller = label_parsed) +
theme_bw() +
scale_colour_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
scale_fill_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
labs(
  x = paste0("Change/delta in composition (mins)"),
  y = paste0("Ratio of odds-ratios (after reallocation:before reallocation)")
) +
theme(legend.position = "none")
```




```
ggsave(
  filename = "fig/lbp_occur_logistic_odds.png",
  dpi = 600, # print quality
  width = 10,
  height = 10
)
```

5.2 Note for outcomes 1 to 2

The dataset for the remain outcomes will be limited to people who responded:

```
bpd_yes <- bpd %>% dplyr::filter(LBP_sufferer == "yes")
nrow(bpd)
```

```
[1] 2333
```

```
nrow(bpd_yes)
```

```
[1] 1660
```

```
bpd_clean_yes <- bpd_clean %>% dplyr::filter(LBP_sufferer == "yes")
```

5.3 Outcome 1: LBP_frequency_year

5.3.1 Model fit

```
(this_outcome <- outcs[1])
```

```
[1] "LBP_frequency_year"
```

```
# (mod_form_null <-as.formula(paste0(this_outcome, " ~ ", rhs_formula)))
(mod_form_ilrs <-as.formula(paste0(this_outcome, " ~ ", rhs_formula, " + ilr")))
```

```
LBP_frequency_year ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```
table(bpd_yes[, this_outcome], useNA = "ifany")
```

```
LBP_frequency_year
        0days        1-7days       8-30days      31-90days 91+_not_evday
            0            760            451            146           203
     everyday
          100
```

```
bpd_yes[[this_outcome]] <- fct_drop(bpd_yes[[this_outcome]])
table(bpd_yes[, this_outcome], useNA = "ifany")
```

```
LBP_frequency_year
      1-7days       8-30days      31-90days 91+_not_evday       everyday
          760            451            146           203            100
```

```
## model without ilrs
# bpd_ordinal_null <- polr(mod_form_null, data = bpd, Hess = TRUE, method = "logistic")
# summary(bpd_ordinal_null)


## model __with__ ilrs
```

```r
bpd_ordinal_ilrs <- polr(mod_form_ilrs, data = bpd_yes, Hess = TRUE, method = "logistic")
summary(bpd_ordinal_ilrs)
```

```
Call:
polr(formula = mod_form_ilrs, data = bpd_yes, Hess = TRUE, method = "logistic")

Coefficients:
                     Value Std. Error t value
age2_middle        0.33918    0.10482  3.2360
age3_older         0.84204    0.16404  5.1331
sex2_male         -0.28829    0.11085 -2.6006
bmi2_normal       -0.02595    0.34969 -0.0742
bmi3_overweight    0.03744    0.35410  0.1057
stress2_stressed   0.57164    0.09929  5.7575
smoking2_nonsmoker -0.17602   0.11788 -1.4932
education2_higher -0.11931    0.10479 -1.1386
ses2_middle       -0.34624    0.14759 -2.3460
ses3_higher       -0.49685    0.20599 -2.4120
ilrilr(++--)      -0.30211    0.10322 -2.9268
ilrilr(+-..)      -0.56557    0.17197 -3.2888
ilrilr(..+-)       0.19891    0.07352  2.7055

Intercepts:
                         Value   Std. Error t value
1-7days|8-30days         -0.3467  0.3935    -0.8812
8-30days|31-90days        0.8660  0.3939     2.1987
31-90days|91+_not_evday   1.3945  0.3953     3.5277
91+_not_evday|everyday    2.6782  0.4038     6.6327

Residual Deviance: 4392.209
AIC: 4426.209
```

```r
Anova(bpd_ordinal_ilrs)
```

```
Analysis of Deviance Table (Type II tests)

Response: LBP_frequency_year
          LR Chisq Df Pr(>Chisq)
age         27.596  2  1.018e-06 ***
```

```
sex           6.822  1   0.009005 **
bmi           0.429  2   0.806925
stress       33.255  1  8.084e-09 ***
smoking       2.218  1   0.136374
education     1.294  1   0.255262
ses           6.813  2   0.033158 *
ilr          11.658  3   0.008651 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# pr <- profile(bpd_ordinal_ilrs)
# confint(pr)
# plot(pr)
# pairs(pr)


est_ci_df <- cbind(est = coef(bpd_ordinal_ilrs), confint(bpd_ordinal_ilrs)) # profiled CIs
```

```
Waiting for profiling to be done...
```

```
kable(est_ci_df, digits = 3)      # these are the log-odds scale estimates (and CI)
```

	est	2.5 %	97.5 %
age2_middle	0.339	0.134	0.545
age3_older	0.842	0.520	1.164
sex2_male	-0.288	-0.506	-0.072
bmi2_normal	-0.026	-0.704	0.675
bmi3_overweight	0.037	-0.649	0.747
stress2_stressed	0.572	0.377	0.767
smoking2_nonsmoker	-0.176	-0.406	0.056
education2_higher	-0.119	-0.325	0.086
ses2_middle	-0.346	-0.635	-0.056
ses3_higher	-0.497	-0.902	-0.094
ilrilr(++–)	-0.302	-0.505	-0.100
ilrilr(+-..)	-0.566	-0.904	-0.229
ilrilr(..+-)	0.199	0.055	0.343

```r
kable(exp(est_ci_df), digits = 3) # these are the odds ratios (and approx CIs)
```

	est	2.5 %	97.5 %
age2_middle	1.404	1.144	1.725
age3_older	2.321	1.683	3.202
sex2_male	0.750	0.603	0.931
bmi2_normal	0.974	0.495	1.965
bmi3_overweight	1.038	0.523	2.111
stress2_stressed	1.771	1.458	2.152
smoking2_nonsmoker	0.839	0.666	1.057
education2_higher	0.888	0.723	1.090
ses2_middle	0.707	0.530	0.945
ses3_higher	0.608	0.406	0.910
ilrilr(++–)	0.739	0.603	0.905
ilrilr(+-..)	0.568	0.405	0.795
ilrilr(..+-)	1.220	1.057	1.410

Ordinal logistic regression has fit the model:

$$
\begin{aligned}
logit(\hat{P}(Y \leq \texttt{1-7days})) &= \hat{\beta}_{0,\texttt{1-7days|8-30days}} - \hat{\beta}_1 \times (age) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)} \\
logit(\hat{P}(Y \leq \texttt{8-30days})) &= \hat{\beta}_{0,\texttt{8-30days|31-90days}} - \hat{\beta}_1 \times (age) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)} \\
logit(\hat{P}(Y \leq \texttt{31-90days})) &= \hat{\beta}_{0,\texttt{31-90days|91+\_not\_evday}} - \hat{\beta}_1 \times (age) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)} \\
logit(\hat{P}(Y \leq \texttt{91+\_not\_evday})) &= \hat{\beta}_{0,\texttt{91+\_not\_evday|everyday}} - \hat{\beta}_1 (age == \texttt{2\_middle}) - \ldots - \hat{\beta}_p \times \texttt{ilr(..+-)}
\end{aligned}
$$

5.3.2 Model diagnostics

```r
# deviance test
g2 <- deviance(bpd_ordinal_ilrs)
df <- df.residual(bpd_ordinal_ilrs)
1 - pchisq(g2, df)
```

```
[1] 0
```

```r
with(bpd_yes,
  table(
```

```
    LBP_frequency_year,
    as.numeric(LBP_frequency_year),
    useNA = "ifany"
  )
)
```

```
LBP_frequency_year   1   2   3   4   5
     1-7days       760   0   0   0   0
     8-30days        0 451   0   0   0
     31-90days       0   0 146   0   0
     91+_not_evday   0   0   0 203   0
     everyday        0   0   0   0 100
```

```
## checking parallel slopes assumptions can be done by fitting successive logistic regress
## while creating a binary outcome using different thresholds of the ordinal outcome
### (note the rhs/linear predictor is negative so coefs should be approx same
### as main model except negative)

# e.g. this is a single logistic regression
coef(glm(
  I(as.numeric(LBP_frequency_year) <= 1) ~
    age + sex + bmi + stress + smoking + education + ses + ilr,
  family = "binomial",
  data = bpd_yes
))
```

```
      (Intercept)        age2_middle         age3_older           sex2_male
      -0.60603839        -0.25411567        -0.65013781          0.24867275
      bmi2_normal    bmi3_overweight    stress2_stressed smoking2_nonsmoker
       0.17851384         0.11126358        -0.50892555          0.22359339
education2_higher        ses2_middle         ses3_higher        ilrilr(++--)
       0.08669237         0.33663193         0.46257283          0.28067682
     ilrilr(+-..)       ilrilr(..+-)
       0.44988574        -0.16448339
```

```
# this is running multiple logistic regressions
## we want to see the coefficients to be roughly the same (intercepts and negative coefs -

### note that the below shows there may be reason to include an age variable that has
```

```r
### non-constant coefficient for each level of the outcome (or subgroup analyses for each
### we can see this because the age coefs increase/decrease monotonically

foreach(i = 1:(length(levels(bpd_yes$LBP_frequency_year)) - 1), .combine = cbind) %do% {
  log_coefs <-
    coef(glm(
      I(as.numeric(LBP_frequency_year) <= i) ~
         age + sex + bmi + stress + smoking + education + ses + ilr,
      family = "binomial",
      data = bpd_yes
    ))
  log_coefs <- as.data.frame(log_coefs)
  colnames(log_coefs) <- paste0("logit(P(Y<=", i, "))")
  log_coefs
} %>%
  kable(., digits = 2)
```

	logit(P(Y<=1))	logit(P(Y<=2))	logit(P(Y<=3))	logit(P(Y<=4))
(Intercept)	-0.61	1.15	1.99	3.51
age2_middle	-0.25	-0.43	-0.50	-0.80
age3_older	-0.65	-0.99	-1.09	-1.69
sex2_male	0.25	0.33	0.29	0.31
bmi2_normal	0.18	0.00	-0.39	0.01
bmi3_overweight	0.11	0.01	-0.50	-0.25
stress2_stressed	-0.51	-0.65	-0.59	-0.94
smoking2_nonsmoker	0.22	0.12	0.10	-0.19
education2_higher	0.09	0.12	0.14	0.49
ses2_middle	0.34	0.32	0.35	0.31
ses3_higher	0.46	0.51	0.74	0.52
ilrilr(++-)	0.28	0.29	0.37	0.38
ilrilr(+-..)	0.45	0.69	0.71	0.54
ilrilr(..+-)	-0.16	-0.25	-0.25	-0.25

5.3.3 Model predictions

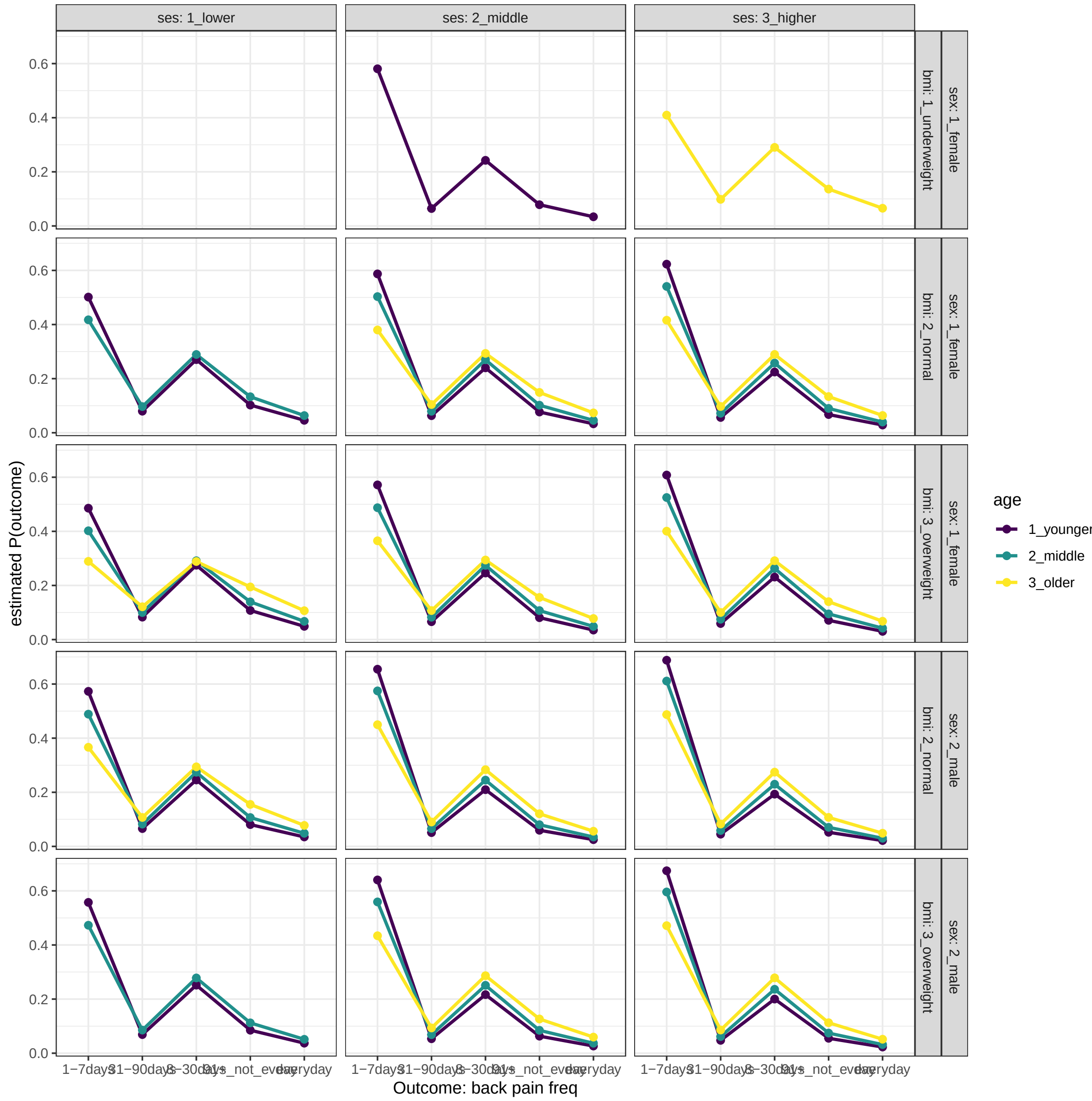
```r
# create dataset for predictions
newdata <-
  bpd_yes %>%
  dplyr::select(all_of(pred_covs), ilr) %>%
```

```r
  distinct(pick(all_of(pred_covs)), .keep_all = TRUE) %>%
  arrange(pick(all_of(pred_covs)))

mean_ilr <- mean(bpd_yes$ilr)
dev_null <- foreach(i = 1:nrow(newdata)) %do% {
  newdata$ilr[i, ] <- mean_ilr
}

# make preds and then put in long format for ggplot
predictions_probs <-
  cbind(
    predict(bpd_ordinal_ilrs, newdata, type = "probs"),
    newdata
  ) %>%
  dplyr::select(-ilr) %>%
  pivot_longer(
    cols = -all_of(pred_covs),
    names_to = "outcome",
    values_to = "P(outc)"
  )

predictions_probs
```

```
# A tibble: 1,030 x 9
   age       sex      bmi       stress smoking education ses   outcome `P(outc)`
   <fct>     <chr>    <fct>     <chr>  <chr>   <chr>     <chr> <chr>       <dbl>
 1 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 1-7days    0.451
 2 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 8-30da~    0.283
 3 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 31-90d~    0.0899
 4 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ 91+_no~    0.120
 5 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  1_lo~ everyd~    0.0558
 6 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 1-7days    0.537
 7 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 8-30da~    0.259
 8 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 31-90d~    0.0727
 9 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ 91+_no~    0.0910
10 1_younger 1_female 1_underw~ 1_nor~ 1_smok~ 2_higher  2_mi~ everyd~    0.0401
# i 1,020 more rows
```

```r
## model predictions for specific values
predictions_probs %>%
```

```r
dplyr::filter(
  # sex == "1_female",
  stress == "1_normal",
  smoking == "2_nonsmoker",
  education == "2_higher",
  # ses == "2_middle"
) %>%
ggplot(., aes(outcome, `P(outc)`)) +
geom_line(aes(colour =  age, group =  age), linewidth = 1) +
geom_point(aes(colour =  age), size = 2) +
facet_grid(sex * bmi ~ ses, labeller = label_both) +
labs(x = "Outcome: back pain freq", y = "estimated P(outcome)") +
theme_bw() +
scale_colour_viridis_d()
```




```
# create a RHS of regression equation dataset for time-reallocation
predict_basis <-
   bpd_yes %>%
   dplyr::select(all_of(pred_covs), all_of(pred_comps)) %>%
   dplyr::filter(
     age == "2_middle",
     sex == "1_female",
```

```r
      stress == "1_normal",
      smoking == "2_nonsmoker",
      education == "2_higher",
      ses == "2_middle",
      bmi == "2_normal"
    )

### continuous situation
# predict_basis$age <- mean(predict_basis$age)

(predict_basis <-
  predict_basis %>%
   distinct(across(all_of(pred_covs)), .keep_all = TRUE) %>%
   as.data.frame())
```

```
      age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   546.4286
  Time_Sedentary Time_LPA Time_MVPA
1       418.2857 435.1429        40
```

```r
# compositional mean: geometric mean to closure
# (comp_mean <- mean(acomp(bpd_yes[, pred_comps])))
(comp_mean <- calc_comp_mean(bpd_yes[, pred_comps], clo_val = 1440))
```

```
   Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
     472.8407       438.4062       502.1666        26.5865
```

```r
predict_basis0 <- predict_basis
predict_basis0[, pred_comps] <- comp_mean

predict_basis0
```

```
      age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
  Time_Sedentary Time_LPA Time_MVPA
1       438.4062 502.1666   26.5865
```

```r
# +15 minutes to Time_MVPA and -15 minutes from Time_Sedentary
comp_mean_changed <- comp_mean
comp_mean_changed["Time_MVPA"] <- comp_mean_changed["Time_MVPA"] + 15
comp_mean_changed["Time_Sedentary"] <- comp_mean_changed["Time_Sedentary"] - 15
# check
comp_mean_changed - comp_mean
```

```
   Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA 
            0            -15              0             15 
```

```r
predict_basis1 <- predict_basis
predict_basis1[, pred_comps] <- comp_mean_changed

pred_df <- rbind(predict_basis0, predict_basis1)
pred_df <- add_ilrs_to_data(pred_df, comp_vars = pred_comps, sbp_matrix = sbp1)
pred_df <- pred_df[, !(colnames(pred_df) %in% pred_comps)] # get rid of compositions
pred_df
```

```
       age      sex      bmi   stress     smoking education      ses      ilr.1
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle 1.37128443
2 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle 1.13019151
       ilr.2      ilr.3
1 0.05346627 2.07785318
2 0.07808340 1.76151343
```

```r
# model.matrix(formula(bpd_ordinal_ilrs), data = cbind(LBP_frequency_year = 0, pred_df))


df <- bpd_yes[, attr(formula(bpd_ordinal_ilrs), "term.labels")]
# this is a list of levels for each factor in the original df (after applying factor functi
xlevs <- lapply(df[,sapply(df, is.character), drop = F], function(j) {
  levels(factor(j))
})

# calling "xlev = " builds out a model.matrix with identical levels as the original df
mm_new <- model.matrix( ~ ., data = pred_df, xlev = xlevs)
colnames(mm_new)
```

```
 [1] "(Intercept)"        "age2_middle"        "age3_older"
 [4] "sex2_male"          "bmi2_normal"        "bmi3_overweight"
 [7] "stress2_stressed"   "smoking2_nonsmoker" "education2_higher"
[10] "ses2_middle"        "ses3_higher"        "ilr1"
[13] "ilr2"               "ilr3"
```

```
mm_new <- mm_new[, -1] # remove intercept


colnames(mm_new)[grepl("^ilr", colnames(mm_new))] <- paste0("ilr", create_ilr_names(sbp1))
# colnames(mm_new)
# don't need intercept  # c("(Intercept)" = 1, coef(bpd_ordinal_ilrs))
betas <- as.matrix(coef(bpd_ordinal_ilrs)) # should be col matrix
# rownames(betas)

if (!all(colnames(mm_new) == rownames(betas))) {
  stop("design and parameter est matrices non-conform")
}

# note as linear predictor is taken from the K intercepts the ratio of odds ratios is flip
# i.e. after:before of odds is calculated as exp(before_log_odds / after_log_odds)
preds <- mm_new %*% betas
exp(preds[1] - preds[2])
```

```
[1] 1.004017
```

```
# check manual calcs agree with model
mm_old <- model.matrix( ~ ., data = df, xlev = xlevs)
mm_old <- mm_old[, -1] # remove intercept
# colnames(mm_old)

# model and manual calcs agree?
# note that bpd_ordinal_ilrs$lp are the eta/linear predictor that is taken
# away from the xi_k intercept
all(abs(as.numeric(mm_old %*% betas) - bpd_ordinal_ilrs$lp) < 1e-9)
```

```
[1] TRUE
```

```r
# bpd_ordinal_ilrs$lp # linear predictor

get_pred_diff <- function(mod, new_dat) {
  betas_ <- as.matrix(coef(mod))
  if (!all(colnames(new_dat) == rownames(betas_))) {
    print(paste(paste(colnames(new_dat), collapse = "|"), "vs", paste(rownames(betas_), co
    stop("design and parameter est matrices non-conform")
  }
  log_odds_pred <- as.numeric(new_dat %*% betas_)
  # note reversal of order (see above)
  odds_ratio_ratio <- exp(log_odds_pred[1] - log_odds_pred[2])
  return(odds_ratio_ratio)
}
(est_v1 <- get_pred_diff(bpd_ordinal_ilrs, mm_new))
```

```
[1] 1.004017
```

```r
fit_mod_boot <- function(data, i, pred_dat) {

  this_dat <- data[i, ]
  this_ordinal <- polr(mod_form_ilrs, data = this_dat, Hess = TRUE, method = "logistic")

  df <- this_dat[, attr(formula(this_ordinal), "term.labels")]
  # this is a list of levels for each factor in the original df (after applying factor fun
  xlevs <- lapply(df[,sapply(df, is.character), drop = F], function(j) {
    levels(factor(j))
  })


  # calling "xlev = " builds out a model.matrix with identical levels as the original df
  mm_new <- model.matrix( ~ ., data = pred_dat, xlev = xlevs)
  mm_new <- mm_new[, -1] # remove intercept
  # make sure ilr colnames are legit/match coeffs
  colnames(mm_new)[grepl("^ilr", colnames(mm_new))] <- paste0("ilr", create_ilr_names(sbp1
  colnames(mm_new)

  est <- get_pred_diff(this_ordinal, new_dat = mm_new)
  return(est)

}
```

```r
### CI method #1 (bootstrapping):
alpha <- 0.05
(ci_v1 <-
    c(
      est = est_v1,
      quantile(
        boot(bpd_yes, fit_mod_boot, R = 100, pred_dat = pred_df)$t,
        c(alpha / 2, 1 - alpha / 2)
      )))
```

```
     est      2.5%     97.5% 
1.004017 0.974810 1.041748 
```

```r
### alternative CI method #2 (Wald approximation - re-transformed):
pred_df[, "ilr"]
```

```
         [,1]       [,2]     [,3]
[1,] 1.371284 0.05346627 2.077853
[2,] 1.130192 0.07808340 1.761513
attr(,"class")
[1] "rmult"
```

```r
diff(pred_df[, "ilr"])
```

```
           [,1]       [,2]       [,3]
[1,] -0.2410929 0.02461713 -0.3163398
attr(,"class")
[1] "rmult"
```

```r
# x_0_red <- matrix(- as.numeric(diff(pred_df[, "ilr"])), nrow = 1)
x_0_red <- matrix(as.numeric(pred_df[1, "ilr"] - pred_df[2, "ilr"]), nrow = 1)
x_0_red
```

```
          [,1]        [,2]      [,3]
[1,] 0.2410929 -0.02461713 0.3163398
```

```r
betas <- coef(bpd_ordinal_ilrs)
nms_kp <- grepl("^ilr", names(betas))
betas_red <- as.matrix(betas[nms_kp])
Sigma <- stats::vcov(bpd_ordinal_ilrs)
nms_kp <- grepl("^ilr", colnames(Sigma))
sigma_red <- Sigma[nms_kp, nms_kp]
sigma_red
```

```
             ilrilr(++--) ilrilr(+-..) ilrilr(..+-)
ilrilr(++--)   0.01065435   0.01320158  -0.00568300
ilrilr(+-..)   0.01320158   0.02957295  -0.00789973
ilrilr(..+-)  -0.00568300  -0.00789973   0.00540513
```

```r
est_red <- x_0_red %*% betas_red
se_red <- sqrt(x_0_red %*% sigma_red %*% t(x_0_red))
z_star <- qnorm(0.975)
(ci_v2 <-
    exp(c(
      est = est_red,
      lo = est_red - z_star * se_red,
      hi = est_red + z_star * se_red
    )))
```

```
      est        lo        hi
1.0040167 0.9717601 1.0373441
```

```r
### alternative CI method #3 (delta method)
# (first order approximation, although still linear combin of param ests):
as.numeric(x_0_red)
```

```
[1]  0.24109292 -0.02461713  0.31633975
```

```r
(g_form <- paste(
  paste(
    as.numeric(x_0_red),
    "*",
    c("`ilrilr(++--)`", "`ilrilr(+-..)`", "`ilrilr(..+-)`")
```

```
  ),
  collapse = " + "
))
```

```
[1] "0.241092921978823 * `ilrilr(++--)` + -0.0246171278023131 * `ilrilr(+-..)` + 0.316339752
```

```
approx_ci <-deltaMethod(bpd_ordinal_ilrs, g_form)
(ci_v3 <-
    exp(c(
      est = approx_ci[["Estimate"]],
      lo = approx_ci[["2.5 %"]],
      hi = approx_ci[["97.5 %"]]
    )))
```

```
      est        lo        hi
1.0040167 0.9717601 1.0373441
```

```
### compare CIs
kable(rbind(ci_v1, ci_v2, ci_v3))
```

	est	2.5%	97.5%
ci_v1	1.004017	0.9748100	1.041748
ci_v2	1.004017	0.9717601	1.037344
ci_v3	1.004017	0.9717601	1.037344

```
do_multi_realloc <- function(mod, basis_data, timeusenames, time_changes, sbp_matrix = sbp


  x0 <- basis_data

  plot_dat <-
    foreach(i = 1:length(timeusenames), .combine = bind_rows) %do% {
      print(paste("i: ", i))
      foreach(j = 1:length(timeusenames), .combine = bind_rows) %do% {
        print(paste("  j: ", j))
        foreach(d = 1:length(time_changes), .combine = bind_rows) %do% { # %dopar%
          print(paste("    d: ", d))
```

```
timeuse_to <- timeusenames[i]
timeuse_from <- timeusenames[j]
change_time <- time_changes[d]

proposed_change_1 <- x0[timeuse_to] + change_time
proposed_change_2 <- x0[timeuse_from] - change_time

if (timeuse_to == timeuse_from) {
  NULL # reallocation exceeds 0 or max time
} else if ((proposed_change_1 < 0) | (proposed_change_1 > 1440)) {
  NULL # reallocation exceeds 0 or max time
} else if ((proposed_change_2 < 0) | (proposed_change_2 > 1440)) {
  NULL # reallocation exceeds 0 or max time
} else {

  x1 <- x0
  x1[timeuse_to] <- x1[timeuse_to] + change_time
  x1[timeuse_from] <- x1[timeuse_from] - change_time

  pred_df <- rbind(x0, x1)
  pred_df <- add_ilrs_to_data(pred_df, comp_vars = timeusenames, sbp_matrix = sb

  ### alternative CI method #3 (delta method)
  # x_0_red <- -as.numeric(diff(pred_df[, "ilr"]))
  x_0_red <- as.numeric(pred_df[1, "ilr"] - pred_df[2, "ilr"])
  # (first order approximation, although still linear combin of param ests):
  (g_form <- paste(
    paste(
      x_0_red,
      "*",
      c("`ilrilr(++--)`", "`ilrilr(+-..)`", "`ilrilr(..+-)`")
    ),
    collapse = " + "
  ))
  approx_ci <-deltaMethod(bpd_ordinal_ilrs, g_form)
  this_ci <-
      exp(c(
        est = approx_ci[["Estimate"]],
        lo = approx_ci[["2.5 %"]],
        hi = approx_ci[["97.5 %"]]
      ))
```

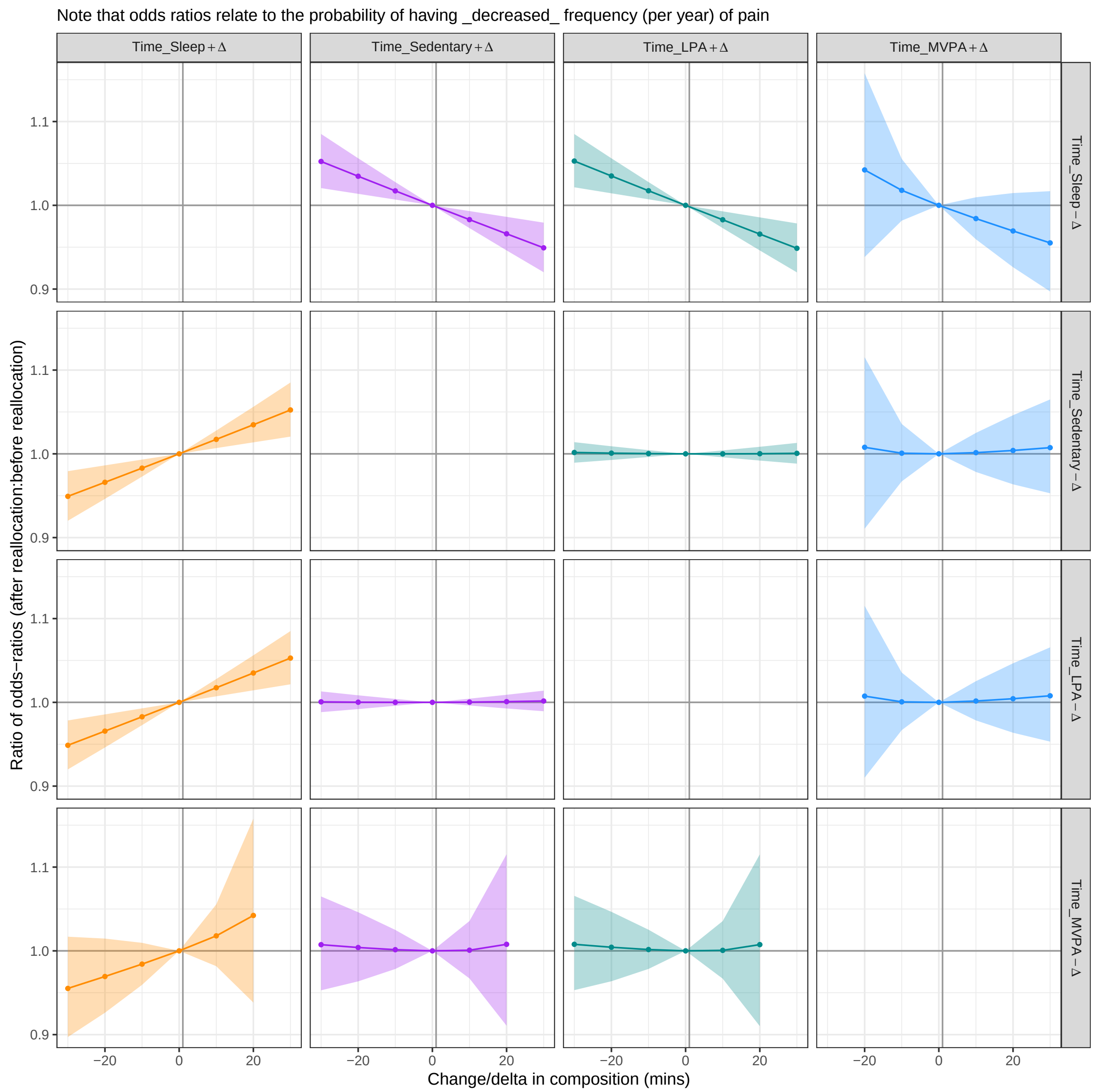
```
          ### bootstrapping takes too long
          # pred_df <- pred_df[, !(colnames(pred_df) %in% timeusenames)] # get rid of co
          #
          # df <- bpd_yes[, attr(formula(bpd_ordinal_ilrs), "term.labels")]
          # # this is a list of levels for each factor in the original df (after applyin
          # xlevs <- lapply(df[,sapply(df, is.character), drop = F], function(j) {
          #   levels(factor(j))
          # })
          #
          # # calling "xlev = " builds out a model.matrix with identical levels as the o
          # mm_new <- model.matrix( ~ ., data = pred_df, xlev = xlevs)
          # mm_new <- mm_new[, -1] # remove intercept
          # # make sure ilr colnames are legit/match coeffs
          # colnames(mm_new)[grepl("^ilr", colnames(mm_new))] <- paste0("ilr", create_il
          #
          # ratio_of_odds_ratios <- get_pred_diff(mod, new_dat = mm_new)
          # bootstrapped_ests <- boot(bpd_yes, fit_mod_boot, R = 10, pred_dat = pred_df)
          # ci_est <- quantile(as.numeric(bootstrapped_ests), c(alpha / 2, 1 - alpha / 2

          tibble(
            to = timeuse_to,
            from = timeuse_from,
            change_time = change_time,
            ratio_of_odds_ratios = this_ci["est"],
            ci_lo = this_ci["lo"],
            ci_hi = this_ci["hi"]
          )
        }
      }
    }
  }

  plot_dat$to <- factor(plot_dat$to, levels = timeusenames)
  plot_dat$from <- factor(plot_dat$from, levels = timeusenames)

  return(plot_dat)

}

set.seed(1234)
```

```r
# takes ~ 3h (single core) for bootstrapping
# takes ~ 4sec (single core) for delta/wald method
tic()

  # # library("doParallel")
  # # no_cores <- detectCores() - 1 # Calculate the number of cores (leave one free)
  # # cl <- makeCluster(no_cores) # Create clusters
  # # registerDoParallel(cl) # and register

# realloc_plot_data <-
#   do_multi_realloc(
#     bpd_ordinal_ilrs,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )

  # # # close para comp
  # # stopCluster(cl)

toc()
```

```
0 sec elapsed
```

```r
# saveRDS(realloc_plot_data, file = "res/ordinal_realloc_boot_res.rda")
# realloc_plot_data <- readRDS(file = "res/ordinal_realloc_boot_res.rda")

# saveRDS(realloc_plot_data, file = "res/ordinal_realloc_wald_res.rda")
realloc_plot_data <- readRDS(file = "res/ordinal_realloc_wald_res.rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
  geom_vline(xintercept = 1, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
  geom_line(aes(x = change_time , y = ratio_of_odds_ratios, col = to)) +
  geom_point(aes(x = change_time , y = ratio_of_odds_ratios, col = to), size = 1) +
```

```
facet_grid(from ~ to, labeller = label_parsed) +
theme_bw() +
scale_colour_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
scale_fill_manual(values = c("darkorange", "purple", "cyan4", "dodgerblue")) +
labs(
  x = paste0("Change/delta in composition (mins)"),
  y = paste0("Ratio of odds-ratios (after reallocation:before reallocation)"),
  subtitle = "Note that odds ratios relate to the probability of having _decreased_ freq
) +
theme(legend.position = "none")
```




```r
ggsave(
  filename = "fig/lbp_freq_ordinal_odds_v1.png",
  dpi = 600, # print quality
  width = 10,
  height = 10
)
```

```r
time_lvls <- gsub("Time_", "", pred_comps)

rep_char <- function(n, char = " ") paste(rep(char, n), collapse = "")
rep_char(3)
```

```
[1] "   "
```

```r
rep_char(0)
```

```
[1] ""
```

```r
rep_char <- Vectorize(rep_char, vectorize.args = "n")
rep_char(0:7)
```

```
[1] ""        " "       "  "      "   "     "    "    "     "   "      "
[8] "       "
```

```r
pd2 <-
  realloc_plot_data %>%
  mutate(
    to = gsub("Time_", "", to),
    from = gsub("Time_", "", from),
    to = gsub("+Delta", "", to, fixed = TRUE),
    from = gsub("-Delta", "", from, fixed = TRUE),
    to_len = nchar(to),
    to_max = max(to_len),
    from_len = nchar(from),
    from_max = max(from_len),
    to_pad =  rep_char(pmax(0, from_max - to_len)),
    from_pad = rep_char(pmax(0, to_max - from_len)),
    to = factor(to, levels = time_lvls),
    from = factor(from, levels = time_lvls),
```

```r
    to_num = as.numeric(to),
    from_num = as.numeric(from)
  ) %>%
  dplyr::filter(to_num > from_num) %>%
  mutate(
    ratio_of_odds_ratios = 1 / ratio_of_odds_ratios,
    tmp = 1 / ci_lo,
    ci_lo = 1 / ci_hi,
    ci_hi = tmp,
    # from_to = paste0("      ", "+", from, rep_char(10), from_pad, "\u2194", to_pad, rep_c
    from_to = paste0("+", from, rep_char(13), from_pad, "", to_pad, rep_char(13), "+", to)
  ) %>%
  arrange(from, to)

unique(pd2$from_to)
```

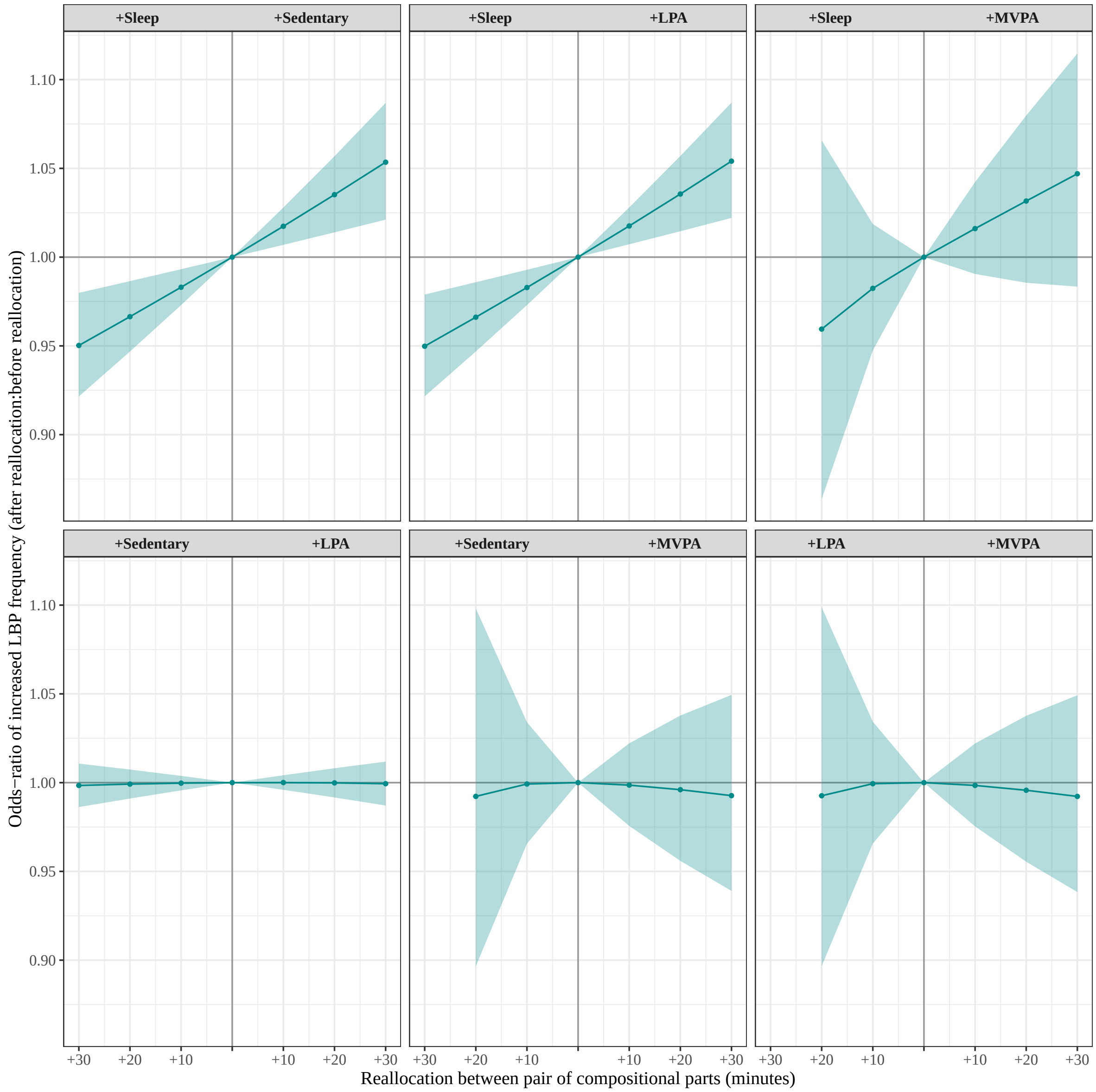
```
[1] "+Sleep                                        +Sedentary"
[2] "+Sleep                                              +LPA"
[3] "+Sleep                                             +MVPA"
[4] "+Sedentary                                          +LPA"
[5] "+Sedentary                                         +MVPA"
[6] "+LPA                                               +MVPA"
```

```r
pd2$from_to <- factor(pd2$from_to, levels = unique(pd2$from_to))

this_breaks <- seq(-30, 30, 10)
this_labs <- sprintf("+%2.0f", abs(seq(-30, 30, 10)))
this_labs[this_labs == "+ 0"] <- ""
this_labs
```

```
[1] "+30" "+20" "+10" ""    "+10" "+20" "+30"
```

```r
ggplot(pd2) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3, co
  geom_line(aes(x = change_time , y = ratio_of_odds_ratios, col = to), col = "cyan4") +
  geom_point(aes(x = change_time , y = ratio_of_odds_ratios, col = to), size = 1, col = "c
  facet_wrap(~ from_to, labeller = label_bquote(.(from_to))) +
```

```
theme_bw() +
scale_x_continuous(breaks = this_breaks, labels = this_labs) +
labs(
  x = paste0("Reallocation between pair of compositional parts (minutes)"),
  y = paste0("Odds-ratio of increased LBP frequency (after reallocation:before reallocat
  # subtitle = "Note that odds ratios relate to the probability of having _increased_ fr
) +
theme(
  legend.position = "none",
  text = element_text(family = "serif"),
  strip.text = element_text(size = 10, face = "bold"),
  axis.text =  element_text(size = 10),
  axis.title =  element_text(size = 12)
)
```




```r
ggsave(filename = "fig/lbp_freq_ordinal_odds_v2.png", width = 14, height = 9, dpi = 600)
# ggsave(filename = "fig/lbp_freq_ordinal_odds.pdf", width = 10, height = 8)



 # ---- outcome1_pred_not_use ----
```

```r
# logitP(Y k x)= _k-
# zeta_{1-7days|8-30days} = -0.2910
# eta = 0.3184 + -0.1786 + -0.1110 + -0.3463 + -0.3017
coef(bpd_ordinal_ilrs)
```

```
     age2_middle         age3_older           sex2_male         bmi2_normal
      0.33918326         0.84203588         -0.28828961         -0.02594625
 bmi3_overweight   stress2_stressed  smoking2_nonsmoker   education2_higher
      0.03744251         0.57164263         -0.17602164         -0.11931436
     ses2_middle         ses3_higher         ilrilr(++--)         ilrilr(+-..)
     -0.34624303        -0.49685106         -0.30210820         -0.56557283
    ilrilr(..+-)
      0.19890657
```

```r
# summary(bpd_ordinal_ilrs)
bpd_ordinal_ilrs$zeta
```

```
     1-7days|8-30days      8-30days|31-90days 31-90days|91+_not_evday
           -0.3467297               0.8659799               1.3944591
91+_not_evday|everyday
            2.6782000
```

```r
# bpd_ordinal_ilrs$lp
# p_0 <- predict(bpd_ordinal_ilrs, pred_df, type = "prob")
# (lodr <- log(p_0 / (1-p_0)))
# # ratio of odds ratios
# exp(apply(lodr, 2, diff))
# # predicted class  argmin_k{abs(zeta_k - eta)}?
# predict(bpd_ordinal_ilrs, type = "class")[1:3]
# p_m <- matrix(rep(bpd_ordinal_ilrs$lp, 5), ncol = 5)
# co_m <- matrix(rep(c(bpd_ordinal_ilrs$zeta, 0), nrow(p_m)), ncol = 5, byrow = TRUE)
# apply(abs(p_m - co_m), 1, which.min)[1:3]
#
# table(
#   predict(bpd_ordinal_ilrs, type = "class"),
#   apply(abs(p_m - co_m), 1, which.min) # + 1) %% 5
# )
```

5.4 Outcome 2: `LBP_intensity_year`

5.4.1 Model fit

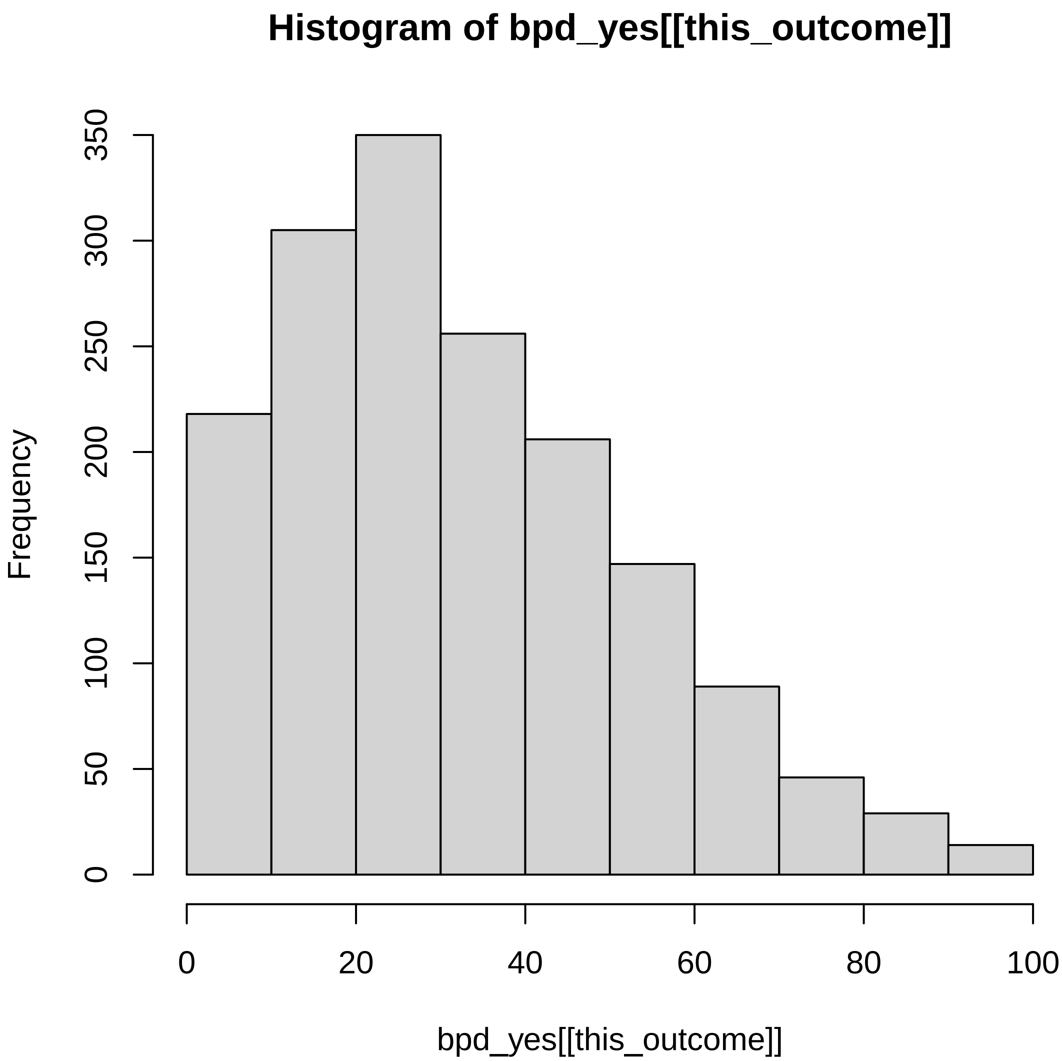
```
(this_outcome <- outcs[2])
```

```
[1] "LBP_intensity_year"
```

```
# (mod_form_null <-as.formula(paste0(this_outcome, " ~ ", rhs_formula)))
(mod_form_ilrs <- as.formula(paste0(this_outcome, " ~ ", rhs_formula, " + ilr")))
```

```
LBP_intensity_year ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```
hist(bpd_yes[[this_outcome]])
```




```r
lbp_intensity_lm <- lm(mod_form_ilrs, data = bpd_yes)
summary(lbp_intensity_lm)
```

```
Call:
lm(formula = mod_form_ilrs, data = bpd_yes)

Residuals:
    Min      1Q  Median      3Q     Max
```

```
-42.94 -14.78  -3.03  12.16  69.47

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)         40.3475     4.3139   9.353  < 2e-16 ***
age2_middle          0.5727     1.1059   0.518 0.604605    
age3_older           7.2422     1.7224   4.205 2.75e-05 ***
sex2_male           -3.5483     1.1587  -3.062 0.002231 ** 
bmi2_normal         -0.5308     3.8464  -0.138 0.890251    
bmi3_overweight      2.4503     3.8931   0.629 0.529177    
stress2_stressed     4.4659     1.0541   4.237 2.39e-05 ***
smoking2_nonsmoker   1.3548     1.2786   1.060 0.289471    
education2_higher   -3.2982     1.1148  -2.959 0.003134 ** 
ses2_middle         -4.2698     1.6008  -2.667 0.007722 ** 
ses3_higher         -7.4351     2.1993  -3.381 0.000740 ***
ilrilr(++--)        -2.0302     1.0869  -1.868 0.061949 .  
ilrilr(+-..)        -6.9931     1.7974  -3.891 0.000104 ***
ilrilr(..+-)        -0.4219     0.7671  -0.550 0.582430    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 19.81 on 1646 degrees of freedom
Multiple R-squared:  0.06251,   Adjusted R-squared:  0.0551 
F-statistic: 8.442 on 13 and 1646 DF,  p-value: < 2.2e-16
```

```r
car::Anova(lbp_intensity_lm)
```

```
Anova Table (Type II tests)

Response: LBP_intensity_year
          Sum Sq   Df F value    Pr(>F)    
age         7709    2  9.8229 5.744e-05 ***
sex         3680    1  9.3784  0.002231 ** 
bmi         3279    2  4.1782  0.015489 *  
stress      7043    1 17.9495 2.394e-05 ***
smoking      441    1  1.1228  0.289471    
education   3435    1  8.7534  0.003134 ** 
ses         4605    2  5.8679  0.002888 ** 
ilr        11652    3  9.8986 1.810e-06 ***
Residuals 645859 1646                      
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
### THis is logodds transform of outcome, not a good fit
# # move extreme values off boundary
# bpd_yes$intensity <- bpd_yes$LBP_intensity_year
# bpd_yes$intensity[bpd_yes$LBP_intensity_year < 0.5] <- 0.5
# bpd_yes$intensity[bpd_yes$LBP_intensity_year > (100 - 0.5)] <- 100 - 0.5
# bpd_yes$logodds_intensity <- with(bpd_yes, log((intensity / 100) / (1 - intensity / 100)
# bpd_yes$intensity <- NULL
# (mod_form_logodds_ilrs <- as.formula(paste0("logodds_intensity ~ ", rhs_formula, " + ilr
# lbp_intensity_logodds_lm <- lm(mod_form_logodds_ilrs, data = bpd_yes)
# summary(lbp_intensity_logodds_lm)
# car::Anova(lbp_intensity_logodds_lm)
# check_model(lbp_intensity_logodds_lm)

lbp_intensity_pois <- glm(mod_form_ilrs, family = "poisson", data = bpd_yes)
summary(lbp_intensity_pois)
```

```
Call:
glm(formula = mod_form_ilrs, family = "poisson", data = bpd_yes)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-9.3235  -2.7749  -0.5123   1.9990  10.0312

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
(Intercept)         3.693194   0.037276  99.077  < 2e-16 ***
age2_middle         0.016658   0.009724   1.713 0.086684 .
age3_older          0.205646   0.014486  14.197  < 2e-16 ***
sex2_male          -0.105547   0.010228 -10.319  < 2e-16 ***
bmi2_normal        -0.017978   0.033512  -0.536 0.591644
bmi3_overweight     0.069579   0.033867   2.054 0.039929 *
stress2_stressed    0.129912   0.009082  14.305  < 2e-16 ***
smoking2_nonsmoker  0.040861   0.011211   3.645 0.000268 ***
education2_higher  -0.095585   0.009529 -10.031  < 2e-16 ***
ses2_middle        -0.112664   0.013084  -8.611  < 2e-16 ***
ses3_higher        -0.209084   0.019007 -11.000  < 2e-16 ***
ilrilr(++--)       -0.057087   0.009171  -6.225 4.83e-10 ***
ilrilr(+-..)       -0.202889   0.015413 -13.164  < 2e-16 ***
ilrilr(..+-)       -0.014136   0.006519  -2.168 0.030126 *
```

```
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for poisson family taken to be 1)

    Null deviance: 21235  on 1659  degrees of freedom
Residual deviance: 19981  on 1646  degrees of freedom
AIC: 28412

Number of Fisher Scoring iterations: 5
```

```r
# check the goodness of fit test not significant,
# p > 0.05: indicates model fit the data
# p < 0.05: indicates model DOES NOIT fit the data
with(
  lbp_intensity_pois,
  cbind(
    res.deviance = deviance,
    df = df.residual,
    p = pchisq(deviance, df.residual, lower.tail = FALSE)
  )
)
```

```
     res.deviance   df p
[1,]     19981.33 1646 0
```

```r
lbp_intensity_nb <- glm.nb(mod_form_ilrs, data = bpd_yes)
summary(lbp_intensity_nb)
```

```
Call:
glm.nb(formula = mod_form_ilrs, data = bpd_yes, init.theta = 2.48631581,
    link = log)

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-3.8080  -0.7834  -0.1351   0.5009   2.3253

Coefficients:
                    Estimate Std. Error z value Pr(>|z|)
```

```
(Intercept)          3.679463   0.143204  25.694  < 2e-16 ***
age2_middle          0.016056   0.036738   0.437 0.662083
age3_older           0.200337   0.057057   3.511 0.000446 ***
sex2_male           -0.104393   0.038509  -2.711 0.006710 **
bmi2_normal         -0.003187   0.127766  -0.025 0.980099
bmi3_overweight      0.086268   0.129300   0.667 0.504650
stress2_stressed     0.133104   0.034966   3.807 0.000141 ***
smoking2_nonsmoker   0.042290   0.042475   0.996 0.319415
education2_higher   -0.099687   0.036963  -2.697 0.006998 **
ses2_middle         -0.105312   0.052931  -1.990 0.046636 *
ses3_higher         -0.207810   0.072984  -2.847 0.004409 **
ilrilr(++--)        -0.053990   0.036021  -1.499 0.133915
ilrilr(+-..)        -0.199774   0.059664  -3.348 0.000813 ***
ilrilr(..+-)        -0.019407   0.025432  -0.763 0.445391
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for Negative Binomial(2.4863) family taken to be 1)

    Null deviance: 1983.2  on 1659  degrees of freedom
Residual deviance: 1897.9  on 1646  degrees of freedom
AIC: 14547

Number of Fisher Scoring iterations: 1

              Theta:  2.4863
          Std. Err.:  0.0942

 2 x log-likelihood:  -14517.0260
```

```
car::Anova(lbp_intensity_nb)
```

```
Analysis of Deviance Table (Type II tests)

Response: LBP_intensity_year
          LR Chisq Df Pr(>Chisq)
age        14.0426  2  0.0008927 ***
sex         7.2582  1  0.0070576 **
bmi         6.8347  2  0.0327987 *
stress     14.4891  1  0.0001410 ***
smoking     0.9800  1  0.3221987
```

```
education    7.2378  1  0.0071383 **
ses          8.0144  2  0.0181840 *
ilr         23.4695  3  3.223e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
(est <- cbind(Estimate = coef(lbp_intensity_nb), confint(lbp_intensity_nb)))
```

```
Waiting for profiling to be done...
```

```
                      Estimate       2.5 %       97.5 %
(Intercept)        3.679462775  3.40299626  3.969011870
age2_middle        0.016055946 -0.05598109  0.087856035
age3_older         0.200337071  0.08850561  0.313532033
sex2_male         -0.104393094 -0.17957684 -0.028571870
bmi2_normal       -0.003187056 -0.26156507  0.239011069
bmi3_overweight    0.086268083 -0.17496158  0.331713392
stress2_stressed   0.133103716  0.06446472  0.201989018
smoking2_nonsmoker 0.042290260 -0.04175439  0.125095866
education2_higher -0.099687445 -0.17285762 -0.026981253
ses2_middle       -0.105311819 -0.21064524 -0.001965261
ses3_higher       -0.207809938 -0.35179957 -0.063592059
ilrilr(++--)      -0.053989622 -0.12392266  0.015888706
ilrilr(+-..)      -0.199773802 -0.31581509 -0.083907349
ilrilr(..+-)      -0.019407373 -0.06865999  0.029729611
```

```r
exp(est)
```

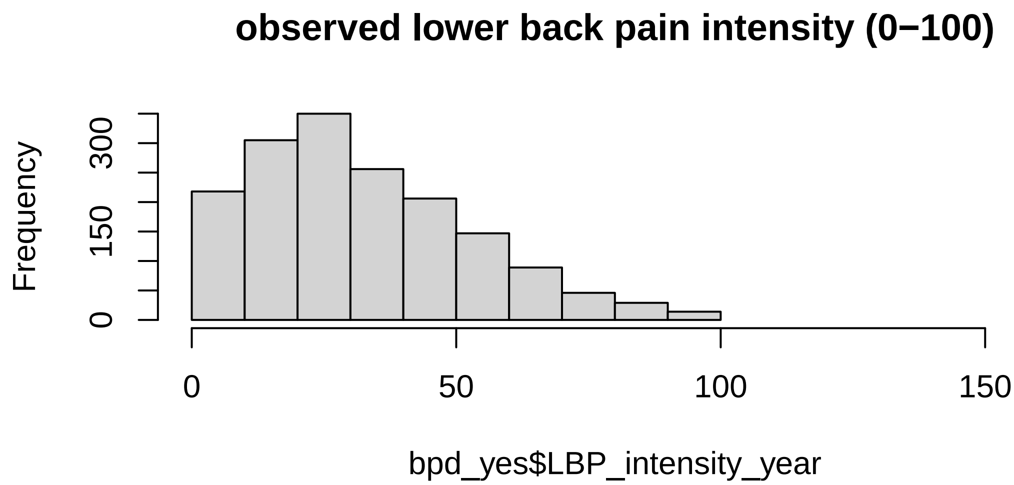
```
                    Estimate      2.5 %     97.5 %
(Intercept)       39.6251008 30.0540149 52.9322011
age2_middle        1.0161855  0.9455570  1.0918309
age3_older         1.2218145  1.0925404  1.3682493
sex2_male          0.9008711  0.8356237  0.9718324
bmi2_normal        0.9968180  0.7698458  1.2699926
bmi3_overweight    1.0900985  0.8394893  1.3933534
stress2_stressed   1.1423685  1.0665879  1.2238346
smoking2_nonsmoker 1.0431972  0.9591053  1.1332571
education2_higher  0.9051203  0.8412574  0.9733795
ses2_middle        0.9000438  0.8100614  0.9980367
```

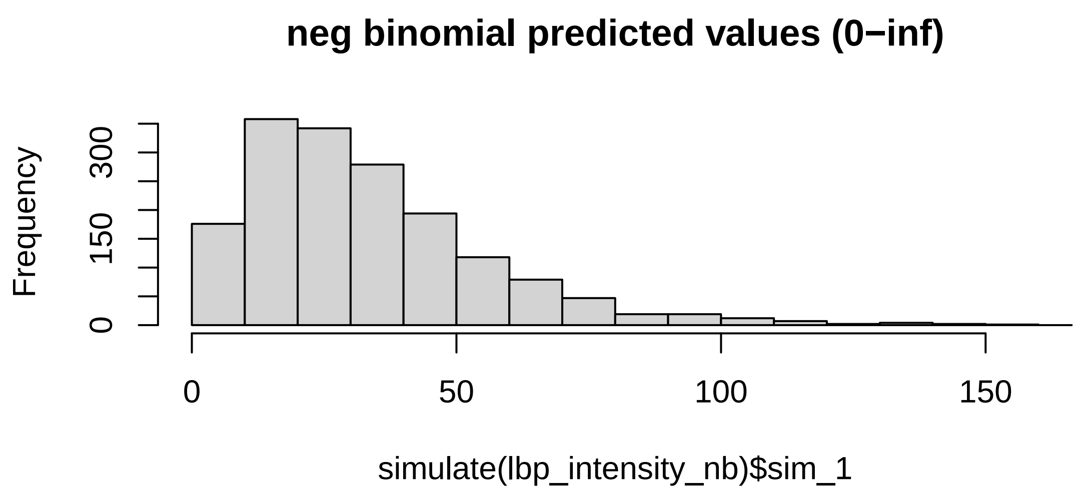
```
ses3_higher          0.8123614  0.7034211  0.9383877
ilrilr(++--)         0.9474419  0.8834482  1.0160156
ilrilr(+-..)         0.8189160  0.7291943  0.9195164
ilrilr(..+-)         0.9807797  0.9336441  1.0301759
```

```r
# likelihood ratio test
# dispersion parameter check is it equal to zero (if not, NB mod preferred)
pchisq(
  2 * (logLik(lbp_intensity_nb) - logLik(lbp_intensity_pois)),
  df = 1,
  lower.tail = FALSE
)
```

```
'log Lik.' 0 (df=15)
```

```r
par(mfrow = c(2, 1))
hist(bpd_yes$LBP_intensity_year, xlim = c(0, 160), breaks = 10,
     main = "observed lower back pain intensity (0-100)")
hist(simulate(lbp_intensity_nb)$sim_1, xlim = c(0, 160), breaks = 20,
     main = "neg binomial predicted values (0-inf)")
```






```r
par(mfrow = c(1, 1))


# Pain intensity could be categorised as (Boonstra et al., 2014):
# - no pain (0)
# - mild pain (1-38)
# - moderate pain (39-57)
# - severe pain (58-100)
```

```r
bpd_yes$intens_ord <-
  cut(
    bpd_yes[[this_outcome]],
    breaks = c(-1, 0, 38, 57, 101),
    labels = c(
      "no pain (0)", "mild pain (1-38)",
      "moderate pain (39-57)", "severe pain (58-100)"
    )
  )
class(bpd_yes$intens_ord)
```

```
[1] "factor"
```

```r
table(
  bpd_yes$intens_ord,
  cut(bpd_yes[[this_outcome]], breaks = c(-1, 0, 38, 57, 101)),
  useNA = "ifany"
)
```

```
                        (-1,0] (0,38] (38,57] (57,101]
  no pain (0)               45      0       0        0
  mild pain (1-38)           0    974       0        0
  moderate pain (39-57)      0      0     401        0
  severe pain (58-100)       0      0       0      240
```

```r
(mod_form_ord_ilrs <-as.formula(paste0("intens_ord ~ ", rhs_formula, " + ilr")))
```

```
intens_ord ~ age + sex + bmi + stress + smoking + education +
    ses + ilr
```

```r
## model __with__ ilrs
bpd_intens_ord_ilrs <- polr(mod_form_ord_ilrs, data = bpd_yes, Hess = TRUE, method = "logi
summary(bpd_intens_ord_ilrs)
```

```
Call:
polr(formula = mod_form_ord_ilrs, data = bpd_yes, Hess = TRUE,
    method = "logistic")

Coefficients:
                      Value Std. Error t value
age2_middle         0.05704    0.11183  0.5100
age3_older          0.60483    0.16942  3.5700
sex2_male          -0.41189    0.11909 -3.4585
bmi2_normal         0.14649    0.38765  0.3779
bmi3_overweight     0.47744    0.39207  1.2177
stress2_stressed    0.43625    0.10534  4.1416
smoking2_nonsmoker  0.15893    0.12884  1.2335
education2_higher  -0.29335    0.11024 -2.6612
ses2_middle        -0.43164    0.15546 -2.7766
ses3_higher        -0.79524    0.22275 -3.5701
ilrilr(++--)       -0.14674    0.10909 -1.3451
ilrilr(+-..)       -0.60042    0.18119 -3.3138
ilrilr(..+-)       -0.04818    0.07715 -0.6245

Intercepts:
                                          Value   Std. Error t value
no pain (0)|mild pain (1-38)              -4.0583  0.4556    -8.9078
mild pain (1-38)|moderate pain (39-57)     0.1189  0.4312     0.2757
moderate pain (39-57)|severe pain (58-100)  1.4918  0.4332     3.4438

Residual Deviance: 3330.714
AIC: 3362.714
```

```
Anova(bpd_intens_ord_ilrs)
```

```
Analysis of Deviance Table (Type II tests)

Response: intens_ord
          LR Chisq Df Pr(>Chisq)
age        13.8442  2  0.0009858 ***
sex        12.1816  1  0.0004826 ***
bmi        10.7801  2  0.0045617 **
stress     17.1554  1  3.444e-05 ***
smoking     1.5339  1  0.2155292
education   7.0539  1  0.0079094 **
```

```
ses         13.2177  2  0.0013484 **
ilr         22.0152  3  6.476e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# profiled CIs
est_ci_df <- cbind(est = coef(bpd_intens_ord_ilrs), confint(bpd_intens_ord_ilrs))
```

```
Waiting for profiling to be done...
```

```r
kable(est_ci_df, digits = 3)      # these are the log-odds scale estimates (and CI)
```

	est	2.5 %	97.5 %
age2_middle	0.057	-0.162	0.277
age3_older	0.605	0.272	0.937
sex2_male	-0.412	-0.647	-0.180
bmi2_normal	0.146	-0.595	0.932
bmi3_overweight	0.477	-0.273	1.271
stress2_stressed	0.436	0.230	0.643
smoking2_nonsmoker	0.159	-0.092	0.413
education2_higher	-0.293	-0.509	-0.077
ses2_middle	-0.432	-0.736	-0.126
ses3_higher	-0.795	-1.234	-0.360
ilrilr(++–)	-0.147	-0.361	0.067
ilrilr(+-..)	-0.600	-0.957	-0.246
ilrilr(..+-)	-0.048	-0.199	0.103

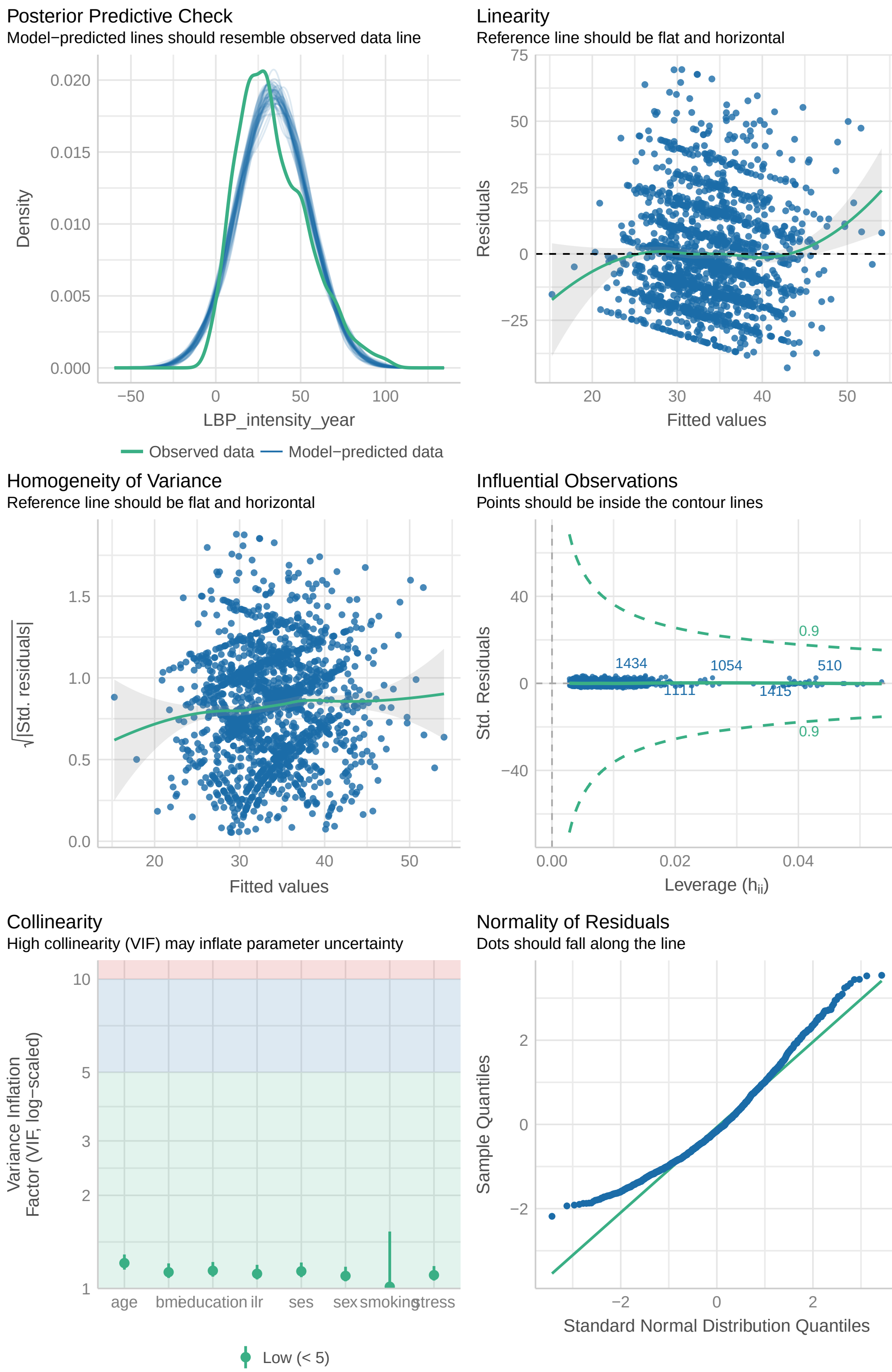
```r
kable(exp(est_ci_df), digits = 3) # these are the odds ratios (and approx CIs)
```

	est	2.5 %	97.5 %
age2_middle	1.059	0.851	1.319
age3_older	1.831	1.313	2.551
sex2_male	0.662	0.524	0.835
bmi2_normal	1.158	0.551	2.539
bmi3_overweight	1.612	0.761	3.564
stress2_stressed	1.547	1.258	1.902

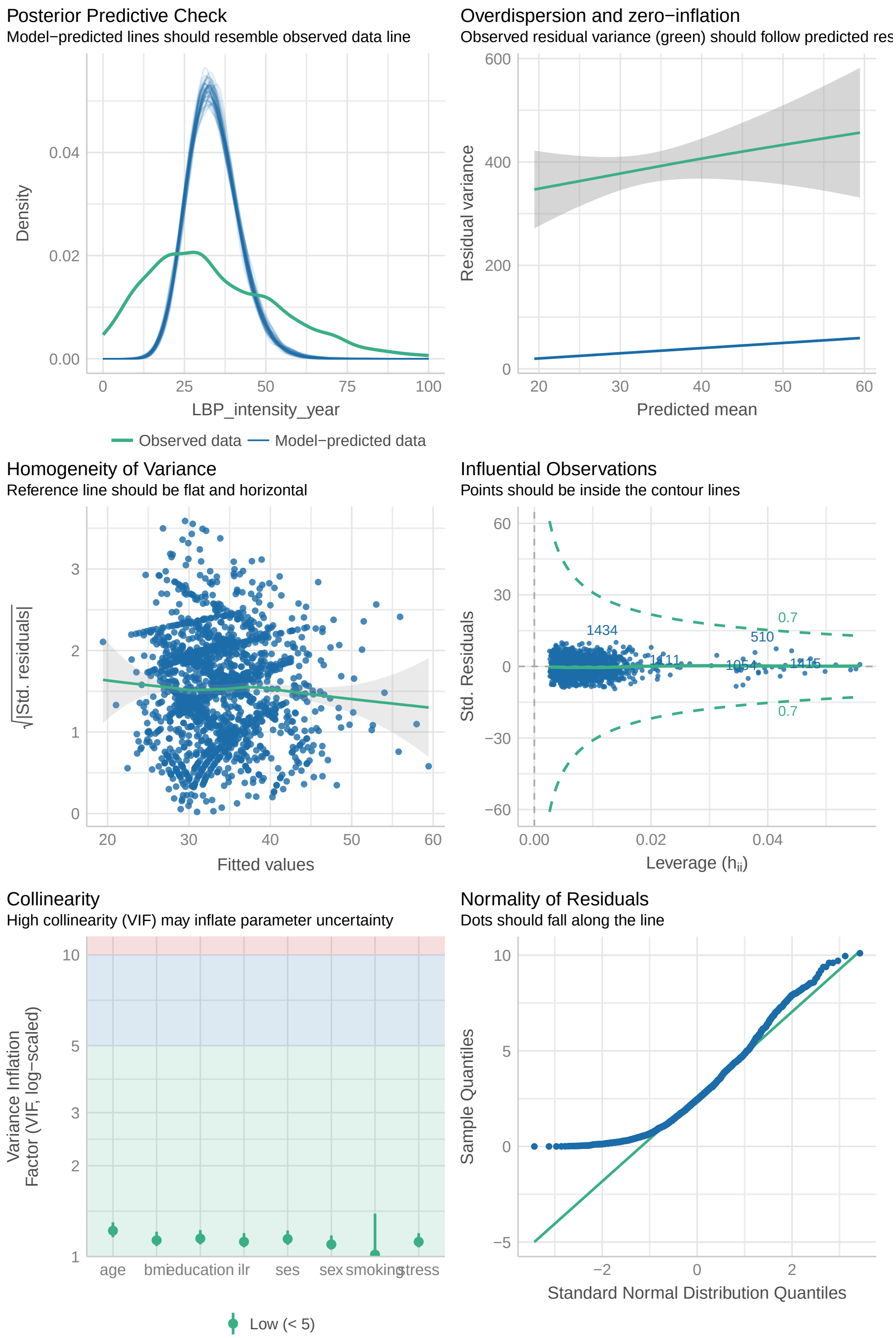
	est	2.5 %	97.5 %
smoking2_nonsmoker	1.172	0.912	1.512
education2_higher	0.746	0.601	0.926
ses2_middle	0.649	0.479	0.882
ses3_higher	0.451	0.291	0.697
ilrilr(++–)	0.864	0.697	1.069
ilrilr(+-..)	0.549	0.384	0.782
ilrilr(..+-)	0.953	0.819	1.109

5.4.2 Model diagnostics

```
## plain linear model
check_model(lbp_intensity_lm) # acceptable?
```

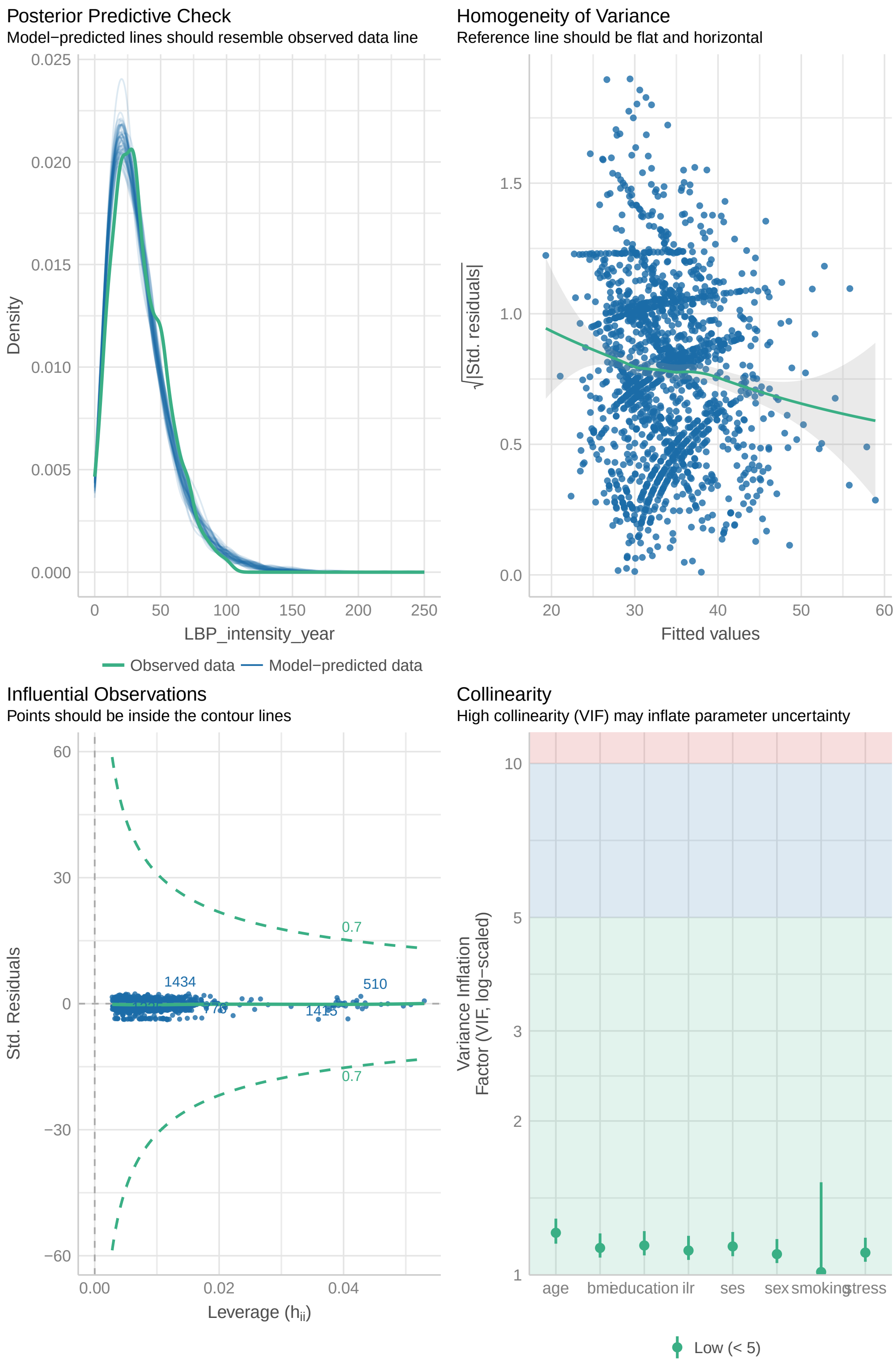

Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
LBP_intensity_year
Observed data
Model−predicted data
Linearity
Reference line should be flat and horizontal
Residuals
Fitted values
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
Leverage (hii)
1434
1054
510
1111
1415
0.9
0.9
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation Factor (VIF, log−scaled)
age
bmi
education
ilr
ses
sex
smoking
stress
Low (< 5)
Normality of Residuals
Dots should fall along the line
Sample Quantiles
Standard Normal Distribution Quantiles

```r
## Poisson regression (bad)
check_model(lbp_intensity_pois) # horrible
```



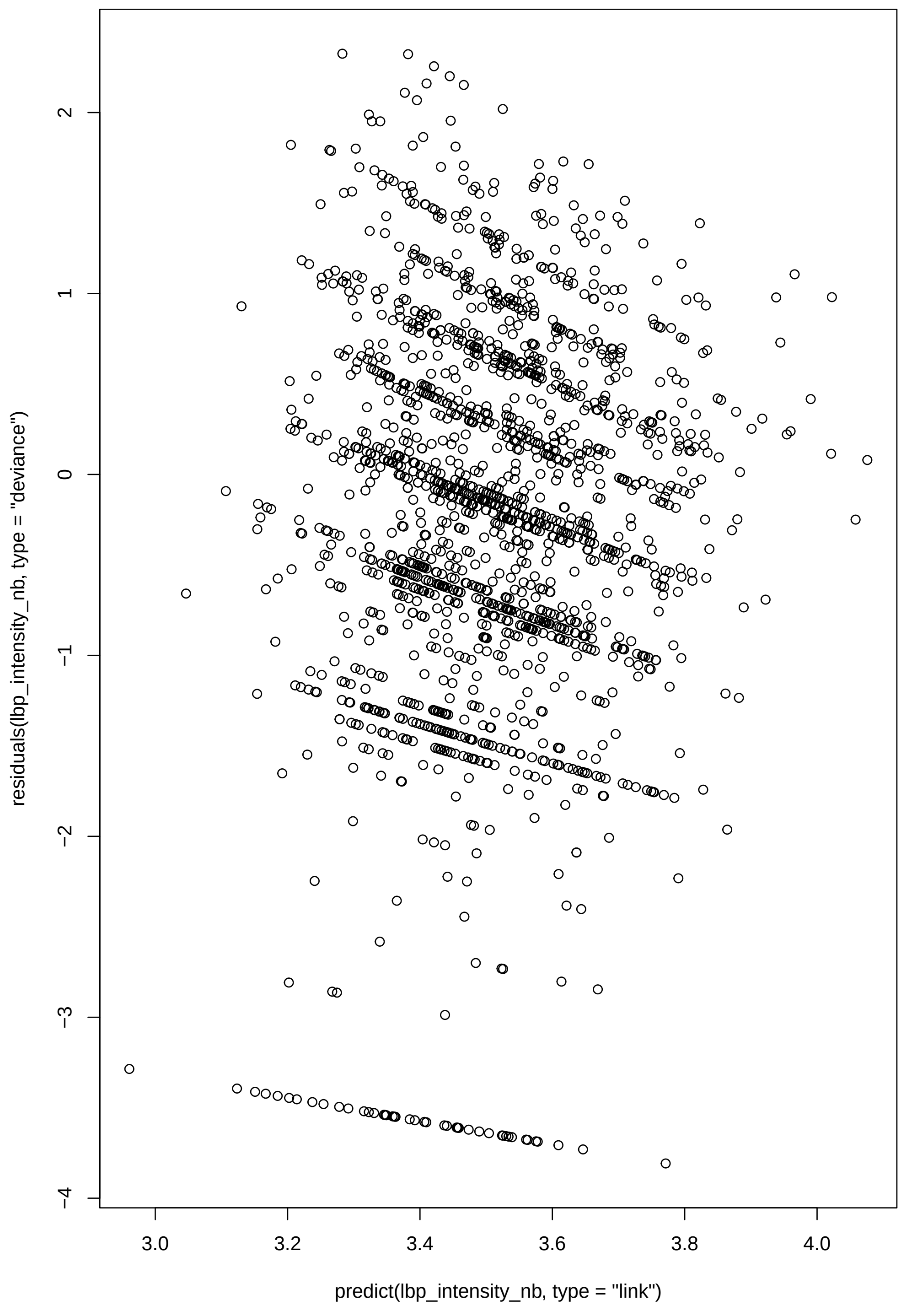
Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
LBP_intensity_year
Observed data
Model−predicted data
Overdispersion and zero−inflation
Observed residual variance (green) should follow predicted res
Residual variance
Predicted mean
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
Leverage (h_{ii})
1434
510
0.7
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation Factor (VIF, log−scaled)
age
bmi
education ilr
ses
sex
smoking
stress
Low (< 5)
Normality of Residuals
Dots should fall along the line
Sample Quantiles
Standard Normal Distribution Quantiles

```r
## Negative Binomial regression
check_model(
  lbp_intensity_nb,
  check = c("pp_check", "homogeneity", "outliers", "vif")
)
```



Posterior Predictive Check
Model−predicted lines should resemble observed data line
Density
LBP_intensity_year
Observed data
Model−predicted data
Homogeneity of Variance
Reference line should be flat and horizontal
√|Std. residuals|
Fitted values
Influential Observations
Points should be inside the contour lines
Std. Residuals
Leverage (h_{ii})
1434
510
0.7
Collinearity
High collinearity (VIF) may inflate parameter uncertainty
Variance Inflation Factor (VIF, log−scaled)
age
bmi
education
ilr
ses
sex
smoking
stress
Low (< 5)

```r
plot(
  predict(lbp_intensity_nb, type = "link"),
  residuals(lbp_intensity_nb, type = "deviance")
)
```


residuals(lbp_intensity_nb, type = "deviance")
predict(lbp_intensity_nb, type = "link")
2
1
0
−1
−2
−3
−4
3.0
3.2
3.4
3.6
3.8
4.0

```r
deviance(lbp_intensity_nb)/lbp_intensity_nb$df.residual
```

```
[1] 1.153043
```

```r
## Ordinal logistic regression (looks ok)
# this is running multiple logistic regressions
## we want to see the coefficients to be roughly the same EXCEPT for the
## (intercept) values
foreach(i = 2:length(levels(bpd_yes$intens_ord)), .combine = cbind) %do% {
  log_coefs <-
    coef(glm(
      I(as.numeric(intens_ord) >= i) ~
        age + sex + bmi + stress + smoking + education + ses + ilr,
      family = "binomial",
      data = bpd_yes
    ))
  log_coefs <- as.data.frame(log_coefs)
  colnames(log_coefs) <- paste0("logit(P(Y>=", i, "))")
  log_coefs
} %>%
  kable(., digits = 2)
```

	logit(P(Y>=2))	logit(P(Y>=3))	logit(P(Y>=4))
(Intercept)	5.80	-0.17	-1.95
age2_middle	-0.28	0.08	0.16
age3_older	0.49	0.62	0.65
sex2_male	-0.76	-0.38	-0.41
bmi2_normal	0.56	0.06	0.28
bmi3_overweight	0.73	0.38	0.68
stress2_stressed	0.52	0.46	0.40
smoking2_nonsmoker	0.34	0.09	0.37
education2_higher	-0.21	-0.36	-0.07
ses2_middle	-0.40	-0.34	-0.70
ses3_higher	-1.14	-0.68	-0.96
ilrilr(++–)	0.23	-0.17	-0.01
ilrilr(+-..)	-0.67	-0.57	-0.38
ilrilr(..+-)	-1.03	0.03	-0.07

5.4.3 Model predictions

```
# create dataset for predictions
newdata <-
  bpd_yes %>%
  dplyr::select(all_of(pred_covs), ilr) %>%
  distinct(pick(all_of(pred_covs)), .keep_all = TRUE) %>%
  arrange(pick(all_of(pred_covs)))

(mean_ilr <- mean(bpd_yes$ilr))
```

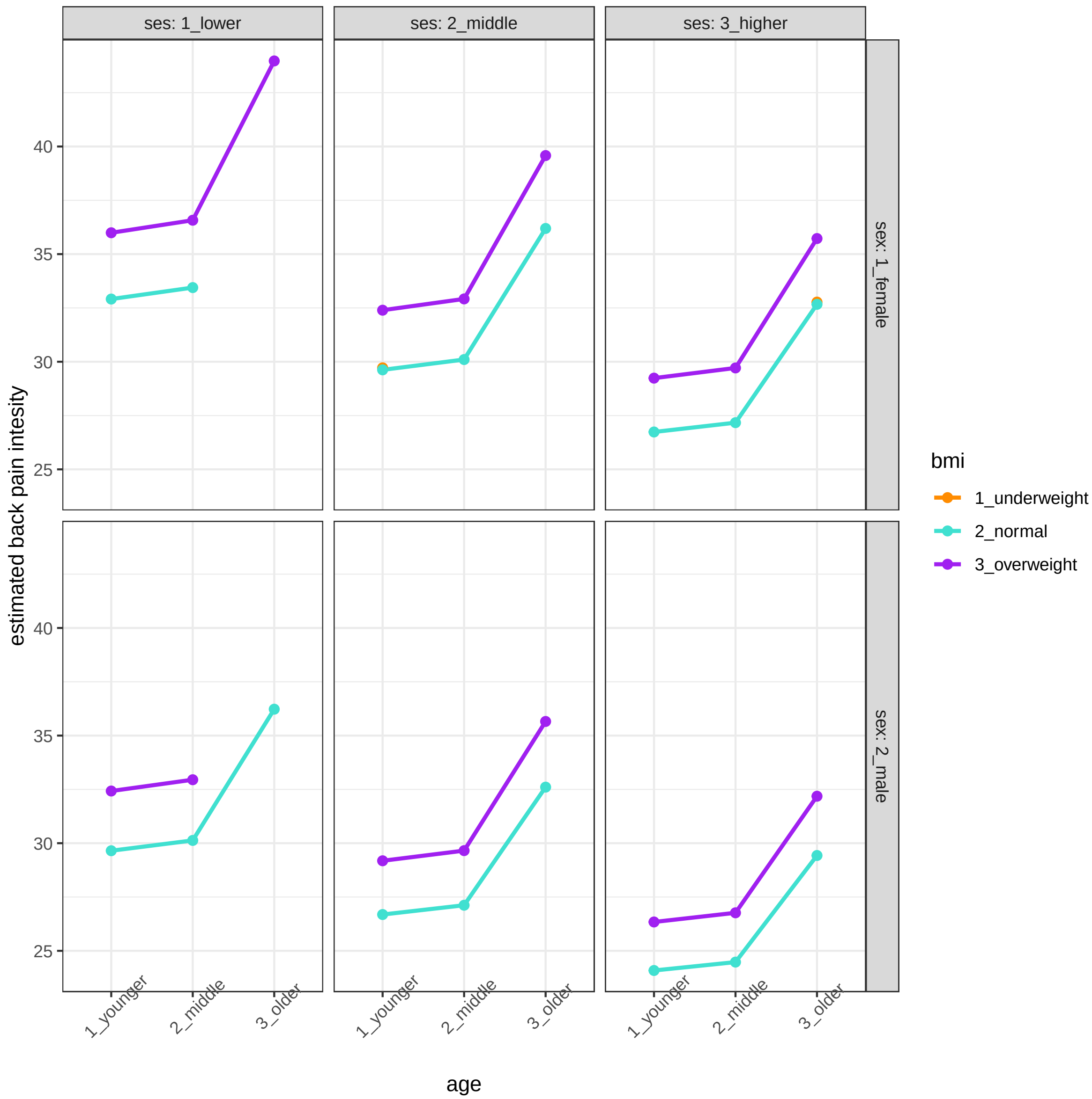
```
 ilr(++--)  ilr(+-..)  ilr(..+-)
1.37128443 0.05346627 2.07785318
attr(,"class")
[1] "rmult"
```

```
dev_null <- foreach(i = 1:nrow(newdata)) %do% {
  newdata$ilr[i, ] <- mean_ilr
}

# make preds and then put in long format for ggplot
predictions_intens <-
  cbind(
    pain_intens = predict(lbp_intensity_nb, newdata, type = "response"),
    newdata
  ) %>%
  dplyr::select(-ilr)

head(predictions_intens)
```

```
  pain_intens       age      sex           bmi     stress     smoking education
1    31.64985 1_younger 1_female 1_underweight   1_normal    1_smoker  2_higher
2    28.48625 1_younger 1_female 1_underweight   1_normal    1_smoker  2_higher
3    36.47806 1_younger 1_female 1_underweight   1_normal 2_nonsmoker   1_lower
4    32.83186 1_younger 1_female 1_underweight   1_normal 2_nonsmoker   1_lower
5    29.71678 1_younger 1_female 1_underweight   1_normal 2_nonsmoker  2_higher
6    32.54180 1_younger 1_female 1_underweight 2_stressed    1_smoker  2_higher
      ses
1  1_lower
```

```
2 2_middle
3  1_lower
4 2_middle
5 2_middle
6 2_middle
```

```
# newdata2 <- cbind(newdata2, predict(lbp_nb, newdata2, type = "link", se.fit=TRUE))
# newdata2 <- within(newdata2, {
#   lbp_pred <- exp(fit)
#   LL <- exp(fit - 1.96 * se.fit)
#   UL <- exp(fit + 1.96 * se.fit)
# })



predictions_intens %>%
  dplyr::filter(
    # sex == "1_female",
    stress == "1_normal",
    smoking == "2_nonsmoker",
    education == "2_higher",
    # ses == "2_middle"
  ) %>%
  ggplot(., aes(age, pain_intens, group = bmi)) +
  geom_line(aes(colour = bmi), linewidth = 1) +
  geom_point(aes(colour = bmi), size = 2) +
  facet_grid(sex~ ses, labeller = label_both) +
  labs(x = "age", y = "estimated back pain intesity") +
  theme_bw() +
  scale_color_manual(values = c("darkorange", "turquoise", "purple")) +
  theme(axis.text.x = element_text(angle = 45))
```




```
# create a RHS of regression equation dataset for time-reallocation
(predict_basis <-
    bpd_yes %>%
    dplyr::select(all_of(pred_covs), all_of(pred_comps)) %>%
    dplyr::filter(
      age == "2_middle",
      sex == "1_female",
```

```r
        stress == "1_normal",
        smoking == "2_nonsmoker",
        education == "2_higher",
        ses == "2_middle",
        bmi == "2_normal"
      ) %>%
      distinct(across(all_of(pred_covs)), .keep_all = TRUE) %>%
      as.data.frame())
```

```
      age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   546.4286
  Time_Sedentary Time_LPA Time_MVPA
1       418.2857 435.1429        40
```

```r
# compositional mean: geometric mean to closure
# (comp_mean <- mean(acomp(bpd_yes[, pred_comps])))
(comp_mean <- calc_comp_mean(bpd_yes[, pred_comps], clo_val = 1440))
```

```
    Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA
      472.8407       438.4062       502.1666        26.5865
```

```r
predict_basis0 <- predict_basis
predict_basis0[, pred_comps] <- comp_mean

predict_basis0
```

```
      age      sex      bmi   stress     smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
  Time_Sedentary Time_LPA Time_MVPA
1       438.4062 502.1666   26.5865
```

```r
# +15 minutes to Time_MVPA and -15 minutes from Time_Sedentary
comp_mean_changed <- comp_mean
comp_mean_changed["Time_MVPA"] <- comp_mean_changed["Time_MVPA"] + 15
comp_mean_changed["Time_Sedentary"] <- comp_mean_changed["Time_Sedentary"] - 15
# check
comp_mean_changed - comp_mean
```

```
   Time_Sleep Time_Sedentary       Time_LPA      Time_MVPA 
            0            -15              0             15 
```

```r
predict_basis1 <- predict_basis
predict_basis1[, pred_comps] <- comp_mean_changed

pred_df <- rbind(predict_basis0, predict_basis1)
pred_df <- add_ilrs_to_data(pred_df, comp_vars = pred_comps, sbp_matrix = sbp1)
pred_df
```

```
       age      sex      bmi   stress      smoking education      ses Time_Sleep
1 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
2 2_middle 1_female 2_normal 1_normal 2_nonsmoker  2_higher 2_middle   472.8407
  Time_Sedentary Time_LPA Time_MVPA      ilr.1      ilr.2      ilr.3
1       438.4062 502.1666   26.5865 1.37128443 0.05346627 2.07785318
2       423.4062 502.1666   41.5865 1.13019151 0.07808340 1.76151343
```

```r
predict(lbp_intensity_nb, pred_df, type = "link")
```

```
       1        2 
3.404581 3.418819 
```

```r
# exponentiate difference in the log back pain intensity (ratio of back pain preds)
exp(diff(predict(lbp_intensity_nb, pred_df, type = "link")))
```

```
      2 
1.01434 
```

```r
# abs difference in the mean back pain intensity
diff(predict(lbp_intensity_nb, pred_df, type = "response"))
```

```
        2 
0.4316527 
```

```r
(p_0 <- predict(lbp_intensity_nb, pred_df, type = "response"))
```

```
       1          2 
30.10167 30.53332 
```

```r
# % increase in pain intensity
(p_0[2]  - p_0[1]) / p_0[1]
```

```
         2 
0.01433983 
```

```r
# ratio version
get_pred_diff_rat <- function(mod, new_dat) {
  log_ratio_pred <- predict(mod, new_dat, type = "link")
  ratio_outc <- exp(log_ratio_pred[2] - log_ratio_pred[1])
  return(ratio_outc)
}
get_pred_diff_rat(lbp_intensity_nb, pred_df)
```

```
      2 
1.01434 
```

```r
# absolute difference version
get_pred_diff_abs <- function(mod, new_dat) {
  log_ratio_pred <- predict(mod, new_dat, type = "response")
  ratio_outc <- log_ratio_pred[2] - log_ratio_pred[1]
  return(ratio_outc)
}
get_pred_diff_abs(lbp_intensity_nb, pred_df)
```

```
        2 
0.4316527 
```

```r
# wrapper:
get_pred_diff <- function(mod, new_dat, type = "abs") {
  if (type == "abs") {
    return(get_pred_diff_abs(mod = mod, new_dat = new_dat))
  } else if (type == "rat") {
    return(get_pred_diff_rat(mod = mod, new_dat = new_dat))
```

```r
  } else {
    stop("'type' must be 'abs' (absolute differnce) or 'rat' (ratio)")
  }
}
get_pred_diff(lbp_intensity_nb, pred_df, type = "abs")
```

```
        2
0.4316527
```

```r
get_pred_diff(lbp_intensity_nb, pred_df, type = "rat")
```

```
      2
1.01434
```

```r
fit_mod_boot <- function(data, i, pred_dat, type = "abs") {

  this_dat <- data[i, ]
  this_nbr <- glm.nb(mod_form_ilrs, data = this_dat)
  est <- get_pred_diff(this_nbr, new_dat = pred_dat, type = type)
  return(est)

}
alpha <- 0.05
quantile(boot(bpd_yes, fit_mod_boot, R = 10, pred_dat = pred_df)$t, c(alpha / 2, 1 - alpha
```

```
     2.5%     97.5%
0.2111926 0.6932640
```

```r
do_multi_realloc <- function(mod, basis_data, timeusenames, time_changes, sbp_matrix = sbp


  x0 <- basis_data

  plot_dat <-
    foreach(i = 1:length(timeusenames), .combine = bind_rows) %do% {
      print(paste("i: ", i))
      foreach(j = 1:length(timeusenames), .combine = bind_rows) %do% {
```

```r
print(paste("  j: ", j))
foreach(d = 1:length(time_changes), .combine = bind_rows) %do% {
  print(paste("    d: ", d))

  timeuse_to <- timeusenames[i]
  timeuse_from <- timeusenames[j]
  change_time <- time_changes[d]

  proposed_change_1 <- x0[timeuse_to] + change_time
  proposed_change_2 <- x0[timeuse_from] - change_time

  if (timeuse_to == timeuse_from) {
    NULL # reallocation exceeds 0 or max time
  } else if ((proposed_change_1 < 0) | (proposed_change_1 > 1440)) {
    NULL # reallocation exceeds 0 or max time
  } else if ((proposed_change_2 < 0) | (proposed_change_2 > 1440)) {
    NULL # reallocation exceeds 0 or max time
  } else {

    x1 <- x0
    x1[timeuse_to] <- x1[timeuse_to] + change_time
    x1[timeuse_from] <- x1[timeuse_from] - change_time

    pred_df <- rbind(x0, x1)
    pred_df <- add_ilrs_to_data(pred_df, comp_vars = timeusenames, sbp_matrix = sb

    outc_ratio <- get_pred_diff(mod, pred_df)

    bootstrapped_ests <- boot(bpd_yes, fit_mod_boot, R = 1000, pred_dat = pred_df)
    ci_est <- quantile(as.numeric(bootstrapped_ests), c(alpha / 2, 1 - alpha / 2))

    tibble(
      to = timeuse_to,
      from = timeuse_from,
      change_time = change_time,
      outc_ratio = outc_ratio,
      ci_lo = ci_est[1],
      ci_hi = ci_est[2]
    )
  }
}
```

```r
      }
    }

  plot_dat$to <- factor(plot_dat$to, levels = timeusenames)
  plot_dat$from <- factor(plot_dat$from, levels = timeusenames)

  return(plot_dat)

}

set.seed(1234)

# takes ~60 min (single core) for bootstrapped CIs (R = 1000)
# takes ~ 6 min (single core) for bootstrapped CIs (R = 100)
tic()
# realloc_plot_data <-
#   do_multi_realloc(
#     lbp_intensity_nb,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )
toc()
```

```
0 sec elapsed
```

```r
# saveRDS(realloc_plot_data, file = "res/negbin_realloc_boot_res(abs).rda")



set.seed(1234)

fit_mod_boot <- function(data, i, pred_dat, type = "rat") {

  this_dat <- data[i, ]
  this_nbr <- glm.nb(mod_form_ilrs, data = this_dat)
  est <- get_pred_diff(this_nbr, new_dat = pred_dat, type = type)
  return(est)

}
```

```r
alpha <- 0.05
quantile(boot(bpd_yes, fit_mod_boot, R = 10, pred_dat = pred_df)$t, c(alpha / 2, 1 - alpha
```

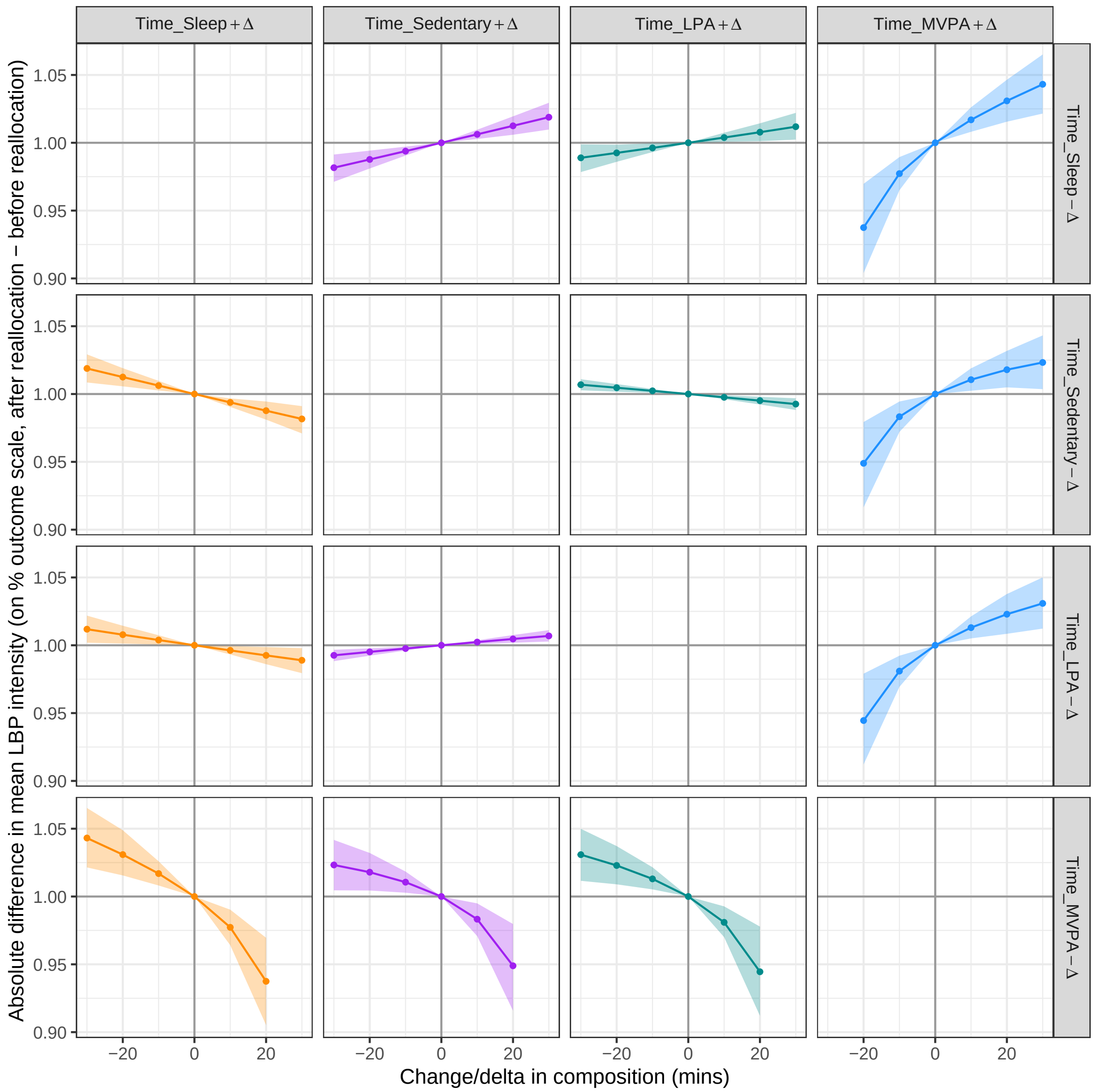
```
   2.5%     97.5%
1.009664 1.026014
```

```r
set.seed(1234)

# takes ~60 min (single core) for bootstrapped CIs (R = 1000)
# takes ~ 6 min (single core) for bootstrapped CIs (R = 100)
tic()
realloc_plot_data <-
#   do_multi_realloc(
#     lbp_intensity_nb,
#     predict_basis0,
#     pred_comps,
#     seq(-30, 30, by = 10)
#   )
toc()
```

```
0 sec elapsed
```

```r
# saveRDS(realloc_plot_data, file = "res/negbin_realloc_boot_res(rat).rda")









realloc_plot_data <- readRDS(file = "res/negbin_realloc_boot_res(abs).rda")


levels(realloc_plot_data$to) <- paste0(levels(realloc_plot_data$to), "+Delta")
levels(realloc_plot_data$from) <- paste0(levels(realloc_plot_data$from), "-Delta")

ggplot(realloc_plot_data) +
  geom_vline(xintercept = 0, col = "grey60") +
```

```
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3) +
  geom_line(aes(x = change_time , y = outc_ratio, col = to)) +
  geom_point(aes(x = change_time , y = outc_ratio, col = to), size = 1) +
  facet_grid(from ~ to, labeller = label_parsed) +
  theme_bw() +
  scale_colour_manual(values = c("darkorange","purple","cyan4", "dodgerblue")) +
  scale_fill_manual(values = c("darkorange","purple","cyan4", "dodgerblue")) +
  labs(
    x = paste0("Change/delta in composition (mins)"),
#     y = paste0("Ratio of back pain intensity (after reallocation:before reallocation)")
    y = paste0("Absolute difference in mean LBP intensity (on % outcome scale, after reall
  ) +
  theme(legend.position = "none")
```




```r
ggsave(
  filename = "fig/lbp_intens_negbin_abs_v1.png",
  dpi = 600, # print quality
  width = 10,
  height = 10
)
```

```r
pd2 <-
  realloc_plot_data %>%
  mutate(
    to = gsub("Time_", "", to),
    from = gsub("Time_", "", from),
    to = gsub("+Delta", "", to, fixed = TRUE),
    from = gsub("-Delta", "", from, fixed = TRUE),
    to_len = nchar(to),
    to_max = max(to_len),
    from_len = nchar(from),
    from_max = max(from_len),
    to_pad =  rep_char(pmax(0, from_max - to_len)),
    from_pad = rep_char(pmax(0, to_max - from_len)),
    to = factor(to, levels = time_lvls),
    from = factor(from, levels = time_lvls),
    to_num = as.numeric(to),
    from_num = as.numeric(from)
  ) %>%
  dplyr::filter(to_num > from_num) %>%
  mutate(
    # from_to = paste0("     ", "+", from, rep_char(10), from_pad, "\u2194", to_pad, rep_c
    from_to = paste0("+", from, rep_char(13), from_pad, "", to_pad, rep_char(13), "+", to)
  ) %>%
  arrange(from, to)

unique(pd2$from_to)
```

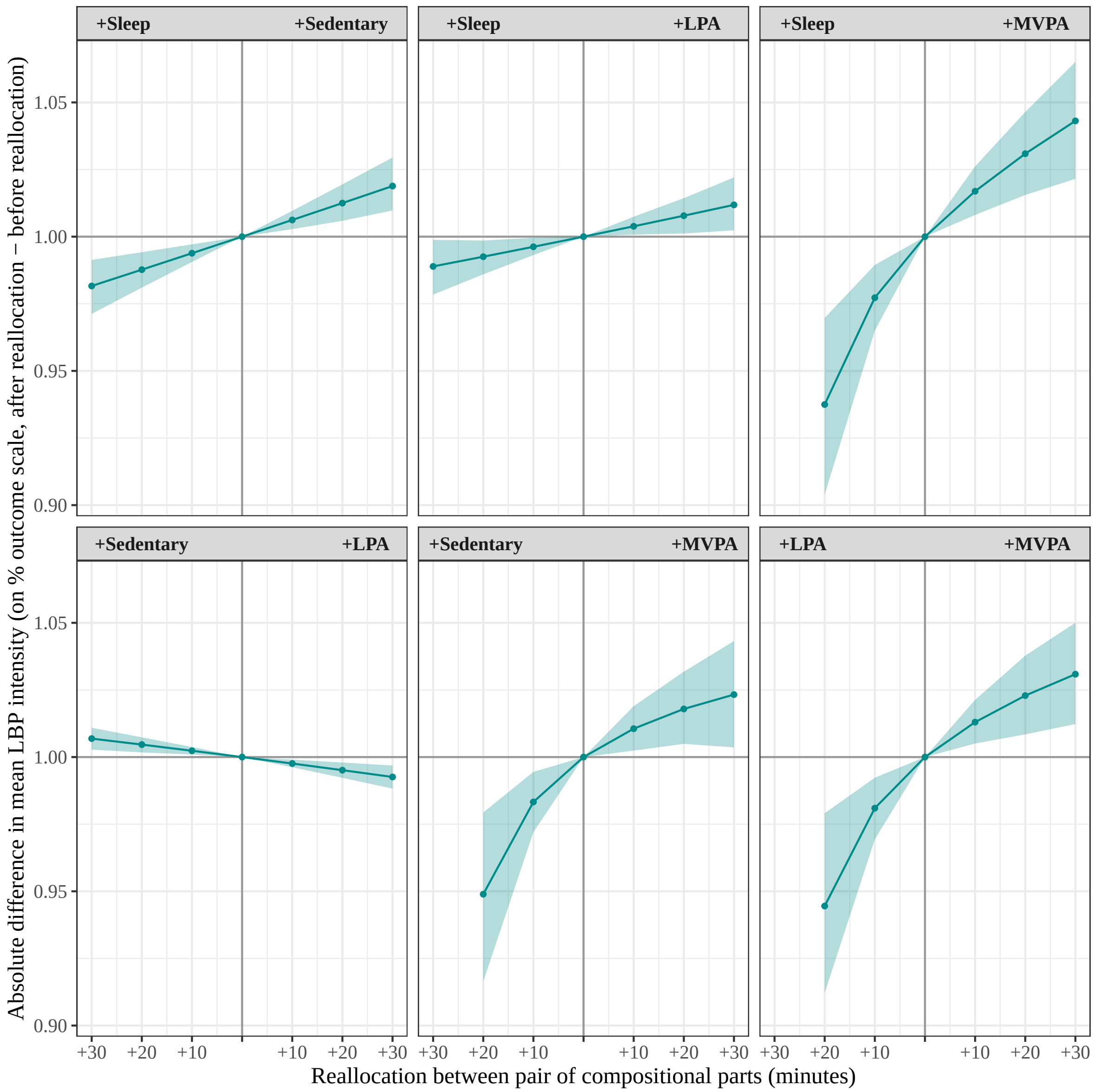
```
[1] "+Sleep                              +Sedentary"
[2] "+Sleep                                    +LPA"
[3] "+Sleep                                   +MVPA"
[4] "+Sedentary                                +LPA"
[5] "+Sedentary                               +MVPA"
[6] "+LPA                                     +MVPA"
```

```r
pd2$from_to <- factor(pd2$from_to, levels = unique(pd2$from_to))

this_breaks <- seq(-30, 30, 10)
this_labs <- sprintf("+%2.0f", abs(seq(-30, 30, 10)))
this_labs[this_labs == "+ 0"] <- ""
```

```
this_labs
```

```
[1] "+30" "+20" "+10" ""    "+10" "+20" "+30"
```

```
ggplot(pd2) +
  geom_vline(xintercept = 0, col = "grey60") +
  geom_hline(yintercept = 1, col = "grey60") +
  geom_ribbon(aes(x = change_time, ymin = ci_lo, ymax = ci_hi, fill = to), alpha = 0.3, co
  geom_line(aes(x = change_time , y = outc_ratio, col = to), col = "cyan4") +
  geom_point(aes(x = change_time , y = outc_ratio, col = to), size = 1, col = "cyan4") +
  facet_wrap(~ from_to, labeller = label_bquote(.(from_to))) +
  theme_bw() +
  scale_x_continuous(breaks = this_breaks, labels = this_labs) +
  labs(
    x = paste0("Reallocation between pair of compositional parts (minutes)"),
#    y = paste0("Ratio of mean LBP intensity (after reallocation:before reallocation)")
    y = paste0("Absolute difference in mean LBP intensity (on % outcome scale, after reall
    # subtitle = "Note that odds ratios relate to the probability of having _increased_ fr
  ) +
  theme(
    legend.position = "none",
    text = element_text(family = "serif"),
    strip.text = element_text(size = 10, face = "bold"),
    axis.text =  element_text(size = 10),
    axis.title =  element_text(size = 12)
  )
```




```r
ggsave(filename = "fig/lbp_intens_negbin_abs_v2.png", width = 14, height = 9, dpi = 600)
# ggsave(filename = "fig/lbp_intens_negbin_ratio.pdf", width = 10, height = 8)
```

6 Session information

```
format(Sys.time(), '%d %b %Y')
```

```
[1] "26 Sep 2023"
```

```
sessionInfo()
```

```
R version 4.2.2 (2022-10-31 ucrt)
Platform: x86_64-w64-mingw32/x64 (64-bit)
Running under: Windows 10 x64 (build 19045)

Matrix products: default

locale:
[1] LC_COLLATE=English_Australia.utf8  LC_CTYPE=English_Australia.utf8
[3] LC_MONETARY=English_Australia.utf8 LC_NUMERIC=C
[5] LC_TIME=English_Australia.utf8

attached base packages:
[1] stats     graphics  grDevices utils     datasets  methods   base

other attached packages:
 [1] tictoc_1.1            boot_1.3-28           foreach_1.5.2
 [4] knitr_1.42            car_3.1-1             carData_3.0-5
 [7] mice_3.15.0           performance_0.10.4    zCompositions_1.4.0-1
[10] truncnorm_1.0-8       NADA_1.6-1.1          survival_3.4-0
[13] MASS_7.3-58.1         compositions_2.0-5    GGally_2.1.2
[16] ggplot2_3.4.1         forcats_1.0.0         readr_2.1.4
[19] tidyr_1.3.0           dplyr_1.1.2

loaded via a namespace (and not attached):
 [1] bit64_4.0.5        vroom_1.6.1        jsonlite_1.8.4     viridisLite_0.4.1
 [5] splines_4.2.2      datawizard_0.7.1   tensorA_0.36.2     ggrepel_0.9.3
 [9] bayestestR_0.13.1  yaml_2.3.7         robustbase_0.95-0  pillar_1.9.0
[13] backports_1.4.1    lattice_0.20-45    glue_1.6.2         digest_0.6.31
[17] RColorBrewer_1.1-3 colorspace_2.1-0   htmltools_0.5.6    Matrix_1.5-3
[21] plyr_1.8.8         pkgconfig_2.0.3    broom_1.0.3        purrr_1.0.1
[25] patchwork_1.1.2    scales_1.2.1       tzdb_0.3.0         tibble_3.2.1
```

```
[29] mgcv_1.8-41        generics_0.1.3     farver_2.1.1       ellipsis_0.3.2
[33] withr_2.5.0        cli_3.6.0          magrittr_2.0.3     crayon_1.5.2
[37] evaluate_0.20      fansi_1.0.4        nlme_3.1-160       textshaping_0.3.6
[41] tools_4.2.2        hms_1.1.2          lifecycle_1.0.3    see_0.7.4
[45] munsell_0.5.0      compiler_4.2.2     systemfonts_1.0.4  rlang_1.1.1
[49] grid_4.2.2         iterators_1.0.14   rstudioapi_0.14    labeling_0.4.2
[53] rmarkdown_2.20     gtable_0.3.1       codetools_0.2-18   abind_1.4-5
[57] reshape_0.8.9      R6_2.5.1           bayesm_3.1-5       fastmap_1.1.1
[61] bit_4.0.5          utf8_1.2.3         ragg_1.2.5         insight_0.19.1
[65] parallel_4.2.2     Rcpp_1.0.10        vctrs_0.6.3        DEoptimR_1.0-11
[69] tidyselect_1.2.0   xfun_0.37
```