Volatility Targeting Is Trendy: How Trend Following Explains Alpha in Volatility-Managed Strategies

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Key Findings

- Volatility targeting, or volatility management, has been demonstrated in the literature to have alpha over a buy-and-hold strategy; we find that this alpha is better described as exposure to trend following.
- Controlling for exposure to trend following reduces the alpha to volatility targeting on equity assets by around 2/3, with an estimated regression beta to trend of 0.35 for a panel of 14 global equity index futures contracts.
- The loading of volatility targeting on trend following appears to be an equity-specific phenomenon arising from the "leverage effect," and does not occur for most commodity, fixed income, and currency futures.

Abstract

Why do volatility targeting/management strategies tend to outperform simple buy-and-hold positions in the same assets? We test the hypothesis that this outperformance is mainly due to a loading on trend following that arises because of the negative correlation between return direction (trend) and magnitude (volatility), the so-called "leverage effect." When controlling for trend exposure, alpha to volatility targeting is shown to mainly accrue to trend for both a long equity history and a set of 14 global equity index futures contracts. By contrast, this is not true for commodity, fixed income, or currency futures, where the leverage effect is not present. We further discuss the mechanical relationship between volatility targeting and trend following, creating a point of connection between these two seemingly different branches of research.

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Introduction

Volatility targeting, or volatility management, is a technique that appears widely across the financial industry. While not quite as common within retail strategies like mutual funds and ETFs, it has large purchase within the insurance industry, where structures like annuities must be hedged – if a reference index has a fixed volatility profile, the cost of hedging that index with options will be steadier.

Another common place to find volatility targeting techniques is in hedge funds and sophisticated asset managers with an institutional investor base. Strategies managed by these funds are often decoupled from a fixed leverage perspective; once we have a long-short equity strategy, for instance, it begs the question of exactly how much leverage should be applied, a question that can be answered by targeting a level of risk rather than exposure. Finally, some strategies, like risk parity, explicitly use risk targeting as the fundamental allocation methodology for the investment.

The basic concept of volatility targeting is straightforward. Volatility varies over time, a stylized fact well known since at least Engle (1982) and Bollerslev (1986). As a result, a fixed-leverage portfolio, like a fully-invested S&P 500 allocation or a 60/40 stock/bond portfolio, is necessarily a variable-risk portfolio. Volatility targeting is an attempt to invert this relationship, resulting in a fixed-risk portfolio that employs variable leverage.

A hypothetical example illuminates: suppose an investor that is comfortable taking 20% annualized risk (in this case, measured by the standard deviation of returns) invests fully in an asset that has a long-run risk level of 20%. This appears to be a good match at first glance, but if the asset achieves that 20% long-run risk level by sometimes experiencing only 10% risk and at other times experiencing 40% risk, then this investor is likely to be unhappy with the investment. Instead, a basic volatility targeting strategy invests only 50% of notional in periods of elevated risk and invests 200% notional in periods of reduced risk. The resulting strategy "targets" 20% volatility consistently, rather than swinging back and forth between over- and under-exposed from a risk perspective.[2]

This investor should, in theory, be much happier with the steadier risk profile of the volatility-targeted strategy. Bollerslev et al. (2018) estimate that a typical mean-variance optimizing investor would pay as much as 48bps to access a sophisticated volatility-targeting strategy when compared to a passive investment. Importantly in their paper, that 48bps of benefit is entirely due to the steadier risk provided by volatility targeting; the authors assume that there is no outperformance or alpha to volatility targeting as a strategy. In other words, volatility targeting appears to be a considerable boon to investors based on the disutility of time-varying risk alone.

But recent literature suggests there may be significant and economically meaningful return outperformance to volatility targeting strategies as well when compared to simple, buy-and-hold investments on the same assets. Moreira and Muir (2017) estimate alpha of 3.85% per year to volatility targeting over simple buy-and-hold returns for US equities back to the 1920's. Harvey et al. (2018) report Sharpe ratios that increase from 0.40 for buy-and-hold to 0.51 for a volatility targeted strategy over the

[2] $50\% * 40\% = 200\% * 10\% = 20\%$ constant risk for the volatility targeted strategy. This simple example of course obscures considerable complexity. How do fund managers know if they are in a high- or low-risk regime? Are their estimates of stock price volatility correct? Are they able to efficiently access 2x leverage in the low-risk state of the world? Many of these questions are served by a deep literature on volatility estimation and prediction, and are outside the scope of this paper. We will intentionally stick to simple, easy-to-replicate strategies for parsimony.

same period for US equities, but interestingly the opposite result for US bonds from 1963 onwards. They hypothesize (correctly, in our view) that this can be explained by the negative correlation between returns and volatility that is most prominent in equity markets, creating a connection between volatility targeting and trend following. Trend following has been shown by Moskowitz et al. (2012), Hurst et al. (2013), and Hurst et al. (2017), among others, to have a positive risk premium, which may be a source of the observed outperformance.

We pick up where these papers left off. Replicating the results from Moreira and Muir (2017) – with some small upgrades to make the simulation more realistic – we demonstrate that the majority of that large alpha to volatility targeting can be explained by a simple trend following strategy, and the remaining alpha is no longer significant. This finding is, to our knowledge, the first direct test of cross exposure between the strategies. Investing in the volatility targeting strategy over this long historical period maps, according to our regressions, to a buy-and-hold portfolio with an added 30% exposure to simple trend on the asset.

We explain this as arising from a mechanical relationship between volatility targeting and trend following positions, showing that these strategies will be linked whenever return direction is strongly negatively correlated with return magnitude (i.e. volatility). But for assets where this negative correlation does not appear, returns to these two different strategies are unlikely to be connected. This mechanical relationship between the positions of the two strategies suggests that there is a strong and substantial linkage between the volatility targeting and trend following literatures that is currently being missed.

Finally, we test our hypotheses using a more-recent, broad set of 50 futures contracts across four asset classes: commodities, equities, fixed income, and currencies. Though we no longer find statistically significant alpha given the smaller date range for the broad data set, results are directionally and economically similar to the long history data set for equities (where we expect a connection), and small or non-existent in other asset classes (where we don't).

Together, these results tell a simple and intuitive story: when return direction and return magnitude are negatively correlated, a volatility targeting strategy will align positioning with a trend following strategy. Given the well-documented trend risk premium, we should expect meaningful outperformance for volatility targeting strategies in those assets.

A Motivating Example

We are strongly motivated in this paper by the interesting (and replicable) results of Moreira and Muir. They demonstrate that volatility management appears to have alpha over a buy-and-hold investment in equity factors – most importantly, the market factor – over a long history dating back to the 1920's. Further, they demonstrate that the alpha found in volatility targeting the market factor cannot be explained by the other two Fama French cross-sectional equity factors, SMB (size) and HML (value).[3]

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[3] Moreira and Muir primarily focuses on *variance* targeting, which in our experience as practitioners is exceedingly uncommon in practice. They further discuss scaling by the inverse of volatility as a robustness check, and the main results hold in that case as well. We will focus exclusively on volatility targeting, as we find it to be the most directly applicable to real-world use-cases.

This result, perhaps surprising at first glance, makes some intuitive sense for an equity investment. Recall that a volatility targeting strategy is going to be underweight the base asset relative to a buy-and-hold strategy when volatility is high, and overweight the base asset when volatility is low. Anecdotally, the reader can likely imagine a handful of recent times when equity volatility was particularly high – the 2008 financial crisis, the early onset of the COVID-19 crisis, etc. – and quickly identify them as periods when equity investors were experiencing negative returns. Thus, a strategy like volatility targeting that cuts positions early in these crises might outperform the underlying asset itself.

However, a clear corollary to the idea that a volatility targeting strategy might be underweight the risky asset in a crisis is that it is also very likely to be underweight if the crisis reaches a turning point and the risky asset begins to rebound. Thus, in order for the volatility targeting strategy to significantly outperform the buy-and-hold strategy of the same asset over the long run, the crisis likely needs to deepen long enough for the volatility targeting strategy to outperform the buy-and-hold strategy over a full crisis and recovery cycle.

Similarly, periods where equity market volatility was well below average loom in our memories as mostly good times for equity investors. By applying leverage to scale up the asset during calm markets, the volatility targeting strategy is betting that it is preferable to be overweight exposure relative to the buy-and-hold strategy at these times. As with the previous example, volatility targeting is likely to benefit if the good times continue to roll.

This appeal to intuition holds a hint at what we will demonstrate is driving the Moreira-Muir result for the market factor: trend following. Recall that volatility targeting is essentially a *timing* strategy, changing positioning in the single asset over time as the volatility environment changes. It should not be surprising to find that any alpha to this timing strategy survives controlling for *cross-sectional* factor strategies like SMB and HML. While it is hypothetically possible that a timing strategy like volatility targeting might load on a cross-sectional strategy, it is more likely that it would load on a timing strategy where the signals share some common component. Given the intuition that volatility targeting is more likely to outperform a buy-and-hold strategy if existing trends continue, trend following appears to be an excellent candidate for explaining the alpha found by Moreira and Muir. This is precisely what we examine in this section.

Methodology

We begin by replicating the main result of Moreira and Muir, with moderate changes intended to make the strategy more like that utilized by hedge funds, insurance companies, and other asset managers. We compare the returns to the Fama-French market factor to those from a simple volatility targeting strategy on the same factor. Our analysis on the *Mkt-Rf* factor runs from 5/4/1927 – 12/31/2023, the longest history available from Kenneth French's website while holding out twelve month's returns needed to form all of the strategies we will discuss in this paper. The volatility targeting strategy will invest in the underlying asset proportional to the inverse of the trailing 3-month volatility estimate. We assume a 1-day implementation lag to enter and exit positions in the underlying asset: a volatility estimate that uses data through the close of business on date t will be used to size positions as of the close of date $t+1$, earning its first daily return marked on the close of business on date $t+2$. Additionally, we assume a 2bp transaction cost for all transactions. Appendix A replicates results from this section using cost assumptions that range from 0-10bps for completeness. Finally, we use the full period buy-and-hold strategy return standard deviation as our target for volatility targeting.[4] The position function of the volatility targeting strategy is provided in Exhibit 1.[5]

EXHIBIT 1A: Strategy Descriptions

Strategy	Signal Description	Position Function
Buy-And-Hold	None	1
Volatility Targeting	Scales buy-and-hold returns by the inverse of trailing 3-month volatility; targets the full period buy-and-hold volatility for ease of interpretation	$\frac{target}{\sigma_{3m}}$, where $target$ is the full period buy-and-hold volatility
Simple Trend	Takes long or short positions in the buy-and-hold asset based on the average sign of the trailing 1-, 3-, and 12-month asset return, as in Hurst et al (2013)	$\frac{1}{3}[sign(R_{1m}) + sign(R_{3m}) + sign(R_{12m})]$

Notes: This table details the three main strategies we discuss in this paper – Buy-and-Hold, Volatility Targeting, and Simple Trend.

EXHIBIT 1B: Strategy Statistics for Equity Market (Mkt-Rf) Factor

	Buy-and-Hold	Volatility Targeting	Trend
Return	7.6%	10.2%	4.9%
Standard Deviation	17.2%	18.7%	13.2%
Sharpe Ratio	0.44	0.54	0.37
3m RMSE From Target	9.3%	4.5%	9.9%
Skewness	-0.2	-0.9	-0.7
Excess Kurtosis	16.4	8.5	24.9

Notes: This table reports basic performance statistics for the three main strategies we discuss in this paper – Buy-and-Hold, Volatility Targeting, and Simple Trend – applied to the Fama-French equity market (Mkt-Rf) factor. Statistics are calculated using daily returns. 3m RMSE From Target is defined as the square root of the mean square error (RMSE) of the strategy's rolling 3-month annualized standard deviation from the target standard deviation; the target is set equal to the full-period standard deviation of the Buy-and-Hold strategy. Excess Kurtosis is defined as full-sample kurtosis minus 3, the kurtosis of a normal random variable. Date range is 1927-2023. Results for other Fama-French factors size (SMB),

[4] This requires knowledge of the full period buy-and-hold return volatility in advance of running the volatility targeting simulation. However, the only thing that the volatility target impacts is the size of the betas in our regressions, and choosing a target equal to the full period risk of the buy-and-hold strategy makes for easier interpretation of those betas; t-stats would be identical if we targeted, say, 10% volatility instead. By targeting a particular level of volatility (rather than ex-post scaling as Moreira and Muir do – see Footnote 5), we add a slight bit of additional realism into the simulation, as any bias in the risk model or volatility targeting process will still be present in our methodology.

[5] Our methodology differs from Moreira-Muir in a few ways that we would like to highlight for clarity. As mentioned, we are scaling by the inverse of volatility rather than variance, as is much more common in practical applications of the strategy. Further they use a monthly rebalance schedule, where volatility is estimated using the previous month's daily returns, and then the inverse of that volatility is used to scale returns for the subsequent month, also rare in practice in our experience. Finally, rather than targeting the full period standard deviation of the buy-and-hold strategy, they ex-post scale the returns to the exact same standard deviation as buy-and-hold. We successfully replicated their results initially using their same methodology. Our results are robust to either methodology, without any meaningful change in results, so we report results using our more practically applicable methodology throughout this paper.

value (HML), and momentum (MOM) are found in Appendix B. We assume a 2bp transaction cost for all trades; results based on other transaction cost assumptions are found in Appendix A.

To test the hypothesis that outperformance for volatility targeting is related to the performance of trend following, we will need a trend following strategy, which we refer to as "simple trend". Based on Hurst et al. (2013), the simple trend strategy will take long or short positions in the underlying asset based on the average sign of the trailing 1-, 3-, and 12-month return. For example, if the return to the equity market factor is positive (negative) for all three horizons, then the simple trend strategy for that asset would be fully long (short) invested. But if only the 1- and 3-month trailing returns were positive with the 12-month trailing return negative, then the simple trend strategy would take a long position of 33.33% of notional. As with volatility targeting, we assume a 1-day implementation lag. The position function of the simple trend strategy is also provided in Exhibit 1.[6]

Basic Volatility Targeting Performance

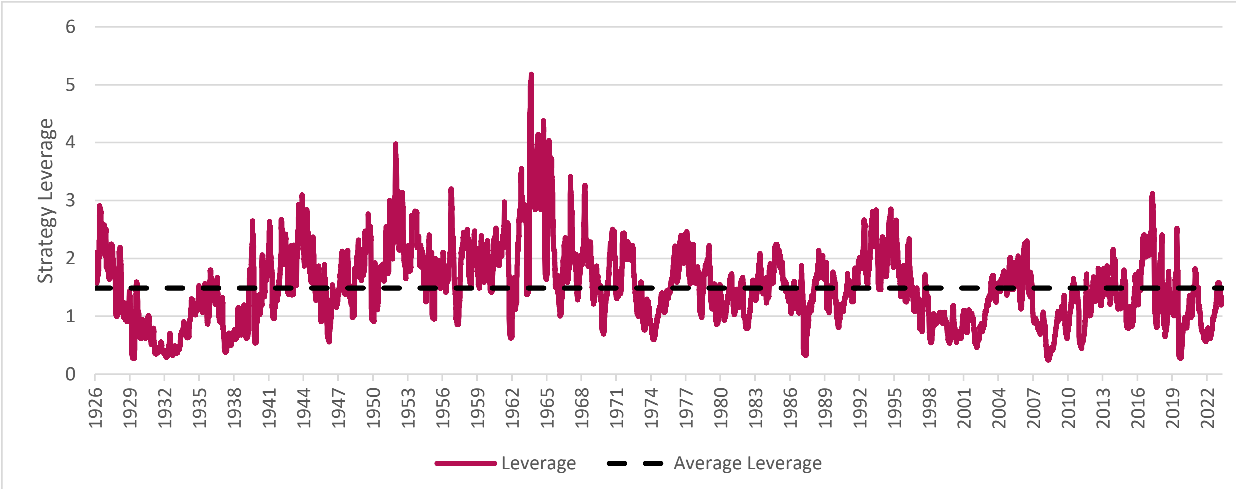
The volatility targeting strategy outperforms the buy-and-hold strategy on both an absolute and risk-adjusted basis, as shown in Exhibit 1. Further, we find that measures of dispersion in realized volatility, like root mean squared error (RMSE) of rolling 3-month volatility from it's target and return kurtosis, are roughly halved for the volatility targeting strategy, despite slightly over-realizing full period volatility.

Volatility targeting does, however, require substantial use of leverage. Exhibit 2 plots the time-varying leverage used by this implementation of volatility targeting, which averages 1.49x (compared to the buy-and-hold strategy that always uses 1x leverage) but gets as high as 5.18x in 1964 and as low as 0.24x in 2008.[7]

[6] We find that results throughout the paper are robust to different choices of trend model and are not particular to the strategy definition used here. Given the ease of replication, we present results using the Hurst et al. (2013) methodology. See Appendix C for tables based on alternative trend strategy formulations.

[7] The leverage of the strategy scales linearly with the volatility target used. Leverage shown in Exhibit 2 corresponds to a volatility target of 17.2%, set to match the full-period volatility of the buy-and-hold strategy. If we were to double the volatility target, strategy leverage would double; if we were to cut the target in half, strategy leverage would be cut in half.

EXHIBIT 2: Leverage, Volatility Targeted Equity Market (Mkt-Rf) Factor



Notes: This table shows the time-varying leverage employed by the volatility targeting strategy on the Mkt-Rf factor. Leverage is defined here to be the total investment of the strategy; an unlevered strategy such as the buy-and-hold strategy used in this paper, would have a fixed leverage of 1. Date range is 1927-2023.

Many investors find themselves unwilling to take as much leverage as would occasionally be required at this volatility target level. One common remedy for this concern is to impose a leverage cap on the strategy. In Exhibit 3, we provide basic volatility targeting strategy statistics for the buy-and-hold strategy on the Mkt-Rf factor, along with four different leverage implementations of the volatility targeting strategy: a 1x leverage cap, a 1.5x leverage cap, a 2.5x leverage cap, and uncapped.[8] Naturally, as positive leverage is needed to balance out periods of reduced exposure, implementing a tight leverage cap results in the strategy under-realizing its volatility target.

To visualize the performance of both the buy-and-hold strategy and the volatility targeting strategy, we also show in Exhibit 3 the cumulative total return of the each strategy on a log scale, along with the same for the leverage capped volatility targeting strategies as well. As expected based on return statistics, the uncapped volatility targeting strategy clearly outperforms the buy-and-hold strategy on a total return basis. Leverage capped volatility targeting strategies generally sit between the buy-and-hold performance and the full uncapped volatility targeting performance, with the 1x cap strategy slightly lagging buy-and-hold and the 2.5x cap strategy falling almost exactly in line with the uncapped strategy.[9] The similarities between the 2.5x cap strategy and uncapped suggest that the 5.5% of the time the strategy wants to take leverage above 2.5x have not been particularly consequential for performance over this period. The remainder of the paper will use the uncapped strategy for volatility targeting.

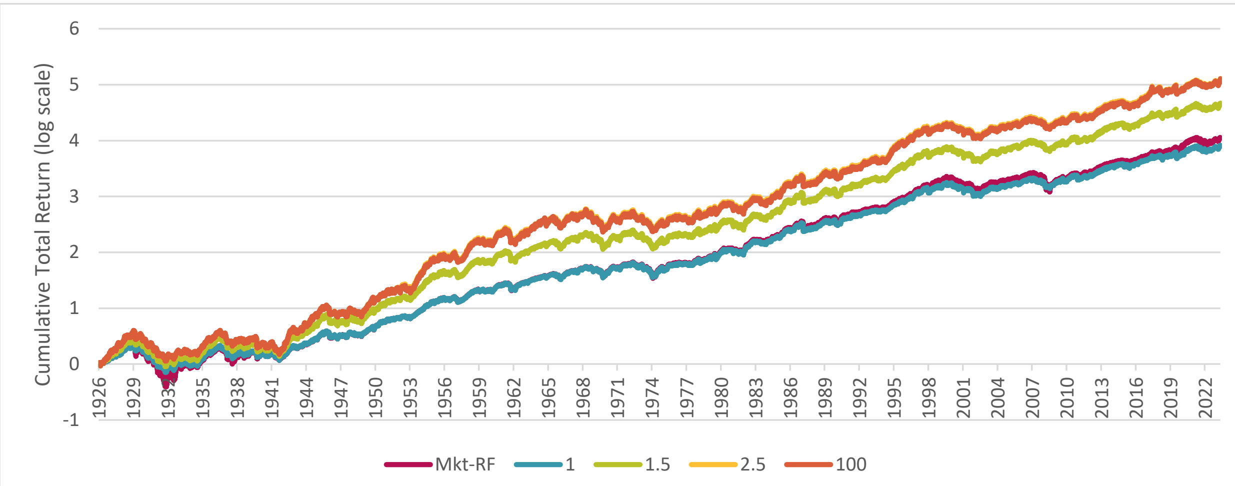
[8] The 1x leverage cap strategy only cuts positions relative to the buy-and-hold strategy. It is unable to increase leverage when volatility is below its target; it can only reduce positions when volatility is above its target.
[9] The 1x capped volatility targeting strategy lags the buy-and-hold in return specifically, but actually outperforms in Sharpe ratio, 3m RMSE from target, and kurtosis, while realizing only about 80% of the risk. In that sense, it behaves like a fairly successful "defensive" strategy.

EXHIBIT 3A: Strategy Statistics for Equity Market (Mkt-Rf) Factor Using Leverage Caps

	Buy-and-Hold	Volatility Targeting With...			
		1x Cap	1.5x Cap	2.5x Cap	Uncapped
Return	7.6%	6.7%	8.9%	10.2%	10.2%
Standard Deviation	17.2%	13.5%	16.6%	18.5%	18.7%
Sharpe Ratio	0.44	0.50	0.53	0.55	0.54
3m RMSE From Target	9.3%	5.0%	4.7%	4.5%	4.5%
Skewness	-0.2	-0.7	-0.8	-0.9	-0.9
Excess Kurtosis	16.4	12.9	9.8	8.8	8.5

Notes: This table reports basic performance statistics for the Buy-and-Hold and Volatility Targeting strategies applied to the Fama-French equity market (Mkt-Rf) factor. Volatility targeting strategies have leverage caps ranging from 1x (only reduces exposure in high volatility periods) to completely uncapped (our standard volatility targeting strategy used throughout the paper). Statistics are calculated using daily returns. 3m RMSE From Target is defined as the square root of the mean square error (RMSE) of the strategy's rolling 3-month annualized standard deviation from the target standard deviation; the target is set equal to the full-period standard deviation of the Buy-and-Hold strategy. Excess Kurtosis is defined as full-sample kurtosis minus 3, the kurtosis of a normal random variable. Date range is 1927-2023.

EXHIBIT 3B: Cumulative Log Total Returns of Buy-and-Hold and Volatility Targeting



Notes: This exhibit shows the evolution of full period cumulative log returns of Buy-and-Hold and Volatility Targeting strategies applied to the Fama-French equity market (Mkt-Rf) factor. Volatility targeting strategies have leverage caps ranging from 1x (only reduces exposure in high volatility periods) to completely uncapped (our standard volatility targeting strategy used throughout the paper). The 2.5x cap level is mostly indistinguishable from the uncapped strategy in this plot. Date range is 1927-2023.

Simple Mkt-Rf Regressions

As in Moreira and Muir, we first regress returns to the volatility-targeting strategy on returns from the buy-and-hold strategy for the equity market factor.[10] The first column of Exhibit 4 details the results from

[10] All single-series (ie, non-panel) regressions in the paper are standard OLS regressions with Newey-West heteroskedasticity and autocorrelation (HAC) robust standard errors with 10 lags. In results not presented here, we found as a robustness check that increasing the number of lags to 21 (a full month of business days) had only small

this basic regression. We find an economically meaningful and statistically significant alpha to volatility targeting over buy-and-hold of 3.07% per year. This impressive outperformance is exactly what caught our attention. The buy-and-hold strategy explains approximately three-quarters of the variation in volatility-managed equity market returns.

EXHIBIT 4: Regression Results for Equity Market (Mkt-Rf) Volatility Targeting Returns versus Buy-and-Hold, Simple Trend

		Buy and Hold Only	Including Trend
Beta	**Buy and Hold**	0.94	0.98
	t-stat	39.9	47.5
	Simple Trend		0.30
	t-stat		12.0
Alpha	**Intercept (annualized)**	3.07%	1.25%
	t-stat	2.9	1.3
	r^2 (adjusted)	0.75	0.79
	Leverage Effect	-0.31	

Notes: This exhibit shows regression output from two different regressions used to explain exposures present in a simple volatility-targeting strategy on the Fama-French Mkt-Rf market factor: a "buy and hold only" regression where the volatility targeting returns are regressed against the un-targeted returns (buy-and-hold) and a constant; and a second "including trend" regression where the volatility targeting returns are regressed against both buy-and-hold returns and returns to a simple trend following strategy, along with a constant. The reported t-stats are estimated using Newey-West standard errors with 10 lags. Leverage effect is defined as the full-period correlation between the 21-day return and the 21-day change in volatility estimate for the base asset. Date range is 1927-2023. Results for other Fama-French factors size (SMB), value (HML), and momentum (MOM) are found in Appendix B.

We then run the same regression a second time, but this time we include returns to the equity market simple trend strategy as an explanatory variable. The findings are easy to interpret: when we control for exposure to simple trend in the regression, the intercept (alpha) falls by 59% to 1.25% and loses its statistical significance. The inclusion of simple trend in this regression absorbs almost three-fifths of the alpha we see from volatility targeting. Further, the simple trend strategy has an estimated beta of 0.30; from the perspective of the regression, volatility targeting is seen as similar to a roughly full investment in the underlying asset plus a 30% allocation to trend following on that same asset.

It is important to note that this reduction in alpha from including trend following *doesn't* mean that volatility targeting isn't outperforming buy-and-hold! We continue to find outperformance to the strategy, even as the coefficient on buy-and-hold rises slightly. We simply demonstrate that a better name for the majority of that "alpha" is really "trend following."

impacts on estimates of standard errors and no impact on the significance of any of the variables presented in Exhibit 4.

The Leverage Effect and the Mechanical Relationship Between Volatility Targeting and Trend Positioning

The result that the majority of equity volatility-targeting alpha is better described as exposure to trend following is interesting, and not immediately obvious. Thinking back to our intuition about when we might expect volatility targeting to outperform, and as hypothesized and evidenced in Harvey et al. (2018), the stark connection between volatility targeting and simple trend likely occurs because of the negative correlation between return direction and return volatility for equity markets, the so-called *leverage effect*.[11]

For the purposes of this paper, we define the leverage effect as the correlation between 1-month return and the 1-month change in our volatility estimate. And indeed in Exhibit 4 we see a meaningfully negative leverage effect for the Mkt-Rf equity market factor of -0.31. The path by which a volatility targeting strategy can pick up positive return from subtle but significant exposure to trend following is exactly this leverage effect.

The position function for a single simple trend signal takes approximately the functional form

$$trendPosition_t(k) = sign\left(\sum_{i=1}^{k} R_{t-i}\right)$$

for lookback window *k* as of date *t*. Compare that to the position function for a simple volatility targeting strategy that takes the approximate functional form

$$vtPosition_t(k) = \frac{target}{\sqrt{\frac{1}{k}\sum_{i=1}^{k} R_{t-i}^2}}$$

where *k* is the window length used to estimate volatility and *target* is the volatility target of the strategy.[12]

When should we expect these position functions to be connected, i.e. when will $trendPosition_t(k) \approx vtPosition_t(k)$? The exact mathematical connection, while arising from the presence of trailing returns R_{t-i} in both functions, is extremely non-linear: in the way stands a sign operator, a "one-over", a square-root outside of an average, and an exponent inside of that same average. The mathematical proof required here is beyond the scope of this paper (and our mathematical abilities, evidently).

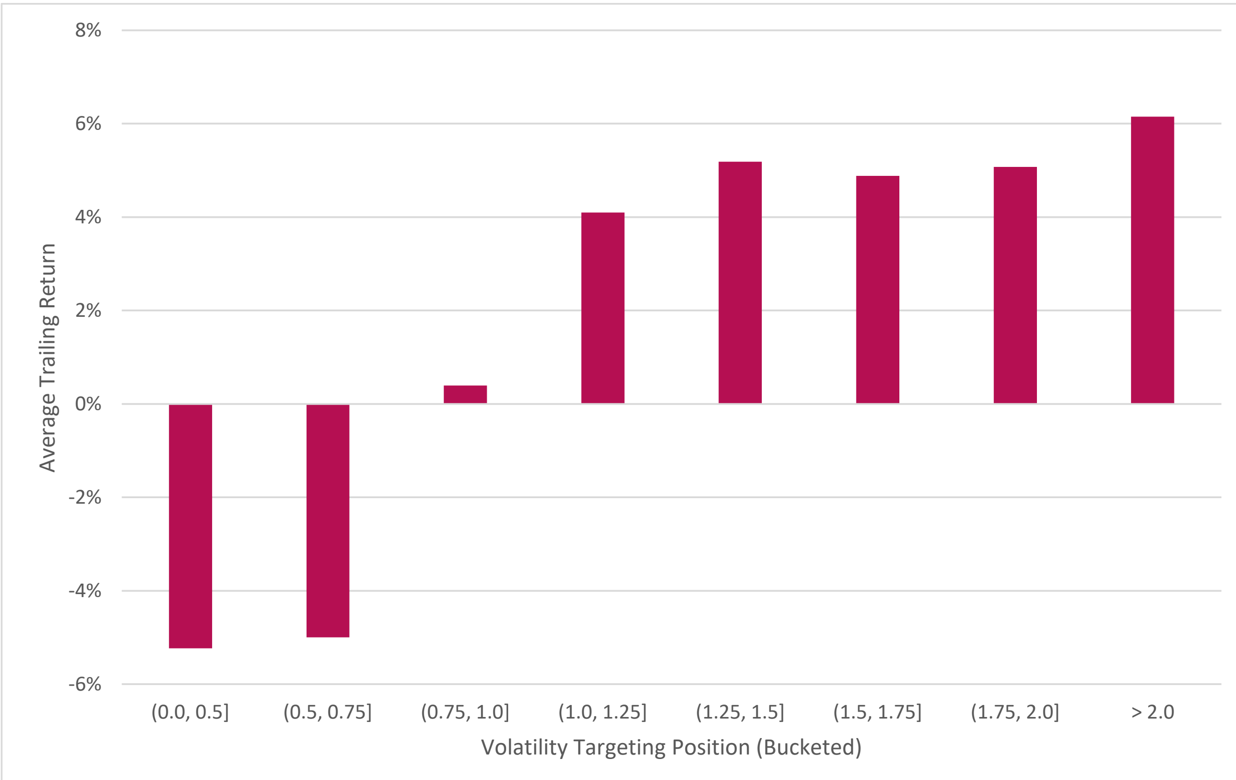
But the intuition is fairly straightforward. The trend signal direction will positively relate to the *direction* of recent returns, and the volatility targeting position will negatively relate to the *magnitude* of recent

[11] Black (1976) and Geske (1979) attribute the leverage effect in equities to increasing likelihood of default, and hence increasing risk, as asset prices fall.

[12] We are suppressing some complexity here that has no real effect on the intuition. Simple trend uses *compound* returns (so sums of *log* returns), and we are using a centered second moment for the volatility estimate in the volatility targeting strategy. Neither of these specific design choices would have a meaningful effect on the results in the paper, as these are both close approximations when using daily returns over relatively short horizons, and the simplification makes the mathematical connection here easier to see for the reader.

returns. Where return direction and magnitude are negatively related, we should expect to see common directionality in the positions of the two strategies.[13]

EXHIBIT 5: Average 1-, 3-, and 12-Month Trailing Return by Volatility Targeting Position, for the Fama-French Mkt-Rf Factor



Notes: This exhibit shows the average 1-, 3-, and 12-month trailing return bucketed by the volatility targeting position for the Mkt-Rf factor. The average trailing return tends to be more positive (negative) on average, suggesting a long (short) position for simple trend, when volatility targeting positions are larger (smaller). This is consistent with Mkt-Rf exhibiting a leverage effect, i.e. a negative correlation between return direction and change in volatility. Date range is 1927-2023.

And indeed, in Exhibit 5 we find exactly that position linkage. This exhibit shows the average 1-, 3-, and 12-month trailing return bucketed by the positioning of our volatility targeting strategy for the market

[13] This begins to touch on some of the discussion in Hood et al. (2019), which directly tested whether or not volatility targeting (in the context of a risk parity strategy) loads on the returns to a short volatility (ie, option selling) strategy. The authors found that there were enough non-linearities between the strategy positions that any linkage was destroyed, resulting in approximately zero loading for short volatility on volatility targeting. One large difference between short volatility and trend following is that systematic option selling involves expiration dates of contracts, and those expiration dates provide both routine resetting of option deltas and “clean-up trades” closing out delta hedges that ultimately fight against trading in a similar direction as volatility targeting, unlike trend following which uses continuously-rolling windows without sharp reversals in positioning.

factor.[14] Recall that the buy-and-hold strategy has a position of one all of the time. So, volatility targeting is underweight (relative to buy-and-hold) whenever its position is less than one and is overweight whenever its position is greater than one. Exactly as expected given the negative leverage effect for the market factor, we find that average trailing return is negative (positive) – indicating a likely short (long) position for trend – in buckets where volatility targeting is also short (long) relative to the buy-and-hold strategy.

To summarize, we believe that the mechanism by which trend following impacts the returns of volatility targeting is the leverage effect. If an asset has a negative relationship between return direction and magnitude, then volatility targeted returns will likely contain some exposure to trend; if trend is a positive contributor to returns, then a related strategy should outperform. When both of these line up – when trend following has positive performance *and* a leverage effect transmits that performance to a volatility targeting strategy – then we should expect meaningful outperformance to volatility targeting over its buy-and-hold benchmark alone. Thus, if we fail to take the connection to trend following into account, the volatility targeting strategy for that asset will appear to have alpha as in Moreira and Muir, when in fact it comes from trend.

Empirical Evidence From A Broad Data Set

To further test this hypothesis, we turn to a much broader set of 50 global commodity, equity, fixed income, and currency futures contracts.[15] Data availability limits us to a much shorter period of analysis for this section, with our futures data beginning in April of 2001 for most assets (after setting aside twelve months required to create simple trend signals) and running through the end of 2023.[16] In expanding our analysis to additional asset classes, we rely on historical prices of futures contracts from Refinitiv. These prices are used to generate a single rolled series of returns for each asset, based on a simple front-month contract rolling rule.[17]

Single Asset Regressions

We repeat the exercise from Exhibit 4 for each of the 50 assets individually: creating volatility-targeting and simple trend strategies, regressing volatility-targeted returns on buy-and-hold returns, adding simple

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[14] The average trailing return across those three lookback windows can be thought of as the simple trend position function without the sign operators.

[15] In addition to allowing for a broader data set, futures contracts have the benefit of implicitly accounting for time-variation in the cost of leverage, making this analysis more realistic than previous sections that used returns in excess of the 3-month T-bill rate. As in earlier sections, we continue to apply a 2bp transaction cost assumption for all trades. By accounting for financing and transaction costs, reported returns to futures contracts in this section constitute hypothetically realizable excess returns.

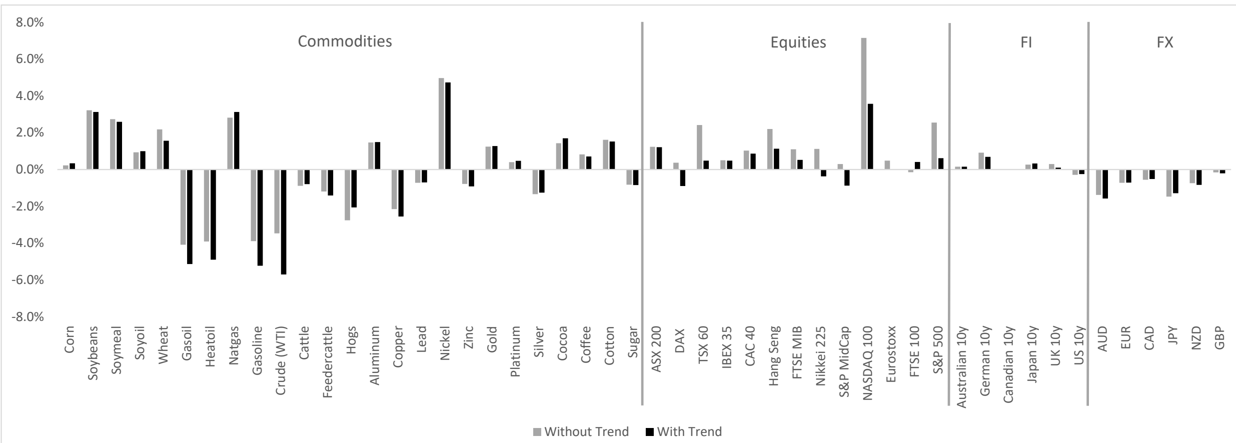
[16] Raw data for all assets begins on April 3, 2000 except ASX 200 (5/3/2000), FTSE MIB (3/23/2004), Nikkei (7/19/2006), and S&P 400 Midcap (12/23/2003). In all cases, we set aside the first 12 months to form signals.

[17] Specifically, we assume that the front month contract is held through 7 days prior to expiration, rolling to the next available contract at the close on that day, for equity, fixed income, and currency contracts. The commodities series follow the same roll rule, but use only contracts that are present in the Bloomberg Commodity Index (BCOM) rolling schedule for liquidity reasons. To calculate returns around the roll, we implicitly assume that the roll happens instantaneously at the close on the roll date; close-to-close returns leading up to and including the roll date are calculated with the old contract, and the close-to-close return for any day after the roll date is calculated using the new contract.

trend returns as explanatory variables, and then analyzing what happens to the coefficients across these regressions.

The first and most obvious question to ask is whether or not volatility targeting appears to have alpha over buy-and-hold for the broad data set. Breaking the 50 assets into distinct asset classes in Exhibit 6, we see that equities have consistent and meaningfully positive alpha prior to adding trend as a regressor.[18] This contrasts with commodities, which are mixed, and fixed income and currencies, which are relatively smaller in magnitude than we find across the suite of global equity index contracts.[19]

EXHIBIT 6: Estimated Single-Asset Regression Alphas Across Four Asset Classes, With and Without Simple Trend Following



Notes: This exhibit shows the estimated single-asset regression alphas for all 50 assets, both with (dark bars) and without (light bars) simple trend returns as an explanatory variable. For most assets, the difference in estimated alpha is very small, but for equities we see a sizeable reduction in alpha once we include simple trend in the regression. Only 3 of 50 estimated alphas without trend as an explanatory variable were statistically significant (had t-stats greater than 2 in absolute value) in this shorter data set, and none of those three maintained statistical significance after inclusion of trend in the regression. Date range is 2000-2023.

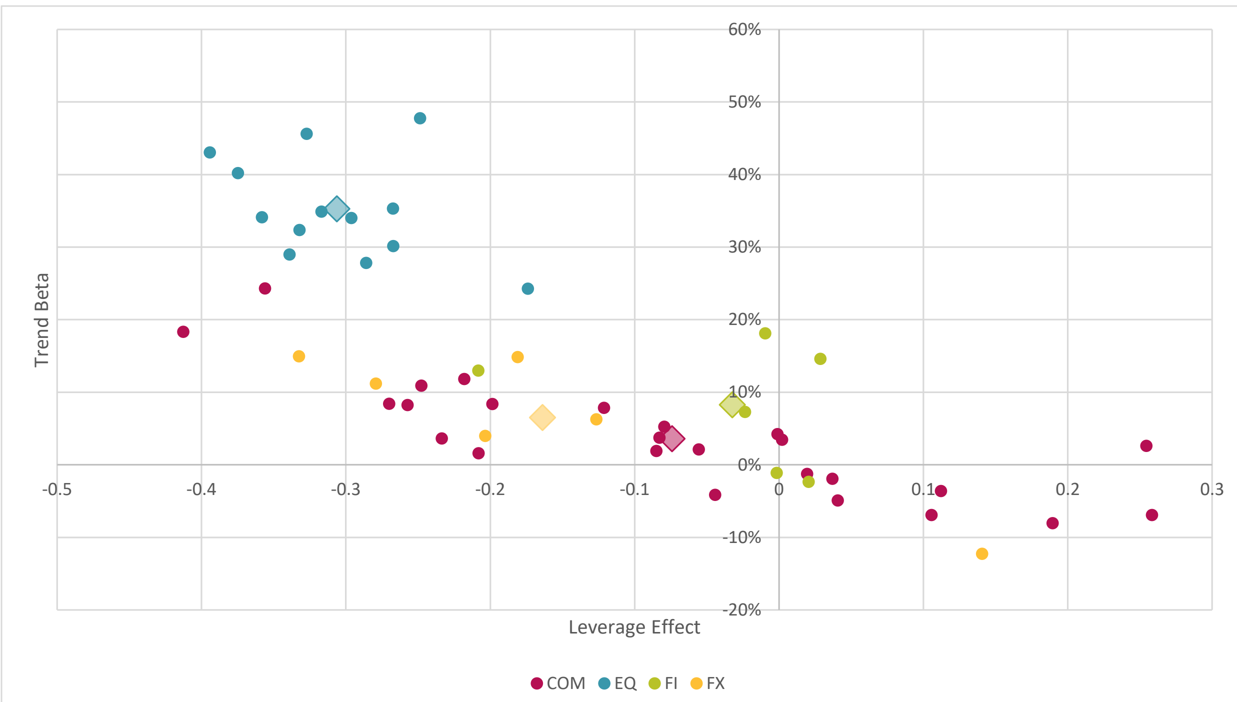
When adding simple trend as an explanatory variable, the average equity alpha to volatility targeting falls by 65%. In contrast, alphas are mostly unchanged for all other assets.[20] To provide the reader a taste of the results from the 50 single-asset regressions, we recreate Exhibits 4 and 5 for two representative assets, S&P 500 e-mini futures (labeled by the contract base "ES") and US 10-year bond futures ("TY"), in

[18] Estimated alphas are generally not statistically significant anywhere in these shorter regressions: only three of the alpha t-stats prior to the inclusion of trend as an explanatory variable were greater than two in absolute value, slightly more than would be expected by pure chance alone. None of these significant alphas maintained that significance after including trend in the regressions.
[19] Currencies showing consistently negative alpha to volatility targeting is a bit of a puzzle to us, particularly as it is often not negligible (-77bps on average across the asset class) and the leverage effect for the asset class is estimated to be slightly negative, shown in Exhibit 8.
[20] Energies (commodities) are an interesting but unsurprising exception here, with negative alphas generally becoming *more* negative, while also exhibiting equity-like leverage effects. The exception to the exception here is, of course, Natural Gas (which is notorious for having many unique characteristics). See Kristoufek (2014) for a discussion of the leverage effect in energy futures.

Appendix D. As expected, ES futures conform to expectations for an asset with a sizeable negative leverage effect – positive alpha to volatility targeting that is subsequently absorbed by a large loading on simple trend when included, decent explanatory power accruing to trend, and average trailing asset returns that positively co-vary with volatility targeting positions – while TY exhibits none of those markers.

As Harvey et al. (2018) suggested might be the case, we also find that the negative leverage effects in equity assets appear to be driving the loading of volatility targeting on trend. Exhibit 7 plots the trend beta as a function of estimated leverage effect for each asset in the sample, color coded by asset class. Every equity asset has a trend loading greater than the highest of the non-equity assets (WTI Crude), and the asset class exhibits the most-consistently negative leverage effect.

EXHIBIT 7: Relationship Between Estimated Leverage Effect and Loading on Simple Trend Following



Notes: This exhibit shows the relationship between estimated leverage effect and the estimated beta to a simple trend strategy for all 50 assets in the broad data set. Diamond markers represent the average observation for each asset class. Equities generally exhibit both a leverage effect (negative correlation between return and change in volatility) and a large, positive loading on simple trend. For other asset classes, there appears to be a relationship between these variables, with reduced or opposite leverage effect pairing with lower or negative trend beta. Date range is 2000-2023.

Even within an asset class like commodities, where we don't see uniform response to adding simple trend returns as an explanatory variable, we see a negative relationship between the leverage effect and estimated loading on trend. 15 of 16 commodities with negative leverage effects also have positive trend betas; 7 of 9 with positive leverage effects also have negative trend betas. While equities as an asset class certainly stand out in Exhibit 7, we can still see a slight negative relationship here between leverage effect and trend exposure to volatility targeting within non-equity asset classes.

Panel Regressions

This result – alpha to volatility targeting absorbed by including simple trend returns as an explanatory variable – appears to be mainly an equity phenomenon. We test this hypothesis by grouping assets together by asset class and running panel versions of our previous regressions.

Panel regression provides a few benefits. First and foremost, it allows us to group assets in a logical manner consistent with our hypothesis, more directly testing it. Second, it suggests potential outcomes from creating a more diversified, global portfolio of volatility targeting series as one might see in a risk parity portfolio. And most importantly, it effectively increases the sample size to each grouping, increasing the power of our statistical analysis.[21]

EXHIBIT 8: Panel Regression Results Across Four Asset Classes

		All Assets		COM		EQ		FI		FX	
		Buy and Hold Only	Including Trend	Buy and Hold Only	Including Trend	Buy and Hold Only	Including Trend	Buy and Hold Only	Including Trend	Buy and Hold Only	Including Trend
Beta	**Buy and Hold**	0.99	0.99	1.00	1.00	0.96	1.06	0.98	0.97	0.98	0.99
	t-stat	152.8	182.5	170.8	177.7	71.2	125.3	128.9	122.8	97.3	107.7
	Simple Trend		0.10		0.04		0.35		0.08		0.08
	t-stat		14.0		5.2		32.4		7.4		6.7
Alpha	**Intercept (annualized)**	0.23%	0.04%	-0.11%	-0.21%	1.57%	0.65%	0.22%	0.17%	-0.87%	-0.91%
	t-stat	0.3	0.1	-0.1	-0.3	1.0	0.5	0.8	0.6	-1.7	-1.8
	r^2 **(adjusted)**	0.87	0.88	0.89	0.89	0.81	0.86	0.88	0.89	0.89	0.89
	Leverage Effect (avg)	-0.14		-0.07		-0.31		-0.03		-0.16	

Notes: This exhibit shows the regression output for panel versions of the "buy-and-hold only" and "including trend" regressions exhibited earlier. Panels are formed by grouping either by asset class or by pooling all 50 assets together. Panel regression standard errors and t-stats are computed using time-cluster-robust covariance estimation techniques as described in Rogers (1993). In general, this results in intercept estimates that are not statistically significant at a 95% confidence level, but are similar in both direction and magnitude to earlier results for equities. Non-equity asset classes are mostly unaffected by the inclusion of simple trend returns. Regression results are consistent with estimated leverage effects across asset classes. Date range is 2000-2023.

Exhibit 8 presents regression statistics for several panel regressions, both with and without trend returns as an exogenous variable, for all 50 assets together as well as each asset class separately. These results certainly rhyme with the findings from earlier in the paper. Equities have the largest and most economically-meaningful alpha to volatility targeting at 1.57% per year, and that is reduced by slightly

[21] We need to be careful here. Panel estimates, if not adjusted for the correlation between assets within an asset class, can falsely inflate the significance of coefficients; estimates from a panel of 14 equity assets, for instance, are unlikely to have standard errors that reflect 14x the sample size if those 14 assets all have correlated returns through time. We compute all t-statistics in this subsection using time-clustering-robust standard errors, as described in Rogers (1993).

less than two-thirds to 0.65% when we include trend in the regression. Other asset classes show smaller (commodities and fixed income) or somewhat negative (currencies) alphas, and these are mostly unchanged by the inclusion of simple trend. Loading on trend is much higher at 0.35 for equities than for other asset classes. And the explanatory power of trend, as measured by the change in adjusted r^2, is isolated to equities alone. Finally, and perhaps unsurprisingly at this point, the estimated leverage effect for equities is much more negative than for other asset classes.

Despite the increase in effective sample size we earn via panel regression, buy-and-hold-only alphas are still not large enough to be statistically significant. This is worth noting here, in our opinion, and we suggest a couple of reasons this may be the case. Most obviously, a simulation that reaches back to the 1920's (as in Moreira and Muir) will have a much larger sample size by which to judge significance, but on a forward-looking basis, the investment horizon for a volatility targeting investor is likely considerably shorter than even our shorter 20+ year sample for the broad data set. Another explanation for this is that volatility targeting on any individual asset will load only on trend following for that same asset, and much of the benefit from trend following comes from pulling together many diversifying assets into a single, more impactful portfolio; loading on a single asset trend strategy can only provide so much return, and will be much more uncertain, resulting in higher standard errors to the buy-and-hold alpha.

Both of these considerations support a viewpoint that we hold: that volatility targeting can outperform buy-and-hold, and the fact that volatility targeting embeds a positive risk premium exposure in trend lends serious weight to that view (at least for equities). But the most salient reason to volatility target, in our view, is the impact that it has on the shape of the return distribution (primarily expressed in higher moments like volatility-of-volatility – or RMSE from target in our case – and kurtosis, as seen in Exhibit 1.B for the market factor and Appendix E for the broad data set) rather than explicitly seeking higher returns.[22] Likewise, an investor hoping to gain exposure to trend following is most likely best off allocating directly to a diversified trend fund alongside an existing equity allocation, rather than seeking to gain trend exposure via volatility targeting.

Conclusion

While other papers have reached the conclusion that volatility targeting has alpha over a buy-and-hold strategy on the same asset, we find that this alpha is better described as exposure to trend following. Using equity market returns dating back to the 1920's, we regress returns to volatility targeting on buy-and-hold and trend returns, and we find a large and meaningful exposure to trend that accounts for around 2/3rds of the total alpha present when we omit trend from the regression.

This exposure to trend is almost certainly caused by the leverage effect, where return direction is correlated with change in volatility; in equity markets in particular, the negative leverage effect creates a positive association between volatility targeting and trend positions. Unsurprisingly, in other asset classes where the leverage effect is not as prominently negative as in equities, we do not find either economically meaningful alpha to volatility targeting (when trend is omitted as an explanatory variable) or economically meaningful exposure of volatility targeting returns to trend following (when trend is

[22] For further evidence of the effect of volatility targeting on the shape of an asset's return distribution, Harvey et. al. (2018) shows meaningful reduction to volatility of volatility and left tail measures when volatility targeting.

included). We test this across four asset classes: equities, where we find the connection, as well as commodities, fixed income, and currencies, where we do not.

This finding functions as an important contribution to our understanding of how these seemingly unconnected strategies behave, binding the volatility targeting and trend following literatures together via a common positioning mechanic that arises from a specific exogenous feature of equity returns: the leverage effect.

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Appendix A: Is Volatility Targeting Trendy With Different Transaction Cost Assumptions?

EXHIBIT A.1: Strategy Statistics for Mkt-Rf using different transaction cost levels

		0bps		1bp		2bps		5bps		10bps	
	Buy-and-Hold	**Volatility Targeting**	**Trend**	**Volatility Targeting**	**Trend**	**Volatility Targeting**	**Trend**	**Volatility Targeting**	**Trend**	**Volatility Targeting**	**Trend**
Return	7.6%	10.3%	5.3%	10.2%	5.1%	10.2%	4.9%	10.0%	4.2%	9.8%	3.1%
Standard Deviation	17.2%	18.7%	13.2%	18.7%	13.2%	18.7%	13.2%	18.7%	13.2%	18.7%	13.2%
Sharpe Ratio	0.44	0.55	0.40	0.55	0.39	0.54	0.37	0.54	0.32	0.52	0.24
3m RMSE From Target	9.3%	4.5%	9.9%	4.5%	9.9%	4.5%	9.9%	4.5%	9.9%	4.5%	9.9%
Skewness	-0.2	-0.9	-0.7	-0.9	-0.7	-0.9	-0.7	-0.9	-0.7	-0.9	-0.7
Excess Kurtosis	16.4	8.5	24.9	8.5	24.9	8.5	24.9	8.5	24.9	8.6	24.9

Notes: This exhibit reports basic performance statistics for the three main strategies we discuss in this paper – Buy-and-Hold, Volatility Targeting, and Simple Trend – applied to the Fama-French Mkt-Rf market factor assuming different transactions costs levels from 0bps to 10bps per trade. The Buy-and-Hold strategy is unaffected by transactions cost assumptions and therefore appears only once in this table. Statistics are calculated using daily returns. 3m RMSE From Target is defined as the square root of the mean square error (RMSE) of the strategy's rolling 3-month annualized standard deviation from the target standard deviation; the target is set equal to the full-period standard deviation of the Buy-and-Hold strategy. Excess Kurtosis is defined as full-sample kurtosis minus 3, the kurtosis of a normal random variable. Date range is 1927-2023.

EXHIBIT A.2: Strategy Statistics for Mkt-Rf using different transaction cost levels

Assumed Transaction Costs		**0bps**		**1bp**		**2bps**		**5bps**		**10bps**	
		Buy and Hold Only	**Including Trend**	**Buy and Hold Only**	**Including Trend**	**Buy and Hold Only**	**Including Trend**	**Buy and Hold Only**	**Including Trend**	**Buy and Hold Only**	**Including Trend**
Beta	**Buy and Hold**	0.94	0.98	0.94	0.98	0.94	0.98	0.94	0.98	0.94	0.98
	t-stat	39.9	47.5	39.9	47.5	39.9	47.5	39.9	47.5	39.9	47.5
	Simple Trend		0.30		0.30		0.30		0.30		0.30
	t-stat		12.0		12.0		12.0		12.0		12.0
Alpha	**Intercept (annualized)**	3.17%	1.21%	3.12%	1.23%	3.07%	1.25%	2.93%	1.30%	2.68%	1.39%
	t-stat	3.0	1.2	3.0	1.3	2.9	1.3	2.8	1.3	2.5	1.4
	r^2 (adjusted)	0.75	0.79	0.75	0.79	0.75	0.79	0.75	0.79	0.75	0.79
	Leverage Effect	-0.31		-0.31		-0.31		-0.31		-0.31	

Notes: Exhibit A.2 shows regression output from two different regressions used to explain exposures present in a simple volatility-targeting strategy on the Fama-French Mkt-Rf market factor: a "buy and hold only" regression where the volatility-targeting returns are regressed against the un-targeted returns (buy-and-hold) and a constant; and a second "including trend" regression where the volatility-targeting returns are regressed against both buy-and-hold returns and returns to a simple trend following strategy, along with a constant. Each pair of regressions is based on returns with transaction cost assumptions that range from 0bps per unit traded to 10bps per unit traded. We use 2bps, the middle assumed level, throughout the paper for all other tables. The reported t-stats are estimated using Newey-West standard errors with 10 lags. Leverage effect is defined as the full-period correlation between the 21-day return and the 21-day change in volatility estimate. Date range is 1927-2023.

Appendix B: Is Volatility Targeting Trendy For Other Fama-French Factors?

EXHIBIT B.1: Strategy Statistics for Fama-French Factor (SMB, HML, and MOM) Returns

	SMB Buy-and-Hold	SMB Volatility Targeting	SMB Trend	HML Buy-and-Hold	HML Volatility Targeting	HML Trend	Mom Buy-and-Hold	Mom Volatility Targeting	Mom Trend
Return	1.1%	0.6%	3.1%	3.8%	5.2%	4.9%	6.2%	15.1%	2.2%
Standard Deviation	9.4%	10.3%	7.2%	9.9%	10.7%	7.8%	12.5%	13.7%	9.7%
Sharpe Ratio	0.12	0.06	0.43	0.38	0.49	0.64	0.50	1.10	0.23
3m RMSE From Target	5.4%	2.4%	5.6%	5.9%	2.2%	6.1%	7.5%	3.4%	7.8%
Skewness	-0.7	-0.6	-0.2	0.6	0.4	-0.4	-1.7	-0.9	-2.6
Excess Kurtosis	20.9	10.6	49.1	14.8	6.7	24.9	27.0	9.4	65.2

Notes: This exhibit reports basic performance statistics for the three main strategies we discuss in this paper – Buy-and-Hold, Volatility Targeting, and Simple Trend – applied to the Fama-French factors SMB, HML, and MOM. Statistics are calculated using daily returns. 3m RMSE From Target is defined as the square root of the mean square error (RMSE) of the strategy's rolling 3-month annualized standard deviation from the target standard deviation; the target is set equal to the full-period standard deviation of the Buy-and-Hold strategy. Excess Kurtosis is defined as full-sample kurtosis minus 3, the kurtosis of a normal random variable. All transaction costs are assumed to be 2bps per unit of notional traded. Date range is 1927-2023.

EXHIBIT B.2: Regression Results for Equity Factor (SMB, HML, and MOM) Volatility Targeting Returns versus Buy-and-Hold, Simple Trend

		Mkt-Rf Buy and Hold Only	Mkt-Rf Including Trend	SMB Buy and Hold Only	SMB Including Trend	HML Buy and Hold Only	HML Including Trend	MOM Buy and Hold Only	MOM Including Trend
Beta	**Buy and Hold**	0.94	0.98	0.93	0.93	0.89	0.89	0.91	0.86
	t-stat	39.9	47.5	36.7	36.4	37.5	37.5	39.5	39.5
	Simple Trend		0.30		0.00		0.00		0.20
	t-stat		12.0		0.1		0.2		7.5
Alpha	**Intercept (annualized)**	3.07%	1.25%	-0.43%	-0.44%	1.82%	1.80%	9.30%	9.16%
	t-stat	2.9	1.3	-0.7	-0.7	2.6	2.5	9.1	9.3
	r^2 (adjusted)	0.75	0.79	0.72	0.72	0.68	0.68	0.69	0.70
	Leverage Effect	-0.31		-0.18		0.17		-0.25	

Notes: Exhibit B.2 shows regression output from two different regressions used to explain exposures present in a simple volatility-targeting strategy on the Fama-French Mkt-Rf market factor, SMB size factor, HML value factor, and MOM momentum factor: a "buy and hold only" regression where the volatility-targeting returns are regressed against the un-targeted returns (buy-and-hold) and a constant; and a second "including trend" regression where the volatility-targeting returns are regressed against both buy-and-hold returns and returns to a simple trend following strategy, along with a constant. The reported t-stats are estimated using Newey-West standard errors with 10 lags. Leverage effect is defined as the full-period correlation between the 21-day return and the 21-day change in volatility estimate. Date range is 1927-2023.

Appendix C: Is Volatility Targeting Trendy With Various Trend Methodologies?

EXHIBIT C.1: Regression Results for Equity Market (Mkt-Rf) Volatility Targeting Returns versus Buy-and-Hold and Trend Using Various Trend Methodologies

		Buy and Hold Only	Simple Trend	CTAmom	1m	3m	6m	9m	12m
Beta	**Buy and Hold**	0.94	0.98	0.99	0.96	0.98	0.96	0.94	0.94
	t-stat	39.9	47.5	44.9	44.0	43.2	43.2	44.9	45.3
	Simple Trend		0.30	0.28	0.10	0.19	0.22	0.22	0.23
	t-stat		12.0	10.0	5.0	8.8	10.7	11.7	13.0
Alpha	**Intercept (annualized)**	3.07%	1.25%	1.24%	2.44%	1.83%	1.57%	1.85%	1.87%
	t-stat	2.9	1.3	1.3	2.4	1.8	1.6	1.9	1.9
	r^2 (adjusted)	0.75	0.79	0.78	0.75	0.78	0.78	0.79	0.79
	Alpha Reduction		59%	60%	21%	40%	49%	40%	39%

Notes: This exhibit recreates Exhibit 4, showing the results of regressing volatility targeted returns on buy-and-hold only (column 1), as well as buy-and-hold and returns to various trend strategies (columns 2-8) for the Mkt-Rf factor. We approximate the weights of the CTAmom strategy from Figure 4: Panel B of Hamill, Rattray, and van Hemert (2016), and further include monthly signals of 1-, 3-, 6-, 9-, and 12-months. Results are robust across these alternative methodologies, with all methodologies resulting in a reduction of alpha to volatility targeting. Simple Trend and CTAmom both reduce alpha by 59% and 60% respectively, while the single month signals of 3-, 6-, 9-, and 12-months range in reduction from 40% to 50% and 1-month reduces alpha 21%. The 1-month signal is the only strategy where the alpha is still significant after including trend in the regression. Reported t-stats are estimated using Newey-West standard errors with 10 lags. Date Range is 1927-2023.

EXHIBIT C.2: Panel Regression Results Across Four Asset Classes with Various Trend Methodologies

				EQ					
		Buy and Hold Only	Simple Trend	CTAmom	1m	3m	6m	9m	12m
Beta	**Buy and Hold**	0.96	1.06	1.06	0.99	1.03	1.05	1.03	1.03
	t-stat	71.2	125.3	118.3	84.1	105.8	128.9	132.3	131.9
	Simple Trend		0.35	0.33	0.09	0.22	0.29	0.30	0.30
	t-stat		32.4	29.1	8.4	23.9	36.2	38.2	37.7
Alpha	**Intercept (annualized)**	1.57%	0.65%	0.67%	1.41%	1.30%	-0.05%	0.14%	0.14%
	t-stat	1.0	0.5	0.5	1.0	1.0	0.0	0.1	0.1
	r^2 (adjusted)	0.81	0.86	0.86	0.81	0.84	0.87	0.88	0.88
	Alpha Reduction		59%	57%	10%	17%	103%	91%	91%

FI

		Buy and Hold Only	Simple Trend	CTAmom	1m	3m	6m	9m	12m
Beta	**Buy and Hold**	0.98	0.97	0.97	0.98	0.97	0.97	0.96	0.96
	t-stat	128.9	122.8	126.8	129.4	127.3	127.9	118.0	115.3
	Simple Trend		0.08	0.08	0.03	0.04	0.06	0.06	0.06
	t-stat		7.4	7.8	4.8	6.3	7.8	7.3	7.0
Alpha	**Intercept (annualized)**	0.22%	0.17%	0.19%	0.21%	0.21%	0.24%	0.15%	0.14%
	t-stat	0.8	0.6	0.7	0.7	0.7	0.9	0.5	0.5
	r^2 (adjusted)	0.88	0.89	0.89	0.88	0.89	0.89	0.89	0.89
	Alpha Reduction		24%	13%	6%	5%	-11%	32%	35%

FX

		Buy and Hold Only	Simple Trend	CTAmom	1m	3m	6m	9m	12m
Beta	**Buy and Hold**	0.98	0.99	0.99	0.98	0.98	0.99	0.99	0.99
	t-stat	97.3	107.7	105.1	98.0	103.0	107.5	111.3	111.6
	Simple Trend		0.08	0.06	0.01	0.04	0.05	0.07	0.08
	t-stat		6.7	5.2	1.4	5.0	6.8	9.7	10.1
Alpha	**Intercept (annualized)**	-0.87%	-0.91%	-0.90%	-0.86%	-0.91%	-0.94%	-1.05%	-1.00%
	t-stat	-1.7	-1.8	-1.8	-1.7	-1.8	-1.9	-2.1	-2.1
	r^2 (adjusted)	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
	Alpha Reduction		-4%	-3%	2%	-4%	-8%	-20%	-15%

COM

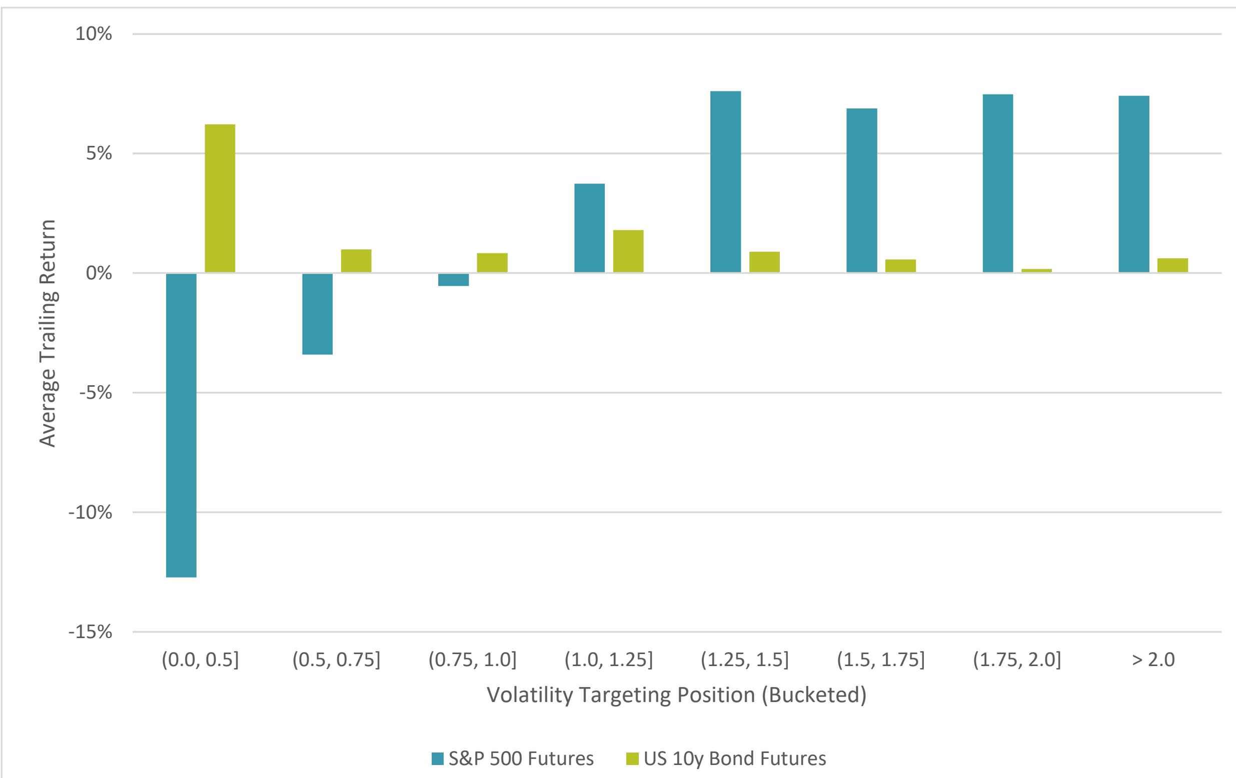
		Buy and Hold Only	Simple Trend	CTAmom	1m	3m	6m	9m	12m
Beta	**Buy and Hold**	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	t-stat	170.8	177.7	180.8	175.5	178.6	176.8	171.8	169.0
	Simple Trend		0.04	0.05	0.02	0.03	0.03	0.02	0.02
	t-stat		5.2	6.0	4.6	6.3	6.0	4.1	3.1
Alpha	**Intercept (annualized)**	-0.11%	-0.21%	-0.24%	-0.14%	-0.19%	-0.16%	-0.19%	-0.17%
	t-stat	-0.1	-0.3	-0.3	-0.2	-0.2	-0.2	-0.2	-0.2
	r^2 (adjusted)	0.89	0.89	0.89	0.89	0.89	0.89	0.89	0.89
	Alpha Reduction		-90%	-118%	-30%	-72%	-46%	-74%	-49%

Notes: This exhibit recreates Exhibit 8, showing the regression output for panel versions of the "buy-and-hold only" and "including trend" regressions exhibited earlier and is extended to compare various trend strategy methodologies. Panels are formed by grouping by asset class. Panel regression standard errors and t-stats are computed using time-cluster-robust covariance estimation techniques as described in Rogers (1993). In general, this results in intercept estimates that are not statistically significant at a 95% confidence level but are similar in both direction and magnitude to earlier results for equities. The alpha reduction for EQ is largest for single month signals 6-, 9-, and 12-months, where the reduction is over 90%, Simple Trend and CTAmom reduce the alpha by 59% and 57% respectively, and 1- and 3-months reduce the alpha by 10% and 17%, respectively. Non-equity asset classes are mostly unaffected by the inclusion of trend returns regardless of the trend methodology. The large negative alpha reduction values for COM are a function of a small negative alpha in the first regression and not reflective of a meaningful change in coefficient estimate. Date range is 2000-2023.

Appendix D: Is Volatility Targeting Trendy for ES and TY?

EXHIBIT D.1: Regression Results and Average 1-, 3-, and 12-Month Trailing Return by Volatility Targeting Position, for ES and TY Futures Contracts

		S&P 500 Futures		US 10y Futures	
		Buy and Hold Only	**Including Trend**	**Buy and Hold Only**	**Including Trend**
Beta	**Buy and Hold**	0.95	1.08	0.98	0.98
	t-stat	18.8	37.7	50.4	48.5
	Simple Trend		0.46		-0.02
	t-stat		11.9		-1.1
Alpha	**Intercept (annualized)**	2.55%	0.62%	-0.29%	-0.25%
	t-stat	1.2	0.4	-0.6	-0.6
	r^2 (adjusted)	0.75	0.85	0.88	0.88
	Leverage Effect	-0.33		0.02	



Notes: The top panel of this exhibit reports regression results for both "ES" S&P 500 e-mini futures and "TY" US 10-year bond futures contracts. As in Exhibit 4, volatility targeting loads significantly on simple trend for ES, with a resulting large reduction in alpha. We see neither of these results for TY futures, where the leverage effect is not present. The bottom panel of this exhibit shows the average trailing 1-, 3-, and 12-month return, bucketed by the volatility targeting position for both ES and TY. As with Exhibit 5, average trailing return tends to positively correlate with volatility targeting position for ES. This is consistent with ES exhibiting a leverage effect, i.e. a negative correlation between return direction and change in volatility. This is not the case for TY futures, where the data does not exhibit a leverage effect. Date range is 2000-2023.

Appendix E: Strategy Statistics for Futures Returns

EXHIBIT E.1: Strategy Statistics for Commodity, Equity, Fixed Income, and Currency Futures Returns

		Corn	Soybeans	Soymeal	Soyoil	Wheat	Gasoil	Heatoil	Natgas	Gasoline	Crude (WTI)	Cattle	Feeder Cattle	Hogs	Aluminum	Copper	Lead	Nickel	Zinc	Gold	Platinum	Silver	Cocoa	Coffee	Cotton	Sugar
Buy-And-Hold	Return	-2.9%	8.2%	12.7%	3.6%	-4.9%	10.5%	9.9%	-17.3%	12.2%	5.0%	-0.6%	1.3%	-7.9%	0.1%	10.6%	10.5%	10.0%	5.3%	7.5%	6.1%	8.7%	7.9%	-4.1%	-1.1%	5.8%
	Standard Deviation	26.6%	23.0%	25.8%	24.6%	30.8%	32.3%	33.9%	48.8%	37.0%	38.4%	15.5%	15.9%	27.2%	21.4%	25.4%	30.2%	39.0%	28.6%	17.2%	24.8%	30.9%	28.8%	33.1%	26.8%	30.8%
	Sharpe Ratio	-0.11	0.36	0.49	0.14	-0.16	0.33	0.29	-0.36	0.33	0.13	-0.04	0.08	-0.29	0.00	0.42	0.35	0.26	0.18	0.44	0.25	0.28	0.27	-0.12	-0.04	0.19
	3m RMSE From Target	8.6%	7.3%	7.1%	7.7%	9.1%	11.6%	11.8%	15.7%	14.3%	16.5%	5.0%	5.2%	8.1%	6.7%	10.1%	11.7%	17.1%	10.6%	5.9%	8.8%	12.1%	7.4%	8.1%	8.0%	8.2%
	Skewness	0.1	-0.1	0.1	0.1	0.2	-0.1	-0.2	0.1	-0.4	-0.3	-0.2	-0.2	0.0	-0.1	0.0	0.0	3.1	0.0	-0.2	-0.2	-0.6	-0.1	0.3	0.0	-0.1
	Excess Kurtosis	2.3	2.3	1.9	2.1	1.9	3.6	4.0	2.5	8.5	12.9	3.2	2.9	1.2	2.3	4.5	3.7	85.0	2.7	5.3	4.2	6.3	2.1	3.2	1.2	1.7
Volatility Targeting	Return	-2.2%	11.7%	15.6%	4.7%	-2.8%	6.1%	5.5%	-16.6%	7.4%	1.1%	-1.4%	0.3%	-10.5%	1.7%	8.3%	9.9%	15.6%	4.1%	8.6%	5.9%	6.6%	9.4%	-2.4%	0.6%	3.6%
	Standard Deviation	28.1%	24.2%	27.0%	25.5%	32.0%	34.0%	36.0%	51.7%	39.1%	41.3%	16.3%	16.7%	28.2%	22.3%	26.6%	31.3%	43.8%	29.8%	18.3%	26.0%	33.0%	29.7%	34.5%	27.9%	32.0%
	Sharpe Ratio	-0.08	0.48	0.58	0.19	-0.09	0.18	0.15	-0.32	0.19	0.03	-0.09	0.02	-0.37	0.08	0.31	0.32	0.36	0.14	0.47	0.23	0.20	0.32	-0.07	0.02	0.11
	3m RMSE From Target	5.4%	4.4%	4.5%	3.5%	4.9%	5.9%	6.9%	10.5%	7.4%	9.4%	2.7%	2.7%	3.9%	3.4%	4.6%	4.6%	15.3%	4.2%	3.6%	4.4%	6.5%	4.1%	5.2%	4.3%	4.3%
	Skewness	0.1	0.1	0.3	0.2	0.3	-0.3	-0.3	0.3	-0.5	-0.7	-0.1	-0.2	0.0	0.0	-0.2	0.0	4.3	0.0	-0.3	-0.3	-0.5	-0.2	0.3	0.0	0.0
	Excess Kurtosis	2.4	1.8	1.9	1.1	1.1	2.5	4.0	2.5	3.4	7.1	1.2	1.1	0.4	1.6	2.3	1.7	125.4	1.4	6.9	2.1	4.5	2.3	2.0	1.0	1.2
Trend	Return	2.4%	4.6%	1.9%	5.0%	-8.9%	12.9%	11.7%	10.7%	6.7%	8.3%	-1.0%	1.8%	-6.8%	0.8%	11.8%	1.0%	11.4%	3.7%	0.8%	-0.5%	-4.0%	-11.1%	-1.4%	2.2%	-0.3%
	Standard Deviation	19.5%	16.6%	18.5%	18.5%	22.6%	24.7%	26.3%	36.7%	28.3%	30.7%	12.1%	12.4%	20.1%	16.4%	19.5%	23.6%	31.3%	21.9%	12.7%	18.3%	23.3%	20.7%	24.3%	19.7%	22.3%
	Sharpe Ratio	0.13	0.28	0.10	0.27	-0.39	0.52	0.45	0.29	0.24	0.27	-0.09	0.14	-0.34	0.05	0.60	0.04	0.36	0.17	0.06	-0.03	-0.17	-0.54	-0.06	0.11	-0.01
	3m RMSE From Target	11.5%	9.9%	10.3%	10.2%	12.9%	15.4%	16.1%	21.4%	18.6%	21.1%	6.7%	7.0%	11.9%	8.8%	12.5%	14.0%	21.9%	12.9%	7.5%	11.2%	14.0%	11.1%	12.4%	11.4%	12.7%
	Skewness	-0.3	-0.1	-0.1	-0.1	-0.1	0.1	0.1	-0.1	0.5	0.3	0.1	0.0	-0.1	-0.4	0.2	-0.4	5.8	-0.2	-0.2	0.0	-0.7	-0.7	0.0	-0.1	-0.5
	Excess Kurtosis	6.1	5.1	4.5	4.5	5.8	9.2	11.6	6.2	21.8	31.9	5.7	6.0	5.5	6.6	11.7	9.6	210.2	6.3	10.5	10.2	11.0	6.1	5.4	4.1	6.3

		ASX 200	DAX	TSX 60	IBEX 35	CAC 40	Hang Seng	FTSE MIB	Nikkei 225	S&P MidCap	NASDAQ 100	Eurostoxx	FTSE 100	S&P 500	Australian 10y	German 10y	Canadian 10y	Japan 10y	UK 10y	US 10y	AUD	EUR	CAD	JPY	NZD	GBP
Buy-And-Hold	Return	5.5%	4.4%	5.6%	4.4%	4.9%	4.5%	6.2%	9.5%	10.1%	8.0%	4.0%	3.5%	6.4%	1.8%	3.1%	2.7%	1.8%	2.2%	3.0%	2.9%	0.3%	0.8%	-2.9%	3.9%	-0.2%
	Standard Deviation	16.3%	22.6%	18.5%	22.9%	22.2%	24.4%	23.1%	24.1%	22.3%	26.9%	23.2%	18.3%	19.5%	7.4%	5.8%	6.0%	3.0%	6.8%	6.0%	12.6%	9.4%	8.6%	9.8%	12.7%	9.4%
	Sharpe Ratio	0.34	0.19	0.30	0.19	0.22	0.19	0.27	0.39	0.46	0.30	0.17	0.19	0.33	0.24	0.53	0.45	0.61	0.33	0.50	0.23	0.04	0.10	-0.30	0.31	-0.02
	3m RMSE From Target	7.3%	10.1%	10.1%	9.6%	9.8%	10.0%	10.2%	10.9%	12.1%	14.3%	10.5%	8.6%	10.2%	2.2%	2.1%	1.7%	1.4%	2.8%	2.1%	5.1%	2.9%	3.1%	3.0%	4.2%	3.1%
	Skewness	-0.5	0.0	-0.4	-0.1	0.0	0.2	-0.4	0.0	-0.5	0.2	0.0	-0.1	0.0	-0.1	0.1	0.0	-0.3	0.3	0.1	-0.5	0.0	0.0	0.3	-0.3	-0.6
	Excess Kurtosis	8.0	6.3	15.6	7.3	6.3	5.5	8.2	14.4	13.8	8.0	6.3	7.6	12.0	2.0	2.8	1.7	6.4	7.3	3.4	8.3	1.7	4.3	4.5	2.8	8.0
Volatility Targeting	Return	6.2%	4.6%	7.6%	5.3%	5.5%	6.7%	7.6%	10.1%	9.4%	15.8%	4.2%	3.1%	8.5%	1.9%	3.9%	2.7%	2.0%	2.5%	2.7%	1.6%	-0.3%	0.2%	-4.2%	3.3%	-0.2%
	Standard Deviation	17.2%	23.9%	19.8%	24.4%	23.5%	25.4%	24.6%	26.1%	24.1%	28.6%	24.6%	19.4%	21.2%	7.7%	6.0%	6.2%	3.3%	7.1%	6.3%	13.1%	9.7%	9.0%	10.4%	13.2%	9.9%
	Sharpe Ratio	0.36	0.19	0.39	0.22	0.23	0.26	0.31	0.39	0.39	0.55	0.17	0.16	0.40	0.24	0.65	0.44	0.61	0.35	0.44	0.12	-0.03	0.02	-0.41	0.25	-0.02
	3m RMSE From Target	3.6%	5.1%	5.1%	5.4%	5.2%	4.6%	5.3%	6.5%	6.0%	6.6%	5.4%	4.2%	5.7%	1.1%	0.8%	0.8%	0.9%	1.0%	1.0%	2.4%	1.4%	1.5%	2.0%	2.1%	1.7%
	Skewness	-0.5	-0.4	-0.8	-0.4	-0.3	-0.1	-0.5	-0.5	-0.6	-0.4	-0.3	-0.4	-0.8	-0.1	-0.1	-0.1	-0.9	0.0	-0.1	-0.4	-0.1	-0.2	0.2	-0.3	-0.4
	Excess Kurtosis	2.9	2.5	7.0	4.5	3.4	1.8	3.6	4.7	3.0	2.9	3.3	2.5	6.6	1.7	1.2	0.9	12.5	1.6	1.7	2.1	1.2	1.4	4.2	1.4	4.3
Trend	Return	-1.1%	2.4%	2.7%	-1.6%	-1.0%	3.4%	0.7%	3.0%	0.3%	4.9%	-0.1%	-2.8%	2.1%	0.2%	2.1%	0.8%	-0.1%	1.1%	2.1%	1.1%	-0.1%	-1.2%	1.4%	1.0%	0.3%
	Standard Deviation	12.6%	17.8%	14.4%	17.5%	17.0%	18.3%	17.7%	19.3%	17.9%	20.0%	18.2%	14.3%	15.4%	5.6%	4.5%	4.6%	2.3%	5.3%	4.6%	9.5%	6.7%	6.5%	7.5%	9.5%	7.1%
	Sharpe Ratio	-0.08	0.14	0.18	-0.09	-0.06	0.18	0.04	0.16	0.02	0.24	0.00	-0.19	0.14	0.04	0.48	0.17	-0.06	0.21	0.47	0.11	-0.02	-0.19	0.19	0.11	0.04
	3m RMSE From Target	8.6%	12.1%	11.3%	11.7%	12.0%	12.7%	12.0%	13.3%	13.8%	15.7%	12.8%	10.2%	11.6%	3.2%	2.5%	2.5%	1.5%	3.5%	2.7%	6.1%	3.9%	4.1%	4.0%	5.6%	4.2%
	Skewness	0.3	-0.5	0.1	0.0	-0.2	-0.4	0.3	-0.4	0.5	-1.0	-0.4	-0.1	-0.8	-0.3	-0.4	-0.2	-0.5	-0.4	-0.1	0.6	-0.4	-0.5	-0.3	0.1	0.3
	Excess Kurtosis	19.4	16.1	38.2	14.2	16.5	15.1	20.5	36.2	33.4	21.8	15.7	20.3	28.8	5.3	5.6	5.2	13.1	18.7	6.1	22.4	5.1	13.2	8.6	7.8	10.8

Notes: This table reports basic performance statistics for the three main strategies we discuss in this paper – Buy-and-Hold, Volatility Targeting, and Simple Trend – applied to the 50 rolled futures contract return series. Statistics are calculated using daily returns. 3m RMSE From Target is defined as the square root of the mean square error (RMSE) of the strategy's rolling 3-month annualized standard deviation from the target standard deviation; the target is set equal to the full-period standard deviation of the Buy-and-Hold strategy. Excess Kurtosis is defined as full-sample kurtosis minus 3, the kurtosis of a normal random variable. All transaction costs are assumed to be 2bps per unit of notional traded. Date range is 2000-2023.