拓扑优化入门——介绍和网络资源

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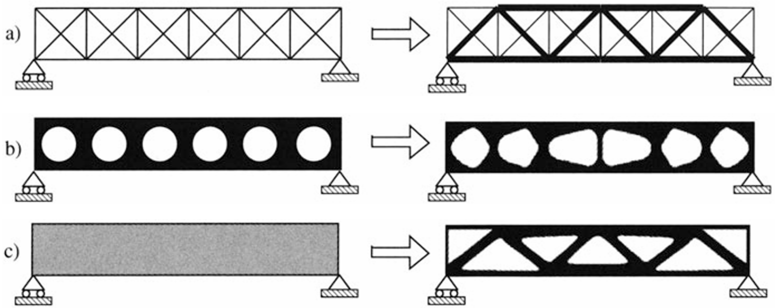
拓扑优化是结构优化方法的一种, 和它并列的有尺寸优化和形状优化.



拓扑优化中的拓扑的意思是在优化过程中允许结构的拓扑发生改变, 从而拓扑优化是自由度最高、设计空间最广的结构优化方法.

结构设计问题一般都可以概括为寻找一个结构在满足某些约束条件（如体积、应力等）的情况下，达到最优的性能指标（如高刚度、高散热效率等）.

传统的设计方法需要设计师根据经验不断试错来调整结构，而拓扑优化等优化方法将结构设计问题建模成优化问题，利用优化算法来寻找局部最优甚至全局最优.

而不管是传统的设计方法还是优化方法，都离不开一个映射，我称之为正向计算

$$\text{结构} \longrightarrow \text{性能}$$

所以要入门一个小方向，首先需要知道关心的性能指标是什么，以及该如何计算.

举一个例子，假设我们想设计一个桥梁使得它在具有较高刚度的同时用料较少.

你需要回答的第一个问题是，数学上该如何去表达桥梁的结构.

可以有体素表达、参数化表达、隐式表达……一般从体素表达入门.

体素表达就是说，将设计区域划分成网格，每个网格值为 0 或 1，分别代表无材料和有材料. 假设一共有 N 个网格，理论上所有可能的结构为 $\{0,1\}^N$.

第二个问题是你关心的性能比如刚度该如何定量刻画. 一般从论文中去学习, 如果没有相关论文, 然后你提出了一个比较合理的刻画方式, 那这是非常有原创性的工作.

我们使用平衡状态时的弹性势能 $F^T U/2$ 去刻画刚度, 认为弹性势能越低刚度越高. 可以类比弹簧去理解, 在相同的外力 F 下, 变化不同的胡可系数 k, 它在稳态下的弹性势能是 $F^2/2k$, 所以用弹性势能去刻画刚度是符合物理直觉的.

第三个问题是给定一个结构如何去计算关心的性能. 这需要去看关心的性能涉及到的物理规律是什么, 或者说关心的量该用什么微分方程去解. 该例子中位移场 U 需要使用线弹性静力学的方程组去求解, 求解的数值方法是有限元.

所以现在可以建模成如下优化问题

$$\min_{x\in\{0,1\}^N} c(x)$$

其中 x 代表体素表达的结构，一共划分了 N 个单元所以一共 N 个设计变量.

除此之外还有约束，其实每个约束也都是一个性能，把哪个放在目标里哪个放在约束里也需要智慧与经验.

该优化问题本身是一个离散优化问题，所以可以用离散优化的算法来做，比如遗传算法. 缺点是计算量大. 也可以松弛成连续变量，利用梯度信息来做，比如 SIMP 方法.

$$
\begin{tikzcd}
 & \text{边界条件} \arrow[dr] & & & \text{约束条件} \arrow[dr] & \\
 & \text{微分方程} \arrow[r] & \text{有限元方程} \arrow[r] & \text{物理场} \arrow[r] & \text{目标函数} \arrow[r] & \text{更新设计变量} \\
\text{设计变量} \arrow[r] & \text{材料属性} \arrow[u] & & & &
\end{tikzcd}
$$

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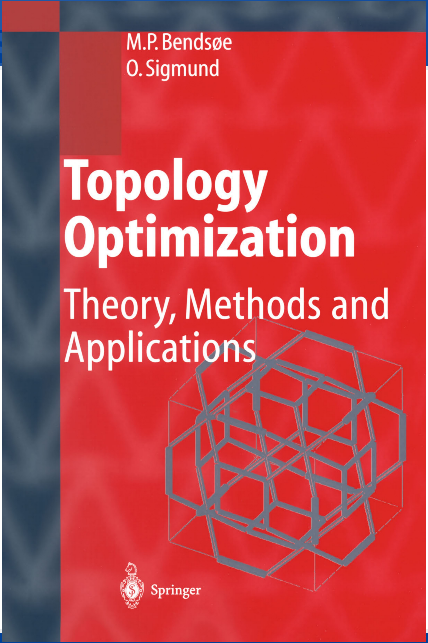
M.P. Bendsøe
O. Sigmund
Topology Optimization
Theory, Methods and Applications
Springer

Educational article

Struct Multidisc Optim 21, 120–127

A 99 line topology optimization code written in Matlab

O. Sigmund

Abstract The paper presents a compact Matlab implementation of a topology optimization code for compliance minimization of statically loaded structures. The total number of Matlab input lines is 99 including optimizer and Finite Element subroutine. The 99 lines are divided into 36 lines for the main program, 12 lines for the Optimality Criteria based optimizer, 16 lines for a mesh-independency filter and 35 lines for the finite element code. In fact, excluding comment lines and lines associated with output and finite element analysis, it is shown that only 49 Matlab input lines are required for solving

fraction and then the program shows the correct optimal topology for comparison.

In the literature, one can find a multitude of approaches for the solving of topology optimization problems. In the original paper Bendsøe and Kikuchi (1988) a so-called microstructure or homogenization based approach was used, based on studies of existence of solutions.

The homogenization based approach has been adopted in many papers but has the disadvantage that the determination and evaluation of optimal microstructures and their orientations is cumbersome if not unresolved (for

Efficient topology optimization in MATLAB using 88 lines of code

Erik Andreassen · Anders Clausen · Mattias Schevenels · Boyan S. Lazarov · Ole Sigmund

Received: date / Accepted: date

Abstract This paper presents an efficient 88 line MATLAB code for topology optimization. It has been developed using the 99 line code presented by Sigmund (2001) as a starting point. The original code has been extended by a density filter, and a considerable improvement in efficiency has been achieved, mainly by preallocating arrays and vectorizing loops. A speed improvement with a factor of 100 is achieved for a benchmark example with 7500 elements. Moreover, the length of the code has been reduced to a mere 88 lines. These improvements have been accomplished without

1 Introduction

MATLAB is a high-level programming language that allows for the solution of numerous scientific problems with a minimum of coding effort. An example is Sigmund's 99 line topology optimization code (Sigmund, 2001). The 99 line code is intended for educational purposes and serves as an introductory example to topology optimization for students and newcomers to the field. The use of MATLAB, with its accessible syntax, excellent debugging tools, and

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https://doi.org/10.1007/s00158-021-03004-z

EDUCATIONAL PAPER

Complementary lecture notes for teaching the 99/88-line topology optimization codes

Ming Zhou[1] · Ole Sigmund[2]

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Abstract
Sigmund's 2001 educational paper with a self-contained 99-line MATLAB code had far-reaching impact to teaching and research of topology optimization. This brief note aims to close the gaps on self-contained content desirable for classroom teaching. The goal is to add clarity to the theoretical foundation, and to enable students' learning of the complete iterative optimization solution with minimum additional effort.

Keywords Topology optimization · Optimality criteria · Convex approximation · Educational paper · Topology optimization code

Structural and Multidisciplinary Optimization (2020) 62:2211–2228
https://doi.org/10.1007/s00158-020-02629-w

EDUCATIONAL PAPER

Check for updates

A new generation 99 line Matlab code for compliance topology optimization and its extension to 3D

Federico Ferrari[1] · Ole Sigmund[1]

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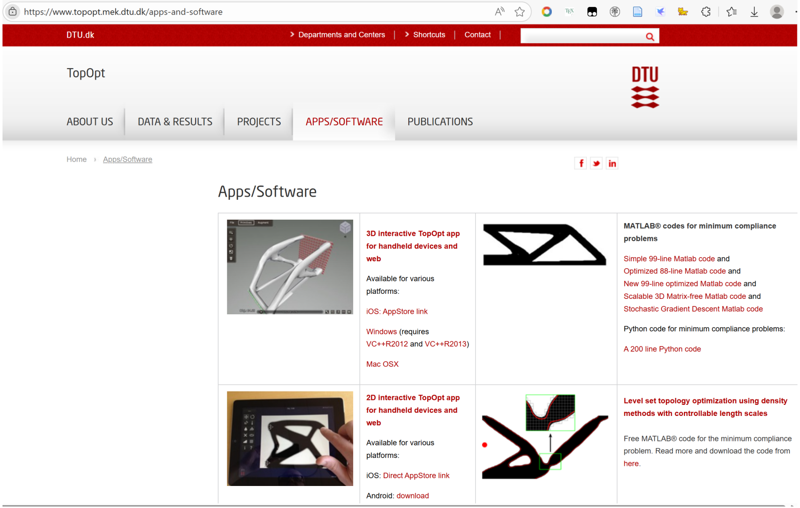
Abstract
Compact and efficient Matlab implementations of compliance topology optimization (TO) for 2D and 3D continua are given, consisting of 99 and 125 lines respectively. On discretizations ranging from $3 \cdot 10^4$ to $4.8 \cdot 10^5$ elements, the 2D version, named `top99neo`, shows speedups from 2.55 to 5.5 times compared to the well-known `top88` code of Andreassen et al. (Struct Multidiscip Optim 43(1):1–16, 2011). The 3D version, named `top3D125`, is the most compact and efficient Matlab implementation for 3D TO to date, showing a speedup of 1.9 times compared to the code of Amir et al. (Struct Multidiscip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