LattSAC: a software for the acoustic modelling of lattice sound absorbers

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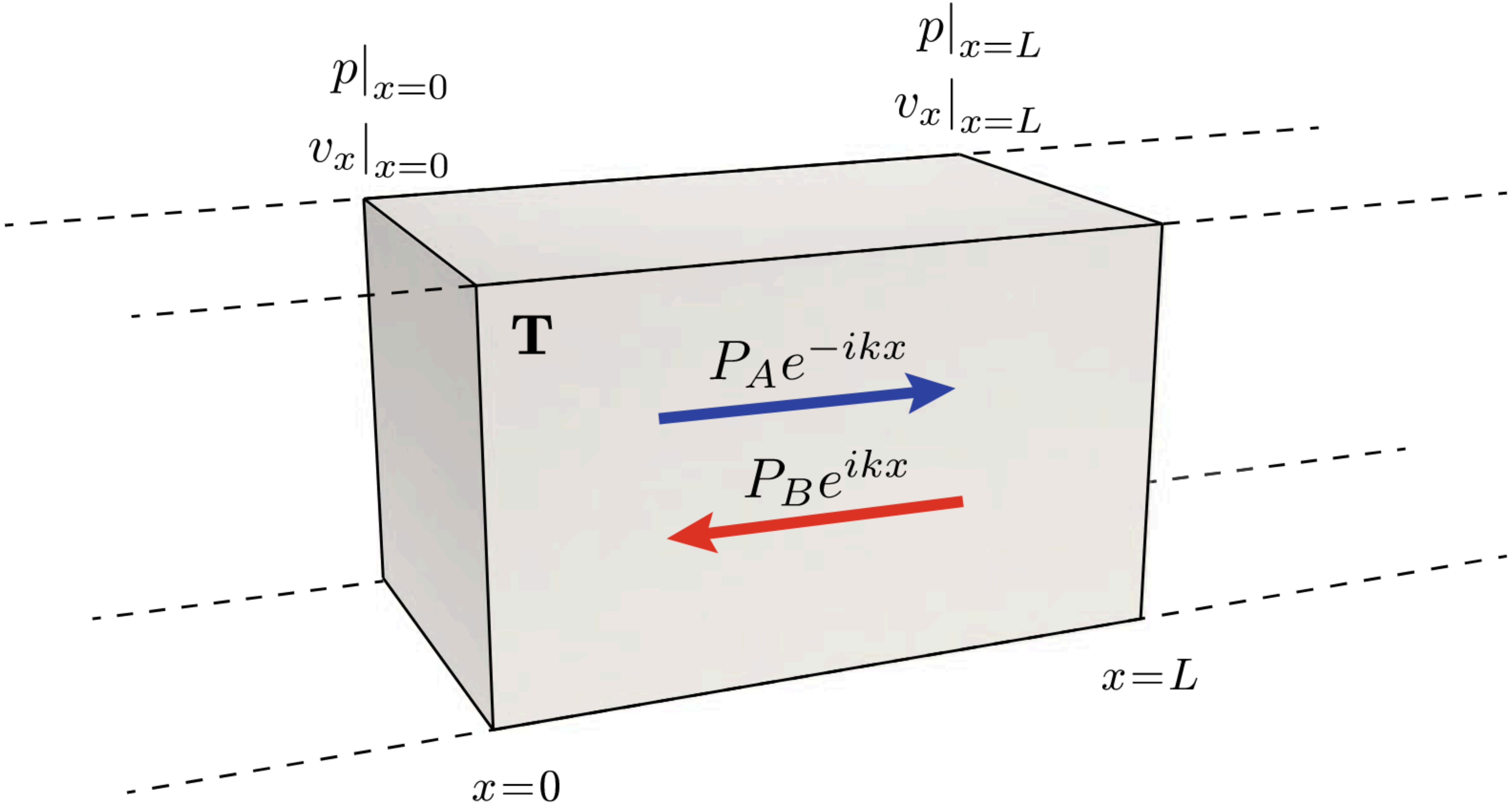
discrete layers of acoustic material[12,27]. The general expression of the TMM for n heterogeneous layers in series is as follows:

$$\begin{aligned}\begin{Bmatrix} p \\ v_y \end{Bmatrix}_{x=0} &= \left[\boldsymbol{T}_{layer\,1}\right]\left[\boldsymbol{T}_{layer\,2}\right]\cdots\left[\boldsymbol{T}_{layer\,n}\right]\begin{Bmatrix} p \\ v_y \end{Bmatrix}_{x=L}, \\ &= \left[\boldsymbol{T}_{total}\right]\begin{Bmatrix} p \\ v_y \end{Bmatrix}_{x=t} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}\begin{Bmatrix} p \\ v_y \end{Bmatrix}_{x=L}\end{aligned} \tag{V}$$

Deriving the basic relations between the acoustic magnitudes evaluated at the boundaries of a layer of homogeneous acoustic material



Assuming that only longitudinal plane waves propagate in the layer and a temporal harmonic dependence of the type eiωt , the total field inside the material is written as the superposition of two waves propagating in opposite directions as

$$p(x) = P_A \mathrm{e}^{-\mathrm{i}kx} + P_B \mathrm{e}^{\mathrm{i}kx}$$

$$v_x(x) = \frac{P_A}{Z}\mathrm{e}^{-\mathrm{i}kx} - \frac{P_B}{Z}\mathrm{e}^{\mathrm{i}kx}$$

where Z = ρc is the characteristic acoustic impedance, k = ω/c is the wavenumber at the angular frequency ω = 2π f , with ρ the density and c the sound speed of the material, and the amplitudes of the two waves are given by PA and PB.

To evaluate these amplitudes we define the pressure and velocity at both sides of the slab. First, at $x = 0$ we obtain

$$p(x)\Big|_{x=0} = P_A + P_B, \tag{4.3}$$

$$Z v_x(x)\Big|_{x=0} = P_A - P_B, \tag{4.4}$$

while at $x = L$ we get

$$p(x)\Big|_{x=L} = (P_A + P_B)\cos(kL) - i\,(P_A - P_B)\sin(kL), \tag{4.5}$$

$$v_x(x)\Big|_{x=L} = \frac{P_A - P_B}{Z}\cos(kL) - i\,\frac{P_A - P_B}{Z}\sin(kL). \tag{4.6}$$

Then, we can relate the acoustic magnitudes at both boundaries by combining (4.3)–(4.4) with (4.5)–(4.6) via

$$p(x)\big|_{x=L} = \cos(kL)\,p(x)\big|_{x=0} - i\,Z\sin(kL)\,v_x(x)\big|_{x=0}, \tag{4.7}$$

$$v_x(x)\big|_{x=L} = \cos(kL)\,v_x(x)\big|_{x=0} - i\,\frac{1}{Z}\sin(kL)\,p(x)\big|_{x=0}. \tag{4.8}$$

Equations (4.7)–(4.8) can be expressed in a matrix form as

$$\begin{bmatrix} p \\ v_x \end{bmatrix}_{x=L} = \begin{bmatrix} \cos(kL) & -i\,Z\sin(kL) \\ \cos(kL) & -i\,\frac{1}{Z}\sin(kL) \end{bmatrix} \begin{bmatrix} p \\ v_x \end{bmatrix}_{x=0}. \tag{4.9}$$

After inversion, we retrieve the basic transfer matrix formulation of a layer of acoustic material, given by

$$\begin{bmatrix} p \\ v_x \end{bmatrix}_{x=0} = \begin{bmatrix} \cos(kL) & iZ\sin(kL) \\ i\frac{1}{Z}\sin(kL) & \cos(kL) \end{bmatrix} \begin{bmatrix} p \\ v_x \end{bmatrix}_{x=L} . \tag{4.10}$$

In this way, the acoustic magnitudes at both sides of the 1D fluid layer are related by a 2×2 matrix which only depends on the impedance and wavenumber in the material. It is interesting to note that additional elements can be introduced into the system in a simple and modular way. This allows to model complex materials and structures using a simple theoretical framework, as we will see below.

Multi-layered micropore-cavity (MMC) model

View the lattices as multiple layers of micropores with air cavities in between.

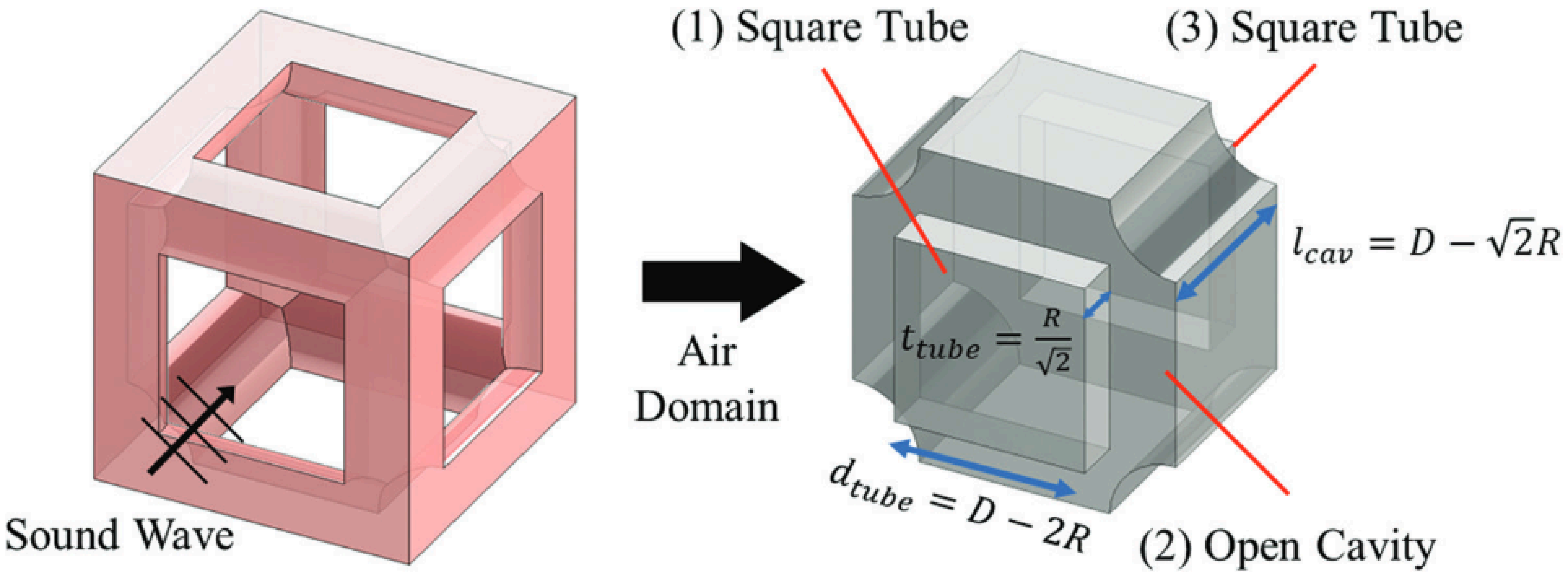


Figure 3. Discretization of the air domain of the SC-Truss unit cell for the acoustic modeling using the MMC model.

By assuming the closed tubes of square cross-section as a set of resonant materials, the transfer matrix is given by[25]

$$\boldsymbol{T}_r = \begin{bmatrix} 1 & Z_r \\ 0 & 1 \end{bmatrix}, \tag{VI}$$

Where Z_r is the characteristic impedance of the closed tube. Based on the works by Maa, Morse and Ingard[21,22] tube. Based on the works by Maa, Morse and Ingard[21,22] on microperforated panel absorbers, Z_r can be expressed as follows:

$$Z_r = \frac{32\eta t_{tube}}{\varepsilon d_{tube}^2}\left(\sqrt{1+\frac{k^2}{32}}+2\delta_1 R_s\right)+j\frac{\omega\rho_0 t_{tube}}{\varepsilon}\left(1+\frac{1}{\sqrt{9+\frac{k^2}{2}}}+\delta_2\frac{d_{tube}}{t_{tube}}\right), \tag{VII}$$

respectively. Thereafter, the transfer matrix of one layer of SC-Truss unit cells is given by,

$$\left[\boldsymbol{T}_{layer\,x}\right]=\boldsymbol{T}_r\boldsymbol{T}_c\boldsymbol{T}_r=\begin{bmatrix}1 & Z_r\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\cos\left(k_0 l_{cav}\right) & jZ_0\sin(k_0 l_{cav})\\ j\dfrac{1}{Z_0}\sin(k_0 l_{cav}) & \cos\left(k_0 l_{cav}\right)\end{bmatrix}\begin{bmatrix}1 & Z_r\\ 0 & 1\end{bmatrix}. \tag{IX}$$