Bottom to up

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Periodic boundary conditions

Bulk modulus

Bottom to up

- Generate microstructures that maximize bulk modulus under specified volume fractions as key microstructures.
- Obtain microstructures for any ρ through interpolation.
- Use these as basic units for macroscopic optimization.

Microstructure interpolation

- Constructing smooth deformations between two key shapes.
- Two differences from traditional shape metamorphosis:
  - The structures have quite similar features.
  - More focused on smooth variations of physical properties.

Hashin-Shtrikman bounds

$$K^{\mathrm{HS}}(\rho) = K_0 - \frac{1-\rho}{\frac{1}{K_0} - \frac{3\rho}{3K_0+4G_0}}, \tag{7}$$

$$G^{\mathrm{HS}}(\rho) = G_0 - \frac{1-\rho}{\frac{1}{G_0} - \frac{6\rho(K_0+2G_0)}{5G_0(3K_0+4G_0)}}. \tag{8}$$

Relationship between E_{ijkl} and ρ

- We have several key microstructures.
- Through interpolation, we have microstructures for each volume fraction, thus obtaining the corresponding E_{ijkl}.
- In practice, we directly fit a curve $E_{ijkl}(\rho)$ based on the data from key microstructures.

$$E^{\text{fit}}_{ijkl}(\rho) = E^{0}_{ijkl} - E^{0}_{ijkl}\frac{1-\rho}{1+a_{ijkl}\rho}.$$

Relationship between E_{ijkl} and ρ

$$E_{ijkl}^{\text{fit}}(\rho) = E_{ijkl}^{0} - E_{ijkl}^{0} \frac{1-\rho}{1+a_{ijkl}\rho}.$$

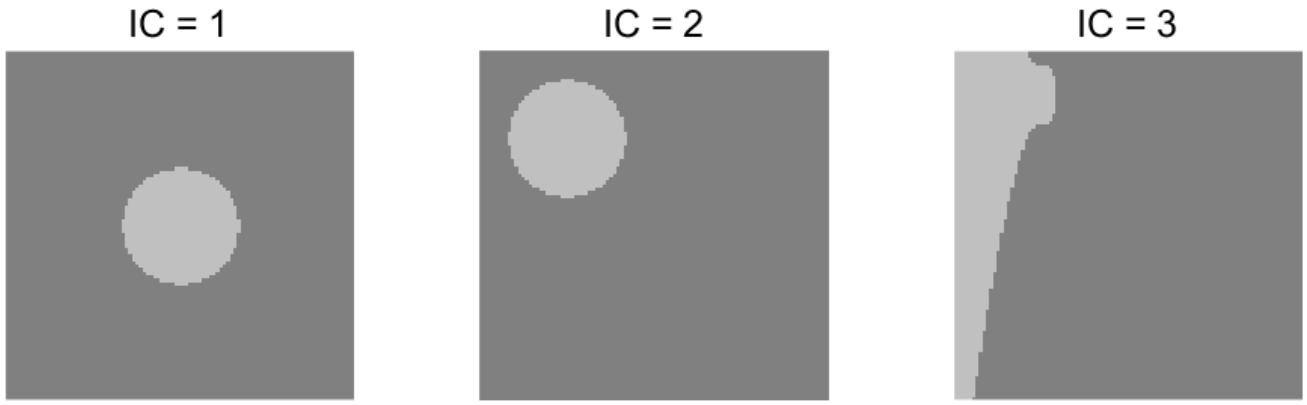
- The functional form is inspired by the H-S bounds.
- a_{ijkl} is obtained through the least squares method.
- This formula replaces SIMP in the workflow.
- Predicts data when ρ is less than the minimum value or greater than the maximum value.

Compatibility

Parameter: CellCount

- The number of key microstructures.
- volfrac = linspace(Vmin,Vmax,CellCount);
  - For example, vlinspace(0.2, 0.6, 3) yields [0.2, 0.4, 0.6].

Parameter: IC



Initialization of the i-th key microstructure

Homogenization of key microstructure

- function CH = homogenize(lx, ly, lambda, mu, phi, x)
- lx = Unit cell length in x-direction.
- ly = Unit cell length in y-direction.
- lambda = Lame's first parameter for materials. Two entries.
- mu = Lame's second parameter for materials. Two entries.
- phi = Angle between horizontal and vertical cell wall. Degrees
- x = Material indicator matrix.

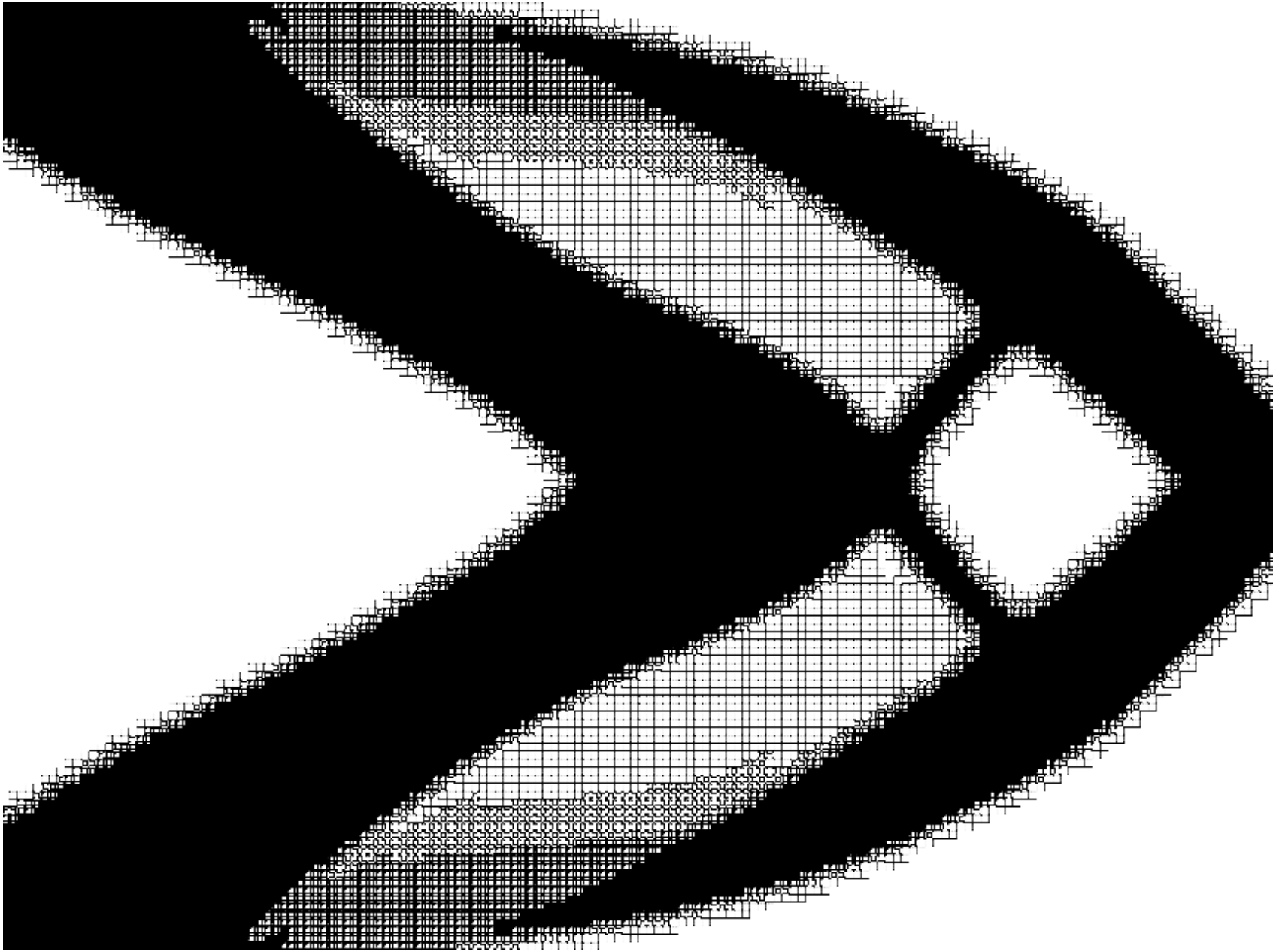
Homogenization of key microstructure

- [nely, nelx] = size(x);
- dx = lx/nelx; dy = ly/nely;
- nel = nelx*nely;

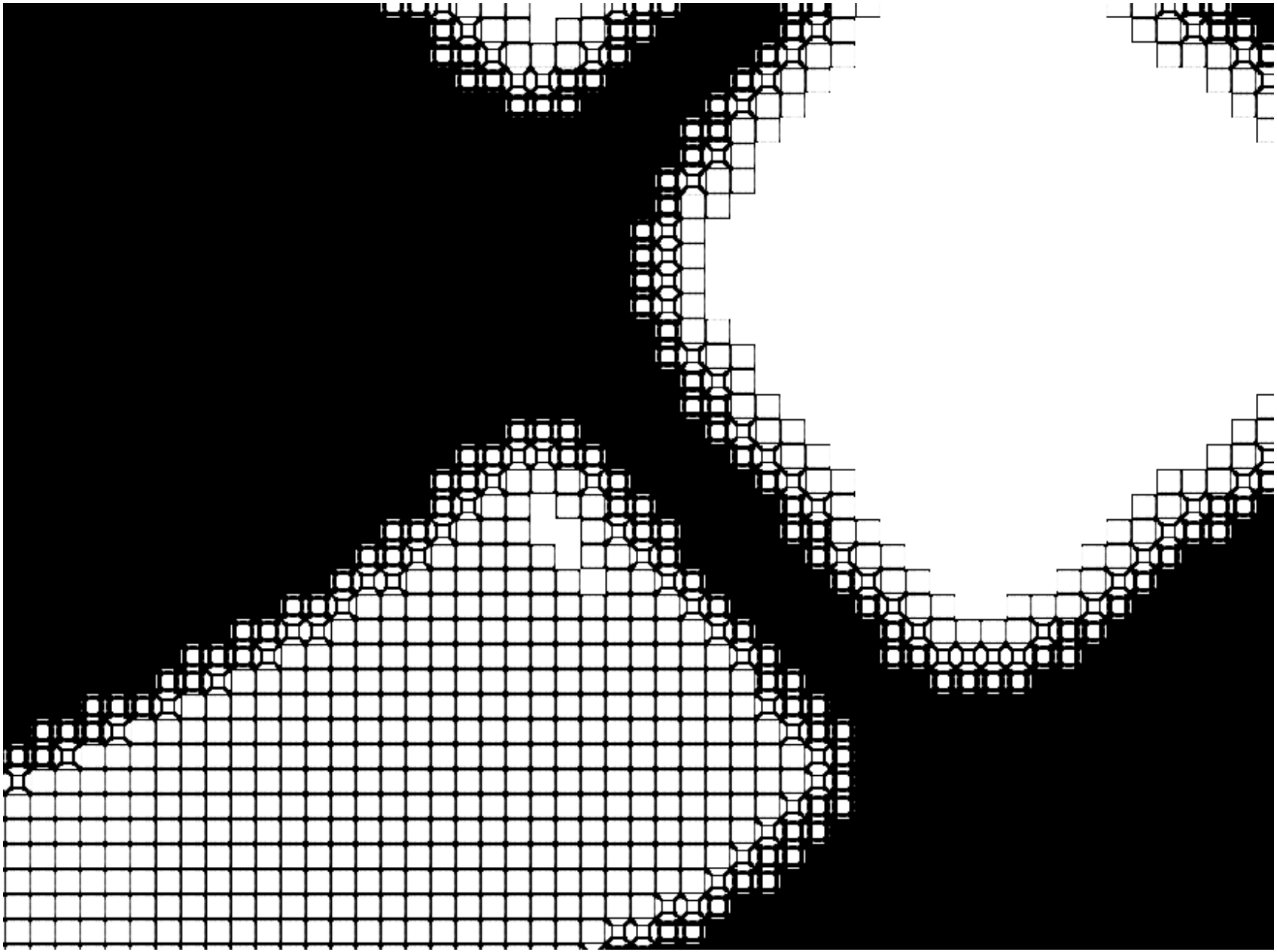
Combine Bottom to up with Compatibility

- Use compatibility method when generating key microstructure.

Results



Results



Results