Topology optimization design for total sound absorption in porous media

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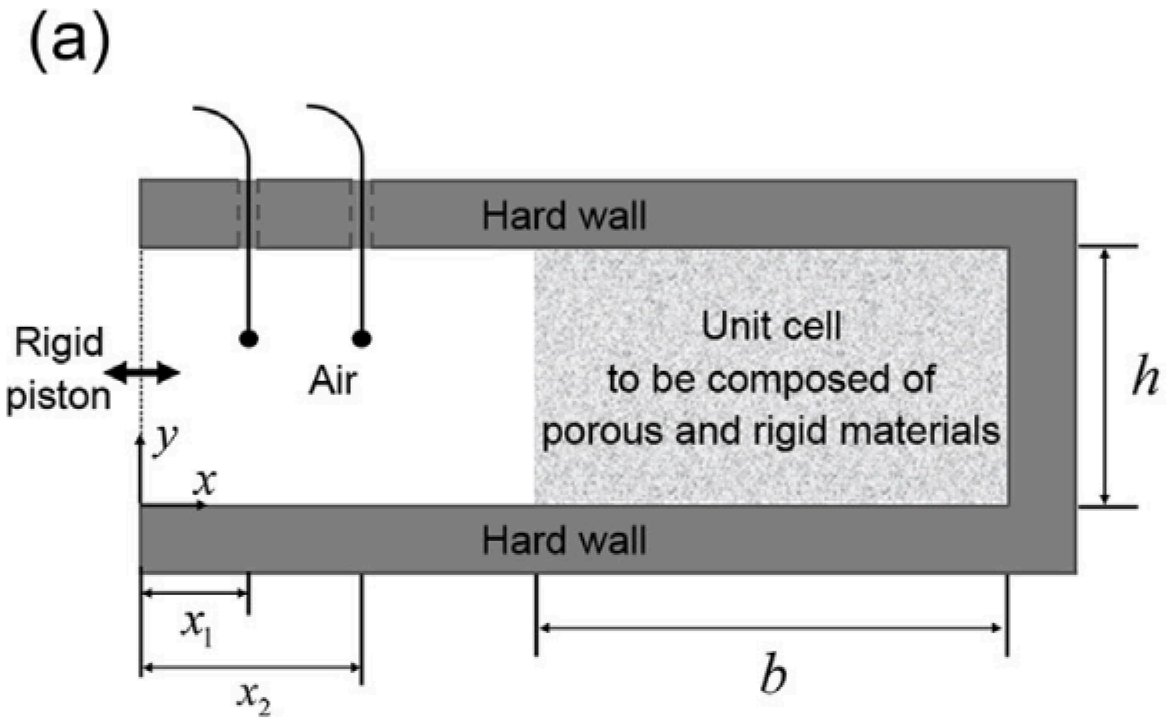
Computer Methods in Applied Mechanics and Engineering

Received 19 February 2019; accepted 30 October 2019

Macroscopic topology optimization, rather than multi-scale topology optimization.

Learn the numerical calculation methods of sound absorption coefficients.

The energy of sound is divided into three parts: reflected, transmitted, and dissipated.

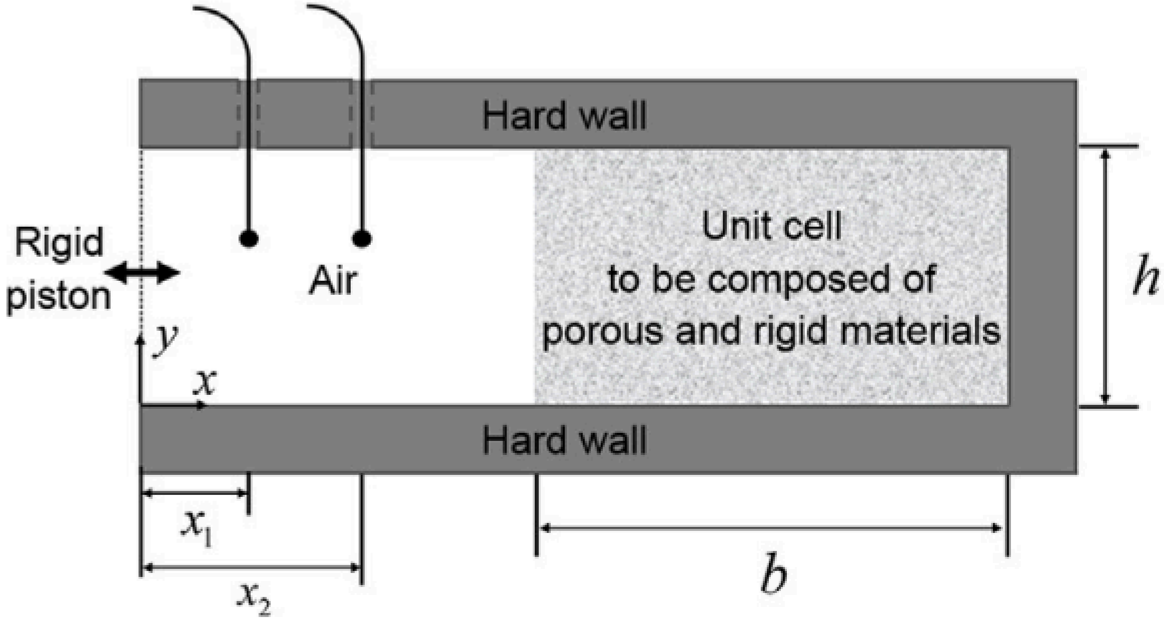


Enclosed by an acoustically "hard wall," with no transmitted portion.

$$P(x) = Ae^{-ikx} + Be^{ikx}, \quad R = \frac{B}{A}, \quad \alpha = 1 - |R|^2$$

$$R(\boldsymbol{\chi}; f) = \frac{-p_2 \cdot \exp(-jk_a x_1) + p_1 \cdot \exp(-jk_a x_2)}{p_2 \cdot \exp(jk_a x_1) - p_1 \cdot \exp(jk_a x_2)}.$$

(a)



To calculate the sound absorption coefficient, it is necessary to solve the equations to determine p1 and p2. The equation for sound propagation is

$$\frac{\partial^2 p}{\partial t^2} = c^2(x)\nabla^2 p.$$

The steady-state acoustic pressure p of an acoustic domain is governed by

$$\rho^{-1}\nabla^2 p + \omega^2\kappa^{-1} p = 0,$$

where the symbols ρ and κ denote the density and bulk modulus of the acoustic medium filled in the domain, respectively.

By applying the variational principle and the divergence theorem to Eq. (1), the weighted residual integral equation (i.e., the weak integral form) of the scalar Helmholtz equation can be written as [29]:

$$\int_{\Omega} \rho^{-1} (\nabla \tilde{p} \cdot \nabla p)\, d\Omega - \omega^2 \int_{\Omega} \kappa^{-1} \tilde{p} p \, d\Omega + \int_{\partial\Omega} j\omega \tilde{p} \, (\mathbf{v} \cdot \mathbf{n})\, d\Gamma = 0, \tag{2}$$

where the symbols $\tilde{p}$ and $\mathbf{v}$ denote the weighting acoustic pressure field and the particle velocity of the acoustic medium, respectively. In Eq. (2), the unit vector $\mathbf{n}$ is defined to be outward normal to the boundary $\partial\Omega$ of the acoustic domain Ω.

Substituting Eqs. (3a,b) into the element version of Eq. (2) yields the following matrix–vector form equation in the eth element Ω_e:

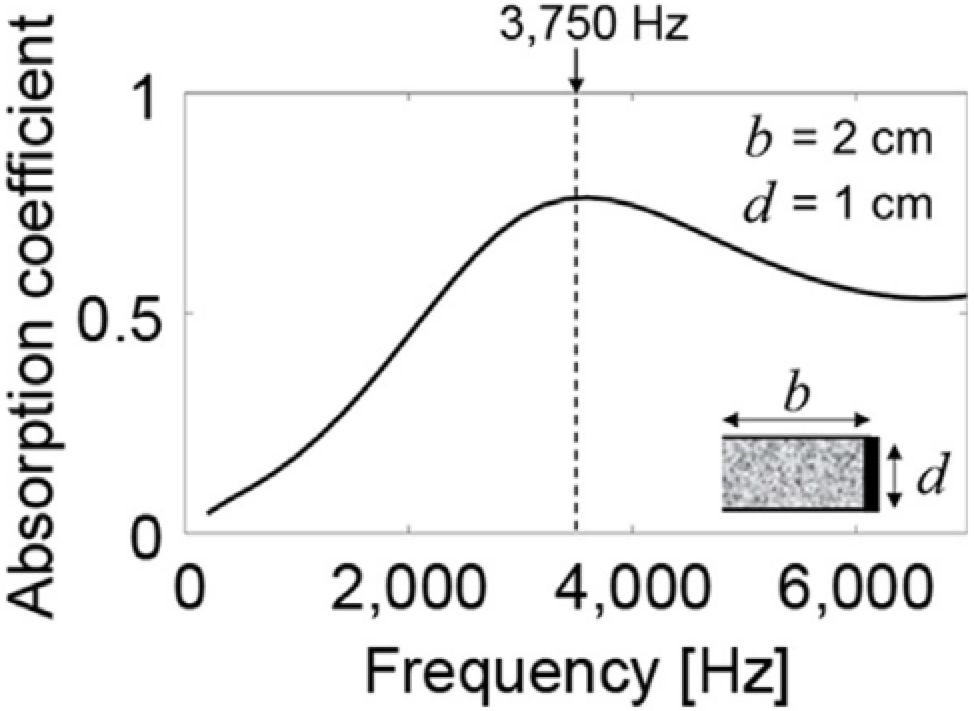
$$[\mathbf{K}_e - \omega^2 \mathbf{M}_e]\mathbf{P}_e = \mathbf{F}_e, \tag{4a}$$

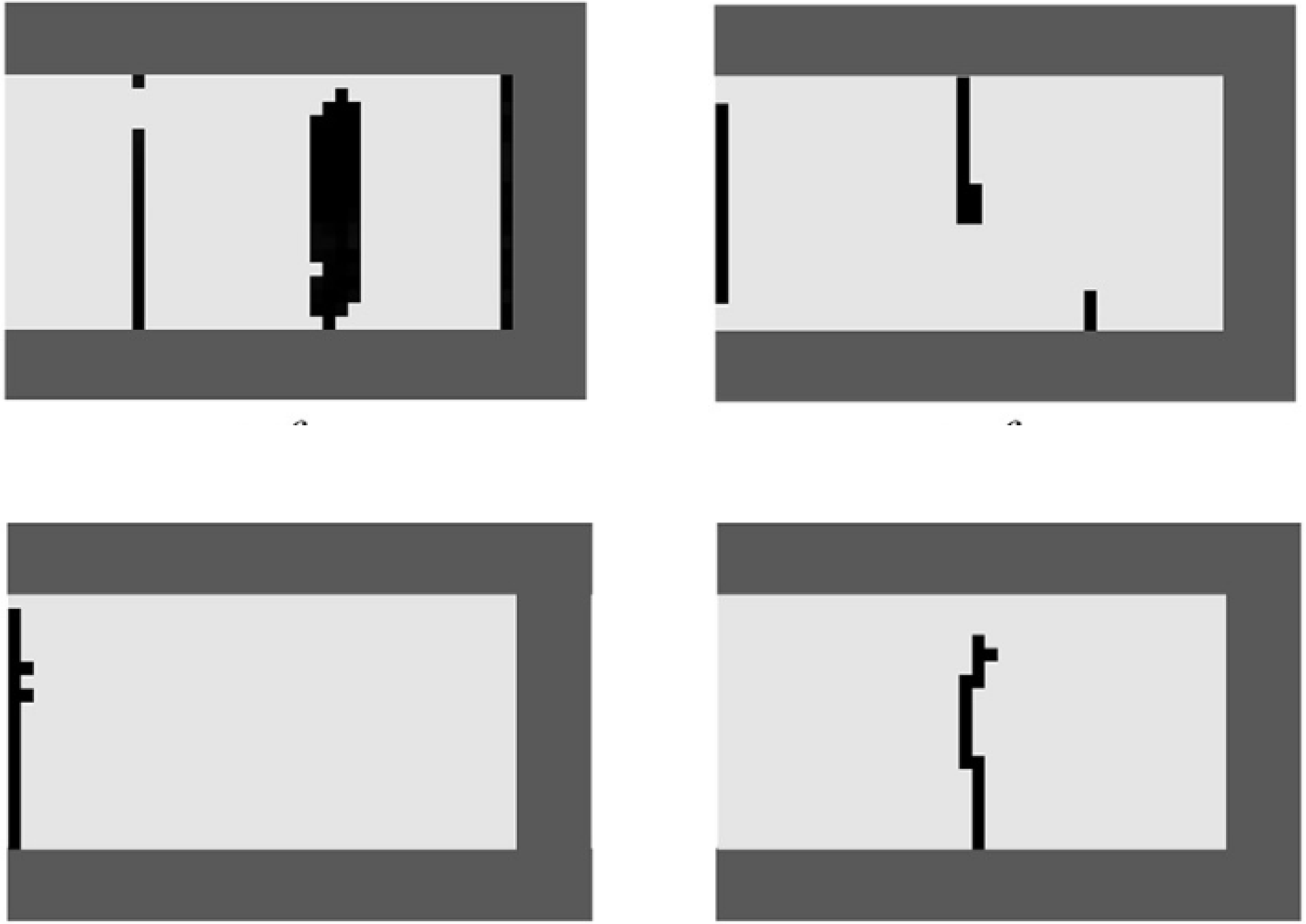
where the element stiffness matrix $\mathbf{K}_e$ and mass matrix $\mathbf{M}_e$ are defined as:

$$\mathbf{K}_e = \int_{\Omega_e} \rho_e^{-1} \mathbf{B}_e{}^T \mathbf{B}_e d\Omega, \;\; \mathbf{M}_e = \int_{\Omega_e} \kappa_e^{-1} \mathbf{N}_e{}^T \mathbf{N}_e d\Omega. \tag{4b,c}$$

In Eq. (4a), $\mathbf{F}_e$ is the nodal acoustic force vector in the element. The symbols ρ_e and κ_e in Eqs. (4b,c) are the density and bulk modulus of an acoustic medium filling the eth element.

Two target materials in unit cells: a porous material and a rigid material





$$\min_{\boldsymbol{\chi}} F(\boldsymbol{\chi}) = \min_{\boldsymbol{\chi}} \left[\frac{1}{N} \sum_{e=1}^{N} \chi_e (1 - \chi_e) \right],$$

$$\text{subject to } \; C(\boldsymbol{\chi}; f_i) = 1 - \alpha(\boldsymbol{\chi}; f_i) \leq \varepsilon \; \text{ with } \; i = 1, 2, \ldots, N_t$$

$$\frac{1}{\rho_e(\chi_e)} = (\chi_e)^p \left\{ \frac{1}{\rho_{porous}} - \frac{1}{\rho_{rigid}} \right\} + \frac{1}{\rho_{rigid}},$$

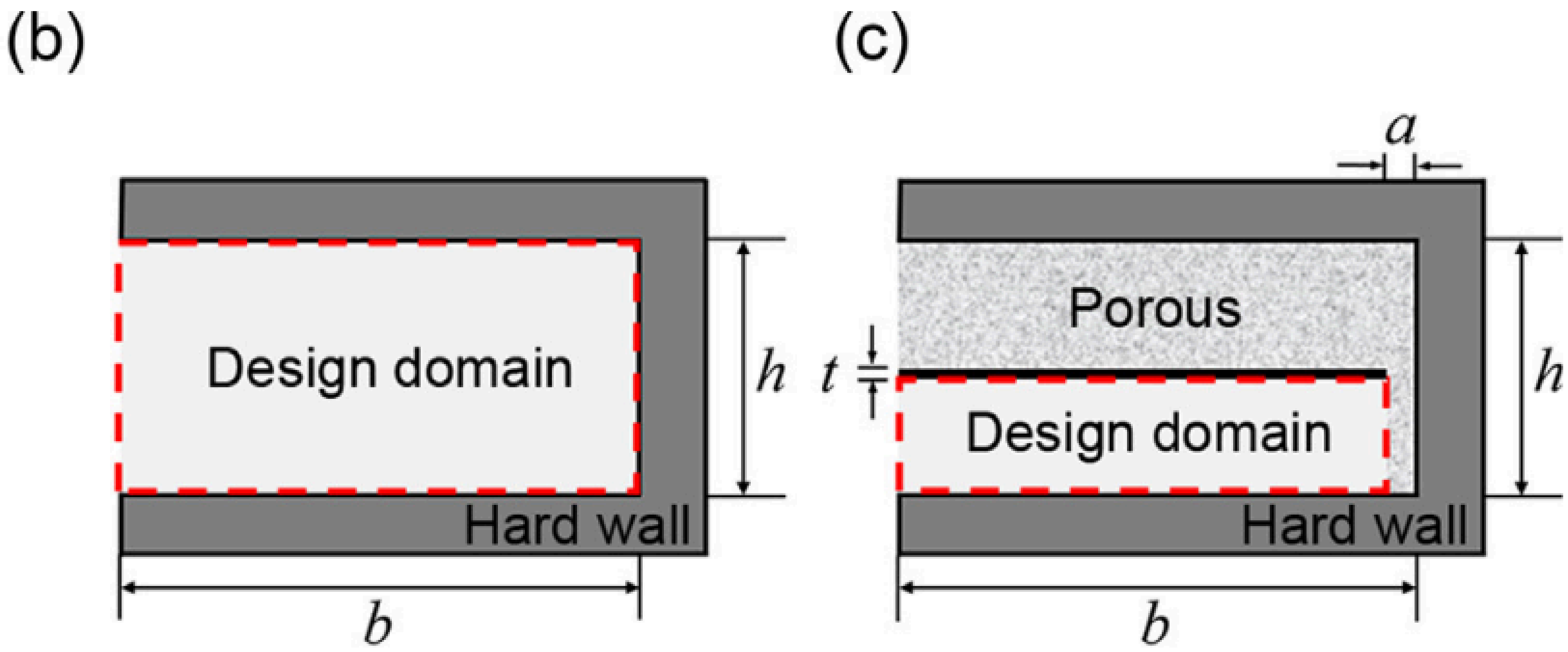
$$\frac{1}{\kappa_e(\chi_e)} = (\chi_e)^q \left\{ \frac{1}{\kappa_{porous}} - \frac{1}{\kappa_{rigid}} \right\} + \frac{1}{\kappa_{rigid}},$$

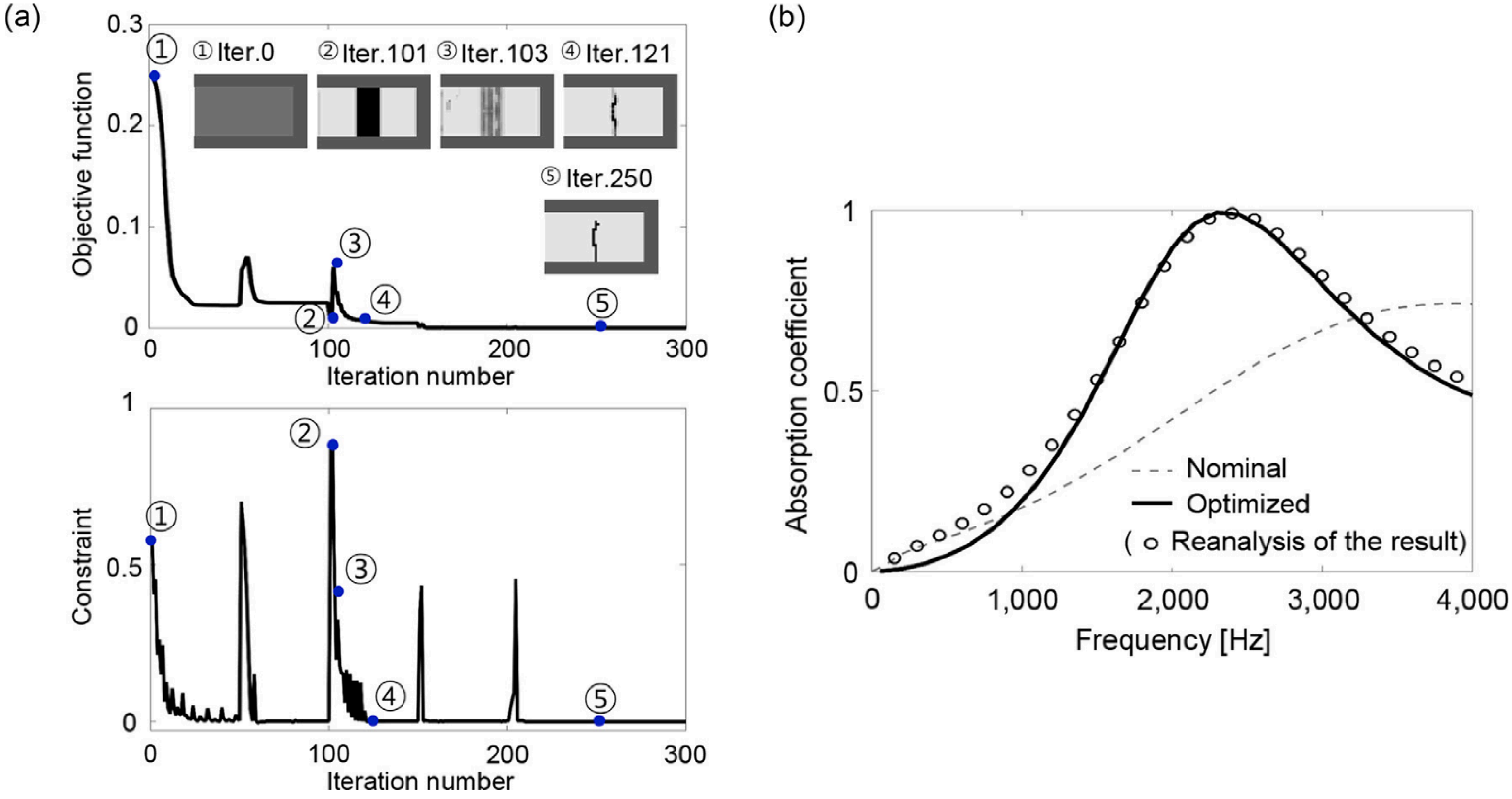
$$\frac{\partial \alpha(\boldsymbol{\chi};\, f_i)}{\partial \chi_e} = -2\,|R(\boldsymbol{\chi};\, f_i)| \cdot \frac{\partial\, |R(\boldsymbol{\chi};\, f_i)|}{\partial \chi_e}, \tag{13a}$$

$$\begin{aligned}\frac{\partial R(\boldsymbol{\chi};\, f_i)}{\partial \chi_e} &= \frac{\left\{-\frac{\partial p_2}{\partial \chi_e} \cdot \exp(-jk_a x_1) + \frac{\partial p_1}{\partial \chi_e} \cdot \exp(-jk_a x_2)\right\} \{p_2 \cdot \exp(jk_a x_1) - p_1 \cdot \exp(jk_a x_2)\}}{\{p_2 \cdot \exp(jk_a x_1) - p_1 \cdot \exp(jk_a x_2)\}^2} \\ &\quad - \frac{\{-p_2 \cdot \exp(-jk_a x_1) + p_1 \cdot \exp(-jk_a x_2)\} \left\{\frac{\partial p_2}{\partial \chi_e} \cdot \exp(jk_a x_1) - \frac{\partial p_1}{\partial \chi_e} \cdot \exp(jk_a x_2)\right\}}{\{p_2 \cdot \exp(jk_a x_1) - p_1 \cdot \exp(jk_a x_2)\}^2},\end{aligned} \tag{13b}$$

where the expression $\partial p / \partial \chi_e$ in Eq. (13b) can be obtained using the system equation in Eq. (5) as:

$$\frac{\partial p}{\partial \chi_e} = [\mathbf{K} - \omega^2 \mathbf{M}]^{-1} \cdot \frac{\partial [\mathbf{K} - \omega^2 \mathbf{M}]}{\partial \chi_e} \cdot \mathbf{P}. \tag{14}$$


(b)
Design domain
Hard wall
h
b
(c)
a
Porous
t
Design domain
Hard wall
h
b



(a)
① Iter.0
② Iter.101
③ Iter.103
④ Iter.121
⑤ Iter.250
Objective function
Iteration number
Constraint
Iteration number
(b)
Absorption coefficient
Frequency [Hz]
Nominal
Optimized
(○ Reanalysis of the result)

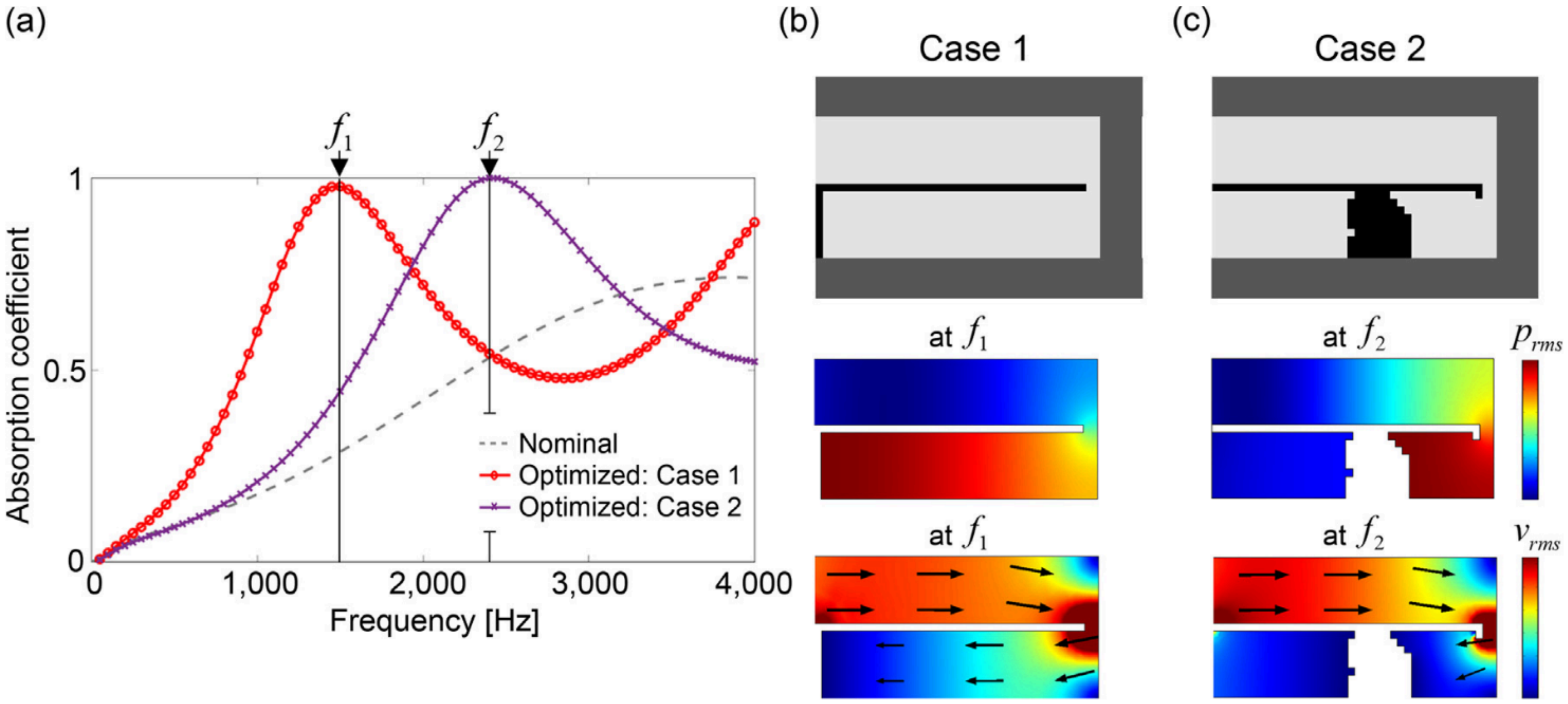
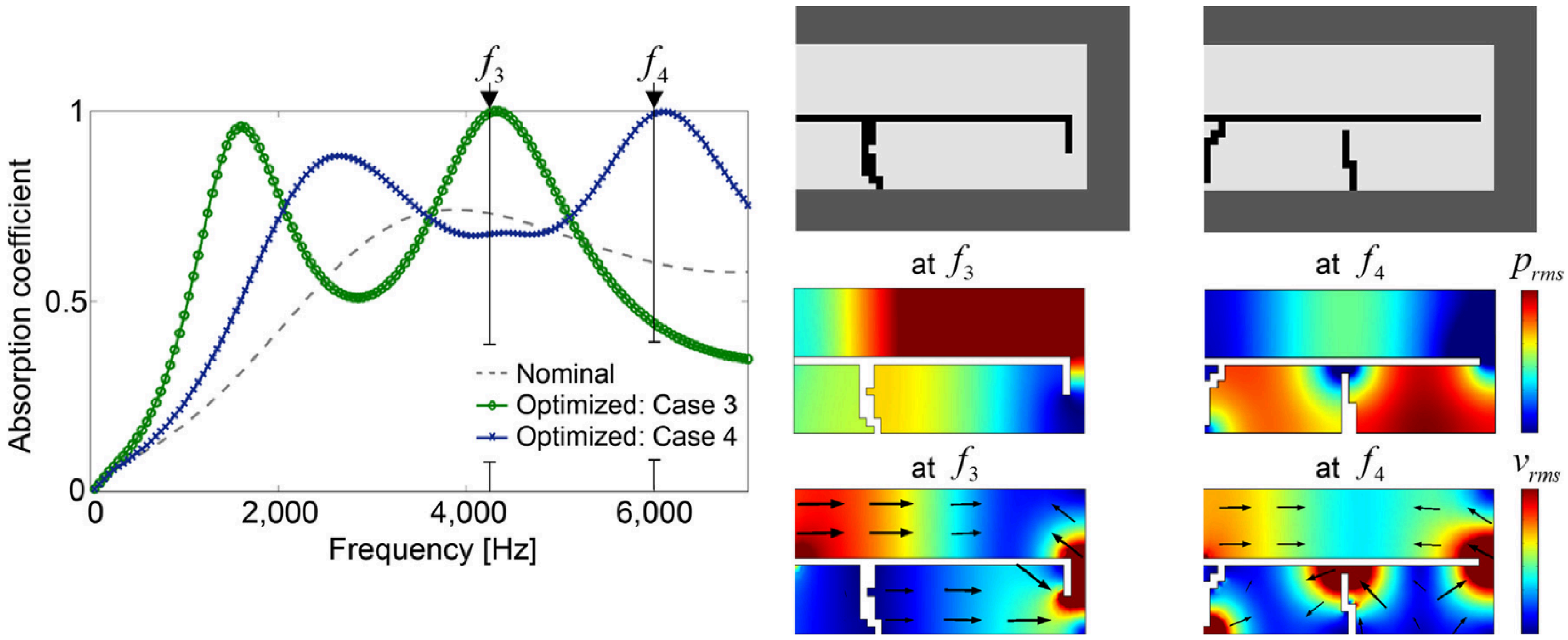


Fig. 4. Topology optimization results at single target frequencies $f_1 = 1500$ Hz (Case 1) and $f_2 = 2400$ Hz (Case2) under the restricted

quarter-wavelength resonance



. **5.** Topology optimization results at single target frequencies $f_3 = 4200$ Hz (Case 3) and $f_4 = 6000$ Hz (Case 4) under the restricted

a three-quarter-wavelength resonance forms to achieve the sound absorption coefficient nearly equal to 1 at f = f3, while the quarter-wavelength resonance makes the first absorption peak at the frequency lower than 2000 Hz.

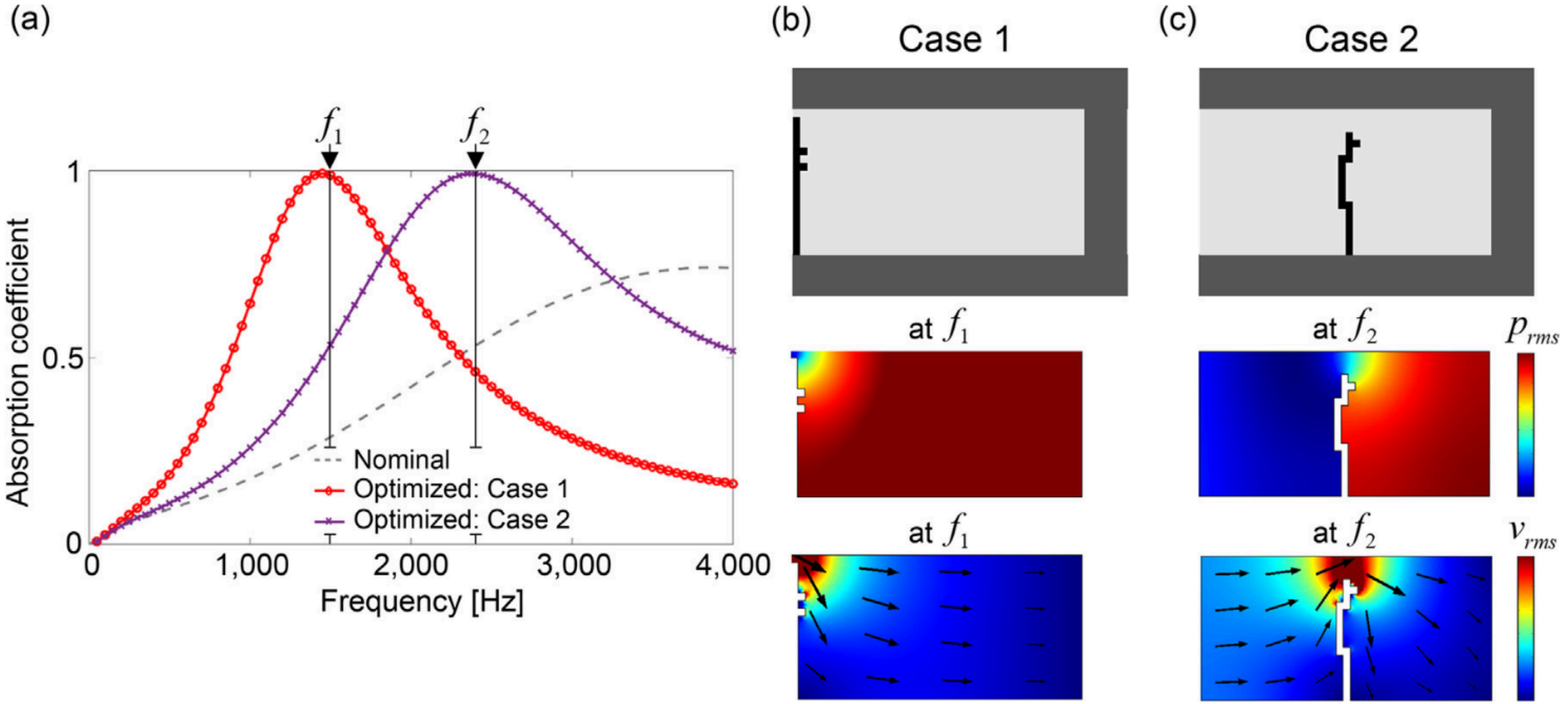


Fig. 6. Topology optimization results at single target frequencies $f_1 = 1500$ Hz (Case 1) and $f_2 = 2400$ Hz (Case2) under the full design

Helmholtz resonance

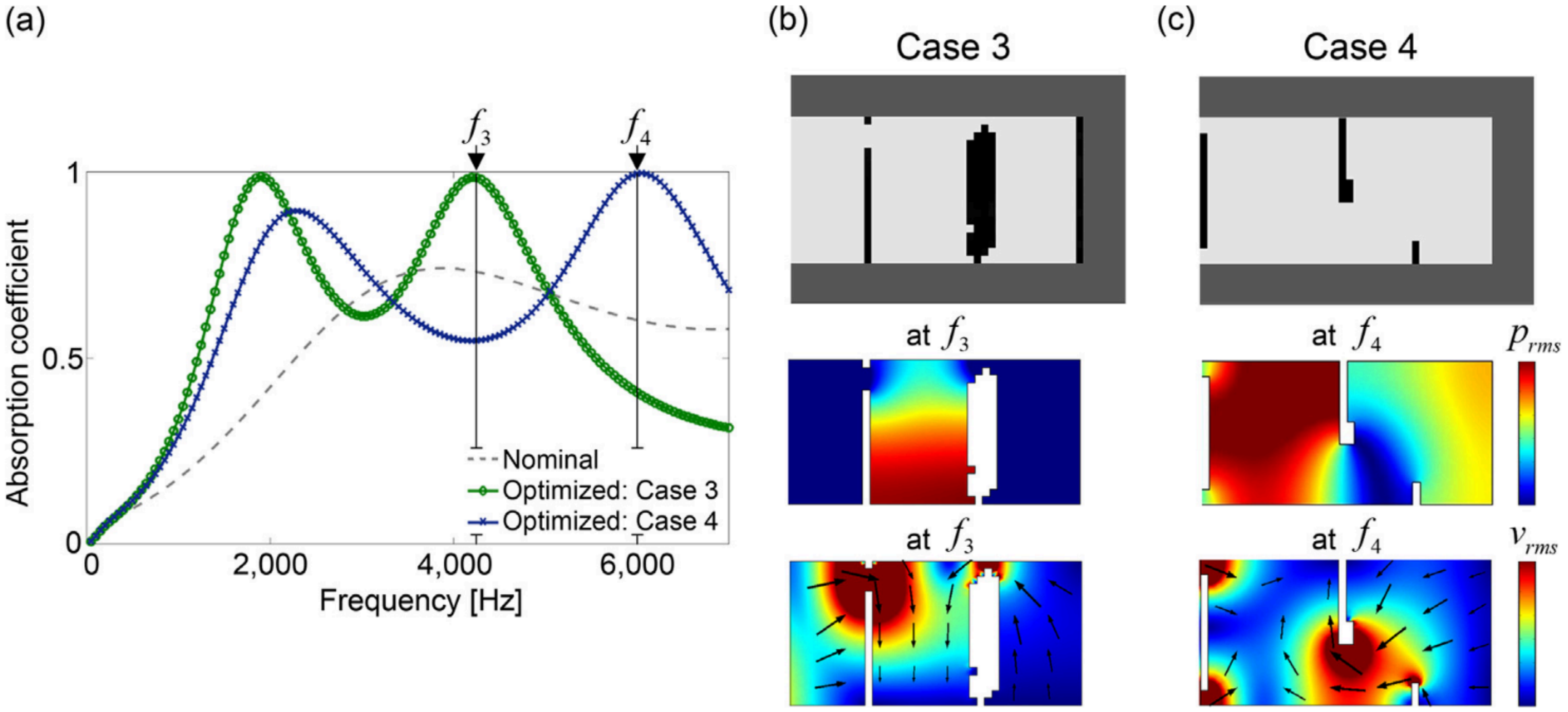
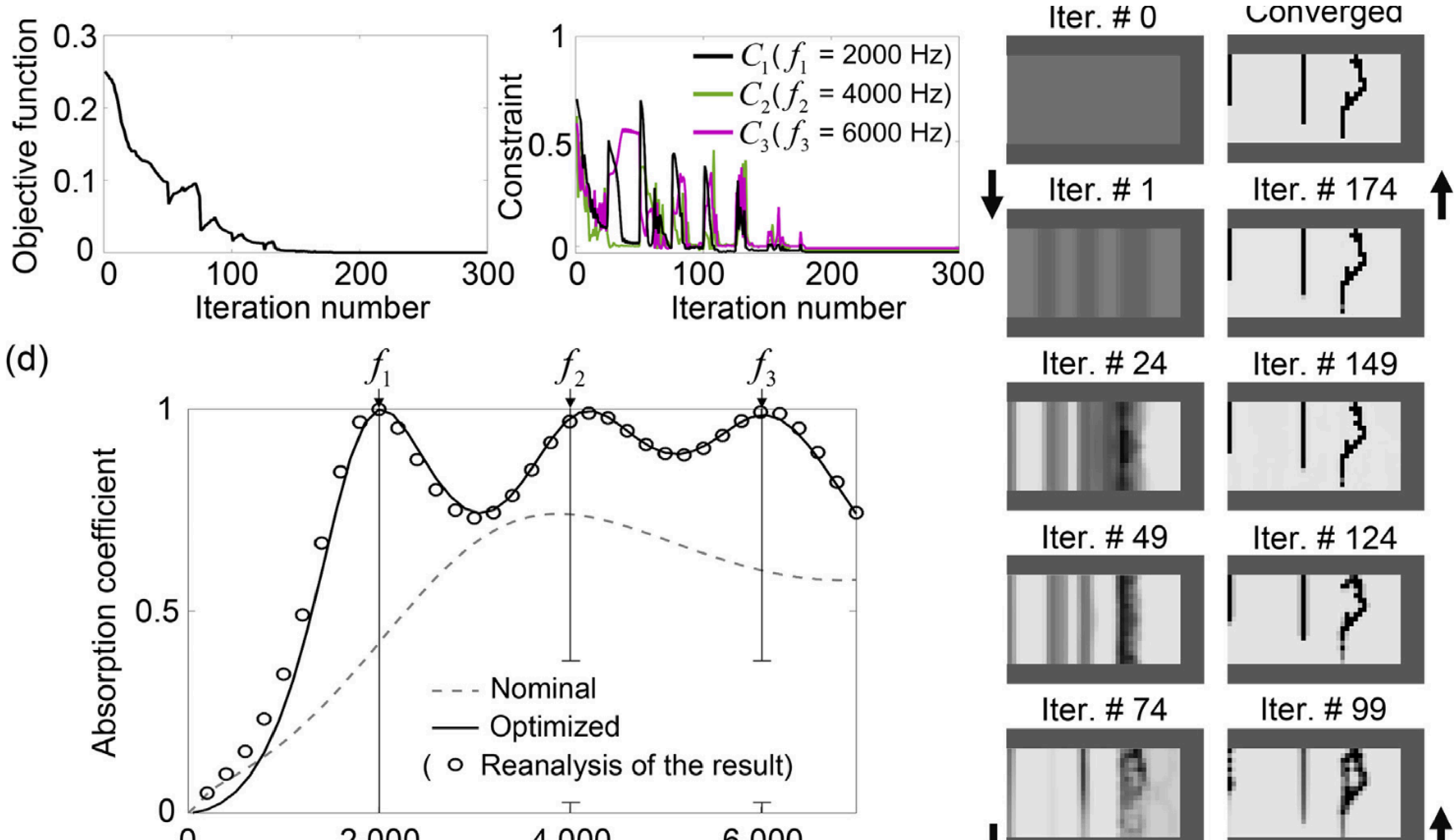
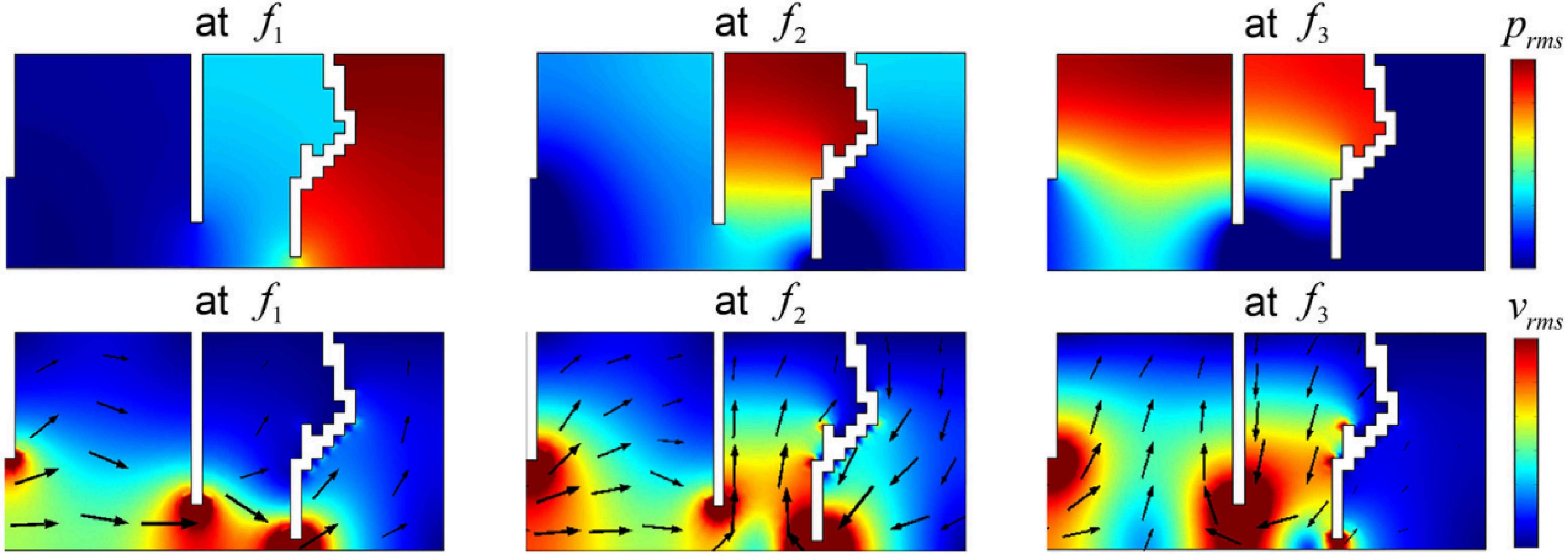


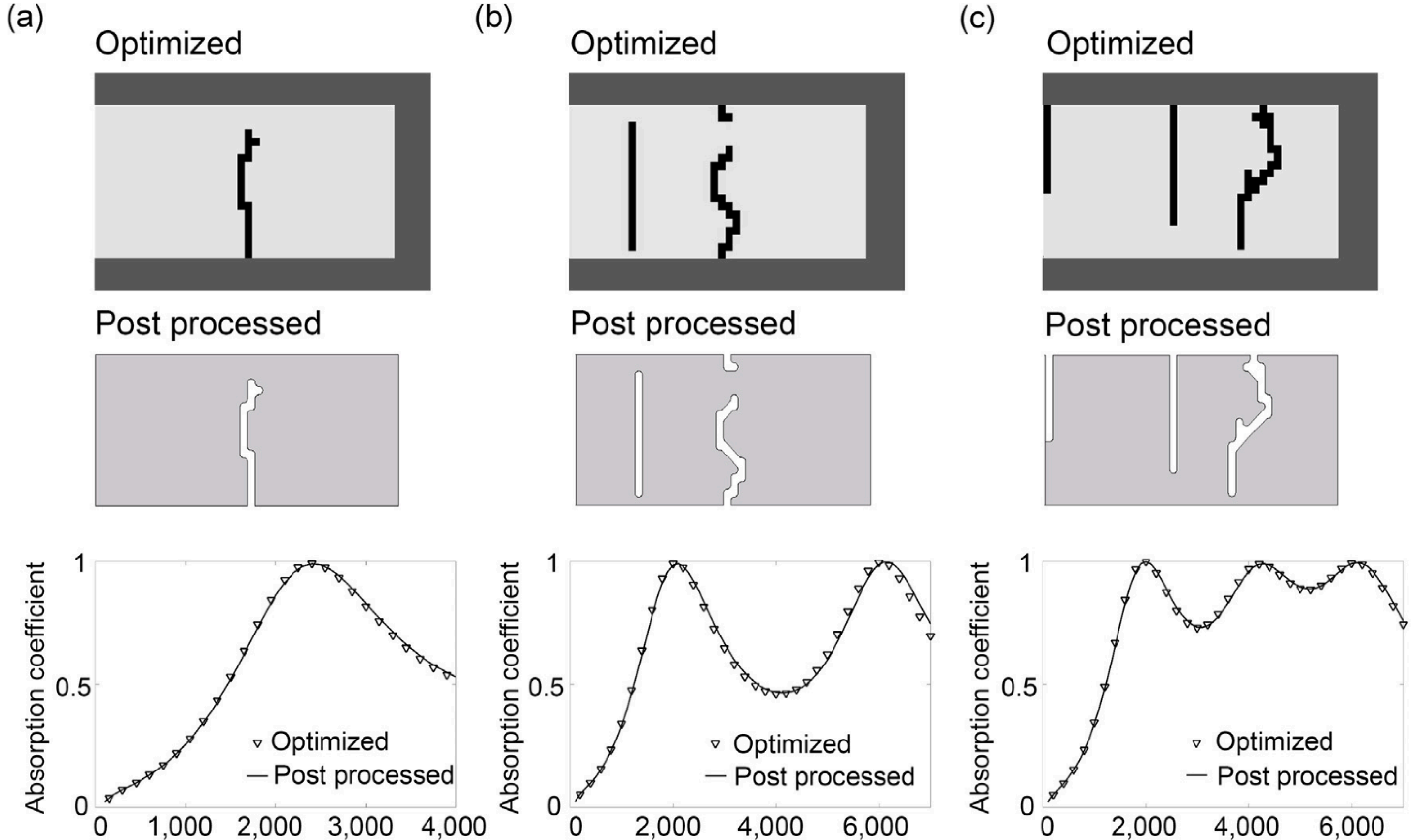
Fig. 7. Topology optimization results at single target frequencies $f_3 = 4200$ Hz (Case 3) and $f_4 = 6000$ Hz (Case 4) under the full design



Objective function
Iteration number
Constraint
Iteration number
C1(f1 = 2000 Hz)
C2(f2 = 4000 Hz)
C3(f3 = 6000 Hz)
Iter. # 0
Converged
Iter. # 1
Iter. # 174
Iter. # 24
Iter. # 149
Iter. # 49
Iter. # 124
Iter. # 74
Iter. # 99
(d)
f1
f2
f3
Absorption coefficient
Nominal
Optimized
(Reanalysis of the result)

(e)





(a)
Optimized
Post processed
Absorption coefficient
1
0.5
0
0
1,000
2,000
3,000
4,000
Optimized
Post processed
(b)
Optimized
Post processed
Absorption coefficient
1
0.5
0
0
2,000
4,000
6,000
Optimized
Post processed
(c)
Optimized
Post processed
Absorption coefficient
1
0.5
0
0
2,000
4,000
6,000
Optimized
Post processed