Problem 13 Three dimensions

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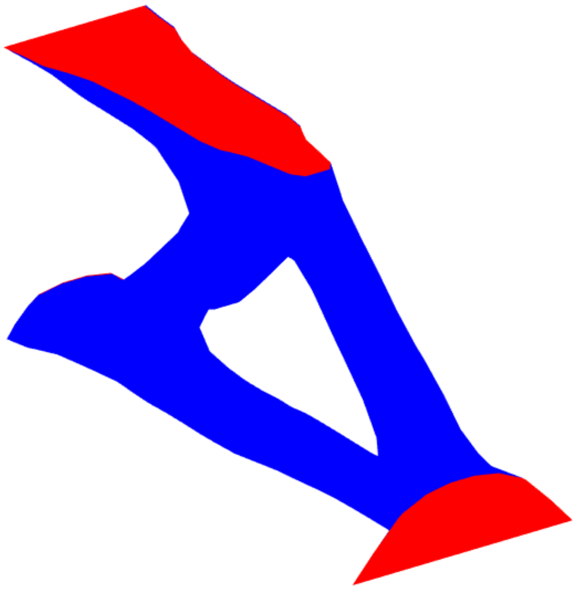
Download the 3D code `top3D125.m` based on the `top99neo.m` code (`https://www.topopt.mek.dtu.dk/Apps-and-software/New-99-line-topology-optimization-code-written-in-MATLAB` - by Ferrrari and Sigmund) and implement some of the exercises in this code.

In this assignment, I successfully run the educational code top3D125.m and test

```
top3D125(24,12,12,0.12,3,sqrt(3),1,'N',0.5,1,0.2,100);
```

and I obtain the following output results.



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Explanation of Key Parameters

This section provides a detailed explanation of the parameters `ftBC`, η, β, and `maxit` used in the topology optimization code.

Filter Boundary Condition (`ftBC`)

The parameter `ftBC` specifies the **boundary condition for the density filter** used to smooth the density field during optimization. Its purpose is to control how the density values near the boundaries of the design domain are treated when applying the filter. The common options for `ftBC` include:

- **`'N'` (Symmetric Boundary Condition)**: When `ftBC = 'N'`, the boundary condition is set to be symmetric (`bcF = 'symmetric'`). This means that the density field values near the boundaries are extended symmetrically beyond the design domain during the filtering process.
- **Other Values (Zero Boundary Condition)**: For any other values (e.g., `'0'` or `0`), the filter uses zero boundary conditions (`bcF = 0`). This indicates that the density values outside the design domain are set to zero during the filtering process.

This parameter impacts the smoothness of the density field at the boundaries and influences the final topology, especially near the domain edges.

Projection Threshold Parameter (η)

The parameter η controls the **threshold value** for the **density projection function**. It determines the cutoff point for transforming intermediate density values into values closer to either 0 or 1. Mathematically, the projection function is defined as:

$$\text{prj}(v, \eta, \beta) = \frac{\tanh(\beta\eta) + \tanh(\beta(v - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$

- **Role of η**: The value of η defines the **transition point** between "solid" and "void" regions. When the density values (v) are less than η, the projection results in values closer to 0 (void), and when they are greater than η, the results move closer to 1 (solid).
- **Optimization Impact**: Adjusting η helps fine-tune the density transition, especially in the presence of a projected density field (`ft > 1`). It ensures that the resulting design achieves a clearer separation between material and void, which is beneficial for manufacturability. During the optimization process, η is often dynamically updated using Newton's method to ensure the design satisfies the volume constraint.

Projection Sharpness Parameter (β)

The parameter β controls the **sharpness** of the **density projection function**. It affects how smoothly or abruptly the projection transitions from 0 to 1. Higher values of β lead to a steeper transition, making the design approach a pure 0-1 solution. The influence of β is expressed in the same projection function:

$$\text{prj}(v, \eta, \beta) = \frac{\tanh(\beta\eta) + \tanh(\beta(v - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$

- **Role of β**: Small values of β (e.g., 1 or 2) result in a **smooth transition** between solid and void regions, allowing for a gradual material distribution. Larger values (e.g., 10 or higher) result in a **sharp transition**, enforcing a clear boundary between solid and void elements.
- **Continuation Scheme**: In practice, β is gradually increased using a continuation scheme during the optimization process to progressively sharpen the density transition. This strategy helps avoid local minima in the early stages and ensures a clear boundary at convergence.

Maximum Iterations (`maxit`)

The parameter `maxit` specifies the **maximum number of iterations** allowed for the optimization loop. It is a control parameter to limit the computational effort and ensure the optimization does not run indefinitely.

- **Role of `maxit`**: The iterative optimization loop stops either when the design change (`ch`) falls below a convergence criterion (e.g., 10^{-6}) or when the number of iterations reaches `maxit`.
- **Setting `maxit`**: For complex problems or finer meshes, `maxit` is set to a higher value (e.g., 100, 200) to ensure sufficient iterations for convergence. For preliminary testing, a smaller value can be used to reduce computational time.
- **Impact**: If `maxit` is set too low, the optimization may stop prematurely, leading to suboptimal solutions. Conversely, a large value provides more opportunity for fine-tuning the design but increases the computational cost.