Problem 8: Robust topology optimization

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Implement the robust design formulation Sigmund (2009), Wang et al. (2011) and test it on the force inverter problem from course Problem 5. The 88-line code already has the Heaviside projection filter built-in so the main challenge consists in implementing the min-max formulation (like in course Problem 7) and solving for the three geometry cases in each iteration.

In the previous work, we designed algorithms based on the problem requirements and obtained the optimized structure. However, during the actual manufacturing process, it is often impossible to perfectly reproduce the designed structure. Due to manufacturing processes or precision limitations, over-etching or under-etching may occur, resulting in the structure being thicker or thinner than expected, which may lead to the loss of the desired performance of the workpiece. **Robust topology** optimization means that when designing the optimized structure, potential over-etching and under-etching are considered in advance. This ensures that the optimized structure still performs well under over-etching or under-etching conditions, although this comes at the cost of sacrificing performance in the ideal scenario.

The primary reference for this assignment is [**?**]. In the article, the author uses the morphological operators 'erode' and 'dilate' to represent the over-etching and under-etching scenarios, respectively. The objective functions for the three cases are optimized using a Min-Max formulation of the MMA algorithm, and the MBB beam, Compliant force inverter, and Compliant gripper are used as examples in the paper.

Morphological Operators

Based on the original image, the new image obtained by the dilate operator is one where, if there is a black point within the radius R neighborhood of a point, that point is also black. In other words, the original image is expanded outward by R. The new image obtained by the erode operator is one where a point is black only if every point in its radius R neighborhood is black. In other words, the original image is contracted inward by R.

These two operators are easy to understand mathematically, but there are two challenges in the practical application to our problem. First, the original image may not be purely black and white. Second, our optimization algorithm is gradient-based, but these two operators are not smooth. Here, we use smoothed Heaviside step functions to overcome these challenges.

First, to describe the black-and-white properties of the points in a point's neighborhood, we calculate the weighted average within the neighborhood of that point,

$$\tilde{\rho}_e = \frac{\sum_{i\in N_e} w(\mathbf{x}_i)\rho_i}{\sum_{i\in N_e} w(\mathbf{x}_i)}, \quad w(\mathbf{x}_i) = R - \|\mathbf{x}_i - \mathbf{x}_e\|.$$

As we can see, this is essentially the familiar density filtering. Next, we approximate the erode and dilate operators by applying increasingly steep Heaviside step functions to ρ_e in the loop.

Min-Max Foumulation

We denote the objective functions in the original, eroded, and dilated cases as c, c_e, c_d, respectively. In other words, we need to solve

$$\min_x \max\{c, c_e, c_d\}$$

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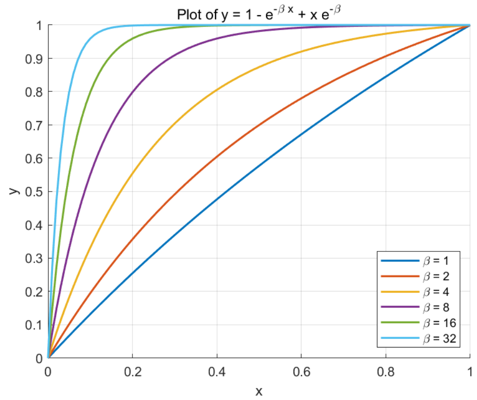
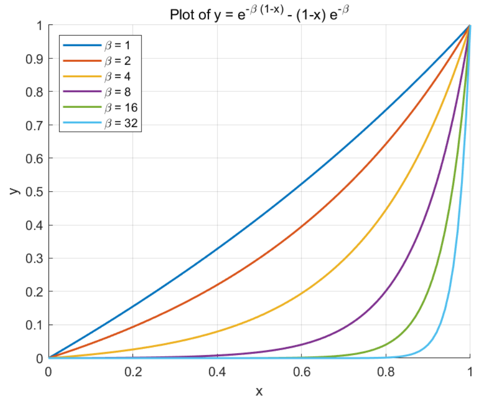




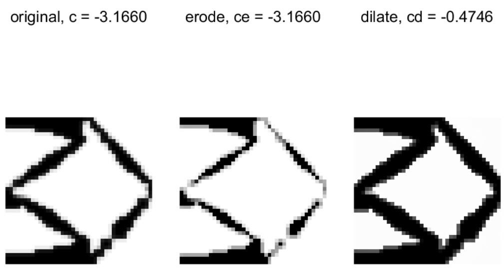
It is worth noting that the problem statement in Assignment 7 is different,

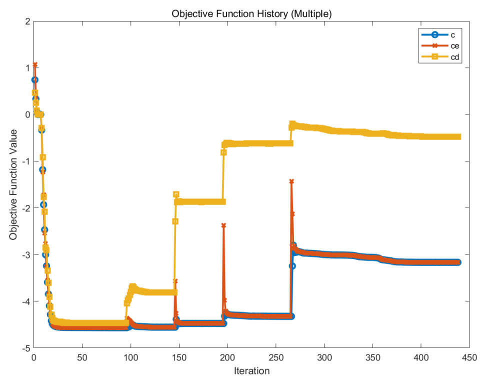
$$\min_x \max_i \{|h_i(x)|\} .$$

Since the functions in our problem do not contain absolute value symbols, the number of required inequalities is halved. It is sufficient to define $h_1(x) = c$, $h_2(x) = c_e$, and $h_3(x) = c_d$ in the MMA algorithm to represent the inequality constraints $z > h_i(x)$, that is, $z = \max_i \; h_i(x)$.

In the actual handling of the Compliant force inverter example, since the value of the objective function is negative and the MMA algorithm has a default constraint $z \geqslant 0$, directly defining $h_i(x)$ as above would cause the optimization to get stuck at $z = 0$ after just a few steps. The solution is to first test each case individually and use the results to determine a lower bound for the objective function, such as -10. Then, we add a constant 10 to each $h_i(x)$. This way, the optimization problem we obtain is equivalent, and we cleverly avoid being stuck by the default $z = 0$ constraint in the MMA algorithm.

Results and Analysis





It can be seen that both the erode and dilate scenarios are well simulated. The original and erode scenarios exhibit similar performance curves, but the performance curve of dilate is significantly worse. If we aim to achieve a more balanced design, we need to find a way to improve the performance of dilate at the expense of some performance in the original and erode scenarios.