

a) $i = 2 \Rightarrow O(1)$

$n = 10 \quad 2, 4 \rightarrow 2 \text{ times}$

$n = 20 \quad 2, 4, 16 \rightarrow 3 \text{ times}$

$n = 30 \quad 2, 4, 16 \rightarrow 3 \text{ times}$

```
int i = 2
while (i < n)
{
    i = i * i
    O(1)
}

for (i = 2; i <= n; i = i * i)
{
    O(1)
}
```

$= 7 \sum_{i=2}^n O(1)$

$n = 10 \quad i = 2, 4$

$n = 20 \quad i = 2, 4, 16$

$n = 30 \quad i = 2, 4, 16$

$\boxed{= \Theta(\log(\log n))}$

b)

$$n = 16$$

$$i = 1$$

\vdots

$$i = 4$$

\rightarrow # times for loop execution: 4^3

\vdots

$$i = 8$$

$\rightarrow 8^3$

\vdots

$$i = 12$$

$\rightarrow 12^3$

\vdots

$$i = 16$$

$\rightarrow 16^3$

$$4^3 + 8^3 + 12^3 + 16^3 = \text{tot. \# of times inner loop executes when } n=16$$

$$= (1 \cdot \sqrt{16})^3 + (2\sqrt{16})^3 + (3\sqrt{16})^3 + (4\sqrt{16})^3$$

$$= \sum_{k=1}^{\sqrt{n}} \Theta(k\sqrt{n})^3$$

$$= \sum_{k=1}^{\sqrt{n}} \Theta(k^3 \sqrt{n}^3)$$

$$= \sqrt{n}^3 \sum_{k=1}^{\sqrt{n}} \Theta(k^3) \rightarrow (\text{math identity})$$

$$= \sqrt{n}^3 \Theta(\sqrt{n})^4$$

$$= \Theta(n^{7/2})$$

[C]

```
void f2(int* A, int n)
{
```

```
    for(int i=1; i <= n; i++) {
```

```
        for(int k=1; k <= n; k++) {
```

```
            if (A[k] == i) {
```

```
                for(int m=1; m <= n; m = m+m) {
```

```
                    //  $\theta(1)$ 
```

$$T(n) = \sum_{i=1}^n \sum_{k=1}^n \left(\theta(1) + \underbrace{\theta(1)}_{\downarrow} \right)$$

iteration 1: $m = 2$

iteration 2: $m = 4$

iteration 3: $m = 8$

iteration 4: $m = 16$

$$2^k = n$$

$$2^k = n$$

$$k = \log(n)$$

$$T(n) = \sum_{i=1}^n \sum_{k=1}^n \left(\Theta(1) + \sum_{k=1}^{\log n} \Theta(1) \right)$$

$$= \sum_{i=1}^n \sum_{k=1}^n \Theta(1) + \sum_{i=1}^n \sum_{k=1}^n \log n$$

$$= \sum_{i=1}^n \Theta(n) + \sum_{i=1}^n \Theta(n \log n)$$

$$= \Theta(n^2) + \Theta(n^2 \log n)$$

$$\boxed{= \Theta(n^2 \log n)}$$

(d)

```
int f(int n)
{
    int *a = new int [10];
    int size = 10;
    for (int i = 0; i < n; i++)
    {
        if (i == size)
        {
            int newSize = 4 * size;
            int *b = new int [newSize];
            for (int j = 0; j < size; j++) b[j] = a[j];
            delete [] a;
            a = b;
            size = newSize;
        }
        a[i] = i * i;
    }
}
```

$$T(n) = \Theta(1) + \Theta(1)$$

$$+ \sum_{i=0}^{n-1} \left[\Theta(1) + \Theta \left(\Theta(1) + \Theta(1) + \sum_{j=0}^{10-1} \Theta(1) + \Theta(1) + \Theta(1) \right) + \Theta(1) \right]$$

$$T(n) = \Theta(2) + \sum_{i=0}^{n-1} \left[\Theta(1) + \overset{\text{only once}}{\cancel{\sum_i (\Theta(5))}} + \sum_{j=0}^9 \Theta(1) \right]$$

$$= \Theta(2) + \sum_{i=0}^{n-1} [\Theta(1) + \Theta(5) + \Theta(10)]$$

$$= \Theta(2) + \sum_{i=0}^{n-1} [\Theta(16)]$$

$$= \Theta(2) + \Theta(n)$$

$$T(n) = \Theta(n)$$