a) $i = 2 \Rightarrow O(1)$

$n=10$ 2, 4 → 2 times

$n=20$ 2, 4, 16 → 3times

$n=30$ 2, 4, 16 → 3 times

```
int i=2
while (i<n)
{ i=i*i
   O(1)
}
```

=>

```
for(i=2; i≤n; i=i*i)
{
    O(1)
}
```

$$\sum_{i=2}^{n} O(1)$$

$n=10$ $i = 2, 4$

$n=20$ $i = 2, 4, 16$

$n=30$ $i = 2, 4, 16$

$$\boxed{=\Theta(\log(\log n))}$$

b) $n = 16$

$i = 1$

$\vdots$

$i = 4$ → # times for loop execution : 4^3

$\vdots$

$i = 8$ → 8^3

$\vdots$

$i = 12$ → 12^3

$\vdots$

$i = 16$ → 16^3

$4^3 + 8^3 + 12^3 + 16^3$ = tot. # of times inner loop executes when $n=16$

$$= (1 \cdot \sqrt{16})^3 + (2\sqrt{16})^3 + (3\sqrt{16})^3 + (4\sqrt{16})^3$$

$$= \sum_{K=1}^{\sqrt{n}} \Theta(K\sqrt{n})^3$$

$$= \sum_{K=1}^{\sqrt{n}} \Theta(K^3 \sqrt{n}^3)$$

$$= \sqrt{n}^{3} \sum_{k=1}^{\sqrt{n}} \Theta(k^{3})$$ → (math identity)

$$= \sqrt{n}^{3} \, \Theta(\sqrt{n})^{4}$$

$$\boxed{= \Theta(n^{7/2})}$$

(c)

```
void f2(int* A, int n)
{
    for(int i=1; i<=n; i++) {
        for(int k=1; k<=n; k++) {
            if ( A[k]==i) {
                for (int m=1; m<=n; m=m+m) {
                    // θ(1)
```

$$T(n) = \sum_{i=1}^{n}\sum_{K=1}^{n}\left(\theta(1) + \underbrace{\theta(\qquad)}_{\downarrow}\right)$$

iteration 1: $m = 2$

iteration 2: $m = 4$

iteration 3: $m = 8$

iteration 4: $m = 16$

$$2^k = n$$

$$2^k = n$$

$$K = \log(n)$$

$$T(n) = \sum_{i=1}^{n}\sum_{k=1}^{n}\left(\Theta(1) + \sum_{k=1}^{\log n}\Theta(1)\right)$$

$$= \sum_{i=1}^{n}\sum_{k=1}^{n}\Theta(1) + \sum_{i=1}^{n}\sum_{k=1}^{n}\log n$$

$$= \sum_{i=1}^{n}\Theta(n) + \sum_{i=1}^{n}\Theta(n\log n)$$

$$= \Theta(n^2) + \Theta(n^2\log n)$$

$$\boxed{= \Theta(n^2\log n)}$$

(d)

```
int f(int n)
{
    int *a = new int [10];
    int size = 10;
    for (int i=0; i<n; i++)
      {
          if (i==size)
            {
                int newsize=3/2*size;
                int *b=new int [newsize];
                for (int j=0;j<size;j++) b[j]=a[j];
                delete [] a;
                a=b;
                size=newsize;
            }
          a[i]= i*i;
      }
}
```

$$T(n) = \Theta(1) + \Theta(1)$$

$$+ \sum_{i=0}^{n-1} \left[\Theta\left(\frac{1}{2}\right) + \Theta\left(\Theta\left(\frac{1}{2}\right) + \Theta\left(\frac{1}{2}\right) + \sum_{j=0}^{10-1} \Theta(1) + \Theta\left(\frac{1}{2}\right) \right) \right]$$

only once

$$T(n) = \Theta(2) + \sum_{i=0}^{n-1} \left[\Theta(1) + \cancel{\sum_i} \left(\Theta\left(\frac{3}{2}\right) + \sum_{j=0}^{9} \Theta(1) \right] \right.$$

$$= \Theta(2) + \sum_{i=0}^{n-1} \left[\Theta(1) + \Theta\left(\frac{3}{2}\right) + \Theta(10) \right]$$

$$= \Theta(2) + \sum_{i=0}^{n-1} \left[\Theta\left(\frac{25}{2}\right) \right] \qquad 11 \quad \frac{22}{2}$$

$$= \Theta(2) + \Theta(n)$$

$$\boxed{T(n) = \Theta(n)}$$