1. c 以 SGD 算出 hypothesis 再求 noise

```
In [1]: import numpy as np

In [2]: steps = 1000
    repeat = 500
    sample = 1000
    lr = 0.01

def target(x):
    return np.exp(x)

w_total = np.zeros(1)

x = np.linspace(0, 2, sample)

for j in range(repeat):
    w = np.zeros(1)
    for i in range(steps):
        k = np.random.randint(sample)
        w = w + lr*(target(x[k]) - w*x[k])*x[k]
        w_total += w

print ((w_total / repeat)[0])

3.1540241762703327

In [3]: print ('wb =', (3+np.exp(1)**2)/8)
    print ('wc =', (3+3*np.exp(1)**2)/8)
    print ('wc =', (3*np.exp(1)**2)/8)
    print ('we =', (3*np.exp(1)**2)/8)
    print ('we =', (3*np.exp(1)**2)/8)
    wb = 1.2986320123663311
    wc = 3.1458960370989937
    wd = 0.9236320123663312
    we = 2.7708966370989937
```

2. c

第一個是一般情況,第三個不可能發生,第二個假設有一個線性可分的分布,有可能找到最佳解使 Ein=Eout=0,但如果產生的 data set 有不好的情況的話,即使是最簡單的 case 也有可能 train 出 Eout 很大的情況,所以以期望值來看 Eout 一定會大於 Ein,2 個選項 always false 故選 c

3. d

$$E(X_{h}^{T}X_{h}) = E(\begin{bmatrix} x_{1} & \dots & x_{N} & x_{1} & \dots & x_{N} \\ \vdots & \ddots & \ddots & \vdots \\ x^{T} & x^{T}x + x^{$$

$$\psi.$$

$$E(X_h^T y_h) = E(\begin{bmatrix} x_1 & \dots & x_n & y_1 & \dots & y_n \\ x_1^T & \dots & x_n & y_1 & \dots & y_n \end{bmatrix} \begin{bmatrix} y \\ y \end{bmatrix})$$

$$= E(X_y^T + X_y^T + \xi y)$$

$$= 2X_y^T y$$

5. d

5. Wreg =
$$\left(Z^{T}Z + \lambda I\right)^{T}Z^{T}y$$

= $\left(Q^{T}X^{T}XQ + \lambda I\right)^{T}QX^{T}y$

$$\chi^{T}x = QTQ^{T}$$

$$\chi^{T}xQ = QT$$

$$Q^{T}x^{T}xQ = T = diag\{Y_{0}, \dots, Y_{d}\}$$

$$Q^{T}x^{T}xQ + \lambda I = diag\{Y_{0} + \lambda, Y_{1} + \lambda, \dots, Y_{d} + \lambda\}$$

$$\left(Q^{T}x^{T}xQ + \lambda I\right)^{T} = diag\{(Y_{0} + \lambda)^{T}(Y_{1} + \lambda)^{T}, \dots, (Y_{d} + \lambda)^{T}\}$$

$$=) Wrey = diag\{(Y_{0} + \lambda)^{T}((Y_{1} + \lambda)^{T}, \dots, (Y_{d} + \lambda)^{T}\}Qx^{T}y$$
when $\lambda = 0$

=)
$$W_{lin} = d_i ag \left(Y_0, Y_1, \dots, Y_d \right) Q \lambda^T y$$

$$\frac{U_i}{V_i} = \frac{w_{reg}}{w_{lin}} = \frac{\left(Y_i + \lambda J^T \right)}{Y_i^{-1}} = \frac{Y_i}{Y_i + \lambda}$$

6. a

6.
$$\|\mathbf{w}^{\dagger}\|^{2} = \mathbf{w}_{reg}^{T} \mathbf{w}_{reg}^{T}$$

$$= \left(\left(\mathbf{z}^{T} \mathbf{z} + \lambda \mathbf{I} \right)^{-1} \mathbf{z}^{T} \mathbf{y} \right)^{T} \left(\mathbf{z}^{T} \mathbf{z} + \lambda \mathbf{I} \right)^{-1} \mathbf{z}^{T} \mathbf{y}$$

$$= \mathbf{y}^{T} \mathbf{z} \left(\left(\mathbf{z}^{T} \mathbf{z} + \lambda \mathbf{I} \right)^{-1} \mathbf{z}^{T} \mathbf{y} \right)$$

$$\left(\mathbf{z}^{T} \mathbf{z} + \lambda \mathbf{I} \right)^{-1} \mathbf{z}^{T} \mathbf{y}$$

$$\left(\mathbf{z}^{T} \mathbf{z} + \lambda \mathbf{I} \right)^{-1} \mathbf{z}^{T} \mathbf{y} + \lambda \mathbf{I} \mathbf{y}^{T} \mathbf{z}^{T} \mathbf{y} \right)$$

$$\left(\mathbf{z}^{T} \mathbf{z} + \lambda \mathbf{I} \right)^{-1} \mathbf{z}^{T} \mathbf{y} \mathbf{z}^{T} \mathbf{y}$$

$$= \mathbf{y}^{T} \mathbf{z} \mathbf{z}^{T} \mathbf{$$

7.d

$$\int_{N} \sum_{1} \sum_{2} (y - y_{n}) = 0 \quad 2y - \frac{2}{N^{2}y} + \frac{44}{N}(y + c) = 0$$

$$2y - \frac{2}{N^{2}y} = 0 \quad y + \frac{2ky}{N} + \frac{2kc}{N} = \frac{1}{N^{2}y}$$

$$= y = \frac{1}{N^{2}y}$$

$$y = \frac{1}{N^{2}y}$$

8.
$$(\tilde{w}^{T} \phi(x_{n}) - y_{n} + \frac{\lambda}{N}(\tilde{w}^{T}\tilde{w}) = (\tilde{w}^{T} P^{T} x_{n} - y_{n})^{2} + \frac{\lambda}{N}(\tilde{w}^{T}\tilde{w})$$

1. Let $\tilde{w}^{*} = a b g n i n + \frac{\lambda}{N} (\tilde{w}^{T} P^{T} x_{n} - y_{n})^{2} + \frac{\lambda}{N} (\tilde{w}^{T}\tilde{w})$
 $(P = P^{T})$

1. Let $\tilde{w}^{*} = P^{T}\tilde{w}$, then $\tilde{w}^{*} P^{2}\tilde{w}^{*} = \tilde{w}^{*} P^{T} P^{T$

9. b

9.
$$X = (x_1, \dots, x_N)^T$$
, $Y = (y_1, \dots, y_N)^T$
 $X' = (\hat{x}) \quad y' = (\hat{y})$
 $=) \min_{X \neq K} \frac{1}{X'} | X' - y| |^2$
 $v = ((x')^T x)^T (x')^T y$
 $= ((\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{y})^T$
 $= ((\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{y})^T$
 $= ((\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{x})^T (\hat{x})^T$
 $= ((\hat{x})^T (\hat{x})^T (\hat{x}$

不管抽掉哪一個,選最多數的 Algorithm 都會產生把抽出的 sample 分到另一個的 hypothesis,使 error=1,每種情況都相同,平均後還是 1,故選 e

11. c

11. First, let N samples be
$$x_1 < X_2 < \cdots < X_N (:: X_1 < X_2 < 0, 0 < X_{N-1} < X_N)$$

if X_i is left for validation and $X_j < 0 < X_j + 1$ for some j

then $(j = j)$
 $0 = \frac{x_{j-1} + x_{j+1}}{2} = \sum_{for X_j} s_i + k_f < 0, x_{f+1} > 0 = 0$
 $0 = \frac{x_{j-1} + x_{j+1}}{2} = \sum_{for X_j} s_i + k_f < 0, x_{f+1} > 0 = 0$

same reason above

for other case would make mistake i, Eloocu < N

12. set
$$h_{o}(x) = v_{o}$$
 $f(x, y_{o}) = v_{o}$
 $f(x, y_{o}) = v$

Solve $\frac{1}{3} \left(4 + \left(\frac{1^{2}}{\rho + 3} \right)^{2} + \left(\frac{1^{2}}{\rho - 3} \right)^{2} \right) = 2$ $x = \pm \left(3\sqrt{9 + 4\sqrt{6}} \right)$ (Q)

Sample mean
$$Var(\bar{X}) = Var\left(\frac{x_1 + \dots + x_n}{n}\right)$$

$$= Var\left(\frac{1}{h}x_1 + \frac{1}{h}x_2 + \dots + \frac{1}{h}x_n\right)$$

$$= \frac{1}{h^2}Var(X_1) + \frac{1}{h^2}Var(X_2) + \dots + \frac{1}{h^2}Var(X_n)$$

$$= \frac{1}{h^2}\left[6^2 + 6^2 + \dots + 6^2\right] - \frac{1}{h^2}\left[h \cdot 6^2\right]$$

$$= \frac{1}{h}6^2$$

$$0 \times 0 \times 0$$
 $\times 0 \times 0$
 $\times 0 \times 0$

:
$$E_{y_1,y_2,y_3,y_4}(\min E_{in}) = \frac{0+2.7}{16} = \frac{8}{64}$$

If
$$p(y=t1)=p$$

then $E_{one}(g)=P(g(x)+y)$
 $=P(y=1)\cdot P(y(x)=-1|y=1)+P(y=-1)\cdot P(g(x)=1|y=-1)$
 $=P\cdot \xi_{+}+(1+P)\xi_{-}$
 $\therefore I+E_{one}(g)=E_{one}(g_{c})$, then $P\xi_{+}+(1-p)\xi_{-}=1-p$
 $=\sum_{k=1}^{\infty}\{\xi_{+}-\xi_{-}+1\}P=1-\xi_{-}=k\}$ $P=\frac{1-\xi_{-}}{\xi_{+}-\xi_{-}+1}$

Data processing for p16 to p20

```
In [11]: with open('foldi_train_set.txt','a') as f:
    writer = csv.writer(f)
    for j in range(40):
        f.write(str(y_train[j].item()) + "")
        for i in range[len(X_train[0])):
            f.write(str(1+1) + ":" + str(X_train[j][i].item()) + "")
        f.write("\n")
    with open('fold2_train_set.txt','a') as f:
    writer = csv.writer(f)
    for j in range(40):
        f.write(str(y_train[j+40].item()) + "")
        for i in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(40):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(40):
            f.write(str(y_train[j+80].item()) + "")
            for i in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train[0])):
            f.write(str(y_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train_set.txt','a') as f:
            writer = csv.writer(f)
        for j in range(len(X_train_set.txt','a
```

```
In [10]: with open("\n" foldd_train_set.txt', a") as f;
writer = csv.writer(f)
for j in range(160):
    f.write(str(ixt) + in'; + str(X_train[j+40][i].item()) + " ")
    f.write(str(ixt) + in'; + str(X_train[j+40][i].item()) + " ")
    f.write("\n")
with open("\n" fold2_train_set.txt', a') as f:
    writer = csv.writer(f)
    for j in range(40):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for i in range(40):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j+80][i].item()) + " ")
        for i in range(100):
        f.write(str(ixt) + in'; + str(X_train[j+80][i].item()) + " ")
        with open("\n" open j in range(80):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(80):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(80):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(80):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(80):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(80):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[j][i].item()) + " ")
        for j in range(100):
        f.write(str(ixt) + in'; + str(X_train[
```

```
with open('lv_fold5_train_set.txt','a') as f:
    writer = csv.writer(f)
    for j in range(160):
        f.write(str(y_train[j].item()) + " ")
        for i in range(1en(X_train[0])):
            f.write(str(i+1) + ":" + str(X_train[j][i].item()) + " ")
        f.write("\n")
```

$$50 = \frac{1}{2.2^*}$$

$$\lambda^* = 10^{-2}$$

$$5 \times 10^5 = \frac{1}{2.24}$$

$$\lambda^* = 10^4$$

```
(base) [r08245012@cluster liblinear]$ ./train -s 0 -c 0.00005 -e 0.000001 train_set.txt init f 6.931e-03 |g| 3.955e-03 iter 1 f 6.924e-03 |g| 4.088e-08 CG 2 step_size 1.00e+00 iter 2 f 6.924e-03 |g| 5.435e-11 CG 2 step_size 1.00e+00 (base) [r08245012@cluster liblinear]$ ./predict test_set.txt train_set.txt.model prediction Accuracy = 51.6667% (155/300)
```

18. e c=50

```
(base) [r08245012@cluster liblinear]$ ./train -c 50 -s 0 -e 0.000001 120_train_set.txt
init f 4.159e+03 |g| 2.864e+03
iter 1 f 2.281e+03 |g| 7.205e+02 CG 3 step_size 1.00e+00
iter 2 f 1.728e+03 |g| 2.224e+02 CG 7 step_size 1.00e+00
iter 3 f 1.632e+03 |g| 5.710e+01 CG 7 step_size 1.00e+00
iter 4 f 1.622e+03 |g| 3.800e+01 CG 8 step_size 1.00e+00
iter 5 f 1.622e+03 |g| 2.226e+00 CG 2 step_size 1.00e+00
iter 6 f 1.622e+03 |g| 1.728e-01 CG 9 step_size 1.00e+00
iter 7 f 1.622e+03 |g| 5.976e-01 CG 8 step_size 1.00e+00
iter 8 f 1.622e+03 |g| 8.347e-02 CG 3 step_size 1.00e+00
iter 9 f 1.622e+03 |g| 1.574e-02 CG 9 step_size 1.00e+00
iter 10 f 1.622e+03 |g| 7.303e-03 CG 3 step_size 1.00e+00
iter 11 f 1.622e+03 |g| 6.187e-03 CG 3 step_size 1.00e+00
iter 12 f 1.622e+03 |g| 1.909e-05 CG 11 step_size 1.00e+00
(base) [r08245012@cluster liblinear]$ ./predict test_set.txt 120_train_set.txt.model prediction
Accuracy = 85.6667% (257/300)
```

19. d 0. 13

```
(base) [r08245012@cluster liblinear]$ ./train -c 50 -s 0 -e 0.000001 train_set.txt
init f 6.931e+03 |g| 3.955e+03
iter 1 f 3.842e+03 |g| 9.887e+02 CG 4 step_size 1.00e+00
iter 2 f 2.901e+03 |g| 4.447e+02 CG 6 step_size 1.00e+00
                                                                                                  4 step_size 1.00e+00
6 step_size 1.00e+00
7 step_size 1.00e+00
iter 1 f 3.842e+03
iter 2 f 2.901e+03
iter 3 f 2.693e+03
iter 4 f 2.667e+03
iter 5 f 2.666e+03
iter 6 f 2.662e+03
iter 7 f 2.662e+03
iter 8 f 2.662e+03
iter 10 f 2.662e+03
iter 11 f 2.662e+03
iter 12 f 2.662e+03
iter 12 f 2.662e+03
                                                             1.326e+02 CG
                                                   g
                                                                                                  8 step_size
2 step_size
2 step_size
9 step_size
5 step_size
                                                             7.797e+01 CG
                                                                                                                                   1.00e+00
                                                   g
                                                             7.387e+00 CG
6.305e+00 CG
                                                                                                                                   1.00e+00
                                                     g
                                                    g
                                                                                                                                   1.00e+00
                                                             5.281e-01 CG
5.527e-01 CG
5.571e-01 CG
                                                    g
                                                                                                                                   1.00e+00
                                                                                                                                   1.00e+00
                                                   g
                                                                                                5 step_size 1.00e+00

2 step_size 1.00e+00

8 step_size 1.00e+00

9 step_size 1.00e+00

3 step_size 1.00e+00

9 step_size 1.00e+00

5 step_size 1.00e+00

11 step_size 1.00e+00
                                                   g
                                                             1.649e+00 CG
                                                   g
            11 f
12 f
13 f
14 f
                                                             9.726e-02 CG
2.778e-02 CG
                                                     g
                                                   g
iter 12 f 2.002e+03
iter 13 f 2.662e+03
iter 14 f 2.662e+03
iter 15 f 2.662e+03
iter 16 f 2.662e+03
                                                             1.687e-02 CG
                                                   g
                                                             2.997e-02 CG
                                                   g
                                                             5.162e-03 CG
1.192e-04 CG
                                                  g
 (base) [r08245012@cluster liblinear]$ ./predict test_set.txt train_set.txt.model prediction
Accuracy = 87\% (261/300)
```

20. c=50

(0.15 + 0.2 + 0.05 + 0.15 + 0.05) / 5 = 0.12

```
uster liblinear]$ ./train -c 50 -s 0 -e 0.000001 lv_fold1_train_set.txt
3.384e+03
          ) [r08245012@cluster liblinear]$ ./train -c 50 -s 0 -e 0.000001 lv_fold1_train_set.txt
f 5.545e+03 |g| 3.384e+03
1 f 3.100e+03 |g| 8.031e+02 CG 3 step_size 1.00e+00
2 f 2.315e+03 |g| 4.649e+02 CG 6 step_size 1.00e+00
3 f 2.144e+03 |g| 1.404e+02 CG 6 step_size 1.00e+00
4 f 2.118e+03 |g| 1.560e+01 CG 7 step_size 1.00e+00
5 f 2.115e+03 |g| 4.904e-01 CG 9 step_size 1.00e+00
6 f 2.115e+03 |g| 6.078e-02 CG 9 step_size 1.00e+00
7 f 2.115e+03 |g| 1.243e-02 CG 9 step_size 1.00e+00
8 f 2.115e+03 |g| 3.836e-03 CG 9 step_size 1.00e+00
9 f 2.115e+03 |g| 8.844e-04 CG 9 step_size 1.00e+00
9 f 2.115e+03 |g| 8.844e-04 CG 9 step_size 1.00e+00
9 f 2.115e+03 |g| 8.844e-04 CG 9 step_size 1.00e+00
9 f 8.844e-04 CG 9 step_size 1.00e+00
9 f 8.844e-04 CG 9 step_size 1.00e+00
9 f 8.844e-04 CG 9 step_size 1.00e+00
iter
iter
iter
iter
iter
(base)
       iter
iter
iter
iter
iter
Accuracy = 80\% (32/40)
```

```
liblinear]$ ./predict fold4_train_set.txt lv_fold4_train_set.txt.model prediction
  Accuracy = 85% (34/40)

(base) [r08245012@cluster liblinear]$ ./train -c 50 -s 0 -e 0.000001 lv_fold5_train_set.txt

init f 5.545e+03 |g| 3.288e+03

iter 1 f 3.191e+03 |g| 8.386e+02 CG 3 step_size 1.00e+00

iter 2 f 2.439e+03 |g| 3.373e+02 CG 7 step_size 1.00e+00

iter 3 f 2.287e+03 |g| 9.570e+01 CG 7 step_size 1.00e+00

iter 4 f 2.270e+03 |g| 8.030e+01 CG 8 step_size 1.00e+00

iter 5 f 2.269e+03 |g| 4.215e+00 CG 2 step_size 1.00e+00

iter 6 f 2.268e+03 |g| 1.201e+00 CG 9 step_size 1.00e+00

iter 7 f 2.268e+03 |g| 3.627e-01 CG 4 step_size 1.00e+00

iter 9 f 2.268e+03 |g| 3.552e-01 CG 2 step_size 1.00e+00

iter 9 f 2.268e+03 |g| 3.574e-01 CG 2 step_size 1.00e+00

iter 10 f 2.268e+03 |g| 3.674e-01 CG 9 step_size 1.00e+00

iter 11 f 2.268e+03 |g| 3.674e-01 CG 9 step_size 1.00e+00

iter 12 f 2.268e+03 |g| 3.674e-01 CG 9 step_size 1.00e+00

iter 13 f 2.268e+03 |g| 2.120e-02 CG 3 step_size 1.00e+00

iter 14 f 2.268e+03 |g| 2.120e-02 CG 3 step_size 1.00e+00

iter 14 f 2.268e+03 |g| 2.50e-02 CG 3 step_size 1.00e+00

iter 15 f 2.268e+03 |g| 2.50e-02 CG 3 step_size 1.00e+00

iter 17 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 18 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.574e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.674e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.674e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.674e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03 |g| 3.674e-01 CG 3 step_size 1.00e+00

iter 19 f 2.268e+03
```