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Exercise 9.4 (Tangent)

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\begin{aligned}
x(t) &= \sum_{i=0}^{n} B_i^n(t) P_i \\
\partial_t :&= \frac{\partial}{\partial_t} \\
\binom{n}{i} &= \frac{n!}{i!(n-i)!} \\
\partial_t x(t) &= \sum_{i01}^{n} \partial_t B_i^n(t) P_i \\
&= \sum_{i=0}^{n} \partial_t \left(\binom{n}{i} t^i (1-t)^{n-i} \right) P_i \\
&= \sum_{i=1}^{n} \left(\binom{n}{i} i t^{i-1} (1-t)^{n-i} \right) P_i - \sum_{i=0}^{n} \left(\binom{n}{i} t^i (n-i)(1-t)^{n-i-1} \right) P_i \\
&= n \left(\sum_{i=1}^{n} \frac{(n-1)!}{(i-1)!(n-i)!} t^{i-1} (1-t)^{n-i} P_i - \sum_{i=0}^{n-1} \frac{(n-1)!}{i!(n-i-1)!} t^i (1-t)^{n-i-1} P_i \right) \\
&= n \left(\sum_{i=0}^{n-1} \frac{(n-1)!}{i!(n-(i+1))!} t^i (1-t)^{n-(i+1)} P_{i+1} - \sum_{i=0}^{n-1} \binom{n-1}{i} t^i (1-t)^{(n-1)-i} P_i \right) \\
&= n \left(\sum_{i=0}^{n-1} \binom{n-1}{i} t^i (1-t)^{(n-1)-i} P_{i+1} - \sum_{i=0}^{n-1} \binom{n-1}{i} t^i (1-t)^{(n-1)-i} P_i \right) \\
&= n \sum_{i=0}^{n-1} B_i^{n-1}(t)(P_{i+1} - P_i) \\
n' &= n - 1 \\
\bar{P}_i &= (P_{i+1} - P_i) n \\
\dot{x}\,(t) &= \sum_{i=0}^{n'} B_i^{n'}\ \bar{P}_i \\
\dot{x}\,(0) &= \bar{P}_0 = n(P_1 - P_0) \\
\dot{x}\,(1) &= \bar{P}_{n'} = n(P_n - P_{n-1})
\end{aligned}
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Tangent in $x(0)$: $g(s)\ = P_0 + \dot{x}\,(0) * l$
Tangent in $x(1)$: $g'(s) = P_n + \dot{x}\,(1) * l$

$P_1 = g(\frac{1}{n}) \Rightarrow$ Tangent in $x(0)$ points in direction of $P_1 - P_0$
$P_{n-1} = g'(-\frac{1}{n}) \Rightarrow$ Tangent in $x(1)$ points in direction of $P_{n-1} - P_n$