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Exercise 11.1 (Normal transformation)

a)

$$\begin{aligned} n' \cdot t' = (n')^T t &\overset{!}{=} n^T t = n \cdot t \\ n^T G^T M t &\overset{!}{=} n^T t \\ \Rightarrow G^T M &= \mathbb{1} \\ G^T &= M^{-1} \\ \Rightarrow \qquad G &= \left(M^{-1}\right)^T \end{aligned}$$

b) The normal must be pointing outside

c) View the ellipsoid as a dragged unit ball:

- ball
  $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $x^2 + y^2 + z^2 = 1$
- ellipsoid
  $\begin{pmatrix} ax \\ by \\ cz \end{pmatrix}$ with $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$

In our case are $a = 4$, $b = 2$, $c = 1$
$\Rightarrow M = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
$\Rightarrow G = \left(M^{-1}\right)^T = \begin{pmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$
normals of ball: $n_B = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$
normal od ellipsoid: $n_e = G n_B = \begin{pmatrix} \frac{x}{4} \\ \frac{y}{2} \\ z \end{pmatrix}$

Exercise 11.2 (Cox-De Boor algorithm)

$$\begin{aligned}
p_l^d &= (1-\alpha_l^d)P_{l-1}^{d-1} + \alpha_l^d P_l^d - 1 \\
&= (1-\frac{t-t_l}{t_{l+1}-t_l})P_{l-1}^{d-1} + \frac{t-t_l}{t_{l+1}-t_l}P_l^{d-1} \\
&= \frac{t_{l+1}-t}{t_{l+1}-t_l})P_{l-1}^{d-1} + \frac{t-t_l}{t_{l+1}-t_l}P_l^{d-1}
\end{aligned} \tag{1}$$

$$\begin{aligned}
x(t) &= \sum_{i=l-d}^{l} B_{i,d}(t)p_i^0 = \sum_{i=l-d}^{l}\left(\frac{t-t_i}{t_{i+d}-t_i}B_{i,d-1}(t) + \frac{t_{i+d+1}-t}{t_{i+d+1}} - t_{i+1}B_{i+1,d-1}(t)\right)p_i^0 \\
&= \sum_{i=l-d}^{l} p_i^0\frac{t-t_i}{t_{i+d}-t_i}B_{i,d-1}(t) + \sum_{i=l-d+1}^{l+1}\frac{t_{i+d}-t}{t_{i+d}-t_i}B_{i,d-1}(t)p_{i-1}^0 \\
&= p_{l-d}^0\frac{t-t_{l-d}}{t_{l-1}-t_{l-d}}\underbrace{B_{l-d,d-1}(t)}_{0} + \sum_{i=l-d+1}^{l} B_{i,d-1}(t)\left(p_{i-1}^0\frac{t_{i+d}-t}{t_{i+d}-t_i} + p_i^0\frac{t-t_i}{t_{i+d}-t_i}\right) \\
&\quad + \frac{t_{l+d+1}-t}{t_{l+d+1}-t_{l+1}}\underbrace{B_{l+1,d-1}(t)}_{0}p_l^0
\end{aligned}$$

$$\begin{aligned}
&\sum_{i=l-d+k}^{l} B_{i,d-k}(t)p_i^k \stackrel{!}{=} \sum_{i=l-d+k+1}^{l} B_{i,d-k-1}(t)p_i^{k+1} \\
&= \sum_{i=l-d+k}^{l}\left(\frac{t-t_i}{t_{i+d-k}-t_i}B_{i,d-k-1}(t) + \frac{t_{i+d-k+1}-t}{t_{i+d-k+1}-t_{i+1}}B_{i+1,d-k-1}(t)\right)p_i^k \\
&= \sum_{i=l-d+k+1}^{l} B_{i,d-k-1}(t)\underbrace{\left(\frac{t-t_i}{t_{i+d-k}-t_i}p_i^k + \frac{t_{i+d-k}-t}{t_{i+d-k}-t_i}p_{i-1}^k\right)}_{p_i^{k+1}}
\end{aligned}$$