9.4 RECIPROCITY AND ANTENNA MEASUREMENTS

The remainder of this chapter is devoted to antenna measurements. This study provides a deeper understanding of antennas, allows one to interpret measured data, and serves as an introduction to those who desire to make antenna measurements. The principles introduced here also apply to broader situations such as scattering measurements, but are directly primarily toward antenna measurements. The primary measured antenna characteristics are radiation pattern, gain, polarization, and impedance. The first three of these are discussed in the following sections. Impedance is usually measured with a network analyzer and was discussed in Secs. 1.9 and 5.3.

In this section, we show that the radiation pattern of an antenna is the same whether it is used as a transmitting antenna or receiving antenna. Reciprocity allows the calculation or measurement of an antenna pattern in either the transmit or receive case, whichever is more convenient. Practical considerations for the measurement of antenna patterns are also discussed in this section.

In order to show that transmit and receive patterns are identical, it is necessary to discuss reciprocity theorems. There are several forms reciprocity theorems take for electromagnetic field problems. We consider two forms of reciprocity for use in antenna problems. The Lorentz reciprocity theorem is discussed first. Let sources $\mathbf{J}_a$ and $\mathbf{M}_a$ produce fields $\mathbf{E}_a$ and $\mathbf{H}_a$ and sources $\mathbf{J}_b$ and $\mathbf{M}_b$ produce fields $\mathbf{E}_b$ and $\mathbf{H}_b$. See Fig. 9-4. The frequencies of all quantities are identical. The Lorentz reciprocity theorem that is derivable from Maxwell's equations (see Prob. 9.4-1) states that for isotropic media,

$$\iiint_{v_a} (\mathbf{E}_b \cdot \mathbf{J}_a - \mathbf{H}_b \cdot \mathbf{M}_a)\, dv' = \iiint_{v_b} (\mathbf{E}_a \cdot \mathbf{J}_b - \mathbf{H}_a \cdot \mathbf{M}_b)\, dv' \tag{9-36}$$

The left-hand side is the reaction (a measure of the coupling) of the fields from sources b on sources a, and the right-hand side is the reaction of the fields from sources a on sources b. This is a very general expression, but it can be put into a more usable form. Let sources b consist of only an ideal electric dipole of vector length $\mathbf{p}$ located at point (x_p, y_p, z_p). Since the ideal dipole can be represented as an infinitesimal source and $\mathbf{M}_b$ is zero, (9-36) becomes[1]

$$\mathbf{E}_a(x_p, y_p, z_p) \cdot \mathbf{p} = \iiint_{v_a} (\mathbf{E}_b \cdot \mathbf{J}_a - \mathbf{H}_b \cdot \mathbf{M}_a)\, dv' \tag{9-37}$$

This expression allows calculation of the electric field from sources a by evaluating the integral using known sources $\mathbf{J}_a$ and $\mathbf{M}_a$ and known ideal dipole fields $\mathbf{E}_b$ and $\mathbf{H}_b$ of (1-69) and (1-68), evaluated at the location of sources a. This can be performed for various orientations $\mathbf{p}$ of the ideal dipole, which is acting as a field probe.

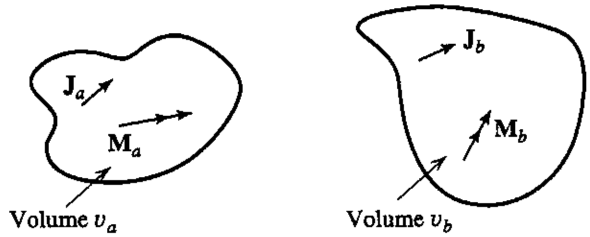


Figure 9-4 Source configuration for the Lorentz reciprocity theorem.

[1]The ideal dipole current could be written as $\mathbf{J}_b = \delta(x - x_p)\,\delta(y - y_p)\,\delta(z - z_p)\mathbf{p}$. This together with $\mathbf{M}_b = 0$ in (9-36) yields (9-37).

The Lorentz reciprocity theorem can also be used to derive a second reciprocity theorem using terminal voltages and currents. Suppose sources a and b are antennas excited with ideal (infinite impedance) current generators I_a and I_b. Since no magnetic sources are present, (9-36) reduces to

$$\iiint_{v_a} \mathbf{E}_b \cdot \mathbf{J}_a \, dv' = \iiint_{v_b} \mathbf{E}_a \cdot \mathbf{J}_b \, dv' \tag{9-38}$$

For perfectly conducting antennas, the electric fields will be zero over the antennas; however, voltages will be produced across the terminals. Taking the current to be constant in the terminal region and using the concept of $\int \mathbf{E} \cdot \mathbf{d\ell} = -V$, we see that (9-37) becomes

$$V_a^{oc} I_a = V_b^{oc} I_b \tag{9-39}$$

where V_a^{oc} is the open circuit voltage across the terminals of antenna a due to the field $\mathbf{E}_b$ generated by antenna b and, similarly, V_b^{oc} is the open circuit voltage at antenna b due to antenna a. Open circuit voltages have been used because of the infinite impedances of the generators. Rearranging (9-39) leads to a statement of reciprocity in circuit form

$$\frac{V_a^{oc}}{I_b} = \frac{V_b^{oc}}{I_a} \tag{9-40}$$

Several factors affect the voltage appearing at one antenna due to another antenna that is excited: the specific antennas used, the medium between the antennas with perhaps other objects present, and the relative orientation of the antennas. We can represent the general situation entirely in terms of circuit parameters using the following, which holds for any linear passive network:

$$V_a = Z_{aa} I_a + Z_{ab} I_b \tag{9-41a}$$

$$V_b = Z_{ba} I_a + Z_{bb} I_b \tag{9-41b}$$

where V_a, V_b, I_a, and I_b are the terminal voltages and currents of antennas a and b. If antenna a is excited with a generator of current I_a, the open circuit voltage appearing at the terminals of antenna b is $V_b|_{I_b=0}$. The transfer impedance Z_{ba} from (9-41b) with I_b zero is

$$Z_{ba} = \left.\frac{V_b}{I_a}\right|_{I_b=0} \tag{9-42}$$

If antenna b is excited with a generator of current I_b, the open circuit voltage appearing at the terminals of antenna a is $V_a|_{I_a=0}$. The transfer impedance Z_{ab} is, from (9-41a) with I_a zero,

$$Z_{ab} = \left.\frac{V_a}{I_b}\right|_{I_a=0} \tag{9-43}$$

Comparing (9-42) and (9-43) to (9-40), we see that

$$Z_{ab} = Z_{ba} = Z_m \tag{9-44}$$

where Z_m is the transfer (or mutual) impedance between the antennas. This can also be shown from the circuit formulation of (9-41) if the individual impedances are linear, passive, and bilateral. (See Probs. 9.4-3 and 9.4-4). This, in turn, is true if the medium and the antennas are linear, passive, and isotropic.

The significance of these results is now explained using the model of Fig. 9-5. If an ideal current source of current I excites antenna a, the open circuit voltage at the terminals of b from (9-42) is

$$V_b\big|_{I_b=0} = IZ_{ba} \tag{9-45}$$

If the same source is now applied to the terminals of antenna b, the open circuit voltage appearing at the terminals of antenna a from (9-43) is

$$V_a\big|_{I_a=0} = IZ_{ab} \tag{9-46}$$

But $Z_{ab} = Z_{ba}$, so the preceding two equations yield

$$V_a\big|_{I_a=0} = V_b\big|_{I_b=0} = V \tag{9-47}$$

Thus, the same excitation current will produce the same terminal voltage independent of which port is excited, as illustrated in Fig. 9-5. In other words, reciprocity states that the source and the measurer can be interchanged without changing the system response. The same is true of an ideal voltage source and short circuit terminal currents. These are familiar results from network theory.

The self-impedances of the antennas from (9-41) are

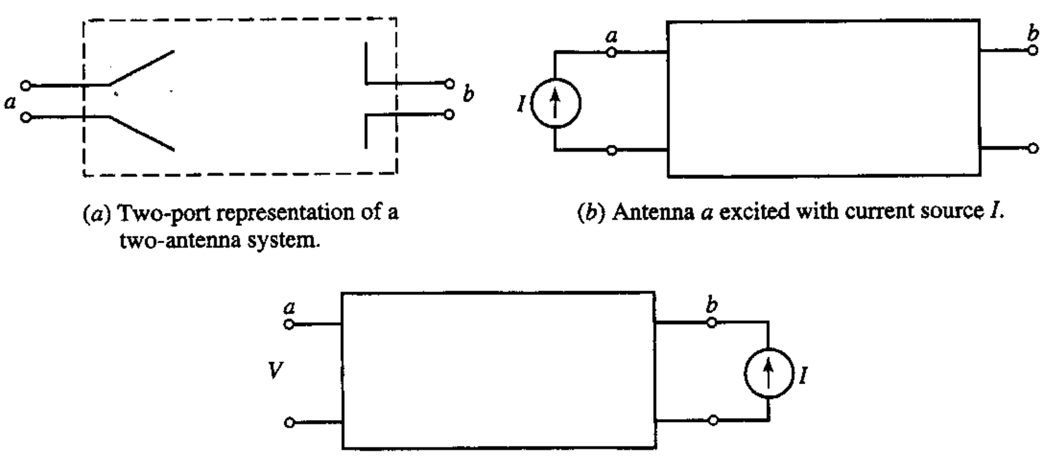
$$Z_{aa} = \frac{V_a}{I_a}\bigg|_{I_b=0} \tag{9-48}$$

$$Z_{bb} = \frac{V_b}{I_b}\bigg|_{I_a=0} \tag{9-49}$$

If antennas a and b are widely separated, which is the usual operating situation, Z_{aa} and Z_{bb} are much greater than $Z_{ab} = Z_{ba} = Z_m$. Thus, the input impedance to antenna a, for example, from (9-41a) is

$$Z_a = \frac{V_a}{I_a} = Z_{aa} + Z_{ab}\frac{I_b}{I_a} \approx Z_{aa} \tag{9-50}$$

Thus, if an antenna is isolated so that all objects including other antennas are far away and the antenna is lossless, the self-impedance equals its input impedance.



(*a*) Two-port representation of a two-antenna system.

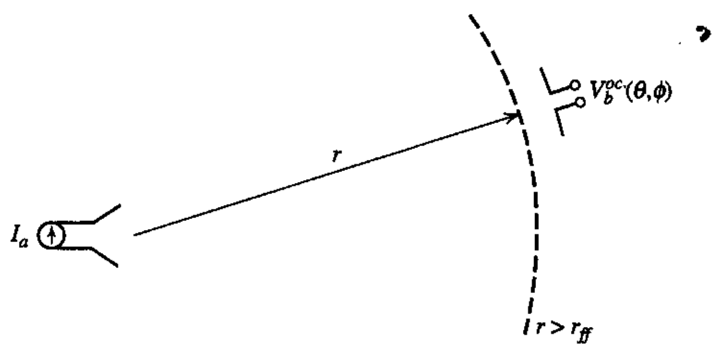
(*b*) Antenna *a* excited with current source *I*.

(*c*) Antenna *b* excited with current source *I*.

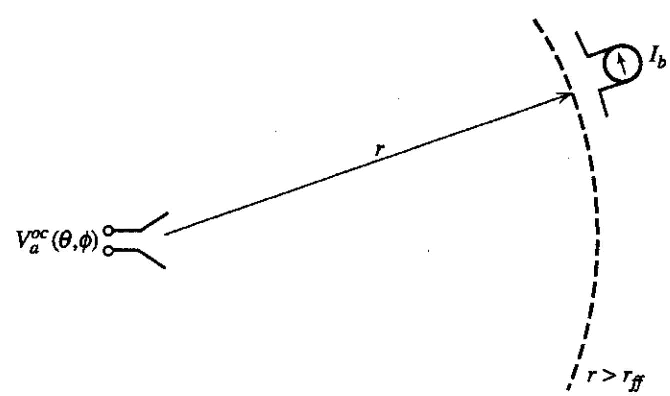
Figure 9-5 Reciprocity for antennas. The output voltage V is the same in (b) and (c) for the same input current I.

Suppose antenna a is excited (i.e., acting as a transmitter) and the voltage prc duced at the terminals of antenna b is measured with an ideal voltmeter. If th antennas are separated so that they are in each other's far field, the transfer im pedance Z_{ba} is actually the far-field (or radiation) pattern of antenna a if antenn b is moved around a on a constant radius as shown in Fig. 9-6a. As antenna b : moved, it is maintained with the same orientation and polarization relative to ar tenna a. The output voltage of b as a function of angle around antenna a gives th relative angular variation of the radiation from antenna a, that is, its radiation pa tern. Examining (9-42), we see that this is really Z_{ba} (I_a is constant). Thus, Z_{ba} as function of angle is the transmitting pattern of antenna a. If now antenna b is excite and antenna a acts as a receiver, the terminal voltage of antenna a is the receivin pattern of antenna a as antenna b is again moved around at a constant distanc from antenna a; see Fig. 9-6b. Thus, Z_{ab} as a function of angle is the receiving patter of antenna a. Since the transfer impedances are identical, we can conclude that *tl transmit and receive patterns of an antenna are identical.* This is an important coı sequence of reciprocity.

The equality of the transmit and receive patterns of an antenna is not an une: pected result. This can be seen through the relation $G(\theta, \phi) = 4\pi A_e(\theta, \phi)/\lambda^2$ ((9-2), which relates the receiving characteristic of the antenna $A_e(\theta, \phi)$ for an iı coming plane wave from angle (θ, ϕ) to the gain pattern value $G(\theta, \phi)$ in the d rection (θ, ϕ) when the antenna transmits. The reciprocal property is of major pra tical importance. It permits the test antenna to be used in either a receive or transm mode during pattern measurements. In practice, pattern measurements are usual made with the test antenna used in reception.



(a) The transmitting pattern of antenna a is $Z_{ba}(\theta,\phi) = V_b^{oc}(\theta,\phi)/I_a$.



(b) The receiving pattern of antenna a is $Z_{ab}(\theta,\phi) = V_a^{oc}(\theta,\phi)/I_b$.

Figure 9-6 Antenna pattern reciprocity. The transmitting an receiving patterns of an antenn: are identical because $Z_{ab}(\theta, \phi)$ $Z_{ba}(\theta, \phi) = Z_m(\theta, \phi)$.

It is important to note that reciprocity, as illustrated in Fig. 9-5 or through (9-44), is a general result. Also, in the case when the antennas are far removed from each other, $Z_m(\theta, \phi)$ is the far-field pattern. Of course, if an antenna contains any nonreciprocal components, reciprocity does not hold. An example is a ferrite isolator included in the antenna system.

9.5 PATTERN MEASUREMENT AND ANTENNA RANGES

An antenna pattern is a graphical representation of the field magnitude at a fixed distance from an antenna as a function of direction. With the antenna at the origin of a spherical coordinate system, radiation fields **E** and **H** are perpendicular to each other and both are transverse to the direction of propagation $\hat{\mathbf{r}}$. Also, the field intensities vary as r^{-1}. In antenna pattern discussions, electric field is used, but magnetic field behavior follows directly since its intensity is proportional to the electric field and its direction is perpendicular to **E** and $\hat{\mathbf{r}}$; see (1-107).

The radiated electric field is both a vector and a phasor. In general, it has two orthogonal components, E_θ and E_ϕ. These components are complex-valued and their relative magnitude and phase determine the polarization; see Sec. 1.10. For simple antennas, only one component is present. For example, the ideal dipole parallel to the z-axis has only an E_θ component as shown in Fig. 1-10. Measurement of the radiation pattern in this case is conceptualized by moving a receiving probe around the antenna as it transmits a constant signal a fixed distance away, r. The probe's orientation is maintained parallel to E_θ as shown in Fig. 9-7. The output of the probe varies in direct proportion to the intensity of the received field component arriving from direction (θ, ϕ). The pattern of the ideal dipole is $\sin\theta$; see Fig. 1-10. In general, antennas will have both E_θ and E_ϕ components and patterns are cut twice, once with the probe oriented parallel to E_θ and once with it parallel to E_ϕ.

Although we have conceptualized the measurement of a radiation pattern by moving a receiver over a sphere of constant radius, this is obviously an impractical way of making such measurements. The important feature is to maintain a constant large distance between the antennas and to vary the observation angle. This is accomplished by rotating the *test antenna*, or *antenna under test* (AUT), as illustrated in Fig. 9-8. By reciprocity, it makes no difference if the test antenna is operated as a receiver or transmitter, but usually the test antenna is used as a receiving antenna and we adopt this convention. The fields from the motionless source antenna provide a constant illumination of the test antenna whose output varies with its angular position. This leads to the rule that *it is the pattern of the rotated antenna that is being measured.*

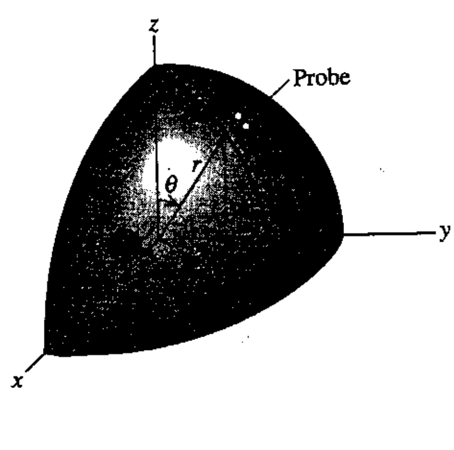


Figure 9-7 Pattern measurement conceptualized by movement of a probe antenna over the surface of a sphere in the far field of the antenna.

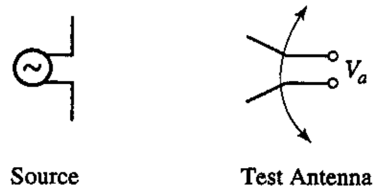
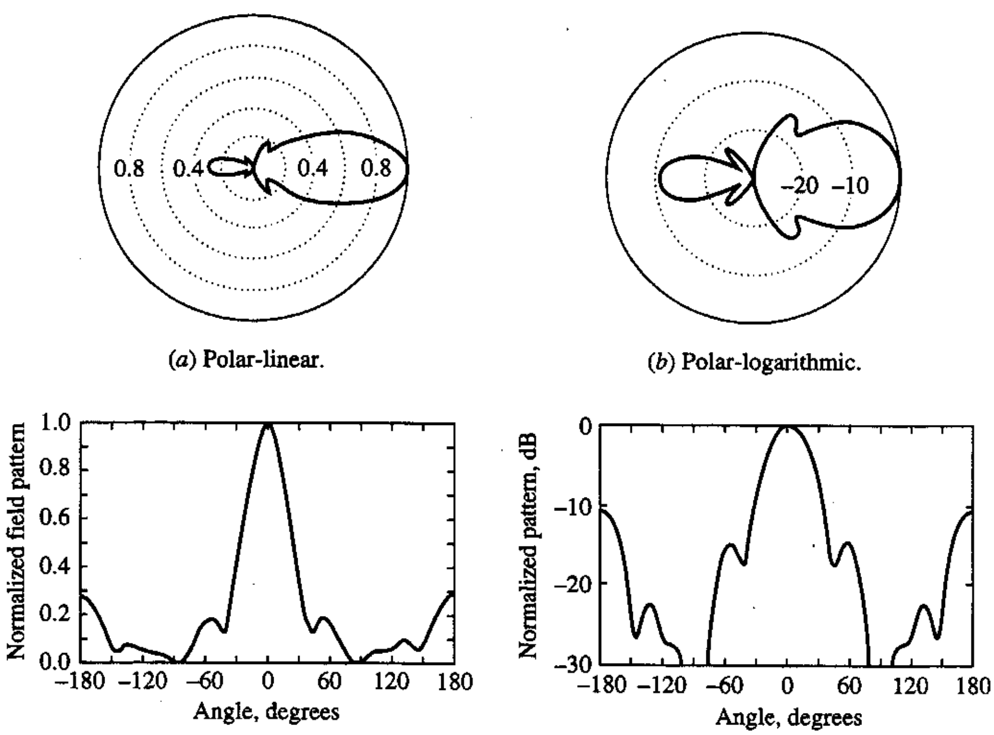


Figure 9-8 Radiation pattern measurement. The patterı of antenna a is proportional to the terminal voltage V_a, which is a function of the angular position of antenna a during rotation.

A complete representation of the radiation properties of an antenna would, course, require measuring the radiation at all possible angles (θ, ϕ). This is rar attempted and fortunately is not necessary. For most applications, the princi plane patterns are sufficient. See Fig. 1-10 for an illustration of the principal pla patterns using an ideal dipole.

There are many ways of displaying antenna patterns. For example, a princi plane pattern could be plotted in polar or rectangular form. In addition, the sc could be either linear or logarithmic (decibel). All combinations of plot type a scale type are used: polar-linear, polar-log, rectangular-linear, and rectangular-l Figure 9-9 shows the same radiation pattern plotted in these four ways. Genera speaking, log plots are used for high-gain, low side-lobe patterns and linear pl are used when the main beam details are of primary interest. These antenna patte representations can be recorded directly using commercially available measuri and recording equipment. When more detailed information is required, the resu of several planar cuts can be put together to make a contour plot. It is import to appreciate that measured patterns are usually not perfectly symmetric ev though the antenna structure appears to be symmetric and also nulls are oft partially filled.



(*a*) Polar-linear. (*b*) Polar-logarithmic.

(*c*) Rectangular-linear. (*d*) Rectangular-logarithmic.

Figure 9-9 Illustration of the four antenna pattern plot types using the same pattern.

A facility used to measure antenna radiation characteristics is referred to as an *antenna range*. Often, the same range can also be used to measure scattering characteristics such as radar cross section. The entire measurement facility consists of the space (indoor or outdoor) for the source and test antennas, antenna positioners, a transmitter, a receiving system, and data display/recording equipment. In this section, we discuss the basic range layouts; see [3] for a complete discussion of antenna measurement techniques.

Table 9-1 lists the types of antenna ranges together with their characteristics and advantages and disadvantages. Most ranges are *free-space ranges* that are designed

Table 9-1 Characteristics of Antenna Ranges

Range Type	Description	Advantages	Disadvantages
FREE SPACE RANGES	Effects of all surroundings are suppressed to acceptable levels.		
Far-field Ranges			
Elevated range	Source and test antenna are placed on towers, buildings, hills, etc.	Low cost.	Requires real estate. May require towers. Outdoor weather.
Slant range	Either the source or test antenna is elevated.	Low cost.	Requires real estate. May require a tower. Outdoor weather.
Anechoic chamber	A room is lined with absorber material to suppress reflections.	Indoors.	Absorber and large room are costly.
Compact Range	The test antenna is illuminated by the collimated near field of a large reflector.	Small space.	A large reflector is required.
Near-field Range	The magnitude and phase of the near field of the test antenna are sampled and the far field is computed.	Very small space.	Accurate probe positioning is required. Accurate amplitude and phase are required. Time-consuming measurements. Computer-intensive.
GROUND REFLECTION RANGE	The ground between the source and test antennas is reflective, enhancing the indirect ray that interferes with the direct ray, giving a smooth test antenna illumination.	Test tower is short. Operates well at low frequencies (VHF).	Outdoor weather.

to have strong direct illumination of the test antenna with weak indirect illumination. First, we consider *far-field ranges* in which the source antenna is far from the test antenna. This can be accomplished by elevating either both or one of the source and test antennas giving an *elevated range* or *slant range*. For all antenna ranges, the site around the test antenna affects pattern measurement accuracy. The guiding principle is to have the line of sight (direct) path between the source and test antenna unblocked and as high above the ground (or floor) as practical. This yields large values for the angles α_t and α_r shown in Fig. 9-10. Then directive antennas will have indirect rays arising from specular reflection from the ground of reduced level because angles α_t and α_r usually correspond to side-lobe directions. In the elevated range of Fig. 9-10, the source and test antennas are approximately the same height, $h_t \approx h_r$. The slant range is similar to the elevated range except that only the source is elevated, leaving the test antenna conveniently located near the ground. When indoor rooms are used for a far-field range, the walls must be lined with absorbing material to reduce reflections. Frequently, the absorber is pyramidal-shaped to eliminate flat surfaces that reflect rays toward the test antenna.

In far-field ranges, the test antenna is located in the far field of the source antenna so that the incoming waves are nearly planar as indicated in Fig. 9-10. In fact, a common goal of all antenna ranges is to provide plane wave illumination of the test antenna. Deviations from uniform field illumination amplitude (i.e., magnitude) and phase across the test antenna aperture add to the inherent aperture taper of the test antenna, causing pattern measurement errors. In far-field ranges, the illumination field amplitude variation is determined by the radiation pattern of the source antenna. The effect of increased amplitude taper imposed on the test antenna aperture is to reduce the measured gain and change the side lobes close to the main beam. If the source antenna pattern peak is centered on the test antenna, as it should be in all cases, and the amplitude taper created by the source antenna pattern is -0.25 dB at the edges of the test antenna aperture, there will be a directivity (and thus, gain) reduction of 0.1 dB [4]. That is, the pattern point at angle $\alpha/2$ is 0.25 dB down from the peak; see Fig. 9-10. This is difficult to achieve for the wide variety of measurement situations on an antenna range, but in all cases the source antenna should be directed toward the test antenna and have a beamwidth that is as small

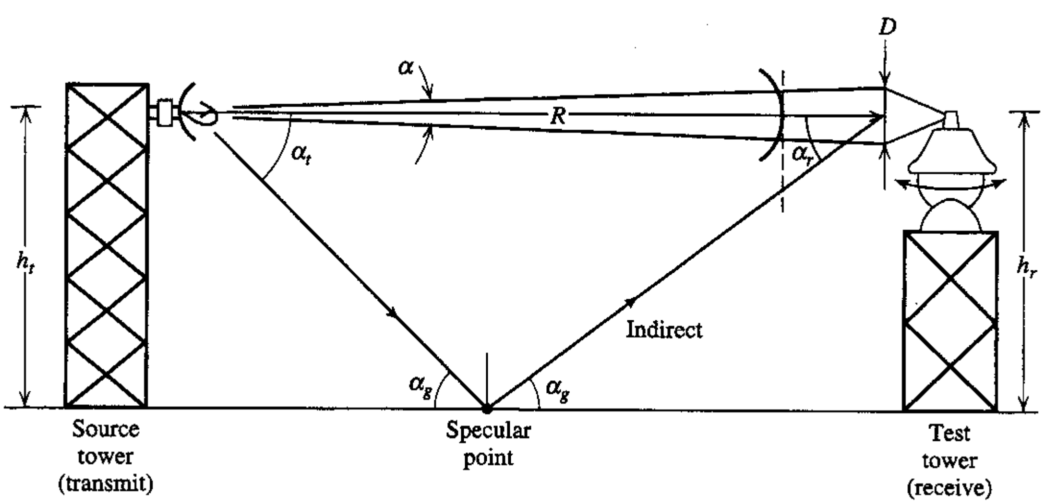


Figure 9-10 The elevated antenna range.

as possible to reduce illumination of the surroundings and to increase the received signal for adequate dynamic range. At the same time, the source antenna should not have a beamwidth that is narrow enough to impose an amplitude taper across the test antenna.

The instrumentation used with an antenna range varies from a simple signal source together with a relative power indicating subsystem to complete commercial systems with automatic data collection and display features. The signal source should be stable in power level and frequency. The receiving system should have a linear dynamic range of at least 40 dB. Both amplitude-only and amplitude-phase receiving systems are available. Also, a network analyzer can be used.

Phase errors are due to the fact that to achieve a planar phase front from a finite-sized source antenna, the source must be an infinite distance away from the test antenna. The spherical waves from the source antenna cause a phase error across the test antenna extent of D that behave exactly as the far-field distance phase error discussed in Sec. 1.7.3. There we found that spherical waves deviate from parallel rays with a 22.5° phase error ($\lambda/16$ distance error) at a distance of $2D^2/\lambda$. Thus, we can say that a phase error of 22.5° is created by the phase front curvature over a test antenna of extent D at a separation distance of

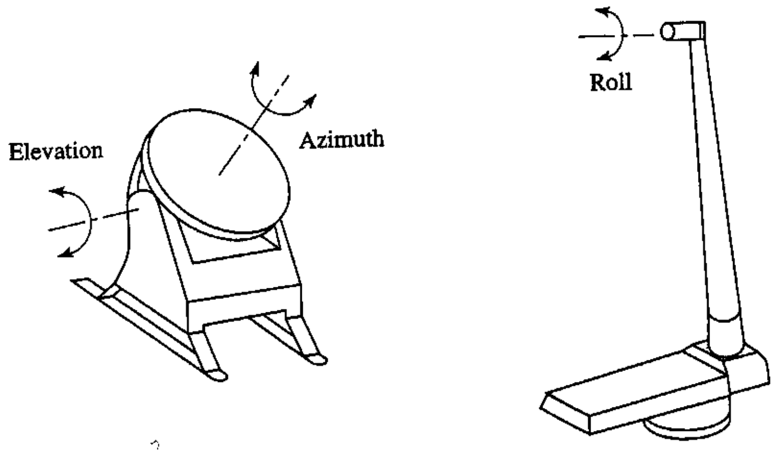
$$r_{ff} = \frac{2D^2}{\lambda} \tag{9-51}$$

The extent of the source antenna need not be included for pattern measurements, but the measurement distance should be more than doubled when the source and test antennas are of the same size to preserve gain accuracy [5].

The measurement distance of (9-51) is adequate for moderate-to-high-gain antennas if high accuracy is not required in the side-lobe levels. In general, the effects of reducing the measurement distance from infinity is to fill in the nulls between side lobes, increase the peak of the side lobes (mainly near the main beam), broaden the main beam, and reduce the main beam peak (implying a directivity reduction) [6]. For example, an antenna with the first side lobe 30 dB below the main beam peak (SLL = −30 dB) when measured at an infinite distance from the source has an error of 3 dB for a measurement distance of $2D^2/\lambda$; that is, the first side lobe is SLL = −27 dB [5, 7]. Gain is reduced about 0.1 dB at the measurement distance of $2D^2/\lambda$ for typical high-gain antennas. In the case of broad main beam antennas, the measurement distance should also be at least that of (9-51) to ensure the accurate measurement of pattern ripple [8].

Electrically large antennas require very large measurement distances. For example, the Deep Space Network 70-m reflector antenna at Goldstone, CA, operating at 2.3 GHz requires a measurement distance from (9-51) of 75 km! Conventional techniques cannot be used to measure such an antenna. However, a source flown in an airplane or available from a satellite can be used. Or, noise from a strong "radio star" can be used as a source together with a radiometer receiver.

The concept for the *compact range* is to place the test antenna close to a reflector antenna as shown in Fig. 9-11. This is possible because the near field of a reflector is collimated, giving a nearly flat phase front and an amplitude taper equal to that across the reflector aperture. Therefore, the phase error problem associated with far-field ranges is traded for an amplitude problem in a compact range. Improved



(*a*) Azimuth-over-elevation. (*b*) Model tower.

Figure 9-13 Antenna positioners for antenna testing.

at any height. Frequently, a *model tower* is used to position the test antenna over the axis of the rotation positioner; see Fig. 9-13*b*. Included with the model tower is a *roll positioner* that rotates the test antenna about its own axis for controlling the pattern cut. The source tower also often has a roll positioner for proper orientation of the source antenna polarization.

9.6 GAIN MEASUREMENT

Pattern measurement discussed in the previous section is a relative measurement that gives the angular variation of the test antenna's radiation. Gain is also needed to fully characterize the radiation properties of a test antenna. It is an absolute quantity and thus is more difficult to measure. Techniques exist to measure the gain of a test antenna with no *a priori* knowledge. However, most gain measurements are made using an antenna of known gain, called a *standard gain antenna* [4, Chap. 12]. The technique is called the *gain comparison* (or *gain transfer*) *method.* A transmitter of fixed input power P_t is connected to a suitable source antenna whose pattern peak is centered on the test antenna. Received power is measured for both the test antenna P_T and the standard gain antenna P_S, as illustrated in Fig. 9-14 by placing each antenna on the test positioner, pointing toward the source for peak output, and recording the received power levels. The gain of the test antenna is

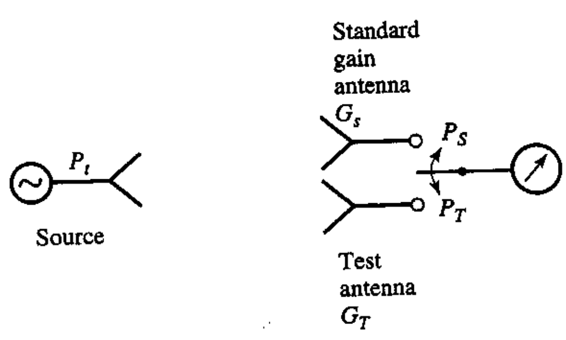


Figure 9-14 Measurement of the gain of a test antenna G_T using the gain comparison method based on the known gain of a standard gain antenna G_S and $G_T = (P_T/P_S)G_S$. From [10] © 1993. Reprinted by permission of Artech House, Inc., Boston, MA.

then easily computed from the gain of the standard gain antenna multiplied by the ratio of the received powers:

$$G_T = \frac{P_T}{P_S} G_S \tag{9-52}$$

This relation is more convenient when expressed in decibels:

$$G_T\,(\text{dB}) = P_T\,(\text{dBm}) - P_S\,(\text{dBm}) + G_S\,(\text{dB}) \tag{9-53}$$

This result is intuitive and simply says that the gain of the test antenna differs from that of the standard antenna by the difference in received powers from the test antenna and the standard antenna. A special case is when the received powers are equal ($P_T = P_S$); then the gain of the test antenna is identical to that of the standard gain antenna. This result is also easily derived by evaluating (2-99) for both the test and standard antenna cases and subtracting; the terms involving distance R, frequency f, and transmit power P_t are constants and drop out, leaving (9-53).

It is obvious from (9-53) that accurate gain measurement requires accurate power measurement. With modern receivers, this is often possible. An approach that does not rely on receiver linearity is the RF substitution method in which a precision attenuator is used to establish the power level change. That is, the attenuator is adjusted to bring the receiver to the same reading in both cases; then the difference in the corresponding attenuator settings equals P_T (dBm) $-$ P_S (dBm). Accuracy also depends directly on a knowledge of the gain of the standard gain antenna. Popular standard gain antennas are the half-wave dipole for UHF frequencies and below and the pyramidal horn for UHF frequencies and above. The gain of the dipole is 2.15 dB (see Fig. 2-6) and manufacturers of standard gain horns supply data on gain across the operating frequency range like Fig. 7-20.

Note that the term *gain* is synonymous with *absolute gain* or *peak gain*. Gain and pattern data can be merged into a *gain pattern* by multiplying gain by the normalized pattern:

$$G(\theta, \phi) = GP(\theta, \phi) = G|F(\theta, \phi)|^2 \tag{9-54}$$

Expressed in decibels (by taking 10 log), the unit of dBi is often used, indicating that the pattern has been referenced to an isotropic antenna.

Gain Measurement by Gain Comparison

Suppose that a standard gain antenna has a gain of 63, or 18, dB. Following the measurement technique illustrated in Fig. 9-14, the measured powers are P_S = 3.16 mW or 5 dBm (5 dB above a milliwatt), and P_T = 31.6 mW, or 15 dBm. The gain of the test antenna is then $G_T = (31.6/3.16)63 = 630$, or in terms of decibels,

$$G_T\,(\text{dB}) = P_T\,(\text{dBm}) - P_S\,(\text{dBm}) + G_S\,(\text{dB}) = 15 - 5 + 18 = 28\text{ dB} \tag{9-55}$$

9.6.1 Gain Measurement of CP Antennas

If good-quality circularly polarized (CP) source and standard gain antennas are available, the gain comparison method of Fig. 9-14 can be used. Frequently, though, the gain of elliptically polarized antennas is measured by using two orthogonal linearly polarized (LP) antennas, or customarily one LP antenna used in two orthogonal orientations. Suppose the gains are measured for vertical and horizontal LP cases. These partial gains, G_{Tv} and G_{Th}, are combined to give the total gain [10, 11]:

$$G_T\text{ (dB)} = 10 \log(G_{Tv} + G_{Th}) \quad \text{[dBic]} \tag{9-56}$$

This is referred to as the *partial gain method.* Any perpendicular orientations can be used because the power in an elliptically polarized wave is contained in the sum of any two orthogonal components. As a side note, we observe that a CP antenna performs this sum instantaneously. Therefore, the gain in (9-56) is relative to an ideal CP antenna. The unit dBic indicates gain relative to an isotropic, perfect CP antenna. Gain measurement accuracy depends on the purity of the source antenna. An LP standard gain antenna usually has an axial ratio of 40 dB or better and does not contribute significantly to gain error.

Calculation of Gain Using the Partial Gain Method

Figure 9-15 gives two patterns measured with an LP source antenna and a nominally CP test antenna, which is a cavity-backed spiral antenna operating at 1054 MHz. Also shown is the pattern of a standard gain horn, which has a gain at 1054 MHz of 14.15 dB based on the manufacturer's gain curve. The receiver gain setting and the source power were constant during these measurements. The peak gains for vertical and horizontal polarizations then are

$$G_{Tv}\text{ (dB)} = 14.15 - 16.1 = -1.95\text{ dB}, \qquad G_{Th}\text{ (dB)} = 14.15 - 13.25 = 0.9\text{ dB} \tag{9-57}$$

because the vertical and horizontal LP pattern peaks are 13.25 and 16.1 dB below the standard gain horn pattern peak, respectively. Then

$$G_{Tv} = 10^{-1.95/10} = 0.64, \qquad G_{Th} = 10^{0.9/10} = 1.23 \tag{9-58}$$

and (9-56) gives

$$G_T\text{ (dB)} = 10 \log(0.64 + 1.23) = 2.71\text{ dBic} \tag{9-59}$$

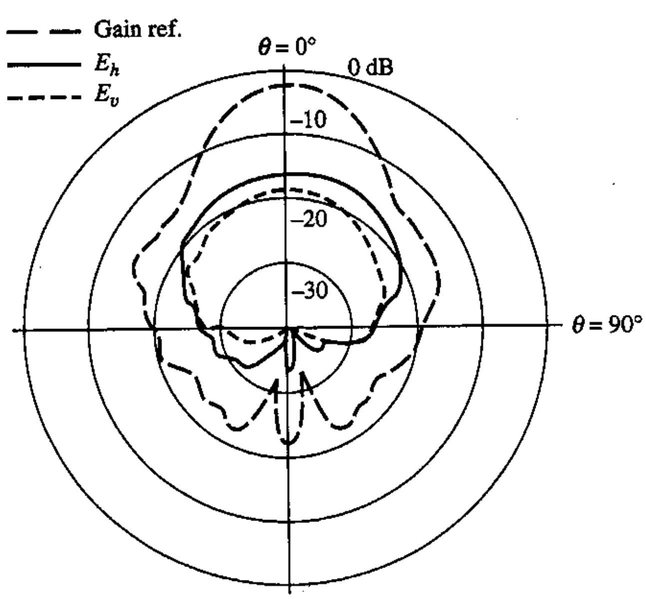


Figure 9-15 Illustration of measurement of the gain of a CP antenna using the partial gain method; see Example 9-7. The patterns are for an LP standard gain horn (long dashed curve) and the nominally CP antenna with the source vertically (solid curve) and horizontally (short dashed curve) polarized. From [10] © 1993. Reprinted by permission of Artech House, Inc., Boston, MA.

9.6.2 Gain Estimation

Frequently, gain can be estimated based on a knowledge of the pattern and the antenna operation. Since directivity is a pattern quantity, we can use pattern data to estimate directivity. That is, directivity from (1-146b) varies inversely with beam solid angle, $D = 4\pi/\Omega_A$. It is possible to record many pattern cuts and numerically integrate the pattern using (1-143) to find Ω_A. A simpler approach is to use the principal plane pattern half-power beamwidths to estimate beam solid angle. To do this, we first find *beam efficiency* from main beam solid angle Ω_M using (7-85):

$$\varepsilon_M = \frac{\Omega_M}{\Omega_A} \tag{9-60}$$

The main beam solid angle is well approximated as the product of the half-power beamwidths in the principal planes:

$$\Omega_M \approx \mathrm{HP}_E \mathrm{HP}_H \tag{9-61}$$

Then directivity can be estimated by combining these results:

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi\varepsilon_M}{\Omega_M} \approx \frac{4\pi\varepsilon_M}{\mathrm{HP}_E\mathrm{HP}_H} = \frac{41{,}253\varepsilon_M}{\mathrm{HP}_{E^\circ}\mathrm{HP}_{H^\circ}} \tag{9-62}$$

where HP_{E° and HP_{H° are the half-power beamwidths in the E- and H-planes expressed in degrees. Often, it is assumed that all power is in the main beam, giving $\varepsilon_M = 1$; see (7-94). Antennas, in practice, have a nonnegligible amount of power in the side lobes; a typical value for ε_M is 0.63. If no loss is present, $e_r = 1$ and the gain for antennas encountered in practice from (9-62) from (7-95) is

$$G = e_r D \approx D \approx \frac{26{,}000}{\mathrm{HP}_{E^\circ}\mathrm{HP}_{H^\circ}} \tag{9-63}$$

See Sec. 7.3 for more discussion on gain. It must be emphasized that this very approximate formula should be used for rough estimates when the only data available are the half-power beamwidths.

9.7 POLARIZATION MEASUREMENT

Quite often, the polarization of an antenna can be inferred from the geometry of the active portion of the antenna. For example, the ideal dipole in Fig. 1-10 is vertical linearly polarized since the radiating element is oriented vertically. For gain and co-polarized pattern measurements, the test antenna should be illuminated with a wave of the expected polarization of the antenna: in this case, a vertical linearly polarized wave. Real antennas always have a certain amount of power in the polarization orthogonal to the intended polarization. For the practical realization of the ideal dipole, there will be a small amount of horizontal linear polarization. Such cross-polarization arises from horizontal currents flowing on the antenna or nearby structures. Thus, a complete antenna measurement set includes characterization of the polarization properties of the test antenna. This is often accomplished by making pattern cuts in the E- and H-planes of the test antenna with it both co-polarized and cross-polarized to the source antenna. This is illustrated in Fig. 9-16 for the case of a nominally LP test antenna and an LP source antenna. Of course, the cross-polarized patterns will be much lower in level than the co-polarized patterns, and

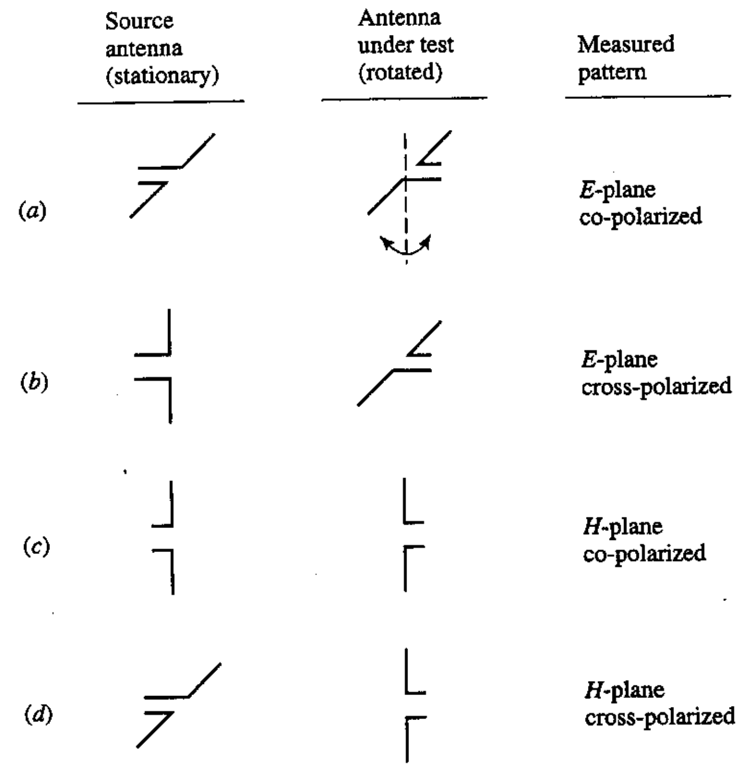


Figure 9-16 Illustration of copolarized and cross-polarized pattern measurement. The source antenna is LP and the test antenna operating in the receiving mode is nominally LP and is rotated about its axis. From [10] © 1993. Reprinted by permission of Artech House, Inc., Boston, MA.

will be zero for a perfect LP test antenna. Co- and cross-polarized patterns for reflector antennas are discussed in Sec. 7.6.5.

There are three measurement techniques used to characterize an antenna that is elliptically polarized but has an axial ratio that is not large (i.e., the polarization state is not close to pure LP). These methods are discussed in the remainder of this section [10].

9.7.1 Polarization Pattern Method

A *polarization pattern* is the amplitude response of an antenna as it is rotated about its roll axis. It can be measured at any fixed pattern rotation angle. The resulting pattern shown in Fig. 9-17 is a polar plot of the response of the test antenna as a function of the relative angle α between the illuminating LP wave orientation and a reference orientation of the antenna. Either the LP source antenna is rolled while the test antenna is stationary or vice versa. It is easier to explain the polarization pattern method with the test antenna operated as an elliptically polarized transmitting antenna and the receiving antenna as a linearly polarized probe. Reciprocity permits us to do this. The tip of the instantaneous electric field vector from the test antenna lies on the polarization ellipse and rotates at the frequency of the wave; that is, the electric vector completes f rotations around the ellipse per second. The output voltage of the LP probe is proportional to the peak projection of the electric field onto the LP orientation line at angle α. This is the distance OP in Fig. 9-17 projected from the tangent point T on the ellipse. The locus of points P as the LP

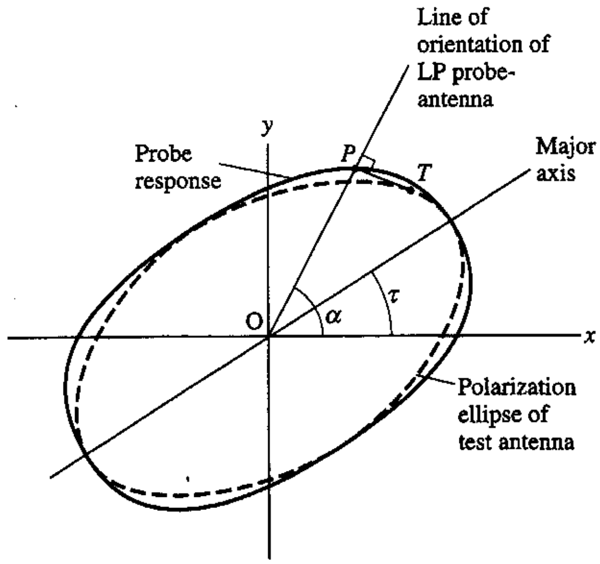


Figure 9-17 Polarization pattern (solid curve) of an elliptically polarized test antenna. It is the response of an LP receiving probe with orientation angle α to a transmitting test antenna with the polarization ellipse shown (dashed curve). From [10] © 1993. Reprinted by permission of Artech House, Inc., Boston, MA.

probe is rotated is fatter than the ellipse, which is also shown in Fig. 1-24. Of course, for a CP test antenna both curves in Fig. 9-17 are circular.

Note that the maximum and minimum of the polarization pattern are identical to the corresponding maximum and minimum of the polarization ellipse when scaled to the same size. Although the measured polarization pattern does not give the polarization ellipse, it does produce the axial ratio magnitude of the antenna polarization. It is also obvious from Fig. 9-17 that the tilt angle of the ellipse is determined as well. *The polarization pattern gives the axial ratio magnitude $|AR|$ and tilt angle τ of the polarization ellipse, but not the sense.* The sense can be determined by additional measurements. For example, two nominally CP antennas that are identical except for sense can be used as receiving antennas with the test antenna transmitting. The sense of the antenna with the greatest output is then the sense of the test antenna.

The polarization pattern method in many cases is a practical way to measure antenna polarization. If the test antenna is nearly circularly polarized, the axial ratio is near unity and measured results are insensitive to the purity of the LP probe. If the test antenna is exactly circular, tilt angle is irrelevant. In the case of a test antenna that is nearly linearly polarized, axial ratio measurement accuracy depends on the quality of the LP probe, which must have an axial ratio much greater than that of the test antenna.

9.7.2 Spinning Linear Method

The *spinning linear* (or *rotating source*) *method* provides a rapid measurement technique for determining the axial ratio magnitude as a function of pattern angle. The test antenna is rotated as in a conventional pattern measurement while an LP probe antenna (usually transmitting) is spun. The spin rate of the LP antenna should be such that the test antenna pattern does not change appreciably during one-half revolution of the LP antenna while the test antenna rotates slowly. An example pattern is shown in Fig. 9-18, which is a pattern of a helix antenna. Superimposed on the antenna pattern are rapid variations representing twice the rotation rate of the probe antenna. For logarithmic (dB) patterns as in Fig. 9-18, the axial ratio is

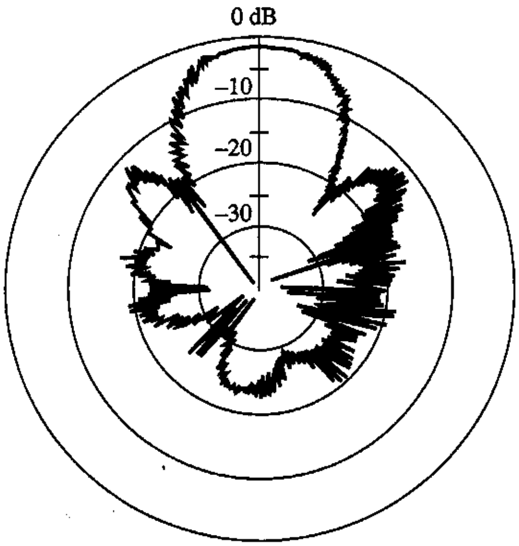


Figure 9-18 Axial ratio measurement as a function of pattern angle using the spinning linear method. The axial ratio is the difference in decibels between adjacent peaks and nulls. The test antenna is a helix antenna operating at X-band and the source is a rotating LP antenna. From [10] © 1993. Reprinted by permission of Artech House, Inc., Boston, MA.

the difference between adjacent maxima and minima at each angle. For example, at a pattern angle of 30° counterclockwise from the main beam axis, the maximum and minimum pattern envelopes are about −8 and −10 dB, corresponding to a 2-dB axial ratio.

Sense cannot be obtained using the spinning linear method. Tilt angle could, in theory, be obtained if probe orientation information were known accurately at the pattern points, but this is usually not done in practice.

9.7.3 Dual-Linear Pattern Method

A method related to the spinning linear method is the *dual-linear pattern method*. In this method, two patterns are measured for orthogonal orientations of the LP probe source antenna so that they align with the major and minor axes of the test antenna polarization ellipse. Figure 9-19 illustrates the resulting patterns for the same sample antenna as in Fig. 9-18 for the spinning linear method. For the same example pattern point at 30° counterclockwise from the beam peak, the two linear

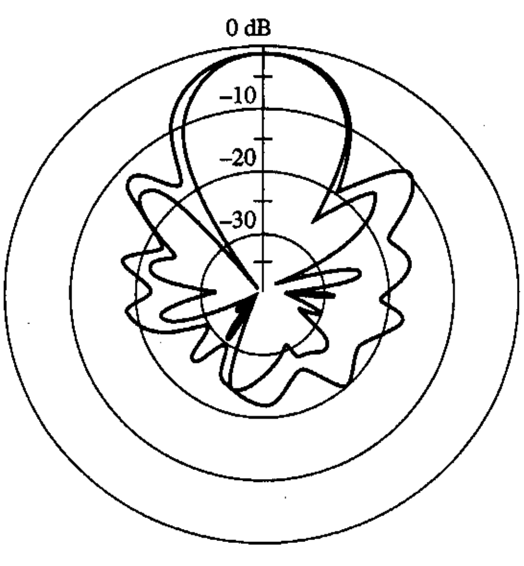


Figure 9-19 Axial ratio measurement as a function of pattern angle using the dual-linear pattern method. The axial ratio is the decibel difference between the two patterns that represent planes containing the major and minor axes of the test antenna polarization ellipse. The test antenna is identical to that in Fig. 9-18. From [10] © 1993. Reprinted by permission of Artech House, Inc., Boston, MA.

pattern values are −8 and −10 dB, again giving a 2-dB axial ratio value. Of course, the gains and other equipment settings must remain constant during the entire measurement period.

9.8 FIELD INTENSITY MEASUREMENTS

A very small receiving antenna can be used as a *field probe*. Probes are used when it is necessary to measure the spatial amplitude distribution of electromagnetic fields. The probe must be small relative to the structure whose fields are being measured in order to minimize the disturbances introduced by the probe itself. The electrically small dipole, in any of its practical forms discussed in Sec. 2.1, is used to probe electric fields. The small loop is used as a magnetic field probe.

Receiving antennas are also used to measure absolute field intensity. For example, it is often necessary to know the field intensity at a fixed distance from a transmit antenna. The antenna pattern can, of course, also be measured by moving a receiving probe around the transmitter at a fixed distance from it in the far field; this gives the relative field intensity variation. Such measurements are often required because the effects of terrain and the real earth surface are difficult to include in calculations. If the gain of the measuring antenna is known (it usually is) and the voltage developed across its terminals is measured, the field intensity incident upon the measuring antenna can be calculated. We now discuss this.

The same model as in Fig. 2-18*b* is used to derive field intensity. The power delivered to the terminating load is

$$P_D = \frac{1}{2}\frac{|V_A|^2}{R_L} = \frac{V_{A,\mathrm{rms}}^2}{R_L} \tag{9-64}$$

where $V_{A,\mathrm{rms}} = |V_A|/\sqrt{2}$ since V_A is a peak quantity. The field form of the delivered power expression from (9-3) and (2-88) is

$$P_D = pqSA_e = pq\frac{(E_{\mathrm{rms}}^i)^2}{\eta}A_e \tag{9-65}$$

Equating these two relations yields

$$(E_{\mathrm{rms}}^i)^2 = \eta\frac{V_{A,\mathrm{rms}}^2}{pqR_L}\frac{1}{A_e} = \eta\frac{V_{A,\mathrm{rms}}^2}{pqR_L}\frac{4\pi}{G\lambda^2} \tag{9-66}$$

where (2-89) was used for A_e. Converting wavelength to frequency using $\lambda = c/f$ and expressing the relation in decibels by taking 10 log of both sides gives

$$\begin{aligned} E_{\mathrm{rms}}^i\ (\mathrm{dB}\mu\mathrm{V/m}) = {} & V_{A,\mathrm{rms}}\ (\mathrm{dB}\mu\mathrm{V}) + 20\log f\ (\mathrm{MHz}) - G\ (\mathrm{dB}) \\ & - 10\log R_L - 10\log p - 10\log q - 12.8 \end{aligned} \tag{9-67}$$

This expression permits easy calculation of electric field intensity E_{rms}^i in decibels relative to 1 μV/m, using the voltage $V_{A,\mathrm{rms}}$ in decibels relative to 1 μV, measured at the terminals of a probe antenna with gain G. Gain loss due to mispointing can also be included. For example, suppose the probe antenna has 6-dB gain and is pointed so that the incoming wave arrives from a direction on the receiving antenna pattern that is 2 dB below its maximum. Then 4-dB gain is used in (9-67) rather than the peak gain of 6 dB.

Sensitivity of an FM Receiver

As an example, suppose the antenna and transmission line input impedances are both 300 Ω. Then (9-67) becomes [11]

$$E^i_{\mathrm{rms}}\ (\mathrm{dB}\mu\mathrm{V/m}) = 20 \log f\ (\mathrm{MHz}) - G\ (\mathrm{dB}) + V_{A,\mathrm{rms}}\ (\mathrm{dB}\mu\mathrm{V}) - 37.57 \quad (9\text{-}68)$$

To be specific, consider a typical FM broadcast receiver with a sensitivity of 1 μV; that is, minimum satisfactory performance is produced when the value of $V_{A,\mathrm{rms}}$ is 1 μV, or 0 dBμV. The most popular antenna for FM receivers is the half-wave folded dipole (see Sec. 5.2) that has a real impedance of about 300 Ω and a gain of 2.15 dB. At a frequency of 100 MHz, the incident field intensity required for minimum satisfactory performance from (9-68) is 0.28 dBμV/m, or 1.03 μV/m.

At frequencies below 1 GHz, antenna measurements are made by illuminating the test antenna with a known field intensity and measuring the terminal voltage. Antenna factor is used to quantify this measurement. *Antenna factor K* is defined as the ratio of the field intensity illuminating the antenna to the received voltage across the antenna terminals:

$$K = \frac{E^i}{V_A} \quad (\mathrm{m}^{-1}) \qquad (9\text{-}69)$$

This is an electric field antenna factor; a corresponding one involving magnetic field intensity is also in use. Antenna factor is often used to determine receiver sensitivity. Then, (9-69) in decibel form using (9-67) becomes

$$E^i_{\mathrm{rms}}\ (\mathrm{dB}\mu\mathrm{V/m}) = \text{receiver sensitivity} = V_{A,\mathrm{rms}}\ (\mathrm{dB}\mu\mathrm{V}) + K\ (\mathrm{dB/m}) \quad (9\text{-}70\mathrm{a})$$

where

$$\begin{aligned} K\ (\mathrm{dB/m}) = {} & 20 \log[f\ (\mathrm{MHz})] - G\ (\mathrm{dB}) - 10 \log R_L \\ & - 10 \log p - 10 \log q - 12.8 \end{aligned} \qquad (9\text{-}70\mathrm{b})$$

It is common to specialize this definition to $R_L = 50\ \Omega$, since that is the normal receiver input impedance. Antenna factor includes impedance mismatch effects and antenna gain. The polarizations of the wave and antenna are usually assumed to be matched (i.e., $q = 0$), which is the customary measurement situation.

Sensitivity of an FM Receiver

We repeat Example 9-9 using antenna factor. Substituting $R_A = Z_o = 300\ \Omega$, $G = 1.64$ and $\lambda = 3$ m in (9-70b) gives

$$K = 20 \log(100) - 2.15 - 10 \log(300) - 0 - 0 - 12.8 = 0.28\ \mathrm{dB/m} = 1.03\ \mathrm{m}^{-1} \quad (9\text{-}71)$$

Then for a 1-μV sensitivity, (9-70a) gives

$$E^i_{\mathrm{rms}} = 0\ \mathrm{dB}\mu\mathrm{V} + 0.28\ \mathrm{dB/m} = 0.28\ \mathrm{dB}\mu\mathrm{V/m} \qquad (9\text{-}72)$$

which is the result we obtained in Example 9-9.

REFERENCES

1. W. L. Flock and E. K. Smith, "Natural Radio Noise—A Mini-review," *IEEE Trans. Ant. & Prop.*, Vol. AP-32, pp. 762–767, July 1984.

2. R. S. Bokulic, "Use Basic Concepts to Determine Antenna Noise Temperature," *Microwaves & RF*, Vol. 30, pp. 107–115, March 1991.
3. G. E. Evans, *Antenna Measurement Techniques*, Artech House, Boston, MA, 1990.
4. *IEEE Test Procedures for Antennas*, IEEE Standard 149-1979, Piscataway, NJ, 1979, p. 19.
5. T. Uno and S. Adachi, "Range Distance Requirements for Large Antenna Measurements," *IEEE Trans. Ant. & Prop.*, Vol. 37, pp. 707–720, June 1989.
6. P. S. Hacker and H. E. Schrank, "Range Distance Requirements for Measuring Low and Ultra Low Sidelobe Antenna Patterns," *IEEE Trans. Ant. & Prop.*, Vol. AP-30, pp. 956–966, Sept. 1982.
7. R. C. Hansen, "Measurement Distance Effects on Low Sidelobe Patterns," *IEEE Trans. Ant. & Prop.*, Vol. AP-32, pp. 591–594, June 1994.
8. D. G. Hundt and W. L. Stutzman, "Pattern Measurement Distance for Broad Beam Antennas," *J. Electromagnetic Waves & App.*, Vol. 8, pp. 221–235, Feb. 1994.
9. D. Slater, *Near-Field Antenna Measurements*, Artech House, Boston, MA, 1991.
10. W. L. Stutzman, *Polarization in Electromagnetic Systems*, Artech House, Boston, MA, 1993. Chap. 4.
11. H. V. Carnagan, "Measure That Field Using Any Antenna," *Microwaves*, Vol. 14, pp. 45–47, July 1975.

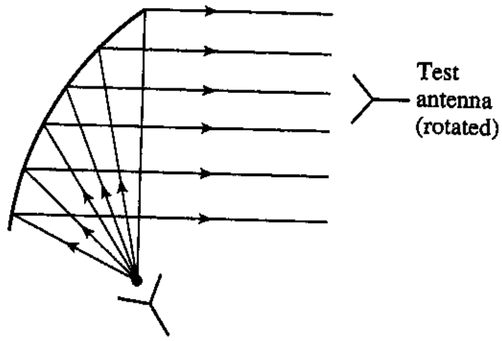


Figure 9-11 The compact range.

performance is possible with a dual reflector. A disadvantage of the compact range is that the reflector must be large—about three times larger than the test antenna.

The final antenna range type is the *near-field range* shown in Fig. 9-12. Here, the test antenna acts as a transmitter and the amplitude and phase are sampled at regular intervals in the near field of the test antenna. The samples are weighted equally by the receiver, providing uniform amplitude and phase across the test antenna as required for accurate measurements. Radiation properties such as the pattern are then computed using a Fourier transform [9]. Accurate probe positioning is required for accurate pattern computation. The near-field range offers the benefit of having the aperture distribution data available for diagnostic use. For example, a dead element in an array antenna can be located.

The operating principle of the *ground reflection range* is completely different from that of free-space ranges. The source and test antenna heights are small and the ground between the towers is constructed to be flat and reflective, which causes the indirect ray to arrive with an amplitude close to that of the direct ray. The indirect ray path distance is not greatly different from that of the direct ray. This gives a slowly varying phase over the test region, which in turn gives a slowly varying interference pattern and a relatively constant field illumination over the test zone. A low test tower height is convenient for large test objects such as antennas on full-scale aircraft.

Rotation positioners are required in most of antenna ranges. Often, a simple *azimuth positioner* (or "turntable") is sufficient. An *elevation-over-azimuth positioner* as illustrated in Fig. 9-13*a* permits alignment with the source antenna placed

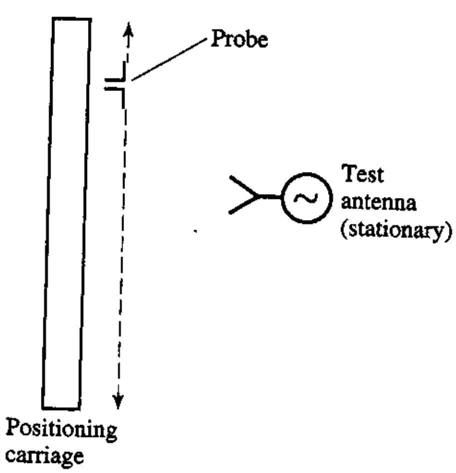


Figure 9-12 The near-field range.