A Note on Constrained Multi-Objective Optimization Benchmark Problems

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***Abstract*—We investigate the properties of widely used constrained multi-objective optimization benchmark problems. A number of Multi-Objective Evolutionary Algorithms (MOEAs) for Constrained Multi-Objective Optimization Problems (CMOPs) have been proposed in the past few years. The C-DTLZ functions and Real-World-Like Problems (RWLPs) have frequently been used for evaluating the performance of MOEAs on CMOPs. In this paper, however, we show that the C-DTLZ functions and widely-used RWLPs have some unnatural problem features. The experimental results show that an MOEA *without* any Constraint Handling Techniques (CHTs) can successfully find well-approximated nondominated feasible solutions on the C1-DTLZ1, C1-DTLZ3, and C2-DTLZ2 functions. It is widely believed that RWLPs are MOEA-hard problems, and finding the feasible solutions on them is a very hard task. However, we show that the MOEA without any CHTs can find feasible solutions on widely-used RWLPs such as the speed reducer design problem, the two-bar truss design problem, and the water problem. Also, it is seldom that the infeasible solution simultaneously violates multiple constraints in the RWLPs. Due to the above reasons, we conclude that constrained multi-objective optimization benchmark problems need a careful reconsideration.**

I. INTRODUCTION

A Constrained Multi-Objective continuous Optimization Problem (CMOP) which frequently appears in engineering problems [1], [2], [3], [4] can be formulated as follows:

$$\begin{aligned} \text{minimize} \quad & f_i(\boldsymbol{x}), i \in \{1, ..., M\} \\ \text{subject to} \quad & g_i(\boldsymbol{x}) \leq 0, i \in \{1, ..., p\} \\ & h_i(\boldsymbol{x}) = 0, i \in \{p+1, ..., N\} \end{aligned} \tag{1}$$

where $\boldsymbol{f} : \mathbb{S} \rightarrow \mathbb{R}^M$ is an objective function vector which consists of M conflicting objective functions, and $\mathbb{R}^M$ is the objective function space. $\boldsymbol{x} = (x_1, ..., x_D)^{\mathrm{T}}$ is a D-dimensional solution vector, and $\mathbb{S} = \Pi_{j=1}^{D}[a_j, b_j]$ is the bound-constrained search space, where $a_j \leq x_j \leq b_j$ for each $j \in \{1, ..., D\}$. The feasible solution satisfies p inequality constraint functions $\{g_1, ..., g_p\}$ and $N-p$ equality constraint functions $\{h_1, ..., j_{N-p}\}$. The set of all the feasible solutions is called the feasible region $\mathbb{F} \subseteq \mathbb{S}$. On the other hand, $\boldsymbol{x} \notin \mathbb{F}$ in $\mathbb{S}$ is the infeasible solution. In the same manner, the set of all the infeasible solutions is called the infeasible region $\overline{\mathbb{F}}$.

On the N inequality and equality constraint functions in Equation (1), the constraint violation value $c_i(\boldsymbol{x})$, $i \in \{1, ..., N\}$ of the solution $\boldsymbol{x}$ can be defined as follows:

$$c_i(\boldsymbol{x}) = \begin{cases} \max(0, g_i(\boldsymbol{x})), & i \in \{1, ..., p\} \\ \max(0, |h_i(\boldsymbol{x}) - \delta|), & i \in \{p+1, ..., N\} \end{cases} \tag{2}$$

where the tolerance value δ is generally used for relaxing the equality constraints to the inequality constraints, and δ should be set to a sufficiently small value (e.g., $\delta = 10^{-6}$). If the solution $\boldsymbol{x}$ is feasible, $\sum_{i=1}^{N} c_i(\boldsymbol{x}) = 0$.

For $\boldsymbol{x}^1$, $\boldsymbol{x}^2 \in \mathbb{F}$, we say that $\boldsymbol{x}^1$ dominates $\boldsymbol{x}^2$ and denote $\boldsymbol{x}^1 \prec \boldsymbol{x}^2$ if and only if $f_i(\boldsymbol{x}^1) \leq f_i(\boldsymbol{x}^2)$ for all $i \in \{1, ..., M\}$ and $f_i(\boldsymbol{x}^1) < f_i(\boldsymbol{x}^2)$ for at least one index i. $\boldsymbol{x}^*$ is a Pareto optimal solution if there exists no $\boldsymbol{x} \in \mathbb{F}$ such that $\boldsymbol{x} \prec \boldsymbol{x}^*$. $\boldsymbol{f}(\boldsymbol{x}^*)$ is also called a Pareto optimal objective function vector. The set of all $\boldsymbol{x}^*$ is the Pareto optimal solution Set (PS), and the set of all $\boldsymbol{f}(\boldsymbol{x}^*)$ is the Pareto Frontier (PF). The goal of CMOPs is finding a set of well-distributed nondominated feasible solutions that close to the PF in the objective function space.

A Multi-Objective Evolutionary Algorithm (MOEA) is one of most promising approaches for solving (C)MOPs [5]. Since MOEAs use a set of individuals (solutions of a given problem) for the search, it is possible that they can find good nondominated feasible solutions in a single run. Unfortunately, as pointed out in [6], researchers in the Evolutionary Computation community have mainly focused on studies of MOEAs for unconstrained or bound-constrained MOPs, and the number of previous studies on CMOPs is much smaller than them. However, some novel, efficient MOEAs for CMOPs has been proposed in the past few years. Recently proposed, representative MOEAs for (C)MOPs are NSGA-III [7], U-NSGA-III [8], I-DBEA [9], MOEA/DD [10], and RVEA [11]. Surrogate-assisted MOEAs for CMOPs have also been studied [12].

In general, theoretically evaluating the performance of MOEAs (and EAs) is hard, and so it is experimentally evaluated by the computational simulation. Suppose that we apply an MOEA to a specific real-world problem and evaluate its performance. In this case, reproducing the experiment by other researchers is very hard in many cases since the computational simulation of real-world problems often requires a special hardware or software. Therefore, for evaluating the performance of MOEAs, artificially designed benchmark functions have been widely used.

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Typical constrained multi-objective benchmark functions are the SRN function [13], the TNK function [14], the OSY function [15], the CTP functions [16], and the CF functions [17]. However, almost all of them are two-objective CMOPs and cannot be set M to an arbitrary number. That is, the benchmark functions described above are not suitable for evaluating the scalability of MOEAs with respect to M. On the other hand, the number of objectives of the recently proposed C-DTLZ functions [7] can be set to an arbitrary number, where the C-DTLZ functions are extended variants of the DTLZ functions [18]. Due to this reason, the C-DTLZ functions have frequently been used for evaluating the performance MOEAs in recent studies [7], [8], [10], [11], [12]. While all the benchmark functions described above are artificially designed functions, Real-World-Like Problems (RWLPs) have also been used in comparative studies. Typical RWLPs are the two-bar truss design problem [19], the car side impact problem [7], and the water problem [20]. It is widely believed that finding the feasible solutions on RWLPs is a very hard task since almost all of them have many complex constraints. Thus, RWLPs have been considered as challenging, difficult problems for MOEAs. In summary, there are many benchmark problems for evaluating the performance of MOEAs on CMOPs. However, as far as we know, their properties have been poorly investigated, and their suitability as benchmark problems for MOEAs have been hardly discussed, except for some previous work such as [16], [21].

In this paper, we investigate the properties of the C-DTLZ functions and widely-used RWLPs, and show that they have some issues. For the C-DTLZ functions, we demonstrate that an MOEA *without* any Constraint Handling Techniques (CHTs) can find well-approximated nondominated feasible solutions on the C1-DTLZ1, C1-DTLZ3, and C2-DTLZ2 functions. As mentioned above, many researchers in the Evolutionary Computation community have considered RWLPs as MOEA-hard problems, and finding the feasible solutions on them is a hard task. However, the experimental results in this paper show that the MOEA without any CHTs can find feasible solutions on the widely-used RWLPs. Nevertheless there are many constraints in the widely-used RWLPs, it is seldom that the infeasible solution simultaneously violates multiple constraints on them. Due to these reasons, although the C-DTLZ functions and the RWLPs have frequently been used for evaluating the performance MOEAs in recent studies [7], [8], [10], [11], [12], they might need to be carefully reconsidered.

This paper is organized as follows: Section II introduces typical constrained multi-objective benchmark problems. Section III describes experimental settings, and we investigate the properties of constrained multi-objective benchmark problems in Section IV. Finally, Section V concludes this paper and discusses our future work.

TABLE I: Properties of typical constrained multi-objective optimization benchmark problems (the number of objectives M, the number of constraint functions N, the dimensionality D, and the feasibility ratio). The feasibility ratio of the search space (i.e., $|\mathbb{F}|/|\mathbb{S}|$) was experimentally estimated by calculating the percentage of feasible solutions in 10^5 uniformly randomly generated solutions as in [22], [23].

Problem	M	N	D	Feasibility ratio
SRN	2	2	2	0.08
TNK	2	2	2	0.03
OSY	2	6	6	0.02
C1-DTLZ1	≥ 2	1	$M+k-1$	0.0
C1-DTLZ3	≥ 2	1	$M+k-1$	0.0
C2-DTLZ2	≥ 2	1	$M+k-1$	0.02
C3-DTLZ1	≥ 2	M	$M+k-1$	0.5
C3-DTLZ4	≥ 2	M	$M+k-1$	0.12
f_{TBTD}	2	3	3	0.0
f_{SRD}	2	11	7	0.043
f_{DBD}	2	5	4	0.319
f_{WB}	2	4	4	0.058
f_{CSI}	3	10	7	0.181
f_{SPD}	3	9	6	0.026
f_{W}	5	7	3	0.920

II. Review of constrained multi-objective optimization benchmark problems

Table I shows some properties of typical constrained multi-objective optimization benchmark problems described in this section. It is widely believed that the feasibility ratio represents the difficulty of finding the feasible solutions [22], [23]. For details of each CMOP in Table I, see the corresponding paper.

The SRN function [13], the TNK function [14], and the OSY function [15] are one of most widely used benchmark functions. The three functions were proposed in $1994 \sim 1995$ and used in much previous work [24], [25], [23], [26]. However, Deb et. al. point out that these functions have the following three issues: (i) the dimensionality D is too small, (ii) finding good solutions is not so hard, and (iii) the difficulty and complexity in optimization cannot be tuned [16].

To address the above issues (i)–(iii), Deb et. al. propose the CTP functions [16]. The CTP can be considered as a general framework for constructing novel constrained multi-objective optimization benchmark problems. Various benchmark functions can be generated by using the CTP framework. In [16], seven CTP functions (CTP1, ..., CTP7) are designed. The CTP framework was also used for constructing the CF functions [17] for the IEEE CEC2009 MOEA Competition[1].

[1]http://dces.essex.ac.uk/staff/zhang/moeacompetition09.htm

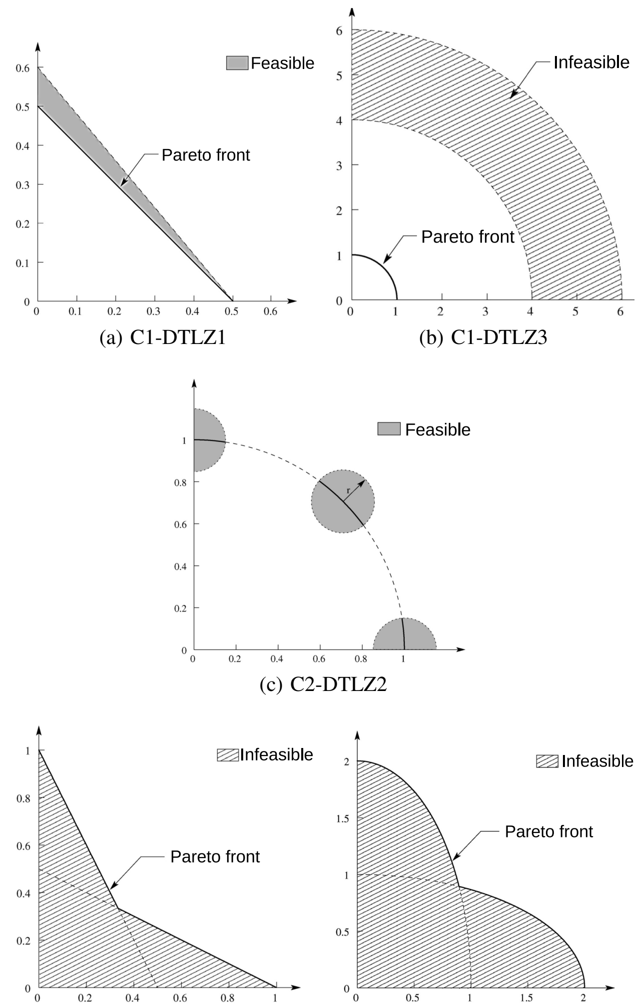
Very recently, Li et. al. propose more difficult variants of the CTP functions [21]. Unfortunately, almost all of them described above are two-objective CMOPs and are not suitable for evaluating the scalability of MOEAs with respect to M^2.

In [7], Jain and Deb propose the five C-DTLZ functions. The C-DTLZ functions are extended variants of the DTLZ functions [18] for benchmarking MOEAs for CMOPs. Unlike the classical benchmark functions described above, the number of objectives of the C-DTLZ functions can be set to an arbitrary number, and so they have frequently been used in recent comparative studies [7], [10], [11], [12]. According to the characteristics of the feasible region in the objective function space, the C-DTLZ functions are classified into the following three categories: Type-1, Type-2, and Type-3 problems.

Figure 1 shows the feasible or infeasible region of each C-DTLZ function in the objective space ($M = 2$). The C2-DTLZ2 and C3-DTLZ4 functions are unimodal, and the C1-DTLZ1, C1-DTLZ3, and C3-DTLZ1 functions are multimodal. In Type-1 constraint C-DTLZ functions (C1-DTLZ$*$), the shape and the position of the PF are same with the original DTLZ functions, but there is "an infeasible barrier" that prevents MOEAs from approaching the PF. Thus, the population has to get over the infeasible barrier to converge to the true PF, and it can be considered as a difficult task. In Type-2 constraint C-DTLZ functions (C2-DTLZ$*$), a part of the PF becomes the infeasible region by introducing the constraint. That is, the shape of the PF of C2-DTLZ$*$ is discontinuous. In general, handling the discontinuously of the PF is hard for MOEAs. Note that nevertheless the constraint is introduced, the position of the PF of C1-DTLZ$*$ and C2-DTLZ$*$ is still unchanged from the original DTLZ functions. On the other hand, in Type-3 constraint C-DTLZ functions (C3-DTLZ$*$), the region of the original PF is infeasible by introducing M linear constraints, and the PF of C3-DTLZ$*$ is a boundary line between the feasible and infeasible region. Note that almost all constraint multi-objective benchmark functions such as SRN, TNK, OSY, and the CTP functions (except for the CTP7 function) can be classified into the Type-3 constraint functions.

Also, Real-World-Like Problems (RWLPs) have frequently been used for comparative studies [19], [27], [23], [26], [7]. Typical RWLPs are the two bar truss design problem (f_{TBTD}) [19], [26], the speed reducer design problem (f_{SRD}) [19], [26], the disc brake design problem (f_{DBD}) [19], [26], the welded beam problem (f_{WB}) [28], [26], the car side impact problem (f_{CSI}) [7], the ship parametric design problem (f_{SPD}) [29], [27], and the water problem (f_{W}) [20], [7]. It is generally believed that finding the feasible solutions on RWLPs is a very hard task since almost all of them have many complex constraints [7]. In fact, as shown in Table I, N of almost all the RWLPs is much larger than N of the artificially designed functions (e.g., f_{CSI} has 11 constraints), and the feasibility ratios of some RWLPs are low. Therefore, RWLPs have been considered as challenging, difficult problems for MOEAs.

[2]The idea for constructing CTP functions with $M \geq 3$ is briefly introduced in [16], but to the best of our knowledge, they have never been realized.



(a) C1-DTLZ1 (b) C1-DTLZ3

(c) C2-DTLZ2

(d) C3-DTLZ1 (e) C3-DTLZ4

Fig. 1: Feasible or infeasible region of each C-DTLZ function in the objective space ($M = 2$). All figures (a)–(e) were derived from [7].

III. Experimental settings

In this paper, we investigate the properties of the two groups of constrained multi-objective optimization benchmark problems (the C-DTLZ functions and the RWLPs) described in Section II. We here describe the experimental settings, and the results will be discussed in Section IV.

A. Problems and performance evaluation methods

We used the two- and three-objective C-DTLZ functions[3]. As suggested in [7], the position parameter k was set to $k = 10$ for the C1-DTLZ3 and C2-DTLZ2 functions, and $k = 5$ for the remaining functions. We also used the seven RWLPs in Table I.

We used the hypervolume (HV) indicator [30] for evaluating the quality of a set of obtained nondominated solutions $\boldsymbol{A}$. Before calculating the HV value, the objective function vector

[3]We used the source code of the C-DTLZ functions implemented by Li. The code was downloaded from http://www.cs.bham.ac.uk/~likw/.

$\boldsymbol{f}(\boldsymbol{x})$ of each $\boldsymbol{x} \in \boldsymbol{A}$ was normalized using the ideal point and the nadir point. The ideal point and the nadir point for all problems were experimentally estimated by using all solutions obtained by all methods for all the 101 runs. The reference point for calculating HV was set to $(1.1, ..., 1.1)^{\mathrm{T}}$. Note that we used only the feasible solutions dominating the reference point for the HV calculation.

Almost all previous work (e.g., [7], [8], [10], [9], [11], [12]) use nondominated solutions in the population at the end of the search for calculating the HV value. In general, an MOEA maintains (nondominated) solutions obtained during the search process in the population, but its size is limited. When using the nondominated solutions in the population for the HV calculation, a monotonic increase of the HV over time (= the number of function evaluations) cannot be ensured [31], [32]. Thus, we cannot exactly evaluate the performance of MOEAs using such traditional evaluation methodology.

To address this issue, we used an *unbounded* external archive as suggested in [32], [33]. The unbounded external archive stores *all* nondominated feasible solutions found during the search process and can be introduced into any MOEAs without any changes in their original algorithms [32], [33]. Solutions in the unbounded external archive are not used for the search of an MOEA. When using the unbounded external archive, the issue described above can be addressed [31], [32]. Since in practice users of MOEAs want to know nondominated solutions as much as possible, we believe that the benchmark methodology using the unbounded external archive is more practice. The unbounded external archive is also adopted to the recently proposed BBOB-biobj benchmark suite [34] in the COCO framework[4]. When the number of obtained nondominated solutions in the unbounded external archive is too large, a method selecting only representative solutions (e.g., [35]) should be applied to them.

B. MOEAs

We analyzed the performance of five variants of (C)NSGA-II [24] since it is one of most widely used MOEAs for CMOPs. CNSGA-II is the NSGA-II algorithm using the constraint-domination $\prec_{\mathrm{const}}$, instead of the Pareto-domination $\prec$ described in Section I. For $\boldsymbol{x}^1, \boldsymbol{x}^2 \in \mathbb{S}$, we say that $\boldsymbol{x}^1$ constraint-dominates $\boldsymbol{x}^2$ and denote $\boldsymbol{x}^1 \prec_{\mathrm{const}} \boldsymbol{x}^2$ if they satisfy one of the following three conditions: (1) $\boldsymbol{x}^1 \in \mathbb{F}$ and $\boldsymbol{x}^2 \notin \mathbb{F}$, (2) $\boldsymbol{x}^1, \boldsymbol{x}^2 \notin \mathbb{F}$ and $C(\boldsymbol{x}^1) < C(\boldsymbol{x}^2)$, (3) $\boldsymbol{x}^1, \boldsymbol{x}^2 \in \mathbb{F}$ and $\boldsymbol{x}^1 \prec \boldsymbol{x}^2$. Where C is a constraint violation function summarizing how the solution $\boldsymbol{x}$ violate N constraints in Equation (1).

In this paper, we investigated the performance of the following five (C)NSGA-II algorithms using different types of C:

1. NSGA-II: The standard NSGA-II using $\prec$, not $\prec_{\mathrm{const}}$.
2. CNSGA-II-S: The CNSGA-II using the sum of the constraint violation values $\{c_1(\boldsymbol{x}), ..., c_N(\boldsymbol{x})\}$ defined in Equation (2) as C, i.e., $C(\boldsymbol{x}) = \sum_{i=1}^{N} c_i(\boldsymbol{x})$.
3. CNSGA-II-NS: The CNSGA-II using the normalized sum of the constraint violation values as C, i.e., $C(\boldsymbol{x}) = \sum_{i=1}^{N}(c_i(\boldsymbol{x}) - c_i^{\min})/(c_i^{\max} - c_i^{\min})$, where $c_i^{\min} = \min_{\boldsymbol{y} \in \boldsymbol{P}}\{c_i(\boldsymbol{y})\}$, $c_i^{\max} = \max_{\boldsymbol{y} \in \boldsymbol{P}}\{c_i(\boldsymbol{y})\}$, and $\boldsymbol{P}$ is the population.
4. CNSGA-II-CD: The CNSGA-II using the Pareto-dominance relationship in the constraint violation value space $\mathbb{R}^N$ [36] as C.
5. CNSGA-II-RR: The CNSGA-II using the Relative Ranking (RR) [27] as C.

For details of CD and RR, see [36], [27] respectively. CNSGA-II-S is identical to the original "CNSGA-II" proposed in [24] and most typical variant in the five (C)NSGA-II algorithms. The aim of the normalizing procedure of CNSGA-II-NS and the sophisticated techniques of CNSGA-II-CD and CNSGA-II-RR is to handle the different scale of constraint violation values. For CNSGA-II-CD and CNSGA-II-RR, if the compared two individuals $\boldsymbol{x}^1$ and $\boldsymbol{x}^2$ have the same rank level, the sum of the constraint violation values as in CNSGA-II-S is used as a tie-breaking method. It is worth noting that as far as we know, there is no previous work analyzing the impact of C on the performance of CNSGA-II.

We used the jMetal[5] source code of NSGA-II. We used the SBX crossover and polynomial mutation as in the original CNSGA-II paper [24]. As suggested in [24], we set the control parameters of the variation operators as follows: $p_c = 1.0$, $\eta_c = 20$, $p_m = 1/D$, $\eta_m = 20$. The population size of the CNSGA-II algorithms was set to 100. For handling the different scale of the objective function values, we introduced the normalization procedure of the objective function values to all the five algorithms. The maximum number of function evaluations was set to 5×10^4, and the 101 independent runs were performed. Owing to a large enough sample size, statistical tests are not necessary.

IV. Experimental results and discussion

Here, we report and discuss the experimental results of the five (C)NSGA-II algorithms on the five C-DTLZ functions and the seven RWLPs listed in Table I. First, we describe the results on the five C-DTLZ functions and seven RWLPs in Section IV-A and IV-B respectively. Finally, in Section IV-C, we generally discuss the experimental results on the C-DTLZ functions and RWLPs.

A. Results on the five C-DTLZ functions

Figure 2 shows the performance comparisons of the five (C)NSGA-II algorithms on the two-objective C-DTLZ functions. Due to space constraints, we do not show the results on three objective functions, but they are similar to Figure 2. Note that the behavior of the four CNSGA-II algorithms (CNSGA-II-S, CNSGA-II-NS, CNSGA-II-CD, and CNSGA-II-RR) on the C1-DTLZ1, C1-DTLZ3, and C2-DTLZ2 functions is exactly same since $N = 1$ in them.

On the Type-1 functions, CNSGA-II performs well on the C1-DTLZ1 function within around the number of functions

[4]http://coco.gforge.inria.fr/

[5]The code was downloaded from http://jmetal.sourceforge.net/

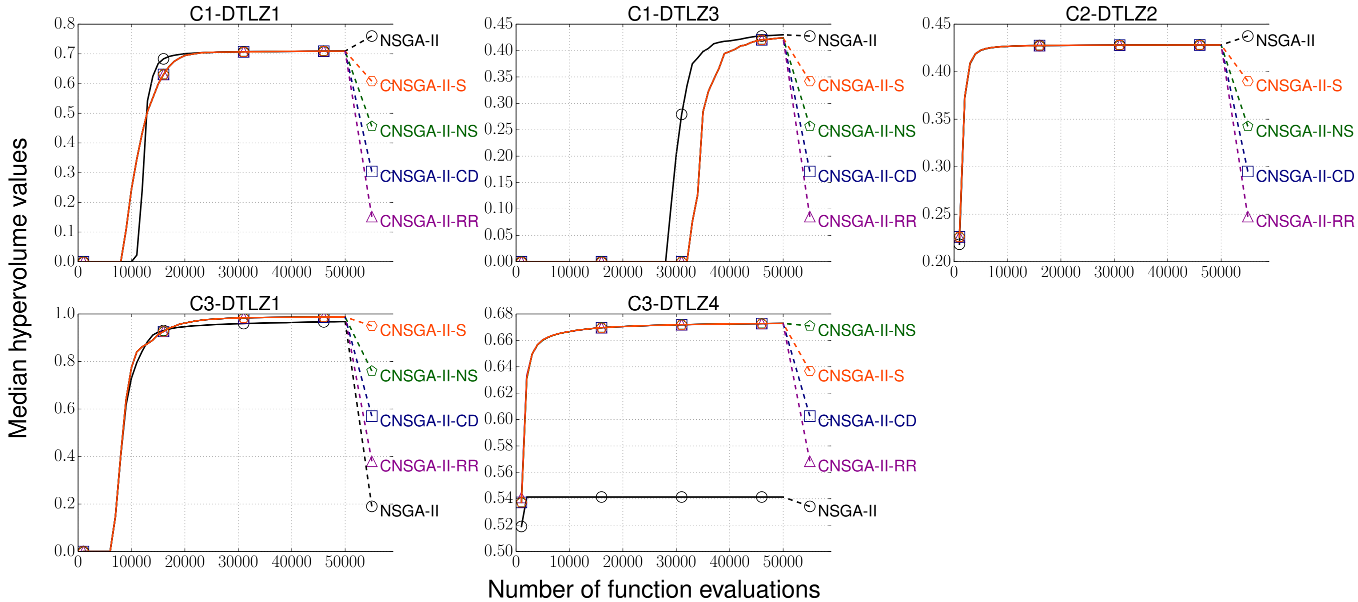


Fig. 2: Convergence behavior of the five (C)NSGA-II algorithms on the five C-DTLZ functions with $M = 2$. We plot the median HV values across all the 101 runs.

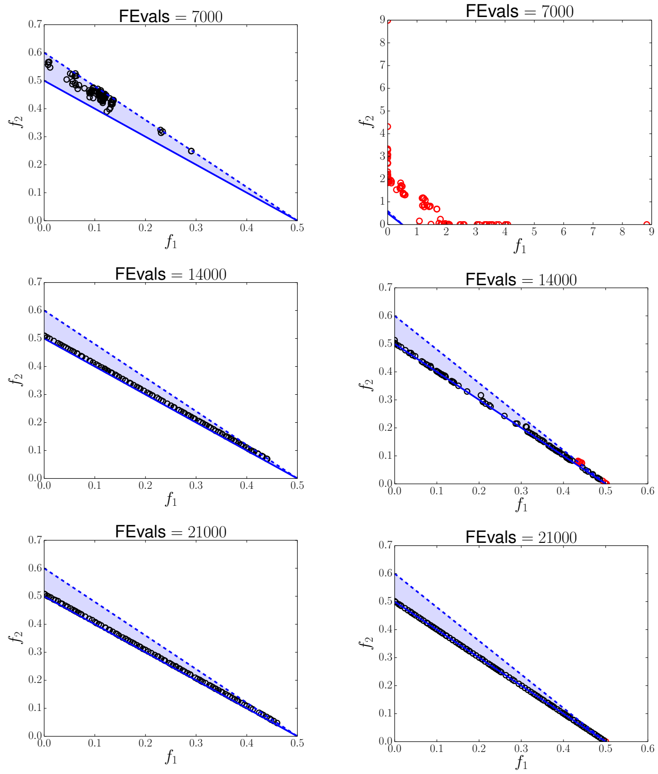
evaluations (FEvals) $= 1.2 \times 10^4$. However, for FEvals $= 1.2 \times 10^4 \sim 2.3 \times 10^4$, the four CNSGA-II algorithms are clearly outperformed by NSGA-II that does not use any Constraint Handling Techniques (CHTs). For FEvals $> 2.3 \times 10^4$, there is no significant difference in the performance of the five algorithms.

For explaining the results on the C1-DTLZ1 function, we show the distribution of individuals in the population of (a) CNSGA-II-S and (b) NSGA-II for FEvals $\in \{7\,000, 14\,000, 21\,000\}$ in Figure 3. As shown in Figure 3, for FEvals $= 7\,000$, CNSGA-II-S converges to the feasible region as well as the PF faster than NSGA-II. The reason for this result is that when all individuals in the population of CNSGA-II-S are infeasible, the environmental selection of CNSGA-II-S is performed based on only the amount of the constraint violation values (see Section III-B). That is, when all the individuals in the population are infeasible, the algorithmic behavior of CNSGA-II-S is identical to a single-objective GA minimizing the sum of constraint violation values $C(\boldsymbol{x}) = \sum_{i=1}^{N} c_i(\boldsymbol{x})$. Due to this reason, we believe that CNSGA-II-S could find good feasible solutions faster than NSGA-II. However, the distribution of the individuals of CNSGA-II is clearly biased to the near of f_2 for FEvals $= 7\,000$. After finding the feasible solutions, the population of CNSGA-II gradually creeps towards the f_1 along the PF. In summary, CNSGA-II-S early finds the feasible solutions, but it takes much FEvals for obtaining good solutions on the PF. On the other hand, the individuals in the population of NSGA-II are widely distributed when reaching the feasible region (FEvals $= 14\,000$). For FEvals $= 21\,000$, NSGA-II finds well-distributed feasible solutions while the population of CNSGA-II-S still moves toward f_1. These are the reason why the HV values of NSGA-II are higher than the one of the CNSGA algorithms for FEvals $= 1.2 \times 10^4 \sim 2.3 \times 10^4$ on the C1-DTLZ1 function.

On the C1-DTLZ3 function, surprisingly, NSGA-II clearly outperforms the four CNSGA-II algorithms for anytime. The reason for this result is that the infeasible barrier of the C1-DTLZ3 function (Figure 1(b)) does not affect the search of an MOEA without any CHTs.

On the C2-DTLZ2 function, there is no performance difference between NSGA-II and CNSGA-II. In the C2-DTLZ2 function, a part of the PF is the infeasible region, but the position of the PF is still unchanged. Recall that in this experiment we used the unbounded external archive that stores all nondominated feasible solutions found during the search process. The shape of the PF of the C2-DTLZ2 function is disconnected, and it is widely believed that handling the discontinuities in the PF is difficult [7]. However, when using the unbounded external archive, the distribution of the individuals in the population is not relevant to the HV value, and so the effect of discontinuity might not be so significant. We also show the results on the CTP7 function [16] in Figure 4. We selected the CTP7 function since a part of its PF is the infeasible region, and it can be classified in to the Type-2 constraint functions. As expected, the results on the CTP7 function are similar to the results on the C2-DTLZ2 function.

Finally, although this is unsurprising results, the four CNSGA-II algorithms perform significantly better than NSGA-II on the C3-DTLZ1 and C3-DTLZ4 functions, especially the C3-DTLZ4 function. We do not show the results here, but the poor performance of NSGA-II can also be seen in the SRN, TNK, and OSY functions. For explaining this result, we show the distribution of individuals in the population



(a) CNSGA-II-S

(b) NSGA-II

Fig. 3: Distribution of the individuals in the population of (a) CNSGA-II-S and (b) NSGA-II on the two-objective C1-DTLZ1 function in a single run (FEvals $\in \{7\,000, 14\,000, 21\,000\}$). The red and black circles represent the infeasible and feasible solutions respectively. The blue bold line is the PF, and the shaded region is the feasible region.

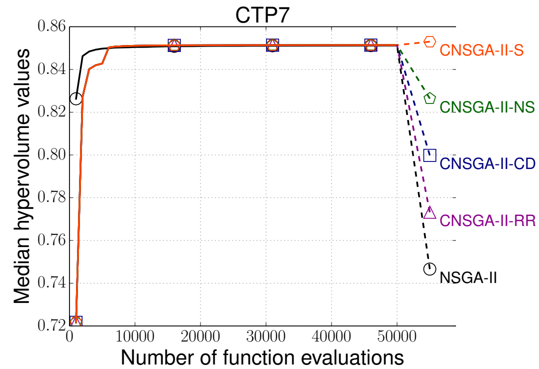


Fig. 4: Convergence behavior of the five (C)NSGA-II algorithms on the two-objective CTP7 function. We plot the median HV values.

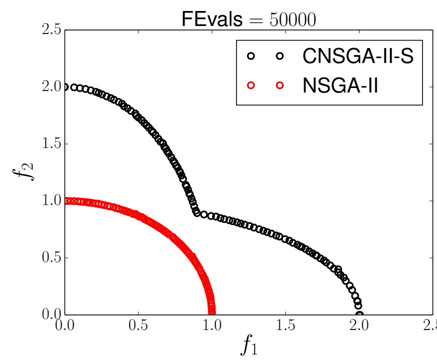


Fig. 5: Distribution of the individuals in the population of CNSGA-II-S and NSGA-II on the two-objective C3-DTLZ4 function in a single run (FEvals $= 50\,000$). All the individuals of CNSGA-II-S and NSGA-II are feasible and infeasible respectively.

of CNSGA-II-S and NSGA-II on the C3-DTLZ4 function in Figure 5. Note that all the individuals of CNSGA-II-S and NSGA-II are feasible and infeasible respectively. On the C3-DTLZ4 function, the original PF is infeasible, and its PF is a boundary line between the feasible and infeasible region. Due to this reason, in Figure 5, NSGA-II that does not use any CHTs passes the true PF and converges to the original PF. As a result, NSGA-II fails to find good feasible solutions.

B. Results on the seven RWLPs

Figure 6 shows the performance comparisons of the five (C)NSGA-II algorithms on the six RWLPs (f_{TBTD}, f_{WB}, f_{SRD}, f_{CSI}, f_{SPD}, and f_{W}). We do not show the result on f_{DBD}, but it is similar to the result on f_{SRD}.

As shown in Figure 6, the results on the RWLPs are similar to the results on the Type-3 constraint C-DTLZ functions – NSGA-II performs significantly worse than the four CNSGA-II algorithms, especially on f_{DBD} and f_{SPD}. On f_{WB}, NSGA-II achieves good feasible solutions only for FEvals $= 10^3$. We believe that the reason of the poor performance of NSGA-II on the RWLPs is the same with the one on the C3-DTLZ4 function described in Section IV-A. It is worth mentioning that NSGA-II finds the feasible solutions on all the seven RWLPs while many researchers consider these problems as difficult problems for finding the feasible solutions. Also, it is widely believed that well-designed CHTs are needed to handle the different scale of each constraint violation value in $\{c_1, ..., c_N\}$ on the RWLPs [36], [27], [23], [7]. However, in this study, we compared the four CNSGA-II algorithms (CNSGA-II-S, CNSGA-II-NS, CNSGA-II-CD, and CNSGA-II-RR), but contrary to intuition, there is no significant difference between their performance. CNSGA-II-S uses the most simple C, and so it should perform worse than the remaining CNSGA-II algorithms using the more sophisticated C.

For analyzing the results, we show the cumulative number of constraints which the infeasible solutions generated by CNSGA-II-S simultaneously violate on f_{SRD}, f_{CSI}, and f_{W} in Figure 7. From seen Figure 7, how many constraint functions are simultaneously violated can be found out. For example, on f_{SRD}, until FEvals $= 8\,000$, CNSGA-II-S generated about the $1\,700$ and 300 infeasible solutions that simultaneously violate one and two constraint functions respectively. Also, it never happened that the infeasible solutions generated by CNSGA-II-S simultaneously violated over four constraints on f_{SRD}. As shown in Table I, there are the 11, 10, and

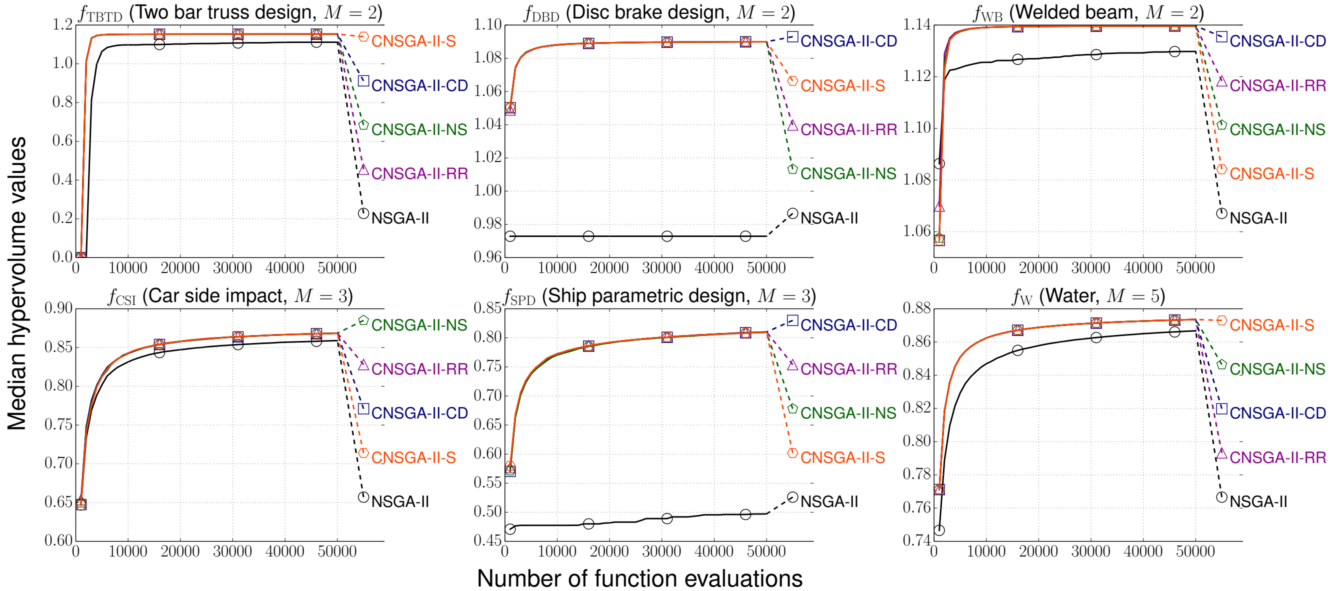


Fig. 6: Convergence behavior of the five (C)NSGA-II algorithms on the six RWLPs ($f_{\rm TBTD}$, $f_{\rm WB}$, $f_{\rm SRD}$, $f_{\rm CSI}$, $f_{\rm SPD}$, $f_{\rm W}$). We plot the median HV values across all the 101 runs.

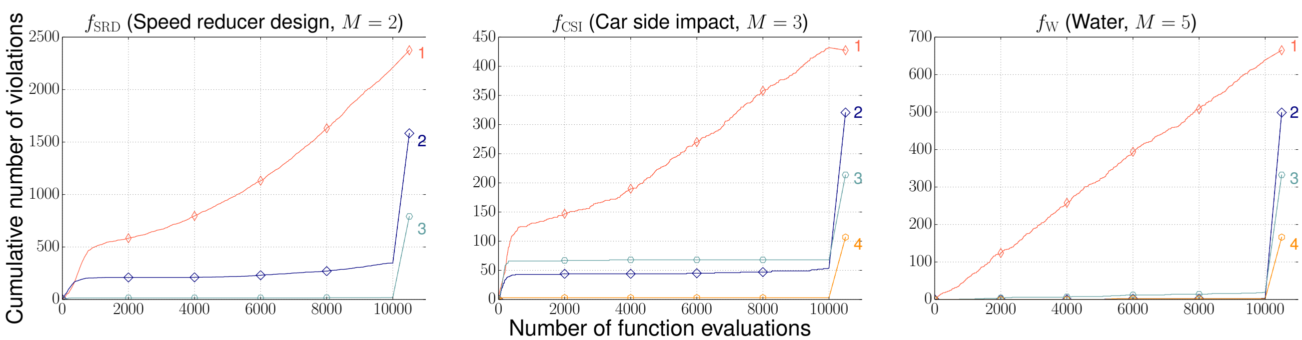


Fig. 7: Cumulative number of constraints which the infeasible solutions generated by CNSGA-II-S simultaneously violate.

seven constraints in $f_{\rm SPD}$, $f_{\rm CSI}$, and $f_{\rm W}$ respectively. Handling many constraints and finding good feasible solutions on such problems seem to be hard. However, as shown in Figure 7, almost all solutions obtained during the search process are feasible, and the infeasible solutions violate only one constraint in many cases. This is the reason why the four CNSGA-II algorithms perform similarly to each other on the seven RWLPs in Figure 6. Although there are many constraints in the seven RWLPs, finding the feasible solutions seems to be relatively easy.

C. Overall discussion on the experimental results on the C-DTLZ functions and RWLPs

From the experimental results described in Section IV-A, we can say that an MOEA without any CHTs (e.g., NSGA-II) can find well-approximated feasible solutions on the Type-1 and Type-2 constraint functions. Since the position of the PF of the Type-1 and Type-2 functions is unchanged from the original functions, improving the solution according to the objective function values is identical to minimizing the constraint violation values (i.e., searching the feasible solutions). The Type-1 and Type-2 constraint functions should probably not be used for evaluating the performance of MOEAs for CMOPs in light of our experimental results.

RWLPs have been considered challenging, difficult problems [7]. However, in Section IV-B, we showed that even NSGA-II without any CHTs can easily find the feasible solutions on the seven RWLPs. Also, the CNSGA-II algorithms with the simple C (i.e., CNSGA-II-S) and the more sophisticated C (i.e., CNSGA-II-NS, CNSGA-II-CD, and CNSGA-II-RR) perform similarly to each other on the RWLPs. Therefore, RWLPs might be not as hard as researchers expected.

We believe that finding good feasible solutions on *actual* real-world problems is a hard task due to their complex constraints [4]. In other words, real-world CMOPs should be much harder than the C-DTLZ functions as well as the widely-used RWLPs. More complex, hard benchmark problems are required for evaluating the performance of MOEAs on CMOPs.

V. Conclusion

We have analyzed the properties of the C-DTLZ functions and the seven Real-World-Like Problems (RWLPs) listed in Table I. The experimental results showed that the Type-1 and Type-2 C-DTLZ functions have some critical issues and might be inappropriate for benchmarking MOEAs for CMOPs. Researchers in the Evolutionary Computation community had considered RWLPs as challenging problems due to their many constraints, but we showed that RWLPs might be not as hard as they expected. In light of our experimental results, it is possible that constrained multi-objective optimization benchmark problems need a careful reconsideration.

There is much room for designing novel, appropriate problems for benchmarking MOEAs for CMOPs. Among the five C-DTLZ functions, the Type-3 functions seem to be one of most appropriate benchmark problems. Therefore, we believe that constructing novel benchmark problems based on the Type-3 DTLZ functions is a promising future direction. The PF of the Type-1 and 2 C-DTLZ functions is identical with the one of the original DTLZ functions, and this causes the problem as shown in this paper. However, the region of the PF of the Type-1 and 2 functions can be infeasible by adding the properties of the Type-3 functions to them. Future work will construct composite functions of the Type-3 functions and the Type-1 and 2 functions.

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References

[1] H. K. Singh, T. Ray, and R. A. Sarker, "Optimum Oil Production Planning Using Infeasibility Driven Evolutionary Algorithm," *Evol. Comput.*, vol. 21, no. 1, pp. 65–82, 2013.

[2] S. Z. Martínez, A. A. Montaño, and C. A. C. Coello, "Constrained multi-objective aerodynamic shape optimization via swarm intelligence," in *GECCO*, 2014, pp. 81–88.

[3] H. K. Singh, K. Alam, and T. Ray, "Use of Infeasible Solutions During Constrained Evolutionary Search: A Short Survey," in *ACALCI*, 2016, pp. 193–205.

[4] A. K. Sharma, R. Datta, M. Elarbi, B. Bhattacharya, and S. Bechikh, "Practical Applications in Constrained Evolutionary Multi-objective Optimization," in *Recent Advances in Evolutionary Multi-objective Optimization*, 2017, pp. 159–179.

[5] B. Li, J. Li, K. Tang, and X. Yao, "Many-Objective Evolutionary Algorithms: A Survey," *ACM Comput. Surv.*, vol. 48, no. 1, p. 13, 2015.

[6] E. Mezura-Montes and C. A. C. Coello, "Constraint-handling in nature-inspired numerical optimization: Past, present and future," *Swarm and Evol. Comput.*, vol. 1, no. 4, pp. 173–194, 2011.

[7] H. Jain and K. Deb, "An Evolutionary Many-Objective Optimization Algorithm Using Reference-Point Based Nondominated Sorting Approach, Part II: Handling Constraints and Extending to an Adaptive Approach," *IEEE TEVC*, vol. 18, no. 4, pp. 602–622, 2014.

[8] H. Seada and K. Deb, "A Unified Evolutionary Optimization Procedure for Single, Multiple, and Many Objectives," *IEEE TEVC*, vol. 20, no. 3, pp. 358–369, 2016.

[9] M. Asafuddoula, T. Ray, and R. A. Sarker, "A Decomposition-Based Evolutionary Algorithm for Many Objective Optimization," *IEEE TEVC*, vol. 19, no. 3, pp. 445–460, 2015.

[10] K. Li, K. Deb, Q. Zhang, and S. Kwong, "An Evolutionary Many-Objective Optimization Algorithm Based on Dominance and Decomposition," *IEEE TEVC*, vol. 19, no. 5, pp. 694–716, 2015.

[11] R. Cheng, Y. Jin, M. Olhofer, and B. Sendhoff, "A Reference Vector Guided Evolutionary Algorithm for Many-Objective Optimization," *IEEE TEVC*, vol. 20, no. 5, pp. 773–791, 2016.

[12] T. Chugh, K. Sindhya, K. Miettinen, J. Hakanen, and Y. Jin, "On Constraint Handling in Surrogate-Assisted Evolutionary Many-Objective Optimization," in *PPSN*, 2016, pp. 214–224.

[13] N. Srinivas and K. Deb, "Multiobjective Optimization Using Nondominated Sorting in Genetic Algorithms," *Evol. Comput.*, vol. 2, no. 3, pp. 221–248, 1994.

[14] M. Tanaka, H. Watanabe, Y. Furukawa, and T. Tanino, "GA-based decision support system for multicriteria optimization," in *IEEE SMC*, 1995, pp. 1556–1561.

[15] A. Osyczka and S. Kundu, "A new method to solve generalized multicriteria optimization problems using the simple genetic algorithm," *Stru. and Multi. Opt.*, vol. 10, no. 2, pp. 94–99, 1995.

[16] K. Deb, A. Pratap, and T. Meyarivan, "Constrained Test Problems for Multi-objective Evolutionary Optimization," in *EMO*, 2001, pp. 284–298.

[17] Q. Zhang, A. Zhou, S. Zhao, P. N. Suganthan, W. Liu, and S. Tiwari, "Multiobjective optimization test instances for the CEC 2009 special session and competition," Univ. of Essex, Tech. Rep. CES-487, 2008.

[18] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler, "Scalable multi-objective optimization test problems," in *IEEE CEC*, 2002, pp. 825–830.

[19] C. A. C. Coello and G. T. Pulido, "Multiobjective structural optimization using a microgenetic algorithm," *Stru. and Multi. Opt.*, vol. 30, no. 5, pp. 388–403, 2005.

[20] T. Ray, K. Tai, and C. Seow, "An evolutionary algorithm for multiobjective optimization," *Eng. opt.*, vol. 33, no. 3, pp. 399–424, 2001.

[21] J. Li, Y. Wang, S. Yang, and Z. Cai, "A comparative study of constraint-handling techniques in evolutionary constrained multiobjective optimization," in *IEEE CEC*, 2016, pp. 4175–4182.

[22] S. Venkatraman and G. G. Yen, "A Generic Framework for Constrained Optimization Using Genetic Algorithms," *IEEE TEVC*, vol. 9, no. 4, pp. 424–435, 2005.

[23] Y. G. Woldesenbet, G. G. Yen, and B. G. Tessema, "Constraint Handling in Multiobjective Evolutionary Optimization," *IEEE TEVC*, vol. 13, no. 3, pp. 514–525, 2009.

[24] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE TEVC*, vol. 6, no. 2, pp. 182–197, 2002.

[25] P. Hingston, L. Barone, S. Huband, and R. L. While, "Multi-level Ranking for Constrained Multi-objective Evolutionary Optimisation," in *PPSN*, 2006, pp. 563–572.

[26] W. Gong, Z. Cai, and Z. Li, "An efficient multiobjective differential evolution algorithm for engineering design," *Stru. and Multi. Opt.*, vol. 38, no. 2, pp. 137–157, 2009.

[27] H. K. Singh, A. Isaacs, T. Ray, and W. Smith, "Infeasibility Driven Evolutionary Algorithm (IDEA) for Engineering Design Optimization," in *AI2008*, 2008, pp. 104–115.

[28] T. Ray and K. M. Liew, "A swarm metaphor for multiobjective design optimization," *Eng. opt.*, vol. 34, no. 2, pp. 141–153, 2002.

[29] M. G. Parsons and R. L. Scott, "Formulation of Multicriterion Design Optimization Problems for Solution With Scalar Numerical Optimization Methods," *J. Ship Research*, vol. 48, no. 1, pp. 61–76, 2004.

[30] E. Zitzler, L. Thiele, M. Laumanns, C. M. Fonseca, and V. G. da Fonseca, "Performance assessment of multiobjective optimizers: an analysis and review," *IEEE TEVC*, vol. 7, no. 2, pp. 117–132, 2003.

[31] M. López-Ibáñez, J. D. Knowles, and M. Laumanns, "On Sequential Online Archiving of Objective Vectors," in *EMO*, 2011, pp. 46–60.

[32] A. Radulescu, M. López-Ibáñez, and T. Stützle, "Automatically Improving the Anytime Behaviour of Multiobjective Evolutionary Algorithms," in *EMO*, 2013, pp. 825–840.

[33] D. Brockhoff, T. Tran, and N. Hansen, "Benchmarking Numerical Multiobjective Optimizers Revisited," in *GECCO*, 2015, pp. 639–646.

[34] T. Tusar, D. Brockhoff, N. Hansen, and A. Auger, "COCO: The Bi-objective Black Box Optimization Benchmarking (bbob-biobj) Test Suite," *CoRR*, vol. abs/1604.00359, 2016.

[35] H. Ishibuchi, Y. Sakane, N. Tsukamoto, and Y. Nojima, "Selecting a small number of representative non-dominated solutions by a hypervolume-based solution selection approach," in *FUZZ-IEEE*, 2009, pp. 1609–1614.

[36] A. Oyama, K. Shimoyama, and K. Fujii, "New Constraint-Handling Method for Multi-Objective and Multi-Constraint Evolutionary Optimization," *T. JSASS*, vol. 50, no. 167, pp. 56–62, 2007.