Challenging Novel Many and Multi-Objective Bound Constrained Benchmark Problems

Hui Li[*], Kalyanmoy Deb[†], Qingfu Zhang[‡], Ponnuthurai Nagaratnam Suganthan[§]

1 An Overview of Current Test Problems

Over the past decades, efforts have been made to solve multi and many-objective optimization problems. The major difficulty of many objective problems is caused by the high number of objective functions. It is well-established that the evolutionary multiobjective optimization (EMO) algorithms based on Pareto dominance cannot guarantee the convergence of moderate sized population towards the Pareto fronts of many-objective optimization problems. In comparison with these algorithms, the decomposition-based EMO algorithms have more advantages in convergence since the diversity of population can be maintained according to different directions. Among the MOEA benchmarks [1, 2, 3, 4], DTLZ [1] and WFG [2], have been widely extended for comparing the performance of EMO algorithms for many-objective optimization. In fact, these test problems can be well solved when the EMO algorithms have good strategies to maintain the diversity of the population by directions. In many cases, the EMO algorithms with small population size (e.g., around 100) can work well on approximating the Pareto fronts of these [1, 2], many-objective optimization problems.

Note that the use of small population in EMO algorithms might not be able to approximate the Pareto fronts of many-objective optimization with more challenging problem difficulties. In fact, some challenging problem difficulties in multi-objective optimization, such as objective scalability, multi-modality, complicated Pareto sets [5], disconnectedness, degeneracy [6, 7], and bias [8], were ignored when dealing with many-objective optimization problems. It is nontrivial that many-objective optimization problems can become more challenging if the above-mentioned problem difficulties are involved. Apart from the diversity strategies, some other issues like selection operators, memory structure of the population and reproduction operators, can also be the key factors for solving many-objective optimization problems satisfactorily.

2 Challenging Test Problems for Many and Multi Objective Optimization

In this work, we construct a set of ten test problems with challenging difficulties based on the framework of DTLZ test problems, where each objective function is the multiplication of distance function and shape function. The general mathematical formulation of these test problems can be

*Hui Li is with the School of Mathematics and Statistics in Xi'an Jiaotong University, China (email:lihui10@xjtu.edu.cn)

†Kalyanmoy Deb is with the Department of Electrical and Computer Engineering, Michigan State University, United States(email: kdeb@msu.edu)

‡Qingfu Zhang is with the Department of Computer Science, City University of Hong Kong, Hongkong (email: qingfu.zhang@cityu.edu.hk)

§Ponnuthurai Nagaratnam Suganthan is with the School of Electrical & Electronic Engineering College of Engineering, Nanyang Technological University, Singapore (email:EPNSugan@ntu.edu.sg)

stated as follows:

$$\begin{array}{lll} \text{minimize} \;\; f_1 & = & r_1 \times \alpha_1(x) \times (1 + g_1(x)) \\ \text{minimize} \;\; f_2 & = & r_2 \times \alpha_2(x) \times (1 + g_2(x)) \\ \cdots\cdots & & \\ \text{minimize} \;\; f_{M-1} & = & r_{M-1} \times \alpha_{M-1}(x) \times (1 + g_{M-1}(x)) \\ \text{minimize} \;\; f_M & = & r_M \times \alpha_M(x) \times (1 + g_M(x)) \end{array} \tag{1}$$

where

- $x = (x_1, \ldots, x_N)$ is the decision vector in $[0, 1]^N$, and $(f_1, \ldots, f_M)$ is the objective vector in R^M;
- r_i, α_i, and $g_i, i = 1, \ldots, M$, are the scaling factor, the shape function, and the distance function for the i-th objective function, respectively.

With the above formulation, we give the detailed description of ten new test problems below.

1. MaOP1 (a variant of DTLZ1)

   - shape function:

$$\begin{array}{lll} \alpha_1 & = & 1 - x_1 x_2 \cdots x_{M-1} \\ \alpha_2 & = & 1 - x_1 x_2 \cdots (1 - x_{M-1}) \\ \cdots\cdots & & \\ \alpha_{M-1} & = & 1 - x_1(1 - x_2) \\ \alpha_M & = & 1 - (1 - x_1) \end{array} \tag{2}$$

   difficulty: inverse of a simplex
   - distance function:

$$g_i(x) = 100 \left[(N - M + 1) + \sum_{j=M}^{N} \left((x_j - 0.5)^2 + \cos(20\pi(x_j - 0.5)) \right) \right]$$

   - difficulty: many local Pareto fronts
   - scaling factor:
$$r_i = 0.1 + 10 \times (i - 1), i = 1, \ldots, M$$
   - difficulty: objective scalability
   - the PF and the PS:

$$PF = \{F(x) \in R^M | f_1/r_1 + f_2/r_2 + \cdots + f_M/r_M = M - 1\}$$

   and
$$PS = \{x \in R^N | x_i \in [0, 1], i = 1, \ldots, M - 1, x_i = 0.5, i = M, \ldots, N\}$$

2. MaOP2 (a variant of DTLZ2)

   - shape function:

$$\begin{array}{lll} \alpha_1 & = & [\cos(0.5x_1\pi)\cos(0.5x_2\pi) \cdots \cos(0.5x_{M-2}\pi)\cos(0.5x_{M-1}\pi))]^{p_1} \\ \alpha_2 & = & [\cos(0.5x_1\pi)\cos(0.5x_2\pi) \cdots \cos(0.5x_{M-2}\pi)\sin(0.5x_{M-1}\pi))]^{p_2} \\ \cdots\cdots & & \\ \alpha_{M-1} & = & [\cos(0.5x_1\pi)\sin(0.5x_2\pi)]^{p_{M-1}} \\ \alpha_M & = & [\sin(0.5x_1\pi)]^{p_M} \end{array} \tag{3}$$

   where
$$p_i = \begin{cases} 2 & \text{if } \mathrm{mod}(i, 2) = 1 \\ 4 & \text{otherwise.} \end{cases} \quad i = 1, \ldots, M$$

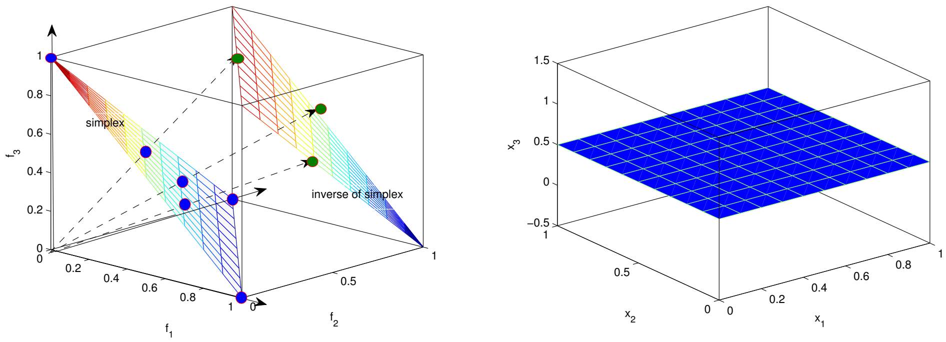

Figure 1: The PF with inverse of a simplex (left) and the PS with a linear plane (right) of MaOP1

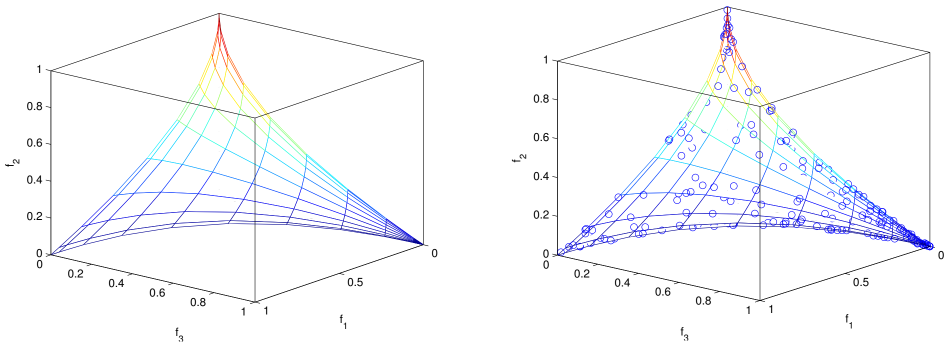

Figure 2: The PF of MaOP2: the distribution of 200 random solutions shows the bias towards the boundary of PF.

- Difficulty: biased
- distance function

$$g_i(x) = \sum_{j=M}^{N} (x_j - y_j)^2, i = 1, \dots, M$$

with

$$y_j = \begin{cases} 0.5 & \text{if } \mathrm{mod}(j,5) \neq 0 \\ \prod_{k=1}^{M-1} \sin(0.5 x_k \pi) & \text{otherwise} \end{cases}$$

- Difficulty: nonlinear Pareto set
- scaling factor: $r_i = 1, i = 1, \dots, M$
- the PF and the PS:

$$PF = \{F(x) \in R^M | f_1^{q_1} + f_2^{q_2} + \dots + f_M^{q_M} = M - 1\}$$

with

$$q_j = \begin{cases} 1 & \text{if } j \text{ is even} \\ 1/2 & \text{otherwise} \end{cases} \quad j = 1, \dots, M$$

and

$$\begin{aligned} PS = &\{x_j \in [0,1] | j \in \{1, \dots, M-1\}\} \cup \{x_j = 0.5 | j \in \{M, \dots, n\} \text{ and } \mathrm{mod}(j,5) \neq 0\} \\ &\cup \{x_j = \prod_{k=1}^{M-1} \sin(0.5 x_k \pi) | j = \{M, \dots, n\} \text{ and } \mathrm{mod}(j,5) = 0\} \end{aligned}$$

3. MaOP3

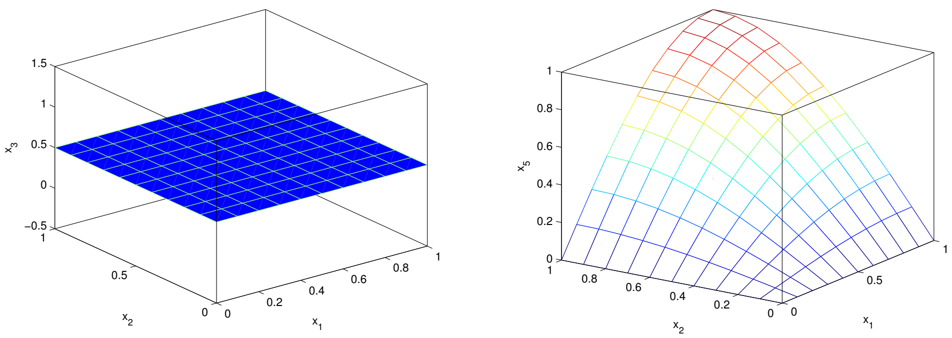

Figure 3: The PSs of MaOP2: left - linear interaction among $x_1 - x_2 - x_3$, right - nonlinear interaction among $x_1 - x_2 - x_5$)

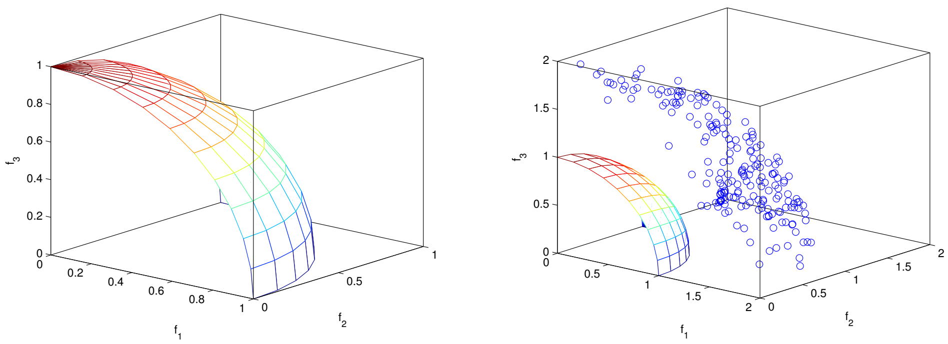

Figure 4: The PF of MaOP3: the distribution of 200 random solutions shows the bias on the convergence towards the PF.

- shape function:

$$\begin{array}{lcl} \alpha_1 & = & \cos(0.5x_1\pi)\cos(0.5x_2\pi)\cdots\cos(0.5x_{M-2}\pi)\cos(0.5x_{M-1}\pi)) \\ \alpha_2 & = & \cos(0.5x_1\pi)\cos(0.5x_2\pi)\cdots\cos(0.5x_{M-2}\pi)\sin(0.5x_{M-1}\pi)) \\ \cdots\cdots & & \\ \alpha_{M-1} & = & \cos(0.5x_1\pi)\sin(0.5x_2\pi) \\ \alpha_M & = & \sin(0.5x_1\pi) \end{array} \tag{4}$$

- distance function:

$$g_i(x) = 10 \times i \times \sum_{j=M}^{N} |x_j - y_j|^{0.1}$$

  with

$$y_j = \begin{cases} 0.5 & \text{if } \mathrm{mod}(j,5) \neq 0 \\ \prod_{k=1}^{M-1} \sin(0.5x_k\pi) & \text{otherwise} \end{cases}$$

- scaling factor: $r_i = 1, i = 1, \ldots, M$
- the PF and the PS:

$$PF = \{F(x) \in [0,1]^M | f_1^2 + f_2^2 + \cdots + f_M^2 = 1\}$$

  and the PS is the same as that of MaOP2.

4. MaOP4

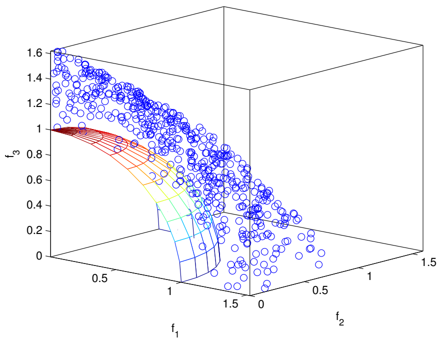

Figure 5: The PF of MaOP4: the distribution of 200 random solutions shows the bias on the convergence towards the PF with $x_1 = 0$ or $x_1 = 1$.

- shape function:

$$\begin{array}{lcl} \alpha_1 & = & \cos(0.5x_1\pi)\cos(0.5x_2\pi)\cdots\cos(0.5x_{M-2}\pi)\cos(0.5x_{M-1}\pi)) \\ \alpha_2 & = & \cos(0.5x_1\pi)\cos(0.5x_2\pi)\cdots\cos(0.5x_{M-2}\pi)\sin(0.5x_{M-1}\pi)) \\ \cdots\cdots & & \\ \alpha_{M-1} & = & \cos(0.5x_1\pi)\sin(0.5x_2\pi) \\ \alpha_M & = & \sin(0.5x_1\pi) \end{array} \tag{5}$$

- distance function:

$$g_i(x) = 2\sin(\pi x_1) \times \sum_{j=M}^{N} [-0.9(x_j - y_j)^2 + |x_j - y_j|^{0.6}]$$

  with

$$y_j = \begin{cases} 0.5 & \text{if } \mathrm{mod}(j,5) \neq 0 \\ \prod_{k=1}^{M-1} \sin(0.5x_k\pi) & \text{otherwise} \end{cases}$$

- scaling factor: $r_i = 1, i = 1, \ldots, M$
- the PF and the PS:
  the PF and the PS is the same as that of MaOP3.
- difficulties: variable bias (x_1) and nonlinear PS

5. MaOP5

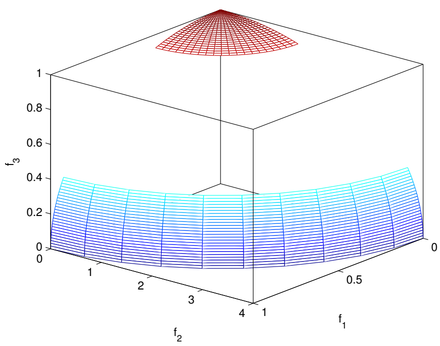

Figure 6: The PF of MaOP5.

- shape function:

$$
\begin{array}{lll}
\alpha_1 & = & \cos(0.5x_1\pi)\cos(0.5x_2\pi) \\
\alpha_2 & = & \cos(0.5x_1\pi)\sin(0.5x_2\pi) \\
\alpha_3 & = & \sin(0.5x_1\pi) \\
\cdots\cdots & & \\
\alpha_i & = & \frac{i}{M}\times\alpha_1(x_1,x_2)+(1-\frac{i}{M})\times\alpha_2(x_1,x_2)+\sin(\frac{0.5i\pi}{M})\alpha_3(x_1,x_2), \quad i=4,\ldots,M
\end{array} \tag{6}
$$

- distance function:

$$
g_i(x)=\begin{cases} \max(0,-1.4\cos(2x_1\pi))+\sum_{j=3}^{N}(x_j-x_1x_2)^2 & i=1,2,3 \\ \exp((x_i-x_1x_2)^2)-1 & i=4,\ldots,M \end{cases}
$$

- difficulty: nonlinear Pareto set
- scaling factor: $r_i=1, i=1,3\ldots,M$ and $r_2=4$.
- the PF and the PS:

$$
PF=\{(f_1,\ldots,f_M)|f_1^2+f_2^2/16+f_3^2=1, f_2\in[0,4\sin(\pi/8)]\cup[4\sin(3\pi/8),4]\}
$$

- difficulties: degeneracy, disconnectedness, nonlinear PS

6. MaOP6

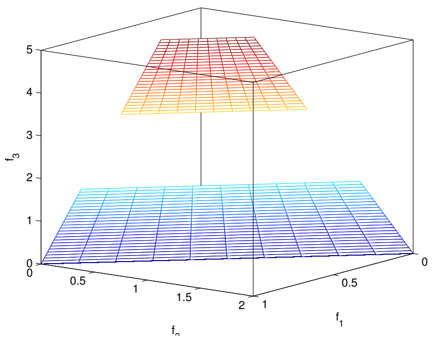

Figure 7: The PF of MaOP6

- shape function:

$$
\begin{array}{lll}
\alpha_1 & = & x_1x_2 \\
\alpha_2 & = & x_1(1-x_2) \\
\alpha_3 & = & (1-x_1) \\
\cdots\cdots & & \\
\alpha_i & = & \frac{i}{M}\times\alpha_1(x_1,x_2)+(1-\frac{i}{M})\times\alpha_2(x_1,x_2)+\mathrm{mod}(i,2)\times\alpha_3(x_1,x_2), \quad i=4,\ldots,M
\end{array} \tag{7}
$$

- difficulty: degeneracy on a minority of objectives and distortion of simplex
- distance function:

$$
g_i(x)=\max(0,1.4\sin(4x_1\pi))+\begin{cases} \sum_{j=1}^{3}|x_j-x_1x_2|^2 & i=1,2,3 \\ \exp(|x_i-x_1x_2|^2)-1 & i=4,\ldots,N \end{cases}
$$

- difficulty: nonlinear Pareto set
- scaling factor: $r_1=2, r_2=6,\ r_i=1, i=3\ldots,M$.

7. MaOP7

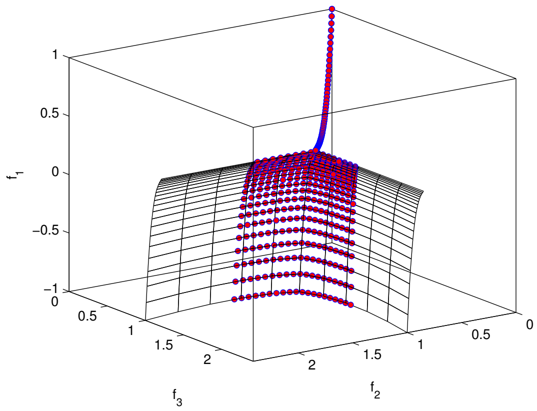

Figure 8: The PF of MaOP7 in $f_1 - f_2 - f_3$: two local 1-D curves, two local 2-D surfaces, connected.

- shape function:

$$\begin{array}{lcl} \alpha_1 & = & (-1) \times (2x_1 - 1)^3 \\ \alpha_{2k}(x_1, x_k) & = & x_1 + \tau u(x_k) + \tau |2x_k - 1|^{0.5+x_1} \\ \alpha_{2k+1}(x_1, x_k) & = & x_1 - \tau u(x_k) + \tau |2x_k - 1|^{0.5+x_1} \\ & & \qquad\qquad\qquad k = 2, \ldots, T+1 \\ \alpha_M & = & \begin{cases} 1 - \alpha_1 & \text{if } M \text{ is even} \\ \alpha_{2T+1} & \text{otherwise} \end{cases} \end{array}$$

  Here, the constant $\tau = \sqrt{2}/2$.

- distance function:

$$g_i(x) = \sum_{j=T+2}^{N} (x_j - y_j)^2, i = 1, \ldots, M$$

  with

$$y_j = \begin{cases} 0.5 & \text{if } \mathrm{mod}(j, 5) \neq 0 \\ \prod_{k=1}^{T+1} \sin(0.5 x_k \pi) & \text{otherwise} \end{cases}$$

- scaling factor: $r_i = 1, i = 1, \ldots, M$.
- the PS and the PF:

$$PS = \Omega_1 \cup \Omega_2$$

  with

$$\Omega_1 = \left\{ x \in [0,1]^n \middle| x_1 \in [0, 0.5], x_k = 0.5, k = 2, \ldots, T+1 \right\}$$

$$\Omega_2 = \left\{ x \in [0,1]^n \middle| x_1 \in (0.5, 1], x_k \in \left[\frac{1 - e(x_1)}{2}, \frac{1 + e(x_1)}{2} \right], k = 2, \ldots, T+1 \right\}$$

  with

$$e(x_1) = (0.5 + x_1)^{\frac{1}{0.5 - x_1}}$$

  The distribution of $PF = F(PS)$ in $f_1 - f_2 - f_3$ is plotted in Fig.8.

8. MaOP8

- shape function:

$$\begin{array}{lcl} \alpha_1 & = & (-1) \times (2x_1 - 1)^3 \\ \alpha_{2k} & = & x_1 + \tau(2x_k - 1) + \tau |2x_k - 1|^{1 - \sin(4x_1\pi)} \\ \alpha_{2k+1} & = & x_1 - \tau(2x_k - 1) + \tau |2x_k - 1|^{1 - \sin(4x_1\pi)} \\ & & \qquad\qquad\qquad k = 2, \ldots, T \\ \alpha_M & = & \begin{cases} 1 - \alpha_1 & \text{if } M \text{ is even} \\ \alpha_{2T+1} & \text{otherwise} \end{cases} \end{array}$$

Here, the constant $\tau = \sqrt{2}/2$.

- distance function: the $g_i, i = 1, \ldots, M$, are the same as those in MaOP2.
- scaling factor: $r_i = 1, i = 1, \ldots, M$.
- the PS and the PF:

$$PS = \Omega_1 \cup \Omega_2$$

with

$$\Omega_1 = \left\{ x \in [0,1]^n \middle| x_1 \in [0, 0.25] \cup [0.5, 0.75], x_k = 0.5, k = 2, \ldots, T+1 \right\}$$

$$\Omega_2 = \left\{ x \in [0,1]^n \middle| x_1 \in (0.25, 0.5) \cup (0.75, 1], x_k \in \left[\frac{1-e(x_1)}{2}, \frac{1+e(x_1)}{2} \right], k = 2, \ldots, T+1 \right\}$$

with

$$e(x_1) = (1 - \sin(4\pi x_1))^{\frac{1}{\sin(4\pi x_1)}}$$

The distribution of $PF = F(PS)$ in $f_1 - f_2 - f_3$ is plotted in Fig.9.

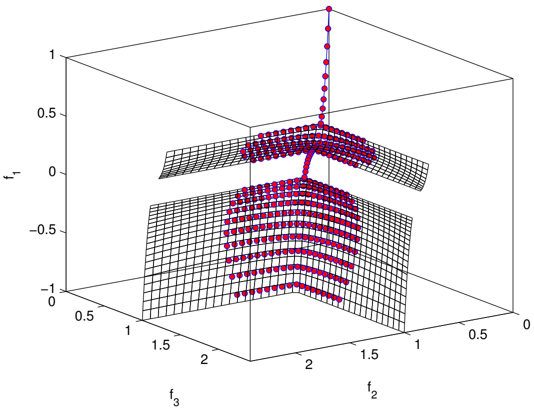

Figure 9: The PF of MaOP8 in $f_1 - f_2 - f_3$: two local 1-D curves, two local 2-D surfaces, connected.

9. MaOP9

- shape function:

$$\begin{array}{lcl} \alpha_1 & = & (-1) \times (2x_1 - 1)^3 \\ \alpha_{2k} & = & x_1 + \tau 2x_k + \tau |2(2x_k - \lfloor 2x_k \rfloor) - 1|^{0.5+x_1} \\ \alpha_{2k+1} & = & x_1 - \tau 2x_k + \tau |2(2x_k - \lfloor 2x_k \rfloor) - 1|^{0.5+x_1} \\ & & \hfill k = 2, \ldots, T \\ \alpha_M & = & \begin{cases} 1 - \alpha_1 & \text{if } M \text{ is even} \\ \alpha_{2T+1} & \text{otherwise} \end{cases} \end{array}$$

Here, the constant $\tau = \sqrt{2}/2$.

- distance function: the $g_i, i = 1, \ldots, M$, are the same as those in MaOP2.
- scaling factor: $r_i = 1, i = 1, \ldots, M$.
- the PS and the PF:

$$PS = \Omega_1 \cup \Omega_2 \cup \Omega_3$$

where

$$\Omega_1 = \left\{ x \in [0,1]^n \middle| x_1 \in [0, 0.5], x_k \in \{0.25, 0.75\}, k = 2, \ldots, T+1 \right\}$$

$$\Omega_2 = \left\{ x \in [0,1]^n \middle| x_1 \in (0.5, 1], x_k \in \left[\frac{1-e(x_1)}{4}, \frac{1+e(x_1)}{4} \right], k = 2, \ldots, T+1 \right\}$$

$$\Omega_3 = \left\{ x \in [0,1]^n \middle| x_1 \in (0.5, 1], x_k \in \left[\frac{3-e(x_1)}{4}, \frac{3+e(x_1)}{4}\right], k = 2, \ldots, T+1 \right\}$$

with

$$e(x_1) = (0.5 + x_1)^{\frac{1}{0.5-x_1}}$$

The distribution of $PF = F(PS)$ in $f_1 - f_2 - f_3$ is plotted in Fig.10.

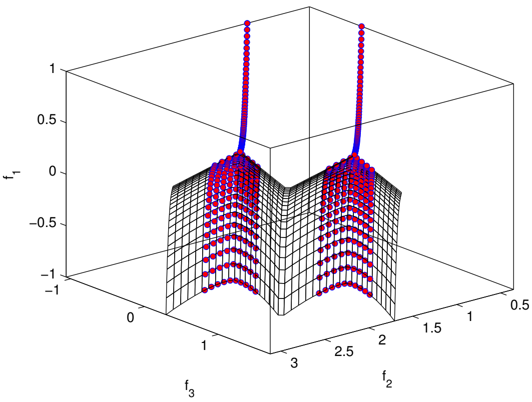

Figure 10: The PF of MaOP9 in $f_1 - f_2 - f_3$: two 1-D curves, two 2-D surfaces, disconnected.

10. MaOP10

- shape function:

$$\begin{array}{lcl}
\alpha_1 & = & (-1) \times (2x_1 - 1)^3 \\
\alpha_{2k} & = & x_1 + \tau 2x_k + \tau |2(2x_k - \lfloor 2x_k \rfloor) - 1|^{p(x_1,x_k)} \\
\alpha_{2k+1} & = & x_1 - \tau 2x_k + \tau |2(2x_k - \lfloor 2x_k \rfloor) - 1|^{p(x_1,x_k)} \\
 & & \qquad\qquad\qquad\qquad k = 2, \ldots, T \\
\alpha_M & = & \begin{cases} 1 - \alpha_1 & \text{if } M \text{ is even} \\ \alpha_{2T+1} & \text{otherwise} \end{cases}
\end{array}$$

  with

$$p(x_1, x_k) = \begin{cases} 0.5 + x_1 & \text{if } \mathrm{mod}(\lfloor 2x_k \rfloor, 2) = 0 \\ 1.5 - x_1 & \text{otherwise} \end{cases}$$

  Here, the constant $\tau = \sqrt{2}/2$.
- distance function: the $g_i, i = 1, \ldots, M$, are the same as those in MaOP2.
- scaling factor: $r_i = 1, i = 1, \ldots, M$.
- the PS and the PF:

$$PS = \Omega_1 \cup \Omega_2 \cup \Omega_3 \cup \Omega_4$$

  where

$$\Omega_1 = \left\{ x \in [0,1]^n \middle| x_1 \in [0, 0.5], x_k = 0.25, k = 2, \ldots, T+1 \right\}$$

$$\Omega_2 = \left\{ x \in [0,1]^n \middle| x_1 \in [0.5, 1], x_k = 0.75, k = 2, \ldots, T+1 \right\}$$

$$\Omega_3 = \left\{ x \in [0,1]^n \middle| x_1 \in [0, 0.5), x_k \in \left[\frac{1-e_1(x_1)}{4}, \frac{1+e_1(x_1)}{4}\right], k = 2, \ldots, T+1 \right\}$$

$$\Omega_4 = \left\{ x \in [0,1]^n \middle| x_1 \in (0.5, 1], x_k \in \left[\frac{3-e_2(x_1)}{4}, \frac{3+e_2(x_1)}{4}\right], k = 2, \ldots, T+1 \right\}$$

  with

$$e_1(x_1) = (0.5 + x_1)^{\frac{1}{0.5-x_1}} e_2(x_1) = (1.5 - x_1)^{\frac{1}{x_1-0.5}}$$

  The distribution of $PF = F(PS)$ in $f_1 - f_2 - f_3$ is plotted in Fig.11.

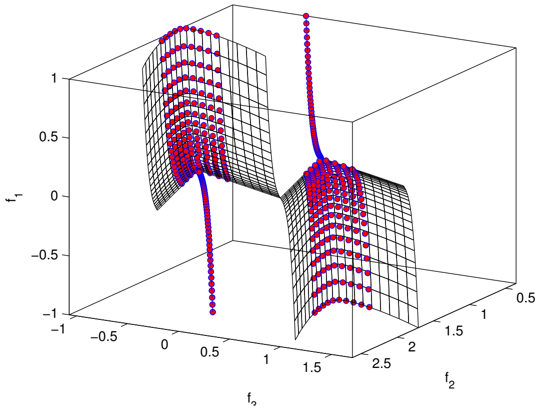

Figure 11: The PF of MaOP10 in $f_1 - f_2 - f_3$. Two 1-D curves and two 2-D surfaces.

3 Algorithms and Test Instances

3.1 Algorithms in Comparison

The participants should present their original contribution(s) by developing novel framework or algorithm or operators for many and multi objective optimization. Since most of the test problems in this benchmark include nonlinear interaction among variables, the differential evolution (DE) and polynomial mutation may produce better solutions during the reproduction. In this benchmarking exercise, the recommended baseline algorithmic framework for many-objective optimization is either MOEA/D or NSGA-III or any of their enhanced variants. By comparing with one of these baseline methods, authors will make the evaluation of their novel contributions easier.

3.2 The settings of test instances and major parameters in algorithms

The number of objectives M in MaOP1-MaOP10 is set as $M = 3, 5, 8, 10$. The number of variables in all ten test problems can be N =20 (small scale) or N =50 (large scale).

The population size in each algorithm is set to $pop = 100 \times M$. The total number of function evaluations is set to $pop \times 500$ for $N = 20$ and $pop \times 1000$ for $N = 50$. This population size restriction is introduced to compare different selection operators in many and multi objective EAs with DE-based reproduction operations.

Authors are also encouraged to present the best results of their novel algorithms by tuning the population size also with the same number of total function evaluations.

4 Performance Indicator

In this competition, all results should be evaluated in terms of inverted generational distance

$$IGD(S_{pf}, S) = \frac{1}{|S_{pf}|} \sum_{y \in S_{pf}} dist(y, S)$$

where $dist(y, S)$ measures the distance between the objective vector y and the set S of objective vectors obtained by some algorithm with following formulation:

$$dist(y, S) = \min_{z \in S} ||y - z||_2$$

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