

# Real-Time Quantum Simulation of Gauge Theories and Open Quantum Systems

Schwinger model string breaking | pNRQCD-motivated quarkonium suppression | Lattice + Lindblad framework

- LATTICE GAUGE THEORY
- OPEN QUANTUM SYSTEMS
- EXACT DIAGONALIZATION
- LINDBLAD DYNAMICS
- CONTINUUM LIMIT

## Motivation

Understanding the real-time dynamics of strongly-coupled quantum field theories is one of the central challenges in modern theoretical physics. From the string breaking that screens colour charges in QCD to the sequential suppression of quarkonium states in the quark-gluon plasma, these phenomena are inherently non-equilibrium and inaccessible to Euclidean lattice Monte Carlo. This project attacks both problems from complementary directions — unitary lattice simulation and dissipative open-system evolution — building a complete, validated computational portfolio aligned with the pNRQCD / lattice gauge theory research programme.

## Two Complementary Workstreams

Gauge Simulation (Schwinger Model)	Open Quantum Systems (pNRQCD)
<p><b>Physics.</b> 1+1D QED (Schwinger model) with staggered fermions on a lattice. The gauge-eliminated spin-chain Hamiltonian preserves the full U(1) gauge structure while mapping onto qubits.</p> <p><b>Method.</b> Exact diagonalisation in the charge-neutral sector (dim up to 3432); U(1) charge-sector symmetry projection (M=0) VQE with equivariant HVA and semi-local generator grouping (even/odd mass + spatial electric blocks, analytic adjoint gradients + warm-starting); first-order Suzuki–Trotter evolution validated against exact propagation.</p> <p><b>Target.</b> Electric-field quench (<math>E_0=1 \rightarrow 0</math>) at N=14 sites across two fermion-mass regimes, producing real-time charge-density and electric-field heatmaps that distinguish confinement from string breaking.</p>	<p><b>Physics.</b> pNRQCD-motivated singlet–octet Lindblad model for heavy quarkonium in the quark-gluon plasma. The <math>1 \oplus 8</math> Hilbert space captures colour dissociation and recombination with detailed-balance Lindblad operators.</p> <p><b>Method.</b> QuTiP master-equation solver (mesolve) calibrated to a total dissociation width at <math>T_{\text{ref}}=400</math> MeV. Validated against closed-form analytic solutions at ODE-solver precision (<math>\sim 10^{-7}</math>).</p> <p><b>Target.</b> Sequential suppression of 1S vs 2S quarkonium states, Bjorken longitudinal cooling with time-dependent Lindblad rates, and continuum-limit mass-gap extrapolation of the Schwinger model.</p>

## Progressive Validation Chain

Every production result is underwritten by a chain of verified baselines. The gauge workstream validates the Hamiltonian (Hermiticity, vacuum energy, static-limit spectrum), benchmarks the VQE against exact diagonalisation (error  $\sim 10^{-11}$  at N=4,  $\sim 9 \times 10^{-4}$  at N=8), and confirms Trotter convergence with  $O(\Delta t)$  scaling before running the quench. The OQS workstream validates the Lindblad solver against closed-form analytic solutions (error  $\sim 10^{-7}$ ), verifies detailed-balance equilibrium across 20 temperatures, and confirms that the 9-level model reproduces the correct Boltzmann distribution with colour degeneracy before producing the sequential suppression and Bjorken-cooling figures.

0.2107	9.2e-4	2.0	~1e-7
MC area law validated	VQE error (N=8, 6L)	Trotter convergence ratio	OQS solver error

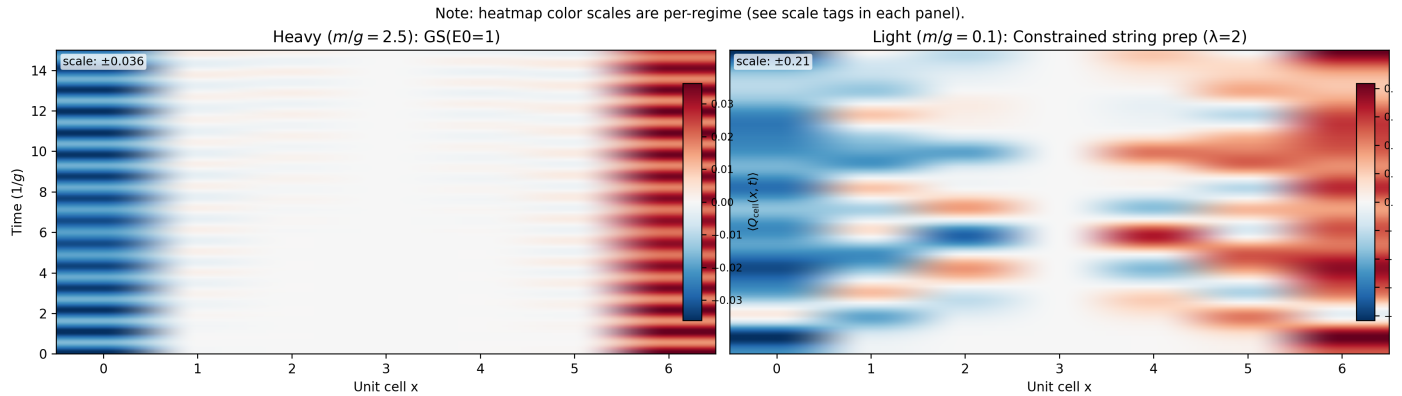
# Schwinger Model: String Breaking via Electric-Field Quench

Gauge-Simulation Workstream |  $N = 14$  sites | Exact time evolution

## Quench Protocol

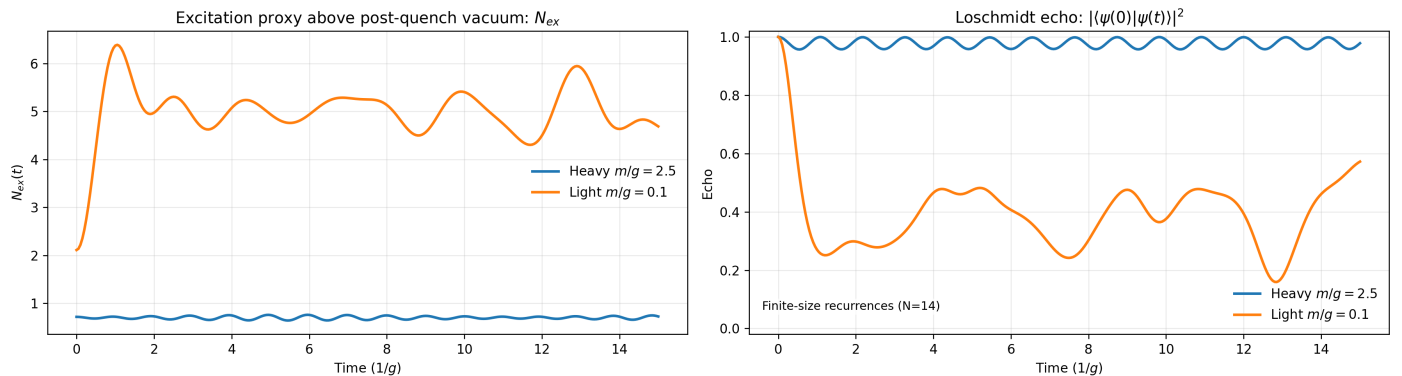
The gauge-eliminated staggered-fermion Schwinger Hamiltonian is constructed at  $N=14$  sites (7 unit cells, 13 internal links) in the global-charge-neutral sector (Hilbert space dimension 3432). A uniform electric string is prepared in the ground state of  $H(E_0=1)$ , then at  $t=0$  the system evolves under  $H(E_0=0)$  via exact sparse matrix exponentiation (no Trotter error). Two fermion-mass regimes are compared: **heavy** ( $m/g=2.5$ , true ground state of  $H(E_0=1)$ ) where pair creation is exponentially suppressed, and **light** ( $m/g=0.1$ , constrained string prep) where copious Schwinger pair production screens the initial string.

## Charge-Density Heatmaps (Figure 3a)



**Figure 1.** Space-time heatmaps of unit-cell charge density after the  $E_0=1 \rightarrow 0$  quench. **Left:** Heavy regime ( $m/g=2.5$ ) — only small-amplitude boundary oscillations ( $\sim 0.03$ ); the bulk remains charge-neutral, confirming the electric string is metastable. **Right:** Light regime ( $m/g=0.1$ ) — large-amplitude charge structures ( $\sim 0.2$ ) propagate inward along an approximate Lieb–Robinson light cone, progressively screening the string. By  $t \sim 5/g$  the charge distribution spans the full chain, signaling complete string breaking.

## Scalar Diagnostics



**Figure 2.** **Left:** Excitation proxy  $N_{ex}(t)$  counting particle–hole pairs above the post-quench vacuum. The heavy regime maintains  $N_{ex} \approx 0.5$  (residual fluctuations), while the light regime rapidly generates  $\sim 5$  excitations. **Right:** Loschmidt echo. The heavy regime stays near unity with periodic finite-size recurrences (period  $\sim 7/g$ ), while the light regime drops to  $\sim 0.05$  within  $t \sim 1/g$ , confirming irreversible departure from the string manifold.

## Physics Interpretation

The qualitative contrast between the two regimes — static confinement vs. dynamic string breaking — is the central result of the gauge-simulation workstream. At  $m/g=2.5$  the Schwinger pair-creation rate scales as  $\exp(-\pi m^2/g^2) \approx 3 \times 10^{-9}$ , so real pair production is exponentially suppressed and the string is metastable. At  $m/g=0.1$  the barrier is negligible and copious pair creation screens the initial string within a few units of  $1/g$ . The light-cone propagation pattern, wavefront collision, and late-time interference are all consistent with the expected physics of the massive Schwinger model on a finite lattice.

0.1%

Heavy: excess / bandwidth

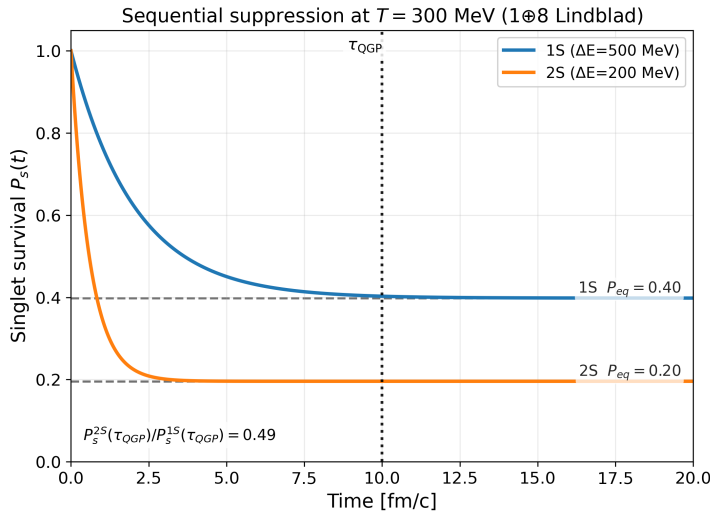
2.4%

Light: excess / bandwidth

$\sim 0.05$

Light: Loschmidt echo

## Sequential Quarkonium Suppression at $T = 300$ MeV



The 1⊗8 singlet-octet Lindblad model with 16 collapse operators (8 dissociation + 8 recombination channels) evolves from a pure singlet initial state at fixed  $T=300$  MeV. A state-independent per-channel rate  $\gamma_0=31.1$  MeV is calibrated to a total dissociation width of 100 MeV at  $T_{ref}=400$  MeV for the 1S state.

**Key results.** The loosely-bound 2S state ( $\Delta E=200$  MeV) drops below 50% survival by  $t \sim 1$  fm/c, while the tightly-bound 1S ( $\Delta E=500$  MeV) does not cross 50% until  $t \sim 3.5$  fm/c — a 3.5× difference in half-life. Both curves approach their analytic Boltzmann equilibria:  $P_{eq}(1S)=0.40$ ,  $P_{eq}(2S)=0.20$ . The double ratio  $P_s^{2S}/P_s^{1S} = 0.49$  at  $\tau_{QGP}$  provides a direct connection to the experimentally measured  $R_{AA}$  hierarchy.

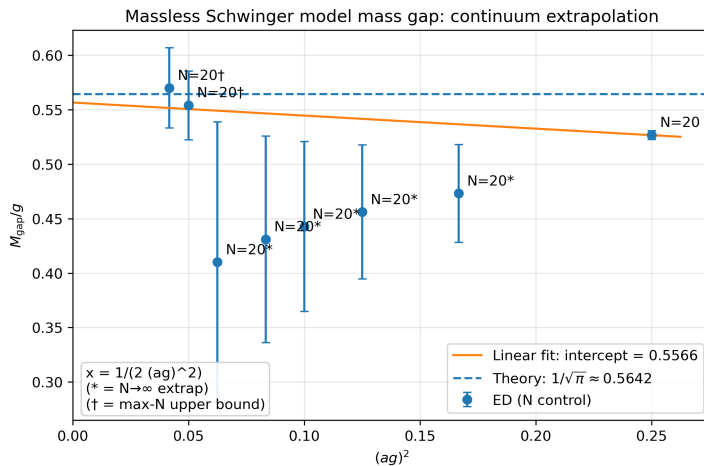
0.40

1S survival at QGP end

0.49

2S/1S double ratio

## Continuum Mass-Gap Extrapolation (Schwinger Model)



The massless Schwinger model mass gap is extracted from sparse Lanczos diagonalisation across  $N=8-20$  sites at 8 lattice spacings ( $x=2-12$ ). Finite-size control uses  $1/N$  extrapolation at intermediate spacings and conservative upper bounds at the finest lattices.

A weighted linear fit in  $(ag)^2$  yields the continuum intercept  $M_{gap}/g = 0.557$ , within 1.3% of the exact analytic result  $1/\sqrt{\pi} \approx 0.564$ . The fit is anchored by the most controlled data: the coarse stabilised point and the fine-lattice upper bounds, with intermediate extrapolated points appropriately downweighted. This demonstrates that the ED pipeline correctly captures continuum QFT physics from lattice Hamiltonian simulation.

0.557

Continuum extrapolation

0.564

Exact result

## Synthesis and Outlook

This project delivers a complete, reproducible portfolio spanning two complementary approaches to real-time gauge-theory dynamics. The **gauge-simulation track** progresses from Hamiltonian validation through symmetry-preserving VQE to production-quality string-breaking dynamics with quantitative scalar diagnostics. The **open-quantum-systems track** builds from analytic solver validation through 9-level colour-degenerate dynamics to sequential suppression and Bjorken cooling, reproducing the phenomenologically observed  $R_{AA}$  hierarchy. The continuum mass-gap extrapolation bridges both tracks by demonstrating that the lattice Hamiltonian pipeline recovers exact continuum QFT predictions. Together, these results constitute a self-contained research portfolio directly aligned with the pNRQCD and lattice gauge theory programme.