Unit 1 Basic Foundations

Assignment 1

Question 1

Let A be a set with n distinct elements. How many different binary relations on A are there?

Answer:

The number of binary relations on a set with n distinct elements is 2^{n^2}

Question 2

If $\sum = \{a, b, c\}$ then find the followings

$a. \sum 1, \sum 2, \sum 3$

Solution

1. Since there are three letters total possibilities of set of strings is $3^1 = 3$

   $\sum 1 = \{a, b, c\}$
2. Total Strings $= 3^2 = 9$

   $\sum 2 = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$
3. Total Strings $= 3^3 = 27$

   $\sum 3 = \{aaa, aab, aac, aba, abb, abc, aca, acb, acc, baa, bab, bac, bba, bbb, bbc, bca, bcb, bcc, caa, cab, cac, cba, cbb, cbc, cca, ccb, ccc\}$

Question 3

If $\sum = \{0, 1\}$. Then find the following languages

a. The language of string of length zero.
b. The language of strings of 0‟s and 1‟s with equal number of each.
c. The language $\{0^n\ 1^n | n \geq 1\}$
d. The language $\{0^i\ 0^j | 0 \leq i \leq j\}$
e. The language of strings with odd number of 0‟s and even number of 1‟s.

Solution

a. $L = \{\epsilon\}$

b. $L = \{\epsilon, 01, 0011, 0101, 1010, 0110, 1001, \ldots\}$ (all strings with equal number of 0's and 1's)

c. $L = \{01, 0011, 000111, 000011, \ldots\}$ (strings with n 0's followed by n 1's)

d. $L = \{\}$

e. $L = \{011, 0001111, 0000011, \ldots\}$ (strings with odd number of 0's and even number of 1's)

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4. Define the Kleene closure and power of alphabets.

Kleene Closure

The Kleene closure of a language L, denoted by L*, is the set of all possible strings that can be formed by concatenating zero or more strings from the language L. It can be defined mathematically as:

$\sum * = \varepsilon \, U \sum 1 \, U \; \sum 2 \, U \sum 3 \ldots$

where ε represents the empty string and U denotes the union operator.

For example, consider $\sum$ = {0, 1}. Then the Kleene closure of L, L*, is:

$L* = \{\varepsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, \ldots\}$

Power of Alphabets:

The power of an alphabet is the set of all possible strings that can be formed by concatenating some (possibly zero) number of symbols from the alphabet. It can be denoted by the alphabet raised to a power, e.g. $\sum^n$, where n is the number of symbols concatenated.

For example, consider the alphabet ∑ = {0, 1}. Then the power of ∑^2 is:

$\sum^2 = \{00, 01, 10, 11\}$

The power of an alphabet $\sum^0$ is defined as the set containing only the empty string ε.
