

Deep Learning Statistical Arbitrage

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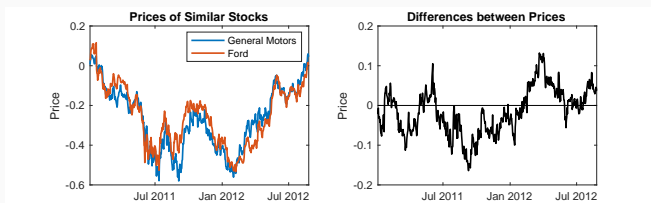
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Motivation

Intuition: Pairs trading (simplest statistical arbitrage)

- Identify two “similar” stocks: e.g. GM and Ford
- Assumption: prices are on average similar
- Exploit temporal price differences between similar assets



Three components of statistical arbitrage:

1. Construct long-short portfolio identifying mispricing: $\epsilon_t = R_t^{\text{GM}} - R_t^{\text{Ford}}$
2. Extract trading signals by statistically modeling ϵ_t
3. Find optimal trading policy given signals: $\max \mathbb{E} [\text{payoff}_T]$

Fundamental Problem

Research questions:

1. How much realistic arbitrage is in the U.S. equity markets?
2. What is the structure of statistical arbitrage?

Key elements of statistical arbitrage:

1. **Arbitrage portfolios**: How to generate long-short portfolios of similar assets?
2. **Arbitrage signal**: What are time-series patterns for temporary price deviations?
3. **Arbitrage allocation**: How to trade given the arbitrage signal?

Challenges:

1. **Large number** of assets with unknown similarities
2. **Complex time-series patterns** in price deviations
3. Optimal trading rules are **complicated** and depend on trading objective

Can machine learning help?

- Machine learning methods very flexible and deal with big data, but ...
- Important to set up the estimation problem correctly: Not a prediction problem!
- We use a **trading objective function on residuals** of asset pricing models

Novel conceptual framework:

- **Provide unified framework** to compare different statistical arbitrage methods:
(1) portfolio generation, (2) signal extraction, (3) allocation decision
- Study each component and compare with conventional models

Our novel method: **Deep learning statistical arbitrage**

1. **Statistical factor** model including characteristics to get arbitrage portfolios
 2. **Convolutional neural network + Transformer** to extract arbitrage signal:
Flexible data driven time-series filter to learn complex time-series patterns
 3. **Neural network** to map signals into allocations:
Generalization of conventional “optimal stopping rules” for investment.
- ⇒ We **integrate and optimize them for global economic objective**:
Maximize risk-adjusted return under constraints.
- ⇒ Most advanced AI for NLP for time-series pattern detection

Contribution: Empirical

Comprehensive out-of-sample study on U.S. equities

- Daily returns for 19 years of 500 largest liquid stocks
- Consider most important risk factor models
- Comparisons include parametric and non-parametric mean-reversion models

Excellent out-of-sample performance:

- Empirically **substantially outperforms** all benchmark approaches out-of-sample
- Our arbitrage strategies achieve annual **Sharpe ratios 4**
- **Annual returns of around 20%** with less than 6% volatility
- Uncorrelated with conventional risk factors and market movements
- Survives **realistic transaction and holding costs**
- Stable over time and robust to tuning parameters

What matters for arbitrage trading?

- **Robust to risk factors** to identify similar assets
- Most **important is time-series signal**; flexible allocation model insufficient
- 4x better than parametric models, 2x better than non-parametric
- **Global objective**: extract time-series model for trading

Insight into the structure of arbitrage trading:

- “Smooth” trend and mean-reversion patterns
- **Asymmetric policies**: fast reaction on downtrends, cautious trading on uptrends

Classical approaches to statistical arbitrage (parametric models)

- PCA + mean-reversion: Avellaneda and Lee (2010), Yeo and Papanicolaou (2017)
- Cointegration: Rad, Low and Faff (2016), Vidyamurthy (2004)
- Stochastic control: Cartea and Jaimungal (2016), Leung and Li (2015)
- Simple pairs trading: Gatev, Goetzmann and Rouwenhorst (2006)
- Intractable parametric models with ML: Mulvey, Sun, Wang, and Ye (2020)

Machine learning for asset pricing (explain risk premium not arbitrage)

- Pricing kernel: Chen, Pelger, Zhu (2019), Bryzgalova, Pelger, and Zhu (2019)
- Return prediction: Gu, Kelly and Xiu (2020), Jiang, Kelly and Xiu (2020), Murray, Xiao and Xia (2021)
- Factor models: Lettau and Pelger (2020), Kelly, Pruitt and Su (2019)

Machine learning for time-series (no trading objective)

- Time-series prediction: Lim and Zohren (2020), Krauss, Doa, and Huck (2017).

Model

Arbitrage portfolios

Excess returns of stocks follow a conditional factor model:

$$R_{n,t} = \beta_{n,t-1}^\top F_t + \epsilon_{n,t} \quad t = 1, \dots, T \text{ and } n = 1, \dots, N_t$$

- K factors F_t capture systematic risk.
- Loadings $\beta_{t-1} \in \mathbb{R}^{N_t \times K}$ are general function of information at time $t - 1$.

Factor models identify similar assets by similar exposures to risk factors

- Define *arbitrage portfolio* as residual portfolios:

$$\epsilon_{n,t} = R_{n,t} - \beta_{n,t-1}^\top F_t$$

- Arbitrage portfolios are only weakly cross-sectionally dependent.
- Arbitrage Pricing Theory implies $\mathbb{E}[\epsilon_{n,t}] = 0$.
- $\beta_{n,t-1}^\top F_t$ is “fair price” of $R_{n,t}$ and $\epsilon_{n,t}$ captures temporary mispricing

Arbitrage portfolios

Residuals with the empirically most important families of factor models:

1. **Observed fundamental factors**: Fama-French factors.
2. **Statistical factors** that explain correlations: PCA factors.
3. **Conditional statistical factors** where loadings are functions of firm characteristics: Instrumented PCA factors (Kelly, Pruitt and Su (2019)).

Factors are projections on returns without loss of generality:

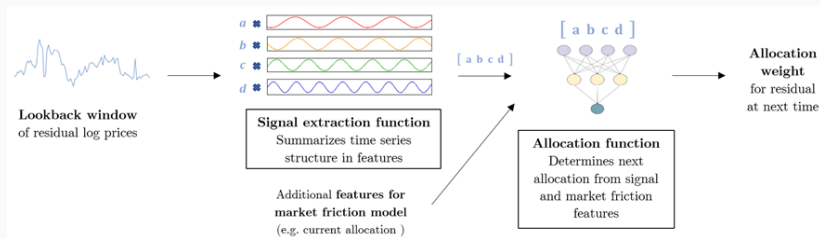
$$F_t = w_{t-1}^F{}^\top R_t.$$

Residuals are traded portfolios for factor implied matrix $\Phi_{t-1} \in \mathbb{R}^{N_t \times N_t}$:

$$\epsilon_t = R_t - \beta_{t-1}^\top F_t = R_t - \beta_{t-1}^\top w_{t-1}^F R_t = \underbrace{\left(I_{N_t} - \beta_{t-1}^\top w_{t-1}^F \right)}_{\Phi_{t-1}} R_t.$$

⇒ Arbitrage portfolios are traded, factor-neutral, weakly correlated and mean-reverting portfolios of all stocks.

Arbitrage Signal and Allocation



Arbitrage trading has 2 steps given a cumulative residual

$$x := \epsilon_t^L := \left(\epsilon_{n,t-L} \quad \sum_{l=1}^2 \epsilon_{n,t-L-1+l} \quad \cdots \quad \sum_{l=1}^L \epsilon_{n,t-L-1+l} \right)$$

1. The **arbitrage signal** function

$$\theta \in \Theta : \epsilon_{n,t-1}^L \mapsto \theta_{n,t-1}$$

models the time series structure using last L cumulative residuals and estimates a sufficient statistic for trading.

2. The **arbitrage allocation** function

$$w^\epsilon \in W : \theta_{n,t-1} \mapsto w_{n,t-1}^\epsilon.$$

assigns investment weights on residuals using only the estimated signal.

Estimation Problem

Estimation: For a given class of models maximize risk-adjusted return:

$$\begin{aligned} \max_{\mathbf{w}^\epsilon \in \mathcal{W}, \theta \in \Theta} \quad & \frac{\mathbb{E} \left[\mathbf{w}_{t-1}^R{}^\top R_t \right]}{\sqrt{\text{Var}(\mathbf{w}_{t-1}^R{}^\top R_t)}} \\ \text{s.t.} \quad & \mathbf{w}_{t-1}^R = \frac{\mathbf{w}_{t-1}^\epsilon{}^\top \Phi_{t-1}}{\|\mathbf{w}_{t-1}^\epsilon{}^\top \Phi_{t-1}\|_1} \quad \text{and} \quad \mathbf{w}_{t-1}^\epsilon = \mathbf{w}^\epsilon(\theta(\epsilon_{t-1}^L)). \end{aligned}$$

- Main objective: **Sharpe ratio**, but we also consider mean-variance objective
- Extension includes **trading costs**
- Stock weights \mathbf{w}_{t-1}^R add up to 1 \Rightarrow implicit leverage constraint
- Many models have separate objective for signal estimation

We consider 3 key model classes for signal θ and allocation \mathbf{w}^ϵ :

1. Parametric model: mean-reversion model with thresholding rule
2. Pre-specified time-series filters and non-parametric allocation
3. Deep-learning arbitrage: data-driven time-series filter and allocation

\Rightarrow We show what are the key elements for profitable arbitrage

Classical mean reversion trading: (Avellaneda and Lee (2010))

- Each residual is modeled as an Ornstein-Uhlenbeck (OU) process

$$dX_t = \kappa(\mu - X_t)dt + \sigma dB_t$$

- The allocation is a threshold rule on the ratio $\frac{X_t - \mu}{\sigma / \sqrt{2\kappa}}$.
- In our framework, this corresponds to

$$\theta^{\text{OU}}(x) = (\hat{\kappa}, \hat{\mu}, \hat{\sigma}, x_L), \quad w^x(\theta^{\text{OU}}) = \begin{cases} -1, & \text{if } \frac{x_L - \hat{\mu}}{\hat{\sigma} / \sqrt{2\hat{\kappa}}} > C_{\text{thres}} \\ 1 & \text{if } \frac{x_L - \hat{\mu}}{\hat{\sigma} / \sqrt{2\hat{\kappa}}} < -C_{\text{thres}} \\ 0 & \text{otherwise} \end{cases}$$

where C_{thres} is chosen optimally.

Limitations: Parametric model might be misspecified (eg. trends, multiple mean reversion frequencies, etc.), restrictive allocation function.

Second class: Pre-specified filter with neural network

Signal θ : General time-series model

- Pre-specified linear filter $\theta_l = \sum_{j=1}^L W_j^{\text{filter}} x_j$ (given matrix $W^{\text{filter}} \in \mathbb{R}^{L \times L}$)
- Includes ARMA models, discretized OU, etc.
- Frequency filters are the most relevant filters for mean reversion patterns:
- We use Fast Fourier Transform (FFT) for a frequency decomposition:

$$x_l = a_0 + \sum_{j=1}^{L/2-1} \left(a_j \cdot \cos\left(\frac{2\pi j}{L} l\right) + b_j \cdot \sin\left(\frac{2\pi j}{L} l\right) \right) + a_{L/2} \cos(\pi l).$$

- Signal are the “loadings” on long and short-term reversal patterns:

$$\theta^{\text{FFT}}(x) = (a_0, \dots, a_{L/2}, b_1, \dots, b_{L/2-1})$$

Allocation w^ϵ : Flexible non-parameteric function with regularization

- g^{FFN} is estimated with feedforward neural network (FFN)

$$w^{\epsilon|\text{FFT}}(\theta^{\text{FFT}}) = g^{\text{FFN}}(\theta^{\text{FFT}}).$$

Limitation: Choice of pre-specified filter limits the time-series patterns.

Third class: Convolutional Network with Transformer

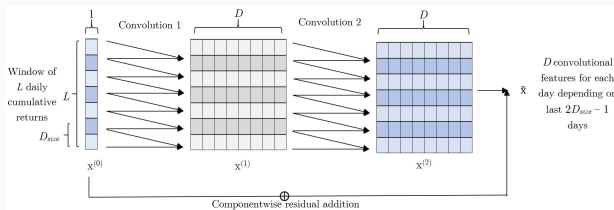
Our novel model: Data driven time-series filter based on most advanced deep learning tools for pattern detection

- **Convolutional neural networks (CNN)** are data-driven non-linear local filters
- **Transformers** learn global dependency patterns between local filters
- CNN+Transformer is a flexible non-linear filter that can learn any time-series pattern
- Examples of global “pattern factors”
 - Mean-reversion: cyclical combination of local curvature patterns
 - Trend: Monotonic combination of local drifts
- Signal $\theta^{\text{CNN+Trans}}(x)$ is the “exposure” to pattern factors
- Allocation function w^ϵ is a flexible FFN:

$$w^{\epsilon|\text{CNN+Trans}}\left(\theta^{\text{CNN+Trans}}\right)=g^{\text{FFN}}\left(\theta^{\text{CNN+Trans}}\right).$$

- Joint estimation of signal and allocation function with trading objective

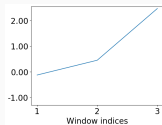
Convolutional Network Intuition



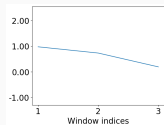
- The network applies to residual time series ϵ_t^L a combination of local estimated linear filters W^{local} followed by non-linear transformations:

$$y_l^{(0)} = \sum_{m=1}^{D_{size}} W_m^{local} x$$

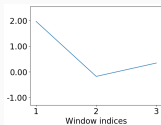
\Rightarrow Represent time-series $x \in \mathbb{R}^L$ in terms of D local patterns $\tilde{x} \in \mathbb{R}^{L \times D}$



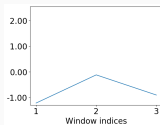
(a) Upward trend



(b) Downward trend

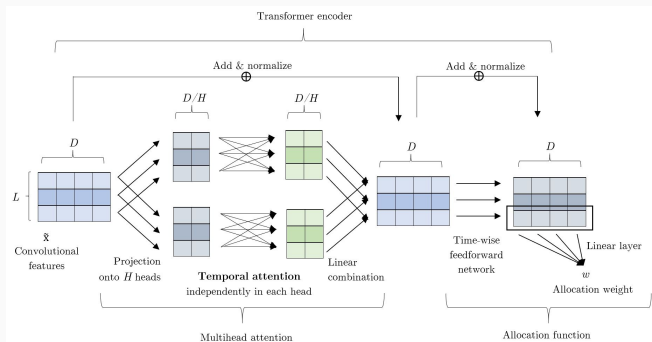


(c) Up reversal



(d) Down reversal

Transformer Network Intuition



Transformer captures temporal dependencies between local patterns

$$h_i = \sum_{l=1}^L \alpha_{i,l} \tilde{x}_l \quad \text{with } \alpha_{i,l} = \alpha_i(\tilde{x}_L, \tilde{x}_l) \text{ for } l = 1, \dots, L \text{ and } i = 1, \dots, H$$

- H global patterns specified by “attention weights” $\alpha_i \in \mathbb{R}^L$.
- Attention heads h_i are “loadings” for a specific “pattern factor” α_i
- Transformer estimate flexible attention weight functions

Empirical Analysis

Out-of-sample analysis on U.S. equity data:

- 19 years of large cap U.S. daily stock returns from Jan 1998 to Dec 2016
- Only stocks with prior month market cap $> 0.01\%$ of total market cap
 $\Rightarrow \sim 550$ large cap stocks/month \approx S&P 500
Most liquid stocks to avoid trading frictions
- For IPCA, supplement with 46 monthly firm characteristics for each stock and month (starting in 1978).

Implementation:

- All results are out-of-sample
- We use $L = 30$ days lookback windows of returns as input for signal.
- We retrain functions every half year using rolling windows of 4 years.
- Factors models are estimated OOS daily on rolling window of 60 days
- Main analysis with Sharpe ratio objective

Arbitrage Portfolios

Residuals with the empirically most important families of factor models:

1. **Fama-French factors** for 1, 3, 5, 8 factors.
market, size, value, investment, profitability, momentum, short-term and long-term reversal
2. **PCA factors** for 1, 3, 5, 8, 10, 15 factors.
3. **IPCA model** of Kelly, Pruitt, and Su (2019), for 1, 3, 5, 8, 10, 15 factors.
4. **"0-factor model"**: original stocks instead of residuals.

Given the residuals, we estimate arbitrage signals and allocations for

1. Ornstein-Uhlenbeck estimation with threshold rule (**OU+Thres**).
2. Fast Fourier Transform with feedforward network (**Fourier+FFN**).
3. Convolutional network with transformer (**CNN+Trans**).

and, for completeness,

4. OU estimation with feedforward network (**OU+FFN**).
5. Just a feedforward network without time-series filter (**FFN**)

OOS Annualized Performance

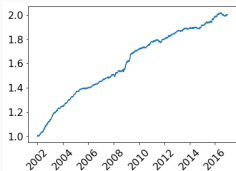
Factors		Fama-French			PCA			IPCA		
Model	K	SR	μ	σ	SR	μ	σ	SR	μ	σ
CNN+	0	1.64	13.7%	8.4%	1.64	13.7%	8.4%	1.64	13.7%	8.4%
Trans	5	3.21	4.6%	1.4%	3.36	14.3%	4.2%	4.16	8.7%	2.1%
FFT+	0	0.36	4.9%	13.6%	0.36	4.9%	13.6%	0.36	4.9%	13.6%
FFN	5	1.66	3.1%	1.8%	1.98	12.4%	6.3%	1.90	7.7%	4.1%
OU+	0	-0.18	-2.4%	13.3%	-0.18	-2.4%	13.3%	-0.18	-2.4%	13.3%
Thres	5	0.38	0.9%	2.3%	0.73	4.4%	6.1%	0.97	3.8%	4.0%

- Arbitrage trading has to be applied to residuals and not returns
- Results do not substantially improve after regressing out 5 factors
- CNN+Transformer strongly dominates all models
- Average return μ is high in spite of leverage constraint
- Arbitrage trading qualitatively robust to choice of factor model
- Fourier+FFN only 50% of CNN+Trans \Rightarrow flexible time-series filter crucial!
- Conventional OU+Thres only 25% of CNN+Trans
 \Rightarrow Too restrictive model!

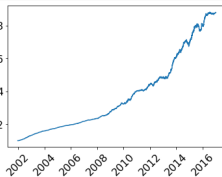
OOS Annualized Performance

Factors		Fama-French			PCA			IPCA		
Model	K	SR	μ	σ	SR	μ	σ	SR	μ	σ
CNN + Trans	0	1.64	13.7%	8.4%	1.64	13.7%	8.4%	1.64	13.7%	8.4%
	1	3.68	7.2%	2.0%	2.74	15.2%	5.5%	3.22	8.7%	2.7%
	3	3.13	5.5%	1.8%	3.56	16.0%	4.5%	3.93	8.6%	2.2%
	5	3.21	4.6%	1.4%	3.36	14.3%	4.2%	4.16	8.7%	2.1%
	8	2.49	3.4%	1.4%	3.02	12.2%	4.0%	3.95	8.2%	2.1%
	10	-	-	-	2.81	10.7%	3.8%	3.97	8.0%	2.0%
	15	-	-	-	2.30	7.6%	3.3%	4.17	8.4%	2.0%
Fourier + FFN	0	0.36	4.9%	13.6%	0.36	4.9%	13.6%	0.36	4.9%	13.6%
	1	0.89	3.2%	3.5%	0.80	8.4%	10.6%	1.24	6.3%	5.0%
	3	1.32	3.5%	2.7%	1.66	11.2%	6.7%	1.77	7.8%	4.4%
	5	1.66	3.1%	1.8%	1.98	12.4%	6.3%	1.90	7.7%	4.1%
	8	1.90	3.1%	1.6%	1.95	10.1%	5.2%	1.94	7.8%	4.0%
	10	-	-	-	1.71	8.2%	4.8%	1.93	7.6%	3.9%
	15	-	-	-	1.14	4.8%	4.2%	2.06	7.9%	3.8%
OU + Thresh	0	-0.18	-2.4%	13.3%	-0.18	-2.4%	13.3%	-0.18	-2.4%	13.3%
	1	0.16	0.6%	3.8%	0.21	2.1%	10.4%	0.60	3.0%	5.1%
	3	0.54	1.6%	3.0%	0.77	5.2%	6.8%	0.88	3.8%	4.3%
	5	0.38	0.9%	2.3%	0.73	4.4%	6.1%	0.97	3.8%	4.0%
	8	1.16	2.8%	2.4%	0.87	4.4%	5.1%	0.91	3.5%	3.8%
	10	-	-	-	0.63	2.9%	4.6%	0.86	3.1%	3.6%
	15	-	-	-	0.62	2.4%	3.8%	0.93	3.2%	3.5%

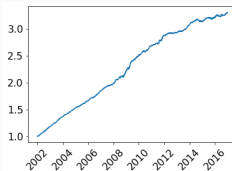
Cumulative OOS Returns of Different Arbitrage Strategies



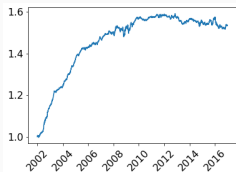
(a) CNN+Trans, Fama-French 5



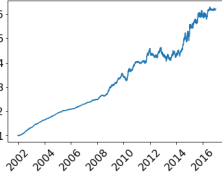
(b) CNN+Trans, PCA 5



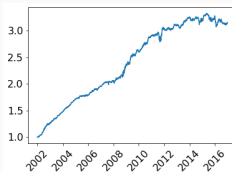
(c) CNN+Trans, IPCA 5



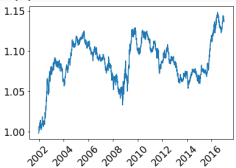
(d) FFT+FFN, Fama-French 5



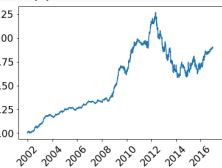
(e) FFT+FFN, PCA 5



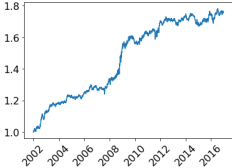
(f) FFT+FFN, IPCA 5



(g) OU+Thresh Fama-French 5



(h) OU+Thresh PCA 5



(i) OU+Thresh IPCA 5

Significance of Arbitrage Alphas

CNN+Trans model						
K	Fama-French		PCA		IPCA	
	0	5	0	5	0	5
α	11.6%	4.5%	11.6%	14.1%	11.6%	8.3%
μ	13.7%	4.6%	13.7%	14.3%	13.7%	8.7%
t_α	6.4***	12***	6.4***	13***	6.4***	16***
t_μ	6.3***	12***	6.3***	13***	6.3***	16***
R^2	30.3%	2.3%	30.3%	1.3%	30.3%	3.9%

- Time-series regression of 8 asset pricing factors:
Fama-French 5 + momentum, short-term and long-term reversal factors
Pricing errors α , t-statistics t_α and R^2 of regression
- Mean μ and corresponding t-statistics t_μ of arbitrage strategies
- CNN+Transformer arbitrage is statistically significant and not subsumed by conventional risk factors
- Arbitrage strategies orthogonal to conventional risk factors
- Conventional mean-reversion trading explained by conventional risk factors.

Significance of Arbitrage Alphas

CNN+Trans model																
	Fama-French					PCA					IPCA					
K	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	
0	11.6%	6.4***	30.3%	13.7%	6.3***	11.6%	6.4***	30.3%	13.7%	6.3***	11.6%	6.4***	30.3%	13.7%	6.3***	
1	7.0%	14***	2.4%	7.2%	14***	14.9%	10***	0.6%	15.2%	11***	8.1%	12***	9.5%	8.7%	12***	
3	5.5%	12***	1.2%	5.5%	12***	15.8%	14***	1.7%	16.0%	14***	8.2%	15***	6.0%	8.6%	15***	
5	4.5%	12***	2.3%	4.6%	12***	14.1%	13***	1.3%	14.3%	13***	8.3%	16***	3.9%	8.7%	16***	
8	3.3%	9.4***	2.1%	3.4%	9.6***	12.0%	12***	0.9%	12.2%	12***	7.8%	15***	5.0%	8.2%	15***	
10	-	-	-	-	-	10.5%	11***	0.7%	10.7%	11***	7.7%	15***	4.0%	8.0%	15***	
15	-	-	-	-	-	7.5%	8.8***	0.5%	7.6%	8.9***	8.1%	16***	4.2%	8.4%	16***	

- Time-series regression of 8 asset pricing factors:
Fama-French 5 + momentum, short-term and long-term reversal factors
Pricing errors α , t-statistics t_{α} and R^2 of regression
- Mean μ and corresponding t-statistics t_{μ} of arbitrage strategies
- CNN+Transformer arbitrage is statistically significant and not subsumed by conventional risk factors
- Arbitrage strategies orthogonal to conventional risk factors
- Conventional mean-reversion trading explained by conventional risk factors.

Mean-Variance Objective

CNN+Trans model, mean-variance objective function										
K	Fama-French			PCA			IPCA			
	SR	μ	σ	SR	μ	σ	SR	μ	σ	
0	0.83	9.5%	11.4%	0.83	9.5%	11.4%	0.83	9.5%	11.4%	
1	3.15	10.5%	3.3%	2.21	27.3%	12.3%	2.83	15.9%	5.6%	
3	2.95	7.8%	2.6%	2.38	22.6%	9.5%	3.13	17.9%	5.7%	
5	3.03	5.9%	2.0%	2.75	19.6%	7.1%	3.21	18.2%	5.7%	
8	2.96	4.2%	1.4%	2.68	16.6%	6.2%	3.18	17.0%	5.4%	
10	-	-	-	2.67	15.3%	5.7%	3.21	16.6%	5.2%	
15	-	-	-	2.20	8.7%	4.0%	3.34	16.3%	4.9%	

Alternative mean-variance objective function:

$$\max_{\mathbf{w}^\epsilon \in \mathcal{W}, \theta \in \Theta} \mathbb{E}[\mathbf{w}_{t-1}^{R \top} R_t] - \gamma \text{Var}(\mathbf{w}_{t-1}^{R \top} R_t)$$

$$\text{s.t.} \quad \mathbf{w}_{t-1}^R = \frac{\mathbf{w}_{t-1}^{\epsilon \top} \Phi_{t-1}}{\|\mathbf{w}_{t-1}^{\epsilon \top} \Phi_{t-1}\|_1} \quad \text{and} \quad \mathbf{w}_{t-1}^{\epsilon} = \mathbf{w}^{\epsilon}(\theta(\epsilon_{t-1}^L)).$$

- Increase mean return while maintaining leverage constraint of $\|\mathbf{w}_{t-1}^R\| = 1$
- Here we set risk aversion to $\gamma = 1$
- Annual returns up to 20% while volatility is only half of market.
- Slightly lower Sharpe ratios

Additional Results

Stability over time:

- Results are **robust to length of local rolling window**
Essentially identical results for $L = 60$ rolling lookback window
- **Constant signal and allocation** function capture most arbitrage information
30% decrease of performance for constant model ($T_{train} = 4$ or 8 years)

Robustness to tuning parameters:

- Results very **robust to all tuning parameters**
- General structure of the problem important, but not number of layers, etc.

Dependency between strategies:

- Between different factor models only weakly correlated (0.2 to 0.45)
- Within factor family model high correlation (0.4 to 0.85)

Unconditional means without allocation:

- Equally weighted residuals have mean returns $< 1\%$
- Need to **apply signal and trading policy to residuals** for profitable trading

Need for time-series signal function:

- **FFN** applied directly to residuals **without time-series model** much worse
- **FFN cannot learn complex dependencies** with limited data

Structure of Arbitrage

Structure of Arbitrage

Arbitrage “factors”:

- Transformer provides 4 global pattern “factors”: fast reversal, early momentum, low-frequency downturn, low frequency momentum
- Asymmetric response of Transformer:
act swiftly during downtrends, stay cautious during uptrends

Which days and patterns matter?:

- Most important basic patterns are smooth trends or local curvature.
- All previous days matter, but on average the most recent 14 days get more attention for trading decisions.

Complexity of arbitrage:

- Simple long-short residual reversal strategies perform substantially lower
- Successful arbitrage trading is more complex than simple reversal patterns.

Diversification and sparse signals:

- Arbitrage portfolios are well-diversified
- Large fraction of profitability can be achieved with a sparse set of assets.
- Arbitrage trading policy does not target specific industries

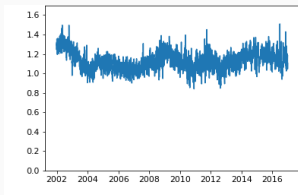
Trading Frictions and Transaction Costs

IPCA factor model						
K	Sharpe ratio			Mean-variance		
	SR	μ	σ	SR	μ	σ
0	0.52	8.5%	16.3%	0.22	2.6%	11.9%
1	0.85	5.9%	6.9%	0.86	5.5%	6.4%
3	1.24	6.6%	5.4%	1.16	6.9%	5.9%
5	1.11	5.5%	5.0%	1.02	5.3%	5.3%
10	0.98	5.1%	5.2%	1.04	5.4%	5.2%
15	0.94	4.8%	5.1%	1.02	5.1%	5.0%

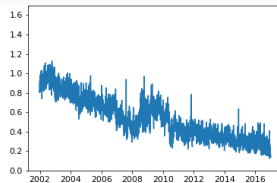
- Include trading costs for high turnover and large short-selling positions:
 $\text{cost}(w_{t-1}^R, w_{t-2}^R) = 0.0005 \|w_{t-1}^R - w_{t-2}^R\|_{L^1} + 0.0001 \| \min(w_{t-1}^R, 0) \|_{L^1}$
5 basis points per transaction and 1 basis point per short position
 - No market impact as we only trade in the largest most liquid stocks
 - Lower bound on profitability: less turnover with sparse factors, etc.
- ⇒ Arbitrage trading retains economic significance in presence of trading costs

Turnover and Short Selling

Turnover with and without trading friction objective:

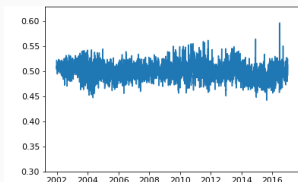


(a) No Trading Friction Objective

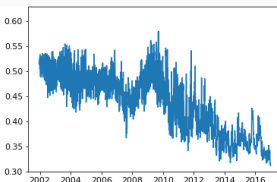


(b) With Trading Friction Objective

Proportion of short allocation weights:



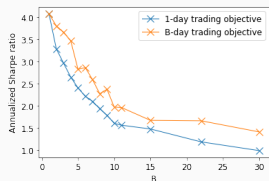
(a) No Trading Friction Objective



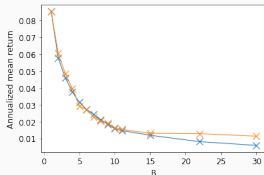
(b) With Trading Friction Objective

⇒ The effect of trading frictions is **time-varying** and our model can **exploit particularly profitable arbitrage** time periods by increasing trading and short positions.

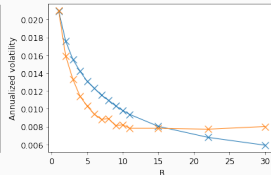
Persistence of Arbitrage



(a) Sharpe ratio



(b) Mean Return

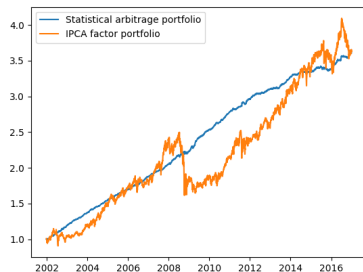


(c) Volatility

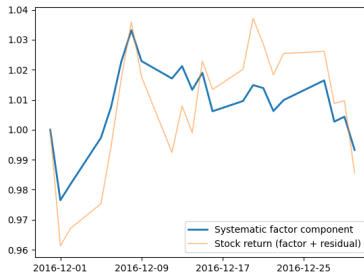
Performance for longer holding periods (Overlapping returns for B days)

- Baseline model CNN+Trans with IPCA-5 residuals.
- Holding for B trading days with 1-day or B trading objective.
- SR half-life is around $B = 7$ trading days: arbitrageurs correct prices
- Large fraction of arbitrage persistent: $SR = 1$ after 1 month
Possible explanations: limited capacity or strategic trading of arbitrageurs, slow expectation adjustments of noise traders
- Similar results for non-overlapping longer holding periods
- B -day trading objective exploits diversification of overlapping strategies

Arbitrage vs risk premium component



(a) Factor and arbitrage component



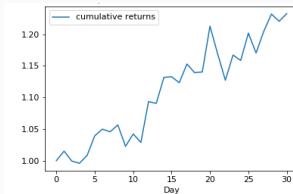
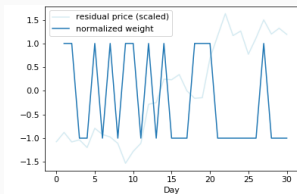
(b) Example stock: factor and residual component

Arbitrage vs. risk premium component

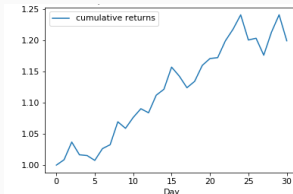
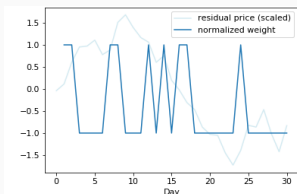
- Benchmark asset pricing model earns a risk premium
 - $SR_{IPCA} = 1.23$: Sharpe ratio of SDF with 5 IPCA factors (MVE portfolio)
 - $SR_{arbitrage} = 4.16$: Sharpe ratio of CNN+Trans on IPCA-5 residuals
 - IPCA 5 portfolio uncorrelated with arbitrage portfolio:
 $R^2 = 0.02$ of regression of arbitrage strategy on IPCA-5 factors:
- ⇒ Arbitrage trading and risk premium can be combined!
- ⇒ Arbitrage trading fundamentally different from earning risk premium

Estimated Structure: Dissecting the CNN+Transformer Model with IPCA-5

Examples of Allocation and Returns of CNN+Transformer Strategy



(a) Example 1: Mean-reversion

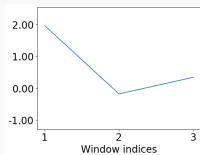


(b) Example 2: Trend

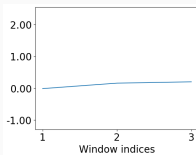
Sample of representative residuals with out-of-sample arbitrage trading

- Positive allocations for positive changes and vice-a-versa
- CNN detects global and local trend and reversion patterns

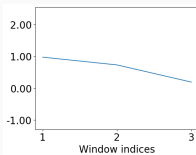
CNN: Local Basic Patterns



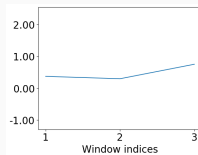
(a) Basic pattern 1



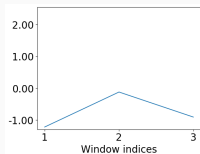
(b) Basic pattern 2



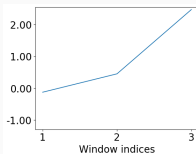
(c) Basic pattern 3



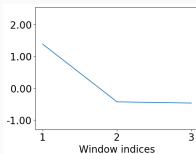
(d) Basic pattern 4



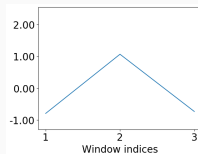
(e) Basic pattern 5



(f) Basic pattern 6



(g) Basic pattern 7

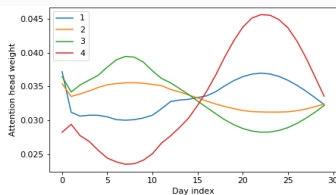
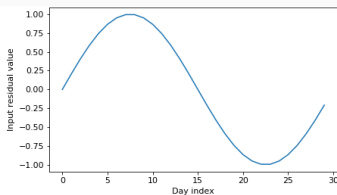


(h) Basic pattern 8

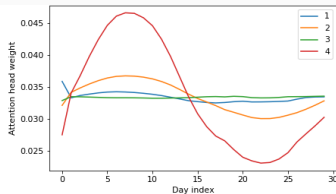
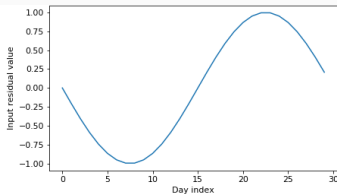
Local filters estimated by CNN to capture relative local patterns

- Basic patterns are “building blocks” for the global patterns
- Visualizations of non-linear 3-dimensional local filters into orthogonal two-dimensional local linear filters
- Sufficient to construct any smooth trend and mean-reversion patterns.

Example Attention Weights for Sinusoidal Residual Inputs



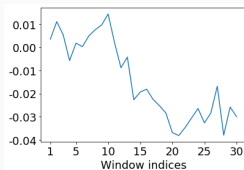
(a) Input residual and attention head weights for $x_l = \sin\left(2\pi \frac{l}{30}\right)$



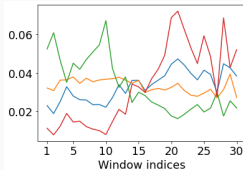
(b) Input residual and attention head weights for $x_l = \sin\left(2\pi \frac{l+15}{30}\right)$

- **Attention head weight 4:** negative reversal factor
- **Attention head weight 3:** early reversal factor
- **Attention head weight 1:** low-frequency downturn factor

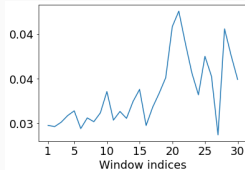
CNN+Transformer Model Structure for Representative Residual



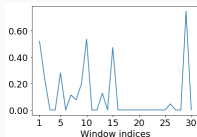
(a) Cumulative residual



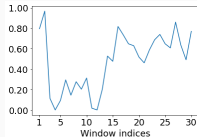
(b) Attention weights per head



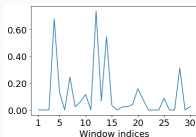
(c) Average attention weights



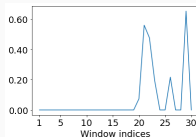
(d) 1st CNN activation



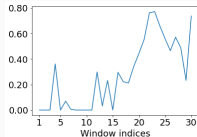
(e) 2nd CNN activation



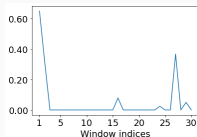
(f) 3rd CNN activation



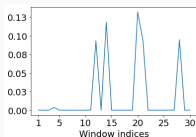
(g) 4th CNN activation



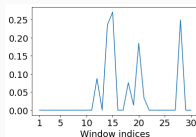
(h) 5th CNN activation



(i) 6th CNN activation

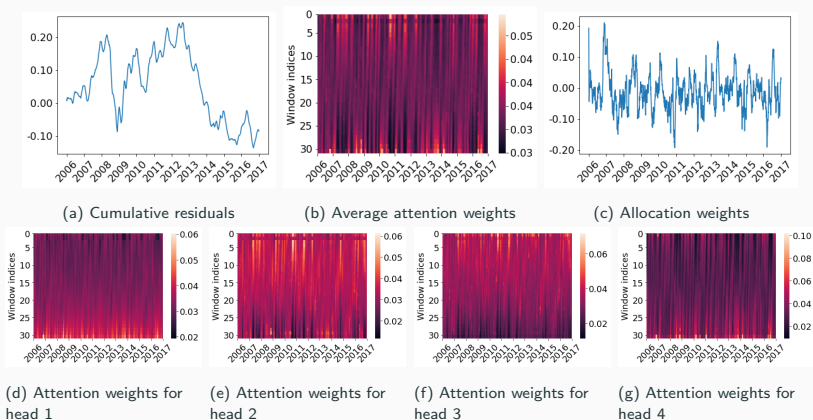


(j) 7th CNN activation



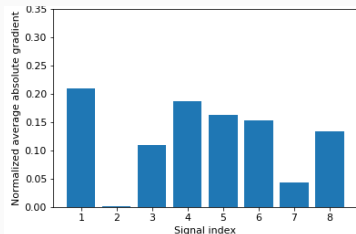
(k) 8th CNN activation

CNN+Transformer Model Structure for Representative Residual Over Time

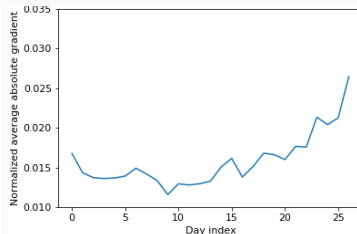


- **Attention head weights 4** highest for down-times in 2009, 2014, middle 2016.
Focuses uniformly on **last 10 days** in 30-day window
- **Attention head weights 3** highest for up-patterns in 2007, 2010, 2012.
Focuses uniformly on **first 20 days** in 30-day window
- Asymmetric response of Transformer:
act swiftly during downtrends, **stay cautious during uptrends**

Variable Importance for Allocation Weight



(a) Importance of Local Basic Patterns



(b) Importance of Residual Days

- Measure importance with average absolute gradient of allocation weight
- Most important basic patterns are trends or local curvature.
Flat basic pattern 2 is negligible.
- All previous days matter, but on average the most recent 14 days get more attention for trading decisions.

Conclusion

Methodology:

- Unifying conceptual framework to compare different approaches:
(1) portfolio generation, (2) signal extraction, (3) allocation decision
- **Novel deep learning statistical arbitrage:**
 1. Conditional latent factors to generate arbitrage portfolios
 2. CNN+Transformer signal: global dependency pattern with local filters
 3. FFN allocation and global trading objective for estimation

Empirical results:

- Comprehensive out-of-sample study on U.S. equities
- CNN+Transformer substantially outperforms benchmark approaches
- Unspanned by conventional risk factors
- Survives realistic transaction and holding costs
- Persistence in arbitrage signals
- Insights into trading policies: asymmetric trend and reversion patterns
- Trading signal extraction is the most challenging and separating element
- **High compensation for arbitrageurs to enforce the law of one price**

Appendix

Firm specific characteristics

Past Returns	Investment	Profitability	Intangibles	Value	Trading Frictions
Momentum	Investment	Operating profitability	Accrual	Book to Market Ratio	Size
Short-term Reversal	Net operating assets	Profitability	Operating accruals	Assets to market cap	Turnover
Long-term Reversal	Change in prop. to assets	Sales over assets	Operating leverage	Cash to assets	Idiosyncratic Volatility
Return 2-1	Net Share Issues	Capital turnover	Price to cost margin	Cash flow to book value	CAPM Beta
Return 12-2		Fixed costs to sales		Cashflow to price	Residual Variance
Return 36-13		Profit margin		Dividend to price	Total assets
		Return on net assets		Earnings to price	Market Beta
		Return on assets		Tobin's Q	Close to High
		Return on equity		Sales to price	Spread
		Expenses to sales		Leverage	Unexplained Volume
		Capital intensity			Variance

46 firm-specific monthly characteristics sorted into six categories.

Importance of Time-Series Signal

Factors		Fama-French			PCA			IPCA		
Model	K	SR	μ	σ	SR	μ	σ	SR	μ	σ
FFN	0	0.57	8.8%	15.3%	0.57	8.8%	15.3%	0.57	8.8%	15.3%
	1	0.60	2.0%	3.3%	0.53	6.2%	11.7%	1.07	6.5%	6.1%
	3	1.02	2.6%	2.6%	1.15	8.2%	7.2%	1.50	7.6%	5.0%
	5	1.32	2.3%	1.7%	1.42	9.8%	6.9%	1.55	7.3%	4.7%
	8	1.31	2.1%	1.6%	0.84	5.1%	6.1%	1.56	7.2%	4.6%
	10	-	-	-	0.70	3.5%	5.0%	1.48	7.0%	4.7%
	15	-	-	-	0.51	2.4%	4.8%	1.68	7.5%	4.5%

Is there a **time-series signal function** actually needed?

- Apply flexible **FFN** directly to residuals **without time-series model**
- Results are substantially worse than Fourier+FFN
- FFN is not efficient enough to learn complex dependencies with limited data

Significance of Arbitrage Alphas

CNN+Trans model																
K	Fama-French					PCA					IPCA					
	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	
0	11.6%	6.4***	30.3%	13.7%	6.3***	11.6%	6.4***	30.3%	13.7%	6.3***	11.6%	6.4***	30.3%	13.7%	6.3***	
1	7.0%	14***	2.4%	7.2%	14***	14.9%	10***	0.6%	15.2%	11***	8.1%	12***	9.5%	8.7%	12***	
3	5.5%	12***	1.2%	5.5%	12***	15.8%	14***	1.7%	16.0%	14***	8.2%	15***	6.0%	8.6%	15***	
5	4.5%	12***	2.3%	4.6%	12***	14.1%	13***	1.3%	14.3%	13***	8.3%	16***	3.9%	8.7%	16***	
8	3.3%	9.4***	2.1%	3.4%	9.6***	12.0%	12***	0.9%	12.2%	12***	7.8%	15***	5.0%	8.2%	15***	
10	-	-	-	-	-	10.5%	11***	0.7%	10.7%	11***	7.7%	15***	4.0%	8.0%	15***	
15	-	-	-	-	-	7.5%	8.8***	0.5%	7.6%	8.9***	8.1%	16***	4.2%	8.4%	16***	

Fourier+FFN model																
K	Fama-French					PCA					IPCA					
	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	
0	2.7%	0.8	8.6%	4.9%	1.4	2.7%	0.8	8.6%	4.9%	1.4	2.7%	0.8	8.6%	4.9%	1.4	
1	3.0%	3.3**	3.3%	3.2%	3.5***	7.4%	2.7**	3.3%	8.4%	3.1**	4.8%	4.0***	16.4%	6.3%	4.8***	
3	3.2%	4.7***	4.2%	3.5%	5.1***	10.9%	6.3***	2.2%	11.2%	6.4***	6.8%	6.4***	13.0%	7.8%	6.9***	
5	2.9%	6.1***	3.5%	3.1%	6.4***	12.1%	7.5***	1.5%	12.4%	7.6***	6.7%	6.9***	13.3%	7.7%	7.4***	
8	3.0%	7.2***	3.2%	3.1%	7.4***	10.0%	7.5***	0.9%	10.1%	7.6***	6.8%	7.0***	13.3%	7.8%	7.5***	
10	-	-	-	-	-	8.0%	6.5***	1.0%	8.2%	6.6***	6.8%	7.1***	12.7%	7.6%	7.5***	
15	-	-	-	-	-	4.7%	4.3***	0.4%	4.8%	4.4***	7.1%	7.6***	12.2%	7.9%	8.0***	

OU+Thresh model																
K	Fama-French					PCA					IPCA					
	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	
0	-4.5%	-1.4	13.4%	-2.4%	-0.7	-4.5%	-1.4	13.4%	-2.4%	-0.7	-4.5%	-1.4	13.4%	-2.4%	-0.7	
1	-0.2%	-0.2	13.5%	0.6%	0.6	0.7%	0.3	6.3%	2.1%	0.8	1.7%	1.4	18.9%	3.0%	2.3*	
3	0.9%	1.2	10.4%	1.6%	2.1*	4.3%	2.5*	4.3%	5.2%	3.0**	2.6%	2.6**	18.8%	3.8%	3.4***	
5	0.5%	0.9	6.8%	0.9%	1.5	3.7%	2.4*	3.2%	4.4%	2.8**	2.8%	3.0**	17.7%	3.8%	3.8***	
8	0.6%	1.2	5.5%	1.0%	1.9	3.9%	3.0**	1.9%	4.4%	3.4***	2.3%	2.6**	17.6%	3.5%	3.6***	
10	-	-	-	-	-	2.6%	2.2*	1.4%	2.9%	2.4*	2.1%	2.5*	17.6%	3.1%	3.3***	
15	-	-	-	-	-	2.1%	2.1*	0.7%	2.4%	2.4*	2.3%	2.8**	18.1%	3.2%	3.6***	

Significance of Arbitrage Alphas with Mean-Variance Objective

CNN+Trans model															
K	Fama-French					PCA					IPCA				
	α	t_α	R^2	μ	t_μ	α	t_α	R^2	μ	t_μ	α	t_α	R^2	μ	t_μ
0	5.8%	2.2*	19.6%	9.5%	3.2**	5.8%	2.2*	19.6%	9.5%	3.2**	5.8%	2.2*	19.6%	9.5%	3.2**
1	9.9%	12***	7.1%	10.5%	12***	26.3%	8.3***	1.6%	27.3%	8.6***	14.0%	11***	23.5%	15.9%	11***
3	7.5%	11***	5.3%	7.8%	11***	22.1%	9.1***	2.2%	22.6%	9.2***	16.6%	12***	17.6%	17.9%	12***
5	5.7%	11***	5.3%	5.9%	12***	19.0%	10***	3.2%	19.6%	11***	16.7%	12***	16.0%	18.2%	12***
8	4.4%	9.8***	3.6%	4.6%	10***	16.3%	10***	1.6%	16.6%	10***	15.5%	12***	18.3%	17.0%	12***
10	-	-	-	-	-	14.8%	10***	1.7%	15.3%	10***	15.2%	13***	20.6%	16.6%	12***
15	-	-	-	-	-	8.5%	8.4***	0.9%	8.7%	8.5***	14.8%	13***	21.6%	16.3%	13***

Fourier+FFN model															
K	Fama-French					PCA					IPCA				
	α	t_α	R^2	μ	t_μ	α	t_α	R^2	μ	t_μ	α	t_α	R^2	μ	t_μ
0	3.2%	0.7	8.4%	5.5%	1.1	3.2%	0.7	8.4%	5.5%	1.1	3.2%	0.7	8.4%	5.5%	1.1
1	2.8%	1.6	1.8%	2.5%	1.5	15.4%	1.7	1.3%	16.6%	1.9	7.9%	1.8	2.6%	9.7%	2.2*
3	4.1%	4.4***	3.4%	4.3%	4.5***	30.3%	1.3	0.1%	32.1%	1.3	17.4%	4.1***	1.9%	17.6%	4.1***
5	2.9%	4.8***	3.1%	3.1%	5.0***	21.0%	1.3	0.1%	22.5%	1.4	15.9%	4.3***	2.6%	17.0%	4.5***
8	3.5%	6.8***	2.3%	3.6%	7.0***	17.4%	2.6**	0.3%	17.2%	2.6**	12.9%	4.3***	4.4%	14.4%	4.7***
10	-	-	-	-	-	7.1%	1.7	0.3%	7.4%	1.8	11.7%	3.9***	3.5%	12.6%	4.1***
15	-	-	-	-	-	5.5%	2.1*	0.1%	5.7%	2.2*	11.3%	4.3***	4.0%	12.1%	4.5***

Dependency between Arbitrage Strategies

Table 1: Correlations between the Returns of the CNN+Transformer Arbitrage Strategies

	FF 3	PCA 3	IPCA 3	FF 5	PCA 5	IPCA 5	PCA 10	IPCA 10
FF 3	1.00	0.32	0.44	0.62	0.25	0.43	0.21	0.44
PCA 3	0.32	1.00	0.32	0.34	0.62	0.35	0.41	0.36
IPCA 3	0.44	0.32	1.00	0.37	0.28	0.81	0.21	0.75
FF 5	0.62	0.34	0.37	1.00	0.28	0.39	0.23	0.40
PCA 5	0.25	0.62	0.28	0.28	1.00	0.29	0.47	0.31
IPCA 5	0.43	0.35	0.81	0.39	0.29	1.00	0.23	0.84
PCA 10	0.21	0.41	0.21	0.23	0.47	0.23	1.00	0.25
IPCA 10	0.44	0.36	0.75	0.40	0.31	0.84	0.25	1.00

Strategies from different factor models have low inter-family correlations

- Inter-family correlations range from 0.21 to 0.44.
- Intra-family correlations range between 0.41 and 0.84.

Importance of Time-Series Signal

Factors		Fama-French			PCA			IPCA		
Model	K	SR	μ	σ	SR	μ	σ	SR	μ	σ
OU + FFN	0	0.50	10.6%	21.3%	0.50	10.6%	21.3%	0.50	10.6%	21.3%
	1	0.34	0.8%	2.3%	0.05	0.7%	11.9%	0.60	4.8%	8.0%
	3	0.16	0.2%	1.4%	0.44	3.4%	7.8%	0.70	4.6%	6.6%
	5	0.17	0.2%	1.2%	0.68	4.7%	7.0%	0.66	4.2%	6.3%
	8	-0.34	-0.3%	1.0%	0.31	2.3%	6.9%	0.61	3.9%	6.2%
	10	-	-	-	0.26	1.3%	5.0%	0.56	3.5%	6.2%
	15	-	-	-	0.31	1.4%	4.3%	0.54	3.3%	6.1%
FFN	0	0.57	8.8%	15.3%	0.57	8.8%	15.3%	0.57	8.8%	15.3%
	1	0.60	2.0%	3.3%	0.53	6.2%	11.7%	1.07	6.5%	6.1%
	3	1.02	2.6%	2.6%	1.15	8.2%	7.2%	1.50	7.6%	5.0%
	5	1.32	2.3%	1.7%	1.42	9.8%	6.9%	1.55	7.3%	4.7%
	8	1.31	2.1%	1.6%	0.84	5.1%	6.1%	1.56	7.2%	4.6%
	10	-	-	-	0.70	3.5%	5.0%	1.48	7.0%	4.7%
	15	-	-	-	0.51	2.4%	4.8%	1.68	7.5%	4.5%

Table 2: OOS Annualized Performance of CNN+Trans for 60 Days Lookback Window

K	Fama-French			PCA			IPCA		
	SR	μ	σ	SR	μ	σ	SR	μ	σ
0	1.50	13.5%	9.0%	1.50	13.5%	9.0%	1.50	13.5%	9.0%
1	2.95	9.6%	3.2%	2.68	15.8%	5.9%	3.14	8.8%	2.8%
3	3.21	8.7%	2.7%	3.49	16.8%	4.8%	3.84	9.6%	2.5%
5	3.23	6.8%	2.1%	3.54	16.0%	4.5%	3.90	9.2%	2.4%
8	2.96	4.2%	1.4%	3.02	12.5%	4.2%	3.93	8.7%	2.2%
10	-	-	-	2.67	9.9%	3.7%	3.98	9.2%	2.3%
15	-	-	-	2.36	8.1%	3.4%	4.24	9.6%	2.3%

Table 3: Significance of Arbitrage Alphas for 60 Days Lookback Window

CNN+Trans Model , Sharpe objective function, $L = 60$ days lookback window															
K	Fama-French					PCA					IPCA				
	α	t_α	R^2	μ	t_μ	α	t_α	R^2	μ	t_μ	α	t_α	R^2	μ	t_μ
0	11.8%	5.6***	19.5%	13.5%	5.8***	11.8%	5.6***	19.5%	13.5%	5.8***	11.8%	5.6***	19.5%	13.5%	5.8***
1	9.1%	11***	7.2%	9.6%	11***	15.5%	10***	1.2%	15.8%	10***	8.2%	12***	10.1%	8.8%	12***
3	8.3%	12***	7.1%	8.7%	12***	16.5%	13***	2.5%	16.8%	14***	9.2%	15***	9.3%	9.6%	15***
5	6.5%	12***	6.0%	6.8%	13***	15.6%	13***	2.2%	16.0%	14***	8.8%	15***	10.3%	9.2%	15***
8	4.1%	11***	3.2%	4.2%	11***	12.2%	11***	1.6%	12.5%	12***	8.3%	15***	8.9%	8.7%	15***
10	-	-	-	-	-	9.7%	10***	1.0%	9.9%	10***	8.8%	15***	8.3%	9.2%	15***
15	-	-	-	-	-	8.1%	9.1***	0.7%	8.1%	9.1***	9.2%	16***	9.3%	9.6%	16***

Mean-Variance Objective

CNN+Trans model, mean-variance objective function									
K	Fama-French			PCA			IPCA		
	SR	μ	σ	SR	μ	σ	SR	μ	σ
0	0.83	9.5%	11.4%	0.83	9.5%	11.4%	0.83	9.5%	11.4%
1	3.15	10.5%	3.3%	2.21	27.3%	12.3%	2.83	15.9%	5.6%
3	2.95	7.8%	2.6%	2.38	22.6%	9.5%	3.13	17.9%	5.7%
5	3.03	5.9%	2.0%	2.75	19.6%	7.1%	3.21	18.2%	5.7%
8	2.96	4.2%	1.4%	2.68	16.6%	6.2%	3.18	17.0%	5.4%
10	-	-	-	2.67	15.3%	5.7%	3.21	16.6%	5.2%
15	-	-	-	2.20	8.7%	4.0%	3.34	16.3%	4.9%

Fourier+FFN model, mean-variance objective function									
K	Fama-French			PCA			IPCA		
	SR	μ	σ	SR	μ	σ	SR	μ	σ
0	0.28	5.5%	19.3%	0.28	5.5%	19.3%	0.28	5.5%	19.3%
1	0.38	2.5%	6.7%	0.48	16.6%	34.8%	0.56	9.7%	17.2%
3	1.16	4.3%	3.7%	0.34	32.1%	93.1%	1.06	17.6%	16.7%
5	1.30	3.1%	2.4%	0.37	22.5%	61.2%	1.17	17.0%	14.5%
8	1.73	3.6%	2.0%	0.67	17.4%	25.9%	1.21	14.4%	11.9%
10	-	-	-	0.45	7.4%	16.4%	1.06	12.6%	11.9%
15	-	-	-	0.56	5.7%	10.2%	1.17	12.1%	10.4%

Constant Model without Re-estimation

Table 4: OOS Annualized Performance of CNN+Trans for Constant Model

$T_{\text{train}} = 4 \text{ years}$										
K	Fama-French			PCA			IPCA			
	SR	μ	σ	SR	μ	σ	SR	μ	σ	
0	1.10	8.5%	7.8%	1.10	8.5%	7.8%	1.10	8.5%	7.8%	
1	1.90	4.5%	2.3%	0.44	3.0%	6.9%	0.94	3.1%	3.3%	
3	1.60	3.6%	2.2%	1.65	8.7%	5.3%	1.82	5.3%	2.9%	
5	1.81	3.0%	1.7%	1.93	9.8%	5.1%	2.09	5.4%	2.6%	
8	1.70	2.5%	1.5%	2.04	9.6%	4.7%	1.89	5.0%	2.6%	
10	-	-	-	2.06	9.1%	4.4%	1.77	4.7%	2.7%	
15	-	-	-	1.82	7.0%	3.9%	2.09	5.5%	2.7%	

$T_{\text{train}} = 8 \text{ years}$										
K	Fama-French			PCA			IPCA			
	SR	μ	σ	SR	μ	σ	SR	μ	σ	
0	1.33	12.0%	9.0%	1.33	12.0%	9.0%	1.33	12.0%	9.0%	
1	2.06	5.0%	2.4%	1.81	15.2%	8.4%	2.02	8.5%	4.2%	
3	2.46	5.3%	2.2%	2.04	13.1%	6.4%	2.47	7.5%	3.0%	
5	1.82	3.2%	1.8%	1.91	11.9%	6.2%	2.64	7.6%	2.9%	
8	1.48	2.5%	1.7%	1.89	10.8%	5.7%	2.71	8.3%	3.1%	
10	-	-	-	1.82	10.0%	5.5%	2.68	8.2%	3.1%	
15	-	-	-	1.38	6.2%	4.5%	2.70	7.8%	2.9%	

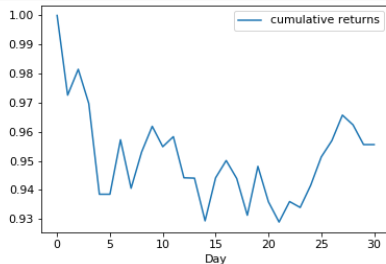
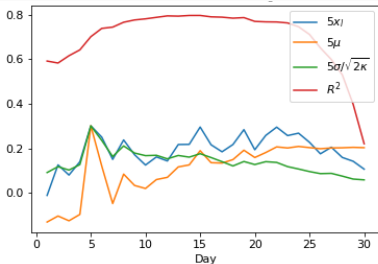
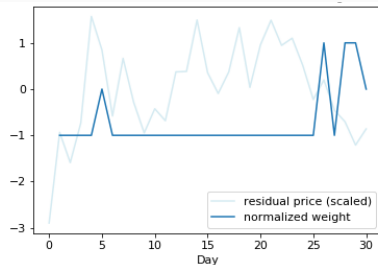
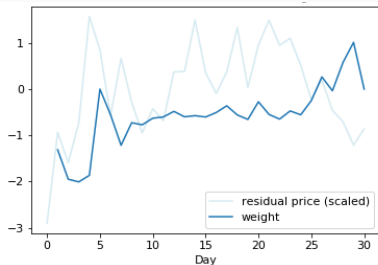
Constant Model without Re-estimation

Table 5: Significance of Arbitrage Alphas for Constant Model

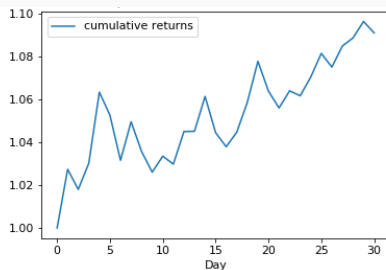
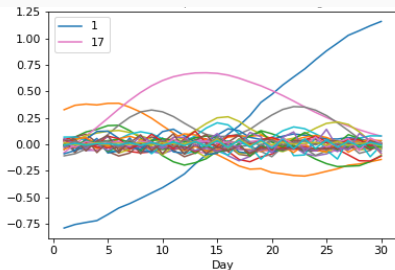
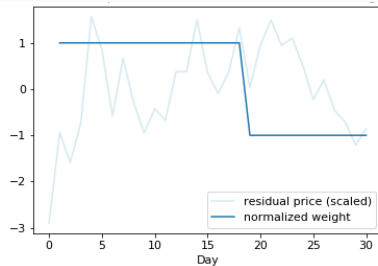
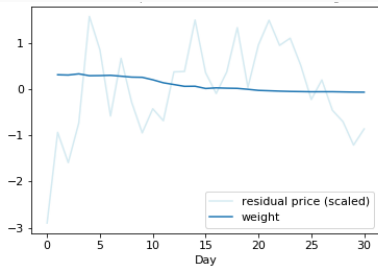
CNN+Trans model, Sharpe objective function, $T_{\text{train}} = 4$ years															
K	Fama-French					PCA					IPCA				
	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}
0	8.4%	4.2***	3.0%	8.5%	4.3***	8.4%	4.2***	3.0%	8.5%	4.3***	8.4%	4.2***	3.0%	8.5%	4.3***
1	4.0%	6.8***	5.9%	4.5%	7.3***	4.1%	2.0*	4.5%	5.2%	2.5*	3.1%	3.7***	1.6%	3.1%	3.6***
3	3.2%	5.7***	4.9%	3.6%	6.2***	8.2%	6.1***	2.7%	8.7%	6.4***	5.3%	7.4***	11.7%	5.3%	7.0***
5	2.8%	6.6***	4.3%	3.0%	7.0***	9.3%	7.1***	1.8%	9.8%	7.5***	5.5%	8.6***	8.3%	5.4%	8.1***
8	2.3%	6.1***	5.1%	2.5%	6.6***	9.0%	7.5***	2.2%	9.6%	7.9***	5.0%	7.7***	8.2%	5.0%	7.3***
10	-	-	-	-	-	8.6%	7.5***	1.9%	9.1%	8.0***	5.1%	8.0***	16.6%	4.7%	6.9***
15	-	-	-	-	-	6.8%	6.8***	1.0%	7.0%	7.1***	5.8%	9.3***	17.6%	5.5%	8.1***

CNN+Trans model, Sharpe objective function, $T_{\text{train}} = 8$ years															
K	Fama-French					PCA					IPCA				
	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}	α	t_{α}	R^2	μ	t_{μ}
0	10.1%	4.1***	18.1%	12.0%	4.4***	10.1%	4.1***	18.1%	12.0%	4.4***	10.1%	4.1***	18.1%	12.0%	4.4***
1	4.4%	6.5***	14.3%	5.0%	6.8***	14.5%	5.8***	2.5%	15.2%	6.0***	7.0%	6.6***	30.6%	8.5%	6.7***
3	4.9%	7.9***	11.6%	5.3%	8.2***	12.8%	6.7***	2.7%	13.1%	6.8***	7.0%	7.9***	8.2%	7.5%	8.2***
5	2.9%	5.8***	12.3%	3.2%	6.0***	11.6%	6.2***	1.6%	11.9%	6.3***	7.1%	8.7***	12.1%	7.6%	8.7***
8	2.3%	4.7***	5.4%	2.5%	4.9***	10.2%	6.0***	3.1%	10.8%	6.3***	7.7%	9.0***	14.6%	8.3%	9.0***
10	-	-	-	-	-	9.4%	5.7***	2.6%	10.0%	6.0***	7.7%	8.9***	11.3%	8.2%	8.9***
15	-	-	-	-	-	6.0%	4.4***	0.9%	6.2%	4.6***	7.4%	8.9***	11.2%	7.8%	8.9***

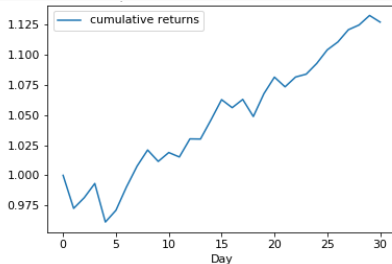
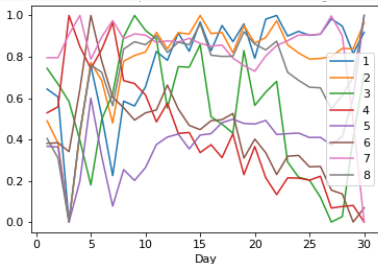
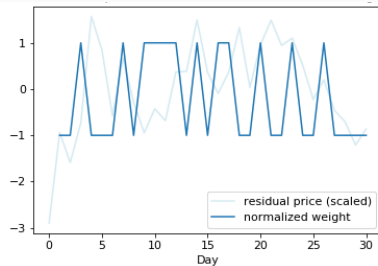
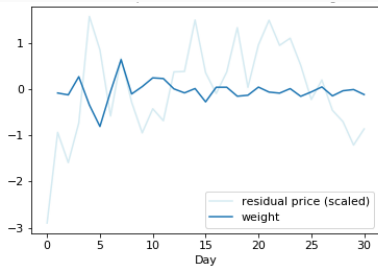
Empirical example: (1) OU+Threshold signals & allocation weights



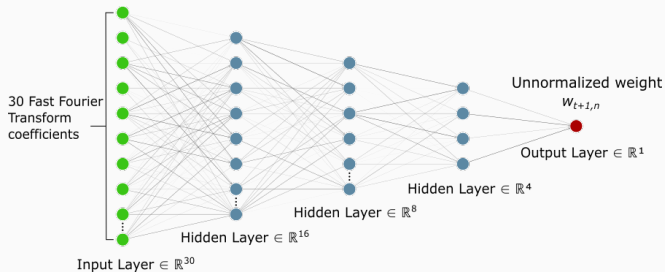
Empirical example: (2) Fourier+FFN signals & allocation weights



Simulation example: (3) CNN+Transformer signals & allocation weights



Fourier+FFN architecture



FFN equations:

$$x^{(l)} = \text{ReLU}(W^{(l-1)}x^{(l-1)} + b^{(l-1)})$$

$$w = W^{(L)}x^{(L)} + b^{(L)}$$

Convolutional network equations

Given

$$\mu_k^{(i)} = \frac{1}{L} \sum_{l=1}^L y_{l,k}^{(i)}, \quad \sigma_k^{(i)} = \sqrt{\frac{1}{L} \sum_{l=1}^L \left(y_{l,k}^{(i)} - \mu_k^{(i)} \right)^2}.$$

Input time series $x^{(0)} \in \mathbb{R}^L$ are passed through $k = 1, \dots, F$ convolutional filters of size f_{size} , normalization, and ReLU:

$$y_{l,k}^{(0)} = b_k^{(1)} + \sum_{m=1}^{f_{size}} W_{k,m}^{(1)} x_{l-m+1}^{(0)}, \quad x_{l,k}^{(1)} = \text{ReLU} \left(\frac{y_{l,k}^{(0)} - \mu_k^{(0)}}{\sigma_k^{(0)}} \right).$$

Output $x_1^{(1)} \in \mathbb{R}^{L \times F}$ passes through $k = 1, \dots, F$ convolutional filters of size $f_{size} \times F$, normalization, and ReLU:

$$y_{l,k}^{(1)} = b_k^{(2)} + \sum_{m=1}^{f_{size}} \sum_{k=1}^F W_{k,m}^{(2)} x_{l-m+1,k}^{(1)}, \quad x_{l,k}^{(2)} = \text{ReLU} \left(\frac{y_{l,k}^{(1)} - \mu_k^{(1)}}{\sigma_k^{(1)}} \right),$$

Finally, residuals are added back to output $x^{(2)} \in \mathbb{R}^{L \times F}$ via residual connection, to compute features $\tilde{x} \in \mathbb{R}^{L \times F}$:

$$\tilde{x}_{l,k} = x_{l,k}^{(2)} + x_l^{(0)}.$$

\Rightarrow All $b^{(i)}$ and $W^{(i)}$ are parameters; all convolutions are left-padded with 0.

Transformer equations

- Features $\tilde{x} \in \mathbb{R}^{L \times F}$ are projected onto $i = 1, \dots, h$ F/h -dimensional subspaces (“heads”):

$$V_i = \tilde{x} W_i^V + b_i^V, \quad K_i = \tilde{x} W_i^K + b_i^K, \quad Q_i = \tilde{x} W_i^Q + b_i^Q \in \mathbb{R}^{L \times F/h}.$$

- Projections V_i are aggregated temporally obtaining hidden states $y_i \in \mathbb{R}^{L \times F/h}$, for

$$y_{i,l} = \sum_{j=1}^L w_{l,j}^{(i)} V_{i,j} \in \mathbb{R}^{F/h}, \quad w_{l,j}^{(i)} = \frac{\exp(K_{i,l} \cdot Q_{i,j})}{\sum_{m=1}^L \exp(K_{i,l} \cdot Q_{i,m})} \in [0, 1].$$

- Final output is $\text{Concat}(y_1, \dots, y_h) W^O + b^O \in \mathbb{R}^{L \times F}$.
- This is passed through time-wise feedforward networks.
- $W_i^V, W_i^K, W_i^Q \in \mathbb{R}^{F \times F/h}$, $b_i^V, b_i^K, b_i^Q \in \mathbb{R}^{F/h}$, $W^O \in \mathbb{R}^{F \times F}$, $b^O \in \mathbb{R}^F$
 \Rightarrow parameters to estimate.

Hyperparameter information

Notation	Model Hyperparameters	Initial	Candidates	Chosen
OU+Thres				
R2T	R^2 filter threshold	0.5	0.25, 0.5, 0.75	0.25
ST	Signal threshold to long/short	1.25	1, 1.25, 1.5	1.25
LKB	Number of days in residual lookback window	30	30	30
DFT+FFN				
HLC	Hidden layer configuration	[16,8,4]	[16,8,4]	[16,8,4]
DRPH	Dropout rate (% removed) in hidden layers	0.25	0.25	0.25
LKB	Number of days in residual lookback window	30	30	30
WDW	Number of days in rolling training window	1000	1000	1000
RTFQ	Number of days of retraining frequency	125	125	125
CNN+Trans				
D	Number of filter channels in CNN	4	4, 8	8
ATT	Number of attention heads	4	2, 4	4
HDN	Number of hidden units in transformer's linear layer	2F	2F, 3F	2F
DRPA	Dropout rate (% removed) in the transformer	0.25	0.5, 0.25	0.25
D_{size}	Filter size in CNN	2	2	2
LKB	Number of days in residual lookback window	30	30	30
WDW	Number of days in rolling training window	1000	1000	1000
RTFQ	Number of days of retraining frequency	125	125	125