Simulation of Solar System using Runge-Kutta Methods

Computer simulations for modelling the Solar System is advantageous because it allows a better understanding of the dynamics of the system including the movements of the planets and other celestial objects. By simulating the interactions between these objects, we can gain insights into complex behaviour of the Solar System. The simulations will account for different variations of starting positions, masses, varying velocities and number of stars in the system. The investigation of external forces such as the effect of a passing star or effect of dark matter) and the long term evolution of the Solar System will be considered.

1. Background

Solar System

The solar system is made up of planets, moons, asteroids, comets, and other objects that revolve around the sun, which is at the centre of the system. Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune are the eight planets in our solar system. The solar system is thought to be about 4.6 billion years old and formed from a massive rotating cloud of gas and dust. Astronomers find it fascinating, and it has sparked human curiosity and imagination for centuries. We can use Newton's laws of motion and gravity to calculate the positions and velocities of these objects over time in Python to simulate the solar system. We can apply these laws to create an accurate simulation of the solar system that can be visualised and investigated further to widen our understanding about our home.

2. Introduction

The midpoint Runge-Kutta Methods of order accuracy Δx^4 will make it ideal to simulate the Dynamics of the Solar System. The simulation provides the conservation of energy, the angular momentum and components displacement which will be used to analyse the orbits stability and time period. I have developed a model that simulates the dynamics of planets up to Neptune. The simulation will be performed with varying initial conditions such as: positions, speeds,masses and with different celestial objects in the system. I will explain my decision choosing the RK4 method instead of Euler methods on more basic systems, such as Start-Planet-Moon or binary star system in addition using the approximation methods for forces.

3. Method

Newton's Gravitational Laws

Sir Isaac Newton defined the forces at work in *Philosophiae Naturalis Principia Mathematica.* Newton determined why the planets move. Newton's laws apply only to particles in an inertial reference frame. His laws of motions are:

1. Everybody is at rest or in uniform motion along a straight line unless acted on by a force

2. The rate of change of linear momentum is equal to the force impressed and is in the same direction as the force.

   $$\overline{F} = \frac{d}{dt}(m\overline{v})$$
   $$\overline{F} = m\overline{a}$$

   Where v is velocity, a is acceleration and m is mass

3. For every force applied there is an equal and opposite reaction force. This gave rise to Universal Law of Gravitation where any two particles attract each other with the force of magnitude of

   $$F = G\frac{m_1 m_2}{r^2}$$

   Where G is universal constant of gravity, r distance between the particles
   This can be represented in vector form. The gravitational force on mass 1 at r1 due to another mass 2 at r2 is given by:

   $$\overline{F_{1,2}} = G\frac{M_1 M_2}{|r_{1,2}|^3}\overline{r_{1,2}}$$

   Where r_{1,2} = r1 - r2

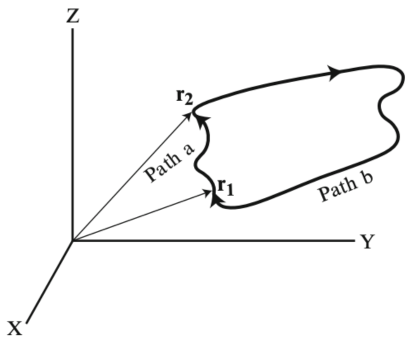
   Circular orbits arise whenever the gravitational force on a mass 2 equals the centripetal force needed to move it with uniform circular motion.

   $$F_c = F_g$$

$$\frac{M_2 v^2}{r} = \frac{G M_1 M_2}{r^2}$$

$$v^2 = \frac{GM_!}{r}$$

Work & Energys & Angular Momentum



If the force **F** acting on a particle at a distance of Δr, the work done is equal to the scalar product $\bar{F} \bullet \Delta \bar{r}$**.** Therefore the total work done going from r1 to r2 will be the sum of a line integral:

$$w_{12} = \int_{r1}^{r1} \bar{F} \bullet d\bar{r}$$

This could be simplified more as the work is equal to the change of kinetic energy.

$$W_{12} = \int_{r1}^{r2} \bar{F} \bullet d\bar{r} = m \int_{t1}^{t2} \bar{a} \bullet \bar{v} dt = \frac{m}{2} \int_{t1}^{t2} \frac{d}{dt} (\bar{v} \bullet \bar{v}) dt = \frac{1}{2} m v_2^{\ 2} - \frac{1}{2} m v_1^{\ 2} = \Delta KE$$

The force F is said to be conservative if the integral of $\bar{F} \bullet d\bar{r}$ is zero over all closed paths.

The potential energy $V(\bar{r})$ is the work done by a conservative force in going from point r1 to a reference point r0

$$V(r_1) = \int_{r1}^{r0} \bar{F} \bullet d\bar{r} + V(\bar{r_0})$$

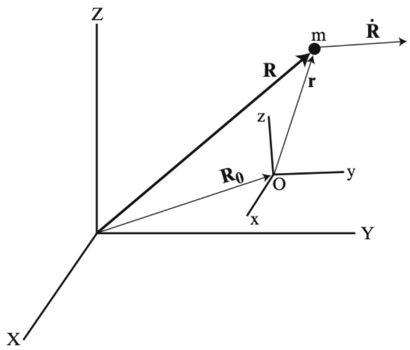
It can also be represented as a derivative of a force function

$$\bar{F} = -\Delta V(\bar{r}) = -\frac{GM_! M_2}{r}$$

The law of Conservation of Total energy:

$$KE_1 + PE_1 = KE_2 + PE_2 = E_{tot}$$

This equation gives the reason why the force is said to be conservative.



The angular momentum $\bar{L}$ of mass m about point O is defined as:

$$\bar{L} = \bar{r} \times m\frac{dR}{dt} = \bar{r} \times m\bar{v}$$

This can be taken further as the R=R0+r and the derivatives V=V0 + v

$$\bar{L} = \bar{r} \times m\frac{dR}{dt} = \bar{r} \times m(\bar{V_0} + \bar{v}) = \bar{r} \times m\frac{dr}{dt} + \bar{r} \times m\frac{dR_0}{dt}$$

The torque is the derivative of the angular momentum. It is found that the torque of the whole system is zero thus L is constant which implies that the angular momentum of a system is conserved if there are no external torques. This doesn't only apply to the single particle example given above but also to a collection of particles

Euler vs Verlet vs Runge-Kutta

Ordinary differential equations (ODEs) are equations for a function (e.g y) of one independent variable (e.g x) and its derivatives with its polynomials. To solve a differential equation the initial conditions have to be specified. Depending on the order of the differential equation the number of initial conditions required is also dependent, for example the gravitational force will require initial position and velocity.

Recalling the definitions of a differential:

$$\frac{df}{dx} = \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

Also, considering the Taylor expansion for a function about a point x:

$$y(x + \delta x) = f(x) + \delta x f'(x) + \frac{\delta x^2}{2!} f''(x) + ...$$

The overall aim of our method is to calculate y(x) for x points such as x1,x2,x3… and this is done using the initial point of x0 evaluating the y function for it and the differential finding then differential of y at point x1 and so on.

One of the basic methods for solving first-order ODE is using Euler's method. This method is quite bad due to its accuracy only to first order. This will be later demonstrated using the basic orbital mechanics examples for comparison. The Euler method starts from the finite differential formula:

$$\frac{dy}{dx} = \frac{Y(x+\Delta x) - y(x)}{\Delta x}$$
$$\Rightarrow y(x + \Delta x) = y(x) + \Delta x \frac{dy}{dx}(x)$$

The general formula for the indexing is:

$$y_{n+1} = y_n + \Delta x \frac{dy}{dx}(x_n, y_n)$$

Where y_{n} is the approximation of the solution at position x_{n}, similarly, y_{n+1} is the approximation of the solution at position x_{n+1}. The delta x is the step and (x_{n},y_{n}) is the derivative of the solution at point x_{n},y_{n}. The error is proportional to the size of the step, which can be represented from the Taylor Expansion:

$$\frac{(\Delta x)^2}{2!}\frac{d^2y}{dx^2} + O(\Delta x^3)$$

Where O indicates the order of the term. To clarify this is an error in a single step, and for the evaluation of all points the error will grow as each step is made from x_{0} to x this will result in a global error of N = (x-x_{0}/ delta x). In the context of mechanics, the euler method can be used for simple orbital mechanics, it can approximate the motion of a celestial body under influence of gravity. As mentioned before, it is very inaccurate for large time steps, also very true for very sensitive initial condition systems because the error will get amplified over time leading to significant inaccuracies. This error is linked to extrapolation.

The algorithm for the computer simulation:

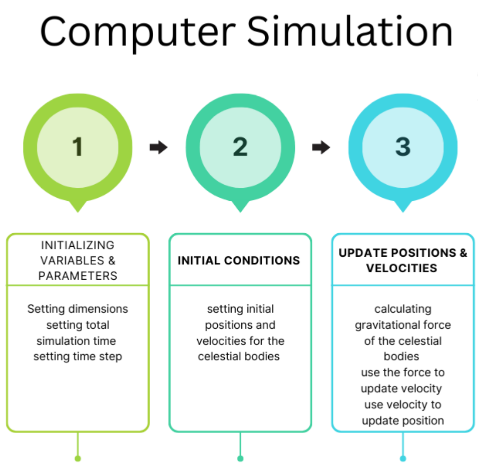


Fig.1 Flow chart explaining the steps of a simple algorithm for Euler Method

The flowing plot demonstrates the error propagation

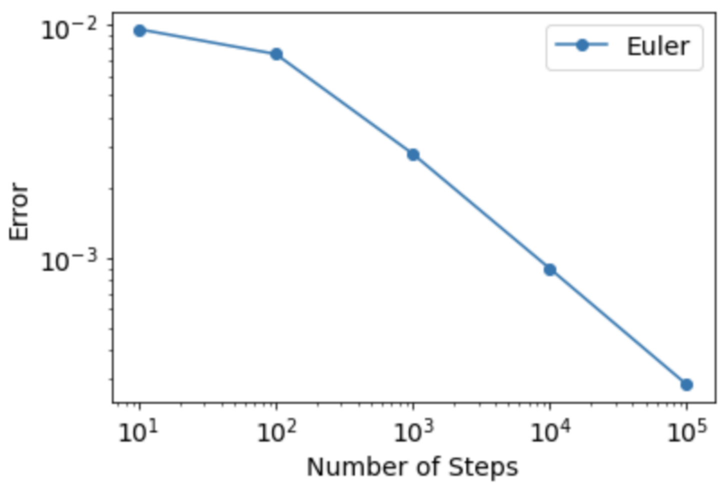


Fig 2. As the number of steps increases the error decreases.

There are numerous methods in order to improve the gradient at the start of the interval, we can reduce the timestep and estimate the gradient.

The other approach is called the Verlet method.The Verlet method works by approximating the position and velocity of an object at discrete time steps. Given the current position and velocity of the object at time t, the method calculates new position and velocity at time t + delta (t) by using objects acceleration at current time and previous time step.

$$x(x + \Delta x) = x(t) + \Delta t v(t) + \Delta t^2 \frac{F(t)}{2m}$$
$$v(v + \Delta t) = v(t) + \Delta t \frac{F(t) + F(t+\Delta t)}{2m}$$

This method is based on an idea that acceleration of an object between two time steps can be approximated as the average of the accelerations at each timestep. Verlet introduces error into simulation at each timestep, as it is second order it means that the error increases quadratically as the time step size increases.

The algorithm for the computer simulation:

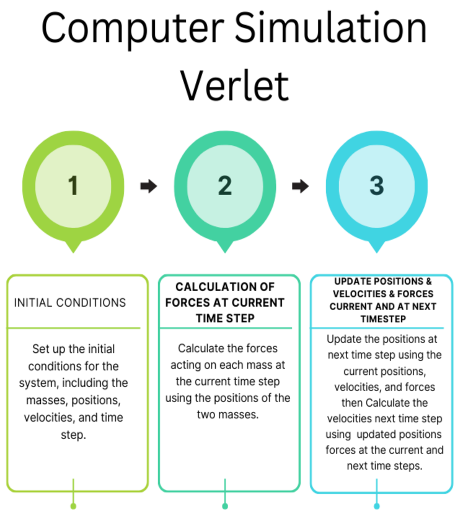
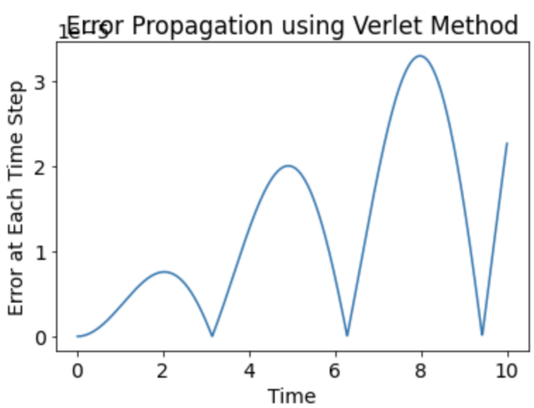


Fig. 3 A flow chart explaining the steps of a Verlet Method Update

To demonstrate the error propagation, I will compare the calculated result for harmonic oscillator with equation $x(t) = A\,cos(wt)$ where A is amplitude and w is frequency of the oscillator with the analytical solution using the Verlet method. I will demonstrate two graphs where I will be showing how error scales up with the number of points and will show the error between each timestep.



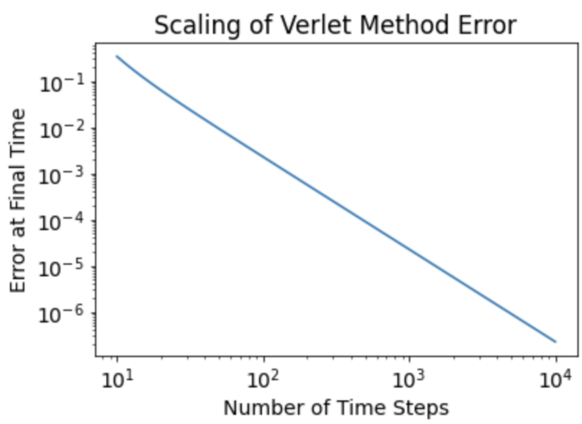


Fig 4. In the second graph, the error has a negative slope which indicates the error is decreasing as the number of steps increases, and it increases very fast as the graph is plotted using the log scale. The fist graph has a very small size which is 10^-5 but it is shown that there are jumps in error, if we zoom out it should show a line gradually increasing.

The other approach is called the midpoint method, where it estimates the gradient half way along the step.

$$k_1 = \Delta x f(x, y(x))$$

$$y(x + \Delta x) = y(x) + \Delta x f(x + \frac{\Delta x}{2}, y(x) + \frac{1}{2}k_1)$$

It is a part of a larger family called Runge-Kutta methods. The RK methods use intermediate gradients which improves the accuracy in each division. The method which i will be using widely in this research will be RK4 which denotes the accuracy of this approach to the 4 order. This requires 4 intermediate divisions of functions to update the overall function.

$$y(x + \Delta x) = y(x) + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

For the array indexing where f(x,y) is a continuous function.

$$k_1 = \Delta x f(x_n, y_n)$$

$$k_2 = \Delta x f(x_n + \frac{\Delta x}{2}, y_n + \frac{k_1}{2})$$

$$k_3 = \Delta x f(x_n + \frac{\Delta x}{2}, y_n + \frac{k_2}{2})$$

$$k_4 = \Delta x f(x_n + \Delta x, y_n + k_3$$

The algorithm for the computer simulation:

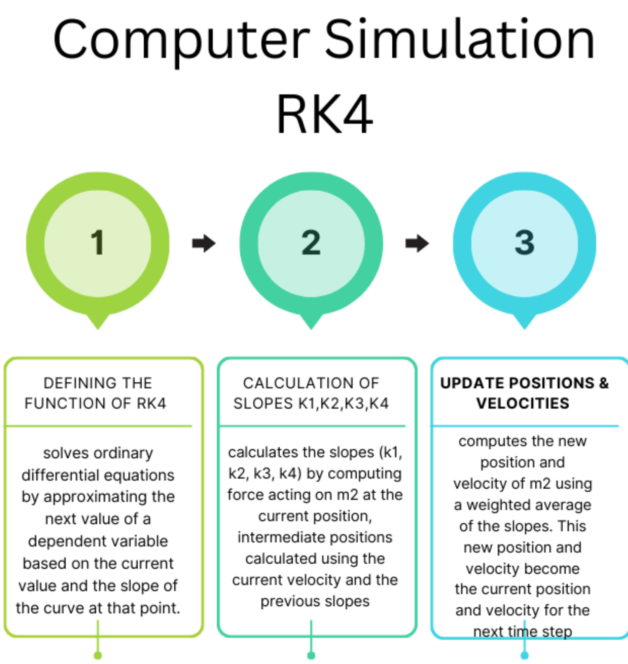


Fig. 5 A flow chart explaining the steps of a RK4 algorithm

I will also demonstrate error scaling and error between each timestep.

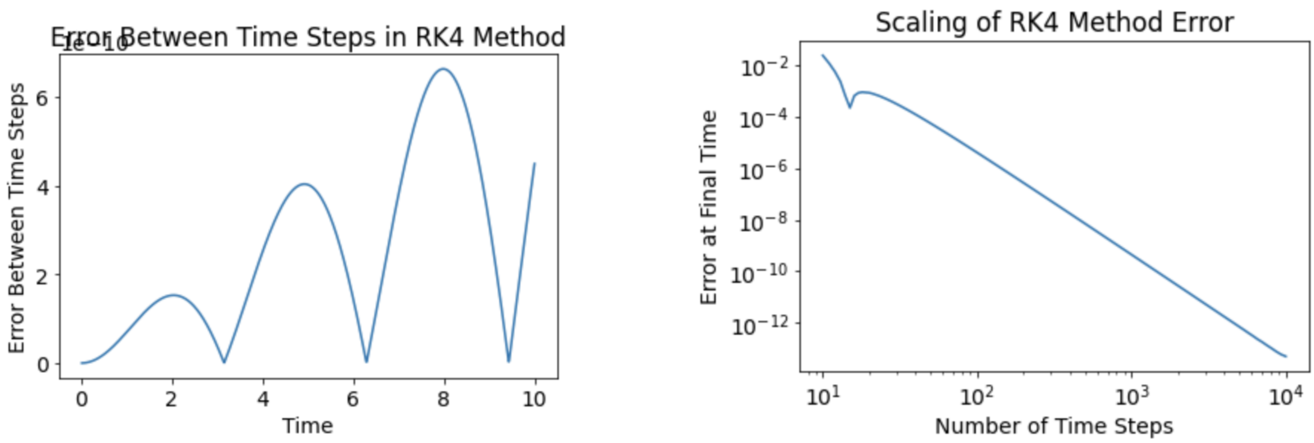


Fig 6. In the first graph, the error between timesteps is very small, in the scale 10^-10 and roughly jumps up as the time increases. This method produces an accurate approximation of the solution. In the second graph, it is shown that as the number of timesteps increases, the error rapidly decreases, the graph is plotted in log scale.

In the next section i will be performing the simulations

Basic Understanding of this Model

The basic theory so that we could simulate the orbits between the systems of planets/ moons/stars is the calculation of the gravitational attraction between each object in the system. The universal law of gravitation states that every object attracts other objects with the force which is proportional to their masses and inversely proportional to the square of the distance between them. For example, one of the basic systems will be one large mass and one small mass orbiting the large one e.g(planets and star). The planet is attracted towards the star due to the gravitational force of the star, which causes the planet to accelerate toward the sun causing the change in velocity and position. This change of velocity and position 'results' in an orbit around the star.

In the following simulations I will be using approximations to simplify the simulations and make them computationally feasible.

1) Point Mass Approximation - celestial bodies approximated as point masses

2) Constant Mass and Trajectory - in reality, celestial bodies can loose or gain mass due to various factors such as collisions, evaporation ect. And Trajectories can be also affected by other forces such as radiation pressure, magnetic fields, drag ect.

3) Center of Mass Approximation - Center of mass is used as the reference point of all calculations. This assumes that the system behaves as a single point mass with all other celestial bodies orbiting.

4) Circular Orbit Approximation - usually, the celestial bodies orbit in elliptical orbits around each other. In this case, I will not be considering elliptical orbit shape.

5) Ignoring effects of perturbations (for several examples) - assumes that the gravitational force between celestial bodies is the only factor affecting their motion. However, there are other gravitational perturbations such as nearby planets/asteroids that can affect the motion. In the first cases this will be ignored but for the last part will be taken into consideration so that I can simulate the solar system dynamics.

More Complex Understanding

There are several more complex theories that simulates the attraction between the celestial bodies. One of them is using General relativity which describes the behaviour of gravity in term of geometrics of spacetime. According to General relativity, gravity is not a force between masses as described by Newton but instead it is a result of curvature of spacetime due to the presence of massive object e.g celestial body. The curvature is determined by distribution of matter and energy which creates gravitational field that is effecting the motion of the celestial bodies around it. General Relativity predicts many things one of which are the gravitational waves caused by the acceleration of massive objects. Overall it is very complex to simulate and I will stick to the basic model with several approximations.

4. Results and Discussion

Two bodies: Different Mass

Euler Method

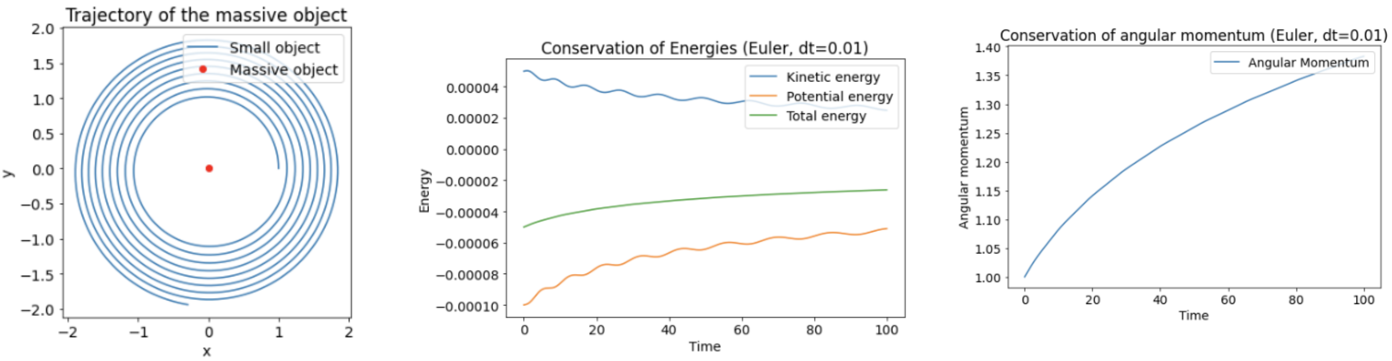


Fig 7.. Visual representation of the orbit paths, energies and the angular momentum for the system of small mass orbiting the massive mass, using the Euler Method of propagation. This is represented for dt = 0.01 for 100 steps.

The orbit of small mass around a massive mass is not stable, it is moving further away with each orbit. This is due to euler methods limitations. If i will be increasing the dt and keeping the number of steps for the propagation the same, i will get orbits more compact together but i will get the 'spinnign out' this is confirmed by the energies (as they are not constant e.g KE is decreasing as it spinns away , the PE increases and total energy is the combination of both and having periodicity line in them) and also by the angular momentum (as it is increasing meaning it is spinning out)). Increasing the number of steps will just increase the number of orbits.

Verlet Method

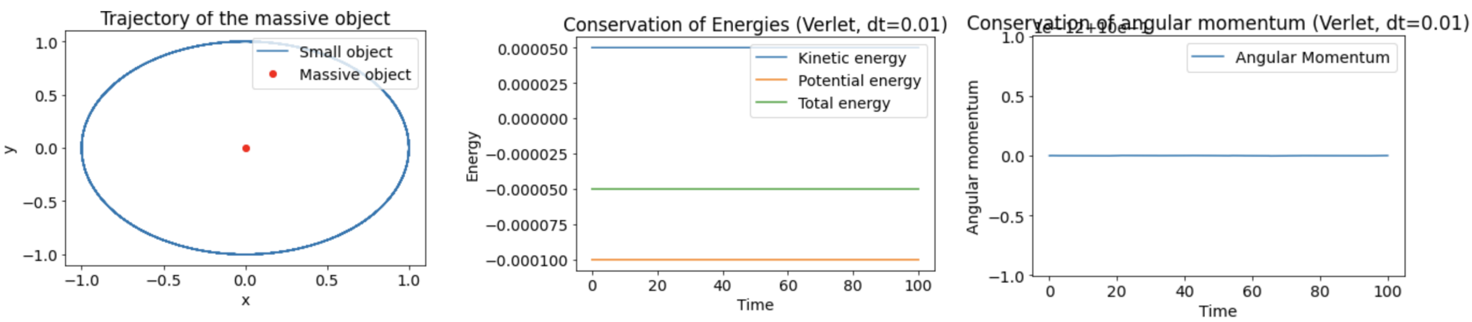


Fig 8.. Visual representation of the orbit paths, energies and the angular momentum for the system of small mass orbiting the massive mass, using the Verlet Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure we can see that the trajectories, energies and angular momentum are stable. This means that the smaller mass is in constant orbit and it will not spin out. This is expected as Verlet is more accurate than Euler Method and as Euler spins out the Verlet keeps its orbit. The kinetic energy is positive meaning the object is moving, the potential is negative meaning that the smaller mass is at a distance form the larger mass. The angular momentum is approximated by Verlet to be a zero, this can be that the orbit is a completely circular path at a constant speed and the direction of motion is constantly changing. As its velocity vector is always perpendicular to the position vector with respect to the centre of mass, in this case its the larger mass. There is no net torque acting on the smaller mass, and thus angular momentum is zero.

RK4 Method

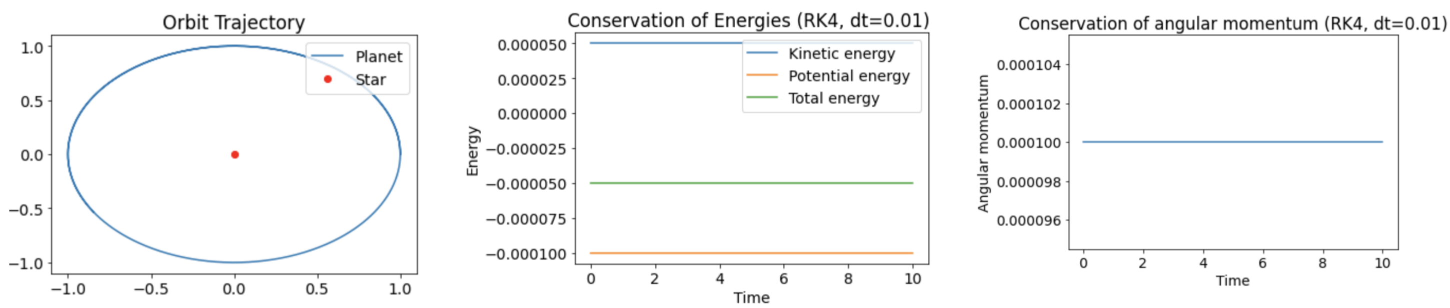


Fig 9.. Visual representation of the orbit paths, energies and the angular momentum for the system of small mass orbiting the massive mass, using the RK4 of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure we can see that the trajectories, energies and angular momentum are stable. This means that the smaller mass is in constant orbit and it will not spin out. The energies look sensible as the Kinetic energy is positive and potential is negative but they are very small. The angular momentum is constant but it is positive. This indicates two things, one is that smaller mass is in a stable orbit around the central body.(To go in more detail, as object gets closer to the central body its speed will increase to compensate for the decrease distance similarly as it moves away as it moves away the speed will decrease so the angular momentum is conserved). Second, the shape of an orbit will be an ellipse. The energy depends on the initial conditions and as we can see its negative due to large negative PE. The energies are conserved, thus the smaller mass will continue to move in the same elliptical orbit. Increasing the number of steps and dt will not change it.

<u>Two Bodies: Similar Mass</u>

Euler method

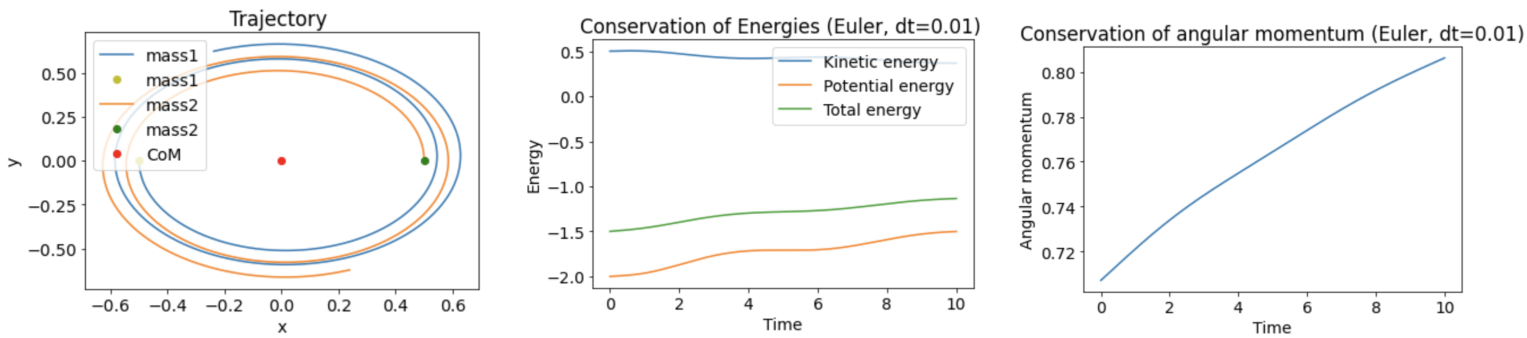


Fig 10.. Visual representation of the orbit paths, energies and the angular momentum for the system of two small masses orbiting their centre of mass located at the origin, using the Euler Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure , we can observe that there is a similar orbital spinning out as in the simulation of the trajectory with one object orbiting another. WE can observe that both move further away with each orbit. The energies are not conserved, as the masses move away the kinetic energy decreases and the potential energy 'increases' due to increasing distance. The angular momentum is not conserved as it is not a stable orbit thus there will not be a constant value.

Verlet Method

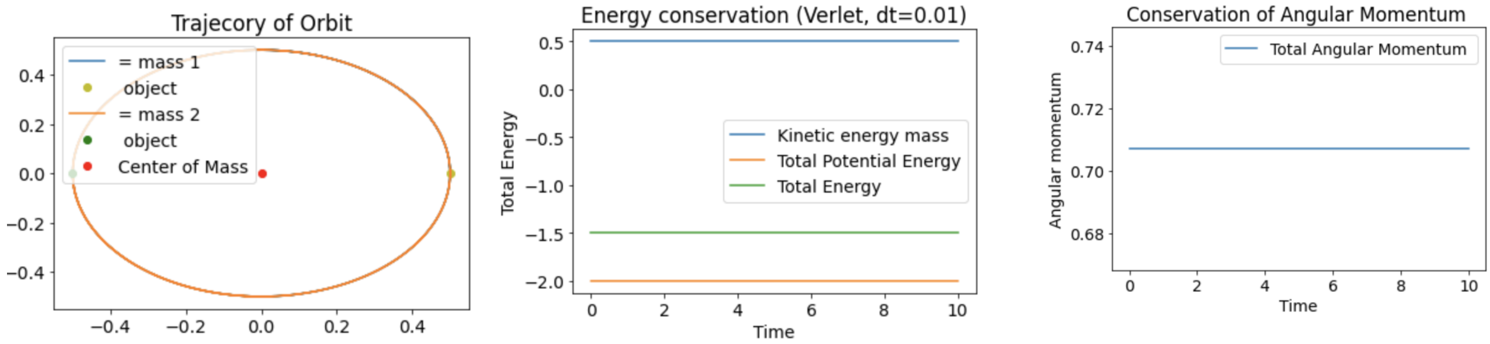


Fig 11.. Visual representation of the orbit paths, energies and the angular momentum for the system of two small masses orbiting their centre of mass located at the origin, using the Verlet Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure, we can observe that the orbit is table, both of the masses orbit their centre of mass. The energies are constant as well and especially the angular momentum is conserved. The angular momentum is positive and constant indicating that there is an elliptical orbit around the centre of mass for both of the masses.

RK4 Method

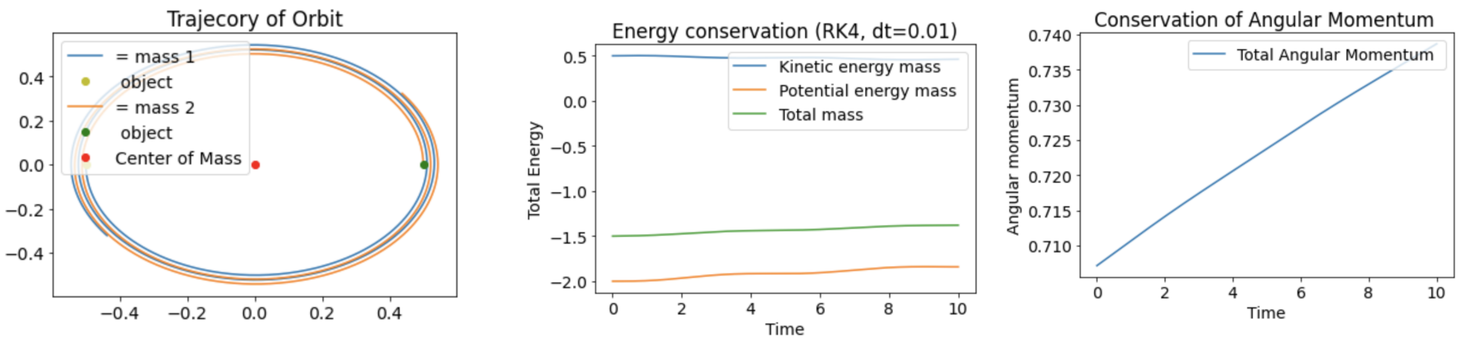


Fig 12.. Visual representation of the orbit paths, energies and the angular momentum for the system of two small masses orbiting their centre of mass located at the origin, using the RK4r Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure we can see that there is no stable orbit, and both planets are spinning out of the initial position with each orbit. The energies are not constant and the angular momentum is not constant either supporting the fact that there is no stable orbit. The reason for this could be due to numerical errors that accumulate over time. Or, because RK4 is not a symplectic integrator (Numerical integration scheme for Hamiltonian systems) meaning that they do not conserve energy in the simulation. The Verlet method may require a much smaller step size than the RK4 method in order to minimise oscillations in solution but the method is symplectic. RK solver will build up an error that is not reduced.

Three Body System
Approximation 1: Ignoring m1 and m3 interaction

For this approximation I used the shifting method, it is described more in detail in my code, but in short I am ignoring the interactions between the moon and the star making this simulation like two systems, one system is the planet orbiting the star and other is the moon orbiting the planet.

Verlet Method

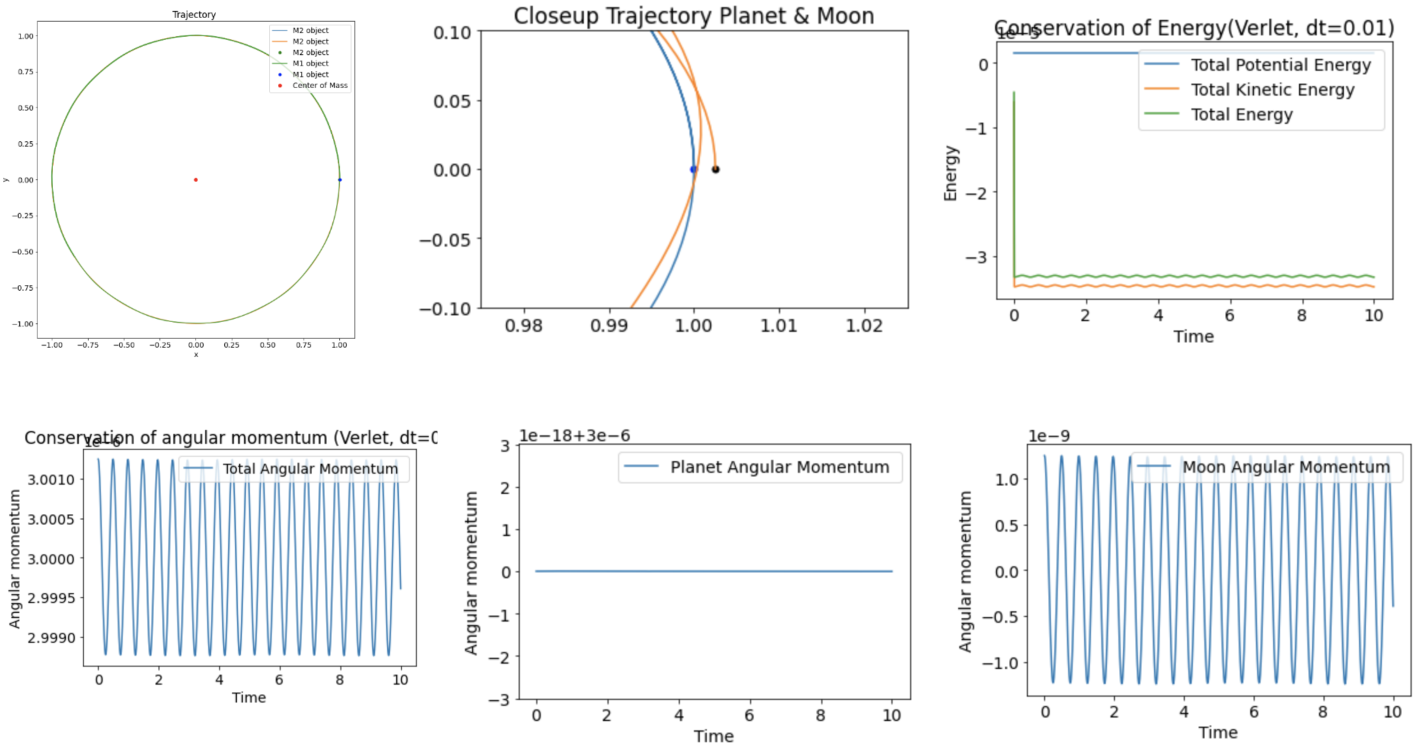


Fig .13. Visual representation of the orbit paths, energies and the angular momentum for the system star, planet and its moon all orbiting around the sun , using the Verlet Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure we can see that there is a stable orbit. Moon is orbiting the planet and the planet is orbiting the sun. From the total energies plot, we see that energies are conserved (ignoring the first point this is due to the system correcting itself after the first step creating a discontinuity in the energy). The plot of the total angular moment is periodic which suggests there is a precession of the orbit of a planet or the moon. Inspecting further, we can see that the planet's angular momentum is constant and at zero meaning it has circular orbit around the sun but, moon has periodic oscillations meaning that the moon's orbit has precession or it can be caused due to wobbling of the axis of rotation. Increasing the number of steps ‘increases’ the number of orbits and increasing the dt causes the energy to get rid of the wobbles that are observed in the figure and it becomes a straight line.

RK4 Method

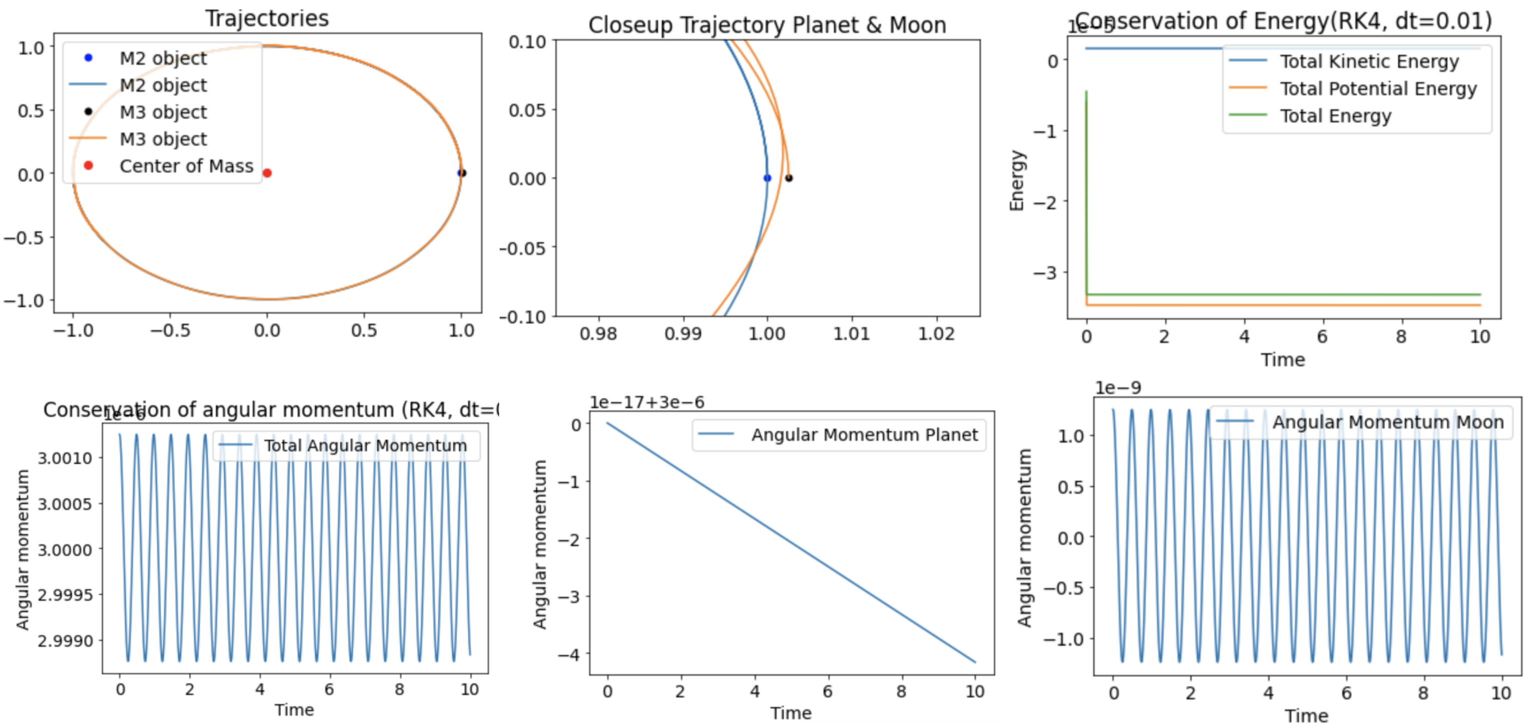


Fig 14.. Visual representation of the orbit paths, energies and the angular momentum for the system star, planet and its moon all orbiting around the sun , using the RK4 Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure, we can observe that the RK4 method also produces a stable orbit where the moon orbits the planet and the planet orbits the sun. The energies are conserved , having a small positive kinetic energy and large negative potential energy, no wobbling showing comparing it to the previous method. The angular momentum becomes more interesting here. The angular momentum for the planet is decreasing and it is negative, this indicates that the planet is experiencing a torque which is causing it to lose angular momentum over time, but it is very small 10^-17. This could be due to errors building up like with the previous system or it could be due to the moon. Increasing the number of steps does not result in the planet to spiral into the star. The moon's angular momentum is periodic, as before, indicating that the moon is experiencing periodic variation in its rotational motion around the planet.

Approximation 2: Interaction of CoMs

For this approximation I will be considering two systems as before but this time I will be considering the centre of masses. Sun and the centre of mass of the planet and the moon orbiting the centre of mass of the whole system, and the second system is the moon and planet orbiting the centre of mass of their system.

Verlet Method

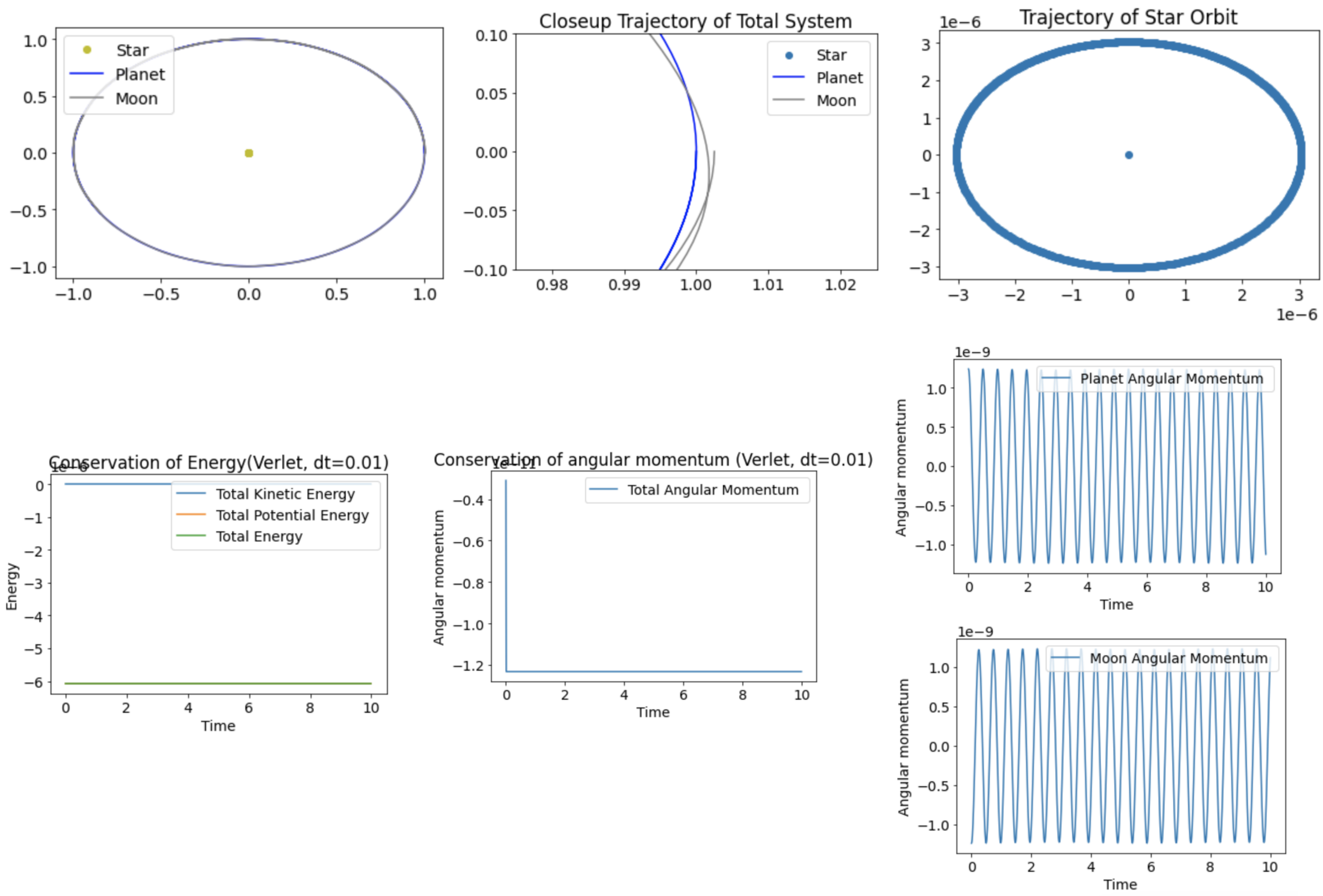


Fig .15. Visual representation of the orbit paths, energies and the angular momentum for the system star, planet, moon and sun all orbiting around their centre of mass , using the Verlet Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure above we can see that with a simulating centre of mass interaction between each other still makes the planet's moon and star have a constant orbit. For this example I have reversed the orbit trajectory, so this time it is going anticlockwise. From the plots we also see that the star is orbiting the centre of mass. From the conservation of energies, we see that energies are conserved, we have total kinetic energy as positive and total potential as large negative . The angular momentum is negative due to the choice of velocity directions, making the orbit going clockwise, the total angular momentum becomes positive meaning there are elliptical orbits. For the angular momentum of the sun, it is a positive constant number indicating that the sun is orbiting the centre of mass with an elliptical orbit. Increasing the number of steps does not make it to increase it orbits or increasing dt doesn't change it either. It is very stable. The angular momentum for the Planet and Moon, they are periodic, indicating that there is a periodic torque causing the orbits to wobble or more generally it experiencing periodic variation in their orbits. Comparing the close up trajectories with the previous example where the com was not considered, they are very different.

RK4 Method

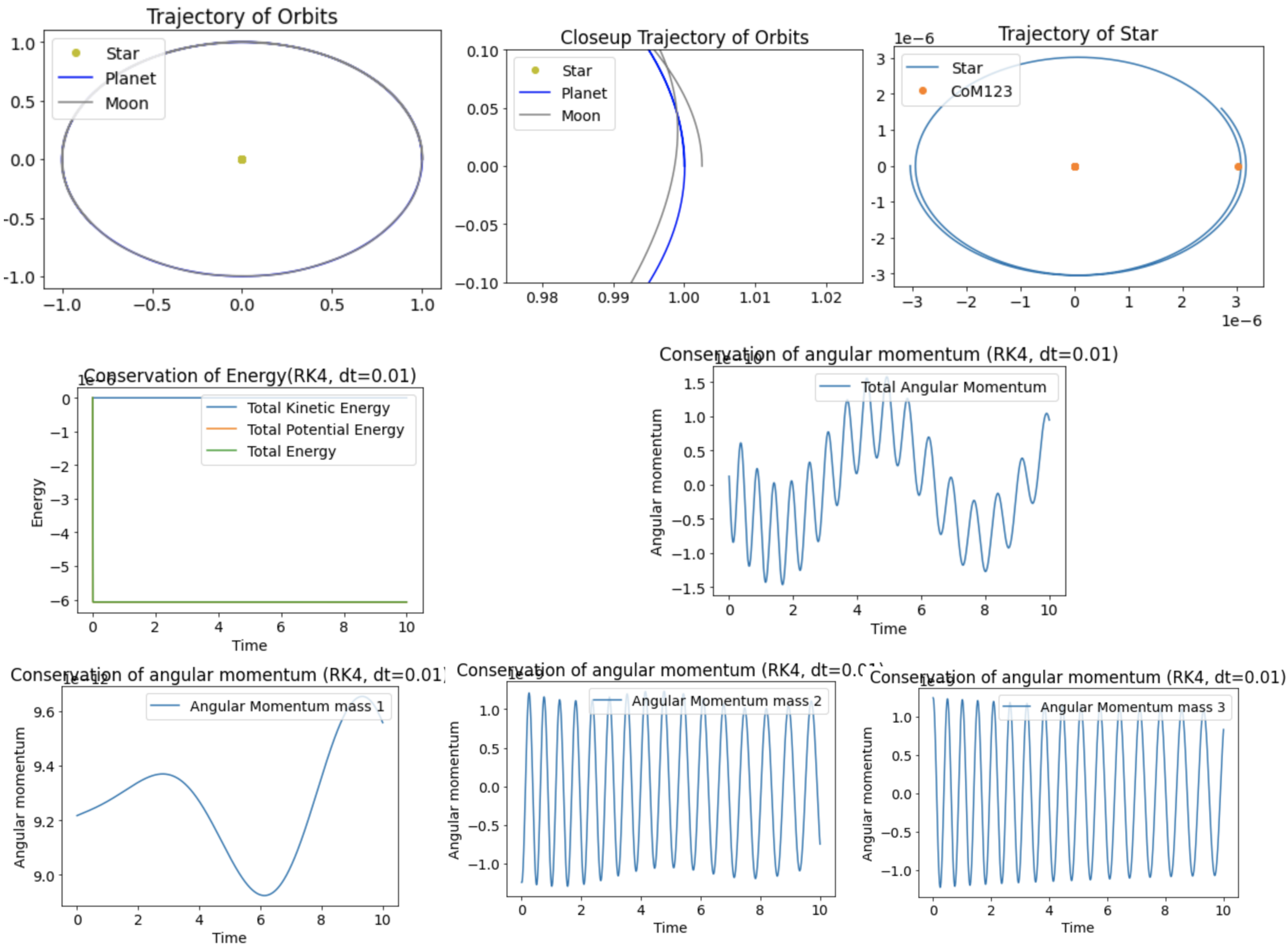


Fig .16. Visual representation of the orbit paths, energies and the angular momentum for the system star, planet, moon and sun all orbiting around their centre of mass , using the RK4 Method of propagation. This is represented for dt = 0.01 for 10 steps.

From the figure, we can observe that there is a stable orbit, the moon is orbiting a planet and a planet is orbiting the sun. The orbit of the sun around the center of mass is not stable but it is very small. Increasing the number of steps shows that the sun is spinning out of orbit in an elliptical manner. But when increasing the dt to 0.001 the sun is not spinning as fast away but very slowly, this can be due to errors building up like in the previous examples. The energies are looking correct with potential being negative and kinetic being positive but very small. The total angular momentum is periodic and itself it is oscillating between negative and positive with each step becoming more spread out. Looking closer at angular momentum of each celestial body, we can observe

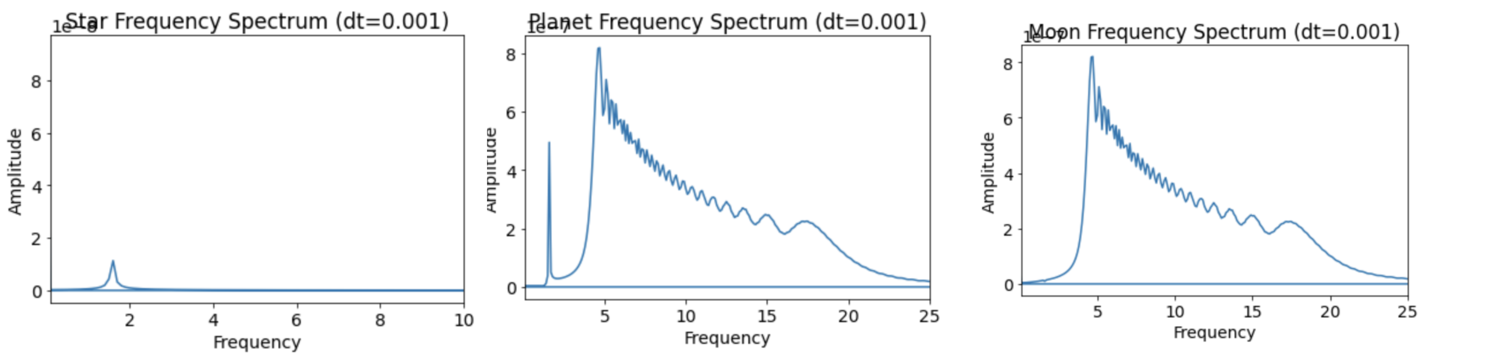


Fig .17. Visual representation of the frequencies in the 3 body telling the power in different frequencies

Here i used fourier transforms for the orbits to find the different frequencies as the system progresses, i can find the time period of the orbit by doing T=1/f

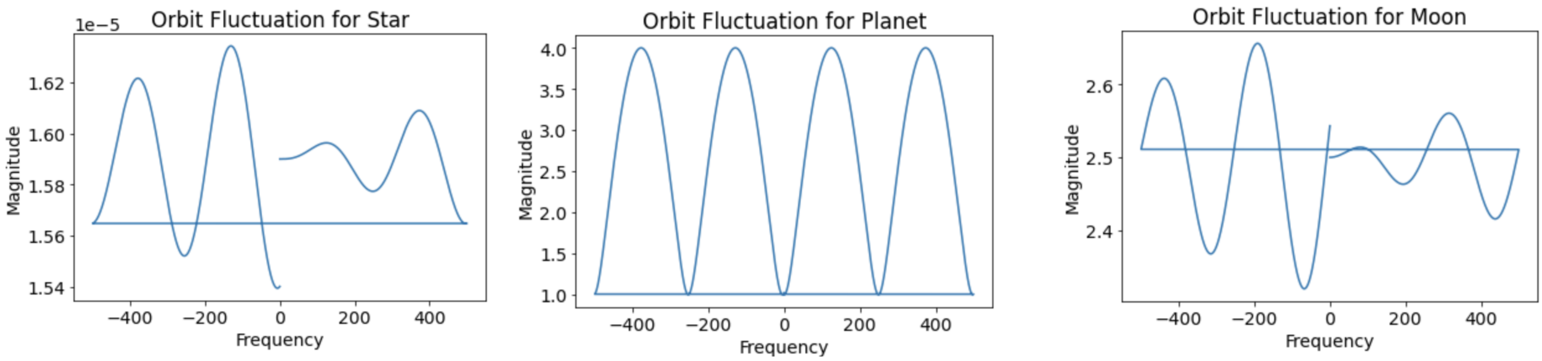


Fig 18.. Visual representation of the of the extraction of the dominant frequencies in the 3 body system and converting it to a timescale were it is mapped to the radial distance

Here i am mapping the dominant frequencies to the radial distance to find if the orbit is periodic and has some sort of the pattern. The earth is orbiting the sun with a specific frequency with a constant radial magnitude, whereas the star and moon are not following the constant path. Both graphs look very familiar at the end. This needs to be explored further.

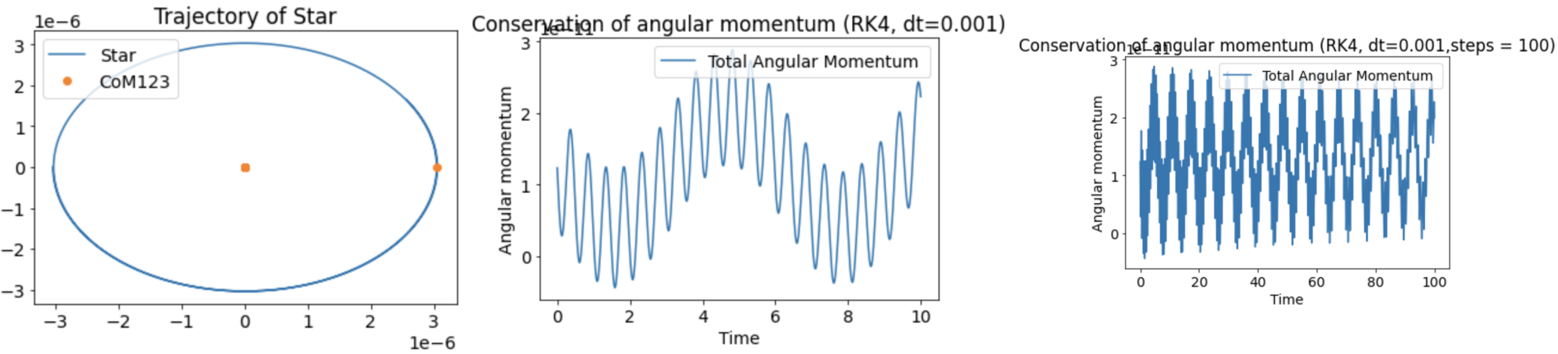


Fig 19.. Visual representation of the orbit paths, energies and the angular momentum for the system star, planet, moon and sun all orbiting around their centre of mass , using the RK4 Method of propagation. This is represented for dt = 0.001 for 10 steps and 100 steps.

The star is orbiting the centre of mass of the whole system. From plotting the angular momentum of the whole system we can see periodicity of angular momentum going from negative to positive. This can be re;ated to the sun's motion, the larger periodicity and the smaller one to the moon. Increasing the number of steps we can see that there is still periodicity which becomes relatively constant.

5. Extension

For my extension i have chosen to simulate the solar system up to mars for the first part using only the forces between the planets and for the second part I will be using the centre of mass principles in order to show trajectories of the planets

Part 1

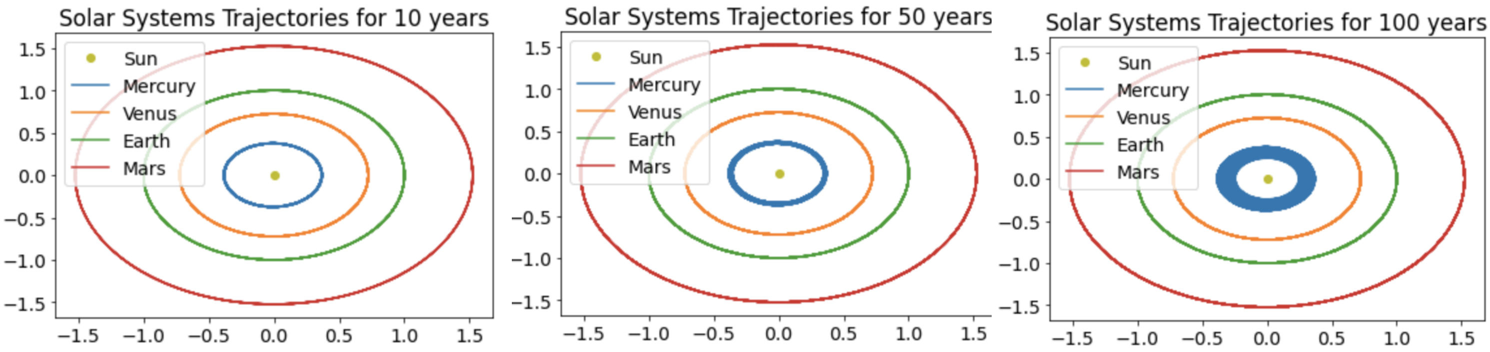


Fig .20. Visual representation of the orbit paths of the solar system over period of years of 10, 50 and 100 years

From the figure, we can observe the structure of the solar system over the course of 100 years. It is shown that mercury has unstable orbits and it spins toward the sun, around the 105 year causing disturbances to over planets in their energies but not affecting their orbit. Over planets have very stable orbits which need to be simulated with a larger time frame which unfortunately my computer cannot handle.

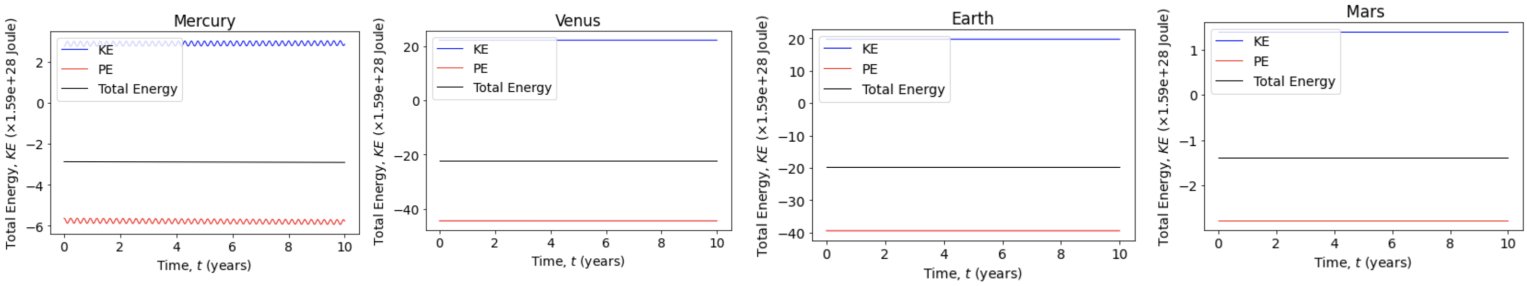


Fig .21. Visual representation of the energies graphs for the solar system bodies over the period of 10 years

We can observe the energies of the planets, mercury has small energies as it is close to the sun, has smaller mass than others but relatively high velocity but overall the total energy is not as large as Venus or earth. Mars has lower total energy than mercury, Mars is 5 times further away from the sun and twice as heavy than mercury and has twice as low velocity than mercury. Due to these factors, Mars appears to have lower total energy than mercury. Overall the energies are looking sensible for the simulation

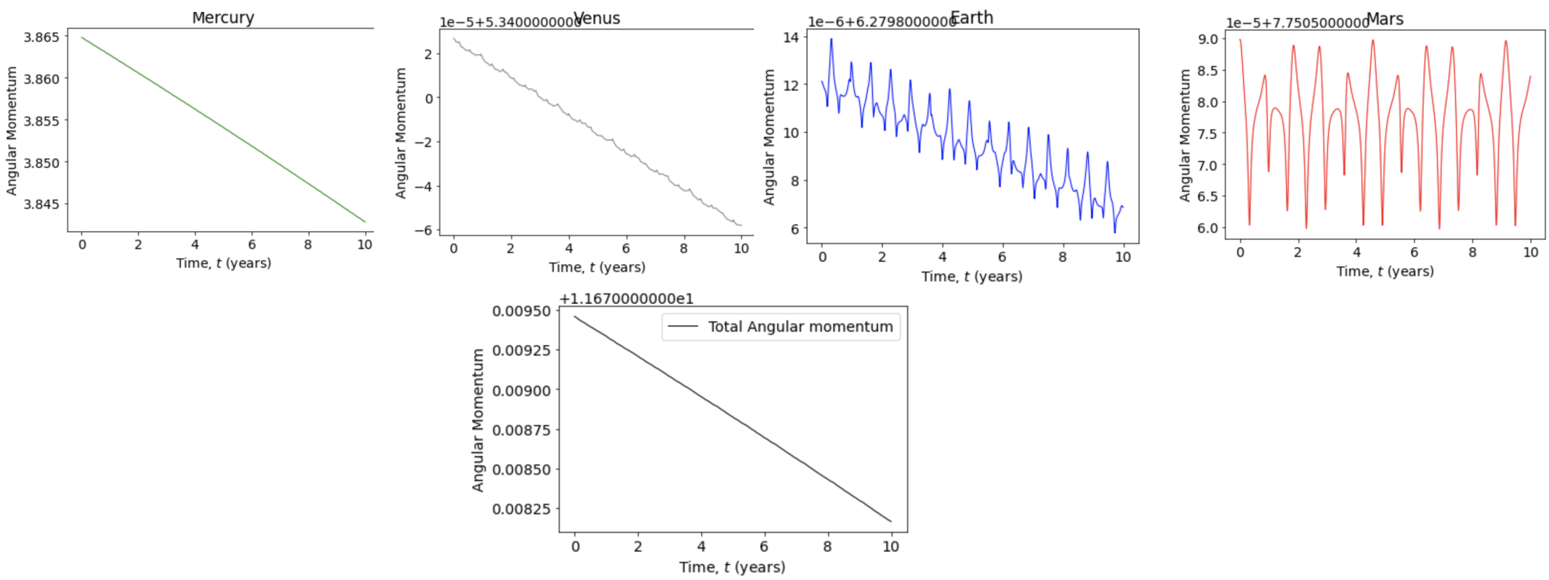


Fig .22. Visual representation of the angular momentum graphs for the solar system bodies over the period of 10 years

From the figure we can see that the overall angular momentum is decreasing, but it should be a constant line. Also it is observed that there is a periodic behaviour in the plants, this is caused by the forces of other celestial bodies orbiting around. The overall decrease , saying that the plants are experiencing 'dissipative forces' causing the orbit to decay, this will eventually lead to the planet colliding with the central body or being ejected from the system altogether. Simulation of 105 years shows us that mercury is spinning inwards to the sun, and seeing from the graph that the angular momentum is quite high. Overall mars is only one which is very stable overall but venus,earth are way more stable than mercury but will eventually spin inwards. Demonstrating this below:

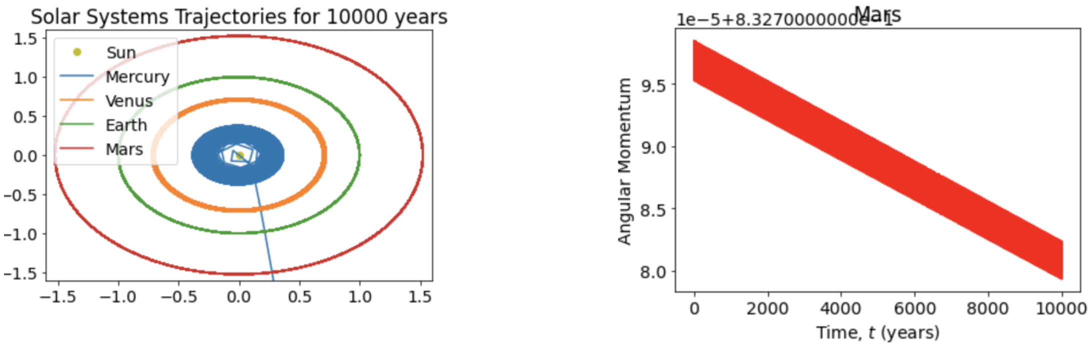


Fig .23. Visual representation of the orbits trajectories and angular momentum of mars after 10000 years of simulation.

Overall, we can see from the figure that orbits became more 'thicker' meaning the system is not stable and everything is spinning inwards towards the sun. The solar system should be stable over a long period of time as a sun becomes a red giant and will 'ingest' all planets up to mars causing the collapse of the solar system. We can observe that mercury spins towards the sun, but then slingshots out. I was not considering the impact of the mercury with the sun, but overall mercury is consumed by the sun.

Also, changing the parameter a bit, for example: what if there was a black hole? Typically, for the star to become a black hole, it needs to have from 3 to 10 times solar mass.

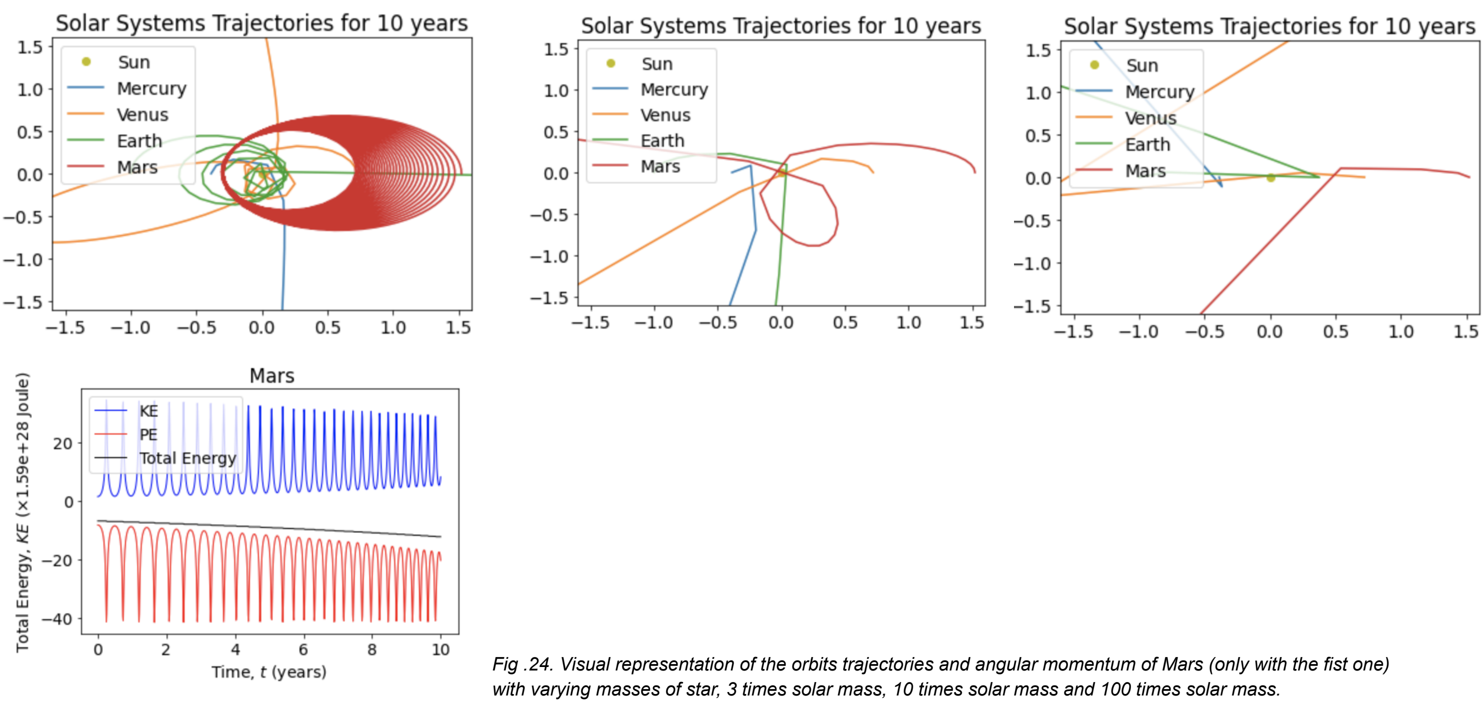


Fig .24. Visual representation of the orbits trajectories and angular momentum of Mars (only with the fist one) with varying masses of star, 3 times solar mass, 10 times solar mass and 100 times solar mass.

Here we can see that orbits because unstable, for 3 time the solar mass we can see that earth and mars are in elliptical orbits which are spinning out (so not constant angular frequency). For other two examples, all planets are consumed by the star/black hole.

I have unfortunately not simulated the whole system, i wished to simulate until the jupyter so then it becomes a sun, but for this example i will make different planets to have solar mass, what will happen?

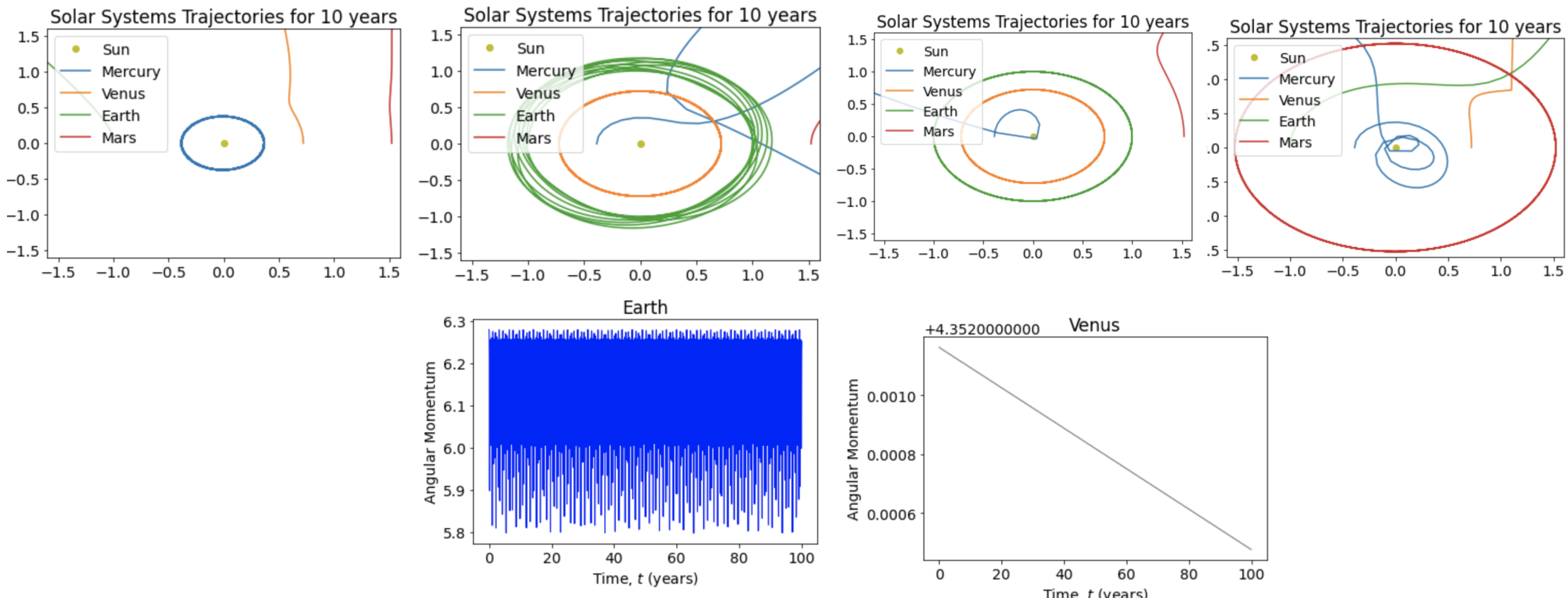


Fig .25. Visual representation
of the orbits trajectories and angular momentums of Earth and Venus, each graph shows the
changed planet mass to solar with each planet starting from mercury and finishing to Mars, solar mass = 1.989e30 kg

From the figures we observe that there are different combinations of orbits emerging. From the first graph we observe that when mercury possesses solar mass with its initial speed all other planets fly away. When Venus possesses solar mass, Mercury and Mars fly away but Earth appears to have a stable orbit. Inspecting the angular momentum graph for earth, we can see that it is periodic, meaning there is a wobble to the orbit which can be seen in the plot, and checking it over 100 years suggests that earth has stable orbit. When Earth has solar mass, mercury and mars fly away but venus appears to have a stable orbit, with closer inspection we see that it is not completely stable and other the course of the time it will spin into the sun. When mars possesses a solar mass, all other planets fly away. When each planet possesses the solar mass it always stays in orbit, this can be connected to the previous simulation made when both similar mass objects orbit, the only two masses considered as main as they are affected more by each other.
What about if each planet possesses one planet's velocity?

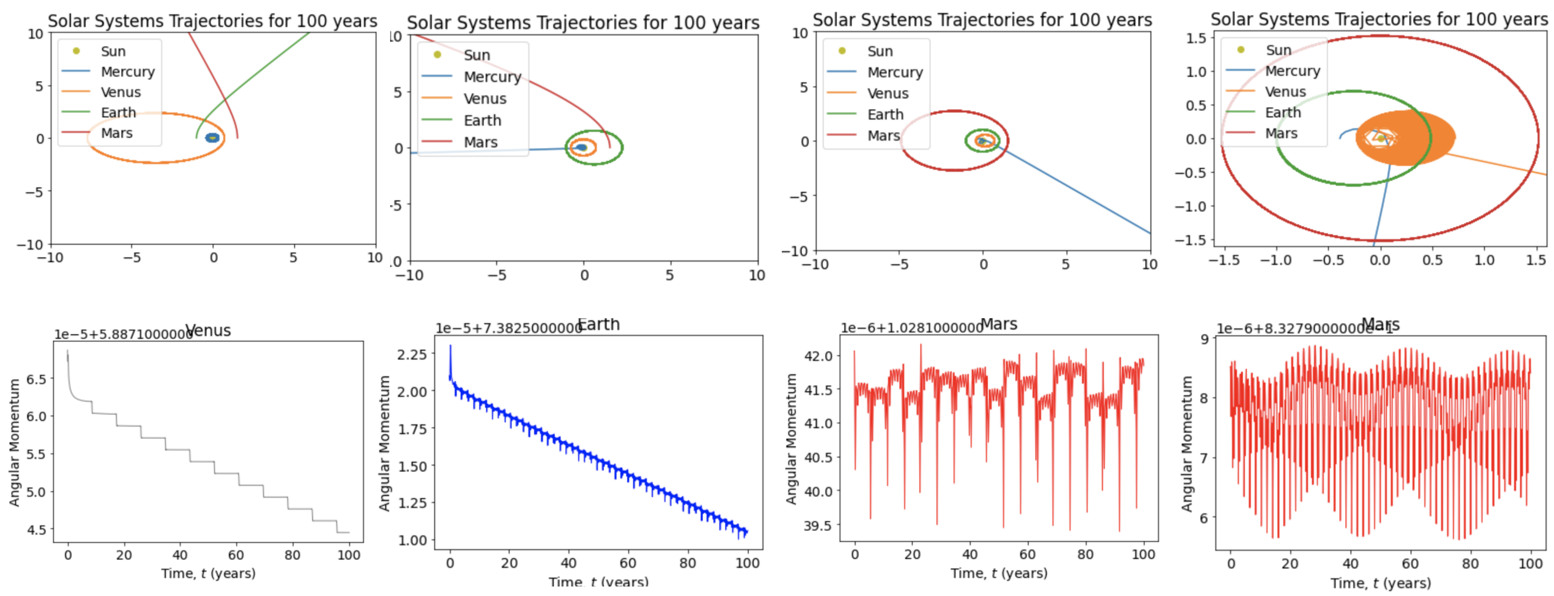


Fig .26. Visual representation of the orbits trajectories and angular momentums of several planets in the combinations. Each graphs shows the velocity of each planets using velocities of mercury,venus,earth and mars gradually

From the graphs we can see that some velocities do work for other planets and others don't. From the first plot, we can see mercury is orbiting with its speed and Venus can orbit with mercury's velocity but all other planets will fly away. The angular momentum is decreasing other times meaning that it is not a stable orbit and it will eventually spin into the sun. Next graph 2, when planets possess Venus velocity, Mercury earth has a relatively stable orbit but will eventually spin into the sun when all other planets will fly away. Next graph 3, when all planets possess Earth velocity Venus and Mars have stable orbit. Looking at venus angular momentum, it is slowly decreasing meaning eventually it will spin into the sun but Mars has signs of a periodic angular momentum which is not overall decreasing other time.Next, graph 4, when all planets posses the velocity of Mars, mercury flies away straight away, venus makes several orbits until it is spinning inwards into the sun and earth has a stable orbit, looking at it angular momentum it is decreasing slowly meaning eventually it will spin into the sun. But also looking into Mars angular momentum, it creates a periodic shape. This can be one of the possible future explorations.

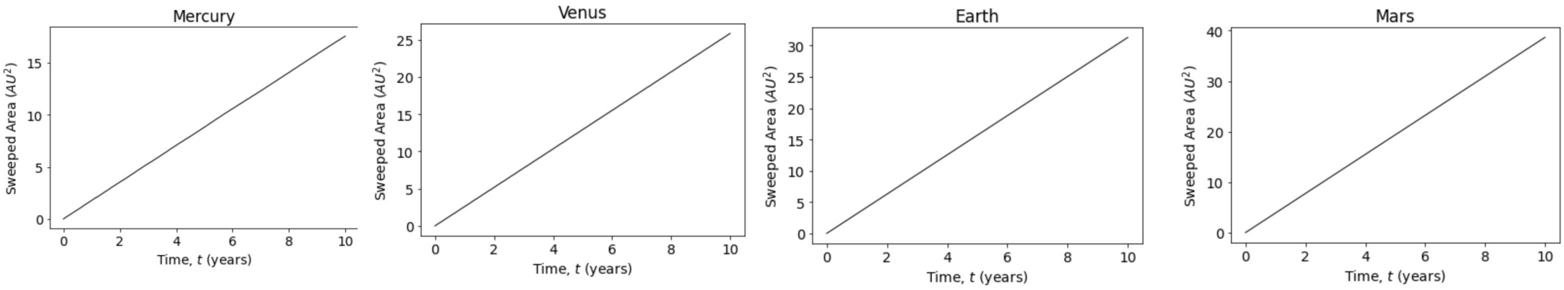
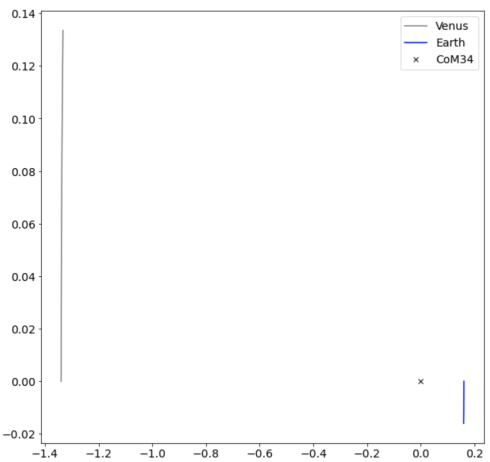


Fig .27. Visual representation of the area swept for the solar system bodies over the period of 10 years

From the figure we can see the sizes of the orbits with mercury having the smallest area as it is the closest to the sun and Mars having the largest as it the furthest away in this simulation. It is a constant gradient line meaning there is no change in their orbit. For mercury if we increase the simulation until 105 years the graph will show a curve (a decrease of the gradient at the top) as it is spinning into the sun and less area is covered as the radius decreases.

Part 2

For part two, I was wishing to simulate the trajectories of planets using their centre of mass used in the previous result to minimise the errors. Unfortunately, the simulation does not have a good outcome. The main idea of the simulation is to think planets as a collection of masses rotation their centre of mass.



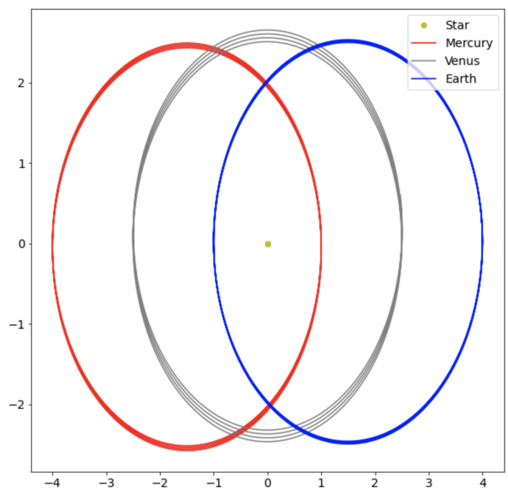


Fig .28. Visual representation of orbital trajectories Mercury, Venus and Earth using their centre of mass.

I have tried to approximate the outermost 2 masses as a system, so it will be earth and venus, finding their centre of mass which will be rotation with Mercury, then finding their centre of mass and updating it will be the sun where they have the centre of mass of the whole system. Using this approach, there was hope for a nice obits shape which will have a very small deviation. The theory may be wrong here, but as one of the future exploration very interesting ways of simulating the orbits.

6. Possible Improvements

I suspect the accuracy of using the RK4 method for simulating the orbits of planets in the solar system. RK4 is a very accurate method for solving differential equations attacking the problems at quite good precision. However, as mentioned before, they are not symplectic (they don't conserve energy). I am also simulating their complex system where errors accumulate very easily. Thus, I need to use more accurate numerical methods which also are symplectic.
I also suspect that not doing the full solar system may affect the predictable results. As the gravitational force of attraction is present between all components of the solar system, everything contributes to the orbital trajectory.

7. Future Exploration

What I covered has just started exploring the capabilities of solar system simulation. There are many interesting topics which could be explored more.

Asteroid Belt & Kuiper Belt

Exploring the asteroid belt, which lies between Mars and Jupiter's orbits, is a fascinating subject for astronomers and space scientists. With recent advancements in space technology and growing interest in space exploration, simulating the asteroid belt in Python can assist us in investigating how these objects affect the solar system. By representing each asteroid as a point in space and applying Newton's laws of motion and gravity, we can create a realistic simulation of the asteroid belt and observe how these objects interact with each other and with the planets in the solar system.This can lead to a better understanding of our solar systems origins as well as insights into the risks and opportunities of asteroid mining and planetary defence.

The Kuiper Belt is a solar system region that extends beyond Neptune's orbit and is home to many icy objects, including dwarf planets for example as Pluto, Eris, Haumea, and Makemake.The Kuiper Belt is important to study because it can reveal information about the formation & evolution of the solar system, and origins of comets. Python simulations of the Kuiper Belt can assist us in modelling the motions and interactions of these objects and investigating the dynamics of this region of the solar system.

Orbits at Angle

Exploring the orbital inclinations of objects in our solar system is very important because it can provide insights into the solar system history and formation. The inclination of a planet's orbit, for example, can reveal information about its formation and evolution, whereas the inclination of an asteroid or comet's orbit can reveal information about its origins and potential impact risks. Simulations of celestial body orbits and inclinations can help us model the complex interactions of these objecs and investigate how they evolve over time.

Simulation of Surface Temperature

Planetary surface temperature simulation is an important area of study for astronomers and planetary scientists.We can use numerical models that incorporate the planet's distance from the sun, albedo, and atmospheric composition to simulate the surface temperature of a planet. These models can account for things like the greenhouse effect, which occurs when certain gases in the atmosphere trap heat and warm the planet's surface.

8. Conclusion

Overall, the model produced reflects the general behavioural experience of the solar system. Firstly, it simulates the orbits of planets simulated from Mercury to Mars with most having a relatively stable orbit (ignoring mercury ofcourse). It demonstrated that small changes to the system will result in major changes in orbit. It demonstrates the small deviations in the angular momentum and how it is affected by other celestial bodies due to their interaction where orbital period and stability can be predicted. It demonstrated the energies of the celestial bodies and their trends.

9. References

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