PID Controller

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APPROACH

For this project I began by implementing the PID controller equation, Equation 1, and then tuned the parameters until I had a result that successfully controlled the vehicle around the track. In order to control the vehicle's steering, a PID controller was used for the center of the road and a proportional controller for the outer edges. The vehicle's throttle was controlled using a PD controller combined with a nonlinear weighting factor based on the vehicles steering angle.

PID Controller

$$u(t) = K_p * CTE_t + K_i * \sum_{n=0}^{t} CTE_n + K_d * (CTE_t - CTE_{t-1})$$

Equation 1: PID Controller where CTE is Cross Track Error and Δt Between Sampling is Assumed to be Constant

In order to develop an understanding of how the terms contribute to the controller, it is useful to examine the four non-trivial possible movements of the car in relation to the centerline of the road. The car can move from the right or left of centerline, either towards or away from the center. Table 1 shows these possible movements with unitary values for simplicity and the responses from the proportional and differential terms.

CTE @ t = i	CTE @ t = i+1	P Response @ i+1	D Response @ i+1
0	-1	-K_p	-Kd
-1	0	0	Kd
1	0	0	-Kd
0	1	K_p	Kd

Table 1: P and D Responses for Possible Car Movements

The proportional term acts in the same direction as the location of the car and therefore K_p should be negative. Additionally, K_p will only be non-zero and will oscillate in discrete time as the vehicle cannot update its steering angle fast enough to asymptotically approach zero without over shooting it. K_d should also be negative to counteract the car's movement away from the centerline and dampen the effects of the proportional term as the car moves towards the centerline. K_d should also be much larger than K_p to account for the smaller value of the differential when compared to CTE.

The integral term is simply the sum of the history of all previous CTEs. This value helps to eliminate errors caused by error in tire alignment. If the car's tires are misaligned, a steering input from [-1, 1] will not translate correctly from input to steering output and therefore favor one side with an outputted steering angle from [-1 + misalignment, 1 + misalignment]. As the car moves through the track the integral term will favor the pulled side and therefore a negative K_i parameter will work towards eliminating that pull.

IMPLEMENTATION

PID Steering Controller

Steering inputs from [-1,1] translate to steering angles outputs of [-25°,25°] for the vehicle. A simple PID controller was implemented following Equation 1 to control the steering input for the vehicle in the center of the lane. In order to help the vehicle steer through turns, a more aggressive steering angle is needed. To compensate for turns, an undamped proportional controller was used for the edges of the road where the absolute value of the CTE was greater than 1.25 m. This allows for smooth control of the vehicle in the straighter areas of the track using the PID controller and more aggressive maneuvers through the turns to return the vehicle to the center of the lane.

The final parameters for the centerline PID controller are shown below in Table 2:

Proportional Constant, K_p	Integral Constant, K_i	Differential Constant, K_d
-0.2	-0.0005	-5

Table 2: PID Constants for the Center of the Road [-1.25m,1.25m]

The parameters were tuned manually using knowledge of each constant's contribution. A larger differential constant was chosen to help dampen the larger swings from the outer edges of the road. Because the integral constant has to incorporate the assumed constant time between each update, it is significantly smaller in order of magnitude than the other two constants. The final proportional constant was chosen as -0.35, which is stronger than the proportional constant for the center of the road producing the desired turns in curves. The steering output from the controllers were then ensured to be within the allowed steering values using min and max logic.

PD Controller Speed

Speed was controlled using the throttle to accelerate or decelerate the vehicle based on CTE and steering angle. A PD controller was used to output a throttle control allowing for faster speed in the center of the lane and slower speeds while turning. This controller essentially maximizes the speeds around the track while safely maneuvering around turns similar to how a human driver would drive.

The final parameters for the PD controller are shown below in Table 3:

Proportional Constant, K_p	Differential Constant, K_d
-.75	-2.5

These parameters safely slowed the vehicle through turns and still allowed for acceleration up to 74 mph through the straightaways. Additionally, the throttle value was then passed through a negative exponential function based on the absolute value of the vehicle's turning angle. This factor reduced the throttle by a factor of up to 2/5 of the original throttle if the throttle was positive. The throttle was also minimized to a value of -0.5 to prevent jerky driving and potentially moving the vehicle in reverse.

RESULTS AND ROOM FOR IMPROVEMENT

The vehicle safely maneuvers the track with speeds ranging from roughly 25 mph to 74 mph. Although the vehicle still oscillates occasionally and would not be a comfortable ride, the vehicle makes it safely around the track while controlling both speed and steering using PID controllers. Improvements could be made by incorporating vehicle dynamics and using nonlinear controllers. Because the PID controller is linear and uses constant coefficients, it is very difficult to tune the parameters so that they successfully control the vehicle in all cases. In order to successfully use the PID controller, segmentation of the track is necessary to essentially create a composite linearization of a nonlinear function to create a smooth ride.