Helicopter Optimization Report

Aureo Zanon, Tristan Zanowic

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Project Description

Paper helicopters are a fun, creative way for child care programs to get children engaged with curiosity and building. Built using a single sheet of paper and other common supplies, these helicopters are designed in a near infinite number of ways in an attempt to gain the most air time.

To determine the optimal paper helicopter specifications in lengthening flight time, a fractional factorial design is conducted along with a response surface experiment in this research. The goal is to analyze the effects of several variables within the helicopter's design to discover what factors lead to the longest flight time, while maintaining a budget of 60 helicopters. With this study, the results can hopefully help children build the best helicopter to stay in the air, keeping them intrigued and curious throughout their learning.

Research Questions

Question 1: Given our variables of interest, what paper helicopter design factors lead to the longest flight time?

Table 1: Variable Table

Variable	Type	Description
Wing length	Explanatory	Length of wings (70mm or 120mm)
Body length	Explanatory	Length of body (70mm or 120mm)
Body width	Explanatory	Width of body (35mm or 50mm)
Paper clips	Explanatory	No. of paper clips (0 or 2)
Folded wings	Explanatory	Are wings folded (no or yes)
Folded body	Explanatory	Is the body folded (no or yes)
Time	Response	Time in air (seconds)

Variables

There are a total of six explanatory variables of interest in predicting flight time, from cutting dimensions such as wing length to add-ons like paper clips. Each variable has two levels to be analyzed (listed in the description column of the table).

Screening Experiment

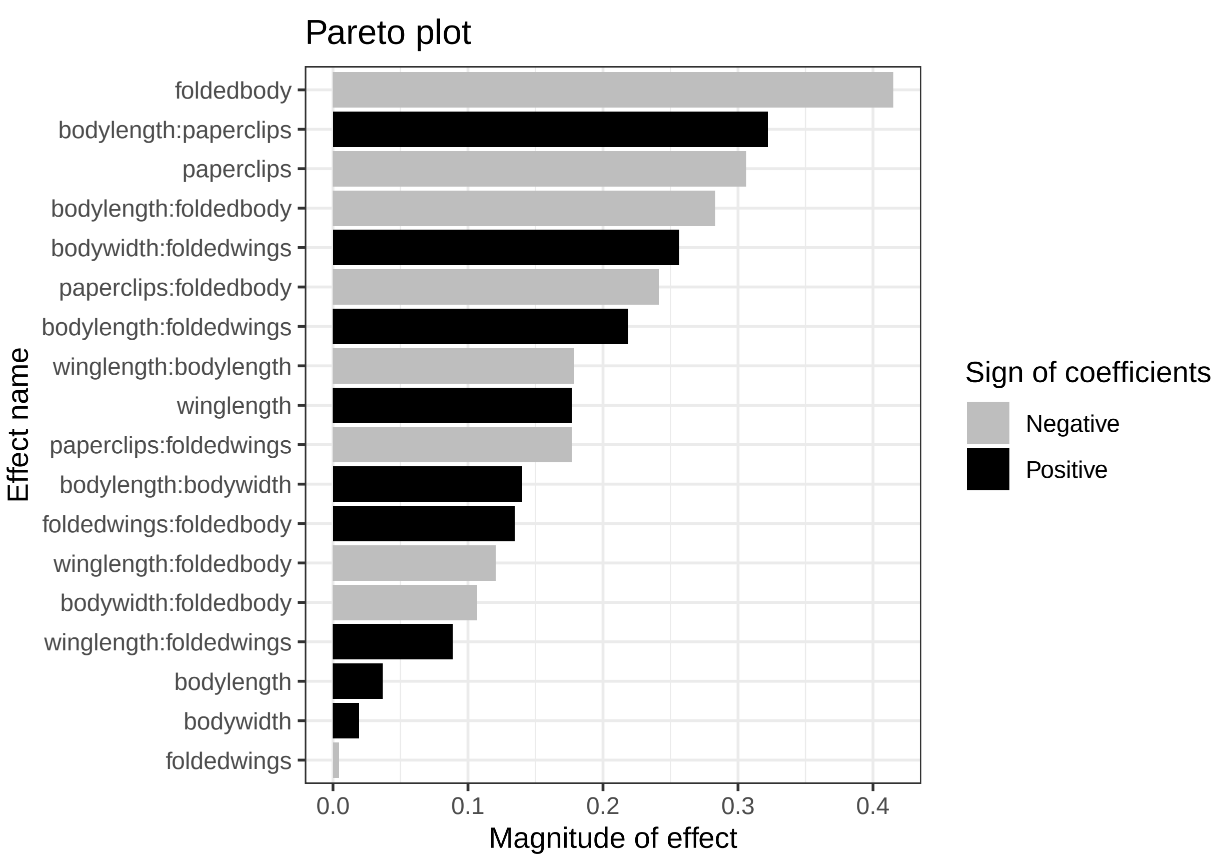
To begin analyzing the six variables of interest, a fractional factorial design is used to screen our predictors. To identify the key variables at play in lengthening paper helicopter flight time, a resolution VI factorial design is used, equating to 32 screening runs, with four additional runs for center points. Within these runs, helicopters were dropped from approximately 15 feet with variations in design specifications across each trial, and the time in air was recorded. One observation that was noted during these trials was the common appearance of tumbling, which seemed to occur more often when paper clips were used. Regardless, no obvious trends related to flight time were observed during the tests.

From these results of these 32 runs, a model can be fitted including our six variables of interest, along with every 2-way interaction. To reduce this complex model, step-wise backwards selection was used to drop the least significant terms, and a summary of this model can be seen in the table below. As seen in this table, the most significant terms appear to be folded body and paper clips, with both having a negative effect on flight time. Many interaction terms (that include folded body and paper clips) being significant as well. Other variables of interest, such as body length, body width, and folded wings, appear to have negligible effects on their own (p-values $> .6$).

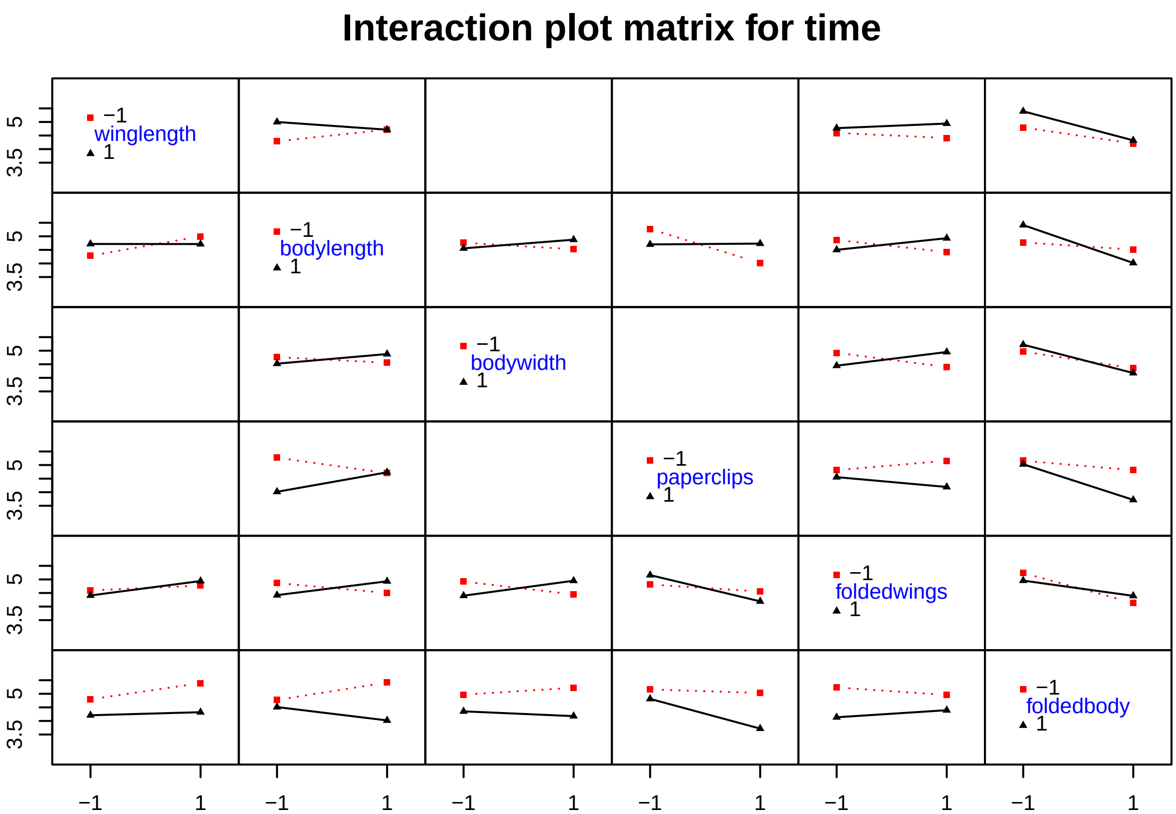
The pareto plot below further emphasizes these effects, showcasing which variables (including interactions) have the greatest effect (both negative and positive) on flight time. In agreement with the model results above, having a folded body had the largest effect on flight time, as the presence of a folded body negatively effected time in the air. Following this, the interaction between body length and paper clips had the largest positive effect, as the inclusion of a longer body (120mm) and two paper clips generally led to longer flight time. However, paper clips on its own had a large negative impact on flight time, the third largest effect.

Table 2: Reduced Fractional Factorial Model Results

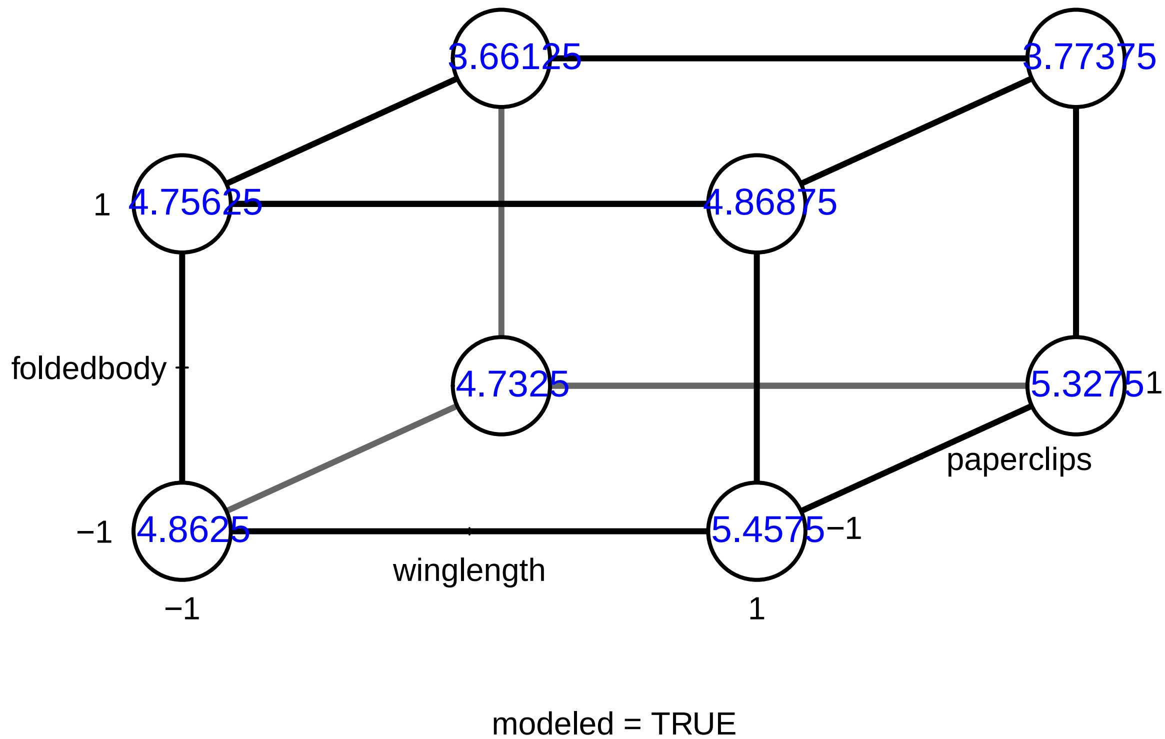
	Coefficient	Standard Error	T-Test Stat	P-value
(Intercept)	4.680	0.086	54.651	0.000
winglength	0.177	0.086	2.065	0.059
bodylength	0.037	0.086	0.431	0.674
bodywidth	0.019	0.086	0.226	0.825
paperclips	-0.306	0.086	-3.576	0.003
foldedwings	-0.004	0.086	-0.051	0.960
foldedbody	-0.415	0.086	-4.846	0.000
winglength:bodylength	-0.179	0.086	-2.087	0.057
winglength:foldedwings	0.089	0.086	1.036	0.319
winglength:foldedbody	-0.121	0.086	-1.409	0.182
bodylength:bodywidth	0.140	0.086	1.635	0.126
bodylength:paperclips	0.322	0.086	3.759	0.002
bodylength:foldedwings	0.219	0.086	2.554	0.024
bodylength:foldedbody	-0.283	0.086	-3.306	0.006
bodywidth:foldedwings	0.256	0.086	2.992	0.010
bodywidth:foldedbody	-0.107	0.086	-1.248	0.234
paperclips:foldedwings	-0.177	0.086	-2.065	0.059
paperclips:foldedbody	-0.241	0.086	-2.817	0.015
foldedwings:foldedbody	0.134	0.086	1.569	0.141



The interaction plot demonstrates several instances of interactions, implying that the interactions do exist between the several different variables that were modified in the experiment. Due to this, in order to ensure a design that includes the most important features, the steepest slope in the interaction lines were selected, which was observed for paperclips with folded body and folded body with wing length. In turn, to optimize the design those three variables will be selected for the optimization run. When looking at the cube plot, it is observed that a combination of +1 wing length with -1 paperclips has the largest value, while the folded body variable doesn't seem to result in very large change when at +1.



Cube Plot for Distance to Optimal Helicopter



Response Surface Experiment

Upon analyzing the results of the screening process, we chose to further optimize the three most significant (not including interactions) explanatory variables: **folded body, paper clips, and wing length**. To do so, a box-behnken design is used for our response surface experiment, equating to 16 optimization runs.

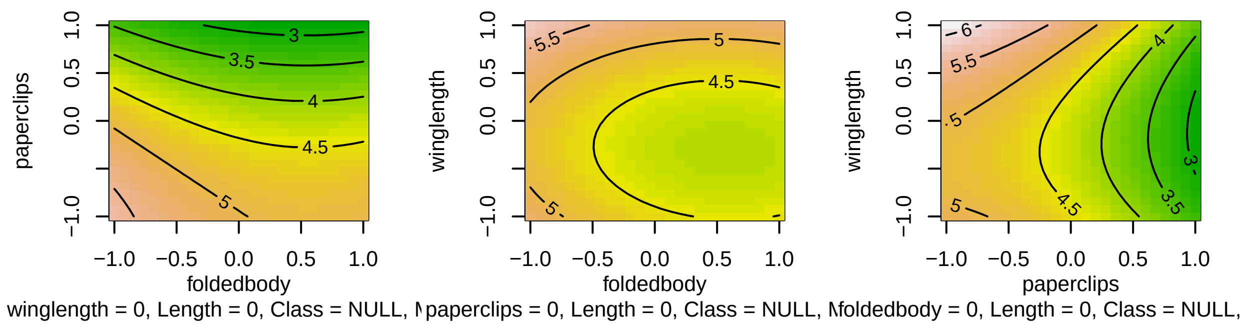
The Box-Behnken design specifically aims to avoid treatment combinations which could be seen as extreme, avoiding the corner and star points typically observed in other designs. The Box-Behnken design also allows for the implementation of a quadratic model, which could better explain the observed variability in the helicopter flight times.

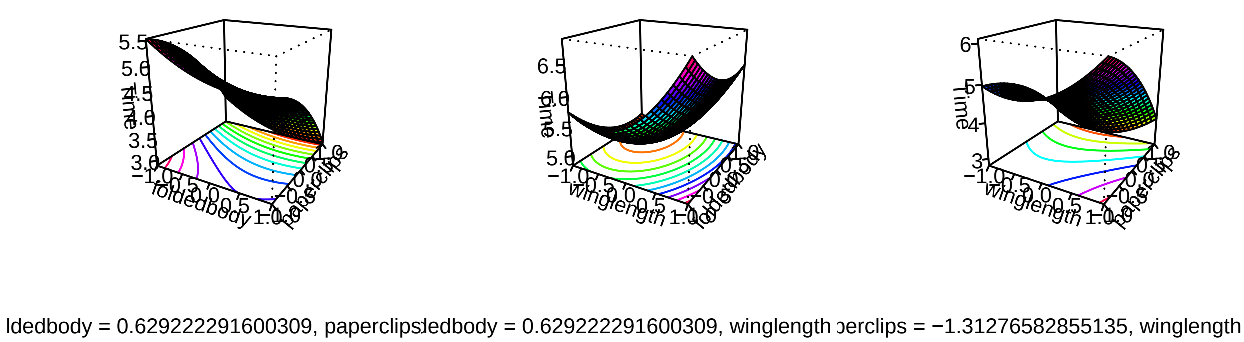
Following measuring the time of these 16 helicopters, we can fit a model to describe the effects of the three variables of interest, their interaction and second order terms. Most significantly (p-value $< .001$), paper clips appeared to have the largest effect on flight time, as the inclusion of paper clips leads to about a two second drop in paper helicopter flight time (since no paper clips is coded as -1), after accounting for folded body and wing length. However, the 2nd order term for paperclips is equal to the threshold of .05, which could imply some moderate significance, although not dramatic. While wing length, wing length squared and folded body factors were significant under a p-value threshold of .05, their interactions can still be noted as not significant they did not fall from this mark. Specifically, the inclusion of a folded body leads to an approximate .628 second decrease in flight time, while longer wings contributes to a .347 second increase in time (as compared to a neutral wing length).

Following analysis of the model we fit for our response surface experiment, contour plots reveal how each of our three variables interacts with each other to maximize flight time. To begin, in the first contour plot with paper clips and folded body, flight time is maximized (highest point on the plane) when the helicopter body is not folded and paper clips are not included. From the other two plots, the results from our model are upheld, showing that the optimal helicopter design has shorter wings, no paper clips, and folded body.

Table 3: Box Behnken Optimization Model Results

	Coefficient	Standard Error	T-Test Stat	P-value
(Intercept)	4.320	0.142	30.442	0.000
foldedbody	-0.314	0.100	-3.127	0.020
paperclips	-1.064	0.100	-10.601	0.000
winglength	0.347	0.100	3.463	0.013
foldedbody:paperclips	0.015	0.142	0.106	0.919
foldedbody:winglength	0.043	0.142	0.299	0.775
paperclips:winglength	-0.198	0.142	-1.392	0.213
foldedbody^2	0.282	0.142	1.991	0.094
paperclips^2	-0.363	0.142	-2.554	0.043
winglength^2	0.610	0.142	4.298	0.005





From the generated response surface designs, the optimal helicopter can be estimated. The optimal helicopter is found to have a folded body value at ~.77, wing length at ~-.55 and paper clips at ~-1.33. The optimal response surface design falls slightly outside the scope of the ranges of the variable, however optimal helicopters can be designed for the confirmation run.

Table 4: Confirmation Runs for Optimal Design

foldedbody	paperclips	winglength	time
1	-1	-0.5	4.70
1	-1	-0.5	5.40
1	-1	-0.5	5.00
1	-1	-0.5	5.20
1	-1	-0.5	4.92

Average = 5.044

Confirmation Run

To confirm that the aforementioned design is truly the optimal design for the longest flight time for a paper helicopter, 5 additional helicopters were dropped to see the flight time comparison to the prediction of the best response. The only differences in the design come when looking at the optimal number of paper clips, as you cannot go beyond zero paper clips at -1. Similarly, the folded body was rounded to 1 since it acted as an indicator. The wing length was set to 82.5 mm as it is the point between -1 (70 mm) and 0 (95 mm).

The confirmation runs show the helicopters flew 5.044 seconds on average, which is close to the predicted value from the response surface design model.

Recommendations

Succinct response to each question laid out in 1.2. This a much shorter version of section 3, and focuses on conclusions rather than the analyses.

Question 1: The optimal paper helicopter design to increase flight time includes shorter wings (82.5mm), a folded body, and no paper clips.

Considerations

From the single optimization run that was performed the total helicopters utilized is equal to 53. This amount is slightly below the limit of 60 which means that there are a small amount of resources left to attempt a further optimization.

One aspect to consider from the optimization design is the fact that there was some extrapolation in looking at the paperclips variable in which it can be observed. The optimal amount of paper clips is found to be coded at -1.33, which is implausible for the design at hand. However, this could mean that the mass of a helicopter could be more readily considered, meaning that removing the equivalent of .33 mass of a paperclip could give the truest optimal design. In a similar fashion, the folding predicted by the optimal design could further imply that a proper angle of the folding may result in a more optimized design.

Limitations

While the results of this study can likely be generalized to a classroom setting, it is important to note some limitations and additional considerations with this research. In particular, there are opportunities for human error that comes along with any building project, but especially with paper helicopters. While cutting and building these helicopters, we tried our best to consistently cut and fold in the same manner over 50+ trials, but obvious limitations in human precision (and scissors) can lead to small amounts of error across helicopters. Similarly, while the helicopters were all dropped from the same spot across each trial, room factors such as the use of HVAC systems could lead to differing environmental conditions across hours or days. Regardless, with many trials, the error introduced with these limitations is ideally negligible.

Technical Appendix

R Script

```r
# Reprinted code chunks used previously for analysis

#variable table
vartable = data.frame(Variable = c("Wing length","Body length","Body width","Paper clips","Folded wings


vartable %>%
  kbl(caption = "Variable Table") %>%
  kable_classic(full_width = F, html_font = "Cambria")

HeliDesign <- read.csv("fracfact.csv")
#store then omit center points
CP <- HeliDesign[33:36,]
HeliDesign <- HeliDesign[1:32,]


# main effects and all 2-way interactions
HeliMod_2way <- lm(time ~(.)^2, data = HeliDesign)

# looking for contributing effects
#pid::paretoPlot(HeliMod_2way)
# reduced model after stepwise backwards selection (alpha = 0.1)**
step <- stepAIC(HeliMod_2way, direction = "both", trace = FALSE)

HeliMod_reduced <- step
sumHeli <- summary(HeliMod_reduced)
screeningtable <- as.data.frame(round(sumHeli$coefficients,3))
names(screeningtable) <- c("Coefficient","Standard Error", "T-Test Stat", "P-value")

screeningtable %>%
  kbl(caption = "Reduced Fractional Factorial Model Results") %>%
  kable_classic(full_width = F, html_font = "Cambria")

#Pareto
pareto <- paretoPlot(HeliMod_reduced)

# effect plots
MEPlot(HeliMod_reduced) # main effects
IAPlot(HeliMod_reduced) # interactions
cubePlot(HeliMod_reduced, eff1 = "winglength", eff2 = "paperclips",
         eff3 = "foldedbody",
         main = "Cube Plot for Distance to Optimal Helicopter")

##boxbehnken design
```

```
bb_design_1 <- bbd(
  k  = 3,             # Number of factors,
  n0 = 4,             # Number of center points,
  block = FALSE,      # Consider blocks or not in the design
  randomize = FALSE)

names <-c("foldedbody","paperclips","winglength")

designbb <- bb_design_1[,3:5]
#setNames(designbb,names)
#fwrite(designbb,"~/Desktop/PSU/SeniorYear/Spring Semester/STAT470W/HeliFFD/BBDesign.csv")

#response surface design
HeliD <- read.csv("BBDesign.csv")
model<-rsm(time~ FO(foldedbody,paperclips,winglength)+TWI(foldedbody,paperclips,winglength)+SO(foldedbo


sumBBHeli <- summary(model)
BBtable <- as.data.frame(round(sumBBHeli$coefficients,3))
names(BBtable) <- c("Coefficient","Standard Error", "T-Test Stat", "P-value")

BBtable %>%
  kbl(caption = "Box Behnken Optimization Model Results") %>%
  kable_classic(full_width = F, html_font = "Cambria")

# countour plots
par(mfrow=c(2,3))
contour(model,~foldedbody+paperclips+winglength,
       image=TRUE, at=summary(model$canonical$xs))

persp(model, ~ foldedbody + paperclips, image = TRUE,
      at = c(summary(model)$canonical$xs, Block="B2"),
      theta=30,zlab="Time",col.lab=33,contour="colors")
persp(model, ~ winglength + foldedbody, image = TRUE,
      at = c(summary(model)$canonical$xs, Block="B2"),
      theta=30,zlab="Time",col.lab=33,contour="colors")
persp(model, ~ winglength + paperclips, image = TRUE,
      at = c(summary(model)$canonical$xs, Block="B2"),
      theta=30,zlab="Time",col.lab=33,contour="colors")

#optimal design for longest time in the air
opt_point <- summary(model)$canonical$xs
opt_point <- t(as.data.frame(opt_point))
opt_point<-as_data_frame(opt_point)

best_response <- predict(
  model,              # Our model
  opt_point              # Data frame with points to be predicted
  )
opt_point$best_response_prediction<-best_response

opt_point %>%
  kbl(caption = "Optimal Helicopter Design to Maximize Time in Air") %>%
```

```r
  kable_classic(full_width = F, html_font = "Cambria")

#Confirmation runs
ConfRun <- read.csv("ConfirmationRuns.csv")
cap <- paste("Average = ",mean(ConfRun$time))

ConfRun %>%
  kbl(caption = "Confirmation Runs for Optimal Design") %>%
  kable_classic(full_width = F, html_font = "Cambria") %>%
  add_footnote(cap,notation = "none")
```