IN2106 Practical Course – Vision-based Navigation: Exercise #3

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1 Batch MAP

Task 1 By taking $k = 0, 1, 2$ into the system model, We can obtain

$$\mathbf{H} = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{1}$$

Task 2 From the system model and Eq. 1, we have,

$$\boldsymbol{e} = \boldsymbol{z} - \mathbf{H}\boldsymbol{x} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} - \begin{pmatrix} x_1 - x_0 \\ x_2 - x_1 \\ x_3 - x_2 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ n_1 \\ n_2 \\ n_3 \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{W}). \tag{2}$$

Obviously, if the noise is independent of each other, we have

$$\mathbf{W} = \mathrm{diag}(Q, Q, Q, R, R, R), \tag{3}$$

otherwise $\mathbf{W}$ could be any covariance matrix with this diagonal terms and other nonzero entries.

Task 3 Yes, if the noise is uncorrelated, then it is independent. In this case there is only one exact solution of $\mathbf{W}$.

2 Iterative Curve Fitting

Task 1 The derivatives of error to parameters could be obtained by

$$\boldsymbol{J}_i = \begin{pmatrix} \frac{\partial e_i}{\partial a} \\ \frac{\partial e_i}{\partial b} \\ \frac{\partial e_i}{\partial c} \end{pmatrix} = \begin{pmatrix} -f(x_i) \cdot x_i^2 \\ -f(x_i) \cdot x_i \\ -f(x_i) \end{pmatrix}, \tag{4}$$

where

$$f(x_i) = \exp(ax_i^2 + bx_i + c). \tag{5}$$

Then we have

$$\begin{aligned} \mathbf{H} &= \sum_i \boldsymbol{J}_i \boldsymbol{J}_i^T, \\ \boldsymbol{b} &= \sum_i -e_i \boldsymbol{J}_i, \\ (\Delta a, \Delta b, \Delta c)^T &= \mathbf{H}^{-1}\boldsymbol{b}. \end{aligned} \tag{6}$$

By implementing this, we can see the results:

```
total cost: 3.19575e+06
total cost: 376785
total cost: 35673.6
total cost: 2195.01
total cost: 174.853
total cost: 102.78
total cost: 101.937
total cost: 101.937
total cost: 101.937
total cost: 101.937
total cost: 101.937
total cost: 101.937
cost: 101.937, last cost: 101.937
estimated abc = 0.890912, 2.1719, 0.943629
```

Task 2 With `Google Ceres`, first we have to implement the $e_i = y_i - f(x_i)$ part in the overloaded `()` function, *i.e.* `operator()(const T *const abc, T *residual) const`, inside the `CURVE_FITTING_COST` object. then we instantiate a `Problem` object, adding the cost function of the `CURVE_FITTING_COST` object into a `CostFunction` object by `AutoDiffCostFunction()` method, and using `Problem::AddResidualBlock()` method to add the residual block we have implemented inside the overloaded `()` function. Finally by the solver setups shown in lecture slides and by the solver function call we have the results:

```
iter      cost      cost_change  |gradient|   |step|    ... total_time
   0  1.597873e+06    0.00e+00    3.52e+06   0.00e+00   ...   1.17e-02
   1  1.884440e+05    1.41e+06    4.86e+05   9.88e-01   ...   1.66e-02
   2  1.784821e+04    1.71e+05    6.78e+04   9.89e-01   ...   1.99e-02
   3  1.099631e+03    1.67e+04    8.58e+03   1.10e+00   ...   2.29e-02
   4  8.784938e+01    1.01e+03    6.53e+02   1.51e+00   ...   2.58e-02
   5  5.141230e+01    3.64e+01    2.72e+01   1.13e+00   ...   2.79e-02
   6  5.096862e+01    4.44e-01    4.27e-01   1.89e-01   ...   2.98e-02
   7  5.096851e+01    1.10e-04    9.53e-04   2.84e-03   ...   3.17e-02
Ceres Solver Report: Iterations: 8, Initial cost: 1.597873e+06,
Final cost: 5.096851e+01, Termination: CONVERGENCE
estimated a,b,c = 0.890908, 2.1719, 0.943628
```

3 Camera Pose Estimation by Gauss-Newton Method

Task 1

Task 2

Task 3