**IN2106 Practical Course – Vision-based Navigation: Exercise #2**

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# 1   Left Jacobian in $SE(3)$

**Proof** From the Taylor's series we have,

$$
\begin{aligned}
\sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\boldsymbol{\phi}^\wedge)^n &= \sum_{n=0}^{\infty} \frac{1}{(n+1)!}(\theta \boldsymbol{a}^\wedge)^n \\
&= \underbrace{\mathbf{I}}_{=\boldsymbol{a}\boldsymbol{a}^T - \boldsymbol{a}^\wedge \boldsymbol{a}^\wedge} + \theta \boldsymbol{a}^\wedge + \frac{\theta^2}{2!}\boldsymbol{a}^\wedge \boldsymbol{a}^\wedge + \frac{\theta^3}{3!}\underbrace{\boldsymbol{a}^\wedge \boldsymbol{a}^\wedge \boldsymbol{a}^\wedge}_{=-\boldsymbol{a}^\wedge} + \cdots \\
&= \boldsymbol{a}\boldsymbol{a}^T + \underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots\right)}_{=\sin\theta}\boldsymbol{a}^\wedge - \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \cdots\right)}_{=\cos\theta}\boldsymbol{a}^\wedge \boldsymbol{a}^\wedge \\
&= \boldsymbol{a}\boldsymbol{a}^T + \sin\theta \boldsymbol{a}^\wedge - \cos\theta \underbrace{\boldsymbol{a}^\wedge \boldsymbol{a}^\wedge}_{=\boldsymbol{a}\boldsymbol{a}^T - \mathbf{I}} \\
&= \cos\theta \mathbf{I} + (1 - \cos\theta)\boldsymbol{a}\boldsymbol{a}^T + \sin\theta \boldsymbol{a}^\wedge.
\end{aligned}
\tag{1}
$$

**Coding validation**

```
given translation vector rho = (1 0 0)^T
given rotation matrix phi =
        0.5 -0.866025          0
   0.866025        0.5          0
          0          0          1
from theta = 1.0472 and axis = (0 0 1)^T
SE3 xi =
        0.5 -0.866025          0          1
   0.866025        0.5          0          0
          0          0          1          0
          0          0          0          1
se3 of xi = (    0.9069 -0.523599          0          0          0     1.0472)^T
its rotation matrix from exponent map =
        0.5 -0.866025          0
   0.866025        0.5          0
          0          0          1
rotation matrix of Rodrigues' =
        0.5 -0.866025          0
   0.866025        0.5          0
          0          0          1
```

# 2 Compare trajectories

**Task 1** The snapshot of plotting trajectories is shown in Fig. 1, where red is for estimated trajectory and the blue the ground truth.
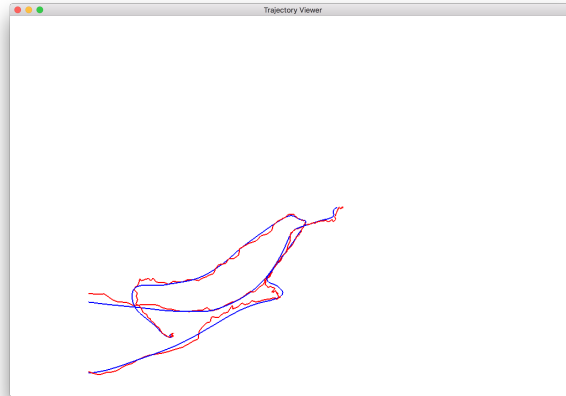


Figure 1: Snapshot of `Pangolin` view of estimated trajectories and the ground truth

**Task 2** The ATE and RMSE are calculated as follows.

```
1  ATE = 0.000533634
2  RMSE = 2.20728
```

# 3 Images, camera intrinsic and extrinsic

**Task 1** The recovered image is shown in Fig. 2. We can see now the straight line of windows and the edges of metal cylinders, which are distort in the original image.
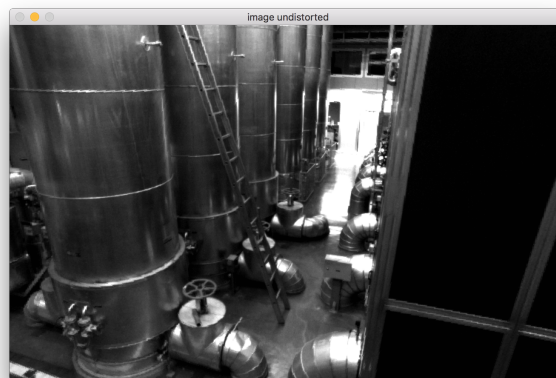


Figure 2: Result of image undistortion

**Task 2** The snapshot of point cloud view of left-eye view is shown in Fig. 3.

**Task 3** The snapshot of point cloud view from RGB-D images is shown in Fig. 4.

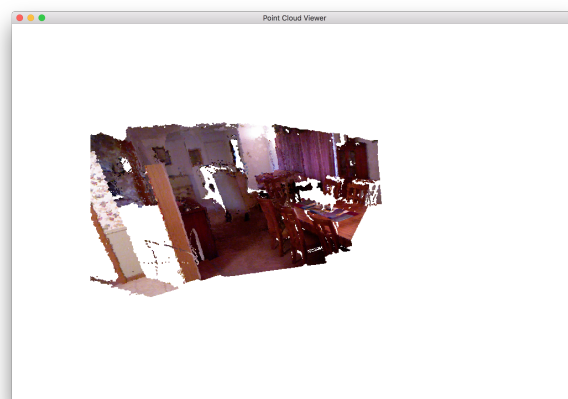Figure 3: Snapshot of `Pangolin` view of left-eye view as point cloud



Figure 4: Snapshot of `Pangolin` view of RGB-D images as point cloud