MATH 305:201, 2020W T2

Homework set 1 — due January 22

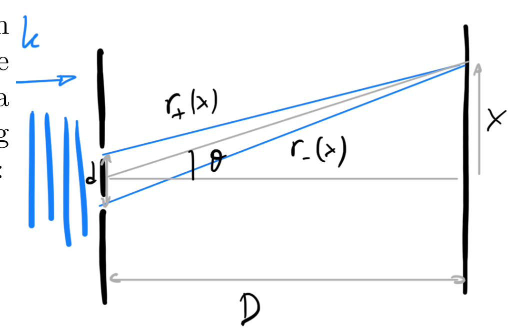
Problem 1. Compute the real and imaginary parts of the following complex numbers:
(i) $(2+3\mathrm{i})-(1-\mathrm{i})$
(ii) $\mathrm{i}^3(1+\mathrm{i})$
(iii) $\frac{2-2\mathrm{i}}{4+3\mathrm{i}}$
(iv) $\frac{2}{\mathrm{i}}+\frac{\mathrm{i}}{2}$
(v) $\frac{2+\mathrm{i}}{1-\mathrm{i}}+\frac{3+2\mathrm{i}}{\mathrm{i}}$

Problem 2. Compute the following:
(i) $|\frac{1-\mathrm{i}}{2+\mathrm{i}}|$
(ii) $|(1-2\mathrm{i})\overline{(1-\mathrm{i})}|$
(iii) $|\frac{(1-\mathrm{i})^{2021}}{\mathrm{i}^{2021}}|$
(iv) $\arg(\pi/2)$ and $\mathrm{Arg}(\pi/2)$
(v) $\arg(\sqrt{3}-\mathrm{i})$ and $\mathrm{Arg}(\sqrt{3}-\mathrm{i})$

Problem 3. Describe geometrically the set of points z in the complex plane defined by:
(i) $|z-\zeta|=2$ where $\zeta\in\mathbb{C}$ is a fixed complex number
(ii) $z^{-1}=\overline{z}$
(iii) $\mathrm{Re}(z)=1/2$
(iv) $\mathrm{Im}(z)-2\mathrm{Re}(z)\leq 3$
(v) $z\overline{z}\geq 1$
(vi) $z^5=1$

Problem 4. Show that
(i) for any complex number z, $\mathrm{Re}(\mathrm{i}z)=-\mathrm{Im}(z)$ and $\mathrm{Im}(\mathrm{i}z)=\mathrm{Re}(z)$,
(ii) for any integer $n\in\mathbb{Z}$, $\mathrm{i}^{4n}=1, \mathrm{i}^{4n+1}=\mathrm{i}, \mathrm{i}^{4n+2}=-1, \mathrm{i}^{4n+3}=-\mathrm{i}$, and compute i^{2021} and i^{-2021},
(iii) the complex numbers $z_1=\mathrm{i}, z_2=-2-3\mathrm{i}$ are solutions of the equation $(\mathrm{i}-1)z^2-4z-1+5\mathrm{i}=0$.

Problem 5. A monochromatic plane wave of wavelength $\lambda=2\pi/k$ hits a screen with just thin slits that are a distance d apart. The diffracted light hits a second screen that is at a distance D, see figure. Describe the diffracted light hitting the screen as the sum of two waves emitted from each slit:



$$u(x,t)=u_+(x,t)+u_-(x,t),\ \text{where } (x,t)\in\mathbb{R}^2 \text{ and}$$
$$u_+(x,t)=\frac{A}{r}\mathrm{e}^{\mathrm{i}(kr_+(x)-\omega t)},\ u_-(x,t)=\frac{A}{r}\mathrm{e}^{\mathrm{i}(kr_-(x)-\omega t)}.$$

Let the intensity be $I(x,t)=|u(x,t)|^2$. Show that

$$I(x,t)=\frac{4A^2}{D^2}\cos^2(\theta)\cos^2\left(\frac{\pi d}{\lambda}\sin(\theta)\right)+\mathcal{O}\left(\frac{d}{D}\right)\qquad\left(\theta\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\right)$$

Conclude that, for small x/D, the distance between two lines of maximal intensity is given by

$$\bar{\lambda}\approx\frac{D}{d}\lambda.$$