MATH 305:201, 2020W T2

Homework set 8 — due March 19

Problem 1. Find the Taylor series of the following functions.
(i) $\frac{1+z}{1-z}$ around $z_0 = \mathrm{i}$. *Hint:* Write $z = \mathrm{i} + (z - \mathrm{i})$ and use the geometric series.
(ii) $z^4 \cos(3z)$ around $z_0 = 0$.
In each case, determine the radius of convergence.

Problem 2. For any $n \in \mathbb{N}$, we define $n!! = n(n-2)(n-4)\cdots$, for example $8!! = 8 \cdot 6 \cdot 4 \cdot 2$. Show that $f(z) = 1 + \sum_{j=1}^{\infty} \frac{1}{(2j)!!} z^{2j}$ is a solution of the differential equation

$$f''(z) - zf'(z) - f = 0$$

such that $f(0) = 1$ and $f'(0) = 0$.
Hint: Plug a Taylor series for f in the equation to obtain a recursion relation for the coefficients.

Problem 3. Compute the four lowest order terms in the Taylor series of $f(z) = (1 + \mathrm{Log}(1-z))^{-1}$ at $z_0 = 0$ and determine its radius of convergence.