ASSIGNEMENT ANSWERS.

QN 1: Given the following English sentences,
C to indicate that Gianni is a climber;
F to indicate that Gianni is fit;
L to indicate that Gianni is lucky:
E to indicate that Gianni climbs Mount Everest.

If Gianni is a climber and he is fit, he climbs Mount Everest. If Gianni is not lucky and he is not fit, he does not climb Mount Everest. Gianni is fit

a) Formalize the above sentences in propositional logic

Answer(s)

$(C \wedge F) \rightarrow E$

$(\neg L \wedge \neg F) \rightarrow E.$

b) Tell if the KB buit in (a) is consistent, and tell if some of the following sets are models for the above sentences: { }; {C, L}; {L, E}; {F, C, E}; {L, F, E}.

Answer(s)

The KB is consistent if it has at least a model, as the following check shows,

i) { } is not a model (it models 1 and 2 but not 3)
ii) {C, L} is not a model (it models 1 and 2 but not 3)
iii) {L, E} is not a model (it models 1 and 2 but not 3)
iv) {F, C, E} is a model,
v) {L, F, E} is a model.

QN 2: What does it mean for propositional logic to be valid, satisfiable, and unsatisfiable? Attempt exercise 7.10 from the main reference book to prove your concepts.

ANSWERS.

- A propositional Logic is valid if it is **true** for all values of its terms.
- A proposition is satisfiable if there is at least one true result in its truth table, valid if all values it returns in the truth table are true.

And the vice versa of the satisfiable is unsatisifiable in which

- A proposition logic is unsatisfiable if there is no at least one true result in its truth table.

Exercise 7.10

Decide whether each of the following sentences is valid, unsatisfiable, or neither. Verify your decisions using truth tables

a. Smoke ⇒ Smoke
b. Smoke ⇒ Fire
c. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

d. Smoke ∨ Fire ∨ ¬Fire
e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))
f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)
g. Big ∨ Dumb ∨ (Big ⇒ Dumb)

Answer(s):

a. Smoke ⇒ Smoke.

Let Smoke be S.

Then Smoke ⇒ Smoke ≡ S ⇒ S ≡ ¬S ∨ S

S	¬S	¬S ∨ S
T	F	T
F	T	F

Therefore Smoke ⇒ Smoke is **VALID.**

b. Smoke ⇒ Fire

Let Smoke be S and Fire be F.

Then S ⇒ F ≡ ¬S ∨ F

S	F	¬S ∨ F
T	T	T
T	F	F
F	T	T
F	F	T

Therefore from the truth above the statement is **Neither.**

c. (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire).

Let Smoke be S, Fire be F. by simplifying the above statement,

(Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire) ≡ (S ⇒ F) ⇒ (¬S ⇒ ¬F) ≡ (¬S ∨ F) ⇒ (S ∨ ¬F),

Then consider the truth table below,

Let

¬S ∨ F be P and S ∨ ¬F be Q, then

S	F	¬S	¬F	¬S ∨ F	S ∨ ¬F	P⇒ Q
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	T	T	T

Then we can conclude that the above table shows that it is **Neither.**

d. Smoke ∨ Fire ∨ ¬Fire

Again Let Smoke be S and Fire be F,

Smoke ∨ Fire ∨ ¬Fire ≡ S ⋁ F ⋁ ¬F

Then consider the truth table below,

Let S ⋁ F be P,

S	F	S ⋁ F (P)	P ⇒ ¬F	¬F
T	T	T	T	F
T	F	T	T	T
F	T	T	F	F
F	F	F	T	T

Then the above table shows that the statement is **Neither.**

e. ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

Let Smoke be S,

Heat be H,

Fire be F,

((S ∧ H) ⇒ F) ↔ ((S⇒ F) ∨ (H⇒ F))

Then consider the truth table below,

S	F	H	S ∧ H (P)	P ⇒ F (Q)	S ⇒ F (R)	H ⇒ F (D)	R ∨ D (E)	Q ↔E
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T	T
T	F	T	T	F	F	F	F	T
T	F	F	F	T	F	T	T	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	F	T	T	F	T	T
F	F	F	F	T	T	T	T	T

The statement is **VALID.**

f. (Smoke ⇒ Fire) ⇒ ((Smoke ∧ Heat) ⇒ Fire)

Let Smoke be S,

Fire be F,

Heat be H, then,

$(S \Rightarrow F) \Rightarrow ((S \land H) \Rightarrow F))$, Then consider the truth table below,

S	F	H	S ⇒ F (P)	S ∧ H (Q)	Q ⇒ F (R)	P ⇒ R
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	F	T
T	F	F	F	F	T	T
F	T	T	T	F	T	T
F	T	F	T	F	T	T
F	F	T	T	F	T	T
F	F	F	T	F	T	T

The conclusion is that the sentence is **VALID.**

g. Big ∨ Dumb ∨ (Big ⇒ Dumb)

Let Big be B,

Dumb be D,

$B \lor D \lor (B \Rightarrow D)$

Then consider the truth table

B	D	B ∨ D (P)	B ⇒ D (Q)	P ∨ D
T	T	T	T	T
T	F	T	F	T
F	T	T	T	T
F	F	F	T	T

The statement above shows that the statement is **VALID.**

3. What does entailment (|=) means in propositional logic? Work on exercise 7.4, 7.5 and 7.6.

Answer(s).

Logical consequence also known as **entailment** is a fundamental concept in **logic**, which describes the **relationship** between **statements** that hold **true** when one statement logically follows from one or more statements.

For that case one can conclude that: a **entails** b if, whenever a is true, b is true.

Models: a entails b if every model of a is a model of b - that is, if $M(a) \subseteq M(b)$.

From exercise 7.4, which of the following are correct?

a. False |= True.

b. True |= False.

c. $(A \land B) \models (A \Leftrightarrow B)$.

d. $A \Leftrightarrow B \models A \lor B$.

e. $A \Leftrightarrow B \models \neg A \lor B$.

f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$.

g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$.

h. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B)$.
i. $(A \lor B) \land (\neg C \lor \neg D \lor E) \models (A \lor B) \land (\neg D \lor E)$.
j. $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable.
k. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable.
l. $(A \Leftrightarrow B) \Leftrightarrow C$ has the same number of models as $(A \Leftrightarrow B)$ for any fixed set of proposition symbols that includes A, B, C.

Answer(s)

a. False |=true is true because False has no models and hence entails every sentence AND because True is true in all models and hence is entailed by every sentence.
b. True |= False is false.
c. $(A \land B) \models (A \Leftrightarrow B)$ is true because the left-hand side has exactly one model that is one of the two models of the right-hand side.
d. $A \Leftrightarrow B \models A \lor B$ is false because one of the models of $A \Leftrightarrow B$ has both A and B false, which does not satisfy $A \lor B$.
e. $A \Leftrightarrow B \models \neg A \lor B$ is true because the RHS is $A \Rightarrow B$, one of the conjuncts in the definition of $A \Leftrightarrow B$.
f. $(A \land B) \Rightarrow C \models (A \Rightarrow C) \lor (B \Rightarrow C)$ is true because the RHS is false only when both disjuncts are false, i.e., when A and B are true and C is false, in which case the LHS is also false. This may seem counterintuitive, and would not hold if $\Rightarrow$ is interpreted as "causes."
g. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$ is true; proof by truth table enumeration, or by application of distributivity.
h. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$ is true; proof by truth table enumeration, or by application of distributivity.
i. $(C \lor (\neg A \land \neg B)) \equiv ((A \Rightarrow C) \land (B \Rightarrow C))$ is true; proof by truth table enumeration, or by application of distributivity
j. $(A \lor B) \land \neg(A \Rightarrow B)$ is satisfiable; model has A and $\neg B$.
k. $(A \Leftrightarrow B) \land (\neg A \lor B)$ is satisfiable; RHS is entailed by LHS so models are those of $A \Leftrightarrow B$.
l. $(A \Leftrightarrow B) \Leftrightarrow C$ does have the same number of models as $(A \Leftrightarrow B)$; half the models of $(A \Leftrightarrow B)$ satisfy $(A \Leftrightarrow B) \Leftrightarrow C$, as do half the non-models, and there are the same numbers of models and non-models.

7.5. Prove each of the following assertions
a. α is valid if and only if True $\models \alpha$.
b. For any α, False $\models \alpha$.
c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid.
e. $\alpha \models \beta$ if and only if the sentence $(\alpha \land \neg\beta)$ is unsatisfiable.

Answer(s).

Remember, $\alpha \models \beta$ if in every model in which α is true, β is also true. Therefore,

a. α is valid if and only if True $\models \alpha$. Forward: If alpha is valid it is true in all models, hence it is true in all models of True. Backward: if True $\models \alpha$ then α must be true in all models of True, i.e., in all models, hence α must be valid.
b. For any α, False $\models \alpha$. False doesn't hold in any model, so α trivially holds in every model of False.
c. $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid. Both sides are equivalent to the assertion that there is no model in which α is true and β is false, i.e., no model in which $\alpha \Rightarrow \beta$ is false.
d. $\alpha \equiv \beta$ if and only if the sentence $(\alpha \Leftrightarrow \beta)$ is valid. Both sides are equivalent to the assertion that α and β have the same truth value in every model.
e. $\alpha \models \beta$ if and only if the sentence $(\alpha \land \neg\beta)$ is unsatisfiable. As in c, both sides are equivalent to the assertion that there is no model in which α is true and β is false.

7.6 Prove, or find a counterexample to, each of the following assertions:

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$
b. If $\alpha \models (\beta \land \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$.
c. If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both)

Answer(s).

a. If $\alpha \models \gamma$ or $\beta \models \gamma$ (or both) then $(\alpha \land \beta) \models \gamma$. True. This follows from monotonicity.

b. If $\alpha \models (\beta \land \gamma)$ then $\alpha \models \beta$ and $\alpha \models \gamma$. True. If $\beta \land \gamma$ is true in every model of α, then β and γ are true in every model of α, so $\alpha \models \beta$ and $\alpha \models \gamma$.

c. If $\alpha \models (\beta \lor \gamma)$ then $\alpha \models \beta$ or $\alpha \models \gamma$ (or both). False. Consider $\beta \equiv A$, $\gamma \equiv \neg A$.

4. Refers to the wumpus world case study on propositional logic in part 7.2, work on exercise 7.1

Answer(s).

To save space, we'll show the list of models as a table (Figure S7.1) rather than a collection of diagrams. There are eight possible combinations of pits in the three squares, and four possibilities for the wumpus location (including nowhere). We can see that KB $\models \alpha 2$ because every line where KB is true also has $\alpha 2$ true. Similarly for $\alpha 3$.