

## Mixed Strategy Nash Equilibrium (MSNE) Tutorial

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**Mixed Strategy Nash equilibrium (MSNE):** In a MSNE, a player chooses a probability for each of their strategies that makes the *expected* payoffs of the **other** player's strategies equal.

*Example:* To find the MSNE, assign complementary probabilities to each player's strategies. In a 2x2 game, we typically assign  $p$  and  $(1-p)$  to the row player's strategies and  $q$  and  $(1-q)$  to the column player's strategies.

		Player 2	
		C $q$	D $(1-q)$
Player 1	A $p$	0 , 5	3 , 4
	B $(1-p)$	1 , 6	2 , 7

Next, for each player, set up the expected payoff for each strategy. Then, set those expected payoffs equal to each other.

**Important Note:** When finding the expected payoff for a strategy, we multiply each of the player's payoffs from that strategy by the **other** player's corresponding probabilities and add them together!

For **Player 1**, we have:

$$\text{Expected Payoff of A} = 0q + 3(1-q) = 1q + 2(1-q) = \text{Expected Payoff of B.}$$

Solving for  $q$ , gives us  $q = 1/2$  because:

$$0q + 3(1-q) = 1q + 2(1-q) \rightarrow 3-3q = 1q + 2-2q \rightarrow 3-3q = 2-q \rightarrow -2q = -1 \rightarrow q = 1/2.$$

Now that we know  $q = 1/2$ , it is easy to find that  $1-q = 1/2$ .

For **Player 2**, we have

$$\text{Expected Payoff of C} = 5p + 6(1-p) = 4p + 7(1-p) = \text{Expected Payoff of D}$$

Solving for  $p$ , gives us  $p = 1/2$  because:

$$5p + 6(1-p) = 4p + 7(1-p) \rightarrow 5p + 6-6p = 4p + 7-7p \rightarrow 6-1p = 7-3p \rightarrow 2p = 1 \rightarrow p = 1/2.$$

Now since we know  $p = 1/2$ , it is easy to find that  $1-p = 1/2$ .

Putting our work together, and referencing our matrix for the strategies that correspond to  $p$  and  $1-p$  for Player 1, and  $q$  and  $1-q$  for Player 2, the **MSNE** is **Player 1 chooses A with probability  $(p=)1/2$  and B with probability  $(1-p=)1/2$ . Player 2 chooses C with probability  $(q=)1/2$  and D with probability  $(1-q=)1/2$ .**