Mixed Strategy Nash Equilibrium (MSNE) Tutorial

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Mixed Strategy Nash equilibrium (MSNE): In a MSNE, a player chooses a probability for each of their strategies that makes the *expected* payoffs of the **other** player's strategies equal.

Example: To find the MSNE, assign complementary probabilities to each player's strategies. In a 2x2 game, we typically assign p and *(1-p)* to the row player's strategies and q and *(1-q)* to the column player's strategies.

Player 2

		C q	D *(1-q)*
	A p	0 , 5	3 , 4
Player 1	B *(1-p)*	1 , 6	2 , 7

Next, <u>for each player</u>, set up the expected payoff for each strategy. Then, set those expected payoffs equal to each other.

Important Note: When finding the expected payoff for a strategy, we multiply each of the player's payoffs from that strategy by the **other** player's corresponding probabilities and add them together!

For Player 1, we have:

Expected Payoff of A = $0q + 3(1-q) = 1q + 2(1-q)$ = Expected Payoff of B.

Solving for q, gives us q = 1/2 because:

$0q + 3(1-q) = 1q + 2(1-q)$ → $3-3q = 1q + 2-2q$ → $3-3q = 2-q$ → $-2q = -1$ → $q = 1/2$.

Now that we know q = 1/2, it is easy to find that 1-q = 1/2.

For Player 2, we have

Expected Payoff of C = $5p + 6(1-p) = 4p + 7(1-p)$ = Expected Payoff of D

Solving for p, gives us p = 1/2 because:

$5p + 6(1-p) = 4p + 7(1-p)$ → $5p + 6-6p = 4p + 7-7p$ → $6-1p = 7-3p$ → $2p = 1$ → $p = 1/2$.

Now since we know p = 1/2, it is easy to find that 1-p = 1/2.

Putting our work together, and referencing our matrix for the strategies that correspond to p and 1-p for Player 1, and q and 1-q for Player 2, the **MSNE** is Player 1 chooses A with probability (p=)1/2 and B with probability (1-p=)1/2. Player 2 chooses C with probability (q=)1/2 and D with probability (1-q=)1/2.