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Thomas Zeuthen

Homework 11T

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Collaborators: None

Q1. Our sample space can be defined as an ordered list of four integers from one to six with the first integer representing the result of the first die, the second the second die, and so on.

Let M be a random variable that represent the greatest die out of all four.

Let D_1 represent the result of the first die, let D_2 represent the result of the second die, and so on up to D_4

Let I_k be an indicator variable that equals 1 if the greatest value of the die rolls is at least k for some integer k between 1 and 6 and one otherwise.

$P(I_k = 1) = P(D_1 \cup D_2 \cup D_3 \cup D_4 \geq k)$.

We can calculate this probability using the complement rule and DeMorgan's law.

$P(D_1 \cup D_2 \cup D_3 \cup D_4 \geq k) = 1 - P(D_1 \cap D_2 \cap D_3 \cap D_4 < k)$

Each die roll is clearly independent since each die roll is fair so knowing the result of one one die will give you a clue as to the result of the other. Therefore we can calculate the probability of the intersection by multiplying the probability of the individual die rolls together.

$P(D_i < k)$ is the probability that a given die roll is between 1 and $k-1$. There are $k-1$ desired outcomes over 6 possible outcomes so $P(D_i < k) = \dfrac{k-1}{6}$

Therefore $P(D_1 \cup D_2 \cup D_3 \cup D_4 \geq k) = 1 - P(D_1 \cap D_2 \cap D_3 \cap D_4 < k) = 1 - (\dfrac{k-1}{6})^4$

$E(I_k) = 0 * P(I_k = 0) + 1 * P(I_k = 1) = 1 - (\dfrac{k-1}{6})^4$

Now we will find $E(M)$.

Consider the scenario where $M = j$ for some integer j between 1 and 6.

If $M = j$ this means the maximum value of the dice rolls is greater than or equal to all of the values from 1 to j including j. So $I_k = 1$ for all values k from one to j, and $I_k = 0$ for all other values of k.

Therefore $\displaystyle\sum_{1}^{6} I_k = j$ so $\displaystyle\sum_{k=1}^{6} I_k = M$ since $M = j$.

Therefore we can use the linearity of expectation to find M

$E(M) = \displaystyle\sum_{k=1}^{6} E(I_k) = \sum_{k=1}^{6} 1 - (\frac{k-1}{6})^4$

Q2. The sample space for this question would be all integers from 10 to 199999 that could be made by this process.

D be a random variable that reprsents the number of digits in the number

Let I_k be a random variable that equals 1 if the kth position has a digit and 0 otherwise. k ranges from 1 to 6. For example for the number 134 I_1 to I_3 would equal 1 and I_4 to I_6 would equal 0.

Next we will find $P(I_k = 1)$

For $k = 1$ and $k = 2$ this is equal to 1 since the first digit is always one and a second digit is always chosen.

For $3 \geq k \geq 6$ $P(I_k = 1)$ is the probability that every digit before position k is odd.

The first digit is always odd since it is one.

For digits 2 to $k-1$, since every digit is randomly chosen and there are an equal number of even and odd digits, the probability the digit is odd is $\frac{1}{2}$.

There are $k - 1 - 2 + 1 = k - 2$ digits between $k - 1$ and 2 inclusive. Since each digit is independently chosen the probability all digits before k are odd is $\frac{1}{2^{k-2}}$ which is the same as $P(I_k = 1)$.

$E(I_k) = 0 * P(I_k = 0) + 1 * P(I_k = 1) = 1$ if $k = 1, 2$ and $\frac{1}{2^{k-2}}$ if $k = 3, 4, 5, 6$

$D = \sum_{k=1}^{6} I_k$ since that gives the total number of digits.

By the linearity of expectation $E(D) = \sum_{k=1}^{6} E(I_k) = 1 + 1 + \sum_{k=3}^{6} \frac{1}{2^{k-2}} = 2 + \sum_{k=3}^{6} \frac{1}{2^{k-2}}$