Crowdsourcing in Smart Cities

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Background

Crowdsourcing

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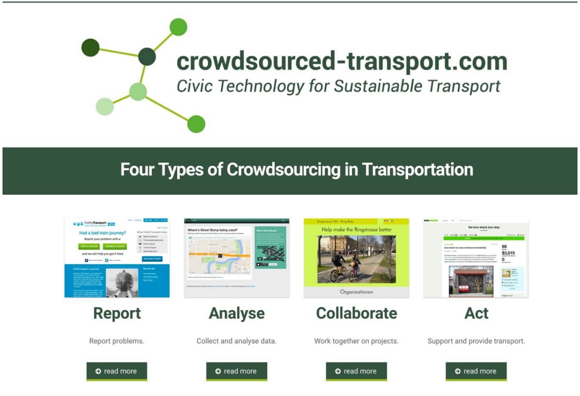
- To utilize the *crowd intelligence* rather than just hiring a small group of experts

Crowdsourcing: Example I

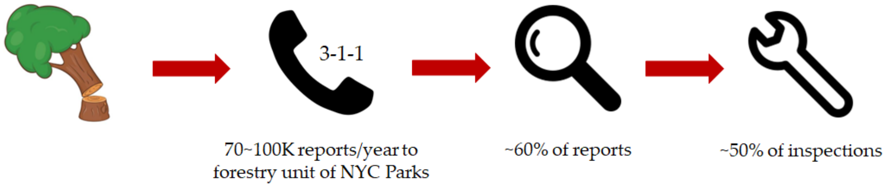


Traffic apps like Waze encourage drivers to report accidents and other roadway incidents to provide real-time, updated information to app users.

Crowdsourcing: Example II

Resident crowdsourcing systems (Liu and Garg 2022):

- 3-1-1 systems in North American cities: potholes, bed bugs, powerlines, and trees. . .
- primary way city government learns about on-the-ground problems
- NYC's 311 system: 2.7 million requests in 2021



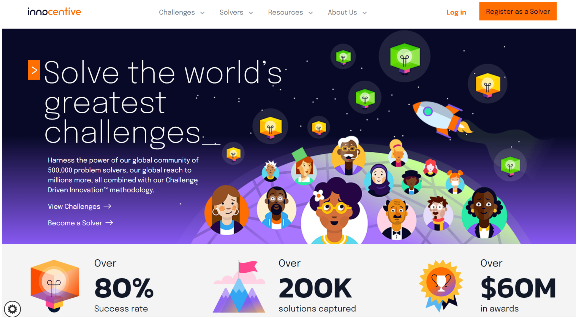
Crowdsourcing: Example III

Crowdsourcing food rescue platforms (Shi, Lizarondo, and Fang 2021):

- Food waste and food insecurity coexist
  - Waste up to 40% of our food globally (¿1.3 billion tons annually)
  - 1 in 8 people go hungry every day
- Food rescue (FR) organizations match food donations to the non-profits
- FR organizations mainly rely on *volunteers* to pick up and deliver the food

(1) Receive push notification (2) Claim on mobile app (3) Pick up from donor (4) Deliver to recipient (5) Success!

Example IV: Crowdsourcing Contests



- In a crowdsourcing contest a requester/sponsor posts a task on a platform and announces a monetary reward that he is willing to pay for a winning solution
- Contestants/players submit solutions on the platform and the requester chooses the best solution (possibly more than one) and awards the prize.

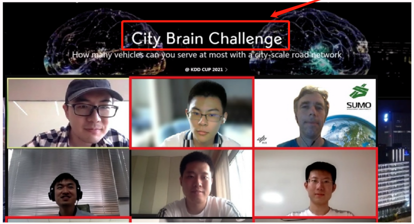
Crowdsourcing Contests Example: City Brain Challenge

有实力！交大学子斩获顶级国际大赛冠军！

请收藏 上海交通大学 2021-09-25 12:20 Posted on 上海

饮水思源 爱国荣校

近日，ACM SIGKDD（国际知识发现与数据挖掘大会）举办的KDD CUP 2021 City Brain Challenge（2021城市大脑挑战赛）在新加坡落下帷幕。电子信息与电气工程学院电子工程系硕士生宋罡、张正、王楚凡及本科生孙泽钰、顾煜程组成“IntelligentLight”团队，在指导教师宫新保的带领下突出重围，**最终从全世界1156支队伍中脱颖而出，将冠军收入囊中，祝贺！**



Elimination Crowdsourcing Contests

Elimination mechanism could filter weak players, and only keep strong players, which, intuitively, will incentive players to exert more efforts.

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Sub-elimination Contests (Moldovanu and Sela 2006)

- In the first-stage, players are randomly divided into to several sub-groups
- The winner of each sub-group is promoted to attend the second-stage contest.
- Example: Science contests among universities, etc.

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Sequential Elimination Contests (SEC) (Sun et al. 2022)

- In the first-stage, all players attend the same contest
- Some top players could proceed to the second-stage contest.
- Example
  - The Voice: Eligible players advance from top 20, top 13, top 11, top 10, top 8 rounds to a top 5 final round eventually.

One Round Crowdsourcing Contests

Model Setup (One-round)

- n: $\#$ of players

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- $V = [V_l : 1 \leq l \leq p]$: multiple prizes satisfying $V_1 \geq \cdots \geq V_p \geq 0$
  - $\sum_{l=1}^{p} V_l = R$, which is the budget for the sponsor to hold the contest. WLOG, $R = 1$.
  - V is pre-specified by the sponsor and commonly known for players
  - The player with highest effort wins the first prize V_1; similarly, the player with second-highest effort wins the second prize V_2, and so on until all prizes are allocated.

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  - Common prior: for $j \ne i$, $A_j \sim F(\cdot)$ with continuous density $f(\cdot)$.

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  - Common prior: for $j \neq i$, $A_j \sim F(\cdot)$ with continuous density $f(\cdot)$.
- e_i: player i's decision variable
  - Assume players' efforts are symmetric mapping from their abilities, i.e. $e_i = b(a_i), \forall i$.
  - Assume $b(\cdot)$ is continuous, differentiable and strictly increasing in a_i.

Model Setup (One-round)

- $g(e_i)/a_i$: the cost of player i by exerting effort e_i
  - $g : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ is a strictly increasing, continuous and differentiable function with $g(0) = 0$
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$$u_i = \sum_{l=1}^{p} V_l P_{i,l} - \frac{g(e_i)}{a_i}$$

  - $P_{i,l}$: player i's winning probability of l^{th} prize.
    - For example: $P_{i,1} = \Pr\left(e_i > e_j, \forall j \neq i\right) = \Pr\left(e_i > b(A_j), \forall j \neq i\right)$

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- The sponsor's problem: find V^* to maximize the expected highest (or total) equilibrium efforts.

Notations

Denote $F_{(l,n)}(\cdot)$ as the distribution function of the l^{th} *largest* order statistic among n IID random variables with distribution $F(\cdot)$.

$$F_{(l,n)}(x) = \sum_{j=n-l+1}^{n} \binom{n}{j} F^j(x)\big(1 - F(x)\big)^{n-j},$$

$$dF_{(l,n)}(x) = \frac{n!}{(n-l)!(l-1)!} F^{n-l}(x)\big(1 - F(x)\big)^{l-1} dF(x).$$

Define $F_{(0,n)}(x) = 0, F_{(n,n-1)}(x) = 0, dF_{(0,n)}(x) = 0, \forall x \in (0,1)$.

Equilibrium Strategies

Proposition (Moldovanu and Sela 2006)

Consider a one-round contest with n_1 players where the sponsor awards $p \leq n_1$ prizes with values $V_1 \geq V_2 \geq \cdots \geq V_p \geq 0$. The symmetric Bayesian Nash equilibrium is

$$b(a_i) = g^{-1}\left(\sum_{l=1}^{p} V_l \int_0^{a_i} x\big(dF_{(l,n-1)}(x) - dF_{(l-1,n-1)}(x)\big)\right). \tag{1}$$

Optimal Reward Structure

Theorem (Moldovanu and Sela 2001)

In terms of the expected total efforts, the optimal reward structure is winner-take-all under linear or concave cost functions.

Theorem (Chawla, Hartline, and Sivan 2019)

In terms of the expected highest efforts, the optimal reward structure is winner-take-all under linear cost functions.

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- a_i: player i's $(1 \leq i \leq n)$ private ability
  - WLOG, $a_i \in (0, 1)$
  - Common prior: for $j \neq i$, $A_j \sim F(\cdot)$ with continuous density $f(\cdot)$.

Model Setup (Sub-elimination)

- e_i^t: player i's decision variable at stage-t ($t = 1, 2$).
  - Assume players' efforts are symmetric mapping from their abilities, i.e. $e_i^t = b^t(a_i)$, for all eligible i at stage-t.
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- Player i's expected utility:

$$u_i = P_{i,1}\left(\alpha + P_{i,2}\left(1 - \alpha n_2\right) - \frac{g(e_i^2)}{a_i}\right) - \frac{g(e_i^1)}{a_i}$$

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  - $P_{i,t}$: player i's winning probability at stage-t given her current stage's belief
- The sponsor's problem: find α^* and n_2^* to maximize the expected highest equilibrium efforts.

Solution

- Essentially, the sub-elimination contest is a dynamic game with incomplete information
- We are playing with the notion of perfect Bayesian equilibrium (PBE)
- In sub-elimination, everything is clear so that we do not need a rigorous definition to work out the solution (Moldovanu and Sela 2006 did not provide a formal definition, either)
  - In later SEC, we will make it rigorous.

Solution

Belief Update

- If player i proceeds to the second-stage, she will perceive her opponent j's ability A_j as the largest order statistic among n_1/n_2 players, i.e. in the second-stage, for player i,

$$A_j \sim G(x) := F_{(1,\frac{n_1}{n_2})} = F^{\frac{n_1}{n_2}}(x) \qquad \forall \text{ eligible } j \neq i$$

  - Every player in the second-stage is the top player among n_1/n_2 players in each sub-group of first-stage.

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Remark. Posterior beliefs are still IID.

Solution

$$\max_{e_i^1, e_i^2} P_{i,1}\left(\alpha + P_{i,2}\left(1 - \alpha n_2\right) - \frac{g(e_i^2)}{a_i}\right) - \frac{g(e_i^1)}{a_i}$$

Backward Induction

Solution

$$\max_{e_i^1, e_i^2} P_{i,1}\left(\alpha + P_{i,2}\left(1-\alpha n_2\right) - \frac{g(e_i^2)}{a_i}\right) - \frac{g(e_i^1)}{a_i}$$

Backward Induction

- Suppose player i has been promoted to the second-stage,

$$\cancel{P_{i,1}}\left(\cancel{\alpha} + P_{i,2}\left(1-\alpha n_2\right) - \frac{g(e_i^2)}{a_i}\right) - \cancel{\frac{g(e_i^1)}{a_i}}$$

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$$\max_{e_i^1, e_i^2} P_{i,1}\left(\alpha + P_{i,2}\left(1 - \alpha n_2\right) - \frac{g(e_i^2)}{a_i}\right) - \frac{g(e_i^1)}{a_i}$$

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- In second-stage, it is equivalent to solve

$$\max_{e_i^2} P_{i,2}\left(1 - \alpha n_2\right) - \frac{g(e_i^2)}{a_i}$$

  - $P_{i,2} = \Pr(e_i^2 > b^2(A_j), \forall \text{ eligible } j \neq i,) \quad A_j \sim G(\cdot) = F^{\frac{n_1}{n_2}}(\cdot)$

Solution

- Put $u_i^2 := \max_{e_i^2} \; P_{i,2}\,(1-\alpha n_2) - \frac{g(e_i^2)}{a_i}$.
- We can obtain $b^1(a_i)$ by solving the following problem:

$$\max_{e_i^2} \; P_{i,1}\left(\alpha + u_i^2\right) - \frac{g(e_i^1)}{a_i}$$

  - $P_{i,1} = \Pr\left(e_i^1 > b^1(A_j), \forall \text{ same-sub-group } j \neq i\right) \qquad A_j \sim F(\cdot)$

Equilibrium Strategies

Proposition (Moldovanu and Sela 2006)

The symmetric Bayesian Nash equilibrium efforts in two-stage sub-elimination is: the second-stage effort is

$$b^2(a_i) = g^{-1}\left((1 - \alpha n_2)\int_0^{a_i} x dG_{(1,n_2-1)}(x)\right);$$

the first-stage effort is

$$b^2(a_i) = g^{-1}\left(\alpha u_i^2 \int_0^{a_i} x dF_{(1,\frac{n_1}{n_2}-1)}(x)\right),$$

where $u_i^2 = (1 - \alpha n_2)G^{n_2-1}(a_i) - \frac{g(b^2(a_i))}{a_i}$.

Sub-elimination VS. One-round

Theorem (Moldovanu and Sela 2006)

Under linear cost function, in terms of the expected highest effort, for a sufficiently high number of players n_1, the optimal two-stage sub-elimination contest is better than optimal one-round contest (winner-take-all).
Specifically, in this case the optimal sub-elimination is to set $\alpha = 0$, i.e. final-stage winner-take-all, and $n_2 = 2$.

Sequential Elimination Crowdsourcing Contests

Motivation

- Sequential elimination contests (SEC) are commonly used in real world.

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- Sequential elimination contests (SEC) are commonly used in real world.
- Intuitively, SEC should be better than sub-elimination contests
  - In sub-elimination, the second strongest player might be eliminated.
  - In SEC, the second strongest player must proceed to the second-stage ($n_2 \geq 2$).

Challenges in SEC

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Intuitively,

- The belief updating rule is not clear (players are Bayesian)
- The posterior beliefs are not identical anymore
  - players' own abilities have an impact on their beliefs about other players' abilities.
- Non-identical posterior beliefs $\overset{?}{\to}$ no symmetric equilibrium strategies
  - It is not easy to derive asymmetric Bayeisan equilibrium.

Model Setup (SEC)

- $\mathcal{I}^1$: a set of $n_1 \in \mathbb{Z}_+$ $(n_1 \geq 2)$ players in the first-stage
  - n_1 is endogenous
- $\mathcal{I}^2$: a set of $n_2 \in \mathbb{Z}_+$ $(2 \leq n_2 \leq n_1)$ players in the second-stage
  - n_2 is pre-specified by the sponsor and commonly known for players
- $\mathcal{I}^t_{-i}$: a set of players except for i $(i \in \mathcal{I}^t)$ at stage-t
- $V = [V_l : 1 \leq l \leq p]$: final-stage multiple prizes satisfying $V_1 \geq \cdots \geq V_p \geq 0$
  - V is pre-specified by the sponsor and commonly known for players
- a_i: player i's private ability
  - WLOG, $a_i \in (0, 1)$
  - Common prior: for $j \neq i$, $A_j \sim F(\cdot)$ with continuous density $f(\cdot)$.
- $a^t_{-i} = [a_j : j \in \mathcal{I}^t_{-i}]$: all players' abilities profile except i

Model Setup (SEC)

- e_i^t: player i's decision variable at stage-t ($t = 1, 2$).
  - Assume players' efforts are symmetric mapping from their abilities, i.e. $e_i^t = b^t(a_i)$, for all eligible i at stage-t.
  - Assume $b^t(\cdot)$ is continuous, differentiable and strictly increasing in a_i.
- $g(e_i^t)/a_i$: the cost of player i by exerting effort e_i^t
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$$u_i = P_i \left(\sum_{l=1}^{p} P_{i,l} V_l - \frac{g(e_i^2)}{a_i} \right) - \frac{g(e_i^1)}{a_i}$$

  - P_i: the probability of a player $i \in \mathcal{I}^1$ advancing to the second stage based on her first-stage belief
  - $P_{i,l}$: probability of a player $i \in \mathcal{I}^2$ winning the l^{th} prize based on her second-stage belief

Model Setup (SEC)

- $s_i^t \in \{1,0\}$: the *signal* of player $i \in \mathcal{I}^1$ at stage t representing whether she is eligible for stage-t contest
  - $s_i^1 = 1 \qquad \forall i \in \mathcal{I}^1$
- $s_{-i}^t = [s_j^t : j \in \mathcal{I}_{-i}^1]$: all players' signals' profile at stage-t except for player i.
- $s^t = [s_i^t, s_{-i}^t]$: all players' signals' profile at stage-t

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- $s^t = [s_i^t, s_{-i}^t]$: all players' signals' profile at stage-t
- $h^t = [s^1, \cdots, s^t]$: the *history* up to stage-t
  - h^t is observable to all players in $\mathcal{I}^t$.
  - For notation brevity, in the following, we sometimes suppress h^1 when appropriate since all n_1 players are able to attend the first-stage contest.

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  - h^t is observable to all players in $\mathcal{I}^t$.
  - For notation brevity, in the following, we sometimes suppress h^1 when appropriate since all n_1 players are able to attend the first-stage contest.
- $\beta_i^t(a_j \mid h^t, a_i)$: player $i \in \mathcal{I}^t$ belief (density) about player $j \in \mathcal{I}_{-i}^t$ conditional on h^t and a_i
- $\beta_i^t\left(a_{-i}^t \mid h^t, a_i\right)$: player $i \in \mathcal{I}^t$ belief (joint density) about all her opponents in $\mathcal{I}_{-i}^t$ as stage-t.
  - IID prior indicates that $\beta_i^1(a_{-i}^1 \mid h^1, a_i) = \prod_{j \in \mathcal{I}_{-i}^1} \beta_i^1(a_j \mid h^1, a_i) = \prod_{j \in \mathcal{I}_{-i}^1} f(a_j)$

Perfect Bayesian Equilibrium (PBE) I

A PBE of the SEC is a tuple of a strategy $[b^1(\cdot), b^2(\cdot)]$ and posterior beliefs $\left[\beta_i^2(\cdot \mid h^2, a_i) : i \in \mathcal{I}^2\right]$ that satisfies the following conditions.

(i) *Bayesian Updating:* for every player $i \in \mathcal{I}^2$, Bayes' rule is used to update her posterior belief $\beta_i^2(\cdot \mid h^2, a_i)$,

$$\beta_i^2(a_{-i}^2 \mid h^2, a_i) = \beta_i^2(a_{-i}^2 \mid s_{-i}^2, s_i^2, a_i) = \frac{\beta_i^1(a_{-i}^2 \mid s_i^2, a_i) \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right)}{\int_{a_{-i}^2} \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right) \beta_i^1(a_{-i}^2 \mid s_i^2, a_i) da_{-i}^2}$$

$$= \frac{\prod_{j \in \mathcal{I}_{-i}^2} f(a_j) \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right)}{\int_{a_{-i}^2} \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right) \prod_{j \in \mathcal{I}_{-i}^2} f(a_j) da_{-i}^2}.$$

The last equation holds because for the conditional density $\beta_i^1(a_{-i}^2 \mid s_i^2, a_i)$, player i does *not* know who else enters the second stage, $\beta_i^1(a_{-i}^2 \mid s_i^2, a_i) = \prod_{j \in \mathcal{I}_{-i}^2} f(a_j)$.

Perfect Bayesian Equilibrium (PBE) II

(ii) *Sequential Rationality:* player $i \in \mathcal{I}^t$'s expected utility starting at any stage $t \in \{1, 2\}$ of the contest is maximized by playing $b^t(a_i)$ given her belief $\beta_i^t(\cdot \mid h^t, a_i)$,

$$b^2(a_i) \in \arg\max_{e_i^2} \sum_{l=1}^{p} P_{i,l} \cdot V_l - \frac{g\left(e_i^2\right)}{a_i},$$

$$b^1(a_i) \in \arg\max_{e_i^1} P_i \cdot u_i^2 - \frac{g\left(e_i^1\right)}{a_i},$$

where $u_i^2 := \max_{e_i^2} \sum_{l=1}^{p} P_{i,l} \cdot V_l - \frac{g\left(b^2(a_i)\right)}{a_i}$; $P_{i,l}$ is the probability of a player $i \in \mathcal{I}^2$ winning the l^{th} prize based on her second-stage belief, and P_i is the probability of a player $i \in \mathcal{I}^1$ advancing to the second stage based on her first-stage belief.

Notation

Define the incomplete beta function $B(x, p, q)$ with parameters $x \in (0, 1), p \in \mathbb{Z}_+, q \in \mathbb{Z}_+$ as

$$B(x, p, q) := \int_0^x t^{p-1}(1-t)^{q-1} dt.$$

Posterior Beliefs

For $i \in \mathcal{I}^2$,

$$\beta_i^2(a_{-i}^2 \mid h^2, a_i) = \frac{\prod_{j \in \mathcal{I}_{-i}^2} f(a_j) \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right)}{\int_{a_{-i}^2} \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right) \prod_{j \in \mathcal{I}_{-i}^2} f(a_j) da_{-i}^2}$$

Posterior Beliefs

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$$\beta_i^2(a_{-i}^2 \mid h^2, a_i) = \frac{\prod_{j \in \mathcal{I}_{-i}^2} f(a_j) \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right)}{\int_{a_{-i}^2} \Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right) \prod_{j \in \mathcal{I}_{-i}^2} f(a_j) da_{-i}^2}$$

With a little notation abuse, put $a_{(1)}^2 := \min_{j \in \mathcal{I}_{-i}^2} a_j$.

$$\begin{aligned}
\Pr\left(s_{-i}^2 \mid a_{-i}^2, s_i^2, a_i\right) &= \begin{cases} 1, & a_{(1)}^2 > a_i, \\ \Pr\left(\max\{A_k : k \in \mathcal{I}^1 \setminus \mathcal{I}^2\} < a_{(1)}^2 \mid a_{-i}^2, s_i^2, a_i\right), & a_{(1)}^2 < a_i. \end{cases} \\
&= \begin{cases} 1, & a_{(1)}^2 > a_i, \\ F^{n_1 - n_2}\left(a_{(1)}^2\right), & a_{(1)}^2 < a_i. \end{cases}
\end{aligned}$$

Posterior Beliefs

Proposition

If player i with ability a_i advances to the second stage of the contest, her belief about the joint density of the other $n_2 - 1$ players' abilities A^2_{-i} in the second stage is

$$\beta_i^2\left(a^2_{-i} \mid h^2, a_i\right) = \begin{cases} \frac{1}{I(a_i,n_2)} \prod\limits_{j \in \mathcal{I}^2_{-i}} f(a_j) & a_i < \min_{j \in \mathcal{I}^2_{-i}} a_j, \\ \frac{F^{n_1-n_2}\left(\min_{j \in \mathcal{I}^2_{-i}} a_j\right)}{I(a_i,n_2)} \prod\limits_{j \in \mathcal{I}^2_{-i}} f(a_j) & a_i > \min_{j \in \mathcal{I}^2_{-i}} a_j, \end{cases} \tag{2}$$

where

$$I(a_i, n_2) := \left(1 - F(a_i)\right)^{n_2-1} + (n_2 - 1) B\left(F(a_i), n_1 - n_2 + 1, n_2 - 1\right).$$

Posterior Beliefs

Proposition

If player i with ability a_i advances to the second stage of the contest, her belief about the joint density of the other $n_2 - 1$ players' abilities A^2_{-i} in the second stage is

$$\beta_i^2\left(a_{-i}^2 \mid h^2, a_i\right) = \begin{cases} \frac{1}{I(a_i,n_2)} \prod\limits_{j\in\mathcal{I}^2_{-i}} f(a_j) & a_i < \min_{j\in\mathcal{I}^2_{-i}} a_j, \\ \frac{F^{n_1-n_2}\left(\min_{j\in\mathcal{I}^2_{-i}} a_j\right)}{I(a_i,n_2)} \prod\limits_{j\in\mathcal{I}^2_{-i}} f(a_j) & a_i > \min_{j\in\mathcal{I}^2_{-i}} a_j, \end{cases} \tag{2}$$

where

$$I(a_i, n_2) := \big(1 - F(a_i)\big)^{n_2-1} + (n_2 - 1)B\big(F(a_i), n_1 - n_2 + 1, n_2 - 1\big).$$

Remark. $\beta_i^2\left(a_{-i}^2 \mid h^2, a_i\right) \neq \prod_{j\in\mathcal{I}^2_{-i}} \beta_i^2\left(a_j \mid h^2, a_i\right)$ when $n_2 < n_1$.

Example: $n_2 = 2$

When $n_2 = 2$, for any player $i \neq j \in \mathcal{I}^2$,

$$\beta_i^2\left(a_j \mid h^2, a_i\right) = \begin{cases} \frac{f(a_j)}{\frac{F^{n_1-1}(a_i)}{n_1-1}+1-F(a_i)}, & a_i < a_j, \\ \frac{f(a_j)F^{n_1-2}(a_i)}{\frac{F^{n_1-1}(a_i)}{n_1-1}+1-F(a_i)}, & a_i > a_j. \end{cases}$$

Example: $n_2 = 2$

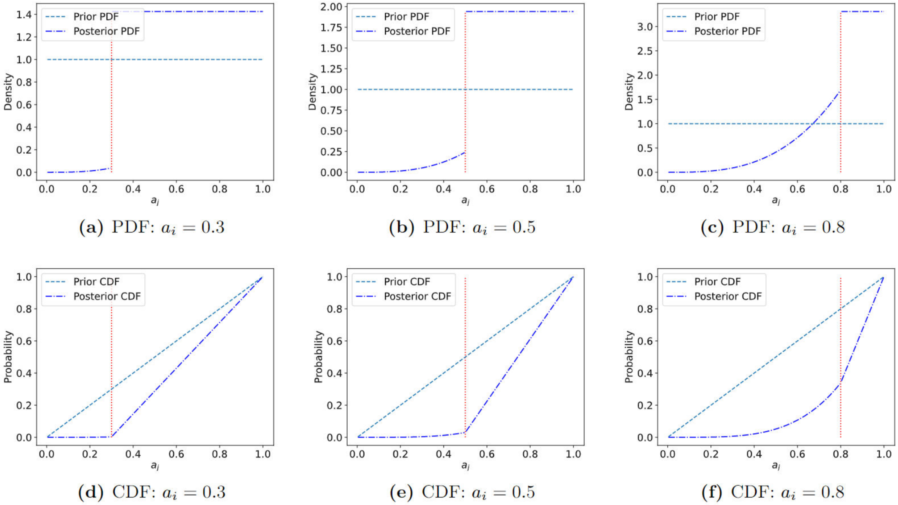


Figure: Comparison between prior and posterior beliefs ($n_1 = 5, n_2 = 2$)

Equilibrium Strategies I

Proposition

There exists a unique symmetric equilibrium strategy as follows in a PBE with posterior beliefs stated in equation (2).
For any player $i \in \mathcal{I}^2$, her second-stage effort is

$$b^2(a_i) = \begin{cases} g^{-1}\left(\sum_{l=1}^{p} V_l \int_0^{a_i} \frac{x}{J(x,n_2)} \left(dF_{(l,n_1-1)}(x) - dF_{(l-1,n_1-1)}(x)\right)\right), & p < n_2, \\ g^{-1}\left(\sum_{l=1}^{p-1} V_l \int_0^{a_i} \frac{x}{J(x,n_2)} \left(dF_{(l,n_1-1)}(x) - dF_{(l-1,n_1-1)}(x)\right) \right. \\ \left. \qquad - V_p \int_0^{a_i} \frac{(n_2-1)x}{I(x,n_2)} \left(1 - F(x)\right)^{n_2-2} dF(x)\right), & p = n_2, \end{cases} \tag{3}$$

where $J(a_i, n_2) := \binom{n_1-1}{n_2-1} \cdot I(a_i, n_2)$;

Equilibrium Strategies II

Proposition (continued)

for any player $i \in \mathcal{I}^1$, her first-stage effort is

$$b^1(a_i) = g^{-1}\left(u_i^2 \sum_{l=1}^{n_2} \int_0^{a_i} x\left(dF_{(l,n_1-1)}(x) - dF_{(l-1,n_1-1)}(x)\right)\right),$$

where u_i^2 is the expected utility of player i starting at the second stage under strategy $b^2(a_i)$,

$$u_i^2 = \begin{cases} \sum_{l=1}^{p} \frac{V_l}{J(a_i,n_2)} \binom{n_1-1}{l-1} \left(1 - F(a_i)\right)^{l-1} F^{n_1-l}(a_i) - \frac{g(b^2(a_i))}{a_i}, & p < n_2, \\ \sum_{l=1}^{p-1} \frac{V_l}{J(a_i,n_2)} \binom{n_1-1}{l-1} \left(1 - F(a_i)\right)^{l-1} F^{n_1-l}(a_i) + \frac{V_p}{I(a_i,n_2)} \left(1 - F(a_i)\right)^{n_2-1} - \frac{g(b^2(a_i))}{a_i}, & p = n_2. \end{cases}$$

SEC VS. One-round Contests I

- Note that, when $p < n_2$, the only difference between $b^2(a_i)$ and $b(a_i)$ is $1/J(a_i, n_2)$.
- This structure provides the opportunity to make the comparison under *any cost function*.

Lemma

For any $n_2 \leq n_1$, $J(a_i, n_2) \geq 1, \ \forall a_i \in (0,1)$, *and the equality holds for all* $a_i \in (0,1)$ *when* $n_2 = n_1$.

SEC VS. One-round Contests II

Theorem (Dominance of Equilibrium Efforts)

Consider a one-round contest and an SEC that adopt the same reward structure $V_1 \geq V_2 \geq \cdots \geq V_p \geq 0$. All players exert weakly lower efforts in the final stage of the SEC compared to those under the one-round contest, regardless of the number of players admitted to the final stage,

$$b^2(a_i) \leq b(a_i), \quad \forall a_i \in (0,1), \, \forall n_2 \leq n_1,$$

and the equality holds if $n_2 = n_1$.

- In terms of the expected highest effort or the expected total effort of the final stage, the optimal two-stage SEC is effectively a one-round contest by letting all n_1 players proceed to the second stage of the contest.

Intuition Behind the Equilibrium Dominance

To simplify the model, in our paper, we assume that

- The sponsor only cares about the second-stage solutions
- The quality of the solution is a deterministic function of the effort, i.e. $q_i = e_i$.

Open questions:

- The first-stage solution could also be beneficial to the sponsor
- However, there might be some noise. In general, the quality should be a stochastic function of the effort, i.e. $q_i = e_i + \epsilon_i$.

Game Theory: An Overview

AI Success and Game Theory

Michael I. Jordan's talk
https://www.youtube.com/watch?v=LM6IygUyNI4

AI (aka Machine Learning) Successes

- First Generation ('90-'00): the backend
  - e.g., fraud detection, search, supply-chain management
- Second Generation ('00-'10): the human side
  - e.g., recommendation systems, commerce, social media
- Third Generation ('10-now): pattern recognition
  - e.g., speech recognition, computer vision, translation
- Fourth Generation (emerging): markets
  - not just one agent making a decision or sequence of decisions
  - but a huge interconnected web of data, agents, decisions
  - many new challenges!

Multi-agent Reinforcement Learning and Game Theory

MARL is to *learn* (numerical results) the equilibrium of the games.

- Analytic representation of the equilibrium might be impossible in complex situations.

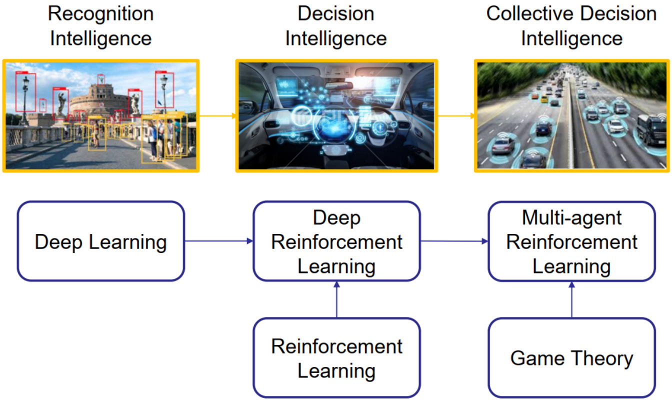
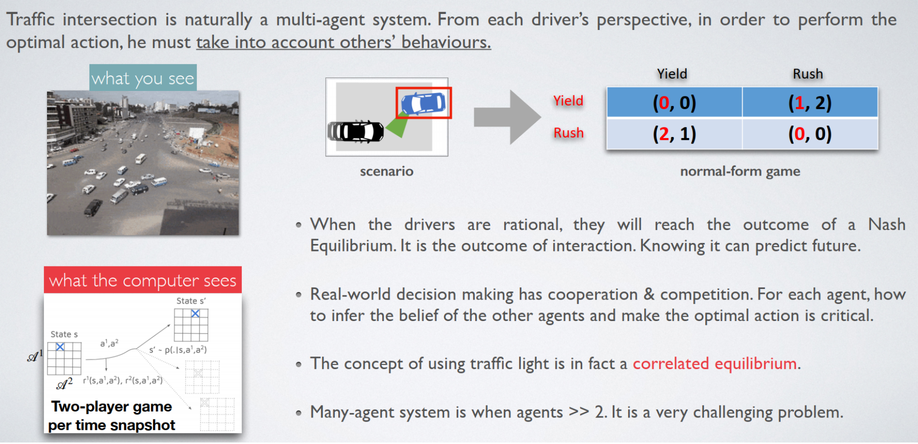


Image source: https://gitee.com/jidiai/rlchina2021/attach_files/843753/download

Example: MARL in Self-driving Cars



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Game Theory: An Overview

Game theory is to study the context where multiple self-interested agents intersect.

- Cooperative games
- Non-cooperative games (conflicting objectives)

		Complete	Incomplete
Static		Normal-form Game, e.g. Prisoner's Dilemma	Bayesian Game, e.g. Auction
Dynamic	Perfect	Extensive-form Game, e.g. Chess	Texas Hold'em Poker
	Imperfect	StarCraft	Mahjong

Dynamic Bayesian game

Figure: Summary of Game Representation

Image source: https://gitee.com/jidiai/rlchina2021/attach_files/843753/download

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