Mini Project 2

ECE4530J - Decision Making in Smart Cities Summer 2022

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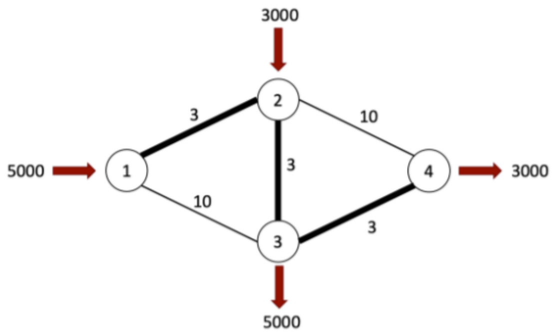
Consider the undirected network in the figure below. Links $(1,2),(2,3),(3,4)$ are highways with a capacity of 6000veh/hr and a travel time of 3 min. Links $(1,3),(2,4)$ are local streets with a capacity of 3000veh/hr and a travel time of 10 min. The traffic demand is indicated in the figure as well (unit: veh/hr). We want to allocate the traffic flows to minimize the average travel time for all vehicles.



a) Suppose that we do not differentiate traffic according to their origin-destination (OD) information. Formulate the min-cost flow problem. Solve it using a coding language of your choice.

b) Now, suppose that every vehicle entering the network at node 1 (resp. 2) must exit through node 3 (resp. 4). Formulate the min-cost flow problem. Solve it using a coding language of your choice.

Answer:

a) Decision variables:

$$f_{12}, f_{13}, f_{23}, f_{24}, f_{34}, f_{21}, f_{31}, f_{32}, f_{42}, f_{43}$$

Objective function:

$$z = 3f_{12} + 10f_{13} + 3f_{21} + 3f_{23} + 10f_{24} + 10f_{31} + 3f_{32} + 3f_{34} + 10f_{42} + 3f_{43}$$

Constraints:

1. flow conservation
2. link capacity
3. non-negativity

$$5000 + f_{21} + f_{31} = f_{12} + f_{13}$$
$$3000 + f_{12} + f_{32} + f_{42} = f_{21} + f_{23} + f_{24}$$
$$f_{23} + f_{13} + f_{43} = 5000 + f_{32} + f_{31} + f_{34}$$
$$f_{24} + f_{34} = 3000 + f_{42} + f_{43}$$
$$0 \leq f_{12} + f_{21} \leq 6000$$
$$0 \leq f_{13} + f_{31} \leq 3000$$
$$0 \leq f_{23} + f_{32} \leq 6000$$
$$0 \leq f_{24} + f_{42} \leq 3000$$
$$0 \leq f_{34} + f_{43} \leq 6000$$

Then, we use MATLAB to solve this linear problem, the code and solution are attached here.

```
A = [1 0 1 0 0 0 0 0 0 0
    -1 0 -1 0 0 0 0 0 0 0
    0 1 0 0 0 1 0 0 0 0
    0 -1 0 0 0 -1 0 0 0 0
    0 0 0 1 0 0 1 0 0 0
    0 0 0 -1 0 0 -1 0 0 0
    0 0 0 0 1 0 0 0 1 0
    0 0 0 0 -1 0 0 0 -1 0
    0 0 0 0 0 0 0 1 0 1
    0 0 0 0 0 0 0 -1 0 1];

b = [6000 0 3000 0 6000 0 3000 0 6000 0];
Aeq = [-1 -1 1 0 0 1 0 0 0 0
      1 0 -1 -1 -1 0 1 0 1 0
      0 1 0 1 0 -1 -1 -1 0 1
      0 0 0 0 1 0 0 1 -1 -1];
beq = [-5000 -3000 5000 3000];

lb = [0 0 0 0 0 0 0 0 0 0 0];
ub = [6000 3000 6000 6000 3000 3000 6000 6000 3000 6000];

f = [3 10 3 3 10 10 3 3 10 3];
x = linprog(f, A, b, Aeq, beq, lb, ub)
```

```
Optimal solution found.


x =

        3000
        2000
           0
        6000
           0
           0
           0
        3000
           0
           0

>> 
```

We could find one optimal solution here.

$$
\begin{aligned}
f_{12} &= 3000 \\
f_{13} &= 2000 \\
f_{21} &= 0 \\
f_{23} &= 6000 \\
f_{24} &= 0 \\
f_{31} &= 0 \\
f_{32} &= 0 \\
f_{34} &= 3000 \\
f_{42} &= 0 \\
f_{43} &= 0 \\
z &= 56000
\end{aligned}
$$

b) In this problem, there are only several links with directions, so we need to modify the variables.

Decision variables:

$$f_{12}^1, f_{13}^1, f_{23}^1, f_{24}^1, f_{34}^1, f_{21}^1, f_{31}^1, f_{32}^1, f_{42}^1, f_{43}^1, f_{12}^2, f_{13}^2, f_{23}^2, f_{24}^2, f_{34}^2, f_{21}^2, f_{31}^2, f_{32}^2, f_{42}^2, f_{43}^2$$

Objective function:

$$z = 3f_{12}^1 + 3f_{21}^1 + 10f_{13}^1 + 10f_{31}^1 + 3f_{23}^1 + 3f_{32}^1 + 10f_{24}^1 + 10f_{42}^1 + 3f_{34}^1 + 3f_{43}^1$$
$$+3f_{12}^2 + 3f_{21}^2 + 10f_{13}^2 + 10f_{31}^2 + 3f_{23}^2 + 3f_{32}^2 + 10f_{24}^2 + 10f_{42}^2 + 3f_{34}^2 + 3f_{43}^2$$

Constraints:

$$\begin{aligned}
&5000 + f_{21}^1 + f_{31}^1 = f_{12}^1 + f_{13}^1 \\
&f_{21}^2 + f_{31}^2 = f_{12}^2 + f_{13}^2 \\
&f_{12}^1 + f_{32}^1 + f_{42}^1 = f_{21}^1 + f_{23}^1 + f_{24}^1 \\
&3000 + f_{12}^2 + f_{32}^2 + f_{42}^2 = f_{21}^2 + f_{23}^2 + f_{24}^2 \\
&f_{23}^1 + f_{13}^1 + f_{43}^1 = 5000 + f_{32}^1 + f_{31}^1 + f_{34}^1 \\
&f_{23}^2 + f_{13}^2 + f_{43}^2 = f_{32}^2 + f_{31}^2 + f_{34}^2 \\
&f_{24}^1 + f_{34}^1 = f_{42}^1 + f_{43}^1 \\
&f_{24}^2 + f_{34}^2 = 3000 + f_{42}^2 + f_{43}^2 \\
&0 \le f_{12}^1 + f_{21}^1 + f_{12}^2 + f_{21}^2 \le 6000 \\
&0 \le f_{13}^1 + f_{31}^1 + f_{13}^2 + f_{31}^2 \le 3000 \\
&0 \le f_{23}^1 + f_{32}^1 + f_{23}^2 + f_{32}^2 \le 6000 \\
&0 \le f_{24}^1 + f_{42}^1 + f_{24}^2 + f_{42}^2 \le 3000 \\
&0 \le f_{34}^1 + f_{43}^1 + f_{34}^2 + f_{43}^2 \le 6000
\end{aligned}$$

Then, we use MATLAB to solve this linear problem, the code and solution are attached here.

```
A = [1  0   1   0   0   0   0   0   0   0   1   0   1   0   0   0   0   0   0   0
-1  0   -1  0   0   0   0   0   0   0   -1  0   -1  0   0   0   0   0   0   0
0   1   0   0   0   1   0   0   0   0   0   1   0   0   0   1   0   0   0   0
0   -1  0   0   0   -1  0   0   0   0   0   -1  0   0   0   -1  0   0   0   0
0   0   0   1   0   0   1   0   0   0   0   0   0   1   0   0   1   0   0   0
0   0   0   -1  0   0   -1  0   0   0   0   0   0   -1  0   0   -1  0   0   0
0   0   0   0   1   0   0   0   1   0   0   0   0   0   1   0   0   0   1   0
0   0   0   0   -1  0   0   0   -1  0   0   0   0   0   -1  0   0   0   -1  0
0   0   0   0   0   0   0   1   0   1   0   0   0   0   0   0   0   1   0   1
0   0   0   0   0   0   0   -1  0   -1  0   0   0   0   0   0   0   -1  0   -1
];

b = [6000 0 3000 0 6000 0 3000 0 6000 0];
Aeq = [1    1   -1  0   0   -1  0   0   0   0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   1   1   -1  0   0   -1  0   0   0   0
-1  0   1   1   1   0   -1  0   -1  0   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   -1  0   1   1   1   0   -1  0   -1  0
0   1   0   1   0   -1  -1  -1  0   1   0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   1   0   1   0   -1  -1  -1  0   1
0   0   0   0   1   0   0   1   -1  -1  0   0   0   0   0   0   0   0   0   0
0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   1   -1  0
];

beq = [5000 0 0 3000 5000 0 0 3000];

lb = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
ub = [6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 6000 ];

f = [3 10 3 3 10 10 3 3 10 3 3 10 3 3 10 10 3 3 10 3];
x = linprog(f, A, b, Aeq, beq, lb, ub)
```

We could find one optimal solution here.

$$f^1_{12} = 3000$$
$$f^1_{13} = 2000$$
$$f^1_{21} = 0$$
$$f^1_{23} = 3000$$
$$f^1_{24} = 0$$
$$f^1_{31} = 0$$
$$f^1_{32} = 0$$
$$f^1_{34} = 0$$
$$f^1_{42} = 0$$
$$f^1_{43} = 0$$
$$f^2_{12} = 0$$
$$f^2_{13} = 0$$
$$f^2_{21} = 0$$
$$f^2_{23} = 3000$$
$$f^2_{24} = 0$$
$$f^2_{31} = 0$$
$$f^2_{32} = 0$$
$$f^2_{34} = 3000$$
$$f^2_{42} = 0$$
$$f^2_{43} = 0$$
$$z = 56000$$

```
>> mp2


Optimal solution found.



x =


        3000
        2000
           0
        3000
           0
           0
           0
           0
           0
           0
           0
           0
           0
        3000
           0
           0
           0
        3000
           0
           0
```