Homework 4

ECE4530J - Decision Making in Smart Cities Summer 2022

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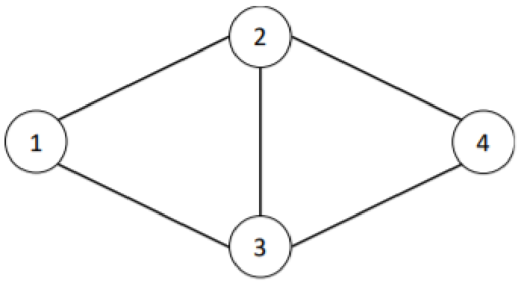
Problem 1

Consider the undirected network in the figure below for the subsequent path planning problems. Assume that every link has a length of 1 . Please clearly define the decision variables, the objective function, and the constraints. You are supposed to enumerate all constraints rather than using generic link/node indices.



a) Formulate the MST problem on this network.

b) Find three distinct MSTs for this network.

c) Formulate the TSP on this network.

d) Formulate the CPP on this network.

Answer:

a) - Decision variable:

$$x_e \in \{0,1\}, \quad e \in E.$$

- Objective function:

$$\min \sum_{e \in E} x_e = \min 0.$$

- Constraint: the links that we select, i.e., the set of links with $x_e = 1$, forms a tree.

$$\begin{aligned}
&\sum_{e \in E} x_e = n - 1, \\
&\sum_{e \in C} x_e \leq |C| - 1, \forall C \in \mathcal{C}, \\
&x_e \in \{0,1\}, e \in E.
\end{aligned}$$

in which $E = \{(1,2),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,2),(4,3)\}$

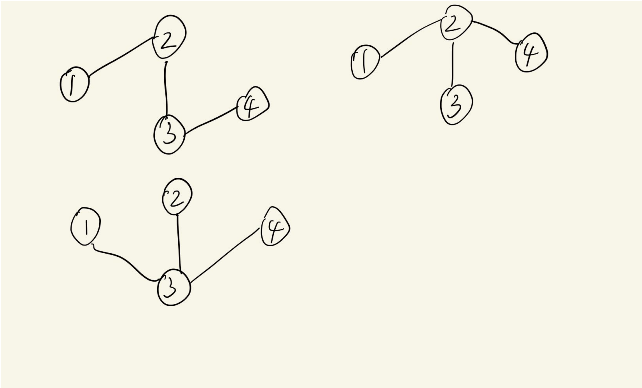
b) 

Figure 1: Figure for Problem 1b

c) - Decision variable:

$$f_{ij} \in \mathbb{Z}_{\geq 0}, (i, j) \in E$$

- Objective function:

$$\min \sum_{(i,j) \in E} f_{ij}$$

- Constraint:

$$\sum_{i \in \text{In}(j)} f_{ij} \geq 1, j \in N$$

$$\sum_{i \in \text{In}(j)} f_{ij} = \sum_{k \in Out(j)} f_{jk}, j \in N$$

in which $E = \{(1,2),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,2),(4,3)\}$

d) - Decision variable:

$$f_{ij} \in \mathbb{Z}_{\geq 0}, (i, j) \in E$$

- Objective function:

$$\min \sum_{(i,j) \in E} c_{ij} f_{ij}$$

- Constraint:

$$f_{ij} \geq 1, (i, j) \in E$$

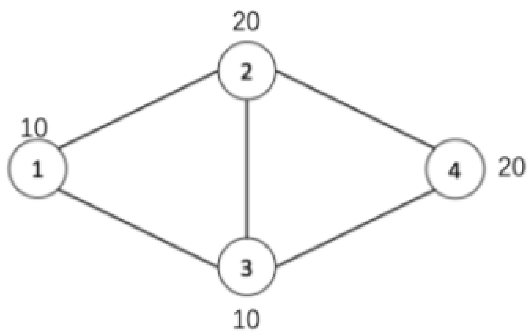
$$\sum_{i \in \text{In}(j)} f_{ij} = \sum_{k \in Out(k)} f_{jk}$$

in which $E = \{(1,2),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,2),(4,3)\}$

Problem 2

Consider the undirected network $G = (N, E)$ in the figure below for the subsequent facility location problems. The demands are indicated in the figure. Assume that every link is 10 unit long. Please clearly define the decision variables, the objective function, and the constraints. You are supposed to enumerate all constraints rather than using generic link/node indices.

a) Suppose that a facility must be located at a node. Specify the distance function $d(x, i)$ for all $x \in N$ and all $i \in N$.



b) Formulate the 2-median problem on this network in terms of $d(x, i)$.

c) Suppose that every facility can serve up to 15 units of demand within a distance of 10 units. Develop a formulation to find the minimal number of facilities needed.

Answer:

a) We can compute the distance (shortest-path) matrix $[d(i, j)]$ for all pairs of nodes, i and j, of the graph.

	1	2	3	4
1	0	10	10	20
2	10	0	10	10
3	10	10	0	10
4	20	10	10	0

b) - Decision variable:

$$X_2 = \left[x_{(1)}, x_{(2)}\right], x_{(1)}, x_{(2)} \in N$$
$$d\left(X_2, j\right) = \min_{x_i \in X_2} d\left(x_i, j\right) \quad \forall j \in N$$
$$J\left(X_2\right) = \sum_{j \in N} h_j d\left(X_2, j\right)$$

- Objective function:

$$\min J\left(X_2\right)$$

- Constraint:

$$d\left(X_2, j\right) = \min_{x_i \in X_2} d\left(x_i, j\right) \quad \forall j \in N$$
$$J\left(X_2\right) = \sum_{j \in N} h_j d\left(X_2, j\right)$$
$$J\left(X_2\right) \leqslant J\left(X_2'\right), \quad X_2' \subset N$$

c)

$$X_k = \left[x_{(1)}, x_{(2)} ... x_{(k)}\right] \subset G$$
$$d\left(X_k, j\right) = \min_{x_i \in X_k} d\left(x_i, j\right) \quad \forall j \in N$$

Distant constraint:

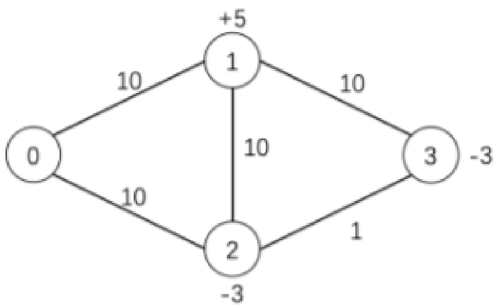
$$m\left(X_k\right) = \max_{j \in X} d\left(X_{k,j}\right) \leq 10$$

Demand Constraint:

$$\forall i \in X_k, j \in N, \quad 0 \leq y_{i,j} \leq 15$$
$$\text{if } d(i,j) > 10, \quad y_{i,j} = 0$$
$$\sum_{j \in N} y_{i,j} \leq 15, \quad \forall i \in X_k$$
$$\sum_{i \in X_k} y_{i,j} = h_j, \quad \forall j \in N$$

Problem 3

Consider the bike sharing rebalancing problem (BRP) on the network below. Node 0 is the depot. The demands as well as the link lengths are indicated in the figure. Suppose that every shipping vehicle can carry no more than 2 bikes. Assume that every vehicle can make only one tour.



a) Suppose that you have infinitely many vehicles to dispatch. Construct a feasible solution to the BRP.

b) With the fleet size you used in part a), formulate the BRP; please clearly indicate which formulation you use.

c) What is the minimal size of the rebalancing fleet? Justify your answer.

d) Find (using any method) the optimal solution to the BRP with the fleet size you identified in part c).

Answer:

a) In this problem, we could use three vehicles to complete this task.

1) First, vehicle 1 starts from node 0 (depot), carrying 2 bikes and travels to 2, puts down one bike, and travels to 3, puts down another bike, then travels to 1, picks two bikes, and then travel to the depot.
2) Second, vehicle 2 starts from node 0 (depot), carrying 2 bikes and travels to 2, puts down one bike, and travels to 3, puts down another bike, then travels to 1, picks two bikes, and then travel to the depot.
3) Third, vehicle 3 starts from node 0 (depot), carrying 2 bikes and travels to 2, puts down one bike, and travels to 3, puts down another bike, then travels to 1, picks one bike, and then travel to the depot.

b) The fleet size m is 3.

- Decision variable:

$$x_{ij} = \begin{cases} 1 & \text{if arc}(i,j) \text{ is used by a vehicle} \\ 0 & o.\text{w.} \end{cases}$$

- Objective function:

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}$$

- Constraint:

$$\sum_{j \in V} x_{ij} = 1, \forall i \in V \backslash \{0\}$$
$$\sum_{j \in V} x_{ji} = 1, \forall i \in V \backslash \{0\}$$
$$\sum_{i \in V} x_{0i} \leq 3$$
$$\sum_{i \in V \backslash \{0\}} x_{i0} = \sum_{i \in V \backslash \{0\}} x_{0i}$$
$$\sum_{i \in S} \sum_{j \in S} x_{ij} \leq |S| - 1, \forall S \subseteq V \backslash \{0\}, S \neq \emptyset,$$
$$\theta_j \geq \theta_i + q_j - M_1 (1 - x_{ij})$$
$$\theta_i \geq \theta_j - q_j - M_2 (1 - x_{ij})$$
$$\max \{0, q_j\} \leq \theta_j \leq \min \{Q, Q + q_j\}, j \in V.$$
$$x_{ij} \in \{0, 1\}, \forall (i,j) \in A.$$

c) Since one vehicle can only visit one node exactly once, and it could carry no more than 2 bikes. If we only use two vehicles, we cannot move all the five bikes in node 1 because two vehicles can only deal with 4 bikes. And part a) is a feasible answer.

d) In part a), the total cost is $31 \times 3 = 93$. We need to prove or disprove it is an optimal solution.

We can easily think about that, in part a), vehicle 3 only carry one bike to the depot, maybe we could increase the efficiency by changing some methods.

1) First, vehicle 1 starts from node 0 (depot), carrying 2 bikes and travels to 2 and travels to 3, puts down two bikes, then travels to 1, picks two bikes, and then travel to the depot.
2) Second, vehicle 2 starts from node 0 (depot), carrying 2 bikes and travels to 2, puts down one bike, and travels to 3, puts down another bike, then travels to 1, picks two bikes, and then travel to the depot.
3) Third, vehicle 3 starts from node 0 (depot), carrying 1 bike and travels to 2, puts down one bike, and travels to 1, picks one bike, and then travel to the depot.

In this method, the total cost is $31 + 31 + 30 = 92$