Quiz 1

ECE4530J - Decision Making in Smart Cities Summer 2022

* Name: Huang Yucheng ID: 519021910885

Problem 1

a. keep a small distance from the front cars so that the road will not be congested

change lane in a high speed with sufficient safety

vehicles typically have an optimal speed, and autonomous driving can track this speed

b. CAVs can communicate with each other and know what's going on in the other car, which can reduce fuel consumption and congestion

CAVs can operate according to the state and actions of the preceding car, making it safer

c.

	Longitudinal	Lateral
Device	throttle, brake, clutch	steering wheel
Action	push, release	turn in different angles
Result	acceleration, speed, position	angular acceleration, angular velocity, direction

d. The mapping is called the control policy. Typically, the mapping is a function.

- A control policy is a function;
- This function maps a state to a control input.

Problem 2

a. - State variable: speed $v[t] \Rightarrow [x[t], v[t]]^T$.
- State Space: $\mathbb{R}^2$
- Control input: acceleration $u[t]$.
- System dynamics: $v[t+1] = v[t] + u[t]\delta \Rightarrow \begin{bmatrix} x[t+1] \\ v[t+1] \end{bmatrix} = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x[t] \\ v[t] \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} u[t]$

b. No, it is unreasonable, because existence can only determine that this one time point conforms to the path, but there is no guarantee that other later time points also shrink to the reference trajectory, so we should use the definition of limit

$$\lim_{t\to\infty} |x[t] - \bar{x}[t]| = 0$$

c. Assume a linear controller: $\mu\left([\tilde{x}[t], \tilde{v}[t]]^T\right) = [-k_1, -k_2] \begin{bmatrix} \tilde{x}[t] \\ \tilde{v}[t] \end{bmatrix}$, and plug it into the above system. Then the system becomes:

$$\begin{bmatrix} \tilde{x}[t+1] \\ \tilde{v}[t+1] \end{bmatrix} = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}[t] \\ \tilde{v}[t] \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} [-k_1, -k_2] \begin{bmatrix} \tilde{x}[t] \\ \tilde{v}[t] \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \delta \\ -k_1\delta & 1-k_2\delta \end{bmatrix} \begin{bmatrix} \tilde{x}[t] \\ \tilde{v}[t] \end{bmatrix} = \begin{bmatrix} 1 & \delta \\ -k_1\delta & 1-k_2\delta \end{bmatrix}^t \begin{bmatrix} \tilde{x}[0] \\ \tilde{v}[0] \end{bmatrix}$$

d. The signs of the coefficients should be negative.

e. The system is stable, so that the system converges when:

$$\lim_{t\to\infty}\left[\begin{array}{cc} 1 & \delta \\ -k_1\delta & 1-k_2\delta \end{array}\right]^t = 0$$

For a square matrix $A, \lim_{t\to\infty} A^t = 0$ if and only if the magnitude of every eigenvalue of A is less than 1 .

For example, choose $k_1 = \frac{1}{\delta^2}, k_2 = \frac{2}{\delta}$. Let $A = \left[\begin{array}{cc} 1 & \delta \\ -k_1\delta & 1-k_2\delta \end{array}\right]$

$$\det(A-\lambda I) = \left|\begin{array}{cc} 1-\lambda & \delta \\ -k_1\delta & 1-k_2\delta-\lambda \end{array}\right| = \left|\begin{array}{cc} 1-\lambda & \delta \\ -\frac{1}{\delta} & -1-\lambda \end{array}\right| = \lambda^2 = 0$$

In this case, the magnitudes of eigenvalues $|\lambda_1| = |\lambda_2| = 0 < 1$ Hence, the linear controller $\mu\left([\tilde{x}[t], \tilde{v}[t]]^T\right) = \left[-\frac{1}{\delta^2}, -\frac{2}{\delta}\right]\left[\begin{array}{c} \tilde{x}[t] \\ \tilde{v}[t] \end{array}\right]$ stabilizes the system.

Problem 3

a. Since the control input that ensures $x(t) = x^*$ and $\dot{x}(t) = 0$. We have $u(t) = -x^2(t) - x(t)$

b. $\dot{x}(t) = x(t) + x^2(t) + u(t)$

c. Taylor series of function $f(x)$ at $x = a$:

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$

Here we can approximate the RHS as follows:

$$\begin{aligned} &\frac{\partial}{\partial x}\left(x+x^2+u\right)_{\substack{x=0 \\ u=0}} x + \frac{\partial}{\partial u}\left(x+x^2+u\right)_{\substack{x=0 \\ u=0}} u \\ &= x+u \end{aligned}$$

Hence, the linearized system is given as: $\dot{x} = x + u$

d. Assume a linear controller: $\mu(x) = -kx$, and plug it into the above linearized system. Then the system becomes:

$$\dot{x} = (1-k)\,x.$$

Any choice of k satisfying $\operatorname{Re}(1-k) < 0$ enables the controller to stabilize the system, i.e., the system converges with the controller plugged in.

e. No, it can only guarantee asymptotic convergence, that is, it gradually converges to the reference value after a certain t1, but global convergence requires all values to be satisfied, and it is possible that some points before t1 are not satisfied.

Problem 4

a. Because it represents a critical position above which there will be traffic congestion, actually it is maximal density.

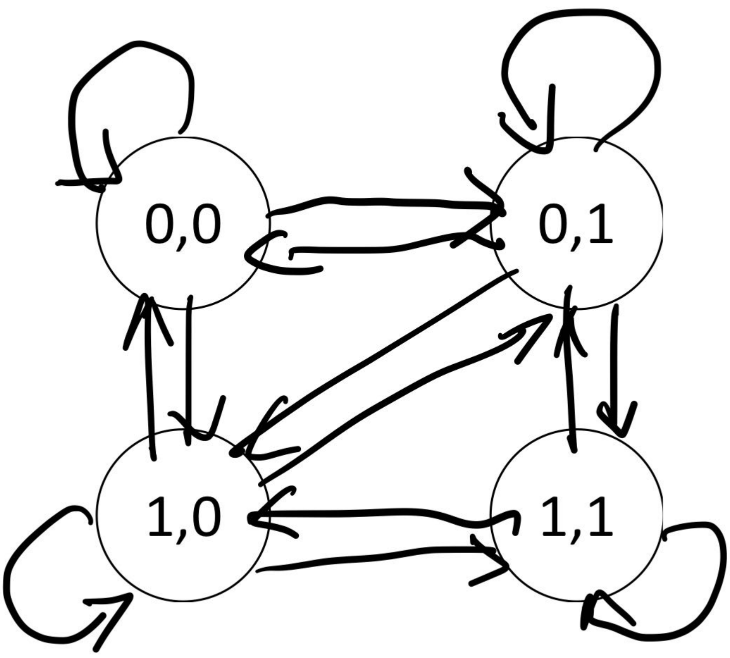
b. When there is a lot of traffic, we find that the speed of the car slows down, which means that the speed of the off-ramp becomes very slow. If the traffic volume has reached the maximum, all cars stop moving, which means that the speed of the off-ramp is 0, so it also has a negative correlation with the critical density.

c. maximal density; congestion wave speed; free-flow speed; Mainline flow ; Off-ramp flow

d. If more and more self-driving cars are added, it means that more cars can drive at a smaller distance at a faster speed, which also means that the maximum traffic density is increased, and the traffic flow f is approximately saturated. case, we can consider γ to be smaller.

Problem 5

a. State: $X[t] = [X_1[t]X_2[t]]^T \in \mathbb{Z}^2_{\geq 0}$



b.

Figure 1: Problem 5.2

c. all of them should be 0.25 because

$$
\begin{aligned}
&p = (1 - p_1)(1 - p_2) = 0.25 \\
&p = p_1 (1 - p_2) = 0.25 \\
&p = (1 - p_1) p_2 = 0.25 \\
&p = p_1 p_2 = 0.25
\end{aligned}
$$

d. On average, Bernoulli routing is less efficient than JSQ routing, because JSQ does not execute certain paths, such as (0,1) to (0,2), and once a path is congested, JSQ will It will be automatically adjusted according to the feedback, which is a closed-loop decision

e.