Shanghai Jiao Tong University
UM Joint Institute ECE4530J
Mini Project 1: Due 2022.6.3

Part 1: Design your own dynamical system

Consider the dynamical system for longitudinal control:

$$\dot{v} = -\frac{\rho C_d}{2M_t}v^2 + \frac{T_e - R_g T_b}{M_t} - \frac{C_r mg}{M_t},$$

$$M_t = \frac{(mR^2 + I_w)R_g}{R}.$$

Please search on the internet for relevant materials, and make your **best guess** for all the parameters involved above, such as the air drag coefficient C_d, the rolling resistance coefficient C_r, etc.
Hint: the decision variables in the system are T_e and T_b.

Part 2(a): Design a linear controller by linearizing the system

Consider the uniform trajectory tracking problem with reference trajectory $\bar{x}[t] = \bar{v}t$.

1. Reformulate the system that you generated in Part 1, with the speed error being the state variable and the total torque T being the control input.
2. **Linearize** the above system at the origin, i.e., use Taylor expansion to linearize the RHS of the dynamical equation in the neighborhood of $v = \bar{v}$.
3. Design a **linear controller** that stabilizes the linearized system.
4. Plug in your controller to the system that you generated in **Part 2a-1**, and reformulate it with the absolute speed v being the state variable again.
5. **Discretize** the above system.
6. Consider the discretized system in the face of additive Gaussian noise. To be specific, a random noise $w[t]$ is added to the RHS of the system:
$$v[t+1] = v[t] + u[t]\delta + \boldsymbol{w[t]}.$$
Here $w[t]$ is a Gaussian random variable with zero mean. Please select the variance of $w[t]$ on your own.

Part 2(b): Directly design a controller for the nonlinear system

Consider the same uniform trajectory tracking problem with reference trajectory $\bar{x}[t] = \bar{v}t$.

1. Consider the reformulated system that you generated in Part 2a-1, directly design a controller for the **nonlinear system**. Hint: recall the method for HW2 P2(c).
2. Plug in your controller to the system, and reformulate it with the absolute speed v being the state variable again.
3. **Discretize** the above system.
4. Consider the discretized system in the face of additive Gaussian noise, which should follow the same distribution as $w[t]$ in Part2a-6.

Part 3: Simulate both controllers with respect to the reference trajectory

Consider the above uniform trajectory tracking problem with reference trajectory $\bar{x}[t] = \bar{v}t$.

a) Please simulate and plot the following three trajectory curves $s[t]$ v.s. t on the same figure:
   1. Consider the system with random noise that you generated in Part 2a-6. Simulate the

actual trajectory $s_1[t]$ with your linear controller applied.

2. Consider the system with random noise that you generated in Part 2b-4. Simulate the actual trajectory $s_2[t]$ with your controller applied.
3. Simulate the reference trajectory $s_{ref}[t]$.

Hint: you may find Python function *random.gauss()* useful.

b) Compare the trajectory tracking performances.

Note: please attach your codes to the end of your document.