Quiz 2

ECE4530J - Decision Making in Smart Cities Summer 2022

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Problem 1

a.

static routing	dynamic routing
time-invariant	time-variant
open-loop policy	closed-loop policy
not respond to actual changes	can respond to instant changes
only calculation at the beginning	remain calculating in real time

b. A businessman, who wanted to visit certain cities in China, asked: The nearest distance, Each city can only be visited once, Starting from a certain city and finally returning to the city

A concert tour manager whose band schedules a series of performances must determine the shortest path for the tour,

An ant wants to collect all the food at multiple points, how can it walk all the way in the shortest time?

c. The traveling cost c_{ij},

At most m uncapacitated vehicles based on a central depot have to visit a set of vertices,

vehicle capacity Q,

an additional continuous variable θ_j, representing the load of a vehicle after it visited node j, for $j \in V$.

d. An optimal solution is a feasible solution x^* such that

$$c^T x^* \geq c^T x$$

for all feasible solution x. Intuitively, an optimal solution is a solution that leads to the optimal objective value; may not be unique! Note: not every feasible LP has an optimal solution.

$\min x$	no optimal
s.t. $x \in \mathbb{R}$	solution

Problem 2

a. Then, the cost on link (i,j) is $c_{ij} f_{ij}$, where $c_{ij} > 0$ is the cost per unit flow

This is a rather simplistic model that ignores any congestion effect.

But when there is a congestion, it will deviate from the original model and the quadratic term is better than linear term.

b. Theorem: Max flow = min cut. Cut = a set of links that partition the nodes into two clusters. We put a cut in 24, 23, 13. And the max flow is 3+6+3=12

c. Decision variables:

$$f_{12}, f_{13}, f_{23}, f_{24}, f_{34}, f_{21}, f_{31}, f_{32}, f_{42}, f_{43}$$

Objective function:

$$z = 3f_{12}^2 + 10f_{13}^2 + 3f_{21}^2 + 3f_{23}^2 + 10f_{24}^2 + 10f_{31}^2 + 3f_{32}^2 + 3f_{34}^2 + 10f_{42}^2 + 3f_{43}^2$$

Constraints: 1. flow conservation 2. link capacity 3. non-negativity

$$\begin{aligned}
&5 + f_{21} + f_{31} = f_{12} + f_{13} \\
&3 + f_{12} + f_{32} + f_{42} = f_{21} + f_{23} + f_{24} \\
&f_{23} + f_{13} + f_{43} = 5 + f_{32} + f_{31} + f_{34} \\
&f_{24} + f_{34} = 3 + f_{42} + f_{43} \\
&0 \le f_{12} + f_{21} \le 6 \\
&0 \le f_{13} + f_{31} \le 3 \\
&0 \le f_{23} + f_{32} \le 6 \\
&0 \le f_{24} + f_{42} \le 3 \\
&0 \le f_{34} + f_{43} \le 6
\end{aligned}$$

d. Recall: $f(x)$ is convex in x if

$$f\left(\lambda x_1 + (1-\lambda)x_2\right) \le \lambda f\left(x_1\right) + (1-\lambda) f\left(x_2\right),$$

$\forall x \in \mathbb{R}^n, \quad \forall \lambda \in [0,1]$ $g(x)$: convex in x

For a problem to be convex:

- Minimization, convex objective function;

. Non-positivity, convex left-hand side.

Obviously, it satisfies the two conditions and it is convex.

Problem 3

a. - Decision variable:

$$x_e \in \{0,1\}, \quad e \in E.$$

- Objective function:

$$\min \sum_{e \in E} x_e = \min 0.$$

b. - Constraint: the links that we select, i.e., the set of links with $x_e = 1$, forms a tree.

$$\begin{aligned}
&\sum_{e \in E} x_e = n - 1 \\
&\sum_{e \in C} x_e \le |C| - 1, \forall C \in \mathcal{C} \\
&x_e \in \{0,1\}, e \in E
\end{aligned}$$

in which $E = \{(01, 02, 12, 13, 24, 34, 35, 45)\}$

To satisfies a tree, we also need $x_{01} + x_{12} + x_{20} \le 2$ and $x_{34} + x_{45} + x_{53} \le 2$

c. - Decision variable:

$$f_{ij} \in \mathbb{Z}_{\geq 0}, (i, j) \in E$$

- Objective function:

$$\min \sum_{(i,j) \in E} f_{ij}$$

d. - Constraint:

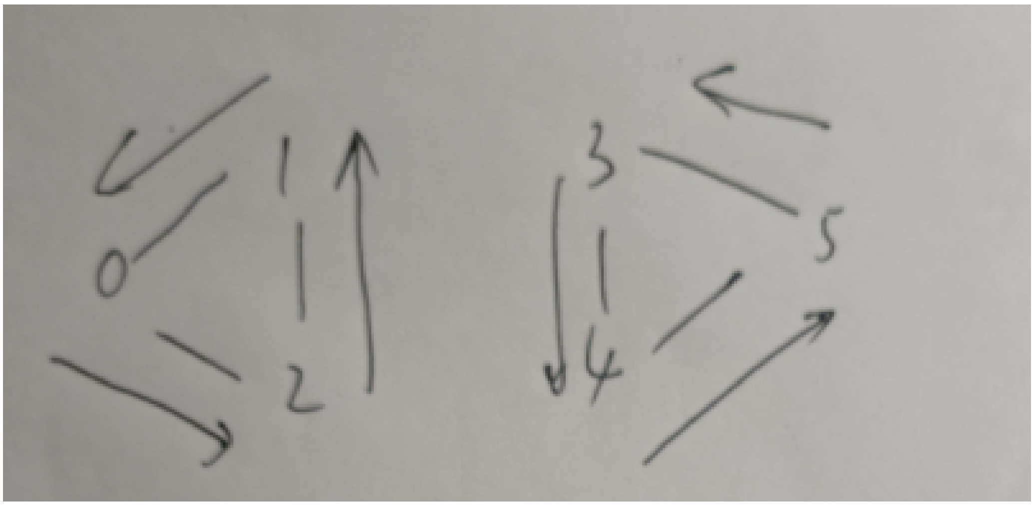
$$\sum_{i \in \mathrm{In}(j)} f_{ij} \geq 1, j \in N$$

$$\sum_{i \in \mathrm{In}(j)} f_{ij} = \sum_{k \in \mathrm{Out}(j)} f_{jk}, j \in N$$

in which $E = \{(01, 02, 12, 13, 24, 34, 35, 45)\}$

For subtour elimination: we also need $x_{01}+x_{12}+x_{20} \leq 2$, $x_{34}+x_{45}+x_{53} \leq 2$, $x_{02}+x_{21}+x_{10} \leq 2$, $x_{35}+x_{54}+x_{43} \leq 2$,

e. As shown in the picture, if we do not consider a subtour elimination, it will let the tour be two discrete tours and one trip cannot satisfy the meet.



Problem 4

a.

	0	1	2	3
0	0	10	10	20
1	10	0	10	10
2	10	10	0	10
3	20	10	10	0

b. If these four points are all residential areas, now we need to set up a charging station for electric vehicles, so that the average time from each residential area to the station is the shortest?

c. Data:

$$(x_k, y_k), d_k, k = 1, 2, \ldots, 4.$$

Decision variables:

(p, q)

Constraints:

$(p, q) \in (x_k, y_k), k = 1, 2, \ldots, 4$

Objective:

$$\min \sum_{k=1}^{N} d_k \sqrt{(x_k - p)^2 + (y_k - q)^2}$$

	0	1	2	3
0	0	10	10	20
1	10	0	10	10
2	10	10	0	10
3	20	10	10	0

$= [d(i, j)]$

	0	1	2	3
0	0	50	30	60
1	10	0	30	30
2	10	50	0	30
3	20	50	30	0

$= h_j[d(i, j)]$

$$D_0 = (0 + 50 + 30 + 60) = 140$$
$$D_1 = (10 + 0 + 30 + 30) = 70$$
$$D_2 = (10 + 50 + 0 + 30) = 90$$
$$D_3 = (20 + 50 + 30 + 0) = 100$$

So, the 1 is the best median.

d. We need to have a fire station that can solve fires in nearby residential areas in an emergency. Then at this time we have to consider the distance of the farthest residential area instead of the average required cost, which is a single-center problem.

Problem 5

a. The main reason is that the decisions made by one party will be the data received by the other party. If the two stages are not considered together, then one stage cannot guarantee optimality if the other stage changes.

b. This is the objective function of first stage: power load balancing, where the goal of the smart grid is to minimize the weighted sum of the cost represented by the ELI and the loss of revenue due to discounts offered to data centers.

The first term is Electric load index (ELI)

$$ELI = \sum_{t=1}^{T} \sum_{i=1}^{N} \left(r_i^t(s)\right)^2 C_i$$

Motivated by the index measurement techniques used for feeder load balancing in distribution system. Minimizing ELI results in balancing the electric load across all the locations.

The second term is

$$\text{Total price } = E_i^t\left(\alpha_i^t+\beta_i\left(E_i^t-s_i^t\right)\right)$$

$$\theta = \text{ tradeoff coefficient between ELI \& discounts}$$

c. It's the Quality of service (delay) constraint. To model the queuing delay, we use queuing theory to analyze the average processing time in data center i when there are x_i^t active servers processing workload λ_i^t with a service rate μ per server, and the average waiting time (delay) is

$$\frac{1}{\mu x_i^t-\lambda_i^t}$$

To meet the QoS requirement, the total time delay experienced by a computing request should satisfy some delay bound D, which is the maximum waiting time that a request can tolerate.

For simplicity, in this paper we will assume homogeneous requests that have the same delay bound D. Therefore, we have the following QoS constraint

$$d_i^t+\frac{1}{\mu x_i^t-\lambda_i^t} \leq D, \mu x_i^t>\lambda^t, 1 \leq i \leq N, 1 \leq t \leq T$$

d. The coefficients are

$$\begin{aligned}
& a=\frac{P_{\text{peak }}-P_{\text{idle}}}{\mu} \\
& b=P_{\text{idle }}+\left(E_{\text{usage }}-1\right) P_{\text{peak}} \\
& c=\xi
\end{aligned}$$

Since μ is a service rate, it should be greater than 0, and P_{peak} should always greater than P_{idle}, then a should be greater than 0.

The term $x_i^t\left(P_{\text{idle }}+\left(E_{\text{usage }}-1\right) P_{\text{peak }}\right)$ represents the base energy consumption, which only depends on the number of active servers. It cannot be negative. b should be greater than 0.

The parameter ξ is an empirical constant. Most of time we will think about that as a positive number. However, we cannot define the constant now. So, the sign is not certain.