Shanghai Jiao Tong University
UM Joint Institute ECE4530J
Homework 2: Due 2022.5.27

Problem 1

Consider the trajectory tracking problem with acceleration being the control input:

$$\begin{bmatrix} x[t+1] \\ v[t+1] \end{bmatrix} = \begin{bmatrix} 1 & \delta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x[t] \\ v[t] \end{bmatrix} + \begin{bmatrix} 0 \\ \delta \end{bmatrix} u[t].$$

Let $\bar{x}[t] = \bar{v}t$ be the reference trajectory to track.

a) Reformulate the state-space model with the position and speed errors being the state.
b) Using the reformulated model, find a linear controller that stabilizes the system.
c) Find a linear controller that destabilizes the system.

Problem 2

Consider a one-dimensional non-linear system

$$\dot{x} = a_1 x + a_2 x^2 + bu + c.$$

a) Use Taylor expansion to linearize the RHS of the dynamical equation in the neighborhood of $x = 0$.
b) For the linearized system, design a linear controller $\mu(x)$ that stabilizes the linearized system. Hint: a linear system $\dot{x} = \tilde{a}x$ is stable if and only if $Re(\tilde{a}) < 0$.
c) For the continuous-time system, design a controller $\mu(x)$ such that, with $u = \mu(x)$, the RHS of the dynamical equation is linear. Hint: do not confuse this part with part a).

Problem 3

Consider a two-vehicle platoon with states $\begin{bmatrix} x_1[t] \\ v_1[t] \end{bmatrix}, \begin{bmatrix} x_2[t] \\ v_2[t] \end{bmatrix}$. Vehicle 1 tracks a pre-specified trajectory $\bar{x}[t] = \bar{v}t,\ t = 0,1,2,\ldots$ Vehicle 2 follows vehicle 1 to keep a spacing of d to vehicle 1. The inputs are the engine torques $\tau_1[t]$ and $\tau_2[t]$. For $i = 1,2$, the propelling force is given by $\alpha\tau_i$, while the resistant force is given by βv_i^2. The vehicle masses are m_1, m_2, respectively.

a) Formulate Newton's second law for both vehicles.
b) Formulate the state-space model for both vehicles using absolute position and speed as the state.
c) Reformulate the model with the tracking/following errors being the state.
d) Construct a trajectory-tracking policy for vehicle 1, i.e., a function that maps tracking errors e_1, y_1 to τ_1. Explain why the policy will work. No mathematical proof needed.
e) [Bonus] Construct a vehicle-following algorithm for vehicle 2, i.e., a function that maps x_1, v_1, x_2, v_2 to τ_2. Explain why the policy will work. No mathematical proof needed.