Shanghai Jiao Tong University
UM Joint Institute ECE4530J
Homework 1: Due 2022.5.18

Problem 1

a) What is a mapping? What is a function?
b) What is asymptotic convergence?

Problem 2

Consider a one-dimensional linear time-invariant system

$$\dot{x} = ax + bu.$$

a) What is the system state? What is the system state space?
b) What is the control input?
c) Given the initial condition $x(0) = x_0$ and a linear controller $\mu(x) = -kx$, find $x(t)$.
d) When does the feedback-controlled system in c) converge?

Problem 3

Consider an n-dimensional linear time-invariant system

$$\dot{x} = Ax + Bu.$$

a) What is the state space?
b) Suppose that $u \in \mathbb{R}^2$. What are the dimensions of matrices A and B?
c) Discretize the system into discrete time with step size δ. You need to express $x\big((k+1)\delta\big)$ in terms of $A, B, \delta, x(k\delta), u(k\delta)$. To make the notation easier to read, you can use $x[k+1]$ instead of $x\big((k+1)\delta\big)$ to denote the discrete-time state and express $x[k+1]$ in terms of $x[k]$; note that you still need to consider the impact of δ.
d) Suppose that we use a linear controller $\mu(x) = -Kx$. Write the difference equation for the discretized system; i.e., write how to obtain $x[k+1]$ from $x[k]$. Find $x[k]$ in terms of A, B, K and the initial condition $x[0] = x_0 \in \mathbb{R}^n$.
e) [**Bonus**] Suppose that $u \in \mathbb{R}^n$. What are the domain and the range for μ? Hint: the range will depend on the rank of K.