VE203 Discrete Math
Spring 2022 — Worksheet 1

February 14, 2022

Exercise 1.1 Set

How many elements does each of these sets have where a and b are distinct elements?

a) $\mathcal{P}(\{a, b, \{a, b\}\})$

b) $\mathcal{P}(\{\emptyset, a, \{a\}, \{\{a\}\}\})$

c) $\mathcal{P}(\mathcal{P}(\emptyset))$

Exercise 1.2 Set

Let $A = \{a, b, c\}, B = \{x, y\}$, and $C = \{0, 1\}$. Find

a) $A \times B \times C$.

b) $C \times B \times A$.

c) $C \times A \times B$.

d) $B \times B \times B$.

Exercise 1.3 Set
Let A, B, and C be sets. Show that
a) $(A - B) - C \subseteq A - C$.
b) $(A - C) \cap (C - B) = \emptyset$.

Exercise 1.4 Set
Show that if A is an infinite set, then whenever B is a set, $A \cup B$ is also an infinite set.

Exercise 1.5 Logic
Determine whether each of these conditional statements is true or false.
a) If $1 + 1 = 3$, then unicorns exist.
b) If $1 + 1 = 3$, then dogs can fly.
c) If $1 + 1 = 2$, then dogs can fly.
d) If $2 + 2 = 4$, then $1 + 2 = 3$.

Exercise 1.6 Logic
Is the assertion "This statement is false" a proposition?

Exercise 1.7 Logic

1. Find the negation of $\forall x \forall y \exists z \; A(x, y, z) \Rightarrow B(x, y, z)$
2. Show that $(\exists x(P(x) \Rightarrow Q(x))) \Leftrightarrow ((\forall x P(x)) \Rightarrow (\exists x Q(x)))$ is a tautology.

Exercise 1.8 Induction
Prove that for every positive integer n,

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1).$$

Reference

1. Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2012.

2. Chengjun Peng, Worksheet 1 for VE203