VE203 Discrete Math
Spring 2022 — Worksheet 3

February 14, 2022

Exercise 3.1 Poset
Which of these are posets?

a) $(\mathbf{Z}, =)$

b) $(\mathbf{Z}, \neq)$

c) $(\mathbf{Z}, \geq)$

d) $(\mathbf{Z}, \not|)$

e) $(\mathbf{R}, =)$

f) $(\mathbf{R}, <)$

g) $(\mathbf{R}, \leq)$

h) $(\mathbf{R}, \neq)$

Exercise 3.2 Partial Order
Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$.

a) Find the maximal elements.

b) Find the minimal elements.

c) Is there a greatest element?

d) Is there a least element?

e) Find all upper bounds of $\{2, 9\}$.

f) Find the least upper bound of $\{2, 9\}$, if it exists.

g) Find all lower bounds of $\{60, 72\}$.

h) Find the greatest lower bound of $\{60, 72\}$, if it exists.

Exercise 3.3 Lattice
Determine whether these posets are lattices.

a) $(\{1, 3, 6, 9, 12\}, 1)$

b) $(\{1, 5, 25, 125\}, |)$

c) $(\mathbf{Z}, \geq)$

d) $(P(S), \supseteq)$, where $P(S)$ is the power set of a set S

Exercise 3.4 Chain and Antichain

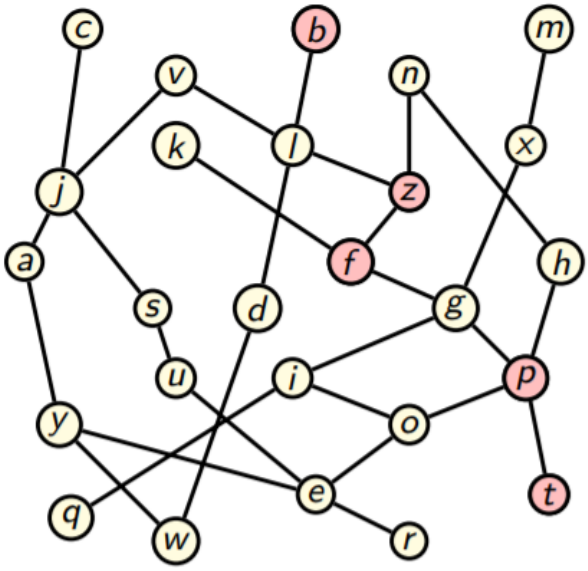

Figure 1: problem 3.4

a) Find a chain

b) Find an antichain

c) Find a maximal chain

d) Find a maximal antichain

Exercise 3.5 Cardinality
Give an example of two uncountable sets A and B such that $A \cap B$ is
a) finite.
b) countably infinite.
c) uncountable.

Exercise 3.6 Cardinality
Show that there is no infinite set A such that $|A| < |\mathbf{Z}^+| = \aleph_0$.

Reference

1. Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2012.

2. https://services.math.duke.edu/ lpereira/Teaching/ApCombSlides/3012_Lecture_18_handout.p