

VE203 Discrete Math

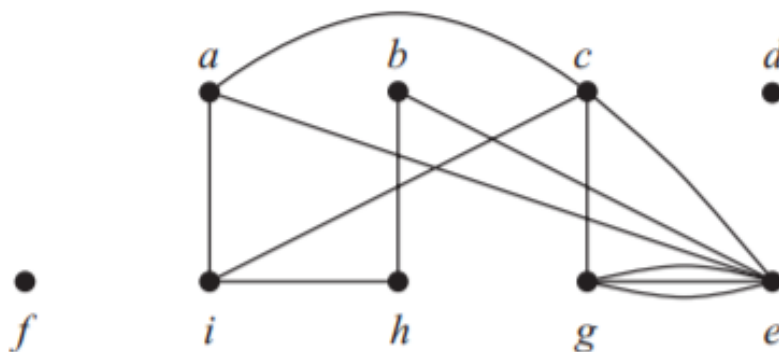
Spring 2022 — Worksheet 7

April 16, 2022



Exercise 7.1 Graph Definition

Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



Exercise 7.2 Graph Definition

How many vertices and how many edges do these graphs have?

- a) K_n
- b) C_n
- c) W_n
- d) $K_{m,n}$
- e) Q_n

Exercise 7.3 K-regular Graph

A simple graph is called regular if every vertex of this graph has the same degree. A regular graph is called n -regular if every vertex in this graph has degree n .

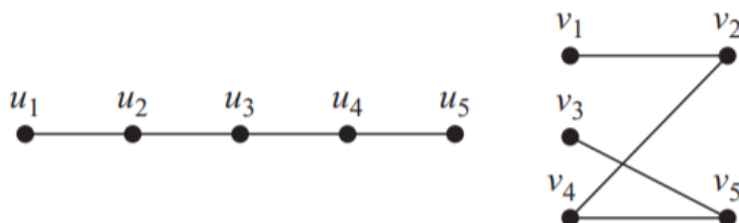
For which values of n are these graphs regular?

- a) K_n
- b) C_n
- c) W_n
- d) Q_n
- e) For which values of m and n is $K_{m,n}$ regular?

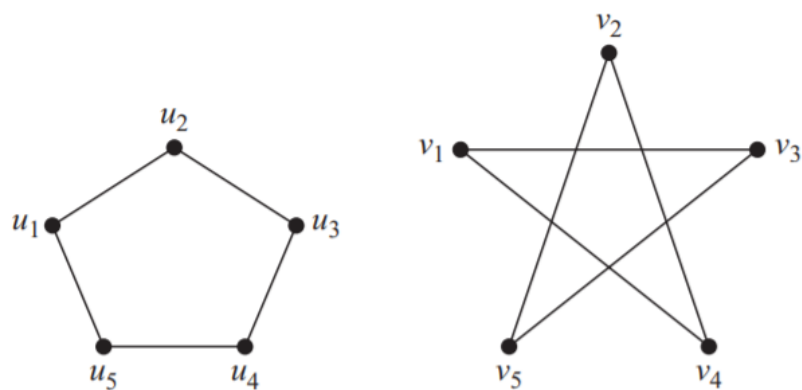
Exercise 7.4 Isomorphism

Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

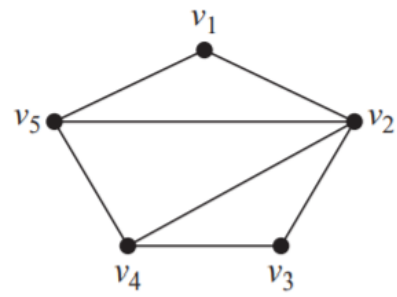
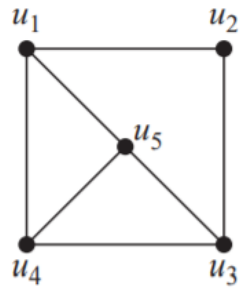
a)



b)



c)



Exercise 7.5 Handshaking Theorem

Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.

Exercise 7.6 Hall's Theorem

Suppose that there are five young women and six young men on an island. Each woman is willing to marry some of the men on the island and each man is willing to marry any woman who is willing to marry him. Suppose that Anna is willing to marry Jason, Larry, and Matt; Barbara is willing to marry Kevin and Larry; Carol is willing to marry Jason, Nick, and Oscar; Diane is willing to marry Jason, Larry, Nick, and Oscar; and Elizabeth is willing to marry Jason and Matt.

- a) Model the possible marriages on the island using a bipartite graph.
- b) Find a matching of the young women and the young men on the island such that each young woman is matched with a young man whom she is willing to marry.
- c) Is the matching you found in part (b) a complete matching? Is it a maximum matching?

Exercise 7.7 Subgraph

1. How many subgraphs with at least one vertex does K_2 have?
2. How many subgraphs with at least one vertex does K_3 have?
3. How many subgraphs with at least one vertex does W_3 have?

Exercise 7.8 Bipartition

Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq v^2/4$.

Reference

1. Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2012.
2. Fraleigh, John B. A first course in abstract algebra. Pearson Education India, 2003.