VE203 Discrete Math
Spring 2022 — Worksheet 4

March 25, 2022

Exercise 4.1 Divisibility

1. Show that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.
2. Show that if a, b, c, and d are integers, where $a \neq 0$, such that $a \mid c$ and $b \mid d$, then $ab \mid cd$.
3. Show that if a, b, and c are integers, where $a \neq 0$ and $c \neq 0$, such that $ac \mid bc$, then $a \mid b$.

Exercise 4.2 Prime

Find the prime factorization of 10!.

Exercise 4.3 Prime

How many zeros are there at the end of 100 !?

Exercise 4.4 Prime

The value of the Euler ϕ-function at the positive integer n is defined to be the number of positive integers less than or equal to n that are relatively prime to n. [Note: ϕ is the Greek letter phi.]

What is the value of $\phi\left(p^k\right)$ when p is prime and k is a positive integer?

Exercise 4.5 Euclidean algorithm

Use the Euclidean algorithm to find

1. $\gcd(1, 5)$.
2. $\gcd(100, 101)$.
3. $\gcd(123, 277)$.
4. $\gcd(1529, 14039)$.
5. $\gcd(1529, 14038)$.
6. $\gcd(11111, 111111)$.

Exercise 4.6 Prime

Adapt the proof in the text that there are infinitely many primes to prove that there are infinitely many primes of the form $3k + 2$, where k is a nonnegative integer. [Hint: Suppose that there are only finitely many such primes $q_1, q_2, \ldots, q_n$, and consider the number $3q_1q_2 \cdots q_n - 1$.]

Exercise 4.7 Goldbach's conjecture

Show that Goldbach's conjecture, which states that every even integer greater than 2 is the sum of two primes, is equivalent to the statement that every integer greater than 5 is the sum of three primes

Reference

1. Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2012.

2. Fraleigh, John B. A first course in abstract algebra. Pearson Education India, 2003.