VE203 Discrete Math
Spring 2022 — Worksheet 9

April 16, 2022

Exercise 9.1 Twelvefold Way

Consider the functions $f : B \to U$, count the number of functions and fill in the blanks below. Express the results in binomial coefficients, factorials, or powers (AVOID double bracket notation for first two rows).

Domain	Codomain	Any	Injective	Surjective
distinguishable	distinguishable			
indistinguishable	distinguishable			
distinguishable	indistinguishable			
indistinguishable	indistinguishable			

where
(i) $B = \{1, 2, 3\}$ and $U = \{1, 2, 3, 4, 5\}$.
(ii) $B = \{1, 2, 3, 4, 5\}$ and $U = \{1, 2, 3\}$.

Exercise 9.2 Multichoosing

Consider

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 100$$

What are the number of integer solutions if
(i) $x_i > 0$ and $=$ holds;
(ii) $x_i \geq 0$ and $=$ holds;
(iii) $x_i > 0$ and $<$ holds;
(iv) $x_i \geq 0$ and $<$ holds;
(v) $x_i \geq 0$.
AVOID double bracket notation in the final solution.

Exercise 9.3 Inclusion-Exclusion Principle

Find the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 = 17$, where $x_i, i = 1, 2, 3, 4$, are nonnegative integers such that $x_1 \leq 3, x_2 \leq 4, x_3 \leq 5$, and $x_4 \leq 8$.

Exercise 9.4 Derangement

What is the probability that none of 10 people receives the correct hat if a hatcheck person hands their hats back randomly?

Exercise 9.5 Derangement

A machine that inserts letters into envelopes goes haywire and inserts letters randomly into envelopes. What is the probability that in a group of 100 letters

a) no letter is put into the correct envelope?

b) exactly one letter is put into the correct envelope?

c) exactly 98 letters are put into the correct envelopes?

d) exactly 99 letters are put into the correct envelopes?

e) all letters are put into the correct envelopes?

Exercise 9.6 Master Method

Find the O or Θ bound of $T(n)$ for the following recurrence relation.

(i) $T(n) = 4T(n/4) + 5n$.

(ii) $T(n) = 4T(n/5) + 5n$.

(iii) $T(n) = 5T(n/4) + 4n$.

(iv) $T(n) = 4T(\sqrt{n}) + \log^5 n$

(v) $T(n) = 4T(\sqrt{n}) + \log^2 n$

Reference

1. Rosen, Kenneth H., and Kamala Krithivasan. Discrete mathematics and its applications: with combinatorics and graph theory. Tata McGraw-Hill Education, 2012.

2. Fraleigh, John B. A first course in abstract algebra. Pearson Education India, 2003.