VE230 Electromagnetics

Chapter 8

September 22, 2022

Exercise 8.1

Obtain the wave equations governing the **E** and **H** fields in a source-free conducting medium with constitutive parameters ϵ, μ, and σ.

Answer:
In a source-free simple medium, Eq. (7-53 b):

$$\bar{\nabla} \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + \epsilon \frac{\partial \bar{E}}{\partial t}$$

Eq. (7-53 a):

$$\bar{\nabla} \times \bar{\nabla} \times \bar{E} = \bar{\nabla}\bar{\nabla} \cdot \bar{E} - \bar{\nabla}^2 \bar{E} = -\mu \frac{\partial}{\partial t}(\bar{\nabla} \times \bar{H})$$

Substituting (1) in (2) and noting that $\bar{\nabla} \cdot \bar{E} = 0$:

$$\bar{\nabla}^2 \bar{E} - \mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\epsilon \frac{\partial^2 E}{\partial t^2} = 0.$$

Similarly for $\bar{H}$.

Exercise 8.2

Prove that the electric field intensity in Eq. (8-22) satisfies the homogeneous Helmholtz's equation provided that the condition in Eq. $(8-23)$ is satisfied.

Answer:
略

Exercise 8.3

A Doppler radar is used to determine the speed of a moving vehicle by measuring the frequency shift of the wave reflected from the vehicle.

a) Assuming that the reflecting surface of the vehicle can be represented by a perfectly conducting plane and that the transmitted signal is a time-harmonic uniform plane wave of a frequency f incident normally on the reflecting surface, find the relation between the frequency shift Δf and the speed u of the vehicle.

b) Determine u both in (km/hr) and in (miles /hr) if $\Delta f = 2.33$(kHz) with $f = 10.5$(GHz).

Answer:
Assume that the vehicle moves with a velocity u in the $+z$ direction, which is the direction of propagation of the incident wave.

a)

$$\bar{E}_i = \bar{a}_x E_0 e^{j(\omega t - kz)}$$

$$E_r = -\bar{a}_x E_0 e^{j'(\omega' t + k' z)}$$

$\bar{E}_i + \bar{E}_r = 0$ must be satisfied on reflecting surface for all t, $z = ut$

$$(\omega - k_u)\, t = (\omega' + k' u)\, t$$

$$\longrightarrow \omega' - \omega = -(k + k')\, u = -\left(\frac{\omega}{c} + \frac{\omega'}{c}\right) u = -(\omega + \omega')\frac{u}{c}$$

$$\longrightarrow \frac{\omega'}{\omega} = 1 - \frac{u}{c}\left(1 + \frac{\omega'}{\omega}\right).$$

$$\longrightarrow \frac{\omega'}{\omega} = \frac{f'}{f} = \frac{1 - u/c}{1 + u/c} \cong 1 - \frac{2u}{c}, \text{ for } u << c.$$

$$\longrightarrow \Delta f = f' - f = -\frac{2u}{c} f.$$

b) For $\Delta f = -2.33 \times 10^3(\text{ Hz})$ and $f = 10.5 \times 10^9(\text{ Hz})$: $u = 120(\text{ km/hr}) =$ 74.6(mi/hr).

Exercise 8.4

For a harmonic uniform plane wave propagating in a simple medium, both **E** and **H** vary in accordance with the factor $\exp(-j\mathbf{k} \cdot \mathbf{R})$ as indicated in Eq. (8-26). Show that the four Maxwell's equations for uniform plane wave in a source-free region reduce to the following:

$$\mathbf{k} \times \mathbf{E} = \omega\mu\mathbf{H}$$

$$\mathbf{k} \times \mathbf{H} = -\omega \in \mathbf{E}$$

$$\mathbf{k} \cdot \mathbf{E} = 0$$

$$\mathbf{k} \cdot \mathbf{H} = 0$$

Answer:

$$\text{Harmonic time dependence: } e^{j\omega t}; \quad \frac{\partial}{\partial t} \to j\omega.$$

Phasors: $\bar{E} = \bar{E}_0 e^{-j\bar{k}\bar{R}}$, $\quad \bar{H} = \bar{H}_0 e^{-j\bar{k}\cdot\bar{R}}$; where $\bar{E}_0$ and $\bar{H}_0$ are constant vectors

Now:

$$\bar{\nabla}\left(e^{-j\bar{k}\cdot\bar{R}}\right) = e^{-j\bar{k}\cdot\bar{R}} \bar{\nabla}(-j\bar{k}\bar{R}) = e^{-j\bar{k}\cdot\bar{R}}\left[-j\nabla\left(k_x x + k_y y + k_z z\right)\right]$$

$$= -j\left(\bar{a}_x k_x + \bar{a}_y k_y + \bar{a}_z k_z\right) e^{-j\bar{k}\cdot\cdot\bar{R}} = -j\bar{k} e^{-j\bar{k}\bar{R}}$$

Maxwell's equations:

$$\bar{\nabla} \times \bar{E} = \bar{\nabla}\left(e^{-j\bar{k}\bar{R}}\right) \times \bar{E}_0 = -j\omega\mu\bar{H} \longrightarrow \bar{k} \times \bar{E} = \omega\mu\bar{H}$$

$$\bar{\nabla} \times \bar{H} = \bar{\nabla}\left(e^{-j\bar{k}\bar{R}}\right) \times \bar{H}_0 = j\omega\epsilon\bar{E} \longrightarrow \bar{k} \times \bar{H} = -\omega\epsilon\bar{E}$$

$$\bar{\nabla} \cdot \bar{E} = \bar{\nabla}\left(e^{-j\bar{k}\cdot\bar{R}}\right) \cdot \bar{E}_0 = 0 \to \bar{k} \cdot \bar{E} = 0; \bar{\nabla} \cdot \bar{H} = \bar{\nabla}\left(e^{-jk\cdot\cdot\bar{R}}\right) \cdot \bar{H}_0 = 0 \to k \cdot \bar{H} = 0$$

Exercise 8.5

The instantaneous expression for the magnetic field intensity of a uniform plane wave propagating in the $+y$ direction in air is given by

$$\mathbf{H} = \mathbf{a}_z 4 \times 10^{-6} \cos\left(10^7 \pi t - k_0 y + \frac{\pi}{4}\right) \quad (\mathrm{A/m}).$$

a) Determine k_0 and the location where H_z vanishes at $t = 3(\text{ ms})$.
b) Write the instantaneous expression for $\mathbf{E}$.

Answer:

$$\bar{H} = \bar{a}_z 4 \times 10^{-6} \cos\left(10^7 \pi t - k_0 y + \frac{\pi}{4}\right) \quad (\mathrm{A/m})$$

a)

$$k_0 = \omega\sqrt{\mu_0 \epsilon_0} = \frac{10^7 \pi}{3 \times 10^8} = \frac{\pi}{30} = 0.105(\mathrm{rad/m})$$
$$\lambda = 2\pi / k_0 = 60(\text{ m})$$

At $t = 3 \times 10^{-3}(\text{ s})$, we require the argument of cosine in $\bar{H}$:

$$10^7 \pi \left(3 \times 10^{-3}\right) - \frac{\pi}{30} y + \frac{\pi}{4} = \pm n\pi + \frac{\pi}{2}, \quad M = 0, 1, 2, \cdots$$
$$\longrightarrow y = \pm 30n - 7.5(\text{ m}) = 22.5 \pm n\lambda/2(\text{ m}).$$

Exercise 8.6

The E-field of a uniform plane wave propagating in a dielectric medium is given by

$$E(t, z) = \mathbf{a}_x 2 \cos\left(10^8 t - z/\sqrt{3}\right) - \mathbf{a}_y \sin\left(10^8 t - z/\sqrt{3}\right) \quad (\mathrm{V/m}).$$

a) Determine the frequency and wavelength of the wave.
b) What is the dielectric constant of the medium?
c) Describe the polarization of the wave.
d) Find the corresponding **H**-field.

Answer:
Phasor:

$$\bar{E} = \bar{a}_x 2 e^{-jz/\sqrt{3}} + \bar{a}_y j e^{-jz/\sqrt{3}} \quad (V/m)$$

a)

$$\omega = 10^8 (\mathrm{rad/s}) \longrightarrow f = 10^8 / 2\pi = 1.59 \times 10^7 (\text{ Hz})$$

$$\beta = 1/\sqrt{3}(\mathrm{rad/m}) \longrightarrow \lambda = 2\pi/\beta = 2\sqrt{3}\pi(\mathrm{m})$$

b)

$$u = \frac{c}{\sqrt{\epsilon_r}} = \frac{\omega}{\beta} \longrightarrow \epsilon_r = \left(\frac{\beta c}{\omega}\right)^2 = 3$$

c) Left-hand elliptically polarized.

d)

$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \frac{120\pi}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{3}}$$

$$\bar{H} = \frac{1}{\eta}\bar{a}_z \times \bar{E} = \frac{\sqrt{3}}{120\pi}\left(\bar{a}_y e^{-jz/\sqrt{3}} - \bar{a}_x j e^{-jz/\sqrt{3}}\right),$$

$$\bar{H}(z,t) = \frac{\sqrt{3}}{120\pi}\left[\bar{a}_x \sin\left(10^8 t - z/\sqrt{3}\right) + \bar{a}_y \cos\left(10^8 t - z/\sqrt{3}\right)\right] \quad (A/m).$$

Exercise 8.7

Show that a plane wave with an instantaneous expression for the electric field

$$\mathbf{E}(z,t) = \mathbf{a}_x E_{10}\sin(\omega t - kz) + \mathbf{a}_y E_{20}\sin(\omega t - kz + \psi)$$

is elliptically polarized. Find the polarization ellipse.

Answer:
Let

$$\alpha = \omega t - kz. \quad \bar{E} = \bar{a}_x E_{10}\sin\alpha + \bar{a}_y E_{20}\sin(\alpha+\psi) = \bar{a}_x E_x + \bar{a}_y E_y$$

$$\frac{E_x}{E_{10}} = \sin\alpha, \quad \frac{E_y}{E_{20}} = \sin(\alpha+\psi) = \sin\alpha\cos\psi + \cos\alpha\sin\psi$$

$$= \frac{E_x}{E_{10}}\cos\psi + \sqrt{1 - \left(\frac{E_x}{E_{10}}\right)^2}\sin\psi.$$

$$\left(\frac{E_y}{E_{20}} - \frac{E_x}{E_{10}}\cos\psi\right)^2 = \left(1 - \frac{E_x}{E_{10}}\right)\sin^2\psi,$$

$$\left(\frac{E_y}{E_{20}\sin\psi}\right)^2 + \left(\frac{E_x}{E_{10}\sin\psi}\right)^2 - 2\frac{E_x E_y}{E_{10}E_{20}}\frac{\cos\psi}{\sin^2\psi} = 1 \qquad (1)$$

which is the equation of an ellipse. In order to find the parameters of the polarization ellipse, rotate the coordinate axes $x - y$ counterclockwise by an angle θ to $x' - y'$. Assume the equation of the ellipse in terms of the new coordinates to be

$$\left(\frac{E'_x}{a}\right)^2 + \left(\frac{E'_y}{b}\right)^2 = 1 \qquad (2)$$

where

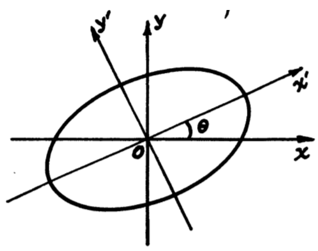
$$E'_x = E_x\cos\theta + E_y\sin\theta \qquad (3)$$

and

$$E'_y = -E_x\sin\theta + E_y\cos\theta \qquad (4)$$

Substituting (3) and (4) in (2) and rearranging:

$$E_x^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) + E_y^2\left(\frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2}\right) - 2E_xE_y\sin\theta\cos\theta\left(\frac{1}{b^2} - \frac{1}{a^2}\right) = 1 \qquad (5)$$



Comparing (1) and (5), we obtain

$$\begin{cases} \frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2} = \frac{1}{E_{10}^2 \sin^2\psi} & (6) \\ \frac{\sin^2\theta}{a^2} + \frac{\cos^2\theta}{b^2} = \frac{1}{E_{20}^2 \sin^2\psi} & (7) \\ \sin\theta\cos\theta\left(\frac{1}{b^2} - \frac{1}{a^2}\right) = \frac{\cos\psi}{E_{10}E_{20}\sin^2\psi} & (8) \end{cases}$$

Eqs. (6), (7), and (8) can be solved for three unknowns:

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{2E_{10}E_{20}\cos\psi}{E_{10}^2 - E_{20}^2}\right),$$

$$a = \sqrt{\frac{1}{\frac{1}{E_{10}^2}(1+\sec 2\theta) + \frac{1}{E_{20}^2}(1-\sec 2\theta)}}\sin\psi,$$

$$b = \sqrt{\frac{1}{\frac{1}{E_{10}^2}{}^2(1-\sec 2\theta) + \frac{1}{E_{20}^2}(1+\sec 2\theta)}}\sin\psi.$$

It particular, if $E_{10} = E_{20} = E_0 : \theta = 45°, a = \sqrt{2}E_0\cos\frac{\psi}{2}, b = \sqrt{2}E_0\sin\frac{\psi}{2}$.

Exercise 8.8

Prove the following:

a) An elliptically polarized plane wave can be resolved into right-hand and left-hand circularly polarized waves.

b) A circularly polarized plane wave can be obtained from a superposition of two oppositely directed elliptically polarized waves.

Answer:

Let an elliptically polarized plane wave be represented by the phasor (with propagation factor e^{-jkz} omitted):

a)

$$\bar{E} = \bar{a}_x E_1 \pm \bar{a}_y E_2 e^{j\alpha}$$

Where E_1, E_2, and α are arbitrary constants.

Right-hand circularly polarized wave:

$$\bar{E}_{rc} = E_{rc}\left(\bar{a}_x - \bar{a}_y j\right)$$

Left-hand circularly polarized wave:

$$\bar{E}_{lc} = E_{lc}\left(\bar{a}_x + \bar{a}_y j\right)$$

If

$$E_{rc} = \frac{1}{2}\left(E_1 \pm jE_2 e^{j\alpha}\right)$$

and

$$E_{lc} = \frac{1}{2}\left(E_1 \mp jE_2 e^{j\alpha}\right)$$

then

$$\bar{E} = \bar{E}_{rc} + \bar{E}_{lc}.$$

b) Right-hand circularly. polarized wave

$$\bar{E}_{rc} = E\left(\bar{a}_x - \bar{a}_y j\right)$$

$$= E\left(\bar{a}_x \frac{1}{2} - \bar{a}_y j2\right) + E\left(\bar{a}_x \frac{1}{2} + \bar{a}_y j\right)$$

$$= \bar{E}_{l+} + \bar{E}_{l-} \text{ where } \bar{E}_{l+} \text{ and } \bar{E}_{l-}$$

are right-hand and left-hand elliptically polarized waves respectively.
Similarly for a left-hand circularly polarized wave:

$$\begin{aligned} E_{lc} = E\left(\bar{a}_x + \bar{a}_y j\right) &= E\left(\bar{a}_x \frac{1}{2} + \bar{a}_y j2\right) + E\left(\bar{a}_x \frac{1}{2} - \bar{a}_y j\right) \\ &= \bar{E}'_{l-} + \bar{E}'_{l+}. \end{aligned}$$

Exercise 8.9

Derive the following general expressions of the attenuation and phase constants for conducting media:

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} \quad (\text{ Np/m}).$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2} \quad (\text{rad/m}).$$

Answer:
For conducting media: $k_c = \beta - j\alpha$.

$$\begin{aligned} k_c^2 &= \beta^2 - \alpha^2 - 2j\alpha\beta \\ &= \omega^2\mu\epsilon_c = \omega^2\mu\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right). \\ \therefore \quad \beta^2 - \alpha^2 &= \mathrm{Re}\left(k_c^2\right) = \omega^2\mu\epsilon, \\ \beta^2 + \alpha^2 &= \left|k_c^2\right| = \omega^2\mu\epsilon\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}. \end{aligned}$$

From (1) and (2) we obtain

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}, \quad \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}$$

Exercise 8.10

Determine and compare the intrinsic impedance, attentuation constant (in both Np/m and dB/m), and skin depth of copper [$\sigma_{\text{cu}} = 5.80 \times 10^7(\text{ S/m})$], silver [$\sigma_{\text{ag}} = 6.15 \times 10^7(\text{ S/m})$], and brass [$\sigma_{\text{br}} = 1.59 \times 10^7(\text{ S/m})$] at the following frequencies:
(a) 60(Hz),
(b) 1(MHz),
(c) 1(GHz).

Answer:
All three metals are good conductors, $\left(\frac{\sigma}{\omega\epsilon}\right)^2 > 1$

$$\alpha = \sqrt{\pi f \mu \sigma}, \quad \delta = \frac{1}{\alpha}, \quad \eta_c = (1+j)\frac{\alpha}{\sigma}.$$

a) $f = 60(\text{Hz})$

	$\eta_c(\Omega)$	$\alpha(Np/m)$	$\alpha(dB/m)$	$\delta(\text{m})$
Copper	$2.02(1+j) \times 10^{-6}$	0.117×10^3	1.02×10^3	8.53×10^{-3}
Silver	$2.08(1+j) \times 10^{-6}$	0.121×10^3	1.05×10^0	8.29×10^{-3}
Brass	$3.86(1+j) \times 10^{-6}$	0.061×10^0	0.53×10^3	16.3×10^{-3}

b) $f = 1(\text{MHz})$

	$\eta_c(\Omega)$	$\alpha\,(N_p/\text{m})$	$\alpha(d\text{ d/m})$	$\delta(\text{m})$
Copper	$2.61(1+j) \times 10^{-4}$	1.51×10^4	1.31×10^5	6.61×10^{-5}
Silver	$2.57(1+j) \times 10^{-4}$	1.58×10^4	1.35×10^5	6.32×10^{-5}
Brass	$4.98(1+j) \times 10^{-4}$	0.79×10^4	0.69×10^5	12.6×10^{-5}

c) $f = 1(\text{GHz})$

	$\eta_c(\Omega)$	$\alpha(\text{Np/m})$	$\alpha(\text{dB/m})$	$\delta(\text{m})$
Copper	$8.25(1+j) \times 10^{-3}$	4.79×10^5	4.16×10^6	2.09×10^{-6}
Silver	$8.01(1+j) \times 10^{-3}$	4.93×10^5	4.28×10^6	2.03×10^{-6}
Brass	$15.8(1+j) \times 10^{-3}$	2.51×10^5	2.18×10^6	3.99×10^{-6}

Exercise 8.11

A 3(GHz), y-polarized uniform plane wave propagates in the $+x$-direction in a nonmagnetic medium having a dielectric constant 2.5 and a loss tangent 10^{-2}.

a) Determine the distance over which the amplitude of the propagating wave will be cut in half.

b) Determine the intrinsic impedance, the wavelength, the phase velocity, and the group velocity of the wave in the medium.

c) Assuming $\mathbf{E} = \mathbf{a}_y 50 \sin(6\pi 10^9 t + \pi/3)\,(\text{V/m})$ at $x = 0$, write the instantaneous expression for $\mathbf{H}$ for all t and x.

Answer:

$$f=3\times 10^9(\text{ Hz}),\quad \epsilon_r=2.5,\quad \tan\delta_c=\frac{\epsilon''}{\epsilon'}=10^{-2}$$

a) Eq. (8-48):

$$\alpha=\frac{\omega\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}}=\frac{\omega}{2}\left(\frac{\epsilon''}{\epsilon'}\right)\frac{\sqrt{\epsilon_r}}{c}=0.497(\text{ N}p/\text{m})$$

$$e^{-\alpha x}=\frac{1}{2}\longrightarrow x=\frac{1}{\alpha}\ln 2=1.395(\text{ m})$$

b) Eq (8-50):

$$\eta_c=\frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}}\left(1+j\frac{\epsilon''}{2\epsilon'}\right)=238(1+j0.005)=238\underline{10.29^\circ}(\Omega)$$

Eq. (8-49):

$$\beta=\omega\sqrt{\mu\epsilon}\left[1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon}\right)^2\right]=31.6\pi(\text{rad/m})$$

$$\begin{aligned}
x&=\frac{2\pi}{\beta}=0.063(\text{ m}),\\
u_p&=\frac{\omega}{\beta}=1.8973\times 10^8(\text{ m/s}),\\
u_g&=\frac{1}{\frac{d\beta}{d\omega}}=\frac{c}{\sqrt{\epsilon_r}}\left[1+\frac{1}{8}\left(\frac{\epsilon''}{\epsilon'}\right)^2\right]=1.8975\times 10^8(\text{ m/s}).
\end{aligned}$$

c) At $x=0$,

$$\begin{aligned}
\bar{E}&=\bar{a}_y e^{j\pi/3}\\
\bar{H}&=\frac{1}{\eta_c}\bar{a}_x\times\bar{E}=\bar{a}_z 0.210 e^{j\left(\frac{\pi}{3}-0.0016\pi\right)}.\\
\bar{H}(x,t)&=\bar{a}_z 0.210 e^{-0.497x}\sin\left(6\pi 10^9 t-31.6\pi x+0.332\pi\right)(\text{A/m}).
\end{aligned}$$

Exercise 8.12

The magnetic field intensity of a linearly polarized uniform plane wave propagating in the $+y$-direction in seawater $[\epsilon_r=80,\mu_r=1,\sigma=4(\text{ S/m})]$ is

$$\mathbf{H}=\mathbf{a}_x 0.1\sin\left(10^{10}\pi t-\pi/3\right)\quad(\text{A/m})$$

at $y=0$.

a) Determine the attenuation constant, the phase constant, the intrinsic impedance, the phase velocity, the wavelength, and the skin depth.

b) Find the location at which the amplitude of H is $0.01(\text{ A/m})$.

c) Write the expressions for $\mathbf{E}(y,t)$ and $\mathbf{H}(y,t)$ at $y=0.5(\text{ m})$ as functions of t.

Answer:

$$\sigma/\omega\epsilon=4/10^{10}\pi\times 80\times\left(\frac{1}{36}\times 10^{-9}\right)=0.18$$

a)

$$\alpha=\omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega t}\right)^2}-1\right]^{1/2}=84(\text{ Np/m})$$

$$\beta=\omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega t}\right)^{2}}+1\right]^{1/2}=300\pi(\mathrm{rad/m})$$

$$\eta_c=\sqrt{\frac{\mu}{\epsilon_c}}=\frac{120\pi}{\sqrt{\epsilon_r}\left[1+(\sigma/\omega\epsilon)^2\right]^{1/4}}e^{\frac{j}{2}\tan^{-1}(\sigma/\omega\epsilon)}=41.8e^{j0.0283\pi}(\Omega)$$

$$u_p=\omega/\beta=33.3\times 10^{6}(\mathrm{~m/s}),\lambda=2\pi/\beta=0.67(\mathrm{~cm}),\delta=\frac{1}{\alpha}=1.19(\mathrm{~cm})$$

b)

$$e^{-\alpha y}=\frac{1}{10},\quad y=\frac{1}{\alpha}\ln 10=2.74(\mathrm{~cm})$$

c)

$$\begin{aligned}\bar{H}(y,t)&=\bar{a}_x 0.1e^{-84\times 0.5}\sin\left(10^{10}\pi t-300\pi\times 0.5-\pi/3\right)\\&=\bar{a}_x 5.75\times 10^{-20}\sin\left(10^{10}\pi t-\pi/3\right)\quad(A/\mathrm{m})\end{aligned}$$

$$\bar{E}(y,t)=\operatorname{Im}\left[\eta_c\bar{H}(y)\times\bar{a}_y\right]e^{j\omega t}=\bar{a}_z 2.41\times 10^{-18}\sin\left(10^{10}\pi t-\frac{\pi}{3}+0.0283\pi\right).(\mathrm{V/m})$$

Exercise 8.13

Given that the skin depth for graphite at 100(MHz) is 0.16(mm), determine (a) the conductivity of graphite, and (b) the distance that a 1(GHz) wave travels in graphite such that its field intensity is reduced by 30(dB).

Answer:
a)

$$\delta=\frac{1}{\sqrt{\pi f\mu\sigma}}\longrightarrow\sigma=\frac{1}{\pi f\mu\delta^2}=0.99\times 10^{5}(\mathrm{~s/m})$$

b)

$$\text{At } f=10^{9}(\mathrm{~Hz}),\quad\alpha=\sqrt{\pi f\mu\sigma}=1.98\times 10^{4}(\mathrm{~Np/m})$$

$$20\log_{10}e^{-\alpha z}=-30(dB)\longrightarrow z=\frac{1.5}{\alpha\log_{10}e}=1.75\times 10^{-4}(\mathrm{~m})=0.175(\mathrm{~mm})$$

Exercise 8.14

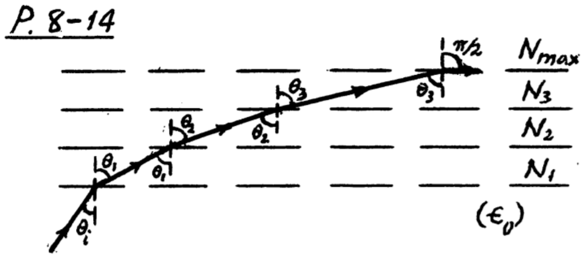
Assume the ionosphere to be modeled by a plasma region with an electron density that increases with altitude from a low value at the lower boundary toward a value $N_{\max}$ and decreases again as the altitude gets higher. A plane electromagnetic wave impinges on the lower boundary at an angle θ_i with the normal. Determine the highest frequency of the wave that will be turned back toward the earth. (Hint: Imagine the ionosphere to be stratified into layers of successively decreasing constant permittivities until the layer containing $N_{\max}$. The frequency to be determined corresponds to that for an emerging angle of $\pi/2$.)

Answer:
Assume the iono.sphere to be stratified into layers having electron densities $N_1<N_2<N_3<\cdots<N_{\max}$

The corresponding equivalent permittivities of the layers are:

$$\epsilon_n = \epsilon_0\left(1 - \frac{f_{pn}^2}{f^2}\right) \text{ with } f_{pn} = \frac{1}{2\pi}\sqrt{\frac{N_n e^2}{m\epsilon_0}}$$

and $\epsilon_i = \epsilon_0 > \epsilon_1 > \epsilon_2 > \epsilon_3 > \cdots > \epsilon_{\min}$ $\left(\begin{array}{l}\text{Corresponding} \\ \text{to } N_{max}\end{array}\right)$



From Snell's law of reflection:

$$\begin{aligned}
\sin\theta_1 &= \sin\theta_i\sqrt{\epsilon_0/\epsilon_i} = \sin\theta_i/\sqrt{1-(f_{p_1}/f)^2}, \\
\sin\theta_2 &= \sin\theta_1\sqrt{\epsilon_1/\epsilon_2} = \sin\theta_i\sqrt{\epsilon_0/\epsilon_2}, \\
\sin\theta_3 &= \sin\theta_2\sqrt{\epsilon_2/\epsilon_3} = \sin\theta_i\sqrt{\epsilon_0/\epsilon_3},
\end{aligned}$$

For total reflection at the layer with $\epsilon_{\text{min,}}$, the angle of refraction $\theta_{\max} = \pi/2$, and $\sin\theta_{\max} = 1 = \sin\theta_i\sqrt{\epsilon_0/\epsilon_{\min}}$.

$$\begin{aligned}
\therefore \quad \epsilon_{\min} &= \epsilon_0\left(1 - f_{p,\max}^2/f^2\right) = \epsilon_0\sin^2\theta_i. \\
\longrightarrow f &= f_{p_{\max}}/\cos\theta_i = 9\sqrt{N_{\max}}/\cos\theta_i.
\end{aligned}$$

Exercise 8.15

Prove the following relations between group velocity u_g and phase velocity u_p in a dispersive medium:

a) $u_g = u_p + \beta\frac{du_p}{d\beta}$

b) $u_g = u_p - \lambda\frac{du_p}{d\lambda}$

Answer:

a) From Eq (8-72):

$$u_g = \frac{d\omega}{d\beta} = \frac{d}{d\beta}(\beta u_p) = u_p + \beta\frac{du_p}{d\beta}$$

b)

$$\lambda = \frac{2\pi}{\beta}, \frac{d\lambda}{d\beta} = -\frac{2\pi}{\beta^2} = -\frac{\lambda}{\beta}$$

$$u_g = u_p + \beta\left(\frac{du_p}{d\lambda}\frac{d\lambda}{d\beta}\right) = u_\beta - \lambda\frac{du_p}{d\lambda}.$$

Exercise 8.16

There is a continuing discussion on radiation hazards to human health. The following calculations will provide a rough comparison.

a) The U.S. standard for personal safety in a microwave environment is that the power density be less than $10\,(\mathrm{mW/cm^2})$. Calculate the corresponding standard in terms of electric field intensity. In terms of magnetic field intensity.

b) It is estimated that the earth receives radiant energy from the sun at a rate of about $1.3\,(\mathrm{kW/m^2})$ on a sunny day. Assuming a monochromatic plane wave (which it is not), calculate the equivalent amplitudes of the electric and magnetic field intensity vectors.

Answer:

a)

$$P_{av} = |E|^2/2\eta_0 = 10^{-2}\left(\mathrm{W/cm^2}\right)$$

$$|E| = \sqrt{0.02\eta_0} = 2.75(\mathrm{V/cm}) = 275(\mathrm{V/m})$$

$$|H| = |E|/\eta_0 = 7.28 \times 10^{-3}(\mathrm{A/cm}) = 0.728(\mathrm{A/m})$$

b)

$$P_{av} = |E|^2/2\eta_0 = 1300\left(\mathrm{W/m^2}\right)$$

$$|E| = 990(\mathrm{V/m}), \quad |H| = 2.63(\mathrm{A/m})$$

Exercise 8.17

Show that the instantaneous Poynting vector of a circularly polarized plane wave propagating in a lossless medium is a constant that is independent of time and distance.

Answer:
Assume circularly polarized plane wave:

$$\bar{E}(z,t) = \bar{a}_x E_0 \cos(\omega t - kz + \phi) + \bar{a}_y E_0 \sin(\omega t - kz + \phi),$$

$$\bar{H}(z,t) = \bar{a}_y \frac{E_0}{\eta} \cos(\omega t - kz + \phi) - \bar{a}_x \frac{E_0}{\eta} \sin(\omega t - kz + \phi).$$

Poynting vector, $\bar{P} = \bar{E} \times \bar{H} = \bar{a}_z \frac{E_0^2}{\eta}\left[\cos^2(\omega t - kz + \phi) + \sin^2(\omega t - kz + \phi)\right] = \bar{a}_z \frac{E_0^2}{\eta}$, a constant independent of t and z.

Exercise 8.18

Assuming that the radiation electric field intensity of an antenna system is

$$\mathbf{E} = \mathbf{a}_\theta E_\theta + \mathbf{a}_\phi E_\phi,$$

find the expression for the average outward power flow per unit area.

Answer:

$$
\begin{aligned}
\bar{E} &= \bar{a}_\theta E_\theta + \bar{a}_\phi E_\phi \\
\bar{H} &= \frac{1}{\eta}\bar{a}_R \times \bar{E} = \frac{1}{\eta}\left(\bar{a}_\phi E_\theta - \bar{a}_\theta E_\phi\right) \\
\bar{P}_{av} &= \frac{1}{2}\operatorname{Re}\left(\bar{E} \times \bar{H}^*\right) = \bar{a}_z \frac{1}{\lambda\eta}\left(|E_\theta|^2 + |E_\phi|^2\right).
\end{aligned}
$$

Exercise 8.19

From the point of view of electromagnetics, the power transmitted by a lossless coaxial cable can be considered in terms of the Poynting vector inside the dielectric medium between the inner conductor and the outer sheath. Assuming that a d-c voltage V_0 applied between the inner conductor (of radius a) and the outer sheath (of inner radius b) causes a current I to flow to a load resistance, verify that the integration of the Poynting vector over the cross-sectional area of the dielectric medium equals the power $V_0 I$ that is transmitted to the load.

Answer:
From Gauss's law, $\bar{E} = \bar{a}_r \frac{\rho}{2\pi\epsilon r}$, where ρ is the line charge density on the inner conductor.

$$
V_D = -\int_b^a \bar{E} \cdot d\bar{r} = \frac{\rho}{2\pi\epsilon}\ln\left(\frac{b}{a}\right) \longrightarrow \bar{E} = \bar{a}_r \frac{V_0}{r\ln(b/a)}.
$$

From Ampère's circuital law,

$$
\bar{H} = \bar{a}_\phi \frac{I}{2\pi r}
$$

Poynting vector,

$$
\bar{\phi} = \bar{E} \times \bar{H} = \bar{a}_z \frac{V_0 I}{2\pi r^2 \ln(b/a)}
$$

Power transmitted over 'cross-sectional area:

$$
P = \int_S \bar{P} \cdot d\bar{s} = \frac{V_0 I}{2\pi \ln(b/a)} \int_0^{2\pi} \int_a^b \left(\frac{1}{r^2}\right) r dr d\phi = V_0 I.
$$

Exercise 8.20

A uniform plane electromagnetic wave propagates in the $+z-$ (downward) direction and impinges normally at $z = 0$ on an ocean surface. Let the magnetic field at $z = 0$ be $\mathbf{H}(0, t) = \mathbf{a}_y H_0 \cos 10^4 t(\text{ A/m})$.

a) Determine the skin depth. (For the ocean: Conductivity $= \sigma$, permeability $= \mu_0$.)
b) Find the expressions for $\mathbf{H}(z, t)$ and $\mathbf{E}(z, t)$.
c) Find the power loss per unit area (in terms of H_0) into the ocean.

Answer:
a)

$$
\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}}, f = 10^4/2\pi
$$

b)

$$
\bar{H}(z, t) = \bar{a}_y H_0 e^{-z/\delta} \cos\left(10^4 t - \frac{z}{\delta}\right)
$$

$$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{1}{\sigma\delta} = \frac{\sqrt{2}}{\sigma\delta}e^{j\pi/4}$$

$$\bar{E}(z,t) = \bar{a}_x \frac{\sqrt{2}}{\sigma\delta} H_0 e^{-z/\delta} \cos\left(10^4 t - \frac{z}{\delta} + \frac{\pi}{4}\right)$$

c)

$$\bar{P}_{av} = \frac{1}{2}\operatorname{Re}\left(\bar{E} \times \bar{H}^*\right) = \bar{a}_z \frac{1}{2}\frac{\sqrt{2}}{\sigma\delta} H_0^2 \cos\frac{\pi}{4}$$
$$= \bar{a}_z \frac{1}{2}\left(\frac{H_0^2}{\sigma\delta}\right) \quad \left(W/m^2\right)$$

Exercise 8.21

A right-hand circularly polarized plane wave represented by the phasor

$$\mathbf{E}(z) = E_0\left(\mathbf{a}_x - j\mathbf{a}_y\right)e^{-j\beta z}$$

impinges normally on a perfectly conducting wall at $z = 0$.

a) Determine the polarization of the reflected wave.

b) Find the induced current on the conducting wall.

c) Obtain the instantaneous expression of the total electric intensity based on a cosine time reference.

Answer:

Given

$$\bar{E}_i = E_0\left(\bar{a}_x - j\bar{a}_y\right)e^{-j\beta z}$$

a) Assume reflected

$$\bar{E}_r(z) = \left(\bar{a}_x E_{rx} + \bar{a}_y E_{ry}\right)e^{j\beta z}$$

Boundary condition at $z = 0$:

$$\bar{E}_i(0) + \bar{E}_r(0) = 0$$

$$\longrightarrow \bar{E}_r(z) = E_0\left(-\bar{a}_x + j\bar{a}_y\right)e^{j\beta z}$$

a left-hand circularly polarized wave in $-z$ direction.

b)

$$\bar{a}_{n2} \times \left(\bar{H}_1 - \bar{H}_2\right) = \bar{J}_s \longrightarrow -\bar{a}_z \times \left[\bar{H}_i(0) + \bar{H}_r(0)\right] = \bar{J}_s. \left(\bar{H}_2 = 0 \text{ in perfect conductor }\right)$$
$$\bar{H}_i(0) = \frac{1}{\eta_0}\bar{a}_z \times \bar{E}_i(0) = \frac{E_0}{\eta_0}\left(j\bar{a}_x + \bar{a}_y\right), \quad \bar{H}_r(0) = \frac{1}{\eta_0}\left(-\bar{a}_z\right) \times \bar{E}_r(0) = \frac{E_0}{\eta_0}\left(j\bar{a}_x + \bar{a}_y\right).$$
$$\bar{H}_1(0) = \bar{H}_i(0) + \bar{H}_r(0) = \frac{2\varepsilon_0}{\eta_0}\left(j\bar{a}_x + \bar{a}_y\right),$$
$$\bar{J}_s = -\bar{a}_z \times \bar{H}_1(0) = \frac{2E_0}{\eta_0}\left(\bar{a}_x - j\bar{a}_y\right).$$

$$c)\bar{E}_1(z,t) = \operatorname{Re}\left[\bar{E}_i(z) + \bar{E}_r(z)\right]e^{j\omega t}$$
$$= \operatorname{Re} E_0\left[\left(\bar{a}_x - j\bar{a}_y\right)e^{-j\beta z} + \left(-\bar{a}_x + j\bar{a}_y\right)e^{i\beta z}\right]e^{j\omega t}$$
$$= \operatorname{Re} E_0\left[-2j\left(\bar{a}_x - j\bar{a}_y\right)\sin\beta z\right]e^{j\omega t}$$
$$= 2E_0 \sin\beta z\left(\bar{a}_x \sin\omega t - \bar{a}_y \cos\omega t\right).$$

Exercise 8.22

A uniform sinusoidal plane wave in air with the following phasor expression for electric intensity

$$\mathbf{E}_i(x, z) = \mathbf{a}_y 10 e^{-j(6x+8z)} \quad \text{(V/m)}$$

is incident on a perfectly conducting plane at $z = 0$.

a) Find the frequency and wavelength of the wave.

b) Write the instantaneous expressions for $\mathbf{E}_i(x, z; t)$ and $\mathbf{H}_i(x, z; t)$, using a cosine reference.

c) Determine the angle of incidence.

d) Find $\mathbf{E}_r(x, z)$ and $\mathbf{H}_r(x, z)$ of the reflected wave.

e) Find $\mathbf{E}_1(x, z)$ and $\mathbf{H}_1(x, z)$ of the total field.

Answer:

Given

$$\bar{E}_i(x, z) = a_y 10 e^{-j(6x+8z)} \quad (V/m)$$

a)

$$k_x = 6, k_z = 8 \longrightarrow k = \beta = \sqrt{k_x^2 + k_z^2} = 10(\text{rad/m})$$

$$\lambda = 2\pi/k = 2\pi/10 = 0.628(\text{ m}); f = c/\lambda = 4.78 \times 10^8(\text{ Hz}); \omega = kc = 3 \times 10^9(\text{rad/s})$$

b)

$$\bar{E}_i(x, z; t) = \bar{a}_y 10 \cos\left(3 \times 10^9 t - 6x - 8z\right) (\text{V/m})$$

$$\begin{aligned}
\bar{H}_i(x, z) &= \frac{1}{\eta_0} \bar{a}_{ni} \times \bar{E}_i \vdots \bar{a}_{ni} = \frac{\bar{k}}{k} = \bar{a}_x 0.6 + \bar{a}_z 0.8. \\
&= \frac{1}{120\pi} \left(\bar{a}_x 0.6 + \bar{a}_z 0.8\right) \times \bar{a}_y 10 e^{-j(6x+8z)} = \left(-\bar{a}_x \frac{1}{15\pi} + \bar{a}_z \frac{1}{20\pi}\right) e^{-j(6x+8z)} \\
\bar{H}_i(x, z; t) &= \left(-\bar{a}_x \frac{1}{15\pi} + \bar{a}_z \frac{1}{20\pi}\right) \cos\left(3 \times 10^9 t - 6x - 8z\right) \quad (\text{A/m}).
\end{aligned}$$

c)

$$\cos\theta_i = \bar{a}_{ni} \cdot \bar{a}_z = \left(\frac{\bar{k}}{k}\right) \cdot \bar{a}_z = 0.8 \longrightarrow \theta_i = \cos^{-1} 0.8 = 36.9^\circ$$

d)

$$\begin{aligned}
\bar{E}_i(x, 0) + \bar{E}_r(x, 0) = 0 &\longrightarrow \bar{E}_r(x, z) = -\bar{a}_y 10 e^{-j(6x-8z)} \\
\bar{H}_r(x, z) &= \frac{1}{\eta_0} \bar{a}_{nr} \times \bar{E}_r(x, z) \\
&= -\left(\bar{a}_x \frac{1}{15\pi} + \bar{a}_z \frac{1}{20\pi}\right) e^{-j(6x-8z)}.
\end{aligned}$$

e)

$$\begin{aligned}
\bar{E}_1(x, z) &= \bar{E}_i(x, z) + \bar{E}_r(x, z) = \bar{a}_y 10 \left(e^{-j8z} - e^{j8z}\right) e^{-j6x} \\
&= -\bar{a}_y j 20 e^{-j6x} \sin 8z (V/m). \\
\bar{H}_1(x, z) &= \bar{H}_i(x, z) + \bar{H}_r(x, z) = -\left(\bar{a}_x \frac{2}{15\pi} \cos 8z + \bar{a}_z \frac{j}{10\pi} \sin 8z\right) e^{-j6x} (\text{ A/m}).
\end{aligned}$$

Exercise 8.23

Repeat Problem P.8-22 for $\mathbf{E}_i(y,z) = 5\left(\mathbf{a}_y + \mathbf{a}_z\sqrt{3}\right) e^{j6(\sqrt{3}y-z)}(\mathrm{V/m})$.

Answer:
Given

$$\bar{E}_i(y,z) = 5\left(\bar{a}_y + \bar{a}_z\sqrt{3}\right) e^{j6(\sqrt{3}y-z)} \quad (\mathrm{V/m})$$

a)

$$k_y = -6\sqrt{3}, k_z = 6 \longrightarrow k = \sqrt{k_y^2 + k_z^2} = 12(\mathrm{rad/m})$$

$$\lambda = 2\pi/k = \pi/6 = 0.524(\mathrm{~m}); f = c/\lambda = 5.73 \times 10^8 (Hz); \omega = kc = 3.60 \times 10^9 (rad/s)$$

b)

$$\bar{E}_i(y,z;t) = 5\left(\bar{a}_y + \bar{a}_z\sqrt{3}\right) \cos\left(3.60 \times 10^9 t + 6\sqrt{3}y - 6z\right) (\mathrm{v/m})$$

$$\begin{aligned}\bar{H}_i(y,z) &= \frac{1}{\eta_0}\bar{a}_{ni} \times \bar{E}_i = \frac{1}{120\pi}\left(-\bar{a}_y\frac{\sqrt{3}}{2} + \bar{a}_z\frac{1}{2}\right) \times 5\left(\bar{a}_y + \bar{a}_z\sqrt{3}\right) e^{j6(\sqrt{3}y-z)} \\ &= \bar{a}_x\left(-\frac{1}{12\pi}\right) e^{j6(\sqrt{3}y-z)} \\ \bar{H}_i(y,z;t) &= \bar{a}_x\left(-\frac{1}{12\pi}\right) \cos\left(3.60 \times 10^9 t + 6\sqrt{3}y - 6z\right) \quad (\mathrm{A/m}).\end{aligned}$$

c)

$$\cos\theta_i = \bar{a}_{ni} \cdot \bar{a}_z = \frac{1}{2} \longrightarrow \theta_i = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

d) Conditions $\bar{a}_{nr} \cdot \bar{E}_r(y,z) = 0$ and $E_{iy}(y,0) + E_{ry}(y,0) = 0$ lead to:

$$E_r(y,z) = 5\left(-\bar{a}_y + \bar{a}_z\sqrt{3}\right) e^{j6(\sqrt{3}y+z)} \quad (V/m)$$

$$\begin{aligned}\bar{H}_t(y,z) &= \frac{1}{\eta_0}\bar{a}_{nr} \times \bar{E}_r(y,z) = \frac{1}{120\pi}\left(-\bar{a}_y\frac{\sqrt{3}}{2} - \bar{a}_z\frac{1}{2}\right) \times 5\left(-\bar{a}_y + \bar{a}_z \cdot \sqrt{3}\right) e^{j6(\sqrt{3}y+z)} \\ &= \bar{a}_x\left(-\frac{1}{12\pi}\right) e^{j6(\sqrt{3}y+z)} \quad (A/m).\end{aligned}$$

e)

$$\bar{E}_1(y,z) = \bar{E}_i(y,z) + \bar{E}_r(y,z) = \left(-\bar{a}_y j10\sin 6z + \bar{o}_z 40\sqrt{3}\cos 6z\right) e^{j6\sqrt{3}y} (\mathrm{~V/m})$$

$$\bar{H}_1(y,z) = \bar{H}_i(y,z) + \bar{H}_r(y,z) = \bar{a}_x\left(-\frac{1}{6\pi}\right)\cos 6z \cdot e^{j6\sqrt{3}y} \quad (A/m)$$

Exercise 8.24

For the case of oblique incidence of a uniform plane wave with perpendicular polarization on a perfectly conducting plane boundary as shown in Fig. 8-11, write (a) the instantaneous expressions

$$\mathbf{E}_1(x, z; t) \quad \text{and} \quad \mathbf{H}_1(x, z; t)$$

for the total field in medium 1, using a cosine reference, and (b) the time-average Poynting vector.

Answer:
a) From Eqs. $(8-113)$ and $(8-114)$:

$$\bar{E}_1(x, z; t) = \bar{a}_y 2E_{i0} \sin(\beta, z\cos\theta_i) \sin(\cos t - \beta_1 x \sin\theta_i)$$

$$\bar{H}_1(x, z; t) = \frac{2E_{i0}}{\eta_t} [-\bar{a}_x \cos\theta_i \cos(\beta_1 z\cos\theta_i) \cos(\cos t - \beta_1 x\sin\theta_i)$$

$$+\bar{a}_z \sin\theta_i \sin(\beta_1 z\cos\theta_i) \sin(\omega t - \beta_1 x\sin\theta_i)]$$

b)

$$\bar{P}_{av} = \frac{1}{2}\operatorname{Re}\left(\bar{E}\times\bar{H}^*\right) = \bar{a}_x \frac{2\epsilon_{i0}^*}{\eta_1} \sin\theta_i \sin^2(\beta_1 z\cos\theta_i)$$

Exercise 8.25

For the case of oblique incidence of a uniform plane wave with parallel polarization on a perfectly conducting plane boundary as shown in Fig. 8-13, write (a) the instantaneous expressions

$$\mathbf{E}_1(x, z; t) \quad \text{and} \quad \mathbf{H}_1(x, z; t)$$

for the total field in medium 1 , using a sine reference, and (b) the time-average Poynting vector.

Answer:
a) From Eqs. $(8-128)$ and $(8-129)$:

$$\bar{E}_1(x, z; t) = -2E_{i0}[\bar{a}_x \cos\theta_i \sin(\beta_1 z\cos\theta_i) \cos(\omega t - \beta_1 x\sin\theta_i)$$
$$+\bar{a}_z \sin\theta_i \cos(\beta_1 z\cos\theta_i) \sin(\omega t - \beta_1 x\sin\theta_i)],$$
$$\bar{H}_1(x, z; t) = \bar{a}_y \frac{2\epsilon_{i0}}{\eta_1} \cos(\beta, z\cos\theta_i) \sin(\omega t - \beta_1 x\sin\theta_i).$$

b)

$$\bar{P}_{av} = \frac{1}{2}\operatorname{Re}\left(\bar{E}\times\bar{H}^*\right) = \bar{a}_x \frac{2E_{i0}^2}{\eta_1} \sin\theta_i \cos^2(\beta_1 z\cos\theta_i)$$

Exercise 8.26

Determine the condition under which the magnitude of the reflection coefficient equals that of the transmission coefficient for a uniform plane wave at normal incidence on an interface between two lossless dielectric media. What is the standing-wave ratio in dB under this condition?

Answer:
For normal incidence: $1+\Gamma=\tau$, where $|\Gamma|\leq 1$.
If $|\tau|=|\Gamma|:\Gamma<0$ and $\eta_1-\eta_2=2\eta_2\rightarrow\eta_1=3\eta_2\rightarrow|\Gamma|=\frac{1}{2}$.

$$\therefore S=\frac{1+|\Gamma|}{1-|\Gamma|}=3 \quad S_{dB}=20\log_{10}3=9.54(dB)$$

Exercise 8.27

A uniform plane wave in air with $\mathbf{E}_i(z)=\mathbf{a}_x 10e^{-j6z}(\text{ V/m})$ is incident normally on an interface at $z=0$ with a lossy medium having a dielectric constant 2.5 and a loss tangent 0.5. Find the following:

a) The instantaneous expressions for $\mathbf{E}_r(z,t),\mathbf{H}_r(z,t),\mathbf{E}_t(z,t)$, and $\mathbf{H}_t(z,t)$, using a cosine reference.

b) The expressions for time-average Poynting vectors in air and in the lossy medium.

Answer:
a) In the lossy medium (medium 2):

$$\bar{E}_t=\bar{a}_x E_{t0}e^{-\alpha_2 z}e^{-j\beta_2 z},$$

where from Problem $8-9$
$$\alpha_2=\omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}-1\right]^{\frac{1}{2}}, \quad \beta_2=\omega\sqrt{\frac{\mu_2\epsilon_2}{2}}\left[\sqrt{1+\left(\frac{\sigma_2}{\omega\epsilon_2}\right)^2}+1\right]^{\frac{1}{2}}$$

$$\text{Given : }\beta_1=6(\text{rad/m})\longrightarrow\omega=\beta_1 c=1.8\times 10^9(\text{rad/s})$$

$$\tan\delta_c=\frac{\sigma_2}{\omega\epsilon_2}=0.5\longrightarrow\alpha_2=2.30(\text{ Np/m}),\beta_2=9.76(\text{rad/m})$$

$$\eta_2=\sqrt{\frac{\mu_2}{\epsilon_2}}=\frac{120\pi}{\sqrt{\epsilon_{r2}}\left(1+\tan^2\delta_c\right)^{1/4}}e^{j\frac{1}{2}\tan^{-1}(\sigma_2/\omega\epsilon_2)}=225e^{j13.3^\circ}.$$

$$\bar{E}_t=\bar{a}_x E_{t0}e^{-2.30z}e^{-j9.76z}, \quad \bar{H}_t=\bar{a}_z\times\frac{\bar{E}_t}{\eta}=\bar{a}_y\frac{E_{t0}}{225}e^{-j13.3^0-2.30z}e^{-j4.76z}$$

Let

$$\bar{E}_r=\bar{a}_x E_{r0}e^{j6z}\longrightarrow\bar{H}_p=-\bar{a}_y\frac{E_{ro}}{120\pi}e^{j6z}\cdot\bar{H}_i=\bar{a}_y\frac{10}{120\pi}e^{-j6z}$$

Boundary conditions

$$\text{for }\bar{E}\text{ and }\bar{H}\text{ at }z=0:\begin{cases}10+E_{r0}=E_{t_0},\\10-E_{r0}=E_{t0}\sqrt{\epsilon_{r2}}\left(1+\tan^2\delta_c\right)^{1/4}e^{-j13.3^\circ}\end{cases}$$

$$\longrightarrow E_{r0}=2.77e^{j157^\circ};\quad E_{t0}=7.53e^{-j172^\circ}$$

$$\begin{aligned}\therefore\quad &\bar{E}_r(z,t)=\bar{a}_x 2.77\cos\left(1.8\times 10^4 t+6z+157^\circ\right)(\text{v/m}),\\&\bar{H}_r(z,t)=-\bar{a}_y 0.073\cos\left(1.8\times 10^9 t+6z+157^\circ\right)(\text{A/m}),\\&\bar{E}_t(z,t)=\bar{a}_x 7.53e^{-2.30z}\cos\left(1.8\times 10^9 t-9.76z-172^\circ\right)(\text{V/m}),\\&\bar{H}_t(z,t)=\bar{a}_y 0.033\mathrm{e}^{-2.30z}\cos\left(1.8\times 10^9 t-9.76z+174.7^\circ\right)(\text{A/m}).\end{aligned}$$

b)

$$\left(\bar{P}_{av}\right)_1 = \bar{a}_z \left(\frac{10^2}{2 \times 120\pi} - \frac{2.77^2}{2 \times 120\pi} \right) = \bar{a}_z 0.122 \cdot \left(\mathrm{~W/m^2} \right)$$

$$\left(\bar{P}_{av}\right)_2 = \bar{a}_z \frac{7.53^2}{2 \times 225} \left(\cos 13.3^{\circ}\right) e^{-4.60z} = \bar{a}_z 0.122 e^{-4.60z} \left(\mathrm{~W/m^2} \right)$$

Exercise 8.28

A uniform plane wave in air with $\mathbf{E}_i(z) = \mathbf{a}_x E_0 \exp\left(-j\beta_0 z\right)$ impinges normally onto the surface at $z = 0$ of a highly conducting medium having constitutive parameters ϵ_0, μ, and $\sigma\left(\sigma/\omega\epsilon_0 \gg 1\right)$.

a) Find the reflection coefficient.

b) Derive the expression for the fraction of the incident power absorbed by the conducting medium.

c) Obtain the fraction of the power absorbed at 1(MHz) if the medium is iron.

Answer:

a)

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_c - \eta_0}{\eta_c + \eta_0}; \quad \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 120\pi, \eta_c = \sqrt{j\omega\mu/\sigma}$$

b)

$$|\Gamma|^2 = \left|\frac{\eta_c - \eta_0}{\eta_c + \eta_0}\right|^2 = \left|\frac{1 - \eta_c/\eta_0}{1 + \eta_c/\eta_0}\right|^2 \cong \left|1 - 2\eta_c/\eta_0\right|^2$$

$$= \left(1 - 2\eta_c/\eta_0\right)\left(1 - 2\eta_c^*/\eta_0\right) \cong 1 - 4R_c\left(\eta_c\right)/\eta_0$$

Fraction of power absorbed,

$$F = 1 - |\Gamma|^2 = \frac{4}{\eta_0} R_e \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$= \frac{4}{\eta_0} \sqrt{\frac{\omega\mu}{2\sigma}}$$

c) $\omega = 2\pi \times 10^6 \left(\mathrm{~Hz_z} \right)$.

$$\text{For iron: } \mu = 4,000 \times \left(4\pi 10^{-7}\right)(H/m),$$

$$\sigma = 10^7 (\mathrm{~S/m})$$

$$F = 4.21 \times 10^{-4}, \text{ or } 0.0421\%$$

Exercise 8.29

Consider the situation of normal incidence at a lossless dielectric slab of thickness d in air, as shown in Fig. 8-15 with

$$\epsilon_1 = \epsilon_3 = \epsilon_0 \quad \text{and} \quad \mu_1 = \mu_3 = \mu_0$$

a) Find E_{r0}, E_2^+, E_2^-, and E_{t0} in terms of E_{i0}, d, ϵ_2, and μ_2.

b) Will there be reflection at interface $z = 0$ if $d = \lambda_2/4$? If $d = \lambda_2/2$? Explain.

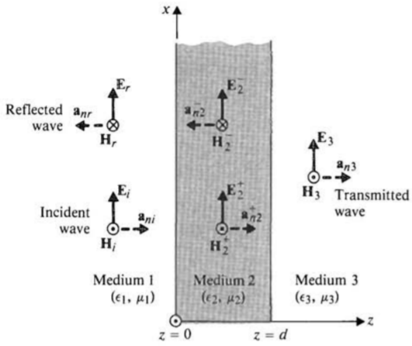


FIGURE 8–15
Normal incidence at multiple dielectric interfaces.

Answer:
From Eqs. $(8-156)$ through $(8-161)$:

$$
\begin{aligned}
\bar{E}_1 &= \bar{a}_x\left(E_{i0}e^{-j_0\beta_0 z}+E_{r0}e^{j\beta_0 z}\right), \quad & \bar{H}_1 &= \bar{a}_y\tfrac{1}{\eta_0}\left(E_{i0}e^{-j\beta_0 z}-E_{r0}e^{j_0\beta_0 z}\right),\\
\bar{E}_2 &= \bar{a}_x\left(E_2^{+}e^{-j\beta_2 z}+E_2^{-}e^{j\beta_2 z}\right), & \bar{H}_2 &= \bar{a}_y\tfrac{1}{\eta_2}\left(E_2^{+}e^{-j'\beta_2 z}-E_2^{-}e^{j\beta_2 z}\right),\\
\bar{E}_t &= \bar{a}_x E_{t0}e^{-j\beta_0 z}, & \bar{H}_t &= \bar{a}_y\tfrac{1}{\eta_0}e^{-j\beta_0 z}.
\end{aligned}
$$

a)

$$
\begin{aligned}
E_{r0} &= -\tfrac{j\left(\eta_0^2-\eta_2^2\right)t_{an}\beta_2 d}{\eta_0\eta_2+j\left(\eta_0^2+\eta_2^2\right)\tan_2\beta_2 d}E_{i0}, & &\text{where}\\
E_2^{+} &= \tfrac{\eta_2(\eta_0+\eta_2)e^{j\beta_2 d}}{\eta_0\eta_2\cos\beta_2 d+j\left(\eta_0^2+\eta_2^2\right)\sin\beta_2 d}E_{i0}, & &\eta_0=\sqrt{\tfrac{\mu_0}{\epsilon_0}}=120\pi,\\
E_2^{-} &= \tfrac{\eta_2(\eta_0-\eta_2)e^{-j\beta_2 d}}{\eta_0\eta_2\cos\beta_2 d+j\left(\eta_0^2+\eta_2^2\right)\sin\beta_2 d}E_{i0}, & &\eta_2=\sqrt{\tfrac{\mu_2}{\epsilon_2}},\\
E_{t0} &= \tfrac{2\eta_0\eta_2 e^{j\beta_0 d}}{\eta_0\eta_2\cos\beta_2 d+j\left(\eta_0^2+\eta_2^2\right)\sin\beta_2 d}E_{i0}, & &\beta_0=\tfrac{\omega}{c},\\
& & &\beta_2=\omega\sqrt{\mu_2\epsilon_2}.
\end{aligned}
$$

b) If $d=\lambda_2/4, \quad \beta_2 d=\pi/2, \quad E_{r0}=-\frac{\eta_0^2-\eta_2^2}{\eta_0^2+\eta_2^2}E_{i0}$.
$\therefore \Gamma=-\frac{\eta_0^2-\eta_2^2}{\eta_0^2+\eta_2^2}\neq 0$ unless $\eta_2=\eta_0$ (a trivial case).
If $d=n\lambda_2/2 \quad (n=1,2,3,\cdots), \tan\beta_2 d=0\rightarrow\Gamma=0$.

Exercise 8.30

A transparent dielectric coating is applied to glass $(\epsilon_r=4,\mu_r=1)$ to eliminate the reflection of red light $[\lambda_0=0.75(\mu\text{m})]$.

a) Determine the required dielectric constant and thickness of the coating.

b) If violet light $[\lambda_0=0.42(\mu\text{m})]$ is shone normally on the coated glass, what percentage of the incident power will be reflected?

Answer:
From Example 8-12:

$$\eta_2=\sqrt{\eta_1\eta_3}\longrightarrow\epsilon_{2r}=\sqrt{\epsilon_{r1}\epsilon_{r3}}=2$$

a) Wavelength of red light:

$$\lambda_2=\frac{0.75}{\sqrt{\epsilon_{r2}}}=0.530(\mu\text{m})\rightarrow d=\frac{\lambda_2}{4}=0.133(\mu\text{m})$$

b) For violet light:

$$\lambda_2' = \frac{0.42}{\sqrt{2}} = 0.297(\mu\text{m})$$

$$\frac{d^2}{\lambda_2'} = 0.447 \longrightarrow \beta_2 d = 0.894\pi.$$

From Eq. (8-173) and using impedances normalized with respect to $\eta_1 = \eta_0$:

$$Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d} = \frac{1}{\sqrt{2}} \frac{\frac{1}{2} + \frac{j}{\sqrt{2}} \tan\beta_2 d}{\frac{1}{\sqrt{2}} + \frac{j}{2} \tan\beta_2 d} = \frac{0.5 - j0.247}{1.0 - j0.247}$$

$$\Gamma = \frac{z_2(0) - 1}{z_2(0) + 1} = 0.317e^{j198^\circ}.$$

Percentage of power reflected $= |\Gamma|^2 \times 100\%$

$$= (0.317)^2 \times 100\% = 10\%$$

Exercise 8.31

Refer to Fig. 8-15, which depicts three different dielectric media with two parallel interfaces. A uniform plane wave in medium 1 propagates in the $+z$-direction. Let Γ_{12} and Γ_{23} denote the reflection coefficients between media 1 and 2 and between media 2 and 3 , respectively. Express the effective reflection coefficient, Γ_0, at $z = 0$ for the incident wave in terms of Γ_{12}, Γ_{23}, and $\beta_2 d$.

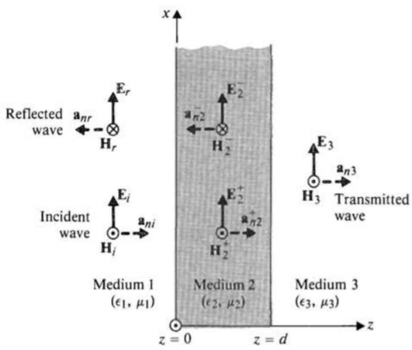


FIGURE 8-15
Normal incidence at multiple dielectric interfaces.

Answer:

$$\Gamma_0 = \frac{Z_2(0) - \eta_1}{Z_2(0) + \eta_1}, Z_2(0) = \eta_2 \frac{\eta_3 + j\eta_2 \tan\beta_2 d}{\eta_2 + j\eta_3 \tan\beta_2 d}.$$

$$\Gamma_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}. \longrightarrow \frac{\eta_1}{\eta_2} = \frac{1 - \Gamma_{12}}{1 + \Gamma_{12}}.$$

$$\Gamma_{23} = \frac{\eta_3 - \eta_2}{\eta_3 + \eta_2}. \longrightarrow \frac{\eta_2}{\eta_3} = \frac{1 - \Gamma_{23}}{1 + \Gamma_{23}}.$$

$$
\begin{aligned}
\therefore \Gamma_0 &= \frac{1 + j\frac{\eta_2}{\eta_3}\tan\beta_2 d - \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3} + j\tan\beta_2 d\right)}{1 + j\frac{\eta_2}{\eta_3}\tan\beta_2 d + \frac{\eta_1}{\eta_2}\left(\frac{\eta_2}{\eta_3} + j\tan\beta_2 d\right)} \\
&= \frac{1 + j\frac{1-\Gamma_{23}}{1+\Gamma_{23}}\tan\beta_2 d - \frac{1-\Gamma_{12}}{1+\Gamma_{12}}\left(\frac{1-\Gamma_{23}}{1+\Gamma_{23}} + j\tan\beta_2 d\right)}{1 + j\frac{1-\Gamma_{23}}{1+\Gamma_{23}}\tan\beta_2 d + \frac{1-\Gamma_{12}}{1+\Gamma_{12}}\left(\frac{1-\Gamma_{23}}{1+\Gamma_{23}} + j\tan\beta_2 d\right)} \\
&= \frac{(\Gamma_{12} + \Gamma_{23}) + j(\Gamma_{12} - \Gamma_{23})\tan\beta_2 d}{(1 + \Gamma_{12}\Gamma_{23}) + j(1 - \Gamma_{12}\Gamma_{23})\tan\beta_2 d}.
\end{aligned}
$$

Exercise 8.32

A uniform plane wave with

$$
\mathbf{E}_i(z,t) = \mathbf{a}_x E_{i0}\cos\omega\left(t - \frac{z}{u_p}\right)
$$

in medium 1 (ϵ_1, μ_1) is incident normally onto a lossless dielectric slab (ϵ_2, μ_2) of a thickness d backed by a perfectly conducting plane, as shown in Fig. 8-22. Find

a) $\mathbf{E}_r(z,t)$
b) $\mathbf{E}_1(z,t)$
c) $\mathbf{E}_2(z,t)$
d) $(\mathscr{P}_{\text{av}})_1$
e) $(\mathscr{P}_{\text{av}})_2$
f) Determine the thickness d that makes $\mathbf{E}_1(z,t)$ the same as if the dielectric slab were absent.

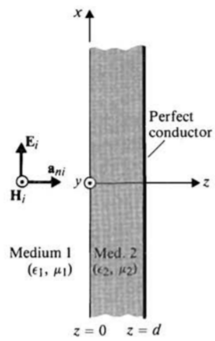


FIGURE 8–22
Plane wave incident normally onto a dielectric slab backed by a perfectly conducting plane (Problem P.8–32).

Answer:

$$
\begin{aligned}
\bar{E}_1 &= \bar{a}_x\left(E_{i0}e^{-j\beta_1 z} + E_{ro}e^{j\beta_1 z}\right) \\
\bar{H}_1 &= \bar{a}_y\frac{1}{\eta_1}\left(E_{i0}e^{-j\beta_1 z} - E_{ro}e^{j\beta_1 z}\right) \\
\bar{E}_z &= \bar{a}_x\left(E_2^+ e^{-j\beta_2 z} + E_2^- e^{j\beta_2 z}\right) \\
\bar{H}_2 &= \bar{a}_y\frac{1}{\eta_2}\left(E_2^+ e^{-j\beta_2 z} - E_2^- e^{j\beta_2 z}\right)
\end{aligned}
$$

At z=d,

$$\bar{E}_2 = 0 \longrightarrow E_2^- = -E_2^+ e^{-j2\beta_2 d}.$$

$$\bar{E}_2 = \bar{a}_x E_2^+ \left[e^{-j\beta_2 z} - e^{j\beta_2(z-2d)}\right].$$

$$\bar{H}_2 = \bar{a}_y \frac{E_2^+}{\eta_2} \left[e^{-j\beta_2 z} + e^{j\beta_2(z-2d)}\right].$$

Boundary conditions at z=0:

$$E_1(0) = E_2(0) \to E_{i0} + E_{ro} = E_2^+ \left(1 - e^{-j2\beta_2 d}\right)$$

$$H_1(0) = H_2(0) \to E_{i0} - E_{r0} = E_2^+ \frac{\eta_0}{\eta_2} \left(1 + e^{-j2\beta_2 d}\right)$$

$$E_2^+ = \frac{2\eta_2 E_{i0}}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2)\, e^{-j2\beta_2 d}},$$

$$E_{r_0} = -\left(\frac{\eta_0 - j\eta_2 \tan\beta_2 d}{\eta_0 + j\eta_2 \tan\beta_2 d}\right) E_{i0}.$$

a)

$$\bar{E}_r(z,t) = \bar{a}_x E_{i0} \cos\left[\omega\left(t - \frac{z}{u_p}\right) + \theta\right], \theta = \pi - 2\tan^{-1}\left(\frac{\eta_2}{\eta_0}\tan_2 d\right)$$

b)

$$\bar{E}_1(z,t) = \bar{a}_x E_{i0} \left\{\cos\omega\left(t - \frac{z}{u_p}\right) + \cos\left[\omega\left(t - \frac{z}{u_p}\right) + \theta\right]\right\}$$

c)

$$\bar{E}_2(z,t) =$$

$$\bar{a}_x \frac{2\eta_2 E_{i0}}{\sqrt{2\left[(\eta_0^2 + \eta_2^2) + (\eta_0^2 - \eta_2^2)\cos 2\beta_2 d\right]}} \left\{\cos\left[\omega\left(t - \frac{z}{u_{p2}}\right) + \psi\right] - \cos\left[\omega\left(t + \frac{2}{u_{p2}}\right) - \frac{2\omega d}{u_{p2}} + \psi\right]\right\}$$

$$\psi = \tan^{-1}\left[\frac{(\eta_0 - \eta_2)\sin 2\beta_2 d}{(\eta_0 + \eta_2) + (\eta_0 - \eta_2)\cos 2\beta_2 d}\right].$$

d)

$$\left(\bar{\mathscr{P}}_{\text{av}}\right), = \frac{1}{2}\operatorname{Re}\left(\bar{E}_1 \times \bar{H}_1^*\right) = 0$$

e)

$$\left(\bar{\mathscr{P}}_{\text{av}}\right)_2 = 0$$

f) Let

$$E_r = -E_{i0} \longrightarrow \tan\beta_2 d = 0 \longrightarrow d = n\lambda_2/2, n = 0, 1, 2, \cdots$$

Exercise 8.33

A uniform plane wave with $\mathbf{E}_i(z) = \mathbf{a}_x E_{i0} e^{-j\beta_0 z}$ in air propagates normally through a thin copper sheet of thickness d, as shown in Fig. 8-23. Neglecting multiple reflections within the copper sheets, find

a) E_2^+, H_2^+
b) E_2^-, H_2^-
c) E_{30}, H_{30}
d) $(\mathscr{P}_{\text{av}})_3 / (\mathscr{P}_{\text{av}})_i$

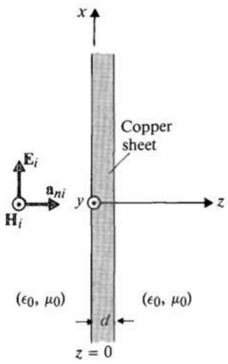


FIGURE 8-23
Plane wave propagating through a thin copper sheet
(Problem P.8-33).

Answer:

$$k_2 = \beta_2 - j\alpha_2 = (1-j)\frac{1}{\delta}, \quad \alpha_2 = \beta_2 = \frac{1}{\delta} = \sqrt{\pi f \mu_2 \sigma_2}.$$

$$\eta_2 = (1+j)\frac{\alpha_2}{\sigma_2} \ll \eta_0 \quad \text{at } 10\,(MHz)\,.$$

a) From Problem $P.8-29$,

$$E_2^+ = \eta_2 H_2^+ \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{e^{\alpha_2 d} e^{j\beta_2 d} E_{i0}}{\sin\left(\beta_2 - j\alpha_2\right)d}$$

b)

$$E_2^- = -\eta_2 H_2^- \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{e^{-\alpha_2 d} e^{-j\beta_2 d} E_{i0}}{\sin\left(\beta_2 - j\alpha_2\right)d}$$

c)

$$E_{30} = E_{t0} = \eta_0 H_{30} \cong -j\left(\frac{\eta_2}{\eta_0}\right)\frac{2e^{j\beta_0 d_0} E_{i0}}{\sin\left(\beta_2 - j\alpha_2\right)d}$$

d)

$$E_{r_0} \cong -\frac{E_{i0}}{1 - j\frac{\eta_2}{\eta_0}\cot\left(\beta_2 - j\alpha_2\right)d}$$

$$\cong -\left(1 + j\frac{\eta_2}{\eta_0}\frac{1 + j\tan\beta_2 d \tanh\alpha_2 d}{\tan\beta_2 d - j\tanh\alpha_2 d}\right)E_{i0}.$$

$$\left(\bar{\mathscr{P}}_{av}\right)_1 = \frac{1}{2}\,\mathrm{Re}\left[\left(\bar{E}_{i0}\times\bar{H}_{i0}^*\right) - \left(\bar{E}_{ro}\times\bar{H}_{ro}^*\right)\right]$$

$$= \bar{a}_z\frac{\alpha}{\eta_0^2\sigma}(A+B),$$

$$\text{where } \frac{1 + j\tan\beta_2 d \tanh\alpha_2 d}{\tan\beta_2 d - j\tanh\alpha_2 d} = A + jB.$$

$$\left(\mathscr{P}_{av}\right)_1 = \frac{\alpha}{\eta_0^2\sigma}\frac{\sin\beta_2 d\cos\beta_2 d + \sinh\alpha_2 d\cosh\alpha_2 d}{\left(\sin\beta_2 d\cosh\alpha_2 d\right)^2 + \left(\cos\beta_2 d\sinh\alpha_2 d\right)^2}.$$

$$\left(\mathscr{P}_{av}\right)_3 = \frac{1}{2\eta_0}|E_{30}|^2 = \frac{1}{\eta_0^3}\left(\frac{\alpha}{\sigma}\right)^2\frac{4E_{i0}^2}{\left(\sin\beta_2 d\cosh\alpha_2 d\right)^2 + \left(\cos\beta_2 d\sinh\alpha_2 d\right)^2}.$$

$$\therefore \quad \frac{(\mathscr{P}_{av})_3}{(\mathscr{P}_{av})_1} = \frac{4}{\eta_0}\left(\frac{\alpha}{\sigma}\right)\frac{1}{\sin\beta_2 d\cos\beta_2 d + \sinh\alpha_2 d\cosh\alpha_2 d}.$$

$$(\mathscr{P}_{av})_i = \frac{1}{2\eta_0}E_{io}^2.$$

$$\frac{(\mathscr{P}_{av})_3}{(\mathscr{P}_{av})_i} = \frac{8}{\eta_0^2}\left(\frac{\alpha}{\sigma}\right)^2\frac{1}{(\sin\beta_2 d\cosh\alpha_2 d)^2 + (\cos\beta_2 d\sinh\alpha_2 d)^2}.$$

At $f = 10^7\,(H_z)\,, \sigma = 5.80\times 10^7(\text{ s/m}), \alpha_2 = \beta_2 = 4.785\times 10^4, d = \delta = \dfrac{1}{\alpha_2}.$

$$\frac{(\mathscr{P}_{\text{av}})_3}{(\mathscr{P}_{\text{av}})_i} = 1.839\times 10^{-11}.$$

Exercise 8.34

A uniform plane wave is incident on the ionosphere at an angle of incidence $\theta_i = 60°$. Assuming a constant electron density and a wave frequency equal to one-half of the plasma frequency of the ionosphere, determine

a) $\Gamma_\perp$ and $\tau_\perp$,

b) $\Gamma_\|$ and $\tau_\|$.

Interpret the significance of these complex quantities.

Answer:

Given $f = f_p/2$ and $\theta_i = 60°$.

$$\longrightarrow \eta_p = \eta_0/\sqrt{1-(f_p/f)^2} = -j\eta_0/\sqrt{3}, \eta_p/\eta_0 = -j/\sqrt{3}$$

$$\text{From Eq. } (8-185): \sin\theta_t = \frac{\eta_p}{\eta_0}\sin\theta_i = -j/2, \cos\theta_t = \sqrt{5}/2, \cos\theta_i = 1/2$$

a)

$$\text{From Eq. } (8-206): \Gamma_\perp = \frac{(\eta_p/\eta_0)\cos\theta_i - \cos\theta_t}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = e^{j209°}$$

$$\text{From Eq. } (8-207): \tau_\perp = \frac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_i + \cos\theta_t} = 0.5e^{-j75.5°}$$

b)

$$\text{From Eq. } (8-221): \Gamma_\| = \frac{(\eta_p/\eta_0)\cos\theta_t - \cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = e^{j26°}$$

$$\text{From Eq. } (8-222); \tau_\| = \frac{2(\eta_p/\eta_0)\cos\theta_i}{(\eta_p/\eta_0)\cos\theta_t + \cos\theta_i} = 0.177e^{-j38°}$$

$|\Gamma_\perp| = |\Gamma_\|| = 1$, but the phase shift of the reflected wave depends on the polarization of the incident wave. There are standing waves in the air and exponentially decaying transmitted waves in the ionosphere.

Exercise 8.35

A 10(kHz) parallelly polarized electromagnetic wave in air is incident obliquely on an ocean surface at a near-grazing angle $\theta_i = 88^\circ$. Using $\epsilon_r = 81, \mu_r = 1$, and $\sigma = 4(\text{ S/m})$ for seawater, find

(a) the angle of refraction θ_t,

(b) the transmission coefficient $\tau_{\|}$,

(c) $(\mathscr{P}_{\text{av}})_t / (\mathscr{P}_{\text{av}})_i$,

(d) the distance below the ocean surface where the field intensity has been diminished by 30(dB).

Answer:

$$k_{2x}^2 + k_{2z}^2 = k_2^2 = \omega^2 \mu_0 \epsilon_2 - j\omega\mu_0\sigma_2 \qquad (1)$$

Continuity conditions at $z = 0$ for all x and y require:

$$k_{2x} = k_{1x} = \omega\sqrt{\mu_0\varepsilon_0}\sin\theta_i = \beta_x = 2.09 \times 10^{-4} \qquad (2)$$

$$k_{2z} = \beta_{2z} - j\alpha_{2z} \qquad (3)$$

Combining (1), (2) and (3), we can solve for α_{2z} and β_{2x} in terms of $\omega, \mu_0, \varepsilon_2, \sigma_2$, and β_x. But, since

$$\beta_x^2 \ll \omega^2 \mu_0 \epsilon_2$$

we have

$$\alpha_2 = \alpha_{2z} \cong \beta_{2z} \cong \frac{1}{\delta} = \sqrt{\pi f \mu_0 \sigma_2} = 0.3974\left(\text{ m}^{-1}\right)$$

a)

$$\theta_t = \tan^{-1}\frac{\beta_x}{\beta_{22}} \cong \tan^{-1}\frac{2.09}{0.3974} \times 10^{-4} \cong 5.26 \times 10^{-4}(\text{rad})$$
$$= 0.03^\circ$$

b)

$$\begin{aligned}
\Gamma_{\|} &= \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_0\cos\theta_i} \\
&= \frac{2 \times 0.0993(1+j)}{0.0993(1+j) + 377\cos 88^\circ} \\
&\cong 0.0151(1+j) = 0.0214e^{j\pi/4} \\
\eta_2 &= \frac{\alpha_2}{\sigma_2}(1+j) = 0.0993(1+i). \\
&\cos\theta_t = \cos 0.03^\circ \cong 1
\end{aligned}$$

c)

$$(\mathscr{P}_{av})_i = \frac{E_{i0}^2}{2\eta_0}$$

$$E_{t0} \cong 2E_{i0}\frac{\eta_2}{\eta_0}, \quad H_{t0} \cong \frac{2E_{i0}}{\eta_0} \cdot \longrightarrow (\mathscr{P}_{av})_e = 2\frac{E_{i0}^2\alpha_2}{\eta_0^2\sigma_2}e^{-2\alpha_2 z}.$$

$$\therefore \frac{(\sigma_{av})_t}{(\theta_{av})_i} = \frac{4\alpha_2}{\eta_0\sigma_2}e^{-2\alpha_2 z} = 1.054 \times 10^{-3}e^{-0.795z}$$

d)

$$20\log_{10} e^{-\alpha_2 z} = -30. \longrightarrow z = \frac{1.5}{\alpha_2\log_{10} e} = 8.69(\text{ m})$$

Exercise 8.36

A light ray is incident from air obliquely on a transparent sheet of thickness d with an index of refraction n, as shown in Fig. 8-24. The angle of incidence is θ_i. Find

(a) θ_t,

(b) the distance ℓ_1 at the point of exit, and (c) the amount of the lateral displacement ℓ_2 of the emerging ray.

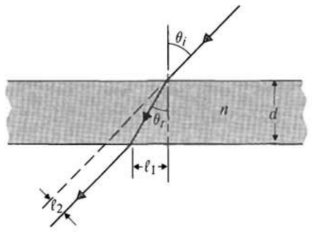
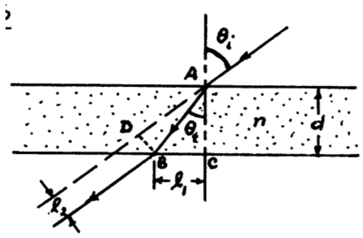


FIGURE 8–24
Light-ray impinging obliquely on a transparent sheet of refraction index n (Problem P.8–36).

Answer:



a) Snell's law:

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{1}{n},$$

$$\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right).$$

b)

$$\cos\theta_t = \sqrt{1 - \left(\frac{1}{n}\sin\theta_i\right)^2}$$

$$l_1 = \overline{BC} = \overline{AC}\tan\theta_t = d\frac{\sin\theta_t}{\cos\theta_t} = \frac{d\sin\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}.$$

c)

$$\begin{aligned}
\ell_2 = \overline{BD} = \overline{AC}\sin(\theta_i - \theta_t) &= \frac{d}{\cos\theta_t}(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t) \\
&= d\sin\theta_i\left[1 - \frac{\cos\theta_i}{\sqrt{n^2 - \sin^2\theta_i}}\right].
\end{aligned}$$

Exercise 8.37

A uniform plane wave with perpendicular polarization represented by Eqs. (8 − 196) and (8-197) is incident on a plane interface at $z = 0$, as shown in Fig. 8-16. Assuming $\epsilon_2 < \epsilon_1$ and $\theta_i > \theta_c$,

(a) obtain the phasor expressions for the transmitted field $(\mathbf{E}_t, \mathbf{H}_t)$,
(b) verify that the average power transmitted into medium 2 vanishes.

Answer:
a)

$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \longrightarrow \sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i > 1 \text{ for } \theta_i > \theta_c$$

$$\cos\theta_t = -j\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}$$

From Eqs. $(8-200)$ and $(8-201)$:

$$\bar{E}_t(x,z) = \bar{a}_y E_{t0} e^{-\alpha_2 z} e^{-j\beta_{2x}x},$$
$$\bar{H}_t(x,z) = \frac{E_{t0}}{\eta_2}\left(\bar{a}_x j\alpha_2 + \bar{a}_z\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right) e^{-\alpha_2 z} e^{-j\beta_{2x}x},$$

where

$$\beta_{2x} = \beta_2\sin\theta_t = \beta_2\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i,$$

$$\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1},$$

$$E_{t0} = \frac{2\eta_2\cos\theta_i E_{i0}}{\eta_2\cos\theta_i - j\eta_1\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1}} \text{ from } E_q(8-207).$$

b)

$$(\mathscr{P}_{av})_{2z} = \frac{1}{2}\operatorname{Re}\left(E_{ty}H_{tx}^*\right) = 0$$

Exercise 8.38

A uniform plane wave of angular frequency ω in medium 1 having a refractive index n_1 is incident on a plane interface at $z = 0$ with medium 2 having a refractive index $n_2\,(< n_1)$ at the critical angle. Let E_{i0} and E_{t0} denote the amplitudes of the incident and refracted electric field intensities, respectively.

a) Find the ratio E_{t0}/E_{i0} for perpendicular polarization.

b) Find the ratio E_{t0}/E_{i0} for parallel polarization.

c) Write the instantaneous expressions of $\mathbf{E}_i(x,z;t)$ and $\mathbf{E}_t(x,z;t)$ for perpendicular polarization in terms of the parameters $\omega, n_1, n_2, \theta_i$, and E_{i0}.

Answer:
Given $\theta_i = \theta_c \longrightarrow \theta_t = \pi/2, \cos\theta_t = 0$
a)

$$\text{From Eq. } (8-207) : (E_{t0}/E_{i0})_\perp = 2$$

b)

$$\text{From Eq. } (8-221) : (E_{t0}/E_{i0})_\parallel = 2\eta_2/\eta_1$$

c)

$$\bar{E}_i(x,z;t) = \bar{a}_y E_{i0}\cos\omega\left[t - \frac{n_1}{c}\left(x\sin\theta_i + z\cos\theta_i\right)\right]$$

$$\bar{E}_t(x, z; t) = \bar{a}_y 2E_{i0} e^{-\alpha z} \cos\omega \left(t - \frac{n_2}{c} x \sin\theta_t\right)$$

$$= \bar{a}_y 2E_{i0} e^{-\alpha z} \cos\omega \left(t - \frac{n_1}{c} x \sin\theta_i\right)$$

where

$$\alpha = \frac{n_2\omega}{c}\sqrt{\left(\frac{n_1}{n_2}\sin\theta_i\right)^2 - 1} = 0$$

when

$$\theta = \theta_c$$

Exercise 8.39

An electromagnetic wave from an underwater source with perpendicular polarization is incident on a water-air interface at $\theta_i = 20°$. Using $\epsilon_r = 81$ and $\mu_r = 1$ for fresh water, find
(a) critical angle θ_c,
(b) reflection coefficient $\Gamma_\perp$,
(c) transmission coefficient $\tau_\perp$,
(d) attenuation in dB for each wavelength into the air.

Answer:
a)

$$\theta_c = \sin^{-1}\sqrt{\epsilon_{r2}/\epsilon_{r1}} = \sin^{-1}\sqrt{1/81} = 6.38°$$

b)

$$\theta_i = 20° > \theta_c. \quad \sin\theta_t = \sqrt{\frac{\epsilon_i}{\epsilon_2}}\sin\theta_i = 3.08, \quad \cos\theta_t = -j2.91$$

$$\Gamma_1 = \frac{\sqrt{\epsilon_{t1}}\cos\theta_i - \cos\theta_t}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_t} = e^{j38°} = e^{j0.66}$$

c)

$$\tau_\perp = \frac{2\sqrt{\epsilon_r}\cos\theta_i}{\sqrt{\epsilon_{r1}}\cos\theta_i + \cos\theta_L} = 1.89e^{j19°} = 1.89e^{j0.33}$$

d) The transmitted wave in air varies as

$$e^{-\alpha_2 z} e^{-j\beta_{2x} x}$$

where

$$\alpha_2 = \beta_2\sqrt{\left(\frac{\epsilon_1}{\epsilon_2}\right)\sin^2\theta_i - 1} = \frac{2\pi}{\lambda_0}(2.91)$$

Attenuation in air for each wavelength

$$= 20\log_{10} e^{-\alpha_2\lambda_0} = 159 \text{ (dB)}.$$

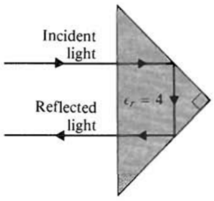


FIGURE 8–25
Light reflection by a right isosceles triangular prism (Problem P.8–40).

Exercise 8.40

Glass isosceles triangular prisms shown in Fig. $8-25$ are used in optical instruments. Assuming $\epsilon_r = 4$ for glass, calculate the percentage of the incident light power reflected back by the prism.

Answer:
When the incident light first strikes the hypotenuse surface, $\theta_i = \theta_t = 0, \tau_1 = \frac{2\eta_2}{\eta_2+\eta_0}$.

$$\frac{(\mathscr{P}_{av})_{t1}}{(\mathscr{P}_{av})_i} = \frac{\eta_0}{\eta_2}\tau_1^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

Total reflections occur inside the prism at both slanting surfaces because

$$\theta_i = 45^\circ > \theta_c = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ.$$

On exit from the prism, $\tau_2 = \frac{2\eta_0}{\eta_2+\eta_0}$.

$$\frac{(\mathscr{P}_{av})_0}{(\mathscr{P}_{\text{av}})_{t1}} = \frac{\eta_2}{\eta_0}\tau_2^2 = \frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}.$$

$$\therefore \quad \frac{(\mathscr{P}_{\text{av}})_0}{(\mathscr{P}_{av})_i} = \left[\frac{4\eta_0\eta_2}{(\eta_2+\eta_0)^2}\right]^2 = \left[\frac{4\sqrt{\epsilon_r}}{\left(1+\sqrt{\epsilon_r}\right)^2}\right]^2 = 0.79.$$

Exercise 8.41

For preventing interference of waves in neighboring fibers and for mechanical protection, individual optical fibers are usually cladded by a material of a lower refractive index, as shown in Fig. $8-26$, where $n_2 < n_1$.

a) Express the maximum angle of incidence θ_a in terms of n_0, n_1, and n_2 for meridional rays incident on the core's end face to be trapped inside the core by total internal reflection. (Meridional rays are those that pass through the fiber axis. The angle θ_a is called the acceptance angle, and $\sin\theta_a$ the numerical aperture (N.A.) of the fiber.)

b) Find θ_a and N.A. if $n_1 = 2, n_2 = 1.74$, and $n_0 = 1$.

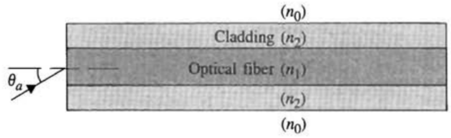


FIGURE 8–26
A cladded-core optical fiber (Problem P.8–41).

Answer:

a)

$$\begin{aligned} n_0 \sin\theta_a &= n_1 \sin(90^\circ - \theta_c) = n_1 \cos\theta_c \\ &= n_1\sqrt{1-\sin^2\theta_c} = n_1\sqrt{1-(n_2/n_1)^2} = \sqrt{n_1^2 - n_2^2}. \\ \sin\theta_a &= \frac{1}{n_0}\sqrt{n_1^2-n_2^2} = \sqrt{n_1^2-n_2^2}. \quad (n_0 = 1) \end{aligned}$$

b)

$$\begin{aligned} \text{N.A. } &= \sin\theta_a = \sqrt{2^2 - 1.74^2} = 0.9861, \\ \theta_a &= \sin^{-1} 0.9861 = 80.4^\circ. \end{aligned}$$

Exercise 8.42

An electromagnetic wave in dielectric medium 1 (ϵ_1, μ_0) impinges obliquely on a boundary plane with dielectric medium 2 (ϵ_2, μ_0). Let θ_i and θ_t denote the incident and refraction angles, respectively, and prove the following:

a) For perpendicular polarization:

$$\Gamma_\perp = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}, \quad \tau_\perp = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)}.$$

b) For parallel polarization:

$$\Gamma_\parallel = \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i}, \quad \tau_{1\parallel} = \frac{4\sin\theta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}.$$

(These four relations are known as Fresnel formulas.)

Answer:

$$E_q(8-185): \quad \frac{\eta_2}{\eta_1} = \frac{\sin\theta_t}{\sin\theta_i}$$

a.)

$$\begin{aligned} E_q(8-206): \Gamma_\perp &= \frac{(\eta_2/\eta_1)\cos\theta_i - \cos\theta_t}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_t} = \frac{\sin\theta_t\cos\theta_i - \cos\theta_t\sin\theta_i}{\sin\theta_t\cos\theta_i + \cos\theta_t\sin\theta_i} \\ &= \frac{\sin(\theta_t-\theta_i)}{\sin(\theta_t+\theta_i)} \end{aligned}$$

$$E_q(8-207): \tau_\perp = \frac{2(\eta_2/\eta_1)\cos\dot{\theta}_i}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_t} = \frac{2\sin\theta_t\cos\theta_i}{\sin(\theta_t+\theta_i)}$$

b)

$$\begin{aligned} E_q(8-221): \Gamma_\parallel &= \frac{(\eta_2/\eta_1)\cos\theta_t - \cos\theta_i}{(\eta_2/\eta_1)\cos\theta_t + \cos\theta_i} = \frac{\sin\theta_t\cos\theta_t - \sin\theta_i\cos\theta_i}{\sin\theta_t\cos\theta_t + \sin\theta_i\cos\theta_i} \\ &= \frac{\sin 2\theta_t - \sin 2\theta_i}{\sin 2\theta_t + \sin 2\theta_i} \end{aligned}$$

$$E_{g.}(8-222): \tau_\parallel = \frac{2(\eta_2/\eta_1)\cos\theta_i}{(\eta_2/\eta_1)\cos\theta_i + \cos\theta_i} = \frac{4\sin\theta_t\cos\theta_i}{\sin 2\theta_t + \sin 2\theta_i}$$

Exercise 8.43

Prove that, under the condition of no reflection at an interface, the sum of the Brewster angle and the angle of refraction is $\pi/2$ for:
a) perpendicular polarization ($\mu_1 \neq \mu_2$),
b) parallel polarization ($\epsilon_1 \neq \epsilon_2$).

Answer:
a) For perpendicular polarization and $\mu_1 \neq \mu_2$.

$$\sin\theta_{B\perp} = \frac{1}{\sqrt{1+\left(\frac{\mu_1}{\mu_2}\right)}}.$$

Under condition of no reflection:

$$\cos\theta_2 = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_{B\perp}} = \frac{1}{\sqrt{1+\left(\frac{\mu_1}{\mu_2}\right)}}$$
$$= \sin\theta_{B\perp} \longrightarrow \theta_t + \theta_{B\perp} = \pi/2.$$

b) For parallel polarization and $\epsilon_1 \neq \epsilon_2$:

$$\sin\theta_{B\|} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$$

$$\cos\theta_t = \sqrt{1-\frac{n_1^2}{n_2^2}\sin^2\theta_{B\|}} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$$

$$= \sin\theta_{B\|} \longrightarrow \theta_t + \theta_{B\|} = \pi/2$$

Exercise 8.44

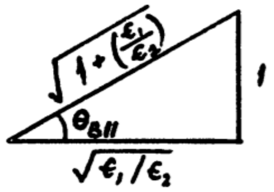
For an incident wave with parallel polarization:
a) Find the relation between the critical angle θ_c and the Brewster angle $\theta_{B\|}$ for non-magnetic media.
b) Plot θ_c and $\theta_{B\|}$ versus the ratio ϵ_1/ϵ_2.

Answer:
a)

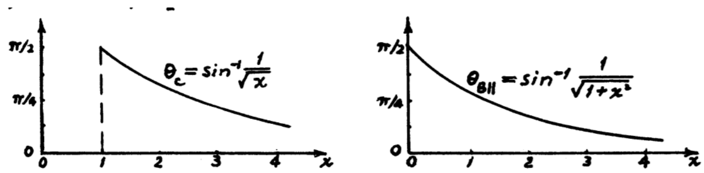
$$\sin\theta_c = \sqrt{\frac{\epsilon_2}{\epsilon_1}}; \sin\theta_{B\|} = \frac{1}{\sqrt{1+\left(\frac{\epsilon_1}{\epsilon_2}\right)}}$$

$$\longrightarrow \tan\theta_{B\|} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}.$$

$$\therefore \sin\theta_c = \tan\theta_{B\|} \cdot \left(\theta_c > \theta_{B\|}\right)$$



b) Let $\epsilon_1/\epsilon_2 = x$



Exercise 8.45

By using Snell's law of refraction,

(a) express Γ and τ in terms of $\epsilon_{r1}, \epsilon_{r2}$, and θ_i;

(b) plot Γ and τ versus θ_i for $\epsilon_{r1}/\epsilon_{r2} = 2.25$ for both perpendicular and parallel polarizations.

Answer:

a)

For perpendicular polarization:

$$\Gamma_L = \frac{\sqrt{\epsilon_{r1}}\cos\theta_i - \sqrt{\epsilon_{r2}}\cos\theta_t}{\sqrt{\epsilon_{r2}}\cos\theta_i + \sqrt{\epsilon_{r2}}\cos\theta_t},$$

$$\sin\theta_t = \sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sin\theta_i, \quad \cos\theta_t = \sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i}.$$

$$\Gamma_\perp = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i - \sqrt{1 - \frac{\varepsilon_r}{\varepsilon_n}\sin^2\theta_i}}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}},$$

$$\tau_\perp = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_2}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i + \sqrt{1 - \frac{\epsilon_{r1}}{\epsilon_{r2}}\sin^2\theta_i}}.$$

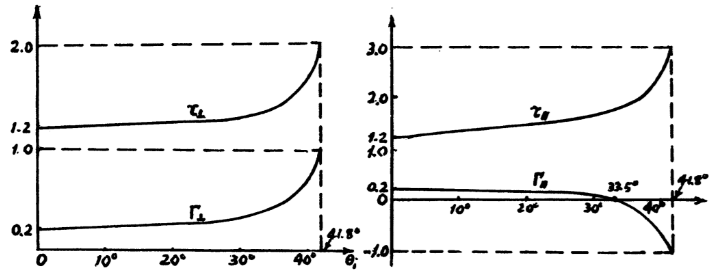
For parallel polarization:

$$\Gamma_{||} = \frac{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} - \cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}$$

$$\tau_{||} = \frac{2\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\cos\theta_i}{\sqrt{\frac{\epsilon_{r1}}{\epsilon_{r2}}}\sqrt{1 - \left(\frac{\epsilon_{r1}}{\epsilon_{r2}}\right)\sin^2\theta_i} + \cos\theta_i}$$

b)

$$\epsilon_{r1}/\epsilon_{r2} = 2.25, \sqrt{\epsilon_{r1}/\epsilon_{r2}} = 1.5 \longrightarrow \theta_c = \sin^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 41.8°$$



Exercise 8.46

A perpendicularly polarized uniform plane wave in air of frequency f is incident obliquely at an angle of incidence θ_i on a plane boundary with a lossy dielectric medium that is characterized by a complex permittivity $\epsilon_2 = \epsilon' - j\epsilon''$. Let the incident electric field be

$$\mathbf{E}_i(x, z) = \mathbf{a}_y E_{i0} e^{-jk_0(x\sin\theta_i - z\cos\theta_i)}.$$

a) Find the expressions of the transmitted electric and magnetic field intensity phasors in terms of the given parameters.

b) Show that the angle of refraction is complex and that $\mathbf{H}_t$ is elliptically polarized.

Answer:
Given:

$$\begin{aligned}
\bar{E}_i(x, z) &= \bar{a}_y E_{i0} e^{-jk_0(x\sin\theta_i - z\cos\theta_i)} \\
\bar{a}_{ni} &= \bar{a}_x \sin\theta_i - \bar{a}_z \cos\theta_i \\
\bar{H}_i(x, z) &= \frac{1}{\eta_0}\bar{a}_{ni} \times \bar{E}_i(x, z) \\
&= \frac{1}{\eta_0}\left(\bar{a}_x \cos\theta_i + \bar{a}_z \sin\theta_i\right) e^{-jk_0(x\sin\theta_i - z\cos\theta_i)}
\end{aligned}$$

$$\begin{aligned}
\epsilon_2 = \epsilon' - j\epsilon'', \quad k_2 &= \omega\sqrt{\mu_0\epsilon_2} = \omega\sqrt{\mu_0\epsilon_0\left(\frac{\epsilon'}{\epsilon_0} - j\frac{\epsilon''}{\epsilon_0}\right)} \\
&= k_0\sqrt{\epsilon_r' - j\epsilon_r''}
\end{aligned}$$

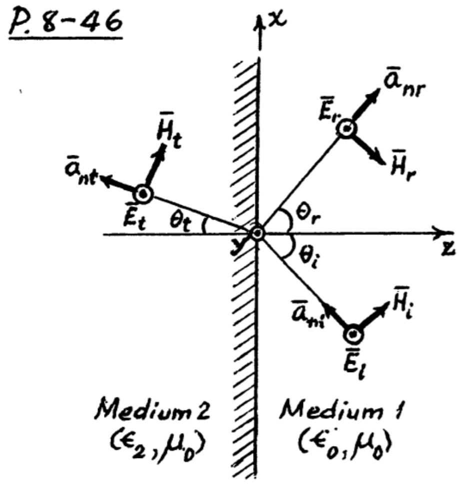
a) From Eq. (8-207):

$$\tau_1 = \frac{2\left(\eta_2/\eta_0\right)\cos\theta_i}{\left(\eta_2/\eta_0\right)\cos\theta_i + \cos\theta_t},$$

where

$$\left(\eta_2/\eta_0\right) = \sqrt{\epsilon_0/\epsilon_2} = 1/\sqrt{\epsilon_r' - j\epsilon_r''}$$

$$\begin{aligned}
\bar{E}_t(x, z) &= \bar{a}_y \tau_\perp E_{i0} e^{-jk_2(x\sin\theta_i - z\cos\theta_i)} \\
\bar{H}_t(x, z) &= \frac{1}{\eta_2}\bar{a}_{nt} \times \bar{E}_t(x, z) = \frac{1}{\eta_2}\left(\bar{a}_x \cos\theta_t + \bar{a}_z \sin\theta_t\right)\tau_\perp \bar{E}_t(x, z)
\end{aligned}$$



b) From Eq. (8-185):

$$\sin\theta_t = \frac{\sin\theta_i}{\sqrt{\epsilon_r' - j\epsilon_r''}} \text{ (complex).}$$

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} \text{ (complex).}$$

The x - and z-components of $\bar{H}_t(x, z)$ in part a) have different amplitudes and are out of phase, indicating that it is elliptically polarized.

Exercise 8.47

In some books the reflection and transmission coefficients for parallel polarization are defined as the ratios of the amplitude of the tangential components of the reflected and transmitted **E** fields, respectively, to the amplitude of the tangential component of the incident **E** field. Let the coefficients defined in this manner be designated $\Gamma_{\|}'$ and $\tau_{\|}'$, respectively.

a) Find $\Gamma_{\|}'$ and $\tau_{\|}'$ in terms of η_1, η_2, θ_i, and θ_t; and compare them with $\Gamma_{\|}$and $\tau_{\|}$in Eqs. $(8-221)$ and $(8-222)$.

b) Find the relation between $\Gamma_{\|}'$ and $\tau_{\|}'$, and compare it with Eq. (8-223).

Answer:

a)

$$\Gamma_{\|}' = \left.\frac{(E_r)_{\tan}}{(E_i)_{\tan}}\right|_{z=0} = \frac{E_{r0}\cos\theta_i}{E_{i0}\cos\theta_i} = \frac{E_{r0}}{E_{i0}} = \Gamma_{\|} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i},$$

$$\tau_{\|}' = \left.\frac{(E_t)_{\tan}}{(E_i)_{\tan}}\right|_{z=0} = \frac{E_{t_0}\cos\theta_t}{E_{i0}\cos\theta_i} = \tau_{\|}\left(\frac{\cos\theta_t}{\cos\theta_i}\right) = \frac{2\eta_2\cos\theta_t}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}.$$

b) From part a) we have

$$1 + \Gamma_{\|}' = \tau_{\|}'$$

This compares with

$$1 - \Gamma_{\|} = \tau_{\|}\left(\frac{\cos\theta_t}{\cos\theta_i}\right) \text{ in } E_{q.}(8-223)$$

Reference

1. Cheng, David Keun. Field and wave electromagnetics. Pearson Education India, 1989.