VE230 Electromagnetics

Chapter 6

September 20, 2022

JOINT INSTITUTE
交大密西根学院

Exercise 6.1

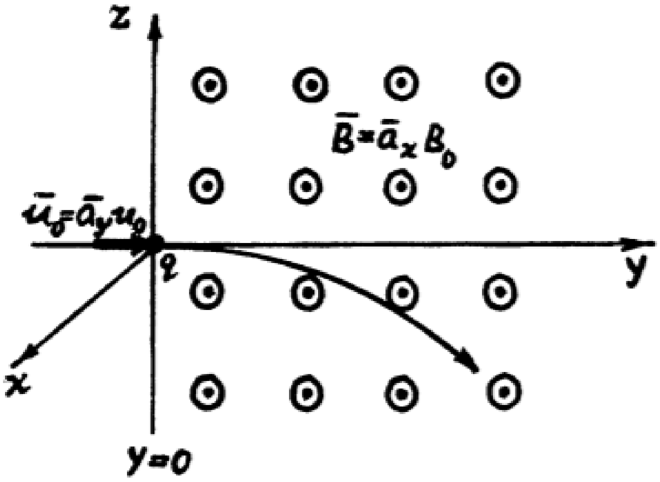
A positive point charge q of mass m is injected with a velocity $\mathbf{u}_0 = \mathbf{a}_y u_0$ into the $y > 0$ region where a uniform magnetic field $\mathbf{B} = \mathbf{a}_x B_0$ exists. Obtain the equation of motion of the charge, and describe the path that the charge follows.

Answer:

$$\frac{du_y}{dt} = \frac{qB_0}{m} u_z = \omega_0 u_z,$$
$$\frac{du_z}{dt} = -\frac{qB_0}{m} u_y = -\omega_0 u_y,$$
$$\omega_0 = qB_0/m.$$

Combining (1) and (2):

$$\frac{d^2 u_z}{dt^2} + \omega_0^2 u_z = 0$$
$$\longrightarrow u_z = A\cos\omega_0 t + B\sin\omega_0 t.$$



At $t = 0, u_z = 0 \to A = 0; u_z = B\sin\omega_0 t$.
Substituting u_z in (2): $u_y = -B\cos\omega_0 t$. At $t = 0, u_y = u_0 \to B = -u_0$.
$\therefore u_y = u_0\cos\omega_0 t \longrightarrow y = \frac{u_0}{\omega_0}\sin\omega_0 t$, $(t = 0, y = 0)$; (3)

$$u_z = -u_0 \sin\omega_0 t \longrightarrow z = \frac{u_0}{\omega_0}\cos\omega_0 t + c_1 \left(t = 0, z = 0 \to c_1 = -\frac{u_0}{\omega_0}\right).$$

$= -\frac{u_0}{\omega_0}(1 - \cos\omega_0 t)$.
From (3) and (4); $y^2 + \left(z - \frac{u_0}{\omega_0}\right)^2 = \left(\frac{u_0}{\omega_0}\right)^2 - E_q$ of a shifted circle.

Exercise 6.2

An electron is injected with a velocity $\mathbf{u}_0 = \mathbf{a}_y u_0$ into a region where both an electric field $\mathbf{E}$ and a magnetic field $\mathbf{B}$ exist. Describe the motion of the electron if

a) $\mathbf{E} = \mathbf{a}_z E_0$ and $\mathbf{B} = \mathbf{a}_x B_0$,

b) $\mathbf{E} = -\mathbf{a}_z E_0$ and $\mathbf{B} = -\mathbf{a}_z B_0$.

Discuss the effect of the relative magnitudes of E_0 and B_0 on the electron paths in parts (a) and (b).

Answer:

$\frac{\partial u}{\partial t} = -\frac{e}{m}(\bar{E} + \bar{u} \times \bar{B})$.

a) $\bar{E} = \bar{a}_z E_0, \bar{B} = \bar{a}_x B_0$.

$$\begin{aligned} \frac{\partial u_x}{\partial t} &= 0, \\ \frac{\partial u_y}{\partial t} &= -\frac{e}{m} B_0 u_z, \\ \frac{\partial u_z}{\partial t} &= -\frac{e}{m}\left(E_0 - B_0 u_y\right). \end{aligned} \qquad \longrightarrow \begin{cases} u_x = 0, \\ u_y = \left(u_0 - \frac{E_0}{B_0}\right) \cos\omega_0 t + \frac{E_0}{B_0}, \\ u_z = \left(\frac{E_0}{R_0} - u_0\right) \sin\omega_0 t; \omega_0 = \frac{e}{m} B_0. \end{cases}$$

If the electron is injected at the origin $(x = y = z = 0)$ at $t = 0$:

$$x = 0, \quad y = \frac{c_2}{\omega_0} \sin\omega_0 t + \frac{E_0}{B_0} t, \quad z = -\frac{c_2}{\omega_0}\left(1 - \cos\omega_0 t\right); \quad c_2 = u_0 - \frac{E_0}{B_0}.$$

$$\text{Eq. of motion: } \left(y - \frac{E_0}{B_0} t\right)^2 + \left(z + \frac{c_2}{\omega_0}\right)^2 = \left(\frac{c_2}{\omega_0}\right)^2$$

If $\frac{E_0}{B_0} = u_0$, $\quad u_x = u_z = 0, u_y = u_0$; $x = z = 0$, and $y = u_0 t$

(b)

$$\begin{aligned} &\bar{E} = -\bar{a}_z E_0, B : \\ &\frac{\partial u_x}{\partial t} = \frac{e}{m} B_0 u_y = \omega_0 u_y, \\ &\frac{\partial u_y}{\partial t} = -\omega_0 u_x, \\ &\frac{\partial u_x}{\partial t} = \frac{e}{m} E_0. \end{aligned}$$

Exercise 6.3

A current I flows in the inner conductor of an infinitely long coaxial line and returns via the outer conductor. The radius of the inner conductor is a, and the inner and outer radii of the outer conductor are b and c, respectively. Find the magnetic flux density $\mathbf{B}$ for all regions and plot $|\mathbf{B}|$ versus r.

Answer:

Application of Ampere's circuital law.

$$0 \leq r \leq a, \quad \bar{B} = \frac{-}{a_\phi} \frac{\mu r I}{2\pi a^2}$$

$$a \leq r \leq b, \quad \bar{B} = \bar{a}_\phi \frac{\mu I}{2\pi r}$$

$$b \leq r \leq c, \quad \bar{B} = \bar{a}_\phi \left(\frac{c^2 - r^2}{c^2 - b^2}\right) \frac{\mu I}{2\pi r}$$

Exercise 6.4

A current I flows lengthwise in a very long, thin conducting sheet of width w, as shown in Fig. 6-35.

a) Assuming that the current flows into the paper, determine the magnetic flux density $\mathbf{B}_1$ at point $P_1(0, d)$.

b) Use the result in part (a) to find the magnetic flux density $\mathbf{B}_2$ at point $P_2(2w/3, d)$.

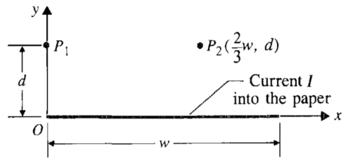


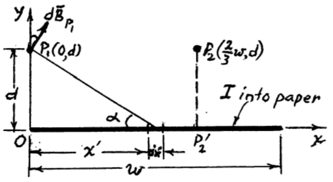
FIGURE 6–35
A thin conducting sheet carrying a current I (Problem P.6–4).

Answer:

$$
\begin{aligned}
&\text{a) Using } E_q(6-33c): \\
d\bar{B}_{p_1} &= \bar{a}_x dB_x + \bar{a}_y dB_y \\
&= \bar{a}_x \left(dB_{p_1} \sin\alpha + \bar{a}_y \left(dB_{p_1}\right)\cos\alpha,\right. \\
dB_{p_1} &= \frac{\mu_0 (I/w) dx'}{2\pi \left(x'^2 + d^2\right)^{3/2}} \\
\sin\alpha &= \frac{d}{\left(x'^2 + d^2\right)^{1/2}}, \cos\alpha = \frac{x'}{\left(x'^2 + d^2\right)^{1/2}} \\
\therefore \bar{B}_{p_1} &= \bar{a}_x B_x + \bar{a}_y B_y
\end{aligned}
$$

where

$$
\begin{aligned}
B_x &= \frac{\mu_0 I d}{2\pi w} \int_0^w \frac{dx'}{x'^2 + d^2} = \frac{\mu_0 I}{2\pi w} \tan^{-1}\left(\frac{w}{d}\right) \\
B_y &= \frac{\mu_0 I}{2\pi w} \int_0^w \frac{x' dx'}{x'^2 + d^2} = \frac{\mu_0 I}{4\pi w} \ln\left(1 + \frac{w}{d}\right).
\end{aligned}
$$



b) To find B at $P_2\left(\frac{2}{3}w, d\right)$, we add vectorially the contributions of the current strips to the right and to the left of point P_2' using the result in part (a)

$$
\begin{aligned}
\bar{B}_{P_2} &= \bar{B}_{2R} + \bar{B}_{2L}. \\
\bar{B}_{2R} &= \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x \tan^{-1}\left(\frac{(w)}{3d}\right) + \bar{a}_y \frac{1}{2} \ln\left(1 + \frac{w^2}{9d^2}\right)\right], \\
\bar{B}_{2L} &= \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x \tan^{-1}\left(\frac{2w}{3d}\right) - \bar{a}_y \frac{1}{2} \ln\left(1 + \frac{4w^2}{9d^2}\right)\right], \\
\therefore \bar{B}_{P_2} &= \frac{\mu_0 I}{2\pi w}\left[\bar{a}_x\left(\tan^{-1}\frac{w}{3d} + \tan^{-1}\frac{2w}{3d}\right) - \bar{a}_y \ln\sqrt{\frac{1 + (2w/3d)^2}{1 + (w/3d)^2}}\right].
\end{aligned}
$$

Exercise 6.5

A current I flows in a $w \times w$ square loop as in Fig. 6-36. Find the magnetic flux density at the off-center point $P(w/4, w/2)$.

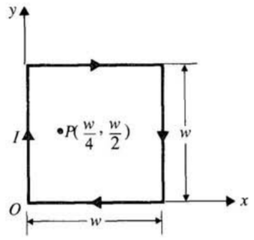
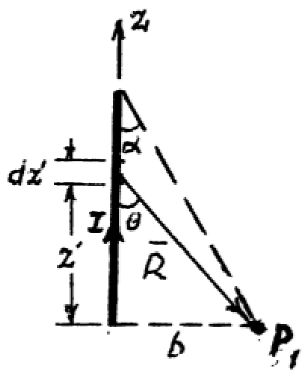
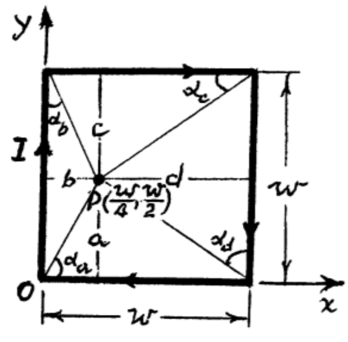


FIGURE 6–36
A square loop carrying a current I (Problem P.6–5).

Answer:





We first find $\bar{B}_{p_1}$ at P_1 flush with one end of a wire carrying a current I and making an angle α with the other end as shown.

$$
\begin{aligned}
d\bar{B}_{P_1} &= \frac{\mu_0 I}{4\pi R^2} d\bar{z}' \times \bar{a}_R & z' = b\cot\theta, dz' = -b\csc^2\theta d\theta, \\
&= \frac{\mu_0 I}{4\pi b}\left(-a_\phi \sin\theta d\theta\right), & R = b\csc\theta, \\
\bar{B}_{P_1} &= -\bar{a}_\phi \frac{\mu_0 I}{4\pi b}\int_{\pi/2}^{\alpha} \sin\theta d\theta & \bar{a}_z \times \bar{a}_R = \bar{a}_\phi \sin\theta. \\
&= \bar{a}_\phi \frac{\mu_0 I}{4\pi b}\cos\alpha.
\end{aligned}
$$

Applying the-above result to the four-sided loop at left, we have
$\bar{B}_p = \bar{a}_z \frac{\mu I}{4\pi}\left(\frac{1}{a}\cos\alpha_a + \frac{1}{b}\sin\alpha_a + \frac{1}{b}\cos\alpha_b + \frac{1}{c}\sin\alpha_b + \frac{1}{c}\cos\alpha_c + \frac{1}{a}\sin\alpha_c + \frac{1}{d}\cos\alpha_d + \frac{1}{a}\sin\alpha_d\right)$.
For this problem,

$$a = c = \frac{w}{2}, b = \frac{w}{4}, d = \frac{3}{4}w$$

$$\alpha_a = \tan^{-1} 2 = 63.4°, \quad \alpha_b = 90° - 63.4° = 26.6°; \alpha_c = \tan^{-1}\frac{2}{3} = 33.7°, \alpha_d = 56.3°$$

$$\bar{B}_p = \bar{a}_z 3.44 \frac{\mu_0 I}{\pi w}.$$

Exercise 6.6

Figure 6-37 shows an infinitely long solenoid with air core having a radius b and n closely wound turns per unit length. The windings are slanted at an angle α and carry a current I. Determine the magnetic flux density both inside and outside the solenoid.

FIGURE 6–37
A long solenoid with closely wound windings carrying a current I (Problem P.6–6).

Answer:
The problem can be decomposed in to two sub-problems (assuming b = radius of solenoid):

1. A cylindrical tube carrying a uniformly distributed longitudinal surface current $2\pi bnI \sin\alpha$.

$$\longrightarrow \bar{B}_1 = \begin{cases} 0, & 0 < r < b, \\ \bar{a}_\phi \frac{bnI}{r}\sin\alpha, & r > b, \end{cases}$$

2. A Solenoid with n turns per unit length carrying a current I $\cos\alpha$.

$$\longrightarrow \bar{B}_2 = \begin{cases} \bar{a}_z \mu_0 nI\cos\alpha, 0 < r < b, \\ 0, r > b. \end{cases}$$

Total $\bar{B} = \bar{B}_1 + \bar{B}_2$.

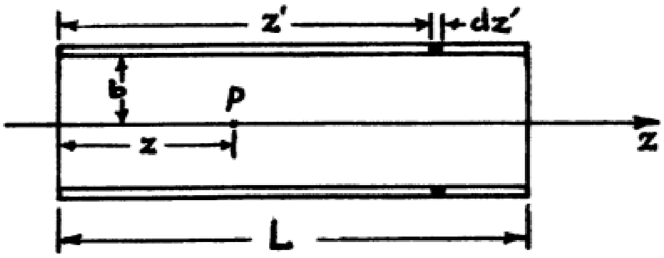
Exercise 6.7

Determine the magnetic flux density at a point on the axis of a solenoid with radius b and length L, and with a current I in its N turns of closely wound coil. Show that the result reduces to that given in Eq. (6-14)

$$B = \mu_0 n I$$

when L approaches infinity.

Answer:



$$\begin{aligned}
dB &= \frac{\mu_0 I b^2}{2\left[(z'-z)^2+b^2\right]^{3/2}}\left(\frac{N}{L}\right)dz', \\
B &= \frac{\mu_0 N I b^2}{2L}\int_0^L \frac{dz'}{\left[(z-z')^2+b^2\right]^{3/2}} \\
&= \frac{\mu_0 N I}{2L}\left[\frac{L-z}{\sqrt{(L-z)^2+b^2}}+\frac{z}{\sqrt{z^2+b^2}}\right] \\
&\longrightarrow \mu_0\left(\frac{N}{L}\right) I \text{ as } L\to\infty.
\end{aligned}$$

Direction of $\vec{B}$ is determined by the right-hand circle.

Exercise 6.8

Starting from the expression for vector magnetic potential $\mathbf{A}$ in Eq. (6-23)

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int_{V'}\frac{\mathbf{J}}{R}dv' \quad \text{(Wb/m)}$$

prove that

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int_{V'}\frac{\mathbf{J}\times\mathbf{a}_R}{R^2}dv'.$$

Furthermore, prove that $\mathbf{B}$ in Eq. (6-222) satisfies the fundamental postulates of magnetostatics in free space, Eqs. (6-6) and (6-7).

$$\begin{aligned}
\nabla\cdot\mathbf{B} &= 0 \\
\nabla\times\mathbf{B} &= \mu_0\mathbf{J}
\end{aligned}$$

Answer:
略

Exercise 6.9

Combine Eqs. $(6-4)$

$$\mathbf{F}_m = q\mathbf{u} \times \mathbf{B}$$

and $(6-33)$

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi}\left(\frac{d\ell' \times \mathbf{R}}{R^3}\right)$$

to obtain a formula for the magnetic force $\mathbf{F}_{12}$ exerted by a charge q_1 moving with a velocity $\mathbf{u}_1$ on a charge q_2 moving with a velocity $\mathbf{u}_2$.

Answer:
略

Exercise 6.10

A very long, thin conducting strip of width w lies in the xz-plane between $x = \pm w/2$. A surface current $\mathbf{J}_s = \mathbf{a}_z J_{s0}$ flows in the strip. Find the magnetic flux density at an arbitrary point outside the strip.

Answer:

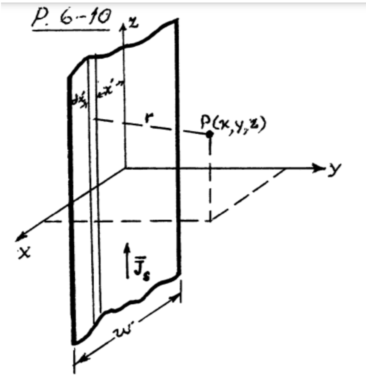
$$\bar{J}_s = \bar{a}_z J_{s0}.$$

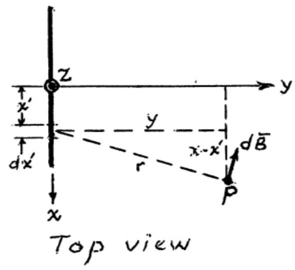
At $P(x, y, z)$ the magnetic flux density due to an infinitely long strip of width of x is

$$d\bar{B} = \frac{\mu_0 J_{s0} dx'}{2\pi r}\left(-\bar{a}_x \frac{y}{r} + \bar{a}_y \frac{x - x'}{r}\right),$$
$$r = \sqrt{(x - x')^2 + y^2}.$$
$$\therefore \quad \bar{B} = \int d\bar{B} = \bar{a}_x B_x + \bar{a}_y B_y,$$

where

$$\begin{aligned}
B_x &= -\frac{\mu_0 J_{s0} y}{2\pi}\int_{-w/2}^{w/2} \frac{dx'}{(x - x')^2 + y^2} \\
&= \frac{\mu_0 J_{s0}}{2\pi}\left[\tan^{-1}\left(\frac{x - \frac{w}{2}}{y}\right) - \tan^{-1}\left(\frac{x + \frac{w}{2}}{y}\right)\right], \\
B_y &= \frac{\mu_0 J_{s0}}{2\pi}\int_{-w/2}^{w/2} \frac{(x - x')\, dx'}{(x - x')^2 + y^2} \\
&= \frac{\mu_0 J_{s0}}{4\pi} \ln \frac{\left(x + \frac{w}{2}\right)^2 + y^2}{\left(x - \frac{w}{2}\right)^2 + y^2}.
\end{aligned}$$





Exercise 6.11

A long wire carrying a current I folds back with a semicircular bend of radius b as in Fig. 6-38. Determine the magnetic flux density at the center point P of the bend.

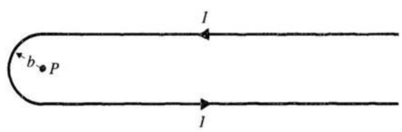


FIGURE 6–38
A very long wire with a semicircular bend (Problem P.6–11).

Answer:

This problem is a superposition of two problems: where $\bar{B} = B_1 + \bar{B}_2$,

1. $\bar{B}_1$ is the magnetic flux density at P due to two

Semi-infinite wires carrying equal and opposite currents. Assuming $\bar{a}_g$ points out of paper:q

$$\bar{B}_1 = \bar{a}_z \frac{\mu_0 I}{2\pi b}.$$

2. $\bar{B}_2$ is the magnetic flux density at P due to a half-circle. Taking one-half of the result in Eg.(6-38) for $z = 0$:

$$\bar{B}_2 = \bar{a}_z \frac{\mu_0 I}{4b}.$$

$$\therefore \quad \bar{B} = \bar{a}_z \frac{\mu_0 I}{2b} \left(\frac{1}{\pi} + \frac{1}{2} \right).$$

Exercise 6.12

Two identical coaxial coils, each of N turns and radius b, are separated by a distance d, as depicted in Fig. 6-39. A current I flows in each coil in the same direction.

a) Find the magnetic flux density $\mathbf{B} = \mathbf{a}_x B_x$ at a point midway between the coils.
b) Show that dB_x/dx vanishes at the midpoint.
c) Find the relation between b and d such that d^2B_x/dx^2 also vanishes at the midpoint. Such a pair of coils are used to obtain an approximately uniform magnetic field in the midpoint region. They are known as Helmholtz coils.

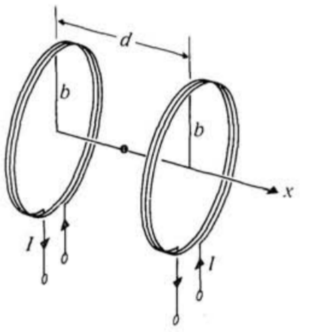


FIGURE 6–39
Helmholtz coils (Problems P.6–12).

Answer:

Use. Eq. (6-38) $$B_x = \frac{N\mu_0 I b^2}{2}\left\{\frac{1}{\left[\left(\frac{d}{2}+x\right)^2+b^2\right]^{3/2}} - \frac{1}{\left[\left(\frac{d}{2}-x\right)^2+b^2\right]^{3/2}}\right\}$$

a) At $x = 0$, $\quad B_x = \frac{N\mu_0 I b^2}{\left[\left(\frac{d}{2}\right)^2+b^2\right]^{3/2}}$.

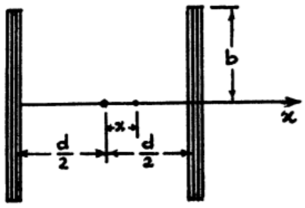
b)

$$\frac{dB_x}{dx} = \frac{N\mu_0 I b^2}{2}\left\{-\frac{3\left(\frac{d}{2}+x\right)^2}{\left[\left(\frac{d}{2}+x\right)^2+b^2\right]^{5/2}}\right.$$
$$\left.+\frac{3\left(\frac{d}{2}-x\right)}{\left[\left(\frac{d}{2}-x\right)^2+b^2\right]^{5/2}}\right\}.$$

At the midpoint, $x = 0$, $\frac{dB_x}{dx} = 0$.

c)

$$\frac{d^2B_k}{dx^2} = -\frac{3N\mu_0 I b^2}{2}\left\{\frac{1}{\left[\left(\frac{d}{2}+x\right)^2+b^2\right]^{5/2}} - \frac{5\left(\frac{d}{2}+x\right)^2}{\left[\left(\frac{d}{2}+x\right)^2+b^2\right]^{7/2}}\right.$$
$$\left.+\frac{1}{\left[\left(\frac{d}{2}-x\right)^2+b^2\right]^{5/2}} - \frac{5\left(\frac{d}{2}-x\right)^2}{\left[\left(\frac{d}{2}-x\right)^2+b^2\right]^{7/2}}\right\}.$$
$$A + x = 0, \frac{d^2B_x}{dx^2} = -3N\mu_0 I b^2\left\{\frac{b^2-4(d/2)^2}{\left[(d/2)^2+b^2\right]^{7/2}}\right\} \to 0, \text{ if } b = d.$$



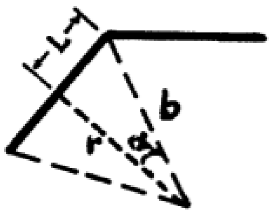
Exercise 6.13

A thin conducting wire is bent into the shape of a regular polygon of N sides. A current I flows in the wire. Show that the magnetic flux density at the center is

$$\mathbf{B} = \mathbf{a}_n \frac{\mu_0 N I}{2\pi b} \tan\frac{\pi}{N},$$

where b is the radius of the circle circumscribing the polygon and $\mathbf{a}_n$ is a unit vector normal to the plane of the polygon. Show also that, as N becomes very large, this result reduces to that given in Eq. $(6-38)$ with $z = 0$.

Answer:



Use Eq. $(6-35)$ for a wire of length $2L$

$$\bar{B} = \bar{a}_\phi \frac{\mu_0 I L}{2\pi r\sqrt{L^2 + r^2}}.$$

In this problem,

$$\alpha = \frac{\pi}{N}, \frac{L}{r} = \tan\alpha$$
$$= \tan\frac{\pi}{N}$$

$$\bar{B} = \bar{a}_n N\left(\frac{\mu_0 I L}{2\pi r b}\right) = \bar{a}_n \frac{\mu_0 N I}{2\pi b} \tan\frac{\pi}{N}$$

When N is very large, $\tan\frac{\pi}{N} \cong \frac{\pi}{N}, \bar{B} \to \bar{a}_n \frac{\mu_0 I}{2b}$
which is the -same as $E_q(6-38)$ with $z = 0$.

Exercise 6.14

Find the total magnetic flux through a circular toroid with a rectangular cross section of height h. The inner and outer radii of the toroid are a and b, respectively. A current

I flows in N turns of closely wound wire around the toroid. Determine the percentage of error if the flux is found by multiplying the cross-sectional area by the flux density at the mean radius.

Answer:

$$B_\phi = \frac{\mu_0 NI}{2\pi r}, \quad \Phi = \int_S B_\phi ds = \frac{\mu_0 NI}{2\pi}\int_a^b \frac{h}{r}dr = \frac{\mu_0 NIh}{2\pi}\ln\frac{b}{a}.$$

If B_ϕ at $r = \frac{a+b}{2}$ is used,

$$\Phi' = \frac{\mu_0 NIh}{\pi}\left(\frac{b-a}{b+a}\right)$$

$$\%\ \text{error} \ = \frac{\Phi'-\Phi}{\Phi}\times 100\% = \left[\frac{2(b-a)}{(b+a)\ln(b/a)} - 1\right]\times 100\%$$

Exercise 6.15

In certain experiments it is desirable to have a region of constant magnetic flux density. This can be created in an off-center cylindrical cavity that is cut in a very long cylindrical conductor carrying a uniform current density. Refer to the cross section in Fig. 6-40. The uniform axial current density is $\mathbf{J} = \mathbf{a}_z J$. Find the magnitude and direction of $\mathbf{B}$ in the cylindrical cavity whose axis is displaced from that of the conducting part by a distance d. (Hint: Use principle of superposition and consider $\mathbf{B}$ in the cavity as that due to two long cylindrical conductors with radii b and a and current densities $\mathbf{J}$ and $-\mathbf{J}$, respectively.)

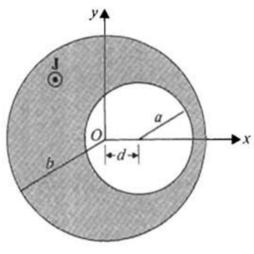


FIGURE 6–40
Cross section of a long cylindrical conductor with cavity (Problem P.6–15).

Answer:

$$\bar{J} = \bar{a}_z J, \quad \oint \bar{B}\cdot d\bar{l} = \mu_0 I.$$

If there is no hole.

$$2\pi r_1 B_{\phi 1} = \mu_0 \pi r_1^2 J$$

$$\longrightarrow B_{\phi 1} = \frac{\mu_0 r_1}{2}J \rightarrow \begin{cases} B_{x1} = -\frac{\mu_0 J}{2}y_1. \\ B_{y1} = +\frac{\mu_0 J}{2}x_1. \end{cases}$$

For - $\bar{J}$ in the hole portion:

$$B_{\phi_2} = -\frac{\mu_0 r_2}{2}J \rightarrow \begin{cases} B_{x_2} = +\frac{\mu_0 J}{2}y_2, \\ B_{y_2} = -\frac{\mu_0 J}{2}x_2. \end{cases}$$

Superposing B_{ϕ_1} and B_{ϕ_2} and noting that $y_1 = y_2$ and $x_1 = x_2 + d$

$$\text{we have } B_x = B_{x_1} + B_{x_2} = 0, \text{ and } B_y = B_{y1} + B_{yz} = \frac{\mu_0 J}{2} d$$

Exercise 6.16

Prove the following:
a) If Cartesian coordinates are used, Eq. (6-18) for the Laplacian of a vector field holds.
b) If cylindrical coordinates are used, $\nabla^2 \mathbf{A} \neq \mathbf{a}_r \nabla^2 A_r + \mathbf{a}_\phi \nabla^2 A_\phi + \mathbf{a}_z \nabla^2 A_z$.

Answer:
略

Exercise 6.17

The magnetic flux density $\mathbf{B}$ for an infinitely long cylindrical conductor has been found in Example 6-1. Determine the vector magnetic potential $\mathbf{A}$ both inside and outside the conductor from the relation $\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}$.

Answer:

$$\bar{B} = \bar{\nabla} \times \bar{A} \longrightarrow \bar{B} = \bar{a}_\phi B = \bar{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) = -\bar{a}_\phi \frac{\partial A_z}{\partial r}.$$

For $0 \leq r \leq b, E_q(6-10)$ gives $\bar{B}_1 = \bar{\alpha}_\phi \frac{\mu_0 I}{2nb^2} r$.
For $r \geq b, E_{q.}(6-11)$ gives $\bar{B}_2 = \bar{\alpha}_\phi \frac{\mu_0 I}{2\pi r}$.
Integrating,

$$\bar{A}_1 = \bar{a}_2 \left[-\frac{\mu_0 I}{4\pi} \left(\frac{r}{b}\right)^2 + c_1 \right], \quad 0 \leq r \leq b$$

$$\bar{A}_2 = \bar{a}_2 \left[-\frac{\mu_0 I}{2\pi} \ln r + c_2 \right], \quad r \geq b.$$

$$\text{At r=b, } \bar{A}_1 = \bar{A}_2 \longrightarrow c_2 = \frac{\mu_0 I}{4\pi}(2 \ln b - 1) + c_1$$

$$\therefore \bar{A}_2 = \bar{a}_z \left\{ -\frac{\mu_0 I}{4\pi} \left[\ln \left(\frac{r}{b}\right)^2 + 1 \right] + c_1 \right\}, r \geq b.$$

Exercise 6.18

Starting from the expression of $\mathbf{A}$ in Eq. (6-34) for the vector magnetic potential at a point in the bisecting plane of a straight wire of length $2L$ that carries a current I :

a) Find $\mathbf{A}$ at point $P(x, y, 0)$ in the bisecting plane of two parallel wires each of length $2L$, located at $y = \pm d/2$ and carrying equal and opposite currents, as shown in Fig. 6-41.

b) Find $\mathbf{A}$ due to equal and opposite currents in a very long two-wire transmission line.

c) Find $\mathbf{B}$ from $\mathbf{A}$ in part (b), and check your answer against the result obtained by applying Ampère's circuital law.

d) Find the equation for the magnetic flux lines in the xy-plane.

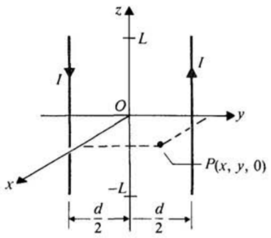


FIGURE 6–41
Parallel wires carrying equal and opposite currents (Problem P.6–18).

Answer:
Eq $E_q(6-34)$ for one wire:

$$\bar{A}=\bar{a}_z\frac{\mu_0 I}{4\pi}\ln\frac{\sqrt{L^2+r^2}+L}{\sqrt{L^2+r^2}-L}$$

For two wires carrying equal and opposite currents:
a)

$$\bar{A}=\bar{a}_z\frac{\mu_0 I}{4\pi}\ln\left[\frac{\sqrt{L^2+r_2^2}+L}{\sqrt{L^2+r_2^2}-L}\frac{\sqrt{L^2+r_1^2}-L}{\sqrt{L^2+r_1^2}+L}\right]=\bar{a}_z\frac{\mu_0 I}{2\pi}\ln\left[\frac{r_1}{r_2}\frac{\sqrt{L^2+r_2^2}-L}{\sqrt{L^2+r_1^2}+L}\right]$$

b) For a very long two-wire transmission line, $L\rightarrow\infty$:

$$\bar{A}=\bar{a}_z\frac{\mu_0 I}{2\pi}\ln\left(\frac{r_1}{r_2}\right)=\bar{a}_z\frac{\mu_0 I}{4\pi}\ln\frac{\left(\frac{d}{2}+y\right)^2+x^2}{\left(\frac{d}{2}-y\right)^2+x^2}.$$

c)

$$\bar{B}=\bar{\nabla}\times\bar{A}=\bar{a}_x\frac{\partial A_z}{\partial y}-\bar{a}_y\frac{\partial A_z}{\partial x}$$

$$=\bar{a}_x\frac{\mu_0 I}{2\pi}\left[\frac{\frac{d}{2}+y}{\left(\frac{d}{2}+y\right)^2+x^2}-\frac{\frac{d}{2}-y}{\left(\frac{d}{2}-y\right)^2+x^2}\right]-\bar{a}_y\frac{\mu_0 I}{2\pi}\left[\frac{x}{\left(\frac{d}{2}+y\right)^2+x^2}-\frac{x}{\left(\frac{d}{2}-y\right)^2+x^2}\right]$$

$$=\frac{\mu_0 I}{2\pi}\left[-\bar{a}_{\phi_1}\frac{1}{r_1}-\bar{a}_{\phi_2}\frac{1}{r_2}\right]$$

d) To find the equation for magnetic flux lines:

$$\frac{dx}{B_x}=\frac{d_y}{B_y}\longrightarrow\frac{\partial A}{\partial x}dx+\frac{\partial A}{\partial y}dy=0$$

$$\longrightarrow dA=0\longrightarrow A=\text{ constant}$$

Thus,

$$\frac{r_1^2}{r_2^2}=\frac{\left(\frac{d}{2}+y\right)^2+x^2}{\left(\frac{d}{2}-y\right)^2+x^2}=K$$

Exercise 6.19

For the small rectangular loop with sides a and b that carries a current I, shown in Fig. 6-42:

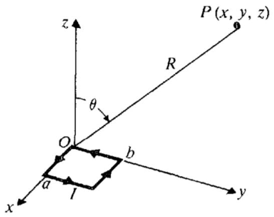


FIGURE 6–42
A small rectangular loop carrying a current I
(Problem P.6–19).

a) Find the vector magnetic potential $\mathbf{A}$ at a distant point, $P(x, y, z)$. Show that it can be put in the form of Eq. (6-45).

b) Determine the magnetic flux density $\mathbf{B}$ from $\mathbf{A}$, and show that it is the same as that given in Eq. $(6-48)$

Answer:
略

Exercise 6.20

For a vector field $\mathbf{F}$ with continuous first derivatives, prove that

$$\int_V (\nabla \times \mathbf{F}) dv = -\oint_S \mathbf{F} \times d\mathbf{s}$$

where S is the surface enclosing the volume V. (Hint: Apply the divergence theorem to $(F \times C)$, where C is a constant vector.)

Answer:
Apply divergence theorem to $(\bar{F} \times \bar{C})$, where $\bar{C}$ is a constant vector.

$$\int_v \bar{\nabla} \cdot (\bar{F} \times \bar{C}) dv = \oint_S (\bar{F} \times \bar{C}) \cdot ds.$$

Now, from problem P.2-33: $\bar{\nabla} \cdot (\bar{F} \times \bar{C}) = \bar{C} \cdot (\bar{\nabla} \times \bar{F}) - \bar{F} \cdot (\bar{\nabla} \times \bar{C})$

$$= \bar{C} \cdot (\bar{\nabla} \times \bar{F});$$

$$\text{from } E_q \cdot (2-19) : (\bar{F} \times \bar{C}) \cdot d\bar{s} = -\bar{C} \cdot (\bar{F} \times d\bar{s})$$

Substituting (2) and (3), in (1):

$$\bar{C} \cdot \int (\bar{\nabla} \times \bar{F}) dv = -c \cdot \oint_S (\bar{F} \times d\bar{s}) \rightarrow \int_v (\bar{\nabla} \times \bar{F}) dv = -\oint \bar{F} \times d\bar{s}$$

Exercise 6.21

A very large slab of material of thickness d lies perpendicularly to a uniform magnetic field of intensity $\mathbf{H}_0 = \mathbf{a}_z H_0$. Ignoring edge effect, determine the magnetic field intensity in the slab:

a) if the slab material has a permeability μ,

b) if the slab is a permanent magnet having a magnetization vector $\mathbf{M}_i = \mathbf{a}_z M_i$.

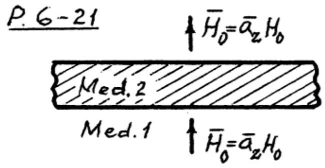
Answer:

a) Given $\bar{B}_2 = \mu_2 \bar{H}_2$

$$B_{2z} = B_{1z} \to \mu_2 H_2 = \mu_0 H_0 \to \bar{H}_2 = \bar{a}_z H_2 = \bar{a}_z \frac{\mu_0}{\mu} H_0.$$

b) Given $\bar{B}_2 = \mu_0 \left(\bar{H}_2 + \bar{M}_i\right)$.

$$B_{2z} = B_{1z} \to \mu_0 \left(H_2 + M_i\right) = \mu_0 H_0 \to \bar{H}_2 = \bar{a}_z \left(H_0 - M_i\right).$$



Exercise 6.22

A circular rod of magnetic material with permeability μ is inserted coaxially in the long solenoid of Fig. 6-4. The radius of the rod, a, is less than the inner radius, b, of the solenoid. The solenoid's winding has n turns per unit length and carries a current I.

a) Find the values of $\mathbf{B}, \mathbf{H}$, and $\mathbf{M}$ inside the solenoid for $r < a$ and for $a < r < b$.

b) What are the equivalent magnetization current densities $\mathbf{J}_m$ and $\mathbf{J}_{ms}$ for the magnetized rod?

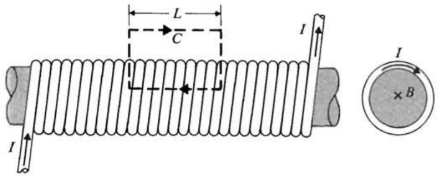


FIGURE 6–4
A current-carrying long solenoid (Example 6–3).

Answer:

$$\text{a) } r < a: \left.\begin{aligned} \bar{H} &= \bar{a}_z nI \\ \bar{B} &= \bar{a}_z \mu nI, \end{aligned}\right\} Eq.(6-14).$$

$$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_z \left(\frac{\mu}{\mu_0} - 1\right) nI.$$

$$a < r < b: \bar{H} = \bar{a}_z nI,$$
$$\bar{B} = \bar{a}_z \mu_0 nI,$$
$$\bar{M} = 0.$$

b) $\bar{J}_m = \bar{\nabla} \times \bar{M} = 0; \quad \bar{J}_{ms} = \bar{M} \times \bar{a}_n = \left(\bar{a}_z \times \bar{a}_r\right) \left(\frac{\mu}{\mu_0} - 1\right) nI = \bar{a}_\phi \left(\frac{\mu}{\mu_0} - 1\right) nI.$

Exercise 6.23

The scalar magnetic potential, V_m, due to a current loop can be obtained by first dividing the loop area into many small loops and then summing up the contribution of these small loops (magnetic dipoles); that is,

$$V_m = \int dV_m = \int \frac{d\mathbf{m} \cdot \mathbf{a}_R}{4\pi R^2},$$

where Prove that

$$d\mathbf{m} = \mathbf{a}_n I ds$$

$$V_m = -\frac{I}{4\pi}\Omega,$$

where Ω is the solid angle subtended by the loop surface at the field point P (see Fig. 6-43).

Answer:
略

Exercise 6.24

Do the following by using Eq. $(6-224)$:

a) Determine the scalar magnetic potential at a point on the axis of a circular loop having a radius b and carrying a current I.

b) Obtain the magnetic flux density $\mathbf{B}$ from $-\mu_0 \nabla V_m$, and compare the result with Eq. (6-38).

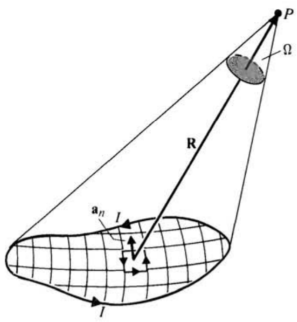


FIGURE 6–43
Subdivided current loop for determination of scalar magnetic potential (Problem P.6–23).

Answer:
a)

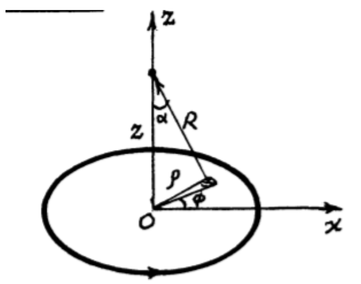
$$V_m = \frac{I}{4\pi}\int \frac{d\bar{s}\cdot\bar{a}_R}{R^2} = \frac{I}{4\pi}\Omega,$$

$$d\bar{s}\cdot\bar{a}_R = (\cos\alpha)\rho d\rho d\phi = \frac{z}{\sqrt{z^2+\rho^2}\rho d\rho d\phi,}$$

$$R = \sqrt{z^2+\rho^2}\cdot$$

$$\therefore V_m = \frac{I}{4\pi}\int_0^{2\pi}\int_0^b \frac{z}{(z^2+\rho^2)^{3/2}}\rho d\rho d\phi$$

$$= \frac{I}{2}\left(1 - \frac{z}{\sqrt{z^2+\rho^2}}\right).$$



b) $\bar{B}=-\mu_0\bar{\nabla}V_m=-\bar{a}_z\mu_0\frac{\partial V_m}{\partial z}=\bar{a}_z\frac{\mu_0 I b^2}{2\left(z^2+b^2\right)^{3/2}}$, which is the same as Eq 6.38

Exercise 6.25

Solve the cylindrical bar magnet problem in Example 6-9, using the equivalent magnetization current density concept.

EXAMPLE 6-9 A cylindrical bar magnet of radius b and length L has a uniform magnetization $\mathbf{M}=\mathrm{a}_z M_0$ along its axis. Use the equivalent magnetization charge density concept to determine the magnetic flux density at an arbitrary distant point.

Answer:

A cylindrical bar magnet having a uniform magnetization $\bar{M}=\bar{a}_z M_0$ is equivalent to a $\bar{J}_m=\bar{\nabla}\times\bar{M}=0$ and a $\bar{J}_{ms}=\bar{M}\times\bar{a}_n=(\bar{a}_z M_0)\times\bar{a}_r=\bar{a}_\phi M_0$ on the cylinder wall. At a distant point, $\bar{B}$ due to this $\bar{J}_{\mathrm{ms}}$ flowing on a cylindrical wall of length L and radius b is the same as that due to a circular loop of radials b carrying a current $I=M_0 L$. It is given by Eq. $(6-44)$, which is the same as Eq. $(6-73)$ obtained in Example 6-9 which the total dipole moment of the cylindrical magnet is $M_T=I\pi b^2=M_0 L\pi b^2$.

Exercise 6.26

A ferromagnetic sphere of radius b is magnetized uniformly with a magnetization $\mathbf{M}=\mathbf{a}_z M_0$.

a) Determine the equivalent magnetization current densities $\mathbf{J}_m$ and $\mathbf{J}_{ms}$.

b) Determine the magnetic flux density at the center of the sphere.

Answer:

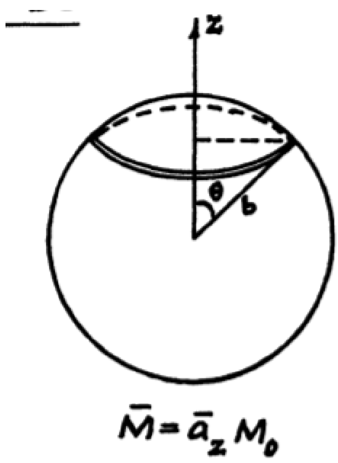
a) $\bar{J}_m=\bar{\nabla}\times\bar{M}=0$.

$$\begin{aligned}\bar{J}_{ms}&=\left(\bar{a}_R\cos\theta-\bar{a}_\theta\sin\theta\right)M\times\bar{a}_R\\&=\bar{a}_\phi M_0\sin\theta.\end{aligned}$$

b) Apply Eq. (6-38) to a loop of radius $b\sin\theta$ carrying a current $J_{m\leq}bd\theta$:

$$\begin{aligned}d\bar{B}&=\bar{a}_z\frac{\mu_0\left(J_{ms}bd\theta\right)(b\sin\theta)^2}{2\left(b^2\right)^{3/2}}\\&=\bar{a}_z\frac{\mu_0 M_0}{2}\sin^3\theta.\end{aligned}$$

$$\bar{B}=\int d\bar{B}=\bar{a}_2\frac{\mu_0 M_0}{2}\int_0^\pi\sin^3\theta d\theta=\bar{a}_2\frac{2}{3}\mu_0 M_0=\frac{2}{3}\mu_0\bar{M}.$$



Exercise 6.27

A toroidal iron core of relative permeability 3000 has a mean radius $R = 80(\text{ mm})$ and a circular cross section with radius $b = 25(\text{ mm})$. An air gap $\ell_g = 3(\text{ mm})$ exists, and a current I flows in a 500 -turn winding to produce a magnetic flux of $10^{-5}(\text{ Wb})$. (See Fig. 6-44.) Neglecting flux leakage and using mean path length, find

a) the reluctances of the air gap and of the iron core,

b) $\mathbf{B}_g$ and $\mathbf{H}_g$ in the air gap, and $\mathbf{B}_c$ and $\mathbf{H}_c$ in the iron core,

c) the required current I.

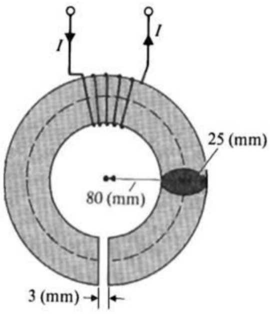


FIGURE 6–44
A toroidal iron core with air gap (Problem P.6–27).

Answer:

$$\text{a) } \phi_g = \frac{l_g}{\mu_0 S} = \frac{3 \times 10^{-3}}{4\pi \times 10^{-7} \times (\pi \times 0.025)^2} = 1.21 \times 10^6 \left(\mathrm{H}^{-1}\right),$$

$$\alpha_c = \frac{2\pi \times 0.08 - 0.003}{3000 \times (4\pi \times 10^{-7}) \times (\pi \times 0.025)^2} = 6.75 \times 10^4 \left(\mathrm{H}^{-1}\right).$$

$$\text{b) } \bar{B}_g = \bar{B}_c = \bar{a}_\phi \frac{10^{-5}}{\pi \times 0.025^2} = \bar{a}_\phi 5.09 \times 10^{-3}(T),$$

$$\bar{H}_g = \frac{1}{\mu_0}\bar{B}_g = \bar{a}_\phi \frac{5.09 \times 10^{-3}}{4\pi \times 10^{-7}} = \bar{a}_\phi 4.05 \times 10^3(\text{ A/m}),$$

$$\bar{H}_c = \frac{1}{\mu_0 \mu_\mu}\bar{B}_c = \bar{a}_\phi \frac{4.05 \times 10^3}{3000} = \bar{a}_\phi 1.35(\text{ A/m}).$$

$$\text{c) NI } = \Phi\left(\phi_c + \phi_g\right), \quad I = \frac{1}{N}\Phi\left(\phi_c + \phi_g\right) = 0.0256(\text{ A}) = 25.6(\text{ mA}).$$

Exercise 6.28

Consider the magnetic circuit in Fig. 6-45. A current of 3 (A) flows through 200 turns of wire on the center leg. Assuming the core to have a constant cross-sectional area of 10^{-3} (m^2) and a relative permeability of 5000 :

a) Determine the magnetic flux in each leg.

b) Determine the magnetic field intensity in each leg of the core and in the air gap.

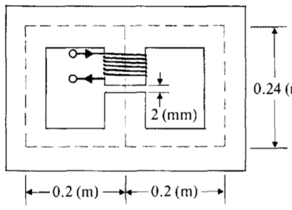


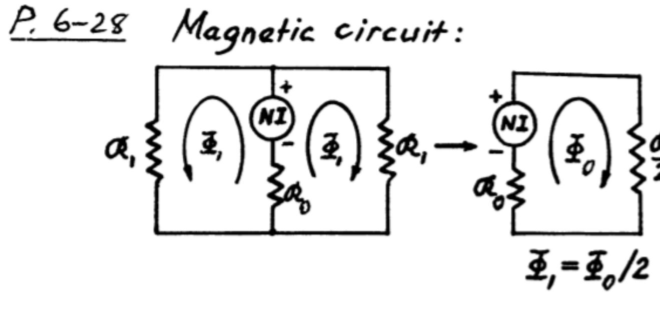
FIGURE 6–45
A magnetic circuit with air gap (Problem P.6–28).

Answer:

$$\frac{1}{\mu_0 S}=\frac{1}{\left(4 \pi 10^{-7}\right) \times 10^{-3}}=7.45 \times 10^5$$

Neglecting leakage flux and assuming constant flux density over s :

$$\begin{aligned}
\phi_0 & =\frac{0.002}{\mu_0 S}-\frac{0.24-0.002}{\mu_0 \mu_r S} \\
& =1.60 \times 10^6\left(H^{-1}\right)
\end{aligned}$$



$$\phi_1=\frac{0.24+2 \times 0.2}{\mu_0 \mu_r S}=0.102 \times 10^6\left(\mathrm{H}^{-1}\right) .$$

a) $\Phi_0=\frac{N I}{\phi_0+\phi_1 / 2}=3.63 \times 10^{-4}(\mathrm{~Wb}) ; \quad \Phi_1=\frac{\Phi_0}{2}=1.82 \times 10^{-4}(\mathrm{~Wb})$.

b) $H_1=\frac{\Phi_1}{\mu_0 \mu_{r,} S}=28.9(\mathrm{~A} / \mathrm{m})$,

$$\begin{aligned}
& \left(H_0\right)_g=\frac{1}{\mu_0 s} \Phi_0=28.9 \times 10^4(\mathrm{~A} / \mathrm{m}) \text { in air gap, } \\
& \left(H_0\right)_c=\left(H_0\right)_g / \mu_r=57.8(\mathrm{~A} / \mathrm{m}) .
\end{aligned}$$

Exercise 6.29

Consider an infinitely long solenoid with n turns per unit length around a ferromagnetic core of cross-sectional area S. When a current is sent through the coil to create a magnetic field, a voltage $v_1 = -nd\Phi/dt$ is induced per unit length, which opposes the current change. Power $P_1 = -v_1 I$ per unit length must be supplied to overcome this induced voltage in order to increase the current to I.

a) Prove that the work per unit volume required to produce a final magnetic flux density B_f is

$$W_1 = \int_0^{B_f} HdB.$$

b) Assuming that the current is changed in a periodic manner such that B is reduced from B_f to $-B_f$ and then is increased again to B_f, prove that the work done per unit volume for such a cycle of change in the ferromagnetic core is represented by the area of the hysteresis loop of the core material.

Answer:

a) Work required per unit length in time dt:

$$P_1 dt = nId\Phi$$

Work per unit volume in d t:

$$dW = \frac{1}{s}P_1 dt = nIdB = HdB$$

Thus,

$$W_1 = \int_0^{B_f} HdB$$

b)

略

Exercise 6.30

Prove that the relation $\mathbf{\nabla} \times \mathbf{H} = \mathbf{J}$ leads to Eq. (6-111)

$$\mathbf{a}_{n2} \times (\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$$

at an interface between two media.

Answer:

略

Exercise 6.31

What boundary conditions must the scalar magnetic potential V_m satisfy at an interface between two different magnetic media?

Answer:

$\bar{H}_1 = -\bar{\nabla} V_{m_1}, \quad \bar{H}_2 = -\bar{\nabla} V_{m_2}.$

Boundary conditions: $\mu_1 H_{1n} = \mu_2 H_{2n} \longrightarrow \mu_1 \frac{\partial V_{m1}}{\partial n} = \mu_2 \frac{\partial V_{m2}}{\partial n}$,

$$H_{1t} = H_{2t} \longrightarrow \begin{array}{c} V_{m_1} = V_{m_2} \\ \text{(assuming absence of current)} \end{array}$$

Exercise 6.32

Consider a plane boundary ($y = 0$) between air (region $1, \mu_{r1} = 1$) and iron (region $2, \mu_{r2} = 5000$)

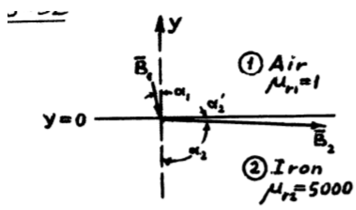
a) Assuming $\mathbf{B}_1 = \mathbf{a}_x 0.5 - \mathbf{a}_y 10(\mathrm{mT})$, find $\mathbf{B}_2$ and the angle that $\mathbf{B}_2$ makes with the interface.

b) Assuming $\mathbf{B}_2 = \mathbf{a}_x 10 + \mathbf{a}_y 0.5(\mathrm{mT})$, find $\mathbf{B}_1$ and the angle that $\mathbf{B}_1$ makes with the normal to the interface.

Answer:

a)

$$\begin{aligned}
\bar{B}_1 &= \bar{a}_x 0.5 - \bar{a}_y 10(\mathrm{mT}), \\
\bar{B}_2 &= \bar{a}_x B_{2x} - \bar{a}_y B_{2y}. \\
H_{2x} &= \frac{B_{2x}}{5000\mu_0} = H_{1x} = \frac{0.5}{\mu_0} \\
&\longrightarrow B_{2x} = 2,500(\mathrm{mT}), \\
B_{2y} &= B_{1y} = -10(\mathrm{mT}). \\
\therefore \bar{B}_2 &= \bar{a}_x 2500 - \bar{a}_y 10(\mathrm{mT}).
\end{aligned}$$



$$\tan\alpha_2 = \frac{\mu_2}{\mu_1}\tan\alpha_1 = 5000\left(\frac{B_{1x}}{B_{1y}}\right) = 250 \longrightarrow \alpha_2 = 89.8^\circ, \alpha_2' = 0.2^\circ.$$

b) If $\bar{B}_2 = \bar{a}_x 10 + \bar{a}_y 0.5(\mathrm{mT})$, $\quad \bar{B}_1 = \bar{a}_x B_{1x} + \bar{a}_y B_{1y}$

$$\begin{aligned}
H_{1x} &= \frac{B_{1x}}{\mu_1} = H_{2x} = \frac{B_{2x}}{\mu_2} \longrightarrow B_{1x} = \frac{1}{\mu_{r2}} B_{2x} = \frac{10}{5000} = 0.002. \\
B_{1y} &= B_{2y} = 0.5. \\
\alpha_1 &= \tan^{-1}\frac{B_{1x}}{B_{1y}} \simeq \frac{0.002}{0.5} = 0.004(\mathrm{rad}) = 0.23^\circ
\end{aligned}$$

$$\therefore \bar{B}_1 = \bar{a}_x 0.002 + \bar{a}_y 0.5(\mathrm{mT})$$

Exercise 6.33

The method of images can also be applied to certain magnetostatic problems. Consider a straight, thin conductor in air parallel to and at a distance d above the plane interface of a magnetic material of relative permeability μ_r. A-current I flows in the conductor.

a) Show that all boundary conditions are satisfied if

i) the magnetic field in the air is calculated from I and an image current I_i,

$$I_i = \left(\frac{\mu_r - 1}{\mu_r + 1}\right) I,$$

and these currents are equidistant from the interface and situated in air;

ii) the magnetic field below the boundary plane is calculated from I and $-I_i$, both at the same location. These currents are situated in an infinite magnetic material of relative permeability μ_r.

b) For a long conductor carrying a current I and for $\mu_r \gg 1$, determine the magnetic flux density $\mathbf{B}$ at the point P in Fig. 6-46.

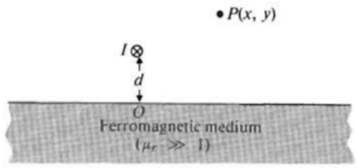


FIGURE 6–46
A current-carrying conductor near a ferromagnetic medium (Problem P.6–33).

Answer:

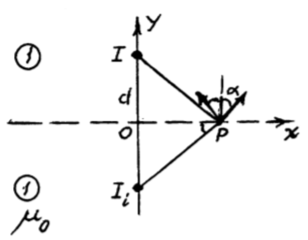
a) Consider two situations: (1) I and I_i both in air; and (2) I and $-I_i$ both in magnetic medium with relative permeability μ_r.

Find B_{1y} and H_{1x} at $P(y=0)$.

$$B_{1y} = \frac{\mu_0}{2\pi r}(I + I_i)\cos\alpha = \frac{\mu_0\mu_r}{\pi(\mu_r+1)}\frac{x}{r^2}I,$$

$$B_{1x} = \frac{\mu_0}{2\pi r}(I - I_i)\sin\alpha = \frac{\mu_0}{\pi(\mu_r+1)}\frac{d}{r^2}I,$$

$$H_{1x} = \frac{B_{1x}}{\mu_0} = \frac{I}{\pi(\mu_r+1)}\frac{d}{r^2}.$$

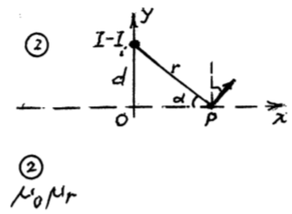


Find B_{2y} and H_{2x} at $P(x=0)$.

$$B_{2y} = \frac{\mu_0\mu_r}{2\pi r}(I - I_i)\cos\alpha = \frac{\mu_0\mu_r}{\pi(\mu_r+1)}\frac{x}{r^2}I,$$

$$B_{2x} = \frac{\mu_0\mu_r}{2\pi r}(I - I_i)\sin\alpha = \frac{\mu_0\mu_r}{\pi(\mu_r+1)}\frac{d}{r^2}I,$$

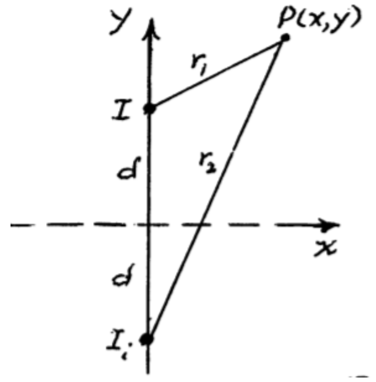
$$H_{2x} = \frac{B_{2x}}{\mu_0\mu_r} = \frac{I}{\pi(\mu_r+1)}\frac{d}{r^2}.$$



$\therefore B_{1y} = B_{2y}$ and $H_{1x} = H_{2x}$ (Boundary conditions satisfied)

b) For $\mu_r \gg 1, \quad I_i = \frac{\mu_r - 1}{\mu_r + 1} I \cong I$. Refer to the following figure.

$$
\begin{aligned}
\bar{B}_I =& \frac{\mu_0 I}{2\pi r_1}\left(-\bar{a}_x \frac{y-d}{r_1} + \bar{a}_y \frac{x}{r_1}\right), \\
\bar{B}_{I_i} =& \frac{\mu_0 I}{2\pi r_2}\left(-\bar{a}_x \frac{y+d}{r_2} + \bar{a}_y \frac{x}{r_2}\right) \\
\therefore \quad \bar{B} =& \bar{B}_I + \bar{B}_{I_i} \\
=& -\bar{a}_x \frac{\mu_0 I}{2\pi}\left[\frac{y-d}{(y-d)^2 + x^2} + \frac{y+d}{(y+d)^2 + x^2}\right] \\
& + \bar{a}_y \frac{\mu_0 I}{2\pi}\left[\frac{1}{(y-d)^2 + x^2} + \frac{1}{(y+d)^2 + x^2}\right]
\end{aligned}
$$



Exercise 6.34

A very long conductor in free space carrying a current I is parallel to, and at a distance d from, an infinite plane interface with a medium.

a) Discuss the behavior of the normal and tangential components of **B** and **H** at the interface:

i) if the medium is infinitely conducting;

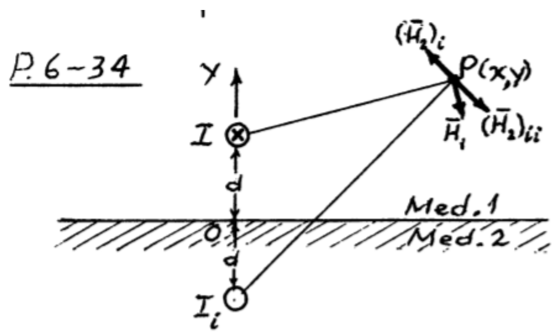
ii) if the medium is infinitely permeable.

b) Find and compare the magnetic field intensities **H** at an arbitrary point in the free space for the two cases in part (a).

c) Determine the surface current densities at the interface, if any, for the two cases.

Answer:

a)



(i) If $\sigma_2 \to \infty$, $\quad \bar{B}_2 = \bar{H}_2 = 0$. B_n continuous $\to B_{1n} = H_{1n} = 0$; $\bar{a}_y \times \bar{H}_1 = \bar{J}_s \longrightarrow \tilde{J}_s = -\bar{a}_z H_{1x}$. Image $I_i(= -I)$ flowing out of the paper.

(ii) If $\mu_2 \to \infty, \bar{H}_2 = 0$, but $\bar{B}_2$ is finite. No surface current. $\longrightarrow H_{1t} = H_{2t} = 0$; B_n continuous $\longrightarrow B_{1n} = B_{2n}$. Image $I_i(= I)$ flowing into the paper.

b)

(i)

$$\bar{H}_p = \bar{H}_1 + \left(\bar{H}_2\right)_i, \text{where } \bar{H}_1 = \frac{I}{2\pi}\left[\bar{a}_x \frac{y-d}{x^2+(y-d)^2} - \bar{a}_y \frac{x}{x^2+(y-d)^2}\right]$$

$$\left(\bar{H}_2\right)_i = \frac{I}{2\pi}\left[-\bar{a}_x \frac{y+d}{x^2+(y+d)^2} + \bar{a}_y \frac{x}{x^2+(y+d)^2}\right]$$

(ii)

$$\bar{H}_p' = \bar{H}_1 + \left(\bar{H}_2\right)_{ii} = \bar{H}_1 - \left(\bar{H}_2\right)_i$$

c)

(i)

$$\bar{J}_s = -\,\bar{a}_z \left(H_p\right)_x\Big|_{y=0} = \bar{a}_z\left(\frac{Id}{x^2+d^2}\right)$$

(ii)

$$\bar{J}_s = 0$$

Exercise 6.35

Determine the self-inductance of a toroidal coil of N turns of wire wound on an air frame with mean radius r_o and a circular cross section of radius b. Obtain an approximate expression assuming $b \ll r_o$.

Answer:

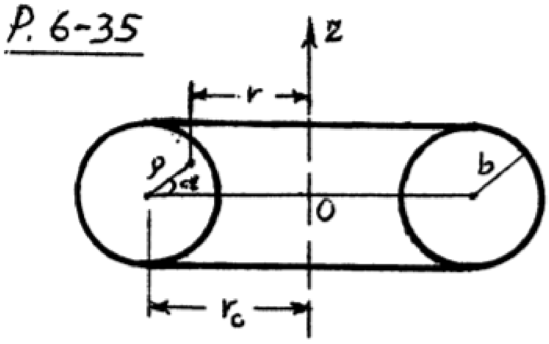
$$\bar{B} = \bar{a}_\phi B_\phi = \bar{a}_\phi \frac{\mu_0 NI}{2\pi r}, \quad r = r_0 - \rho\cos\alpha$$

$$\Phi = \frac{\mu_0 NI}{2\pi}\int_0^b \int_0^{2\pi} \frac{\rho d\alpha d\rho}{r_0 - \rho\cos\alpha} = \mu_0 NI\left(r_0 - \sqrt{r_0^2 - b^2}\right).$$

$$\therefore L = \frac{N\Phi}{I} = \mu_0 N^2\left(r_0 - \sqrt{r_0^2 - b^2}\right).$$

$$\text{If } r_0 \gg b, B_\phi \cong \frac{\mu_0 NI}{2\pi r_0}(\text{constant}).$$

$$\bar{\Phi} \cong B_\phi s = B_\phi\left(\pi b^2\right) = \frac{\mu_0 N b^2 I}{2r_0} \to L \cong \frac{\mu_0 N^2 b^2}{2r_0}.$$



Exercise 6.36

Refer to Example 6-16. Determine the inductance per unit length of the air coaxial transmission line assuming that its outer conductor is not very thin but is of a thickness d.

EXAMPLE 6-16 An air coaxial transmission line has a solid inner conductor of radius a and a very thin outer conductor of inner radius b. Determine the inductance per unit length of the line.

Answer:
For

$$b \leq r \leq (b+d), \bar{B}_3 = \bar{a}_\phi B_{3\phi} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r}\left[1 - \frac{\pi\left(r^2 - b^2\right)}{\pi(b+d)^2 - \pi b^2}\right]$$
$$= \bar{a}_\phi \frac{\mu_0 I}{2\pi r}\left[\frac{(b+d)^2 - r^2}{(b+d)^2 - b^2}\right]$$

Magnetic energy per unit length stored in the outer conductor,

$$W'_m = \frac{1}{2\mu_0}\int_b^{b+d} B_{3\phi}^2 2\pi r dr$$

$$= \frac{\mu_0 I^2}{4\pi}\left\{\frac{(b+d)^4}{\left[(b+d)^2 - b^2\right]^2}\ln\left(1 + \frac{d}{b}\right) + \frac{b^2 - 3(b+d)^2}{4\left[(b+d)^2 - b^2\right]}\right\}$$

From Eqs. $(6-175), (6-176a)$, and $(6-176b)$ we have

$$L' = \frac{2}{I^2}\left(W'_{m_1} + W'_{m_2} + W'_{m_3}\right)$$
$$= \frac{\mu_0}{2\pi}\left\{\frac{1}{4} + \ln\frac{b}{a} + \frac{(b+d)^4}{\left[(b+d)^2 - b^2\right]}\ln\left(1 + \frac{d}{b}\right) - \frac{b^2 - 3(b+d)^2}{4\left[(b+d)^2 - b^2\right]}\right\} (H/m).$$

Exercise 6.37

Calculate the mutual inductance per unit length between two parallel two-wire transmission lines $A - A'$ and $B - B'$ separated by a distance D, as shown in Fig. 6-47. Assume the wire radius to be much smaller than D and the wire spacing d.

Answer:
$\bar{B}$ at a distance r from an infinitely long line carrying a current $I : \bar{B} = \bar{a}_\phi \frac{\mu I}{2\pi r}$.

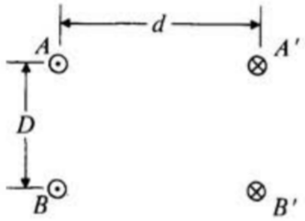


FIGURE 6–47
Coupled two-wire transmission lines (Problem P.6–37).

For a unit length the flux due to I in line A that links with the second line pair $B-B'$ is

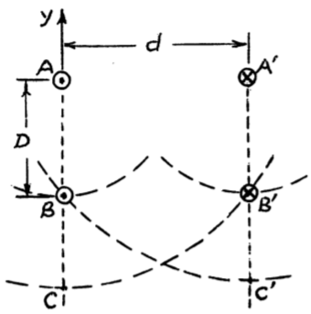
$$\Phi'_A = \frac{\mu_0 I}{2\pi}\int_{AB}^{AC}\frac{dr}{r} = \frac{\mu_0 I}{2\pi}\ln\frac{AC}{AB}.$$

That due to I in line A' is

$$\Phi'_{A'} = \frac{\mu_0 I}{2\pi}\ln\frac{A'C'}{A'B'}$$

Total flux linkage per 4 it length

$$\begin{aligned}\Lambda'_{12} = \Phi'_A + \Phi'_{A'} &= \frac{\mu_0 I}{2\pi}\ln\frac{(AC)\,(A'C')}{(AB)\,(A'B')}.\\ &= \frac{\mu_0 I}{2\pi}\ln\frac{(AB')\,(A'B)}{(AB)\,(A'B')} = \frac{\mu_0 I}{2\pi}\ln\frac{D^2+d^2}{D^2}\end{aligned}$$



$$\therefore M'_{12} = \frac{\Lambda'_{12}}{I} = \frac{\mu_0}{2\pi}\ln\left(1+\frac{d^2}{D^2}\right)$$

Exercise 6.38

Determine the mutual inductance between a very long, straight wire and a conducting equilateral triangular loop, as shown in Fig. 6-48.

Answer:

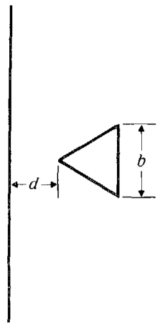


FIGURE 6–48
A long, straight wire and a conducting equilateral triangular loop (Problem P.6–38).

For I in the long straight wire, $\bar{B} = \bar{a}_\phi \frac{\mu_0 I}{2\pi r}$.

$$\Lambda_{12} = \int_S \bar{B} \cdot d\bar{s} = \int B_\phi \frac{2}{\sqrt{3}}(r-d)dr = \frac{\mu_0 I}{\pi T \sqrt{3}} \int_d^{d+\frac{\sqrt{3}}{2}b} \left(\frac{r-d}{r}\right) dr$$

$$= \frac{\mu_0 I}{\pi\sqrt{3}} \left[\frac{\sqrt{3}}{2}b - d\ln\left(1 + \frac{\sqrt{3}b}{2d}\right)\right] \to L_{12} = \frac{\mu_0}{\pi}\left[\frac{b}{2} - \frac{d}{\sqrt{3}}\ln\left(1 + \frac{\sqrt{3}b}{2d}\right)\right]$$

Exercise 6.39

Determine the mutual inductance between a very long, straight wire and a conducting circular loop, as shown in Fig. 6-49.

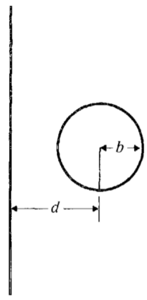


FIGURE 6–49
A long, straight wire and a conducting circular loop (Problem P.6–39).

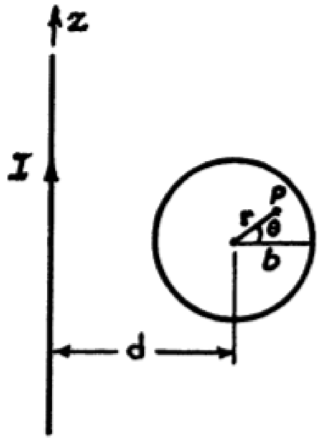
Answer:
Assume a current I.

$$B \text{ at } P(r,\theta) \text{ is } \bar{a}_\phi \frac{\mu_0 I}{2\pi(d + r\cos\theta)}$$

$$\Lambda_{12} = \frac{\mu_0 I}{2\pi} \int_0^b \int_0^{2\pi} \frac{r dr d\theta}{d + r\cos\theta}$$

$$= \frac{\mu_0 I}{2\pi} \int_0^b \frac{2\pi r dr}{\sqrt{d^2 - r^2}} = \mu_0 I \left(d - \sqrt{d^2 - b^2}\right)$$

$$L_{12} = \mu_0 \left(d - \sqrt{d^2 - b^2}\right)$$



Exercise 6.40

Find the mutual inductance between two coplanar rectangular loops with parallel sides, as shown in Fig. 6-50. Assume that $h_1 \gg h_2 \, (h_2 > w_2 > d)$.

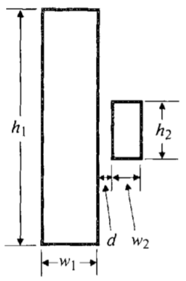


FIGURE 6–50
Two coplanar rectangular loops, $h_1 \gg h_2$ (Problem P.6–40).

Answer:
Approximate the magnetic flux due to the long loop linking with the small loop by that due to two infinitely long wires carrying equal and opposite current I.

$$\Lambda_{12} = \frac{\mu_0 h_2 I}{2\pi} \int_0^{w_2} \left(\frac{1}{d+x} - \frac{1}{w_1+d+x} \right) dx = \frac{\mu_0 h_2 I}{2\pi} \ln \left(\frac{w_2+d}{d} \cdot \frac{w_1+d}{w_1+w_2+d} \right).$$

$$L_{12} = \frac{\Lambda_{12}}{I} = \frac{\mu_0 h_2}{2\pi} \ln \frac{(w_1+d)\,(w_2+d)}{d\,(w_1+w_2+d)}.$$

Exercise 6.41

Consider two coupled circuits, having self-inductances L_1 and L_2, that carry currents I_1 and I_2, respectively. The mutual inductance between the circuits is M.

a) Using Eq. (6-161), find the ratio I_1/I_2 that makes the stored magnetic energy W_2 a minimum.

b) Show that $M \leq \sqrt{L_1 L_2}$.

Answer:
Eq $(6-163): W_2 = \frac{1}{2} L_1 I_1^2 + M I_1 I_2 + \frac{1}{2} L_2 I_2^2$.

a)

$$W_2 = \frac{I_2^2}{2}\left[L_1\left(\frac{I_1}{I_2}\right)^2 + 2M\left(\frac{I_1}{I_2}\right) + L_2\right] = \frac{I_2^2}{2}\left(L_1x^2 + 2Mx + L_2\right), x = \frac{I_1}{I_2}$$

$$\frac{dW_2}{dx} = \frac{I_2^2}{2}\left(2L_1x + 2M\right) = 0, \quad \frac{d^2W_2}{dx^2} = I_2^2L_1 > 0$$

$$\therefore x = \frac{I_1}{I_2} = -\frac{M}{L_1} \text{ for minimum } W_2$$

b)

$$(W_2)_{\min} = \frac{I_2^2}{2}\left(-\frac{M^2}{L_1} + L_2\right) \geq 0 \longrightarrow M \leq \sqrt{L_1L_2}$$

Exercise 6.42

Calculate the force per unit length on each of three equidistant, infinitely long, parallel wires 0.15(m) apart, each carrying a current of 25(A) in the same direction. Specify the direction of the force.

Answer:

$$I_1 = I_2 = I_3 = 25(\text{ A}); \quad d = 0.15(\text{ m}).$$

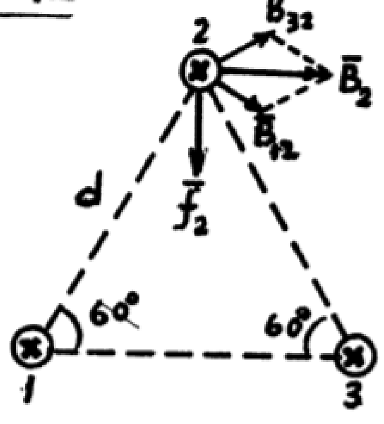
$$\bar{B}_2 = \bar{a}_x 2B_{12}\cos 30^\circ = \bar{a}_x\frac{\sqrt{3}\mu_0 I}{2\pi d}$$

Force per unit length on wire 2 :

$$\begin{aligned}\bar{f}_2 &= -\bar{a}_y IB_2 = -\bar{a}_y\frac{\sqrt{3}\mu_0 I^2}{2\pi d} \\ &= -\bar{a}_y 1150\mu_0 = -\bar{a}_y 1.44\times 10^{-3}(\text{ N/m}).\end{aligned}$$

Forces on all three wires are of equal magnitude and toward the center of the triangle.

P. 6-42



Exercise 6.43

The cross section of a long thin metal strip and a parallel wire is shown in Fig. 6-51. Equal and opposite currents I flow in the conductors. Find the force per unit length on the conductors.

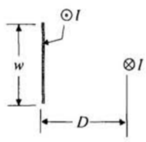


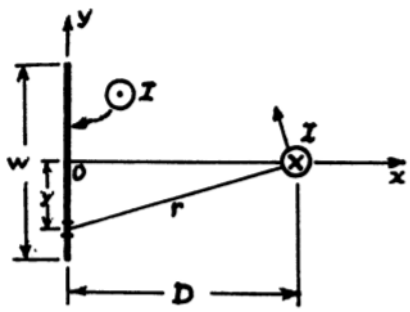
FIGURE 6–51
Cross section of parallel strip and wire conductor (Problem P.6–43).

Answer:
Magnetic field intensity at the wire due to the current $dI = \frac{I}{W}dy$ in an elemental dy is

$$|d\vec{H}| = \frac{dI}{2\pi r} = \frac{Idy}{2\pi w\sqrt{D^2+y^2}}.$$

Symmetry $\longrightarrow$ H at the wire has only a y-component.

$$\begin{aligned}\bar{H} &= \bar{a}_y \int (dH)\cdot\left(\frac{D}{r}\right) = \bar{a}_y 2\int_0^{w/2} \frac{IDdy}{2\pi w\,(D^2+y^2)} \\ &= \bar{a}_y \frac{I}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right).\end{aligned}$$



$$\bar{f}' = \bar{I}\times\bar{B} = (-\bar{a}_z I)\times\left(\mu_0\bar{H}\right) = \bar{a}_x\frac{\mu_0 I^2}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right)$$

Exercise 6.44

Determine the force per unit length between two parallel, long, thin conducting strips of equal width w. The strips are at a distance d apart and carry currents I_1 and I_2 in opposite directions as in Fig. 6-52.

Answer:
From Problem P.6-4 we have the y-component of the magnetic flux density at an arbitrary point $P(d,y)$ on the right-hand strip due to I_1 in the left-hand strip $B_{\mathrm{Py}} = -\frac{\mu_0 I_1}{2\pi w}\left[\tan^{-1}\left(\frac{y}{d}\right) + \tan^{-1}\left(\frac{w-y}{d}\right)\right]$.

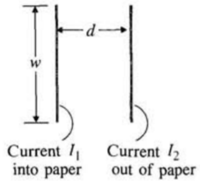


FIGURE 6–52
Cross section of two parallel strips carrying opposite currents (Problem P.6–44).

The x component of the force on a strip of width dy due to I_2 in the right-hand conductor is
$dF'^2_{2x} = \left(\frac{I_2}{w}dy\right) B_{\text{py}}$ (in the $+x$ direction, a repulsive force).

$$\bar{F}'_2 = \bar{a}_x \int dF_{2x} = \bar{a}_x \frac{\mu_0 I_1 I_2}{2\pi w^2} \int_0^w \left[\tan^{-1}\left(\frac{y}{d}\right) + \tan^{-1}\left(\frac{w-y}{d}\right)\right] dy$$

$$= \bar{a}_x \frac{\mu_0 I_1 I_2}{2\pi w^2} \left[2w \tan^{-1}\left(\frac{w}{d}\right) - d \cdot \ln\left(1 + \frac{w^2}{d^2}\right)\right]$$

per unit length.

There is no net force in the y-direction.

Exercise 6.45

Refer to Problem 6-39 and Fig. 6-49. Find the force on the circular loop that is exerted by the magnetic field due to an upward current I_1 in the long straight wire. The circular loop carries a current I_2 in the counterclockwise direction.

Answer:

B due to I, in the straight wire in the z-direction at an elemental arc $bd\theta$ on the circular loop is

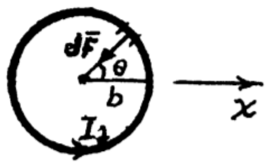
$$\bar{B} = \bar{a}_\phi \frac{\mu_0 I_1}{2\pi(d + b\cos\theta)}.$$

$$\bar{F} = -\bar{a}_x 2 \int_0^\pi (I_2 b d\theta) \cos\theta$$

on loop

$$= -\bar{a}_x \frac{\mu_0 I_1 I_2 b}{\pi} \int_0^\pi \frac{\cos\theta}{d + b\cos\theta} d\theta$$

$$= \bar{a}_x \mu_0 I_1 I_2 \left[\frac{1}{\sqrt{1-(b/d)^2}} - 1\right] \text{ (Repulsive force).}$$



$\bar{F}$ has no net y-component.

Exercise 6.46

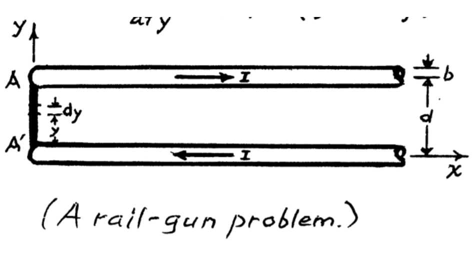
The bar AA' in Fig. 6-53 serves as a conducting path (such as the blade of a circuit breaker) for the current I in two very long parallel lines. The lines have a radius b and are spaced at a distance d apart. Find the direction and the magnitude of the magnetic force on the bar.

FIGURE 6–53
Force on end conducting bar
(Problem P.6–46).

Answer:

$$\bar{B}\ (\text{at } y) = \bar{a}_z \frac{\mu_0 I}{4\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right), \quad d\bar{l} = \bar{a}_y dy$$

$$\begin{aligned}
d\bar{F} &= I d\bar{l} \times \bar{B} \\
&= -\bar{a}_x \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{y} + \frac{1}{d-y}\right) dy \\
\bar{F} &= -\bar{a}_x \frac{\mu_0 I^2}{4\pi}\int_b^{d-b}\left(\frac{1}{y} + \frac{1}{d-y}\right) dy \\
&= -\bar{a}_x \frac{\mu_0 I^2}{2\pi}\ln\left(\frac{d}{b} - 1\right).
\end{aligned}$$



(A rail-gun problem.)

Exercise 6.47

A d-c current $I = 10$ (A) flows in a triangular loop in the xy-plane as in Fig. 6-54. Assuming a uniform magnetic flux density $\mathbf{B} = \mathbf{a}_y 0.5(\text{ T})$ in the region, find the forces and torque on the loop. The dimensions are in (cm).

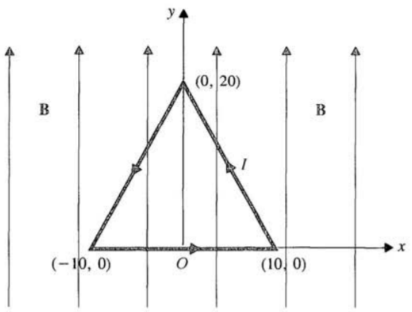


FIGURE 6–54
A triangular loop in a uniform magnetic field (Problem P.6–47).

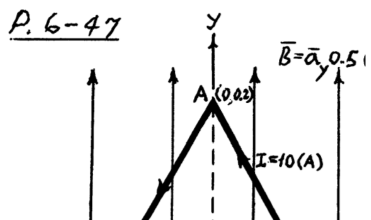
Answer:

$$\bar{B} = \bar{a}_y 0.5(T).$$

$$\begin{array}{llll}
\bar{B}\times & I(\overrightarrow{AB}) & I(\overrightarrow{CA}) & I(\overrightarrow{BC}) \\
(\bar{a}_y 0.5) & \times 10\left(-\bar{a}_x 0.1 - \bar{a}_y \cdot 0.2\right), & 10\left(\bar{a}_x 0.1 + \bar{a}_y 0.2\right), & 10\bar{a}_x 0.2 \\
\text{Force:} & \bar{a}_z 0.5 & \bar{a}_z 0.5 & \bar{a}_z 1.0(N)
\end{array}$$

Torque on loop:

$$\begin{aligned}
\bar{T} &= \bar{m} \times \bar{B} = (\bar{a}_z I S) \times \bar{B} \\
&= \left(\bar{a}_z 10 \times \frac{1}{2} \times 0.2 \times 0.2\right) \times (\bar{a}_y 0.5) = -\bar{a}_x 0.1(\text{ N} \cdot \text{m})
\end{aligned}$$



Exercise 6.48

One end of a long air-core coaxial transmission line having an inner conductor of radius a and an outer conductor of inner radius b is short-circuited by a thin, tight-fitting conducting washer. Find the magnitude and the direction of the magnetic force on the washer when a current I flows in the line.

Answer:
Let x-axis be the center line of the coaxial cable. The magnetic energy stored in a section of length x is

$$W_m = \frac{1}{2}LI^2.$$

$$L = \frac{\Phi}{I} = \frac{x}{I}\int_a^b B_\phi dr = \frac{x}{I}\int_a^b \frac{\mu_0 I}{2\pi r}dr = \frac{\mu_0 x}{2\pi}\ln\frac{b}{a}.$$

$$\bar{F}_I = \bar{a}_x\frac{\partial W_m}{\partial x} = \bar{a}_x\left(\frac{I^2}{2}\right)\frac{\partial L}{\partial x} = \frac{\mu_0 I^2}{4\pi}\ln\frac{b}{a}$$

Exercise 6.49

Assuming that the circular loop in Problem P.6-45 is rotated about its horizontal axis by an angle α, find the torque exerted on the circular loop.

Answer:
Resolve the circular loop into many small loops, each with a magnetic dipole moment

$$d\bar{m} = I_2 d\bar{s}, d\bar{T} = d\bar{m}\times\bar{B}$$

$$\bar{T} = \int d\bar{T} = I_2\int d\bar{s}\times\bar{B} = -\bar{a}_x I_2\sin\alpha\int Bds = -\bar{a}_x\mu_0 I_1 I_2\left(d-\sqrt{d^2-b^2}\right)\sin\alpha$$

from Problem P.6-39. This torque is in the direction of aligning the flux produced by I_2 in the loop with that of $\bar{B}$ due to I_1 in the straight wire.

Exercise 6.50

A small circular turn of wire of radius r_1 that carries a steady current I_1 is placed at the center of a much larger turn of wire of radius r_2 ($r_2 \gg r_1$) that carries a steady current I_2 in the same direction. The angle between the normals of the two circuits is θ and the small circular wire is free to turn about its diameter. Determine the magnitude and the direction of the torque on the small circular wire.

Answer:
$\bar{B}_2$ at the of the large circular turn of wire carrying a current I_2 is (by setting $z=0$ in Eq. 6-38):

$$\bar{B}_2 = \bar{a}_{z_2}\frac{\mu_0 I_2}{2r_2}$$

$$\bar{T} = \bar{m}_1\times\bar{B}_2 \cong \left(\bar{a}_{z_1}I_1\pi {r_1}^2\right)\times\left(\bar{a}_{z2}\frac{\mu_0 I_2}{2r_2}\right) = \left(\bar{a}_{z1}\times\bar{a}_{z2}\right)\frac{\mu_0 I_1 I_2\pi r_1^2}{2r_2}$$

$\longrightarrow$ Magnitude $= \frac{\mu_0 I_1 I_2 \pi r_1^2}{2r_2}\sin\theta$ in a direction to align the magnetic fluxes produced by $I_1 \& I_2$.

Exercise 6.51

A magnetized compass needle will line up with the earth's magnetic field. A small bar magnet (a magnetic dipole) with a magnetic moment 2 ($\mathrm{A} \cdot \mathrm{m}^2$) is placed at a distance 0.15 (m) from the center of a compass needle. Assuming the earth's magnetic flux density at the needle to be 0.1 (mT), find the maximum angle at which the bar magnet can cause the needle to deviate from the north-south direction. How should the bar magnet be oriented?

Answer:
$\bar{B}_m$ (magnetized compass needle)

$$= \frac{\mu_0 m}{4\pi R^3}\left(\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta\right) \text{from}_{Eq,(6-48)}$$

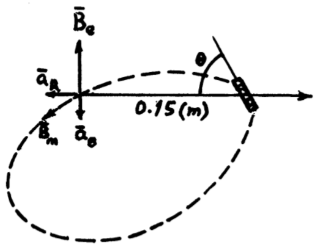
$$= \frac{(4\pi\times 10^{-7})\times 2}{4\pi(0.15)^3}\left(\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta\right)$$

$$= 0.59\times 10^{-4}\left(\bar{a}_R 2\cos\theta + \bar{a}_\theta \sin\theta\right)(T)$$

$\bar{B}_e(\text{ earth }) = -\bar{a}_\theta 10^{-4}(T)$.
Max. deflection occurs when $\left|\frac{B_R}{B_\theta}\right|$ is snax. or when

$$\left|\frac{B_\theta}{B_R}\right| = \left|\frac{(0.59\sin\theta - 1)\times 10^{-4}}{1.18\times 10^{-4}\cos\theta}\right| \text{ is min.}$$

Set $\frac{d}{d\theta}\left(\frac{1-0.59\sin\theta}{1.18\cos\theta}\right) = 0 \longrightarrow \sin\theta = 0.59$, or $\theta = 36.2^\circ$
At $\theta = 36.2^\circ, |B_R/B_\theta| = 1.47$, and $\alpha = \tan^{-1} 1.47 = 55.8^\circ$.



Exercise 6.52

The total mean length of the flux path in iron for the electromagnet in Fig. 6-33 is 3(m), and the yoke-bar contact areas measure 0.01 (m^2). Assuming the permeability of iron to be $4000\mu_0$ and each of air gaps to be 2(mm), calculate the mmf needed to lift a total mass of 100(kg).

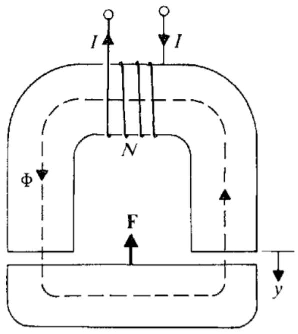


FIGURE 6–33
An electromagnet (Example 6–23).

Answer:

$$F=\frac{\Phi^2}{\mu_0 S}=\frac{(NI)^2}{\mu_0 S\left(\frac{2l_g}{\mu_0 S}+\frac{l_i}{\mu_0\mu_r S}\right)^2}=\frac{(NI)^2\mu_0 S}{\left(2l_g-\frac{l_i}{\mu_r}\right)^2}$$

$$F=100\times 9.8=980(\mathrm{~N}), S=0.01\left(\mathrm{~m}^2\right), l_g=2\times 10^{-3}(\mathrm{~m}),$$

$l_i=3(\mathrm{~m}), \mu_r=4000.$
Solving: mmf $=NI=1.33\times 10^3(\mathrm{~A}\cdot t)$.

Exercise 6.53

A current I flows in a long solenoid with n closely wound coil-turns per unit length. The cross-sectional area of its iron core, which has permeability μ, is S. Determine the force acting on the core if it is withdrawn to the position shown in Fig. 6-55.

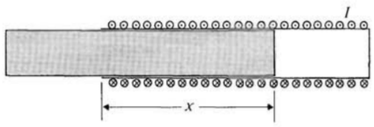


FIGURE 6–55
A long solenoid with iron core partially withdrawn (Problem P.6–53).

Answer:

$$W_m=\frac{1}{2}\int \mu H^2 dv$$

Assume a virtual displacement, Δx, of the iron core.

$$\begin{aligned} W_m(x+\Delta x) &= W_m(x)+\frac{1}{2}\int_{S\Delta x}\left(\mu-\mu_0\right)H^2 dv \\ &= W_m(x)+\frac{1}{2}\mu_0\left(\mu_r-1\right)n^2 I^2 S\Delta x. \end{aligned}$$

$$\left(F_I\right)_x=\frac{\partial W_m}{\partial x}=\frac{\mu_0}{2}\left(\mu_r-1\right)n^2 I^2 S, \text{ in the direction}$$
of increasing x.

Reference

1. Cheng, David Keun. Field and wave electromagnetics. Pearson Education India, 1989.