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FET Single Stage Amplifier

Ve311 Electronic Circuits (Fall 2021)

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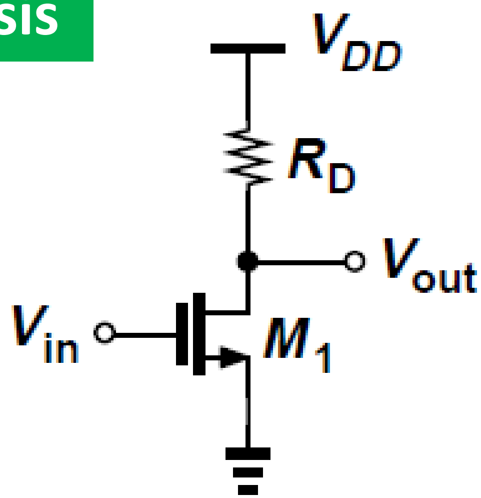
Common-Source with Resistive Load

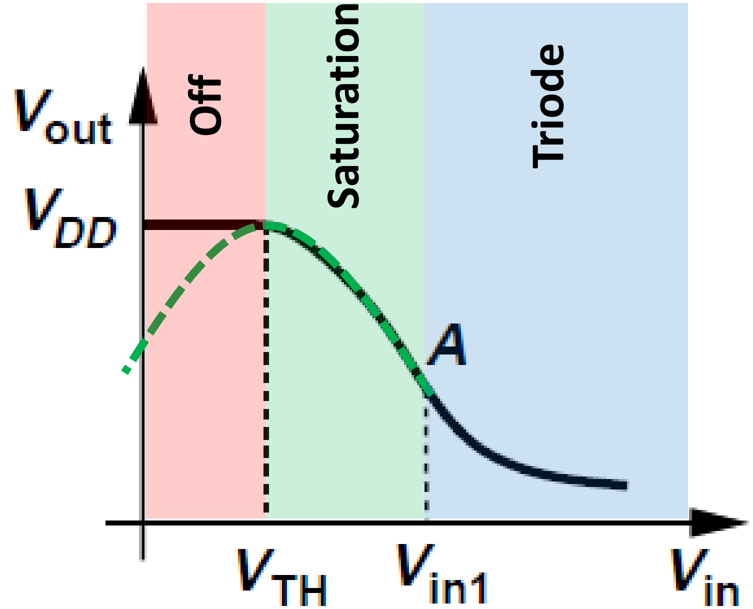
Common-Source with Resistive Load

DC Analysis

$\lambda = 0$

$\gamma = 0$





$$V_{out} = V_{in1} - V_{TH}$$
$$= V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in1} - V_{TH})^2$$

- **$V_{in} < V_{TH}$ → M_1 Off**

  $$V_{out} = V_{DD}$$

- **$V_{in1} > V_{in} > V_{TH}$ → M_1 in Saturation**

  $$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2$$

- **$V_{in} > V_{in1}$ → M_1 in Triode**

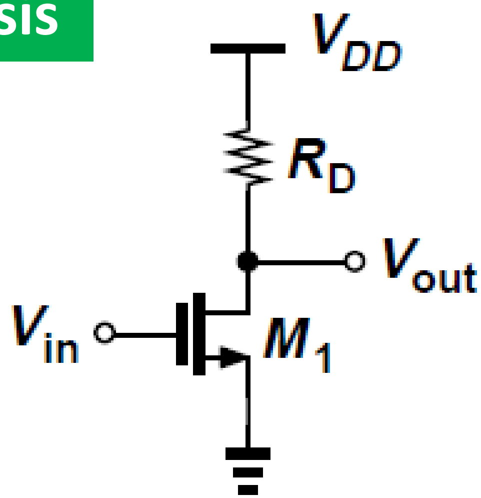
  $$V_{out} = V_{DD} - R_D \mu_n C_{ox} \frac{W}{L} \left[(V_{in} - V_{TH}) V_{out} - \frac{1}{2} V_{out}^2\right]$$

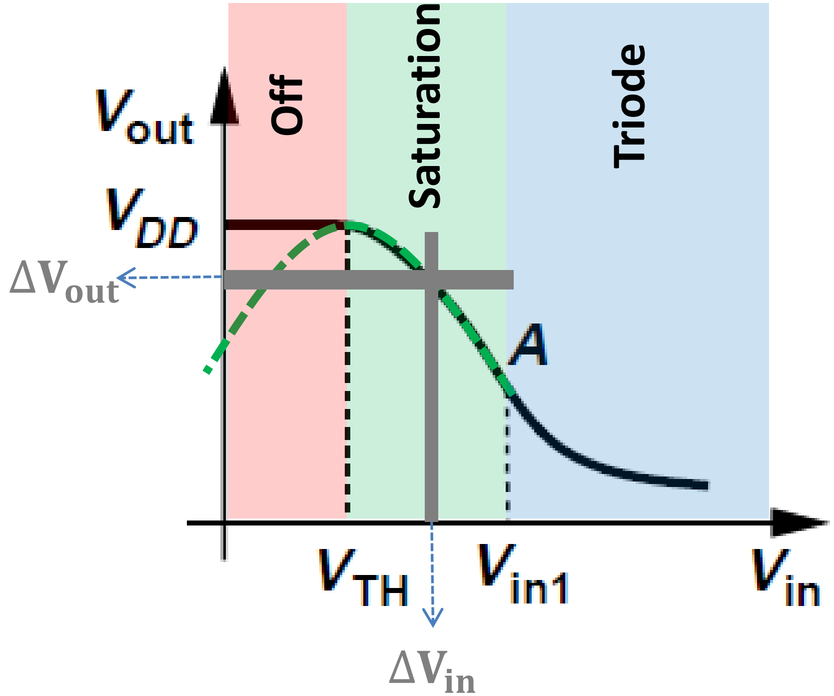
Common-Source with Resistive Load

DC Analysis

$\lambda = 0$

$\gamma = 0$





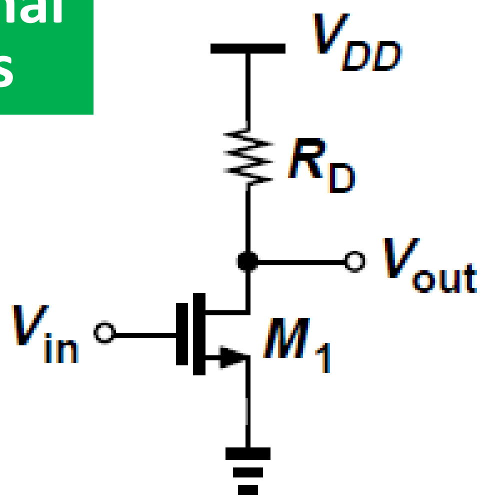
$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = -R_D \mu_n C_{ox} \frac{W}{L}(V_{in} - V_{TH})$$
$$= -gm \cdot R_D$$

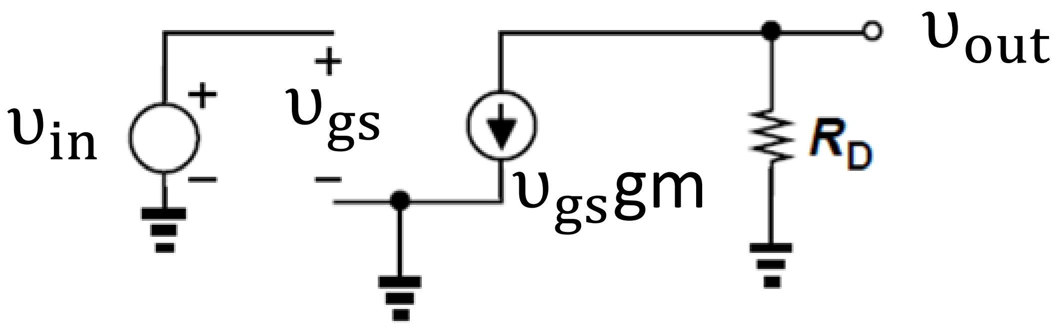
- V_{gs} increases by ΔV_{in} → I_d increases by $\Delta V_{in} \cdot gm$ → V_{out} decreases by $\Delta V_{in} \cdot (gm \cdot R_D)$

Common-Source with Resistive Load

Small-signal Analysis

$\lambda = 0$
$\gamma = 0$





$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm \cdot R_D$$

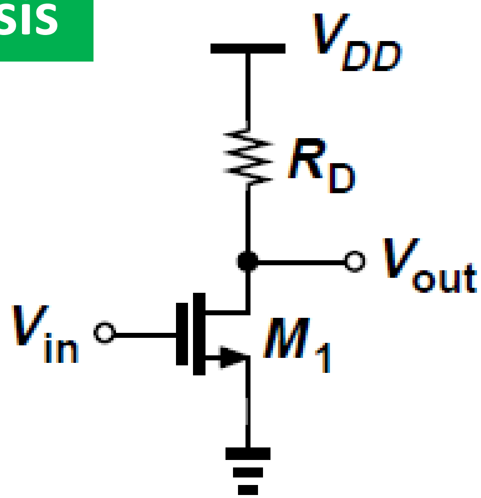
- Small-signal analysis leads to the same result as DC analysis.

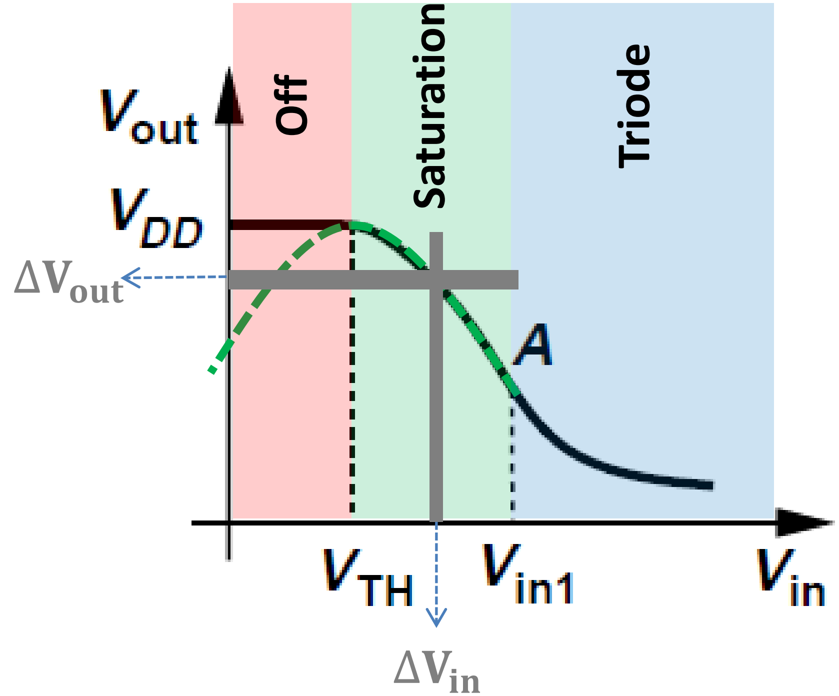
Common-Source with Resistive Load

DC Analysis

$\lambda \neq 0$

$\gamma \neq 0$





$$A_v = \frac{\partial V_{out}}{\partial V_{in}} = \frac{\partial \left[V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2 (1 + \lambda V_{out}) \right]}{\partial V_{in}}$$

$$= -R_D \underbrace{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})(1 + \lambda V_{out})}_{= gm} - R_D \underbrace{\frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{TH})^2}_{\approx I_D} \lambda \underbrace{\frac{\partial V_{out}}{\partial V_{in}}}_{= A_v}$$

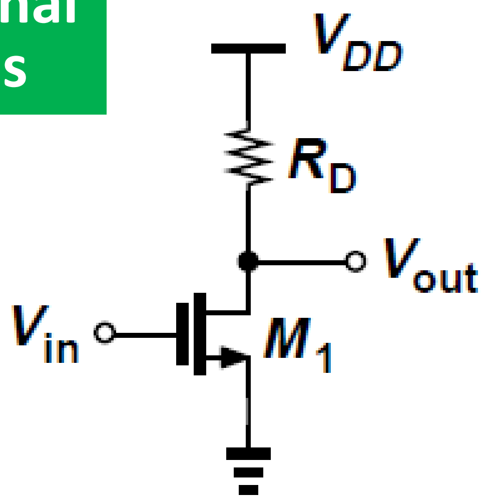
$$A_v = \frac{-gmR_D}{1 + R_D \underbrace{I_D \lambda}_{= 1/r_o}} = -gm \frac{1}{\frac{1}{R_D} + \frac{1}{r_o}} = \boxed{-gm(R_D \parallel r_o)}$$

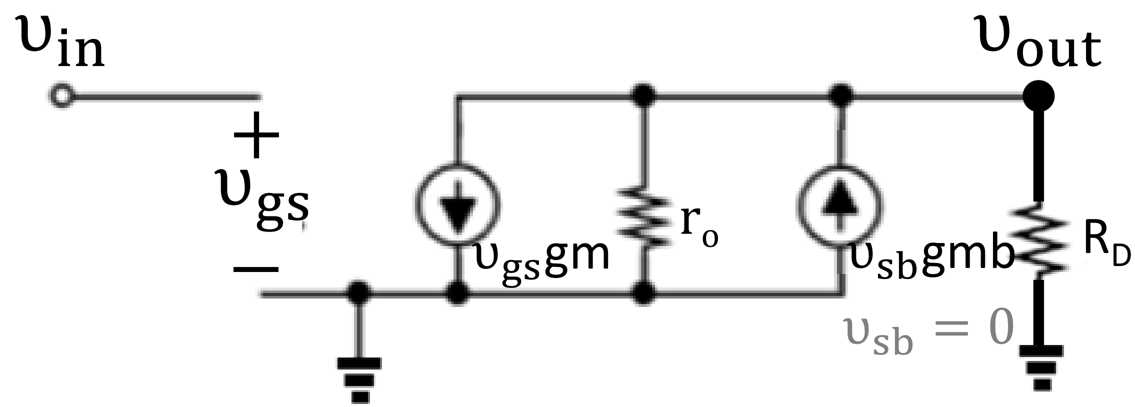
Common-Source with Resistive Load

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



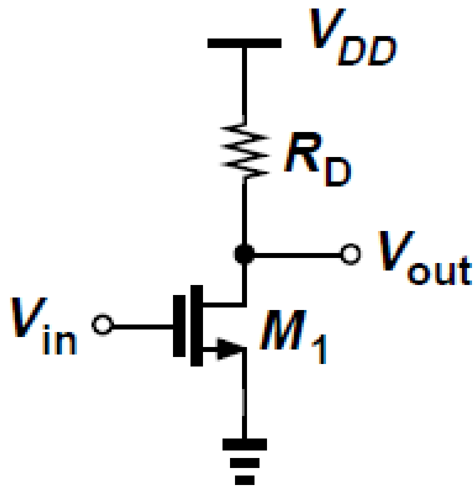


$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm \cdot (R_D \parallel r_o)$$

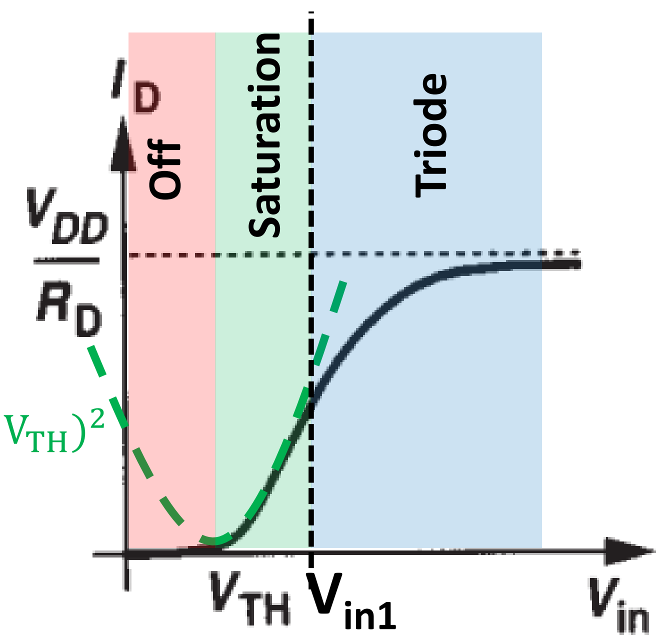
- Small-signal analysis leads to the same result as DC analysis.
- gm is a function of V_{GS} and V_{DS}, while r_o is a function of I_D. → **Nonlinearity**

Example

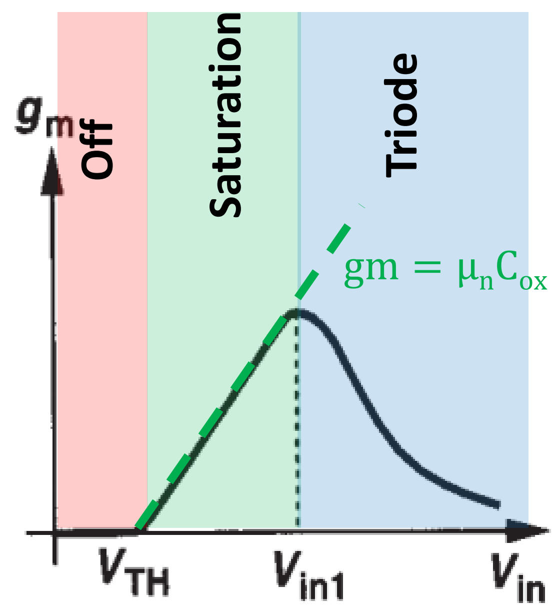
Sketch the drain current and transconductance of M_1 as a function of input voltage. Assume λ = γ = 0.



Solution:



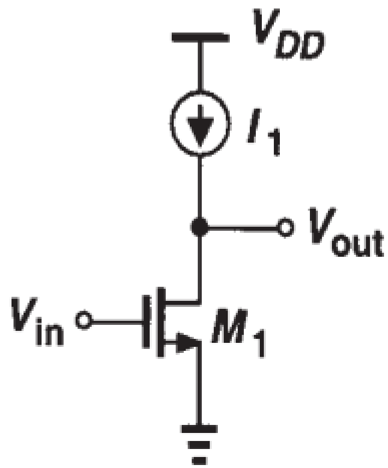
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2$$



$$gm = \mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})$$

Example

Assuming M_1 in saturation, calculate its small-signal gain.



Solution:

- Small-signal Analysis:

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -g m_1 r_{o1}$$

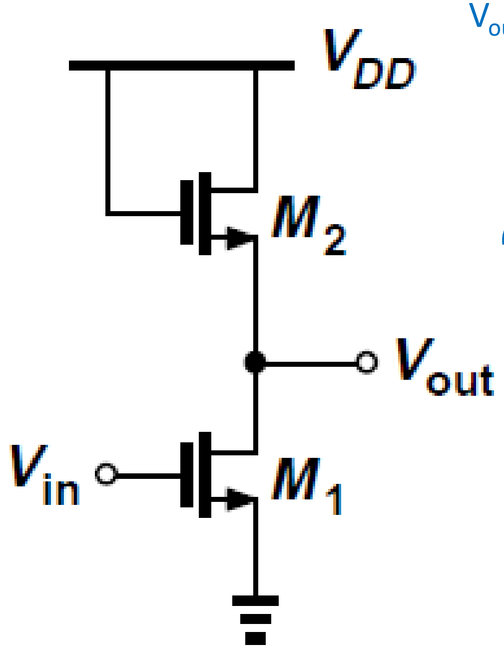
- DC Analysis:

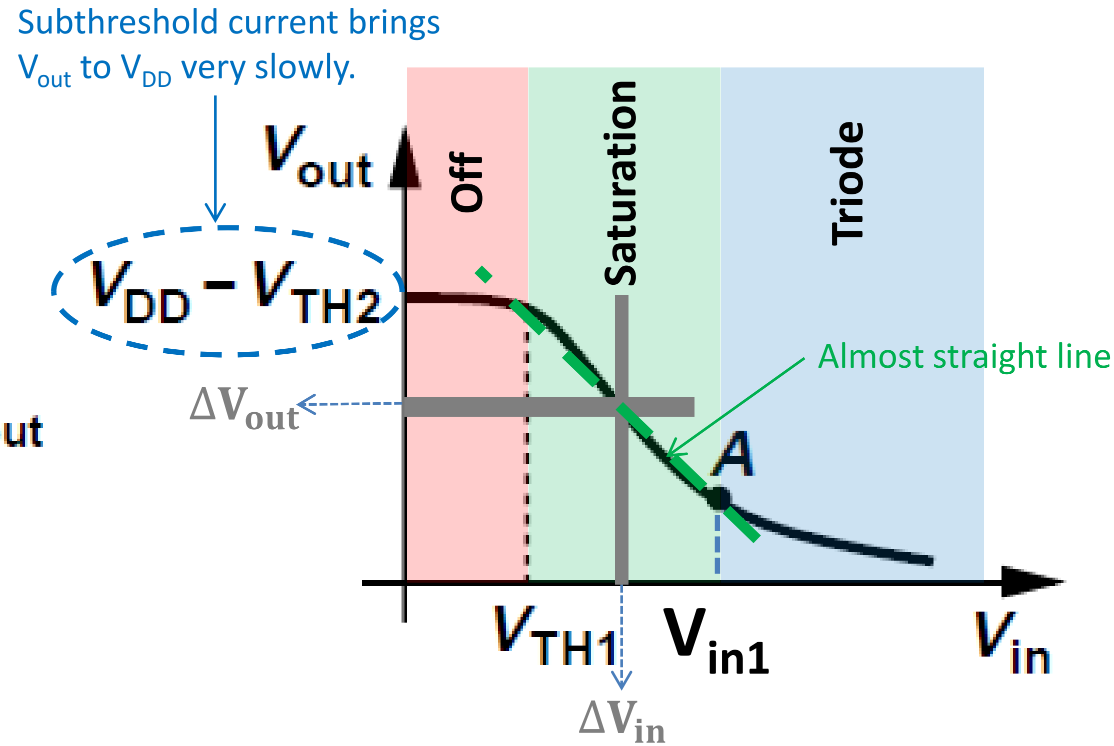
$$I_1 = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{TH})^2(1 + \lambda V_{DS})$$

Common-Source with Diode-Connected Load

Common-Source with Diode-Connected Load

DC Analysis $\lambda = 0 \quad \gamma \neq 0$





$$V_{out} = V_{in1} - V_{TH1}$$

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in1} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 [V_{DD} - (V_{in1} - V_{TH1}) - V_{TH2}]^2$$

- V_{gs} increases by ΔV_{in} → I_d increases by $\Delta V_{in} \cdot gm$ → V_{out} decreases

Common-Source with Diode-Connected Load

DC Analysis $\lambda = 0 \quad \gamma \neq 0$

- **$V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation**

$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - V_{TH1})^2 = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_2 [V_{DD} - V_{out} - V_{TH2}]^2$$

$$\sqrt{\left(\frac{W}{L}\right)_1} (V_{in} - V_{TH1}) = \sqrt{\left(\frac{W}{L}\right)_2} (V_{DD} - V_{out} - V_{TH2})$$

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2}\left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{in}}\right)$$

$$\sqrt{\left(\frac{W}{L}\right)_1} = \sqrt{\left(\frac{W}{L}\right)_2}\left(-\frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH2}}{\partial V_{out}}\frac{\partial V_{out}}{\partial V_{in}}\right)$$

$$= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}$$

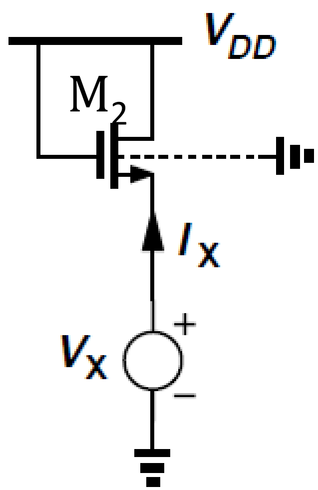
$$\boxed{A_v = \frac{\partial V_{out}}{\partial V_{in}} = -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta}}$$

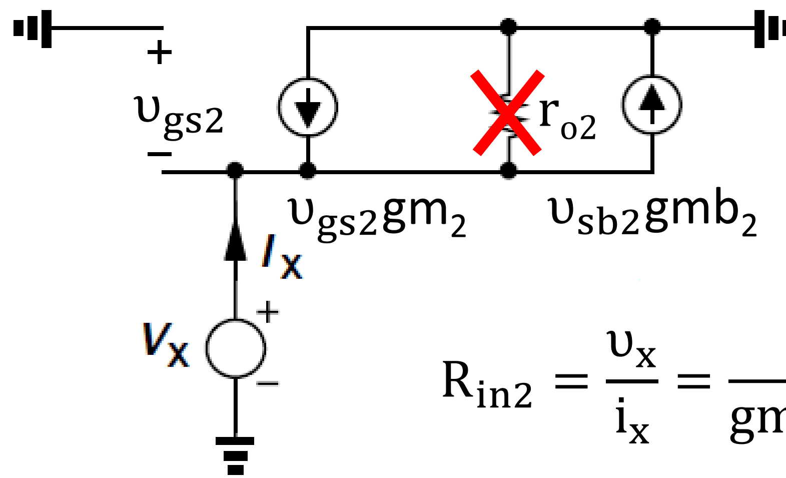
- η is a function of V_{SB}.
- A_v is almost linear for M_1 in saturation.

Common-Source with Diode-Connected Load

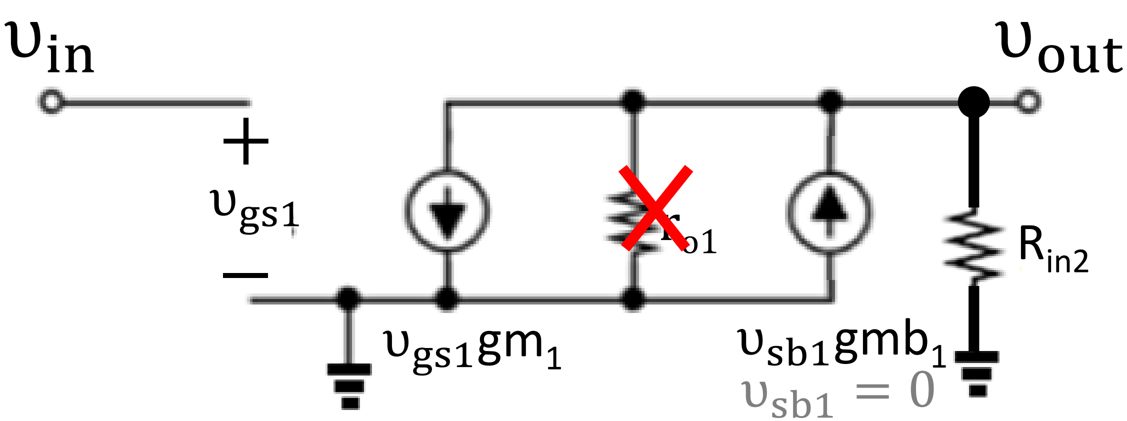
Small-signal Analysis

$\lambda = 0 \quad \gamma \neq 0$





$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{gm_2 + gmb_2}$$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = \frac{-gm_1}{gm_2 + gmb_2}$$

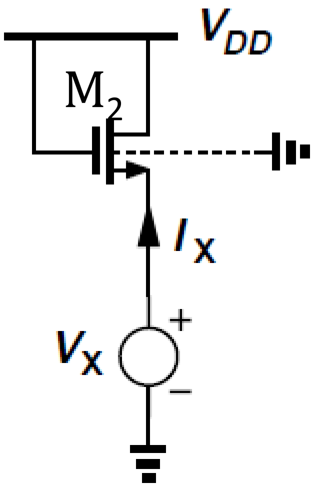
$$= -\sqrt{\frac{(W/L)_1}{(W/L)_2}}\frac{1}{1+\eta}$$

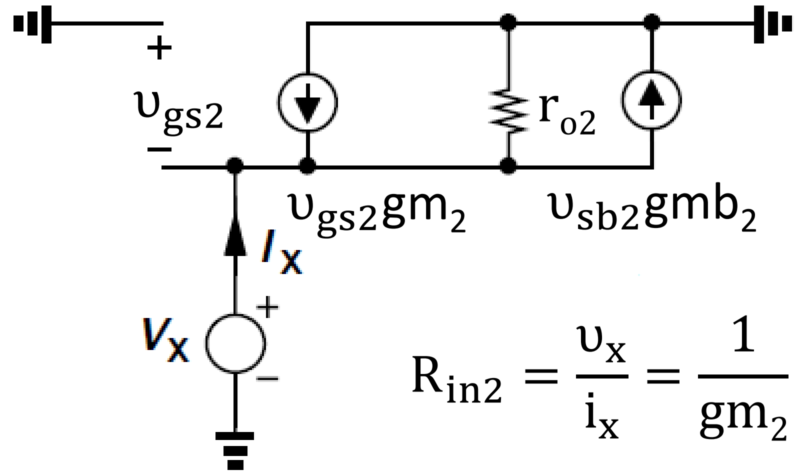
- Small-signal analysis leads to the same result as DC analysis.

Common-Source with Diode-Connected Load

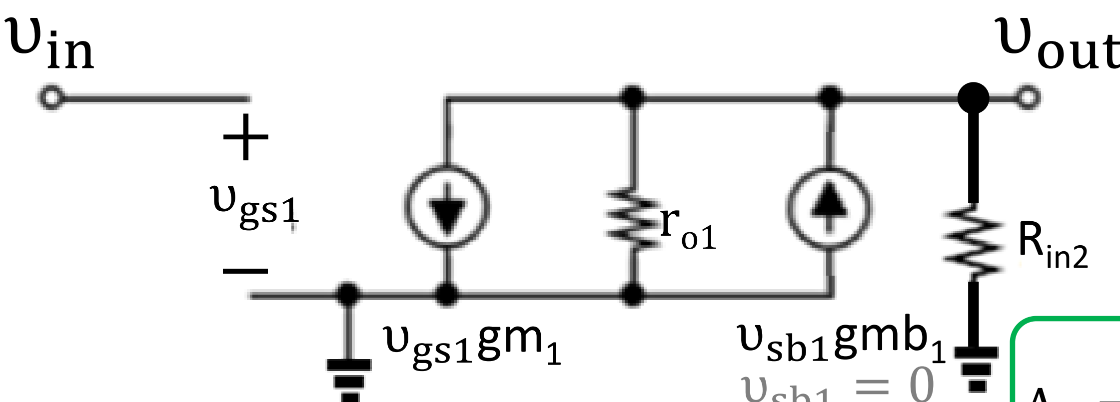
Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$





$$R_{in2} = \frac{\upsilon_x}{i_x} = \frac{1}{gm_2} \parallel \frac{1}{gmb_2} \parallel r_{o2}$$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$

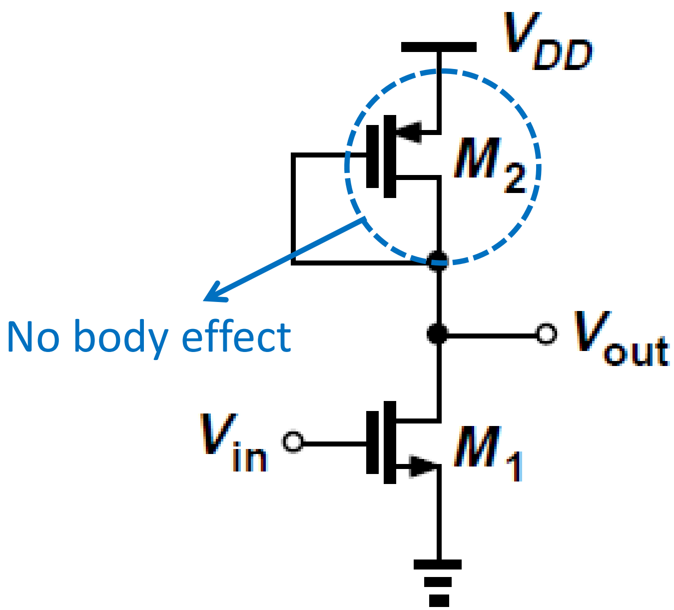
$$= -gm_1 \left(\frac{1}{gm_2} \parallel \frac{1}{gmb_2} \parallel r_{o2} \parallel r_{o1} \right)$$

$r_o >> 1/gm$

Common-Source with Diode-Connected Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$

$$= -gm_1\left(\frac{1}{gm_2} \parallel r_{o2} \parallel r_{o1}\right)$$

$r_o >> 1/gm$

$$\approx -\frac{gm_1}{gm_2}$$

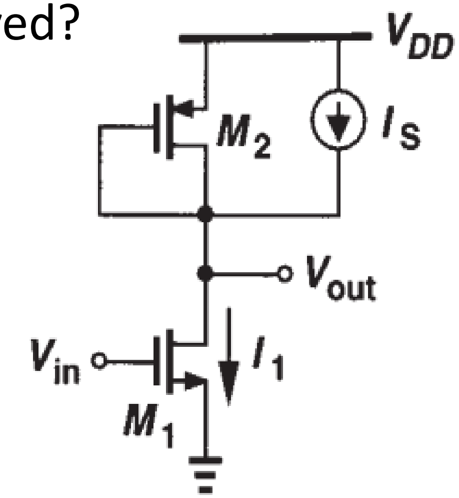
$$= -\sqrt{\frac{\mu_n(W/L)_1}{\mu_p(W/L)_2}}$$

$$= -\frac{V_{SG2} - V_{TH2}}{V_{GS1} - V_{TH1}}$$

- For $A_v = 10$, $(W/L)_1 >> (W/L)_2$ → **Disproportionally large transistor**
- For $A_v = 10$, $(V_{SG2} - V_{TH2}) = 10 \times (V_{GS1} - V_{TH1})$ → **Limited output swing**

Example

M_1 in saturation and $I_S = 0.75 \times I_1$. How do the disadvantages of CS stage with diode-connected load get improved?



Solution:

- Small-signal Analysis ($\lambda = 0$):

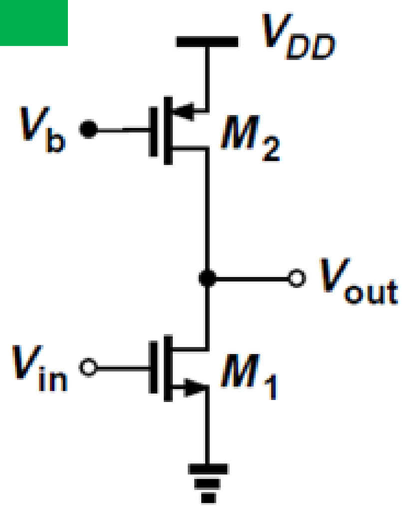
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -\frac{gm_1}{gm_2} = -\frac{\sqrt{2\mu_n C_{ox}\left(\frac{W}{L}\right)_1 I_{D1}}}{\sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right)_2 I_{D2}}} = -\frac{\sqrt{4\mu_n\left(\frac{W}{L}\right)_1}}{\sqrt{\mu_p\left(\frac{W}{L}\right)_2}} = -\frac{4(V_{SG2} - V_{TH2})}{(V_{GS1} - V_{TH1})}$$

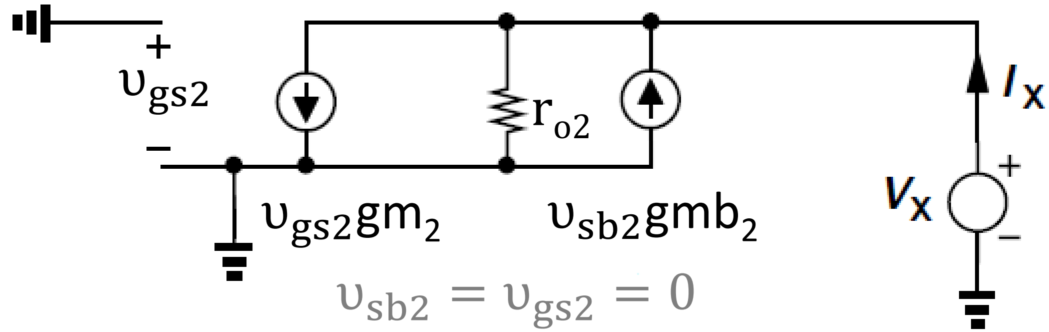
Common-Source with Current-Source Load

Common-Source with Current-Source Load

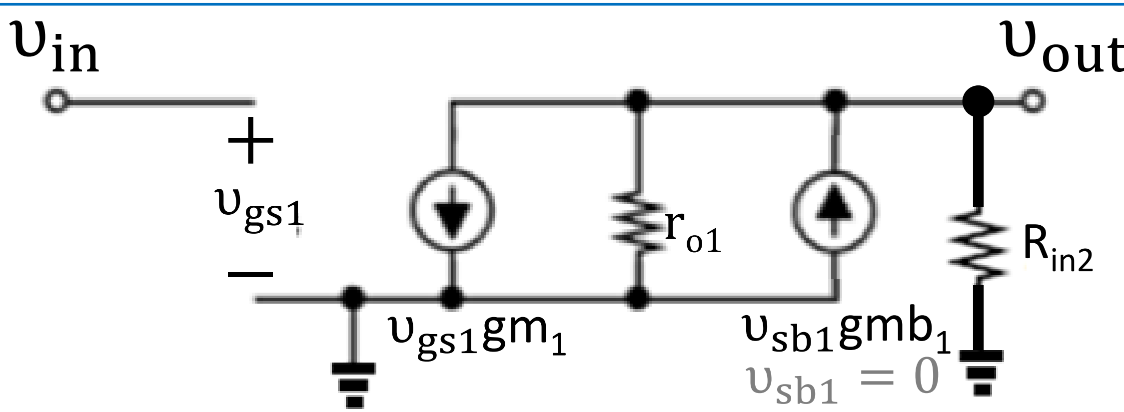
Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$





$$R_{in2} = \frac{\upsilon_x}{i_x} = r_{o2}$$



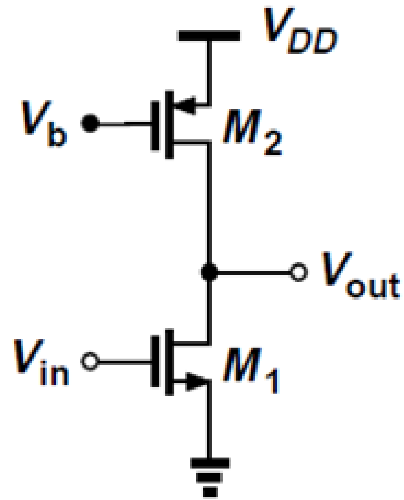
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = -gm_1 (r_{o2} \parallel r_{o1})$$

- To achieve high A_v, the output swing is severely limited in the CS stages with resistive load and diode-connected load.
- Here $V_{out,\,max} = V_{DD} - (V_{SG2} - V_{TH2})$, which can be quite close to V_{DD}.

Common-Source with Current-Source Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



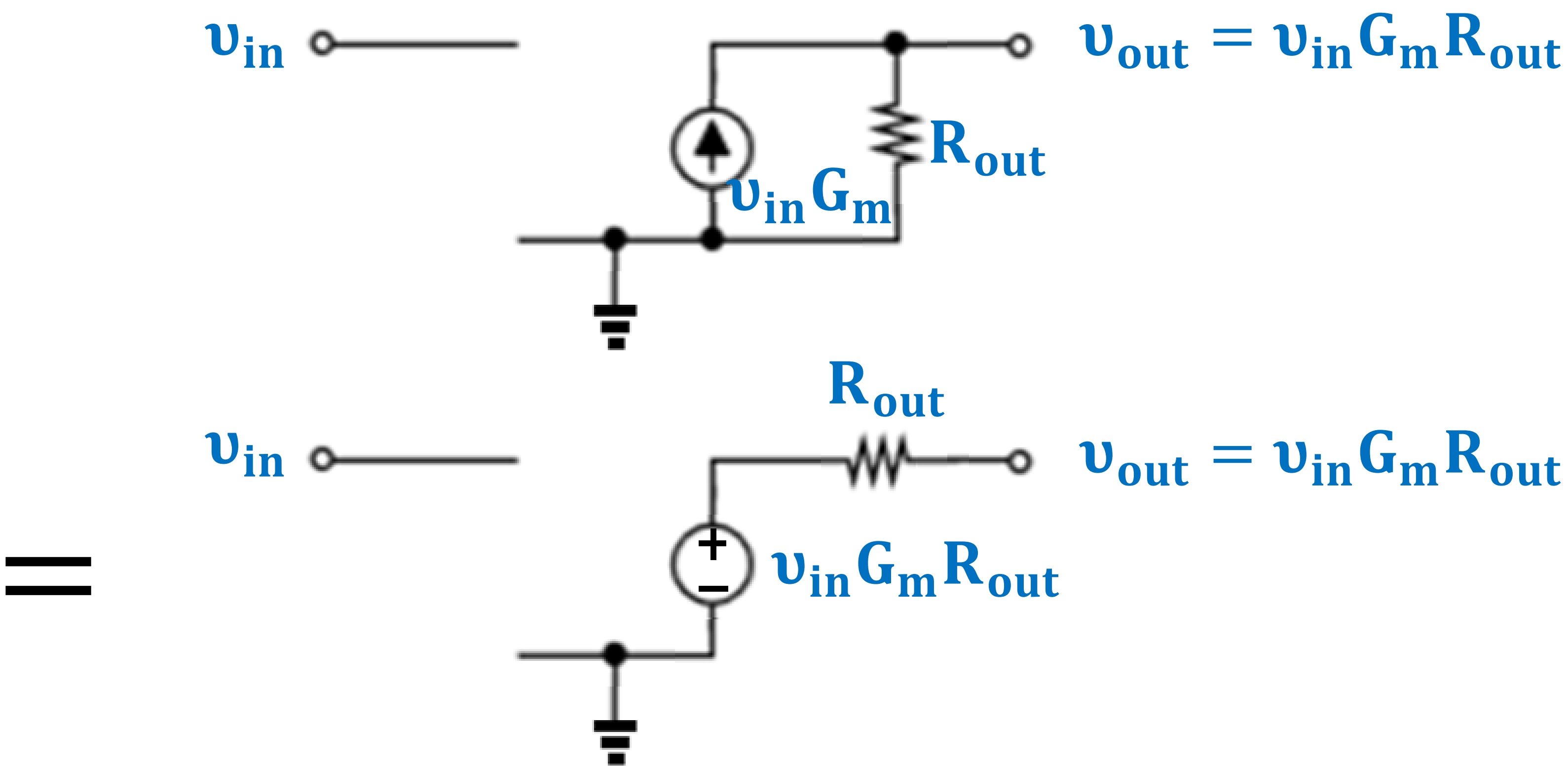
$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}}$$
$$= -gm_1(r_{o2} \parallel r_{o1})$$

$$r_o \approx \frac{1}{\lambda I_D} \quad \lambda \propto \frac{1}{L}$$

- $V_{out,\ max} = V_{DD} - (V_{SG2} - V_{TH2})$
- $V_{out,\ min} = (V_{GS1} - V_{TH1})$
- For high gm_1 and small $(V_{GS1} - V_{TH1})$, W of M_1 needs to be large.
- For high r_{o1} and r_{o2}, L of M_1 and M_2 need to be large and L of M_1 and M_2 needs to be increased proportionally. The cost is the **large parasitic drain junction capacitance** at the output.

Common-Source with Source Degradation

Amplifier Equivalent Circuit

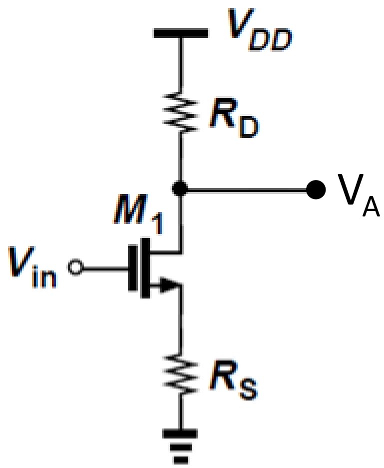


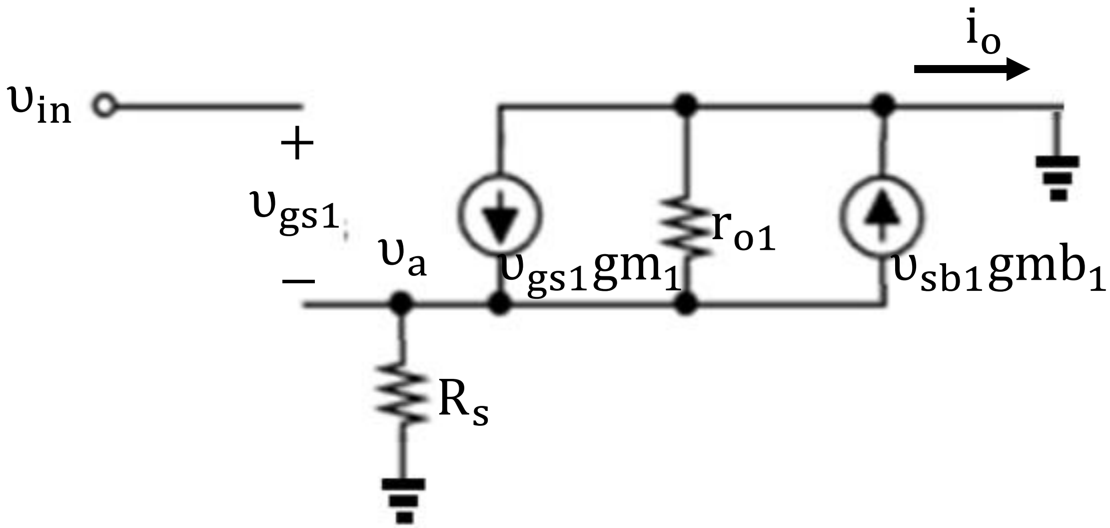
- How to calculate G_m? υ_{out} shorted to ground. $\mathbf{G_m = i_{out}/\upsilon_{in}}$
- How to calculate R_{out}? υ_{in} shorted to ground and υ_{out} connected to υ_{test}. $\mathbf{R_{out} = \upsilon_{test}/i_{test}}$

Common-Source with Source Degradation

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$





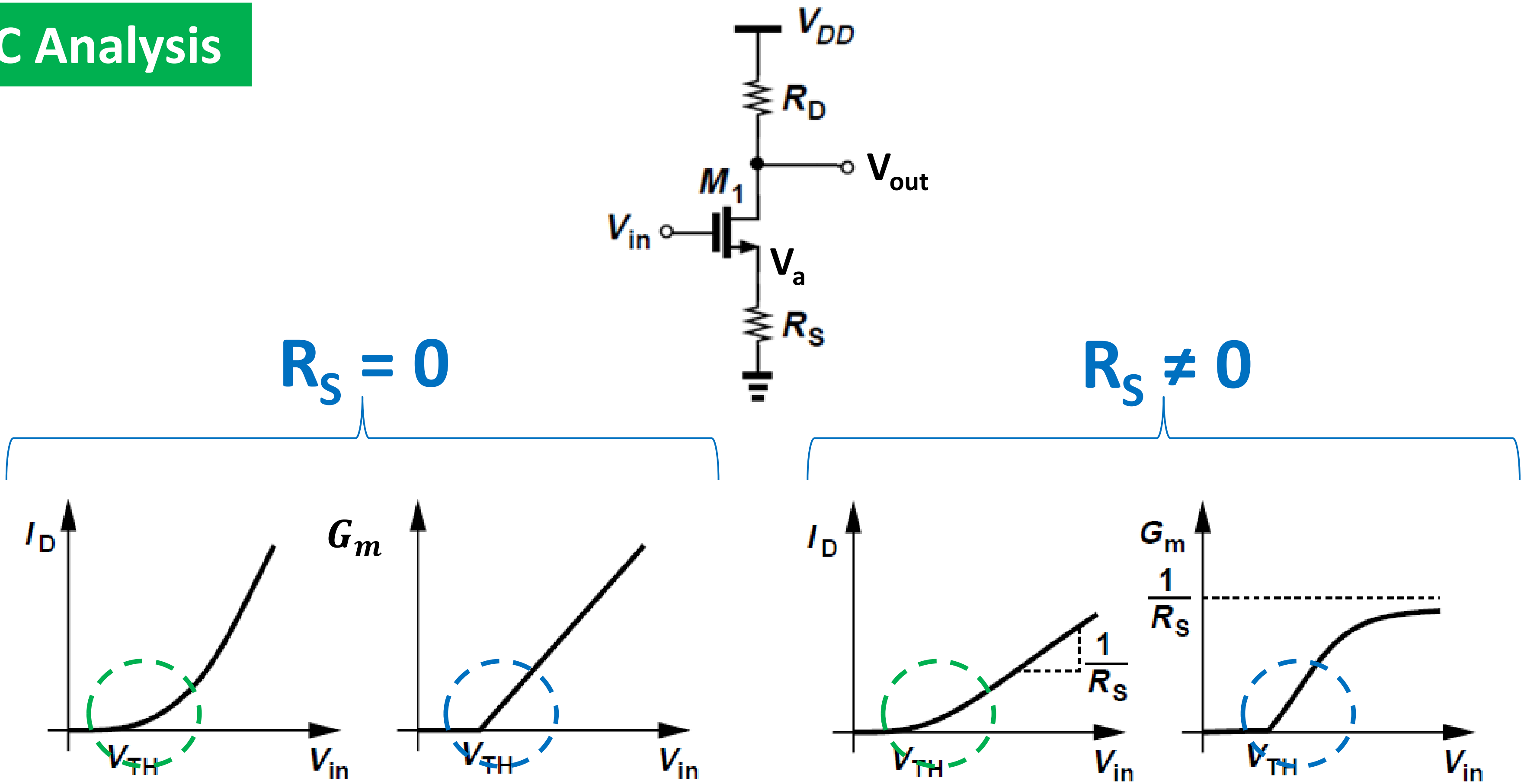
$$\begin{cases} i_o = \dfrac{-\upsilon_a}{R_S} \\ (\upsilon_{in} - \upsilon_a)gm_1 + i_o = \dfrac{\upsilon_a}{r_{o1}} + \upsilon_a gmb_1 \end{cases}$$

$$G_m = \frac{i_o}{\upsilon_{in}} = \frac{-gm_1 r_{o1}}{R_S + r_{o1} + (gm_1 + gmb_1)r_{o1}R_S} \approx -\frac{1}{R_S}$$

If $(gm_1 + gmb_1)r_{o1}R_S$ >> r_{o1} and R_S

Common-Source with Source Degradation

DC Analysis

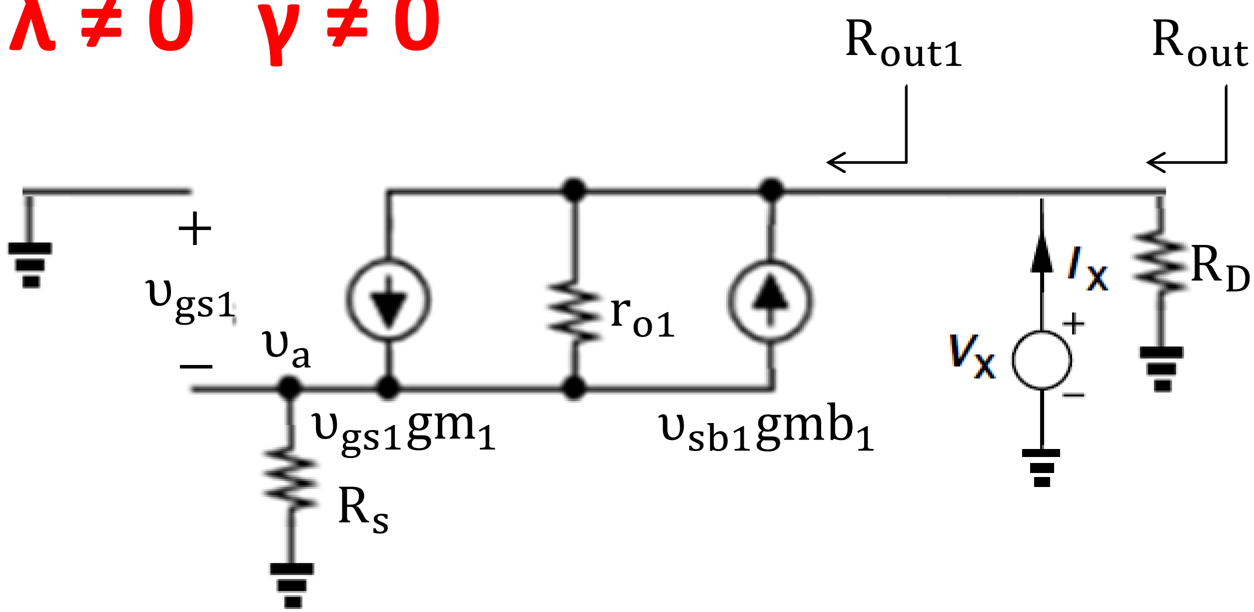


- At low V_{in} (gm small), turn-on behavior of $R_S \neq 0$ is similar to that of $R_S = 0$.
- At large V_{in} (gm large), the effect of R_s, i.e. degradation, becomes more significant.
- V_{in} = 0 V → M_1 off, no current flowing → V_a = 0 V and $V_{out} = V_{DD}$

Common-Source with Source Degradation

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\begin{cases} i_x = \dfrac{\upsilon_a}{R_S} \\ \upsilon_a gm_1 + \upsilon_a gmb_1 + \dfrac{\upsilon_a - \upsilon_x}{r_o} + i_x = 0 \end{cases}$$

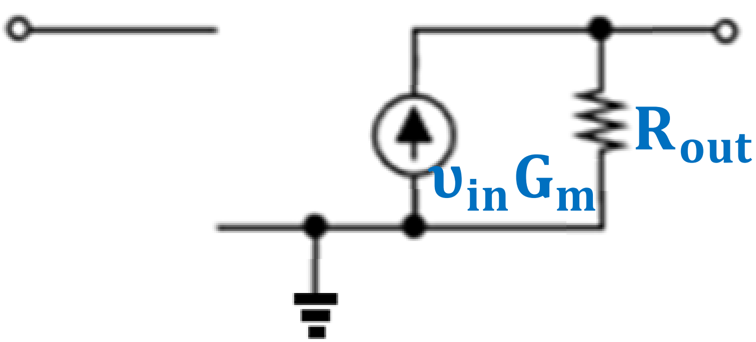
$$\boxed{R_{out} = R_{out1} \parallel R_D = [R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] \parallel R_D \approx R_D}$$

If $(gm_1 + gmb_1) r_{o1} R_S >> R_D$

Common-Source with Source Degradation

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = G_m R_{out}$$

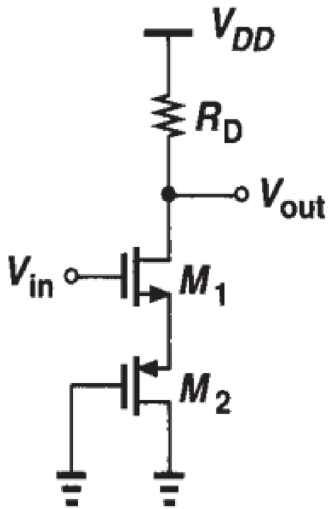
$$= \frac{-gm_1 r_{o1}}{R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S} \cdot \frac{[R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] R_D}{[R_S + r_{o1} + (gm_1 + gmb_1) r_{o1} R_S] + R_D}$$

$$\approx -\frac{R_D}{R_S}$$

If $(gm_1 + gmb_1) r_{o1}$, the intrinsic gain, is large.

Example

Assuming λ = γ = 0, calculate the small signal voltage gain of the circuit below.



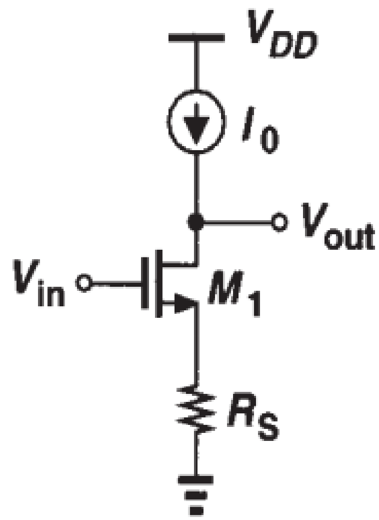
Solution:

$$G_m = -\frac{1}{\frac{1}{gm_1} + \frac{1}{gm_2}} \qquad R_{out} = R_D$$

$$A_v = G_m R_{out}$$

Example

Calculate the small signal voltage gain of the circuit below.



Solution:

$$G_m = \frac{-gm_1 r_{o1}}{r_{o1} + R_S + (gm_1 + gmb_1) r_{o1} R_S}$$

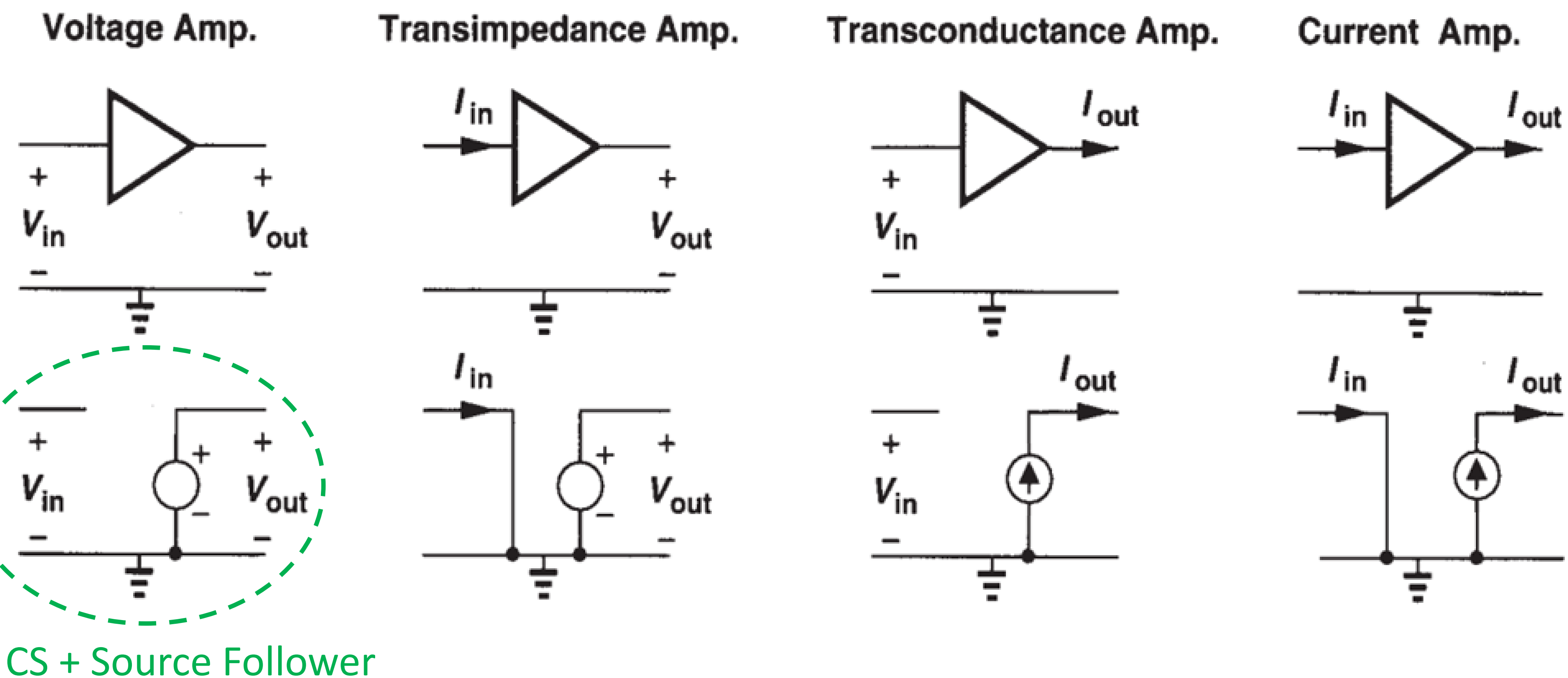
$$R_{out} = r_{o1} + R_S + (gm_1 + gmb_1) r_{o1} R_S$$

$$A_v = G_m R_{out} = -gm_1 r_{o1}$$

- I_o is ideal current source → Voltage across R_s is constant → M_1 source shorted to ground
- R_D replaced by current source → Nonlinearity issue arises again

Source Follower

Ideal Amplifier



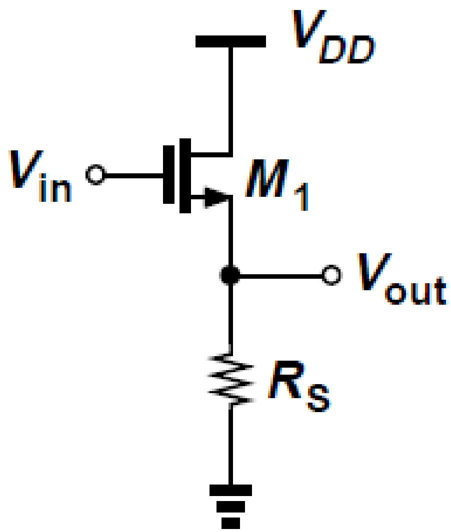
- For driving a low impedance load, source follower, as a buffer, provides **no gain** but **large input impedance** and **low output impedance**.

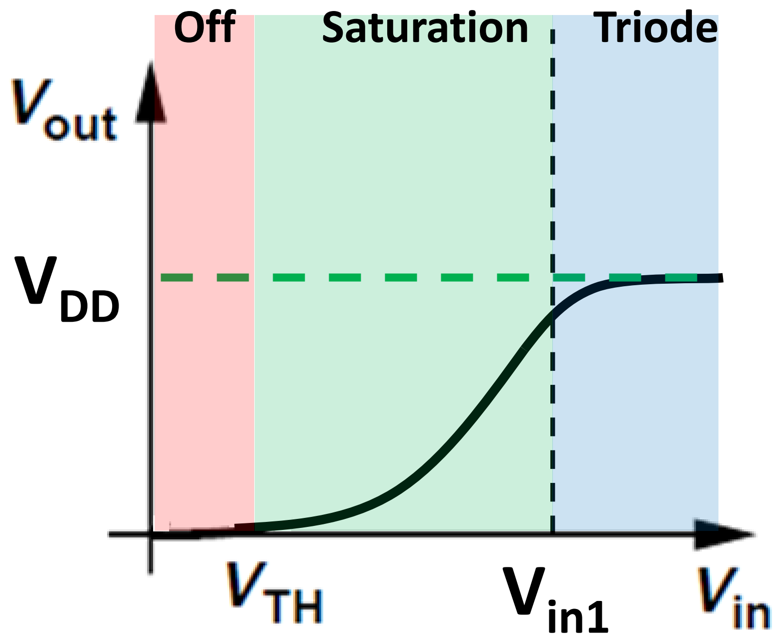
Source Follower

DC Analysis

$\lambda = 0$

$\gamma \neq 0$





$$V_{DD} - V_{out} = V_{in1} - V_{out} - V_{TH}$$

$$\rightarrow V_{in1} = V_{DD} + V_{TH}$$

- **$V_{in} < V_{TH} \rightarrow M_1$ Off**

$$V_{out} = 0$$

- **$V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ in Saturation**

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

- **$V_{in} > V_{in1} \rightarrow M_1$ in Triode**

$$R_S \mu_n C_{ox} \frac{W}{L} \left[(V_{in} - V_{out} - V_{TH})(V_{DD} - V_{out}) - \frac{1}{2}(V_{DD} - V_{out})^2 \right] = V_{out}$$

Source Follower

DC Analysis

$\lambda = 0$

$\gamma \neq 0$

- $V_{in1} > V_{in} > V_{TH} \rightarrow M_1$ **in Saturation**

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})^2 = V_{out}$$

$$R_S \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_{in} - V_{out} - V_{TH}) \left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$R_S \underbrace{\boxed{\mu_n C_{ox} \frac{W}{L} (V_{in} - V_{out} - V_{TH})}}_{= gm} \left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\frac{\partial V_{TH}}{\partial V_{out}}}_{= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}} \frac{\partial V_{out}}{\partial V_{in}}\right) = \frac{\partial V_{out}}{\partial V_{in}}$$

$$\boxed{A_v = \frac{gmR_S}{1 + gmR_S(1+\eta)} = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1+\eta}}$$

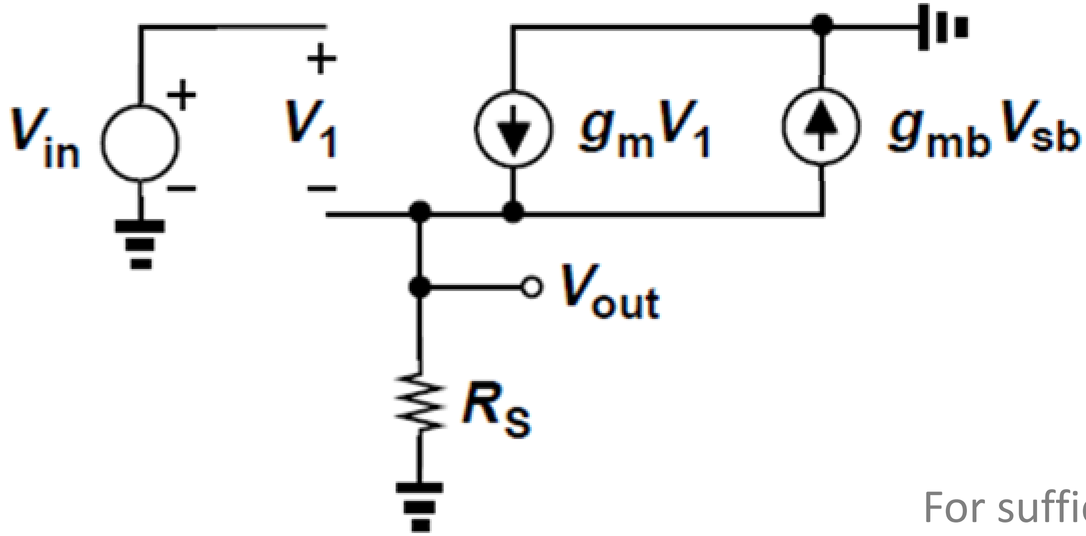
If $(gm + gmb)R_S >> 1$

Source Follower

Small-signal Analysis

$\lambda = 0$

$\gamma \neq 0$



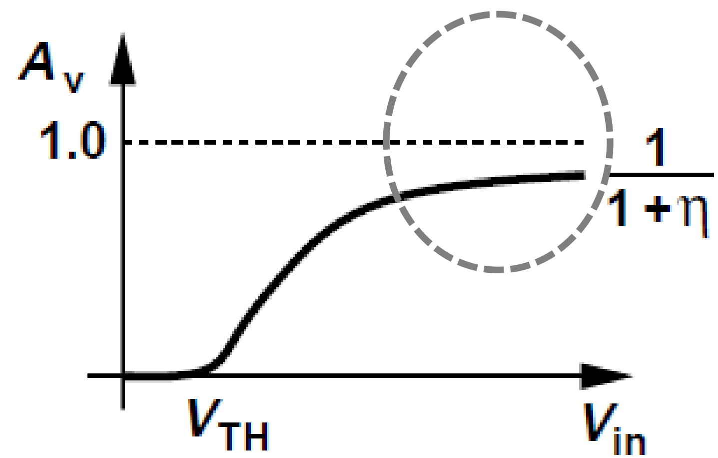
$$G_m = gm$$

$$R_{out} = R_S \parallel \left(\frac{1}{gm + gmb}\right)$$

$$\boxed{A_v = \frac{gmR_S}{1 + (gm + gmb)R_S} \approx \frac{1}{1 + \eta}}$$

If $(gm + gmb)R_S >> 1$

For sufficiently large V_{in}, I_D and thus gm.

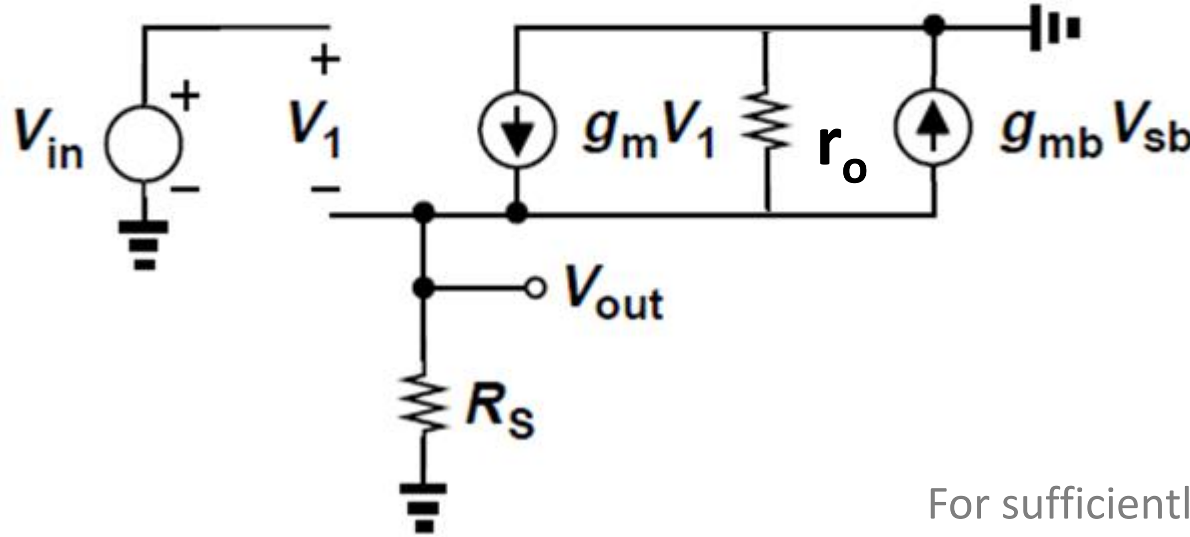


Source Follower

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$

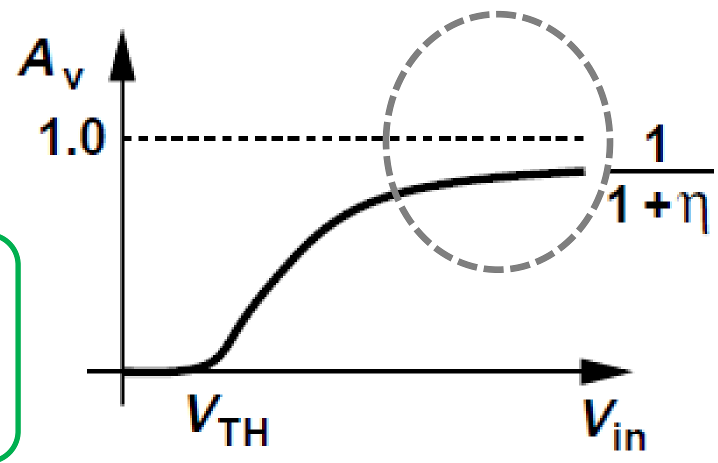


$$G_m = gm$$

$$R_{out} = r_o \parallel R_S \parallel \left(\frac{1}{gm + gmb}\right)$$

$$\boxed{A_v = \frac{gm r_o R_S}{r_o + R_S + (gm + gmb) r_o R_S} \approx \frac{1}{1 + \eta}}$$

For sufficiently large V_{in}, I_D and thus gm.



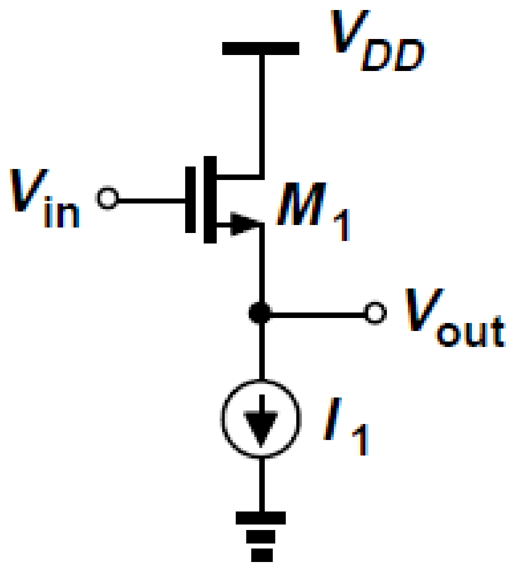
If $(gm + gmb) r_o R_S >> r_o$ and R_S

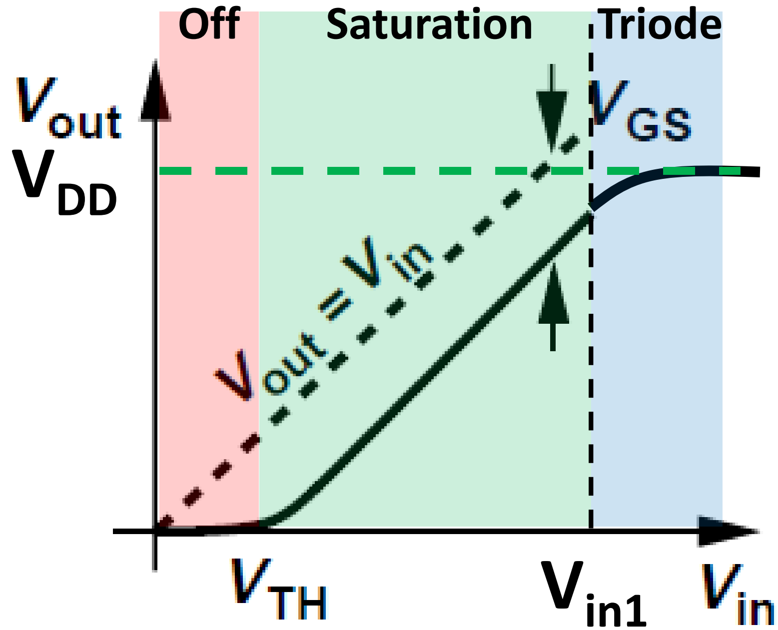
Source Follower with Current Source

DC Analysis

$\lambda = 0$

$\gamma \neq 0$





$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})^2 = I_1$$

$$\frac{1}{2}\mu_n C_{ox}\frac{W}{L}2(V_{in} - V_{out} - V_{TH})\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \frac{\partial V_{TH}}{\partial V_{in}}\right) = 0$$

$$\underbrace{\boxed{\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{TH})}}_{= gm}\left(1 - \frac{\partial V_{out}}{\partial V_{in}} - \underbrace{\boxed{\frac{\partial V_{TH}}{\partial V_{out}}}}_{= \eta}\frac{\partial V_{out}}{\partial V_{in}}\right) = 0$$

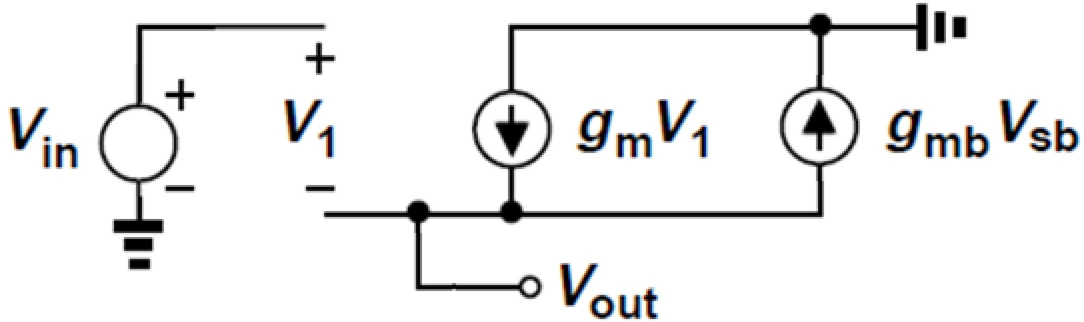
$$\boxed{A_v = \frac{1}{1+\eta} \qquad \text{If } \gamma = 0,\ A_v = 1.}$$

Source Follower with Current Source

Small-signal Analysis

λ = 0

γ ≠ 0

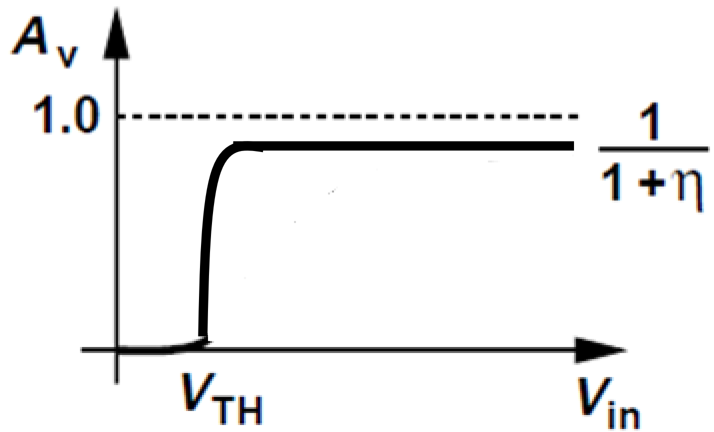


$$G_m = gm$$

$$R_{out} = \frac{1}{gm + gmb}$$

$$A_v = \frac{1}{1+\eta}$$

If $\gamma = 0$, $A_v = 1$.

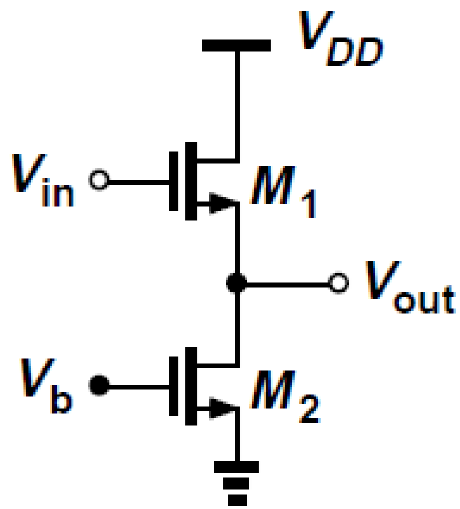


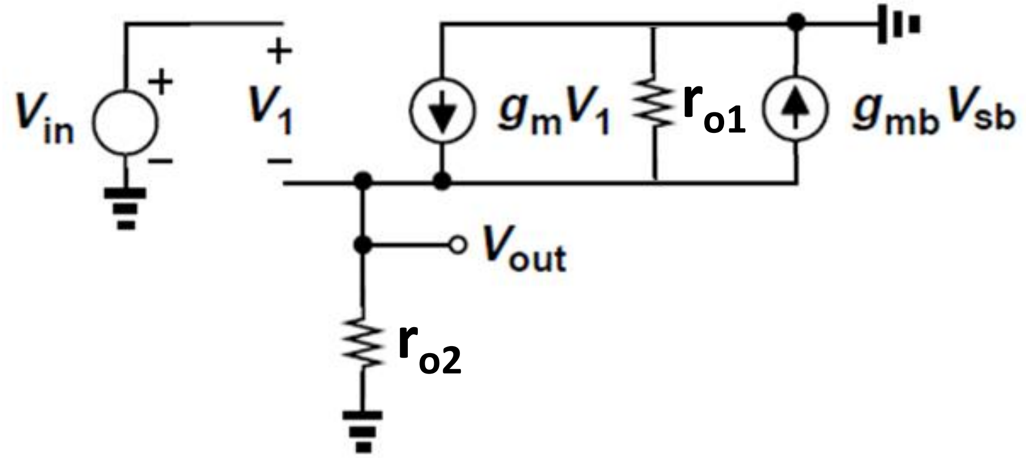
Source Follower with Current Source

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





$$G_m = gm_1$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \left(\frac{1}{gm_1 + gmb_1}\right)$$

$$\boxed{A_v = \frac{gmr_{o1}r_{o2}}{r_{o1} + r_{o2} + (gm + gmb)r_{o1}r_{o2}}}$$

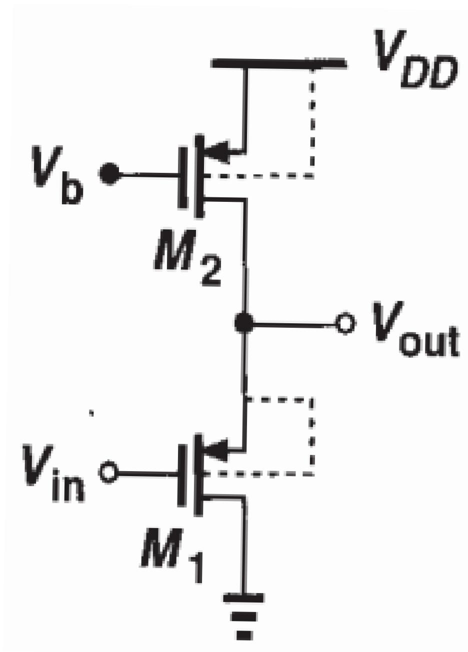
If r_{o1} and r_{o2} large, A_v is linear.

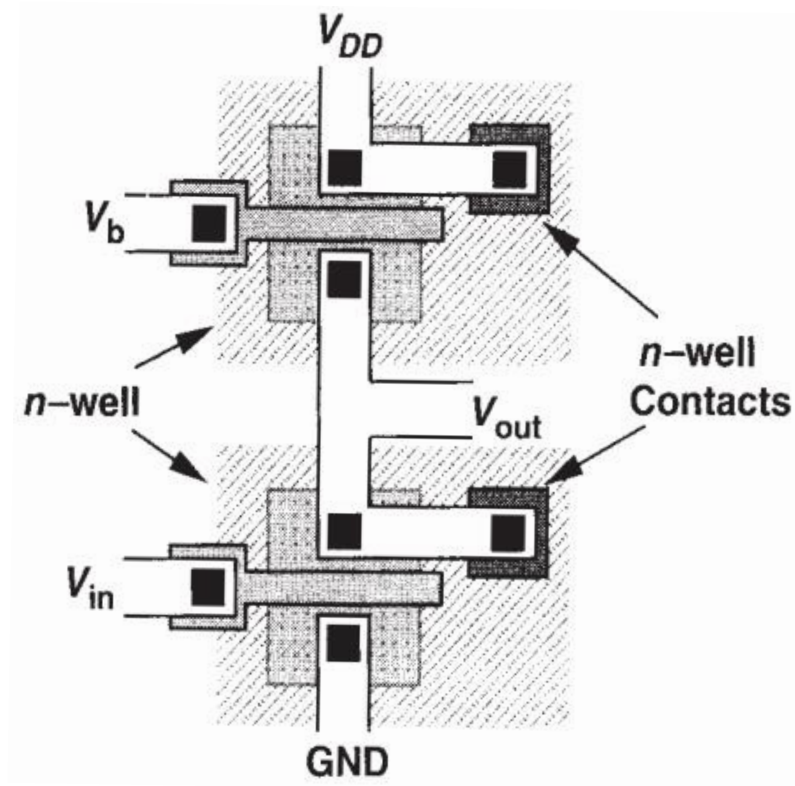
Source Follower with Current Source ($V_{SB} = 0$)

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





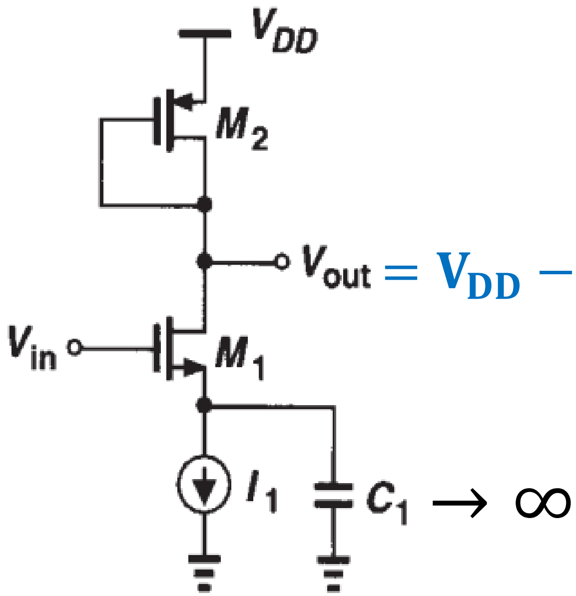
$$G_m = gm_1$$

$$R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{gm_1}$$

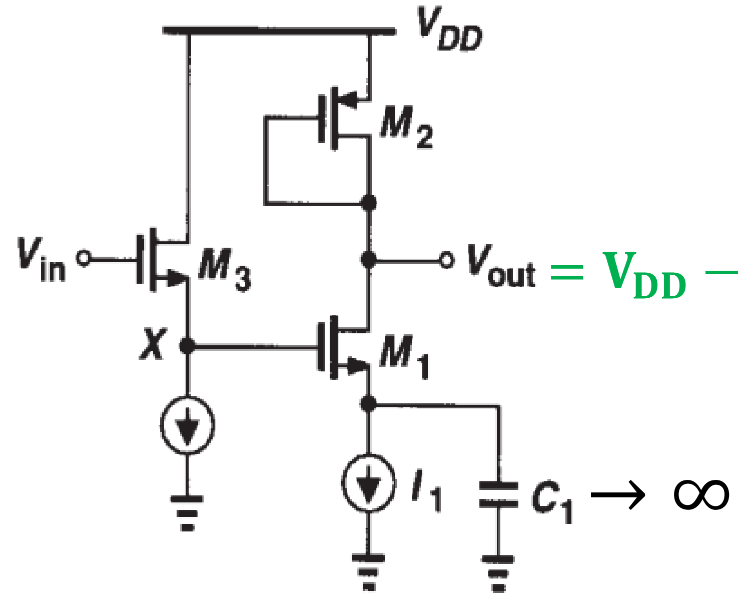
$$\boxed{A_v = \frac{gm_1 r_{o1} r_{o2}}{r_{o1} + r_{o2} + gm_1 r_{o1} r_{o2}}}$$

- The sacrifice here is the higher output impedance due to smaller mobility of holes relative to electrons.

Source Follower as Level Shifter



$V_{out} = V_{DD} - V_{SG2}$



$V_{out} = V_{DD} - V_{SG2}$

$$V_{in} \le V_{DD} - V_{SG2} + V_{TH1}$$

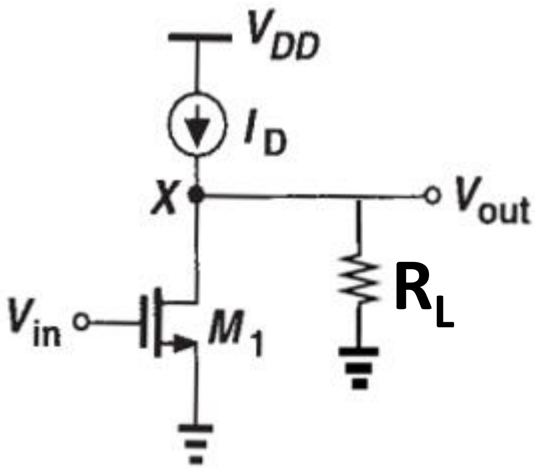
$$V_{in} - V_{GS3} \le V_{DD} - V_{SG2} + V_{TH1}$$

$$\begin{cases} G_m = -gm_1 \\ R_{out} = r_{o1} \parallel r_{o2} \parallel \dfrac{1}{gm_2} \end{cases}$$

$$\begin{cases} G_{m(left)} = gm_3 \\ R_{out(left)} = r_{o3} \parallel \dfrac{1}{gm_3 + gmb_3} \end{cases}$$

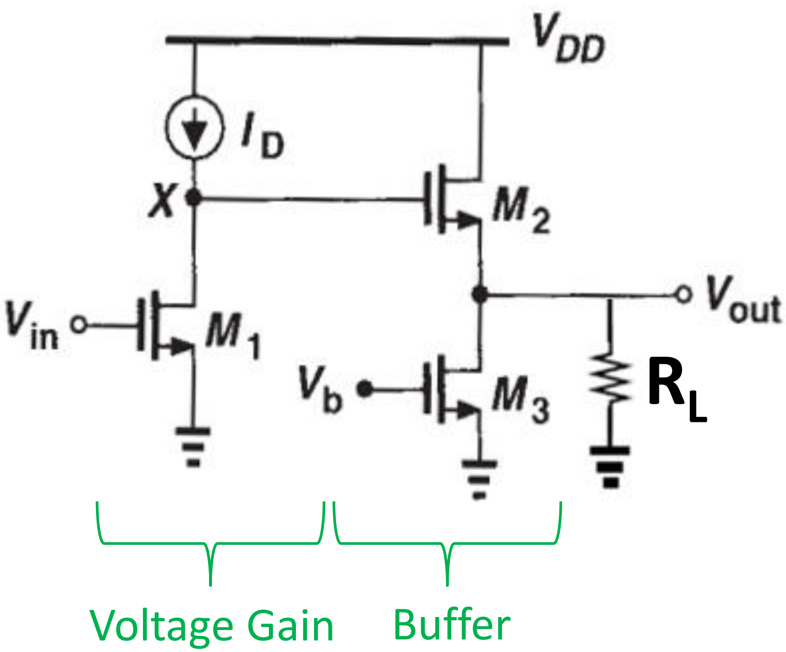
$$\begin{cases} R_{in(right)} = \infty \\ G_{m(right)} = -gm_1 \\ R_{out(right)} = r_{o1} \parallel r_{o2} \parallel \dfrac{1}{gm_2} \end{cases}$$

CS + Source Follower



$$A_v = -gm_1(r_{o1} \parallel R_L)$$

- Voltage gain severely reduced when R_L very small

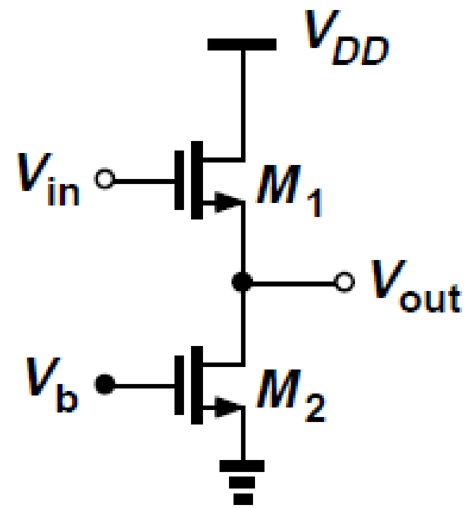


$$A_v = -gm_1 r_{o1} \times gm_2\left(r_{o2} \parallel \frac{1}{gm_2 + gmb_2} \parallel r_{o3} \parallel R_L\right)$$

- Voltage gain maintained when R_L very small

Example

$(W/L)_1 = 20/0.5$, $I_D = 0.2$ mA, $V_{THO} = 0.6$ V, $2\phi_F = 0.7$ V, $\mu_n C_{ox} = 50\ \mu A/V^2$, $\gamma = 0.4\ V^{1/2}$ and $\lambda = 0$. (a) Calculate V_{out} for $V_{in} = 1.2$ V. (b) Minimum $(W/L)_2$ for which M_2 remains saturated.



Solution:

(a)

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in} - V_{out} - V_{THO})^2 \rightarrow V_{out} = 0.153\ \text{V}$$

$$V_{TH1} = V_{THO} + \gamma\left(\sqrt{2\Phi_F + V_{out}} - \sqrt{2\Phi_F}\right) = 0.635\ \text{V} \rightarrow V_{out} \approx 0.118\ \text{V}$$

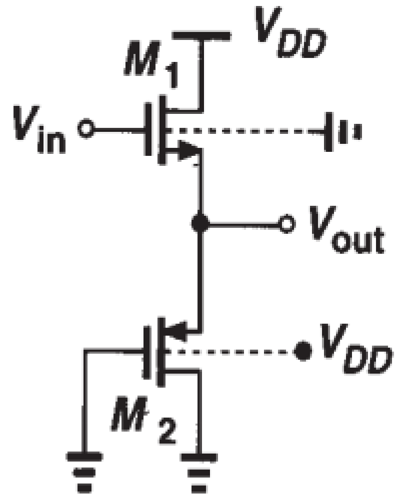
(b)

$$V_{out} = 0.118\ \text{V} \geq V_{GS2} - V_{TH2}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS2} - V_{TH2})^2 \rightarrow \left(\frac{W}{L}\right)_2 \geq \frac{283}{0.5}$$

Example

Calculate the small signal voltage gain of the circuit below.



Solution:

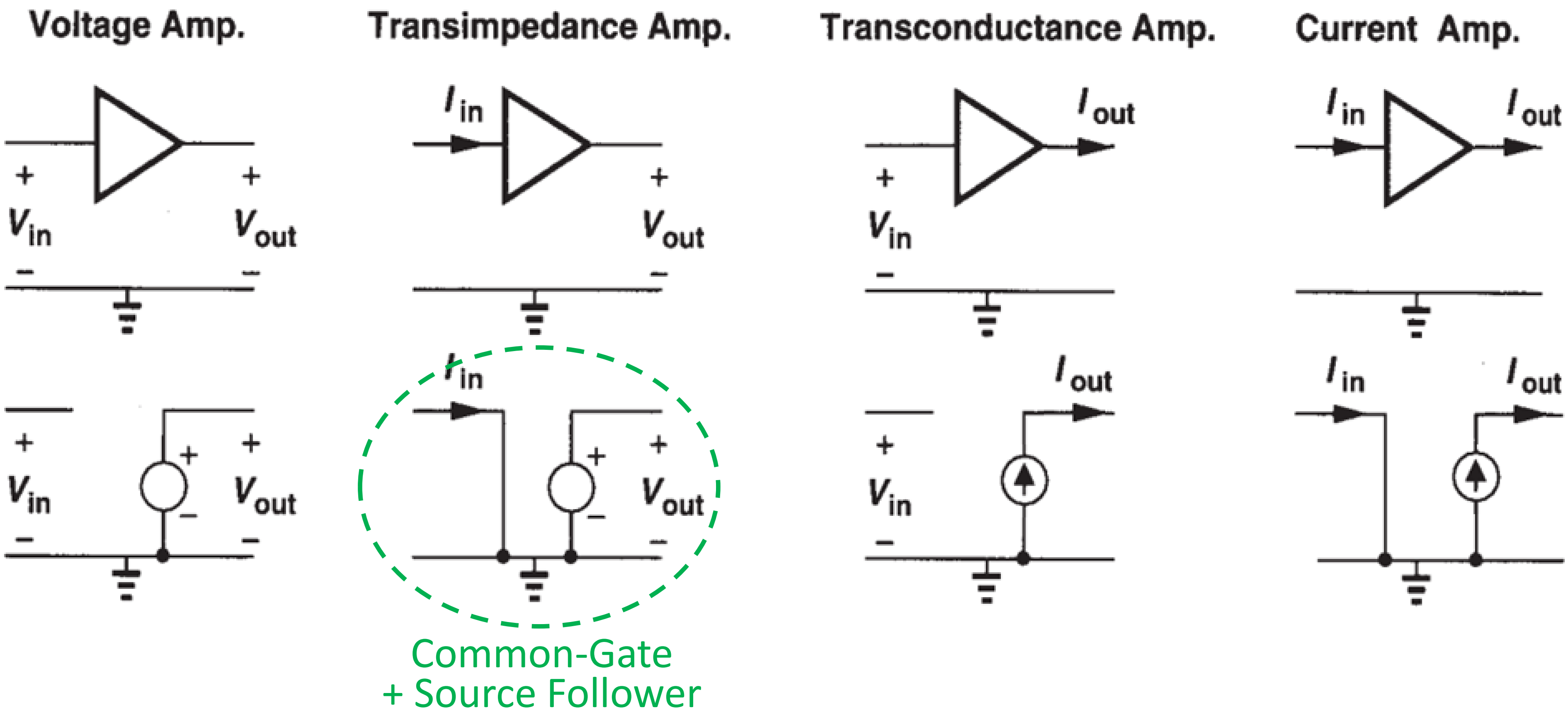
$$G_m = gm_1$$

$$R_{out} = \frac{1}{gm_1 + gmb_1} \parallel r_{o1} \parallel \frac{1}{gm_2 + gmb_2} \parallel r_{o2}$$

$$A_v = G_m R_{out}$$

Common-Gate

Ideal Amplifier



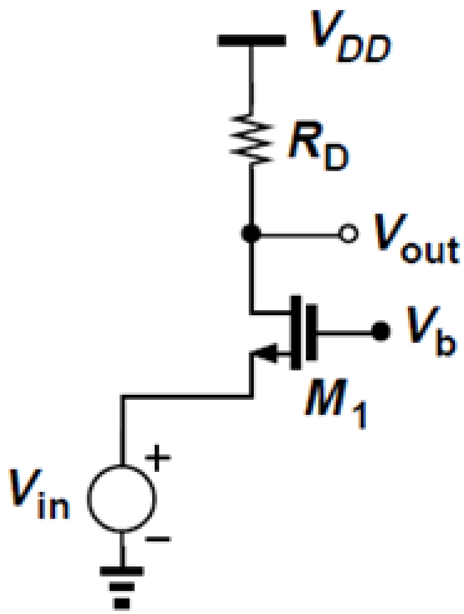
- For converting and amplifying small-signal current to voltages, common-gate provides **low input impedance** and **moderate gain**, but relatively **large output impedance**.

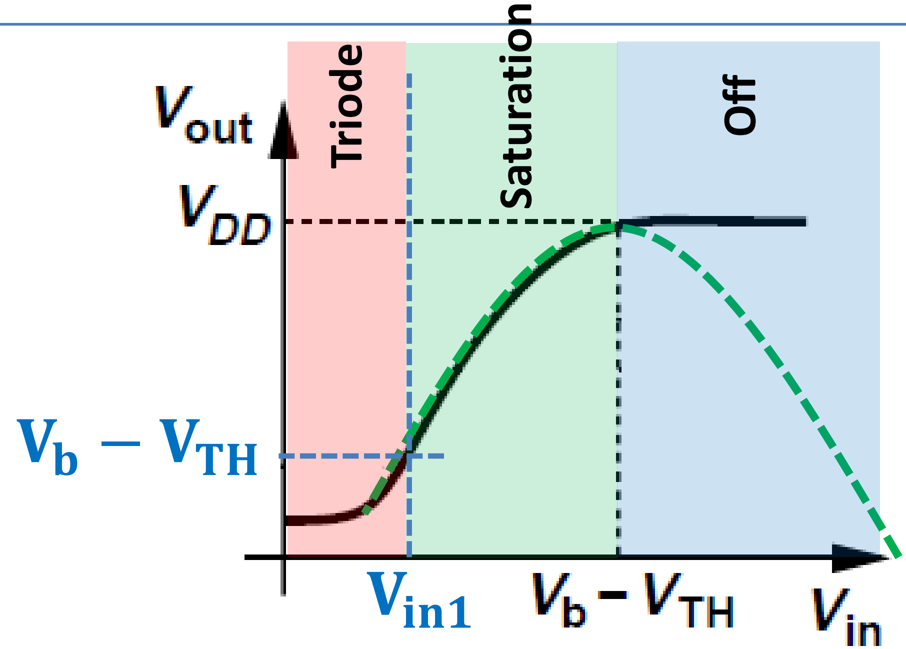
Common-Gate

DC Analysis

$\lambda = 0$

$\gamma \neq 0$





$$V_{out} = V_b - V_{TH}$$
$$= V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in1} - V_{TH})^2$$

- **$V_{in} > V_b - V_{TH}$ → M_1 Off**

  $V_{out} = V_{DD}$

- **$V_b - V_{TH} > V_{in} > V_{in1}$ → M_1 in Saturation**

  $$V_{out} = V_{DD} - R_D \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - V_{in} - V_{TH})^2$$

- **$V_{in} < V_{in1}$ → M_1 in Triode**

  $$V_{out} = V_{DD} - R_D \mu_n C_{ox}\frac{W}{L}\left[(V_b - V_{in} - V_{TH})(V_{out} - V_{in}) - \frac{1}{2}(V_{out} - V_{in})^2\right]$$

Common-Gate

DC Analysis

$\lambda = 0$
$\gamma \neq 0$

- **$V_b - V_{TH} > V_{in} > V_{in1} \rightarrow M_1$ in Saturation**

$$V_{out} = V_{DD} - R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})^2$$

$$\frac{\partial V_{out}}{\partial V_{in}} = -R_D \frac{1}{2} \mu_n C_{ox} \frac{W}{L} 2(V_b - V_{in} - V_{TH}) \left(-1 - \frac{\partial V_{TH}}{\partial V_{in}}\right)$$

$$= R_D \underbrace{\mu_n C_{ox} \frac{W}{L} (V_b - V_{in} - V_{TH})}_{= gm} \left(1 + \underbrace{\frac{\partial V_{TH}}{\partial V_{in}}}_{= \eta = \frac{\gamma}{2\sqrt{2\Phi_F + V_{SB}}}}\right)$$

$$\boxed{A_v = \frac{\partial V_{out}}{\partial V_{in}} = R_D gm(1 + \eta)}$$

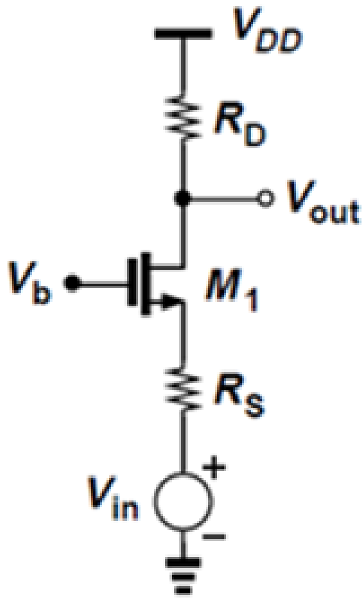
- gm is a function of I_D and η is a function of V_{SB}.
- A_v is not quite linear.

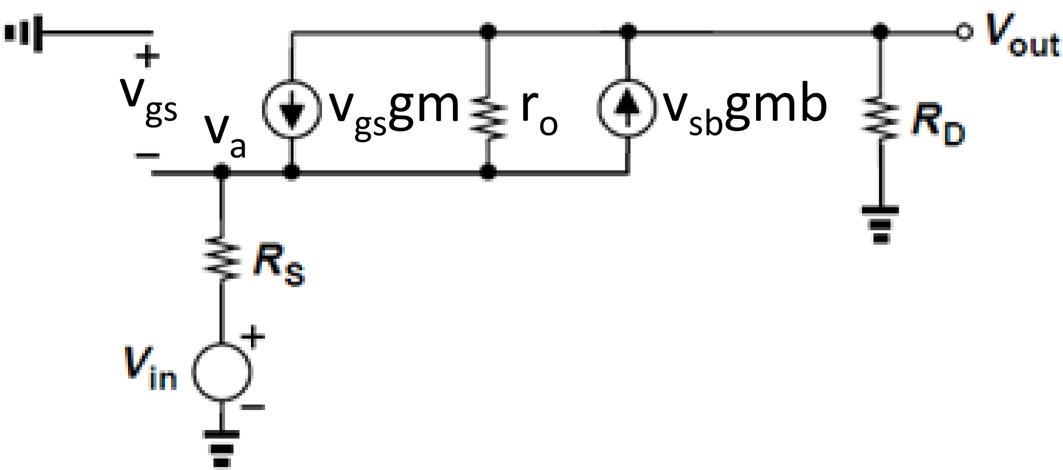
Common-Gate

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$





$$G_m = \frac{(gm + gmb)r_o + 1}{r_o + R_S + (gm + gmb)r_oR_S}$$

$$R_{out} = R_D \parallel [r_o + R_S + (gm + gmb)r_oR_S]$$

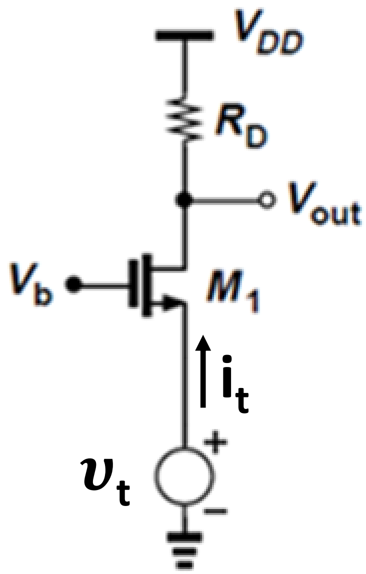
$$\boxed{A_v = \frac{(gm + gmb)r_o + 1}{r_o + R_S + (gm + gmb)r_oR_S + R_D}R_D} \approx R_Dgm(1+\eta)$$

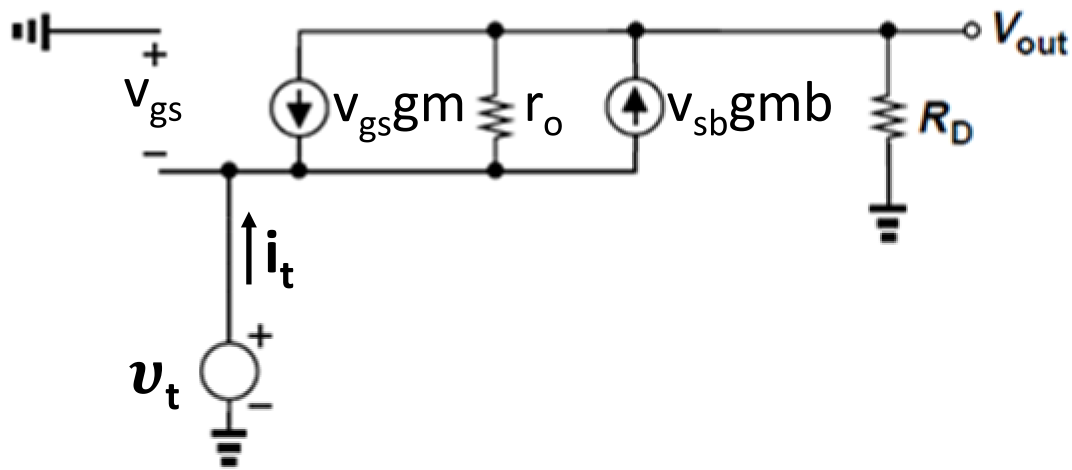
If $R_S = 0$ and $r_o = \infty$

Common-Gate (Input Impedance)

Small-signal Analysis

$\lambda \neq 0$
$\gamma \neq 0$





$$\begin{cases} i_t = \upsilon_t(gm + gmb) + \dfrac{\upsilon_t - \upsilon_{out}}{r_o} \\ \upsilon_{out} = R_D i_t \end{cases}$$

$$\boxed{R_{in} = \frac{R_D + r_o}{1 + (gm + gmb)r_o}}$$

If $R_D = 0$ → $R_{in} = r_o \parallel \dfrac{1}{gm} \parallel \dfrac{1}{gmb}$

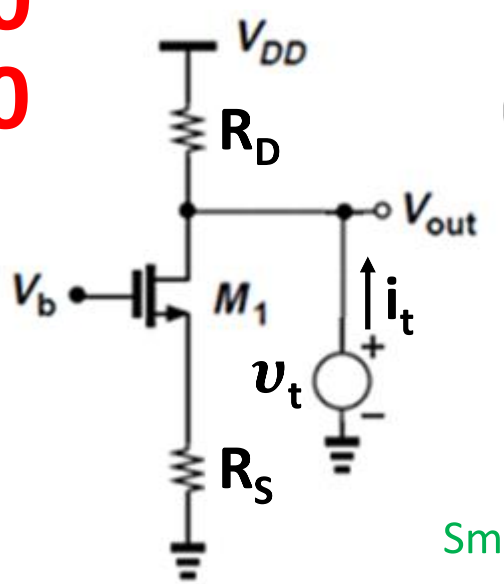
If $R_D = \infty$ → $R_{in} = \infty$

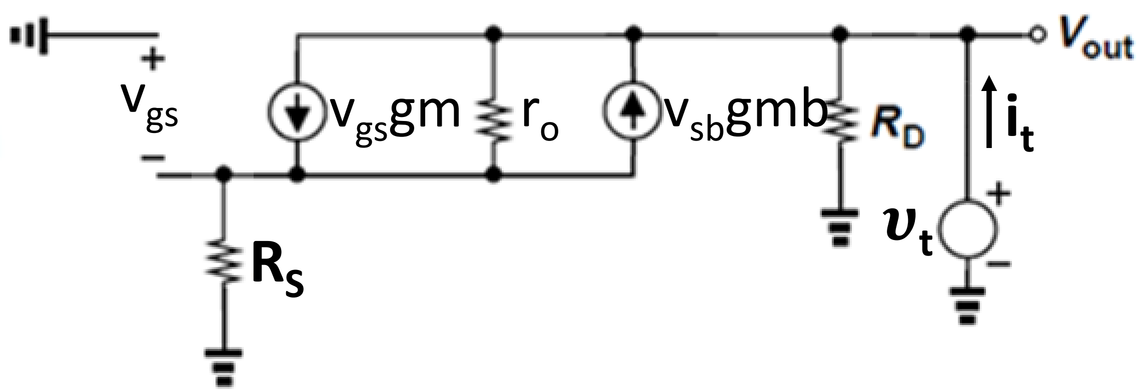
Common-Gate (Output Impedance)

Small-signal Analysis

$\lambda \neq 0$

$\gamma \neq 0$



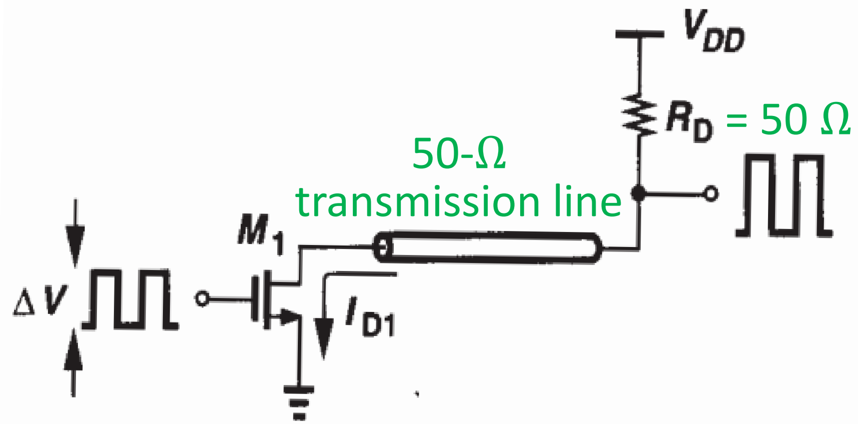


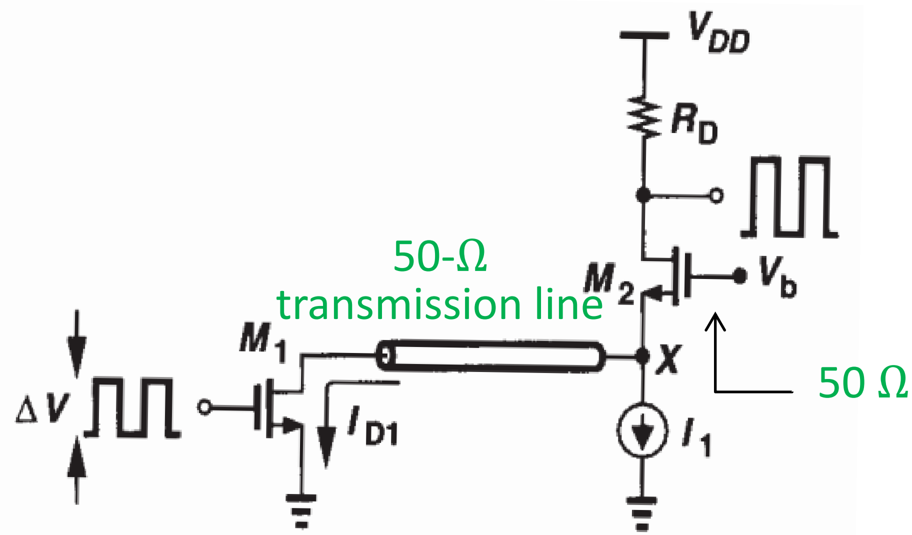
Small-signal model the same as CS with source degradation

$$R_{out} = [R_S + r_{o1} + (gm_1 + gmb_1)r_{o1}R_S] \parallel R_D$$

Example

Calculate the small-signal voltage gain at low frequencies of the circuits below. To minimize wave reflection at point X, the input impedance must be equal to 50 Ω.





$$A_v = -gm_1 R_D$$

= 50 Ω

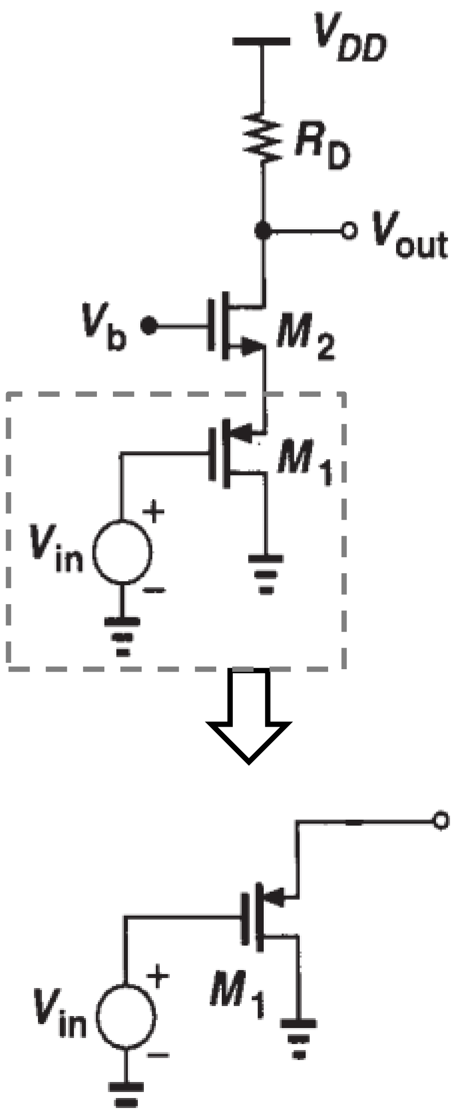
$$A_v = -gm_1 R_D$$

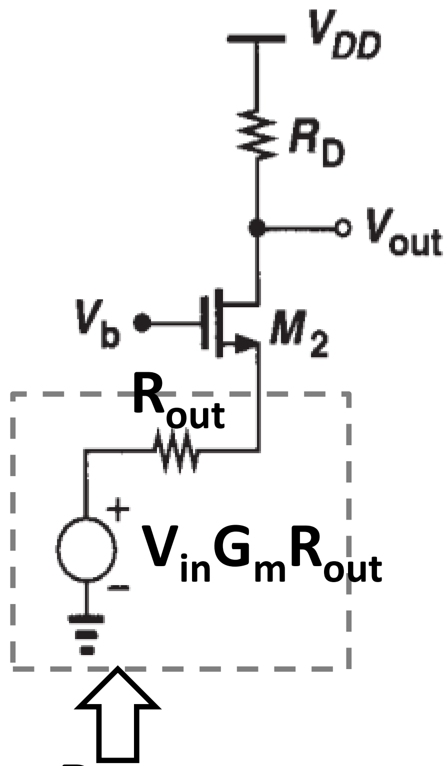
can be much larger than 50 Ω, so as to achieve a much higher gain.

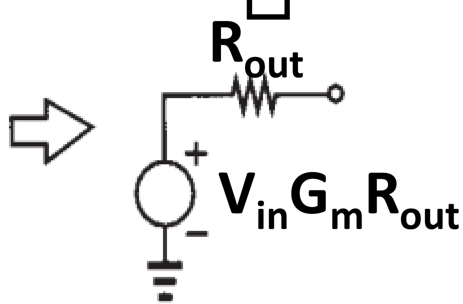
$$R_{in} = \frac{R_D + r_{o2}}{1 + (gm_2 + gmb_2)r_{o2}} = 50\ \Omega$$

Example

Calculate the small-signal voltage gain of the circuit below. ($\lambda \neq 0$, $\gamma \neq 0$)



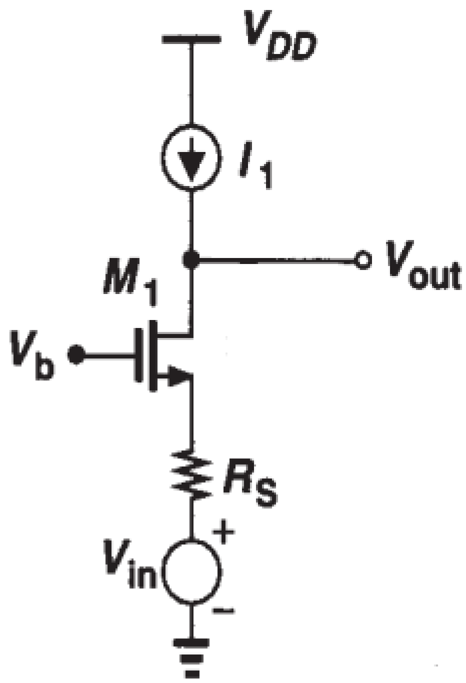


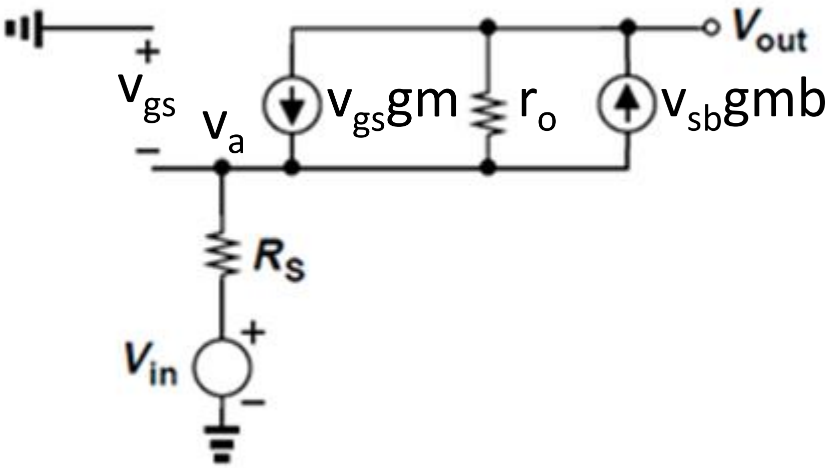


$$\begin{cases} G_m = gm_1 \\ R_{out} = r_{o1} \parallel \dfrac{1}{gm_1} \parallel \dfrac{1}{gmb_1} \end{cases}$$

Example

Calculate the small-signal voltage gain of the circuit below. (λ ≠ 0, γ ≠ 0)





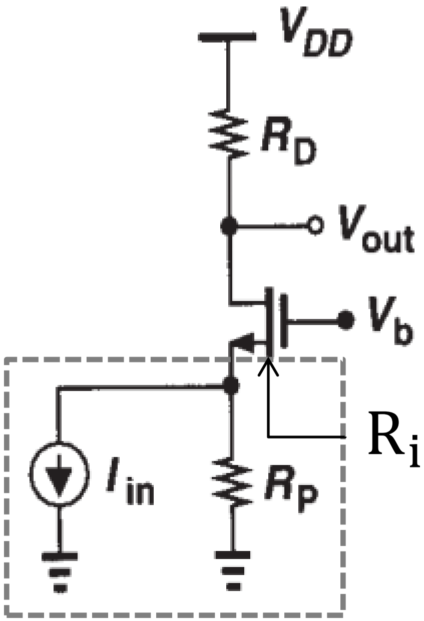
Since no current flowing in Rs, $\upsilon_a = \upsilon_{in}$.

$$\upsilon_{out} - \upsilon_{in}(gm + gmb)r_o = \upsilon_{in}$$

$$A_v = \frac{\upsilon_{out}}{\upsilon_{in}} = 1 + (gm + gmb)r_o$$

Example

Calculate the small-signal transimpedance gain of the circuit below. (λ ≠ 0, γ ≠ 0)



Current Source

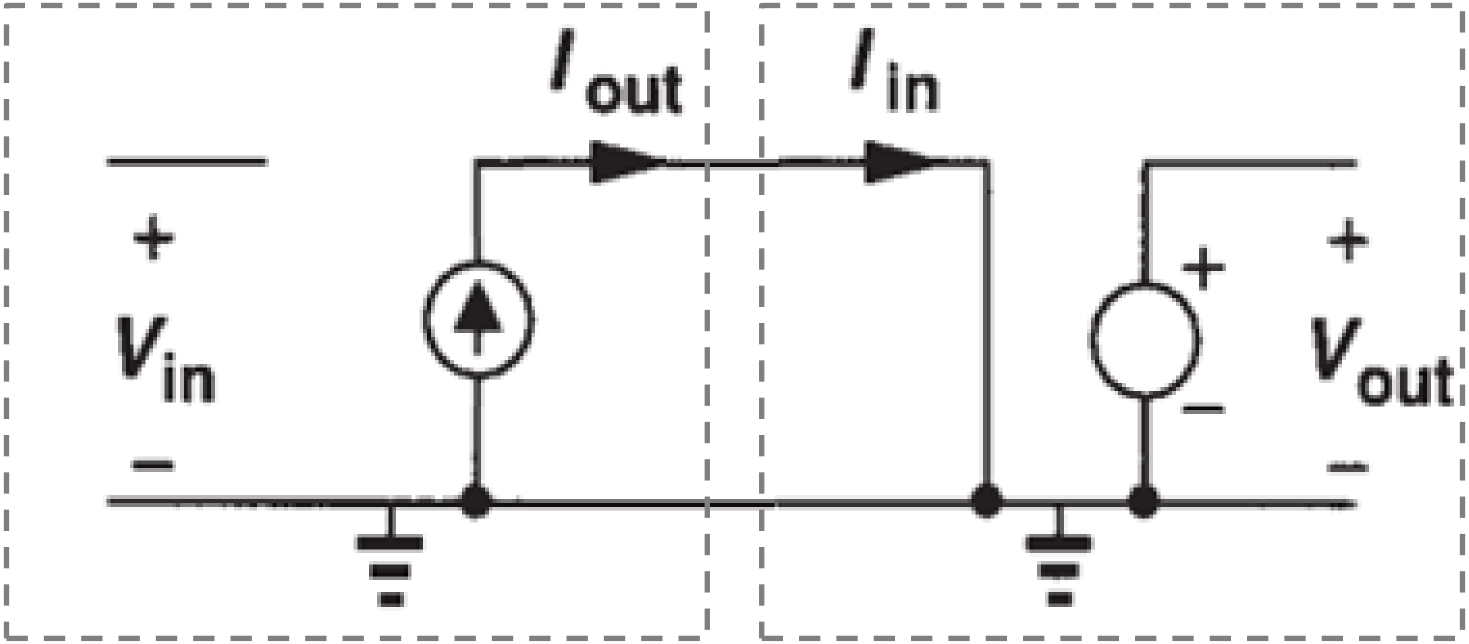
$$R_{in} = \frac{R_D + r_o}{1 + (gm + gmb)r_o}$$

$$-i_{in}\frac{R_P}{R_{in} + R_P}R_D = \upsilon_{out}$$

$$\frac{\upsilon_{out}}{i_{in}} = -\frac{R_P}{R_{in} + R_P}R_D = \frac{-R_P R_D[1 + (gm + gmb)r_o]}{R_D + r_o + R_P + (gm + gmb)r_o R_P}$$

Cascode

Ideal Amplifier



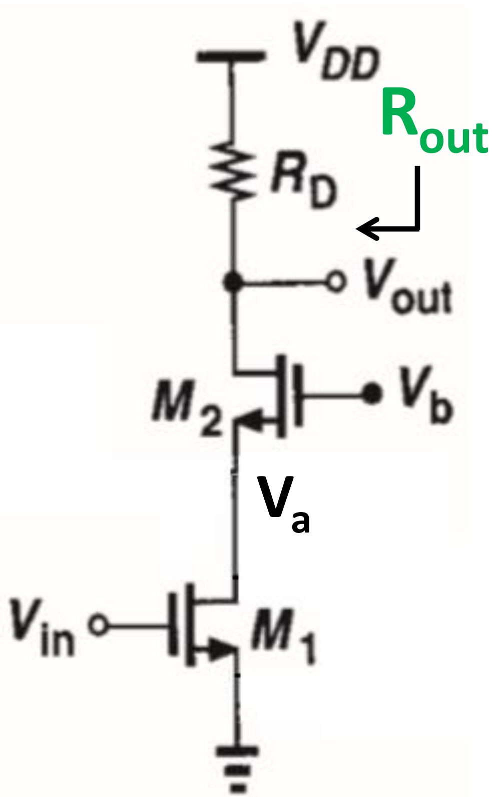
Common-Source

Common-Gate
+ Source Follower

CS + CG with Resistive Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\begin{cases} G_m = -gm_1 \dfrac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\ R_{out} = [r_{o1} + r_{o2} + (gm_2 + gmb_2)r_{o2}r_{o1}] \parallel R_D \end{cases}$$

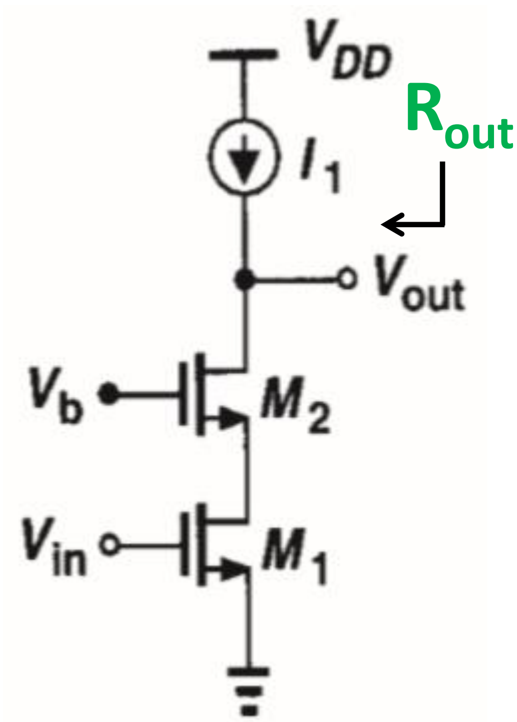
$$A_v = G_m R_{out}$$

$$V_a \geqq V_{in} - V_{TH1}$$
$$V_b - V_{GS2} \geqq V_{in} - V_{TH1}$$
$$V_b \geqq V_{in} - V_{TH1} + V_{GS2}$$
$$V_{out} \geqq V_b - V_{TH2} \geqq (V_{in} - V_{TH1}) + (V_{GS2} - V_{TH2})$$
$$\mathbf{V_{DD} \geqq V_{out} \geqq V_{ov1} + V_{ov2}}$$

CS + CG with Ideal Current Source Load

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



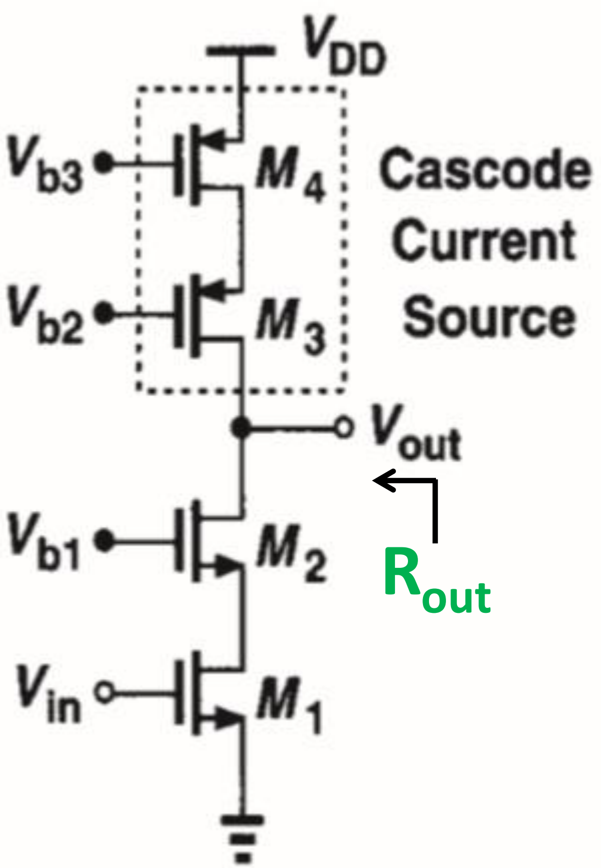
$$\begin{cases} G_m = -gm_1 \dfrac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\ R_{out} = r_{o1} + r_{o2} + (gm_2 + gmb_2) r_{o2} r_{o1} \end{cases}$$

$$A_v = G_m R_{out}$$

CS + CG with Cascode Current Source Load

Small-signal Analysis

λ ≠ 0 γ ≠ 0



$$
\begin{cases}
G_m = -gm_1 \dfrac{r_{o1}}{r_{o1} + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\
R_{out} = [r_{o1} + r_{o2} + (gm_2 + gmb_2) r_{o2} r_{o1}] \\
\qquad \parallel [r_{o3} + r_{o4} + (gm_3 + gmb_3) r_{o3} r_{o4}]
\end{cases}
$$

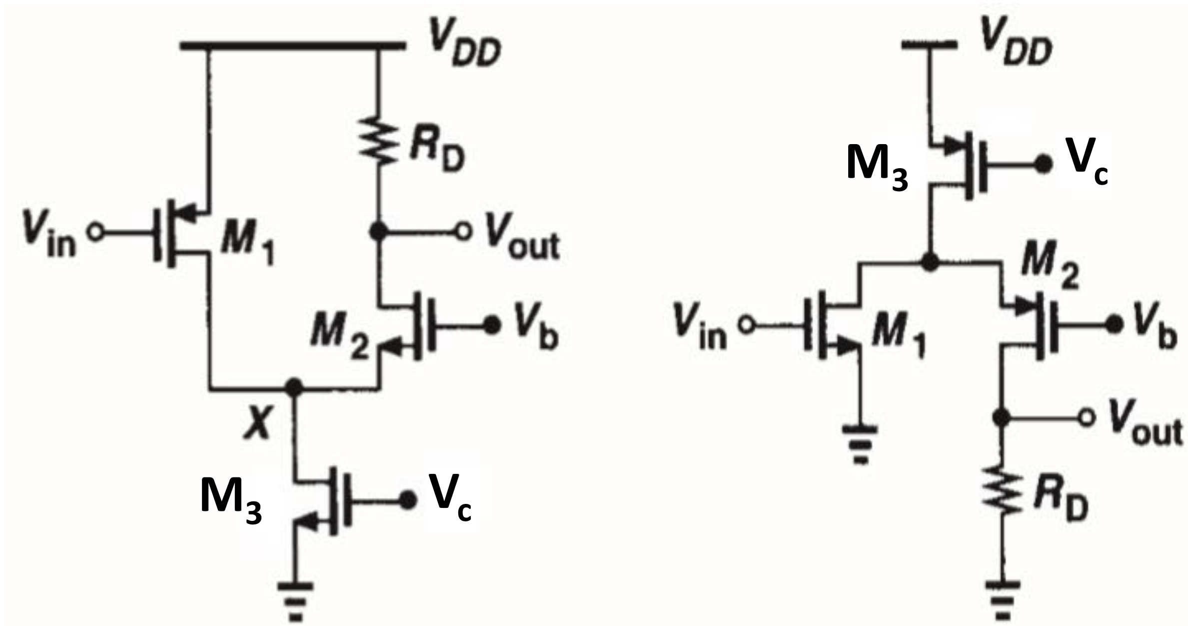
$$A_v = G_m R_{out}$$

$$\mathbf{V_{DD} - V_{ov3} - V_{ov4} \geqq V_{out} \geqq V_{ov1} + V_{ov2}}$$

Folded Cascode

Small-signal Analysis

$\lambda \neq 0 \quad \gamma \neq 0$



$$\begin{cases} G_m = -gm_1 \dfrac{(r_{o1} \parallel r_{o3})}{(r_{o1} \parallel r_{o3}) + \left(r_{o2} \parallel \dfrac{1}{gm_2 + gmb_2}\right)} \\ R_{out} = [(r_{o1} \parallel r_{o3}) + r_{o2} + (gm_2 + gmb_2) r_{o2} (r_{o1} \parallel r_{o3})] \parallel R_D \end{cases}$$

$$A_v = G_m R_{out}$$