Homework 4

VE311 - Electronic Circuits Fall 2021

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4.1

4.1.1

Since it is NMOS and $V_D = V_{DD}$ and $V_S = V_X$. We have

$$
\begin{aligned}
V_{TH} &= 0.7 \text{ V} \\
\mu_n &= 350 \text{ cm}^2/(\text{V}\cdot\text{s}) = 0.035 \text{ m}^2/(\text{V}\cdot\text{s}) \\
C_{OX} &= \frac{\epsilon_o \epsilon_r}{t_{ox}} = \frac{8.85\times10^{-12}\times3.9}{9\times10^{-9}} = 3.835\times10^{-3} \text{ F/m}^2 \\
W &= 2\times10^{-5} \text{ m} \\
L_{eff} &= L_{\text{drawn }} - 2L_D = 1.84\times10^{-6} \text{ m}
\end{aligned}
$$

Since

$$V_{GS} - V_{TH} = 4.3 - V_X$$

When $V_X < 4.3$ V

$$V_{DS} - (V_{GS} - V_{TH}) = 0.7 \text{ V} > 0 \quad \Rightarrow \quad V_{DS} > V_{GS} - V_{TH}$$

in the saturation region.

$$
\begin{aligned}
I_X = I_D &= \frac{1}{2}\mu_n C_{OX}\frac{W}{L_{eff}}(V_{GS}-V_{TH})^2 \\
&= 7.295\times10^{-4}\times(V_{DD}-V_X-V_{TH})^2 \\
&= 7.295\times10^{-4}\times(4.3-V_X)^2
\end{aligned}
$$

Therefore,

$$
I_x = \begin{cases} 7.295\times10^{-4}\times(4.3-V_x)^2 & V_x < 4.3 \text{ V} \\ 0 & V_x > 4.3 \text{ V} \end{cases}
$$

4.1.2

$V_G = 1.9$ V.

$$
\begin{aligned}
V_{TH} &= 0.7 \text{ V}, \\
\mu_n &= 350 \text{ cm}^2/(\text{V}\cdot\text{s}) = 0.035 \text{ m}^2/(\text{V}\cdot\text{s}) \\
C_{OX} &= \frac{\epsilon_o \epsilon_r}{t_{\text{ox}}} = \frac{8.85\times10^{-12}\times3.9}{9\times10^{-9}} = 3.835\times10^{-3} \text{ F/m}^2 \\
W &= 2\times10^{-5} \text{ m} \\
L_{\text{eff }} &= L_{\text{drawn }} - 2L_D = 1.84\times10^{-6} \text{ m}
\end{aligned}
$$

$V_x < 1$ V, Then S and D changes their roles

$$V_D - V_G + V_{TH} = -0.2 \text{ V} \quad \Rightarrow \quad V_{DS} < V_{GS} - V_{TH}$$

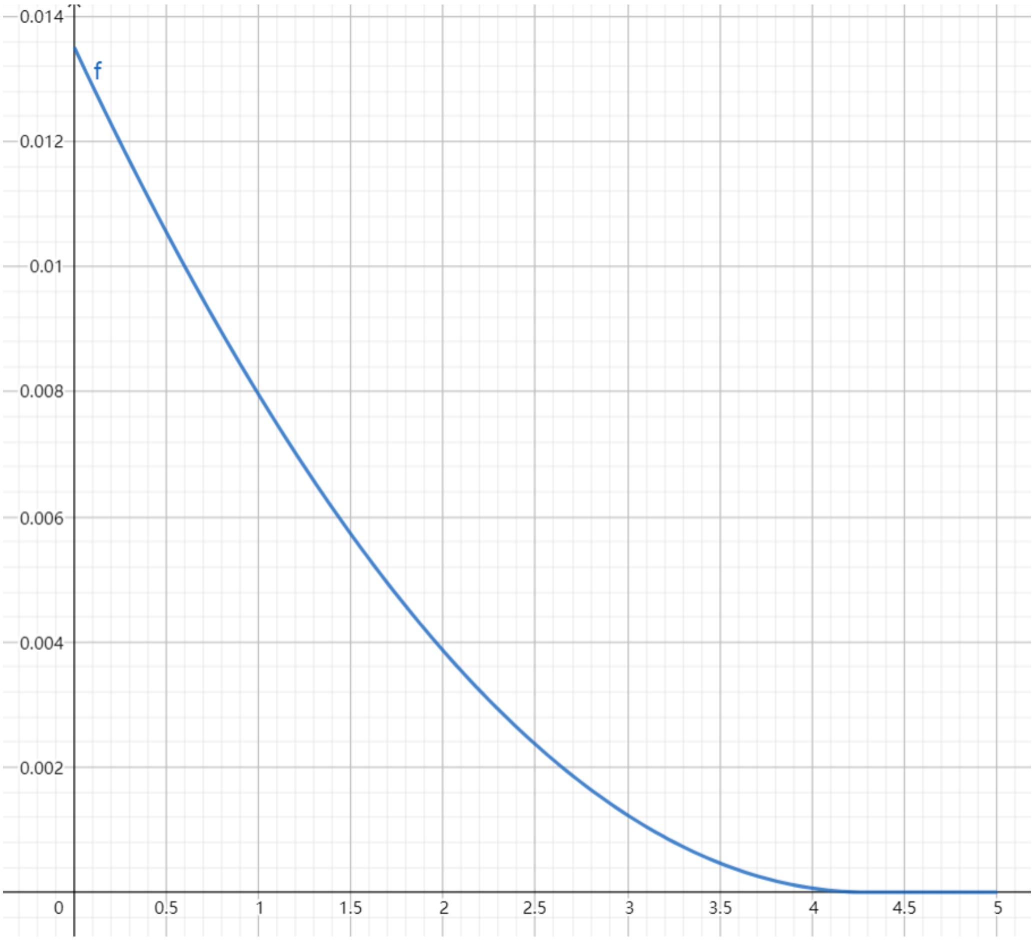

Figure 1: problem 1

it is in the triode region. According to the formula:

$$\begin{aligned} I_X &= -I_D = -\mu_n C_{OX} \frac{W}{L_{eff}} \left[(V_{GS} - V_{TH}) V_{DS} - \frac{1}{2} V_{DS}^2 \right] \\ &= -1.92 \times 10^{-3} \times (1 - V_X)(0.7 - 0.5 V_X) \end{aligned}$$

When $V_x > 1$ V Then, S and D changes their roles again, If $1 \text{ V} < V_x < 1.2$ V, then it is in the triode region, so they are the same.

If $V_x > 1.2$ V, then it is in the saturation region, so we have

$$\begin{aligned} I_X = I_D &= \frac{1}{2} \mu_n \, \text{CoX} \, \frac{W}{L_{\text{eff}}} (V_{GS} - V_{TH})^2 \\ &= 0.7295 \times 10^{-3} \times 0.2^2 \\ &= 2.918 \times 10^{-5} \text{ A} \end{aligned}$$

So,

$$I_X = \begin{cases} 1.92 \times 10^{-3} \times (V_x - 1)(0.7 - 0.5 V_X) & 0 < V_x < 1.2 \text{ V} \\ 2.918 \times 10^{-5} \text{ A} & v_X > 1.2 \text{ V} \end{cases}$$

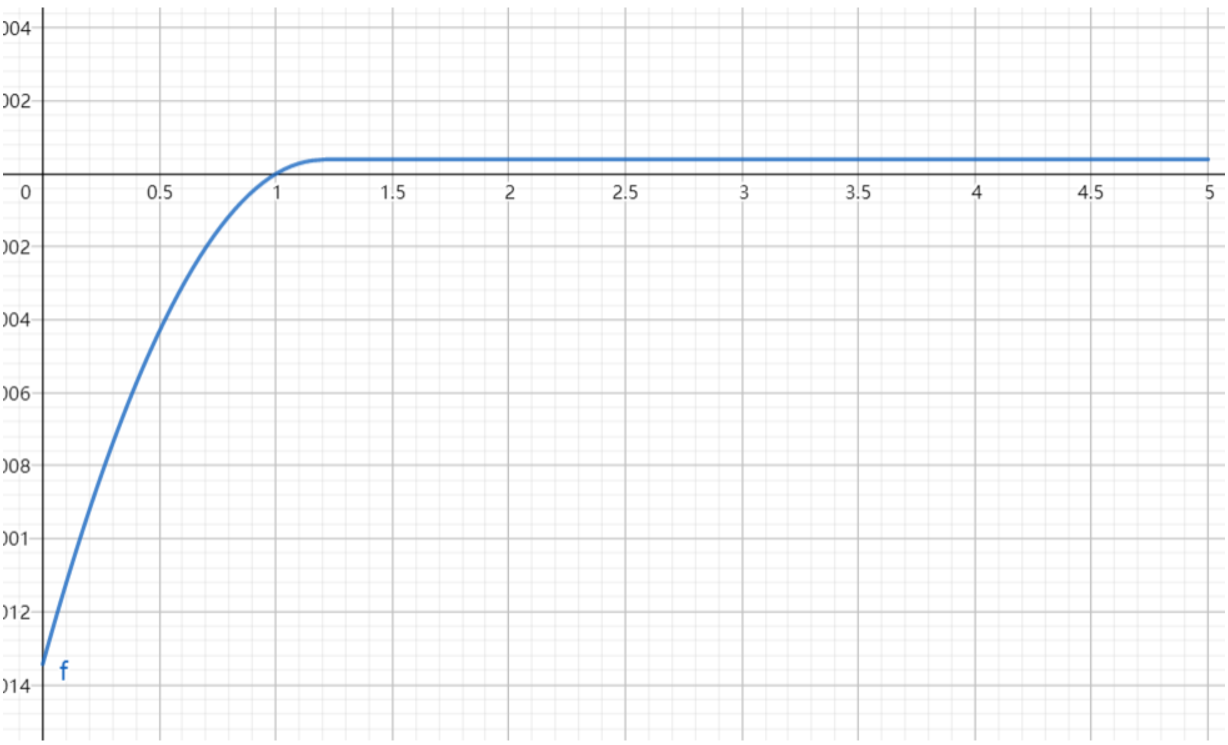

Figure 2: problem 2

4.1.3

$V_G = 1$ V, we have

$$\begin{aligned} V_{TH} &= 0.7 \text{ V} \\ \mu_n &= 0.035 \text{ m}^2/(\text{V}\cdot\text{s}) \\ C_{OX} &= \frac{\epsilon_o \epsilon_r}{t_{ox}} = \frac{8.85\times10^{-12}\times3.9}{9\times10^{-9}} = 3.835\times10^{-3} \text{ F/m}^2 \\ W &= 2\times10^{-5} \text{ m} \\ L_{\text{eff}} &= L_{\text{drawn}} - 2L_D = 1.84\times10^{-6} \text{ m} \end{aligned}$$

When $V_X < 1.9$ V, then S and D changes their roles

$$V_D - V_G + V_{TH} = 1.6 \text{ V} > 0 \quad \Rightarrow \quad V_{DS} > V_{GS} - V_{TH}$$

in the saturation region.

$$\begin{aligned} I_X = -I_D &= -\frac{1}{2}\mu_n \text{Cox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 \\ &= -0.729\times10^{-3}\times(0.3 - V_X)^2 \end{aligned}$$

When $V_X > 1.9$ V, $I_D = 0$. Therefore

$$I_x = \begin{cases} -0.729\times10^{-3}\times(0.3-V_x)^2 & 0 < V_X < 0.3 \text{ V} \\ 0 & V_X > 0.3 \text{ V} \end{cases}$$

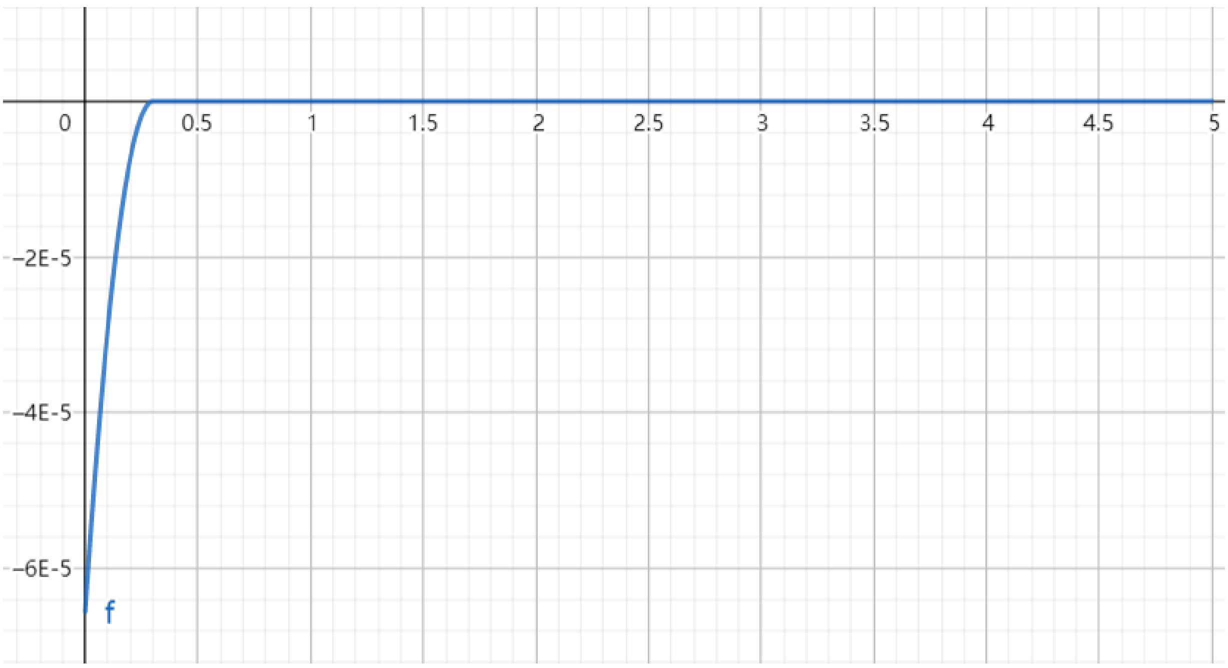

Figure 3: problem 3

4.1.4

$$
\begin{aligned}
V_{THP} &= -0.8 \text{ V} \\
\mu_p &= 100 \text{ cm}^2/(\text{V} \cdot \text{s}) = 0.01 \text{ m}^2/(\text{V} \cdot \text{s}) \\
C_{OX} &= \frac{\epsilon_o \epsilon_r}{t_{\text{ox}}} = \frac{8.85 \times 10^{-12} \times 3.9}{9 \times 10^{-9}} = 3.835 \times 10^{-3} \text{ F/m}^2 \\
W &= 2 \times 10^{-5} \text{ m} \\
L_{eff} &= L_{\text{drawn}} - 2L_D = 1.82 \times 10^{-6} \text{ m}
\end{aligned}
$$

When $V_X < 1.9$ V, then S and D changes their roles

$$V_G - V_D + |V_{THP}| = 1.8 - V_X$$

$V_x < 1.8$ V, then in the saturation region

$$
\begin{aligned}
I_X = -I_D &= -\frac{1}{2}\mu_p C_{OX} \frac{W}{L_{eff}} \left(V_{SG} - |V_{THP}|\right)^2 \\
&= -2.107 \times 10^{-6} \text{ A}
\end{aligned}
$$

1.8 V $< V_X <$ 1.9 V, then it is in the triode region

$$
\begin{aligned}
I_X = -I_D &= -\mu_p C_{OX} \frac{W}{L_{eff}} \left[\left(V_{SG} - |V_{THP}|\right) V_{SD} - \frac{1}{2} V_{SD}^2\right] \\
&= -4.214 \times 10^{-4} \left(0.5V_X - 0.85\right)\left(1.9 - V_X\right)
\end{aligned}
$$

When $V_X > 1.9$ V, it is in the triode region

$$
\begin{aligned}
I_X = I_D &= \mu_p C_{OX} \frac{W}{L_{\text{eff}}} \left[\left(V_{SG} - |V_{THP}|\right) V_{SD} - \frac{1}{2} V_{SD}^2\right] \\
&= 6.321 \times 10^{-4} \left(0.5V_X - 0.85\right)\left(V_X - 1.9\right)
\end{aligned}
$$

Therefore, we can obtain

$$
I_X = \begin{cases} -2.107 \times 10^{-6} \text{ A} & 0 < V_X < 1.8 \text{ V} \\ 4.214 \times 10^{-4} \left(0.5V_x - 0.85\right)\left(V_X - 1.9\right) & 1.8 \text{ V} < V_X < 5 \text{ V} \end{cases}
$$

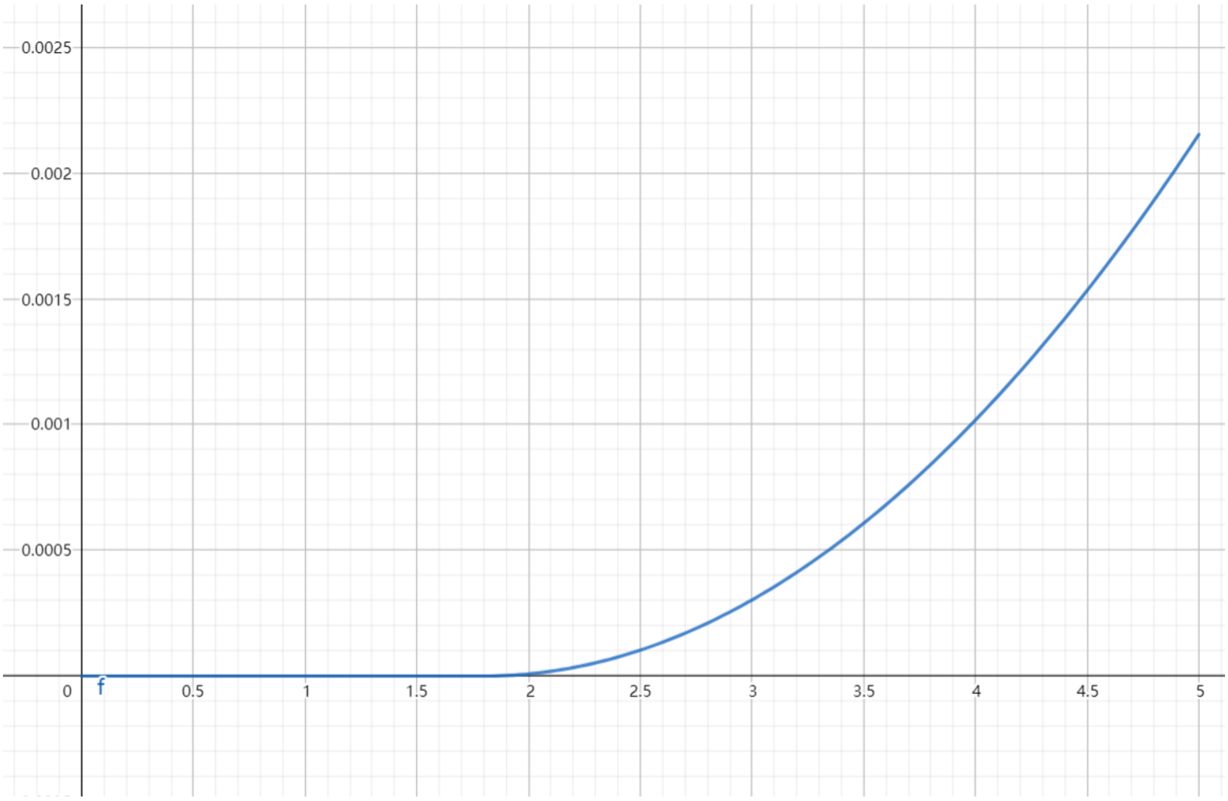

Figure 4: problem 4

4.2

4.2.1

When $V_{GS} = 1$ V, triode area is less than 0.3V, and saturation area is 0.3V-5V. There is no turn off region.

$$r_0 = 1.6 \times 10^5 \Omega$$

Theoratically,

$$r_o = \frac{1}{\frac{1}{2}\mu_n C_{OX} \frac{W}{L_{\text{eff}}} (V_{GS} - V_{TH})^2 \cdot \lambda} = 1.523 \times 10^5 \Omega$$

They are almost the same.

When $V_{GS} = 1.5$ V, triode area is less than 0.6V, and saturation area is 0.6V-5V. There is no turn off region.

$$r_0 = 3 \times 10^4 \Omega$$

Theoratically,

$$r_o = \frac{1}{\frac{1}{2}\mu_n C_{OX} \frac{W}{L_{\text{eff}}} (V_{GS} - V_{TH})^2 \cdot \lambda} = 2.142 \times 10^4 \Omega$$

They are almost the same.

When $V_{GS} = 2$ V, triode area is less than 1.2V, and saturation area is 1.2V-5V. There is no turn off region.

$$r_0 = 8.3 \times 10^3 \Omega$$

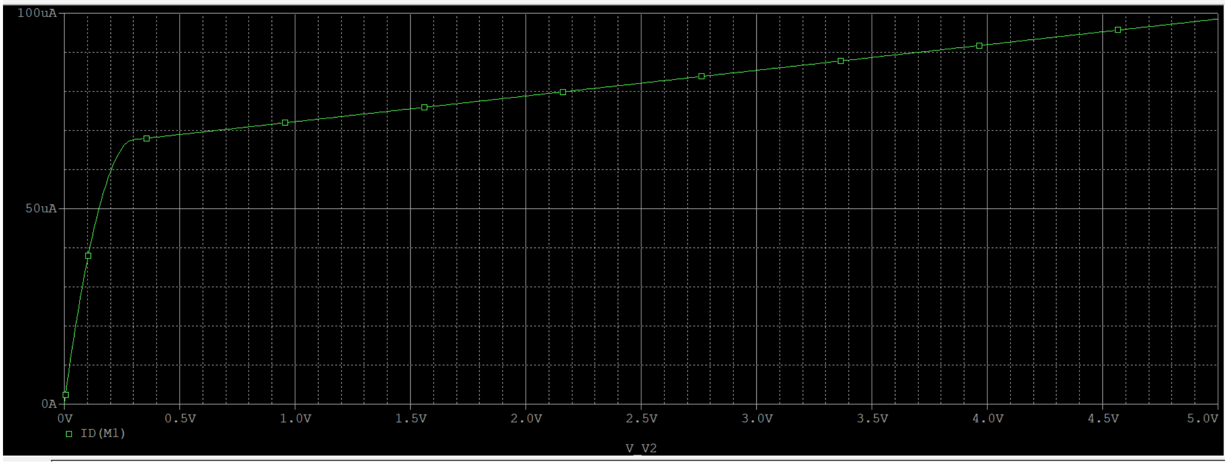

Figure 5: problem 1a

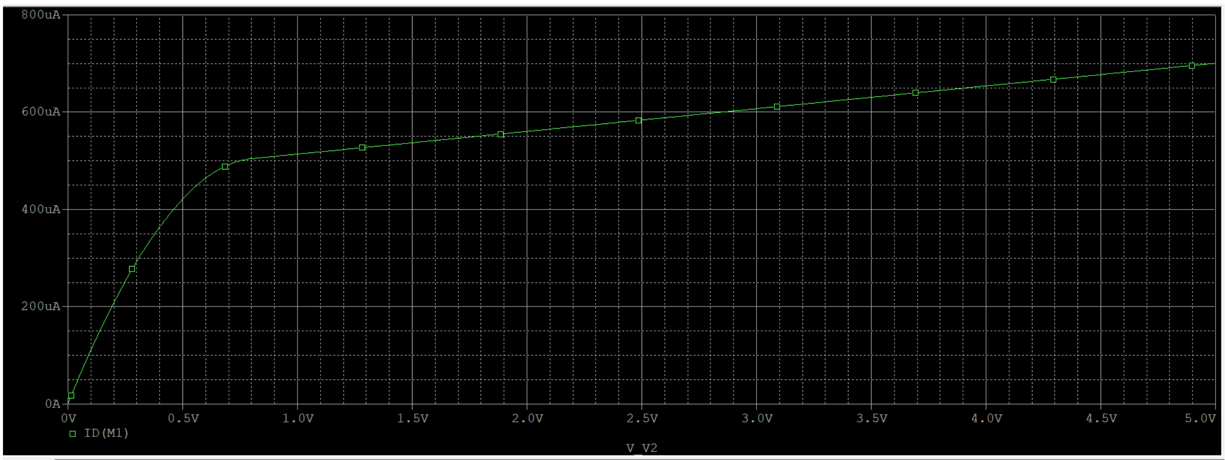

Figure 6: problem 1b

Theoratically,

$$r_o = \frac{1}{\frac{1}{2}\mu_n C_{OX} \frac{W}{L_{\text{eff}}} (V_{GS} - V_{TH})^2 \cdot \lambda} = 8.111 \times 10^3 \Omega$$

They are almost the same.

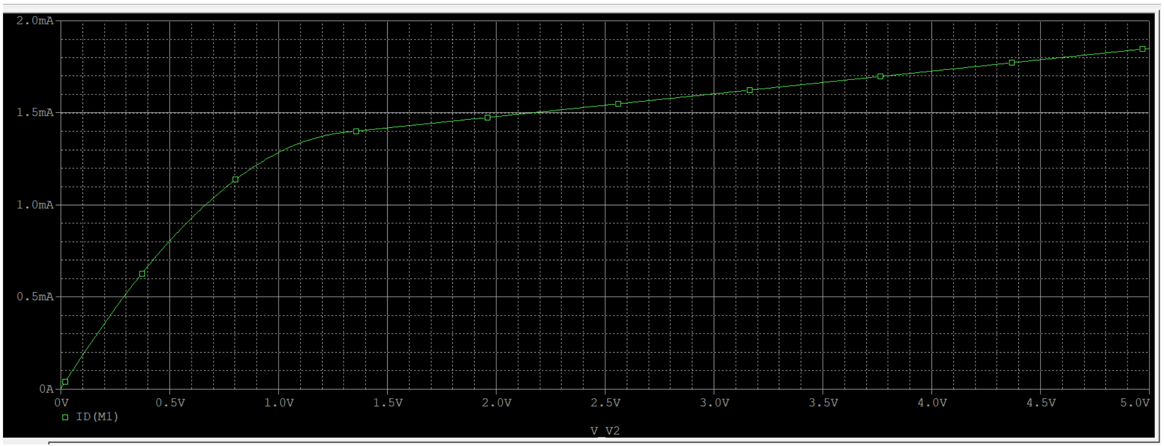

Figure 7: problem 1c

4.2.2

In the figure, I can see the slope as gm.

$$\text{gm} = 2.9 \times 10^{-3} \Omega^{-1}$$

In theory, we can calculate gm as

$$\text{gm} = \mu_n C_{OX} \frac{W}{L_{eff}} (V_{GS} - V_{TH}) (1 + \lambda V_{DS}) = 2.845 \times 10^{-3} \Omega^{-1}$$

They are almost the same.

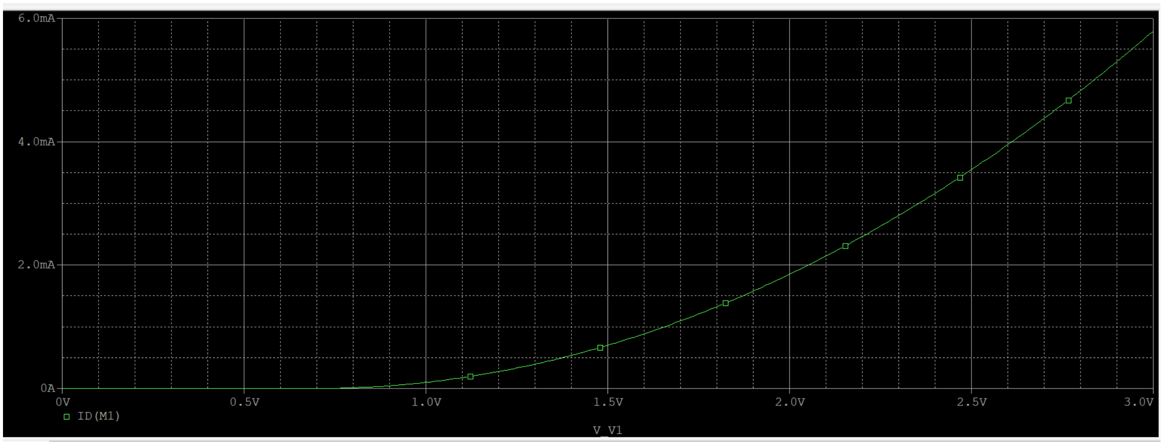

Figure 8: problem 2