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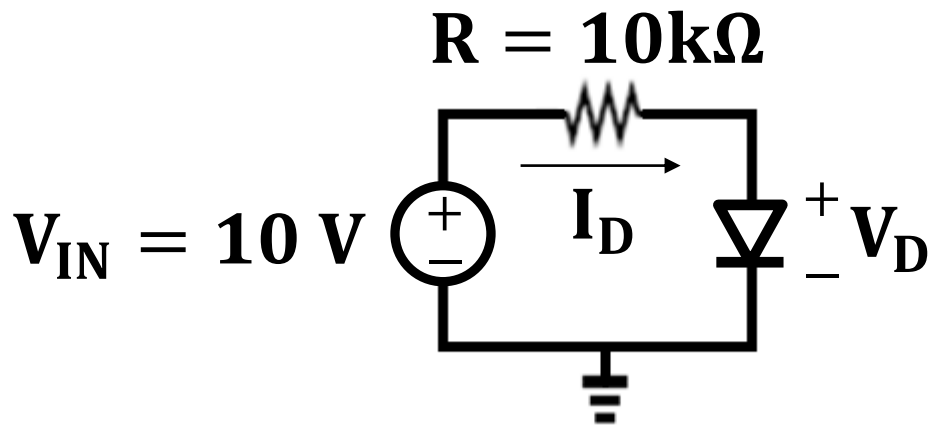
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## Diode Circuit

VE311 Electronic Circuits (Fall 2021)

Dr. Chang-Ching Tu

# Solving Techniques



$$V_{IN} = I_D R + V_D$$

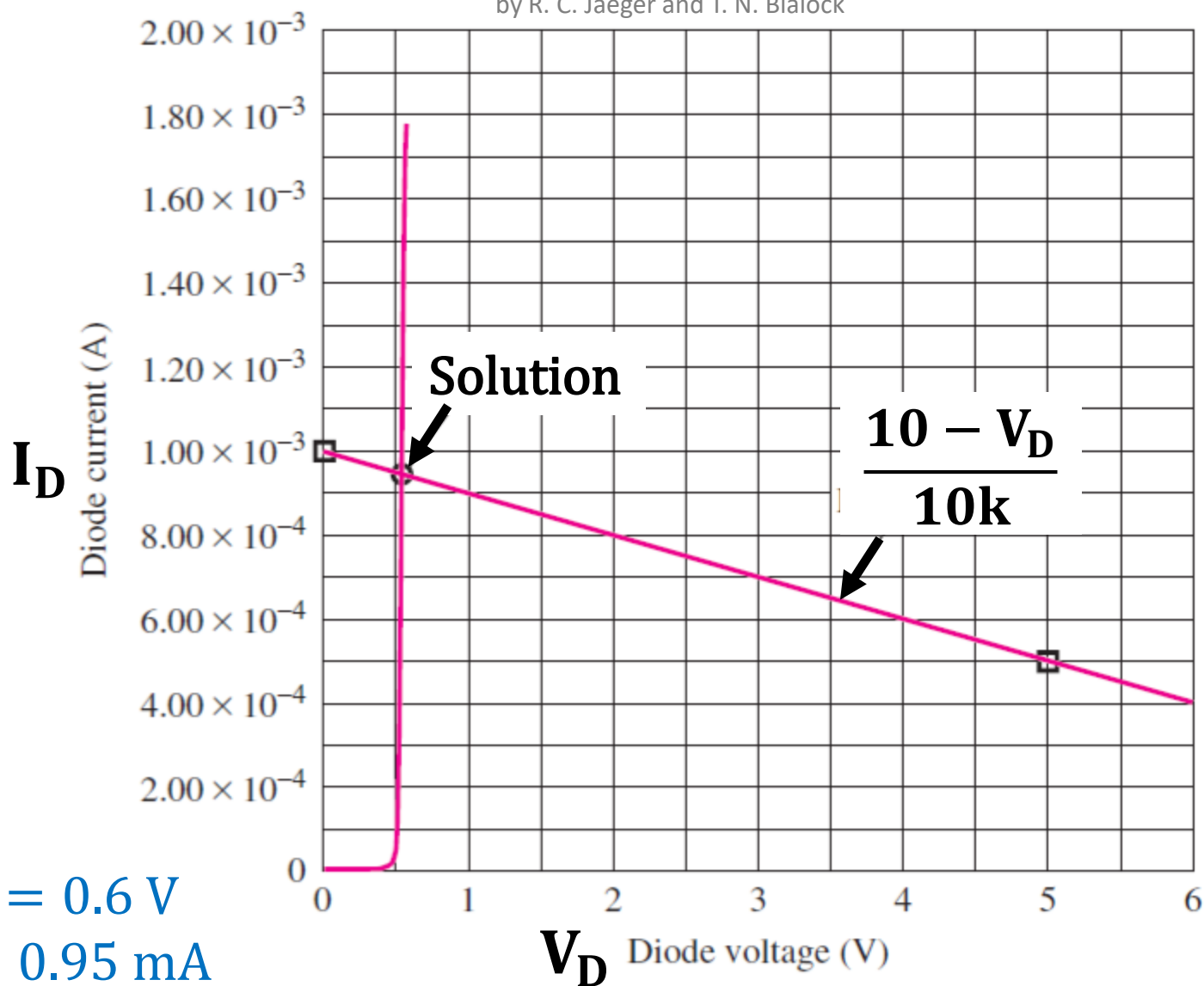
$$I_D = ?$$

$$V_D = ?$$

1. Graphical analysis
2. Mathematical analysis
3. Simplified analysis  
(ideal diode)
4. Simplified analysis  
(constant voltage drop)

# Graphical Analysis

Source: Microelectronic Circuit Design, 4<sup>th</sup> Edition,  
by R. C. Jaeger and T. N. Blalock



$$V_D = 0.6 \text{ V}$$
$$I_D = 0.95 \text{ mA}$$

# Mathematical Analysis

$$V_{IN} = I_D R + V_D$$

$$V_{IN} = \left[ I_S \left( e^{\frac{qV_D}{kT}} - 1 \right) \right] R + V_D$$

$$10 = \left[ 10^{-13} \left( e^{\frac{V_D}{0.0258}} - 1 \right) \right] 10^4 + V_D$$

Above is a transcendental equation which does not have a close-form **analytical solution**. So, through trial and error, we seek a **numerical solution**.

```
function xd = diode(vd)
```

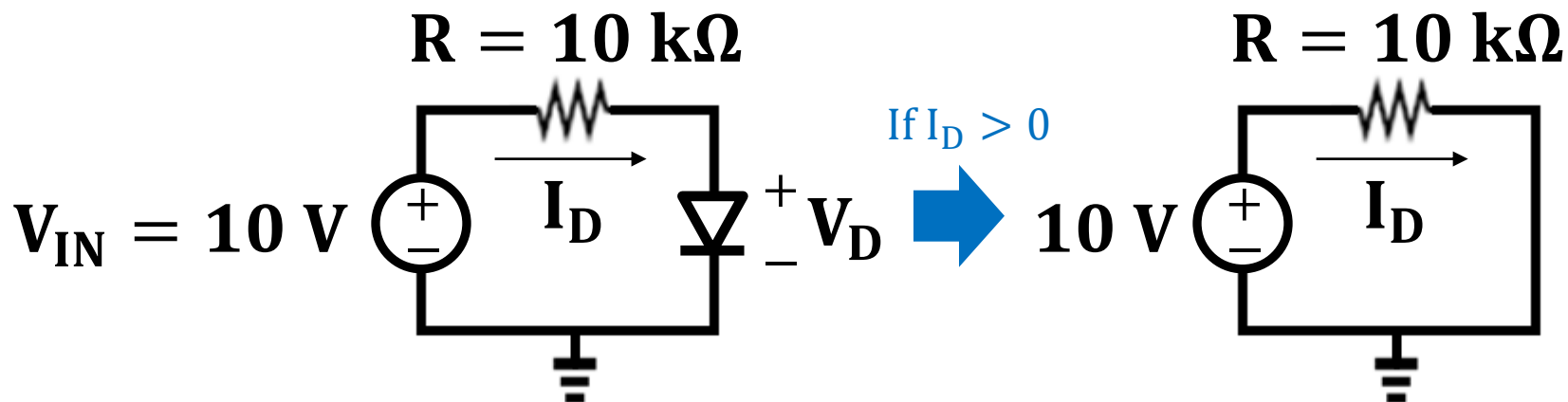
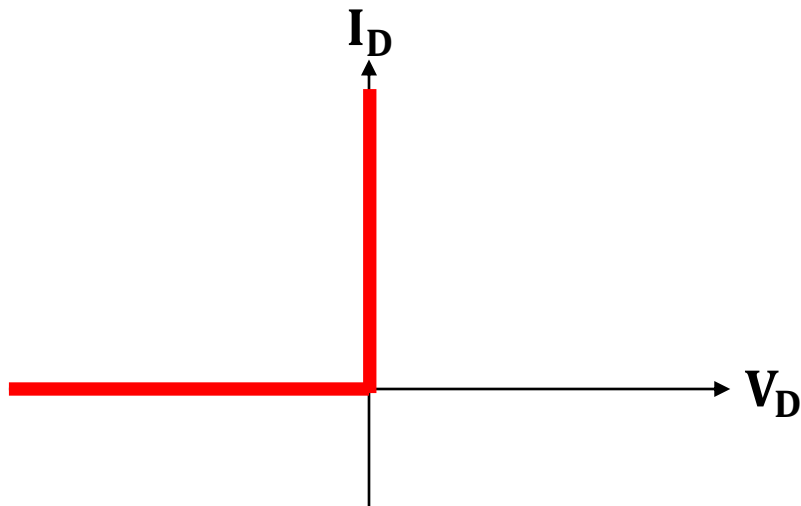
```
xd = 10 - (10-9) * (exp( (vd / 0.0258) ) - 1) - vd
```

Use MATLAB to plot xd as a function of vd, and find out a vd that makes xd closest to zero.

$$V_D = 0.5742 \text{ V}$$

$$I_D = 0.944 \text{ mA}$$

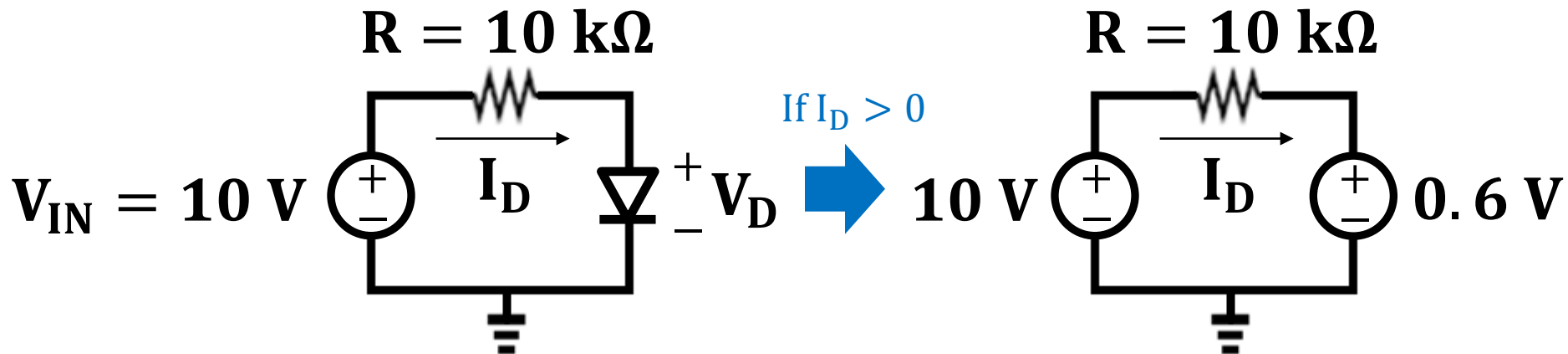
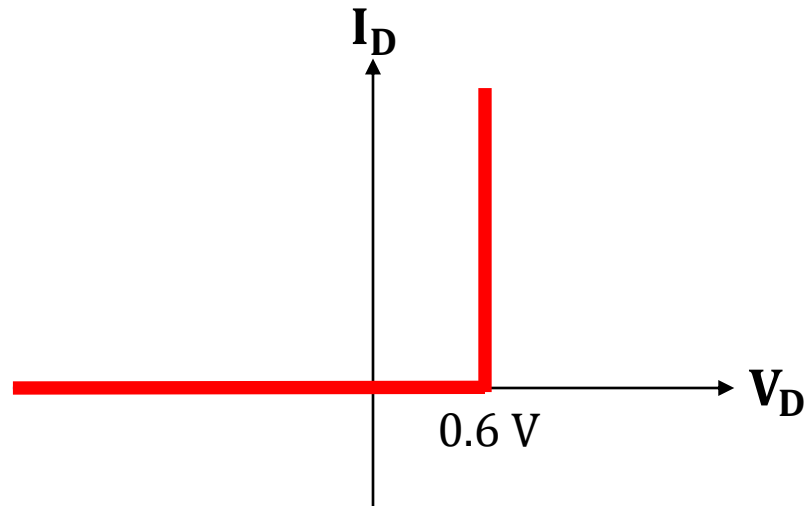
# Simplified Analysis (Ideal Diode)



$$I_D = \frac{V_D = 0 \text{ V} + (10 - 0) \text{ V}}{10 \text{ k}\Omega} = 1 \text{ mA}$$

# Simplified Analysis (Constant Voltage Drop)

6



*$V_D$  a power supply?  
What happens if  $V_{IN} = 0.5$  V?*

$$I_D = \frac{V_D = 0.6 \text{ V}}{(10 - 0.6) \text{ V}} = 0.94 \text{ mA}$$

# Comparison

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	$V_D$	$I_D$
Graphical Analysis	0.6 V	0.95 mA
Mathematical Analysis	0.5742 V	0.944 mA
Ideal Diode Model	0 V	1 mA
Constant Voltage Drop Model	0.6 V	0.94 mA

# Example

Use constant voltage drop model ( $V_{on} = 0.6 \text{ V}$ ) to calculate  $V_D$  and  $I_D$  of each diode.

- Assume no current flowing through  $D_3$

$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k} + \frac{V_B - 0.6 + 10}{10k}$$

$$V_B = -6.27 \text{ V}$$

$$V_C = -6.87 \text{ V} \Rightarrow D_3 \text{ in forward bias}$$

**Assumption NOT valid**

- Assume no current flowing through  $D_2$

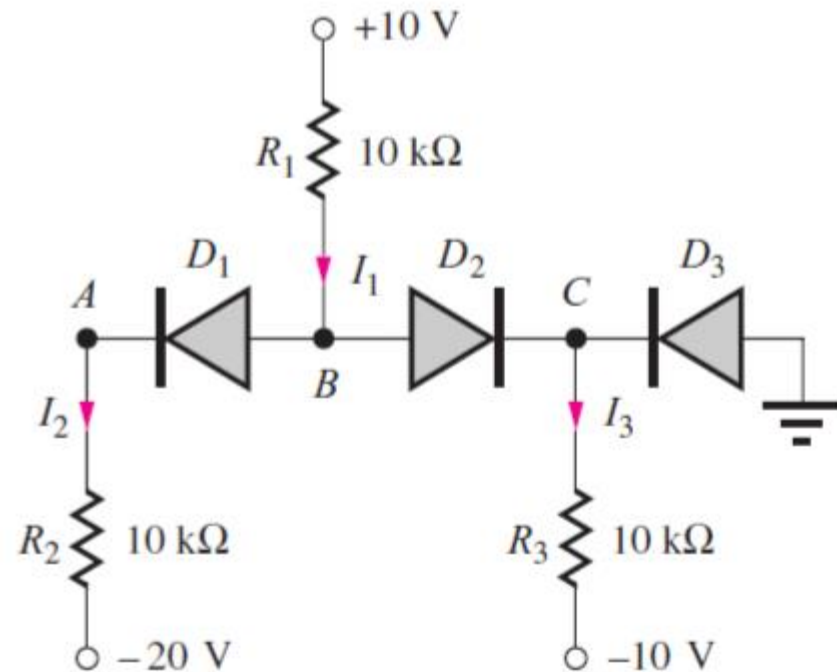
$$\frac{10 - V_B}{10k} = \frac{V_B - 0.6 + 20}{10k}$$

$$V_B = -4.7 \text{ V}$$

$$V_C = -0.6 \text{ V} \Rightarrow D_2 \text{ indeed in reverse bias}$$

**Assumption valid**

$$\begin{array}{lll} V_{D1} = 0.6 \text{ V} & V_{D2} = -4.1 \text{ V} & V_{D3} = 0.6 \text{ V} \\ I_{D1} = 1.47 \text{ mA} & I_{D2} = 0 \text{ mA} & I_{D3} = 0.94 \text{ mA} \end{array}$$





# Example

Use ideal diode model ( $V_{on} = 0 \text{ V}$ ) to calculate  $V_D$  and  $I_D$  of each diode.

- Assume  $D_3$  in reverse bias

$$\frac{10 - V_B}{10\text{k}} = \frac{V_B + 20}{10\text{k}} + \frac{V_B + 10}{10\text{k}}$$

$$V_B = -6.67 \text{ V}$$

$$V_C = -6.67 \text{ V} \Rightarrow D_3 \text{ in forward bias}$$

**Assumption NOT valid**

- Assume  $D_2$  in reverse bias

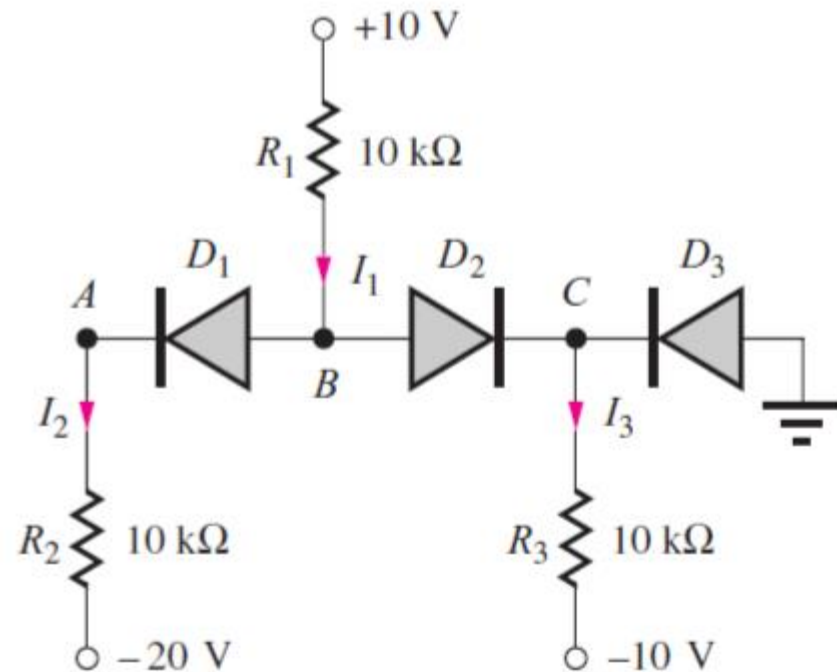
$$\frac{10 - V_B}{10\text{k}} = \frac{V_B + 20}{10\text{k}}$$

$$V_B = -5 \text{ V}$$

$$V_C = 0 \text{ V} \Rightarrow D_2 \text{ indeed in reverse bias}$$

**Assumption valid**

$V_{D1} = 0 \text{ V}$	$V_{D2} = -5 \text{ V}$	$V_{D3} = 0 \text{ V}$
$I_{D1} = 1.5 \text{ mA}$	$I_{D2} = 0 \text{ mA}$	$I_{D3} = 1 \text{ mA}$



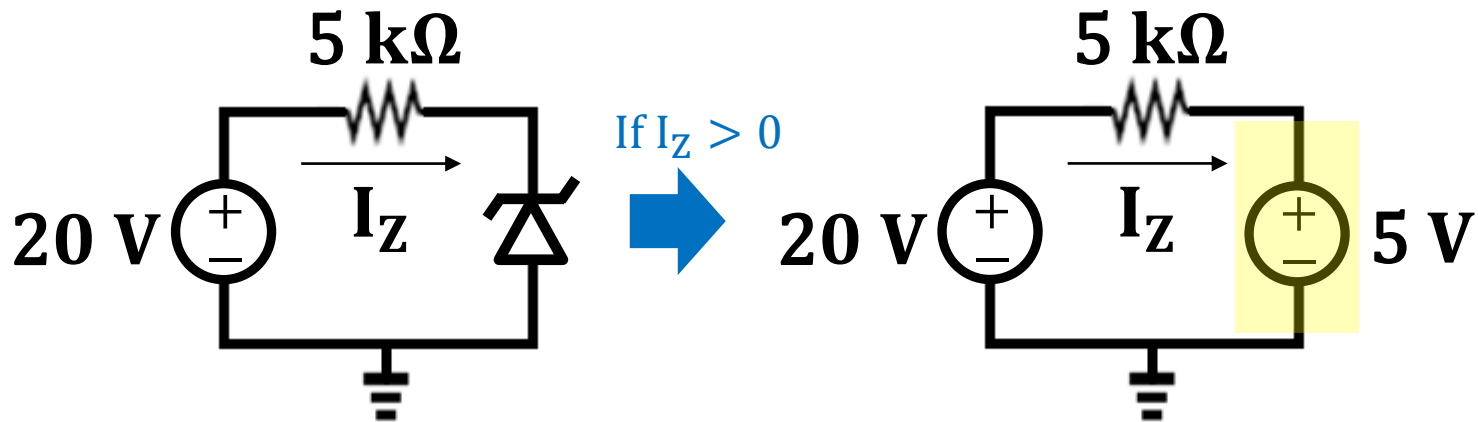
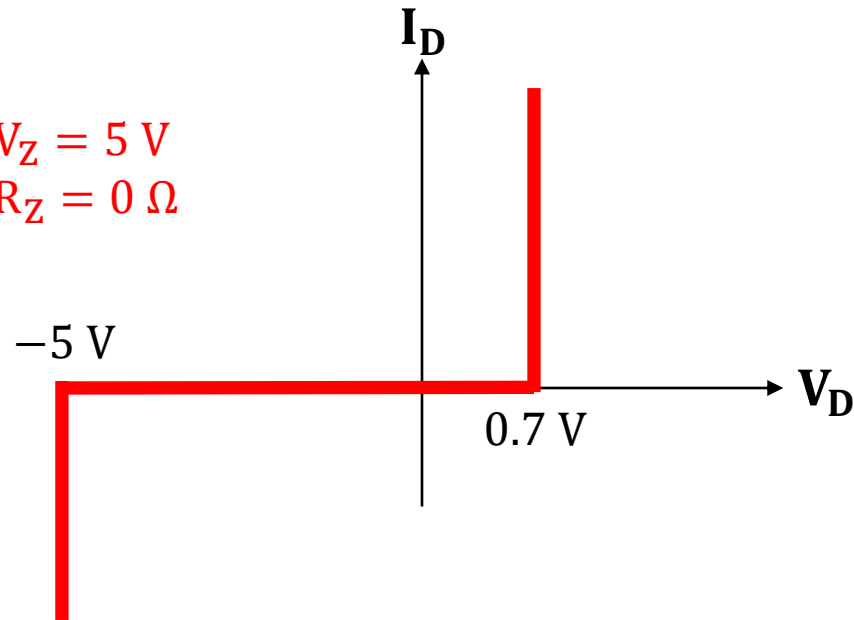
# Zener Diode Circuit

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# Simplified Analysis for Zener Diode ( $R_Z = 0$ )

$$V_Z = 5 \text{ V}$$

$$R_Z = 0 \Omega$$

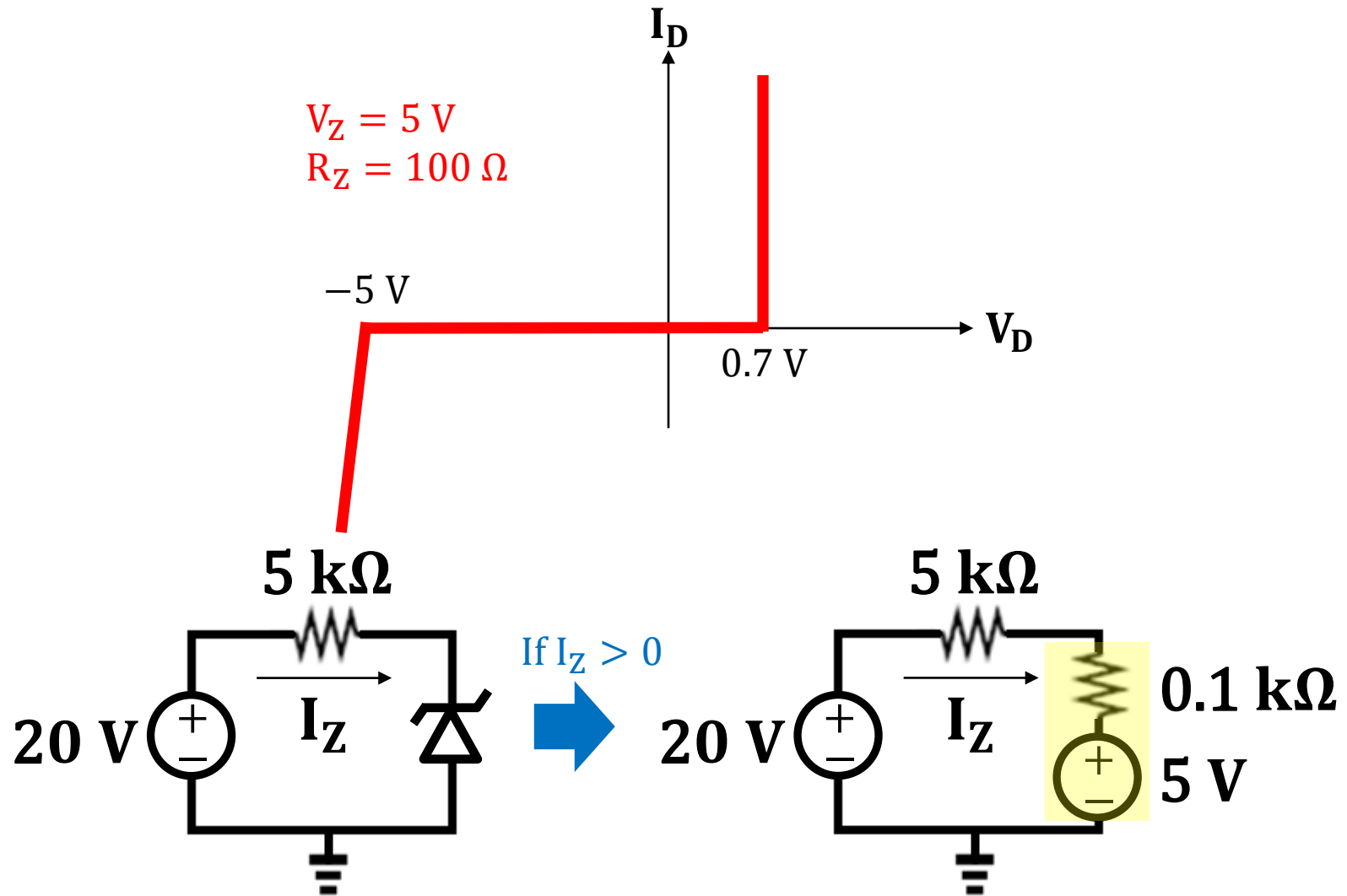


$$I_Z = \frac{20 - 5}{5000} = 3 \times 10^{-3} \text{ (A)}$$

What happens if  $V_{IN} = 1 \text{ V}$ ?

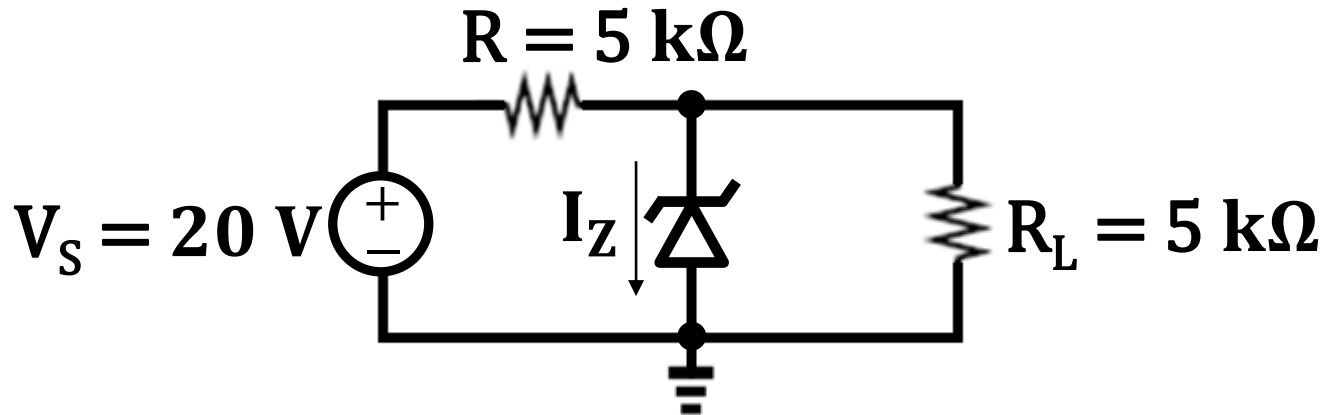
# Simplified Analysis for Zener Diode ( $R_Z \neq 0$ )

12

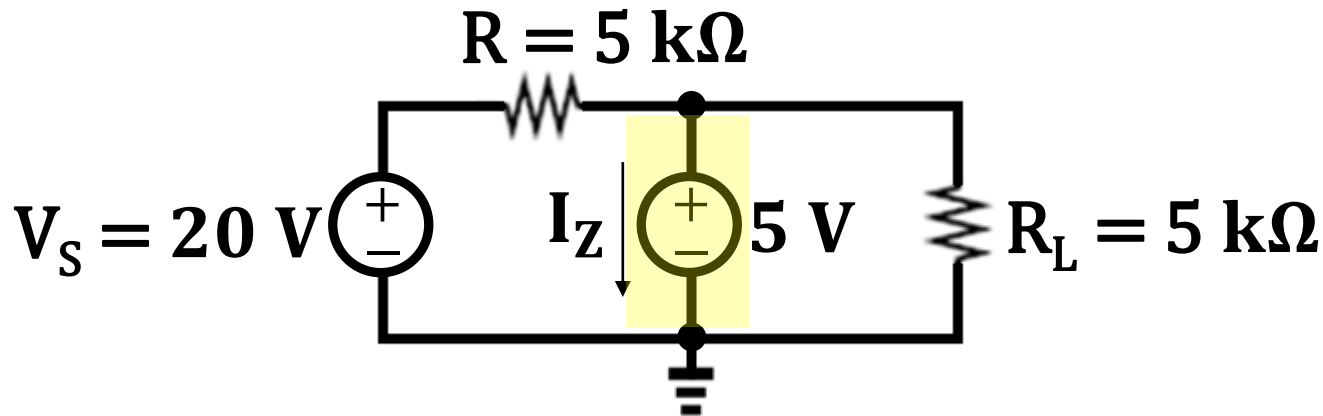


$$I_Z = \frac{20 - 5}{5100} = 2.94 \times 10^{-3} \text{ (A)}$$

# Voltage Regulator Using Zener Diode ( $R_Z = 0$ )<sup>13</sup>



↓ If  $I_Z > 0$

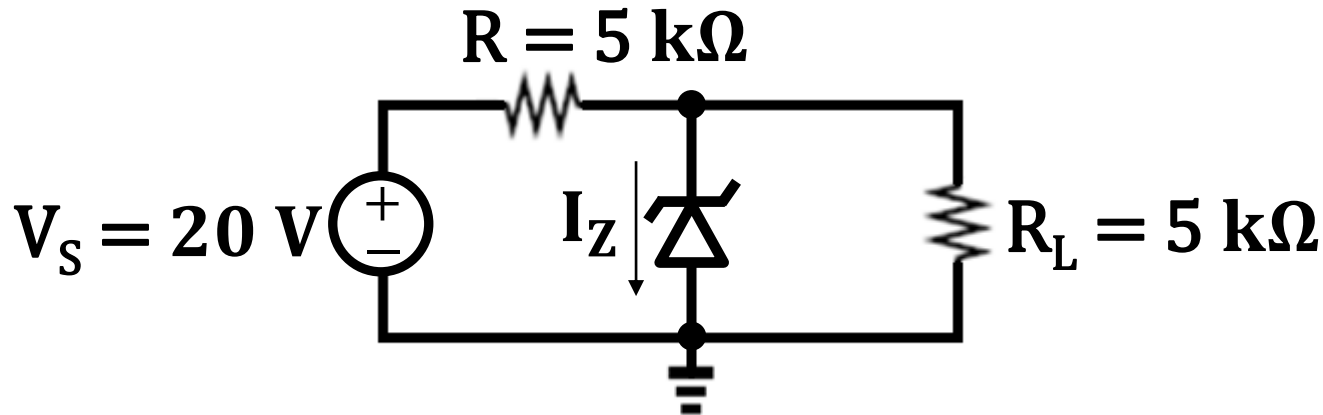


$$I_Z = \frac{20 - 5}{5\text{k}} - \frac{5}{5\text{k}} = 2\text{ mA} > 0$$

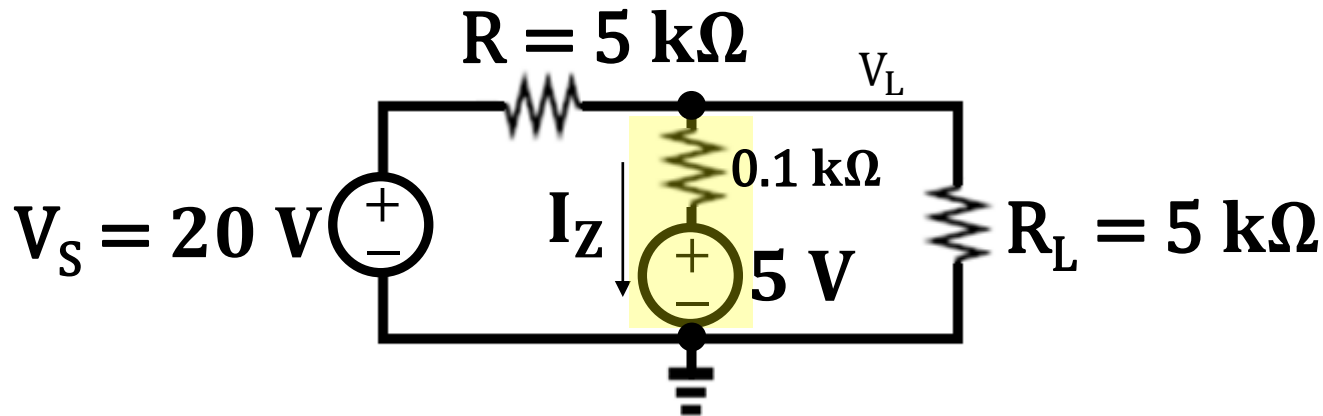
As long as the zener diode operates in reverse breakdown region ( $I_Z > 0$ ), a constant voltage ( $5\text{ V}$ ) appears across  $R_L$ .

What is the smallest  $R_L$  for reverse breakdown to happen? Answer:  $R_{L,\min} = 1.67\text{ k}\Omega$

# Voltage Regulator Using Zener Diode ( $R_Z \neq 0$ )<sup>14</sup>



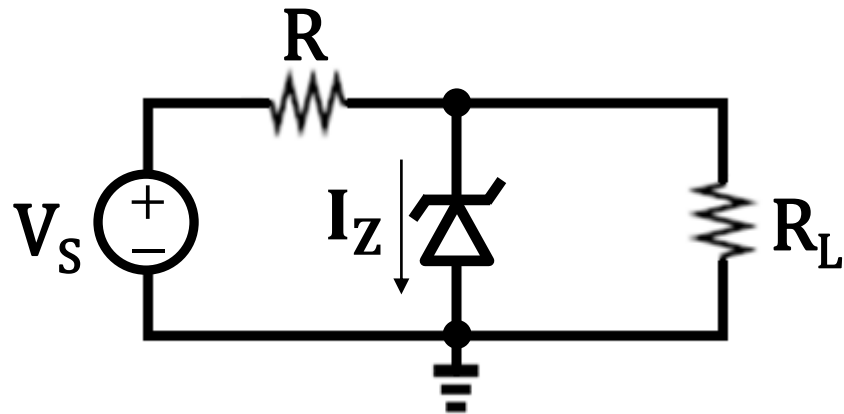
↓ If  $I_Z > 0$



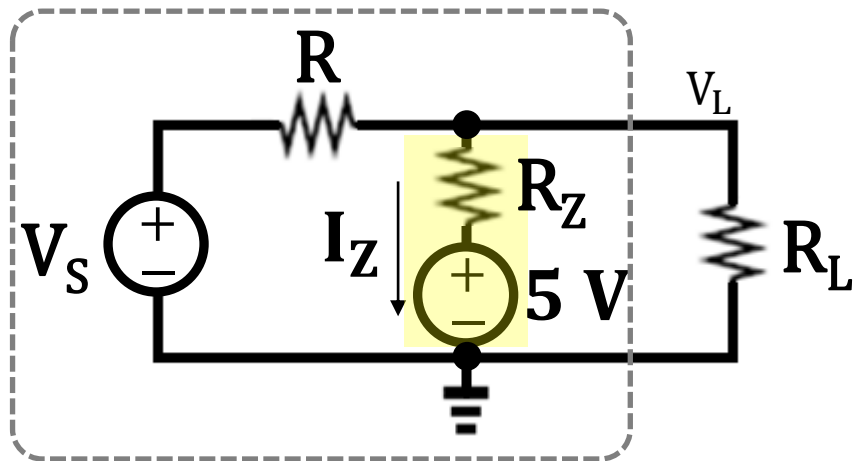
$$\frac{20 - V_L}{5\text{k}} = \frac{V_L - 5}{0.1\text{k}} + \frac{V_L}{5\text{k}} \quad V_L = 5.1923\text{ V}$$

$$I_Z = \frac{5.19 - 5}{0.1\text{k}} = 1.9\text{ mA} > 0$$

# Line Regulation and Load Regulation



If  $I_Z > 0$



Voltage Regulator

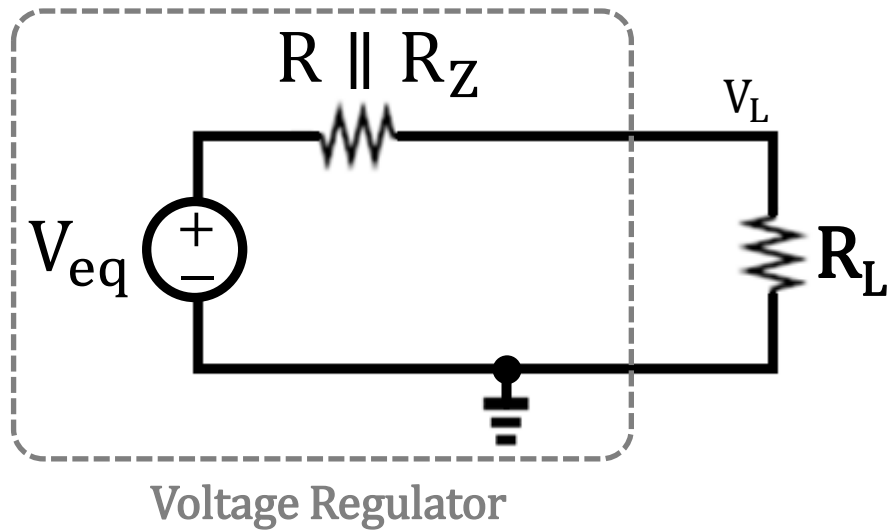
- **Line Regulation:** how sensitive the output voltage ( $V_L$ ) is to input voltage ( $V_S$ ) changes, when  $R_L = \infty$ .

$$\text{Line Regulation} = \frac{dV_L}{dV_S} = \frac{R_Z}{R + R_Z}$$

- **Load Regulation:** output impedance of the voltage regulator.

$$\text{Load Regulation} = \frac{dV_L}{dI_L} = R \parallel R_Z$$

# Thevenin Equivalent Circuit of Voltage Regulator



$$V_{eq} = 5 + \frac{V_S - 5}{R + R_Z} R_Z$$

$$V_L = V_{eq} \frac{R_L}{(R \parallel R_Z) + R_L}$$

Numerical Test:  $V_S = 20 \text{ V}$ ,  $R = 5 \text{ k}\Omega$ ,  $R_Z = 0.1 \text{ k}\Omega$ ,  $V_Z = 5 \text{ V}$ ,  $R_L = 5 \text{ k}\Omega$

$$V_{eq} = 5 + \frac{20 - 5}{5k + 0.1k} 0.1k = 5.2941 \text{ (V)}$$

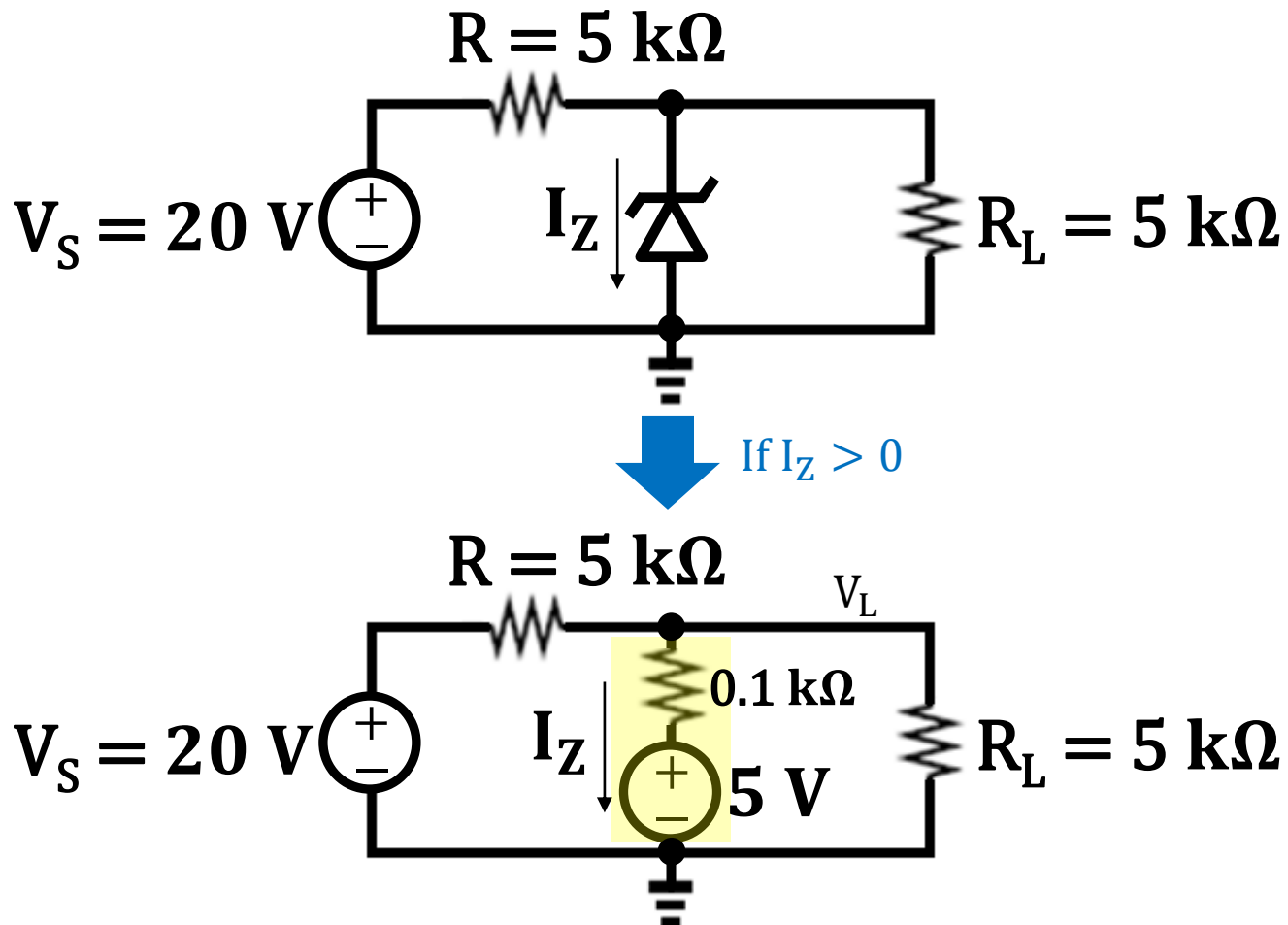
$$V_L = 5.2941 \times \frac{5k}{\frac{5k \times 0.1k}{5k + 0.1k} + 5k} = 5.1923 \text{ (V)}$$

The result exactly the same as on page 14.



# Example

What are the line and load regulations for the circuit below?

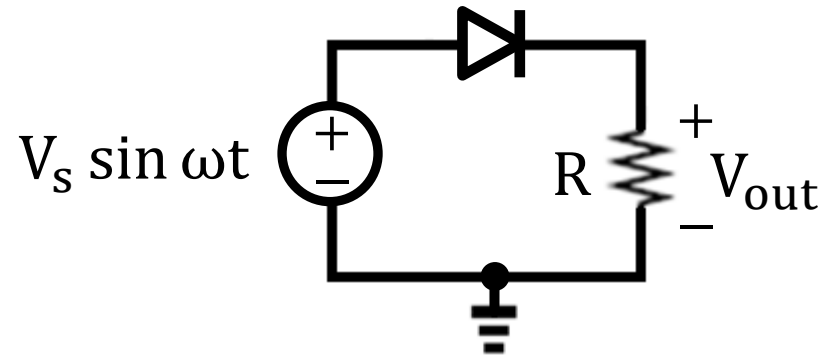


$$\text{Line Regulation} = \frac{0.1\text{k}}{5\text{k} + 0.1\text{k}} = 19.6\text{ mV/V} \quad \text{Load Regulation} = 5\text{k} \parallel 0.1\text{k} = 98\text{ }\Omega$$

# Half-Wave Rectifier

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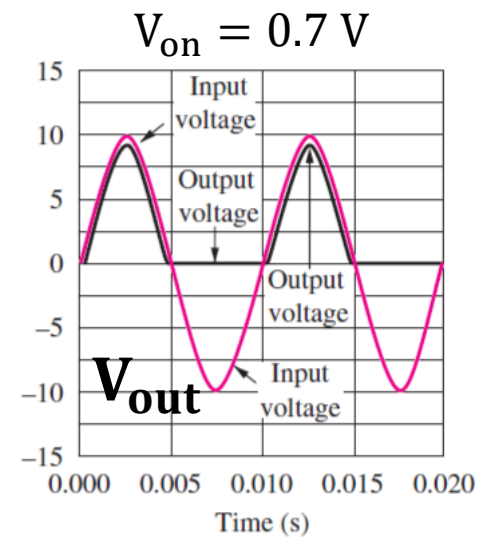
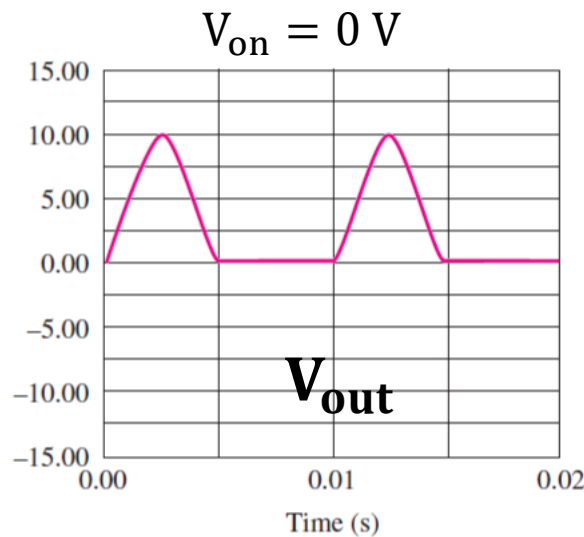
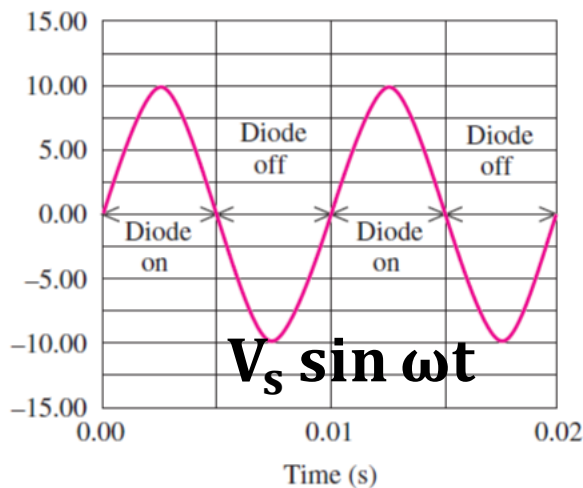
# Half-Wave Rectifier with Resistive Load



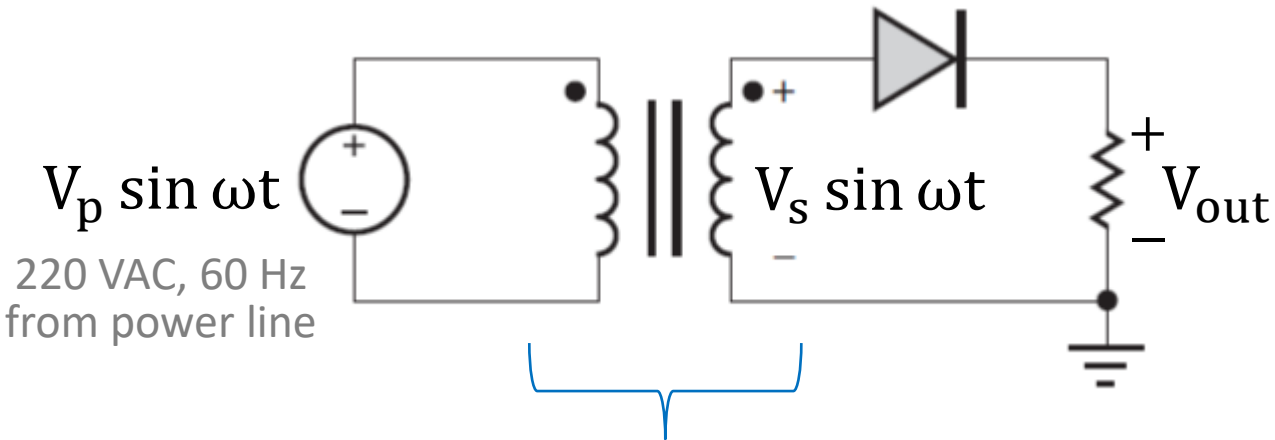
$$V_s = 10 \text{ V}$$

$$\omega = 2\pi f = 2\pi \frac{1}{0.01} = 200\pi \text{ (rad/sec)}$$

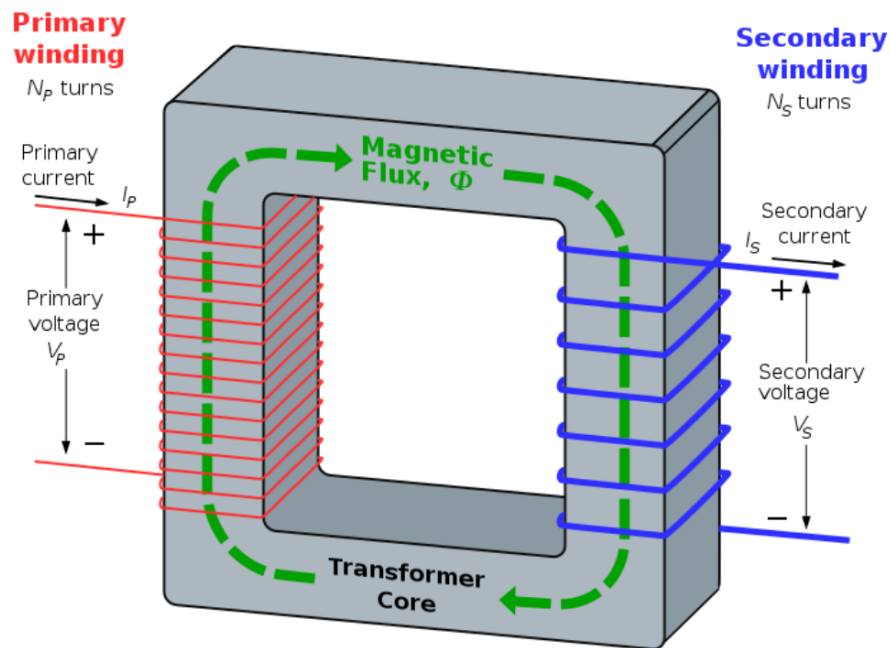
**$V_{out}$  is not DC.**



# Transformer



**The output of an ideal transformer can be represented as an ideal voltage source.**



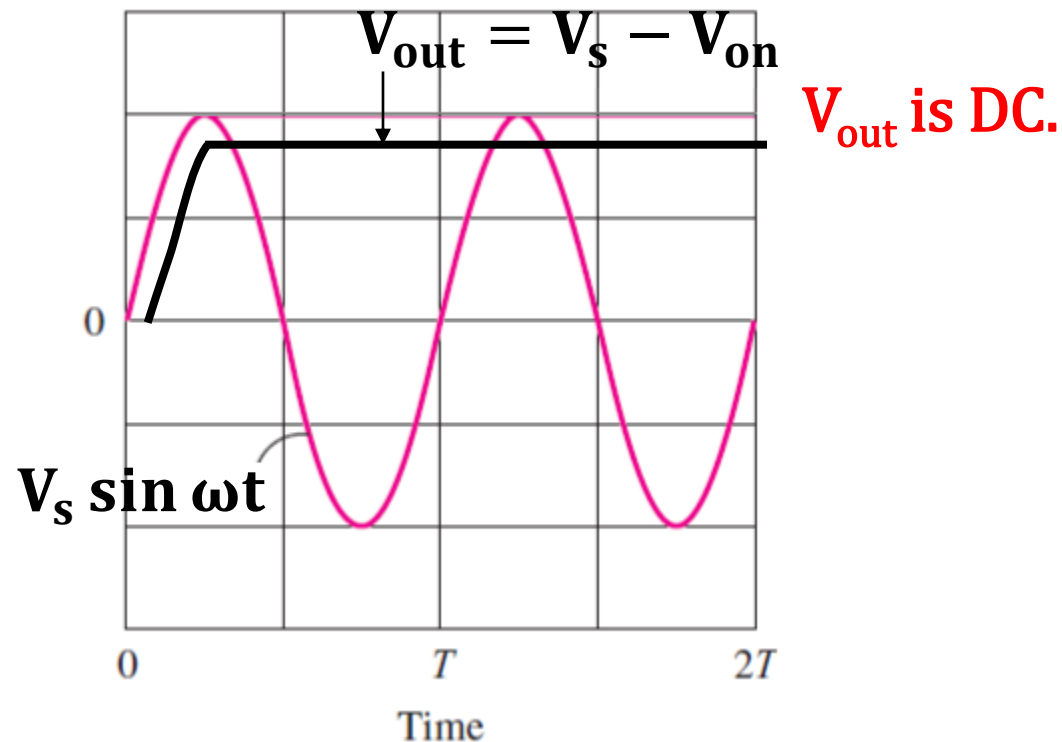
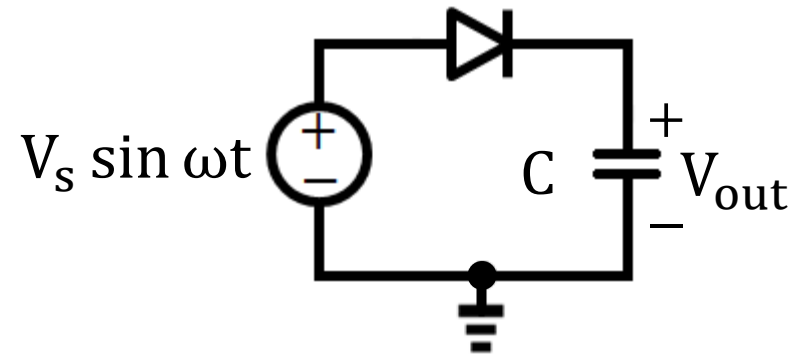
Working Principle:  
By Faraday's law of induction

$$V_s = -N_s \frac{d\phi}{dt}$$

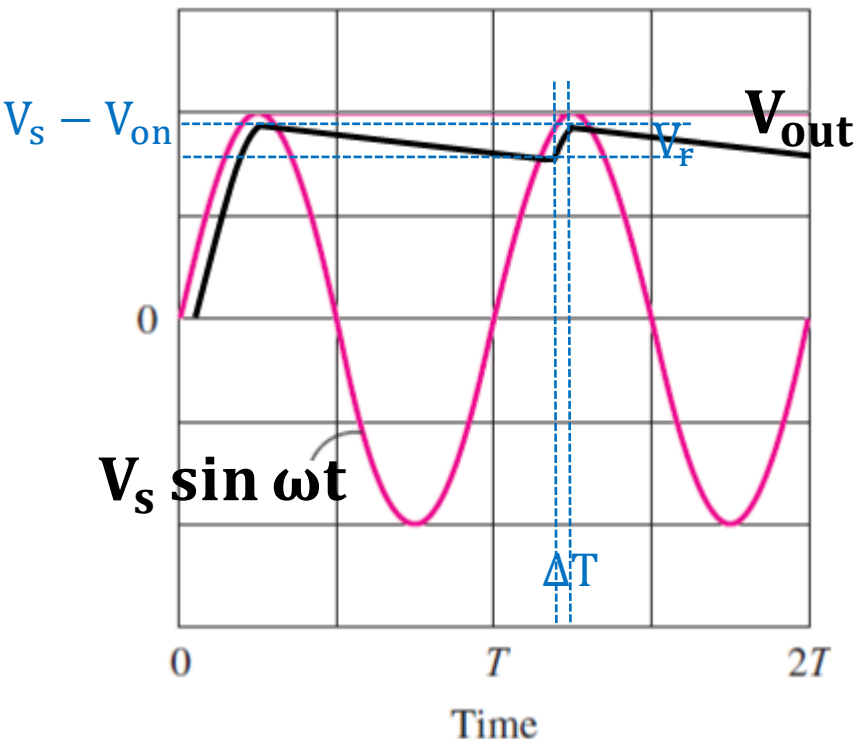
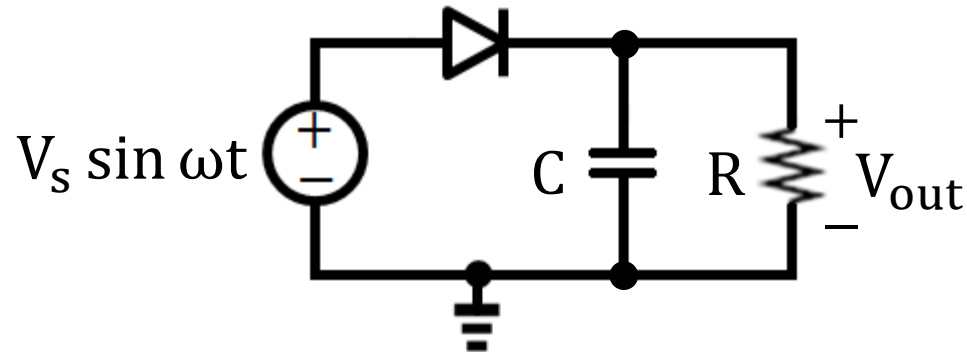
$$V_p = -N_p \frac{d\phi}{dt}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

# Half-Wave Rectifier with Capacitive Load



# Half-Wave Rectifier with RC Load (I)



$$V_{dc} = V_s - V_{on}$$

$$I_{dc} = \frac{V_{dc}}{R}$$

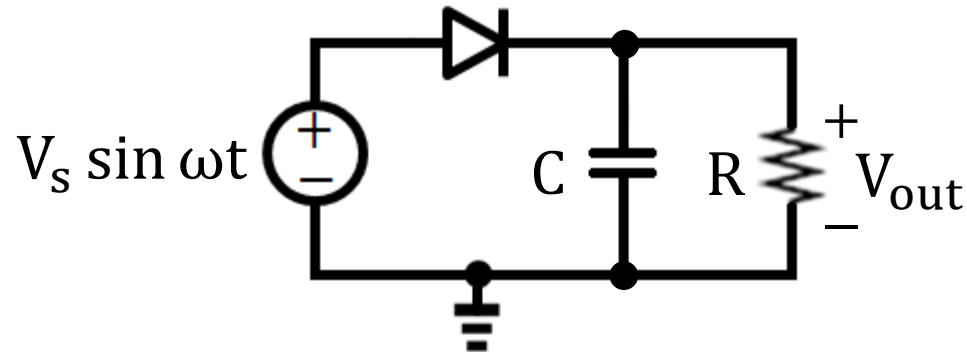
**ripple voltage**

$$V_r = (V_s - V_{on}) \left( 1 - e^{-\frac{T - \Delta T}{RC}} \right)$$

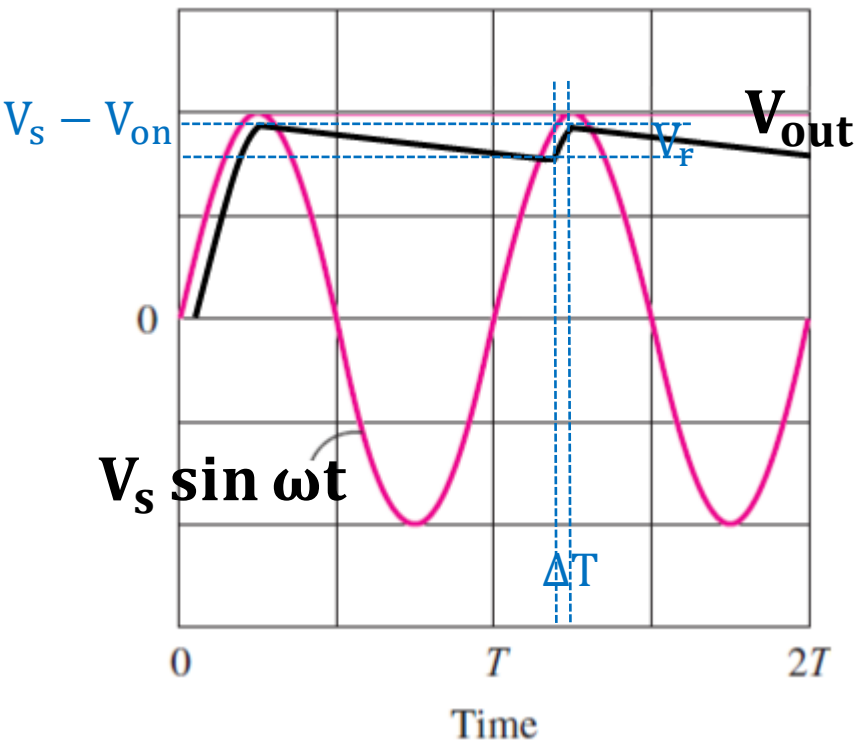
$$\cong (V_s - V_{on}) \left( \frac{T - \Delta T}{RC} \right) \text{ if } (T - \Delta T) \ll RC$$

$$\cong (V_s - V_{on}) \left( \frac{T}{RC} \right) \text{ if } \Delta T \ll T$$

# Half-Wave Rectifier with RC Load (II)



conduction angle and interval



$$V_s \sin \left[ \omega \left( \frac{5T}{4} - \Delta T \right) \right] - V_{on} = (V_s - V_{on}) - V_r$$

$$V_s \sin \left( \frac{5\pi}{2} - \theta_c \right) - V_{on} = (V_s - V_{on}) - V_r$$

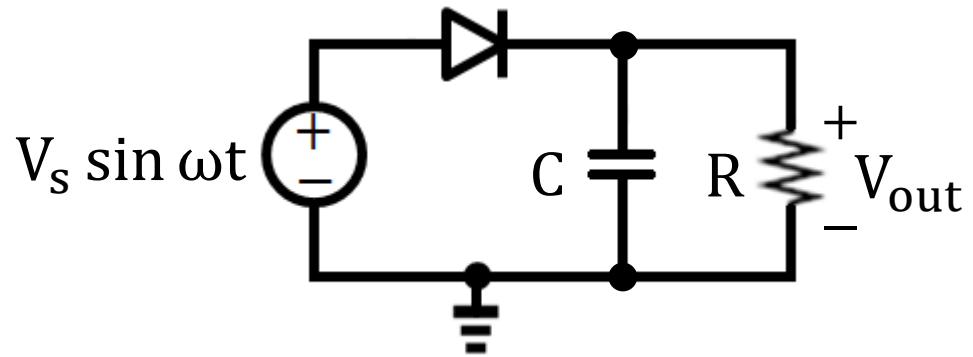
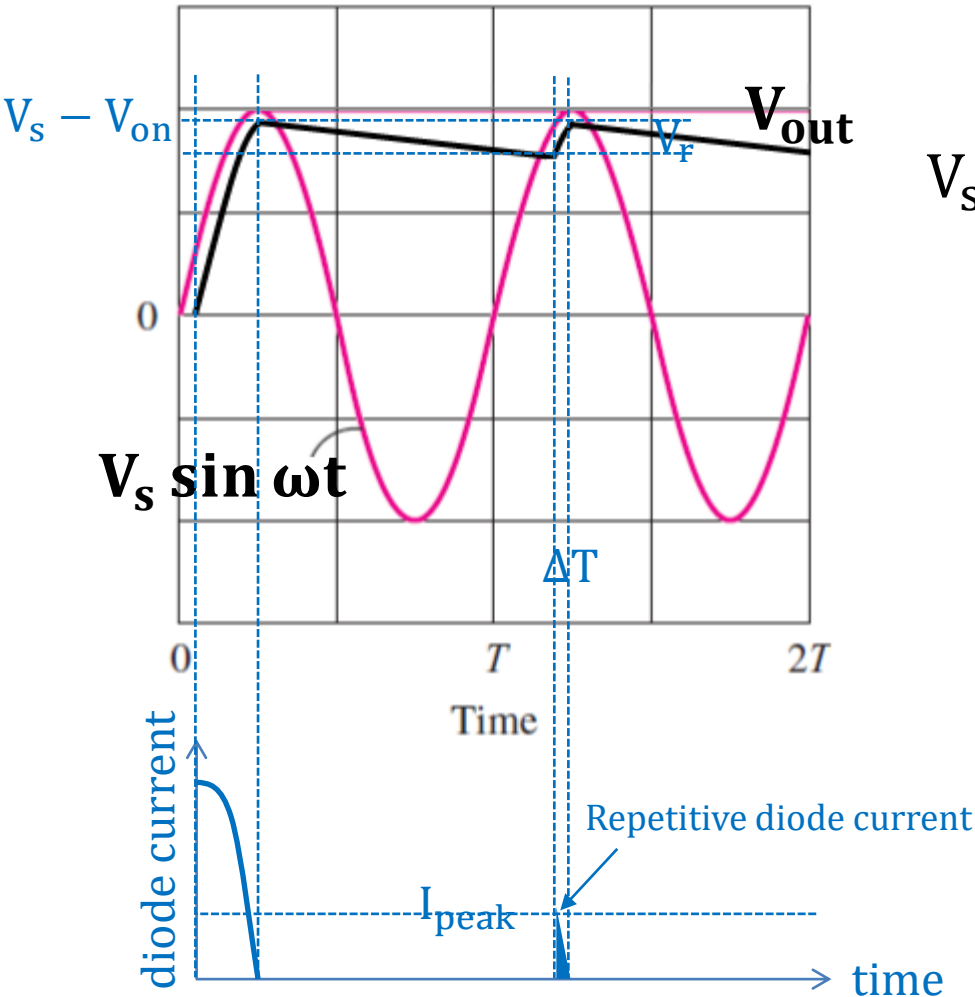
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c^2}{2} \quad \text{if } \theta_c \text{ very small}$$

$$\theta_c = \sqrt{\frac{2V_r}{V_s}}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_s}}$$

# Half-Wave Rectifier with RC Load (III)



The charge filled on  $C$  during  $\Delta T$  is discharged during  $T - \Delta T$ .

$$Q \cong \frac{I_{peak} \Delta T}{2} = I_{dc}(T - \Delta T) \cong I_{dc} T$$

$$I_{peak} = \frac{2I_{dc} T}{\Delta T}$$

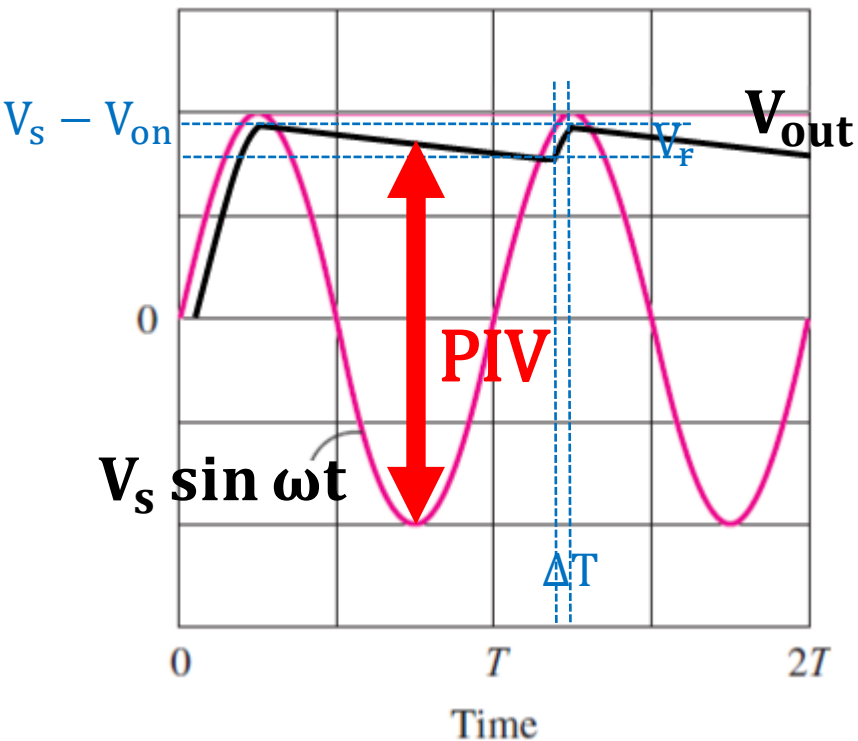
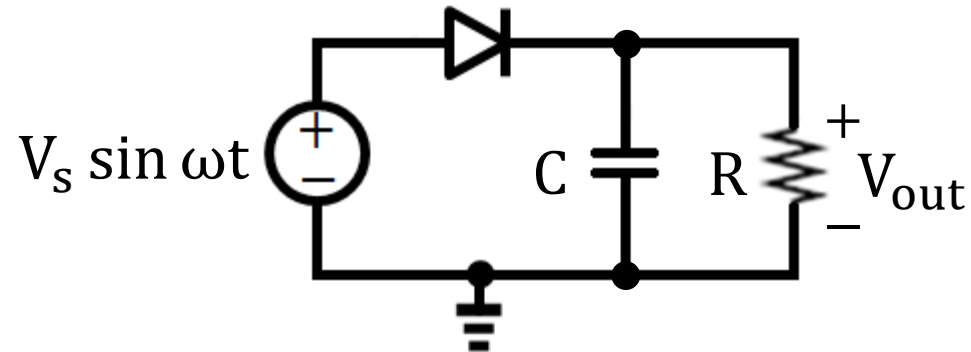
$$I_{surge} = C \frac{d(V_s \sin \omega t - V_{on})}{dt} = \omega C V_s \quad \text{if } t = 0$$

During charging period ( $\Delta T$ ), almost all diode current goes to  $C$ .

$$\left| \frac{1}{SC} \right| = \frac{1}{2\pi \frac{1}{T} C} = \frac{T}{2\pi C} \ll R \quad \text{if } RC \gg T$$



# Half-Wave Rectifier with RC Load (IV)



**Peak-inverse-voltage (PIV)  $\cong 2V_s - V_{on}$**

If too large, the diode breaks down.

# Example

Find the value of the dc output voltage, dc output current, ripple voltage, conduction interval, conduction angle and diode peak current for a half-wave rectifier driven from a transformer having a secondary voltage of  $12.6 \text{ V}_{\text{rms}}$  (60 Hz) with  $R = 15 \Omega$  and  $C = 25,000 \mu\text{F}$ . Assume  $V_{\text{on}} = 1 \text{ V}$ .

$$V_{\text{dc}} = 12.6\sqrt{2} - 1 = 16.8 \text{ (V)}$$

$$I_{\text{dc}} = \frac{16.8}{15} = 1.12 \text{ (A)}$$

$$V_r \cong V_{\text{dc}} \frac{T}{RC} = 16.8 \frac{\frac{1}{60}}{15 \times 25000 \times 10^{-6}} = 0.747 \text{ (V)}$$

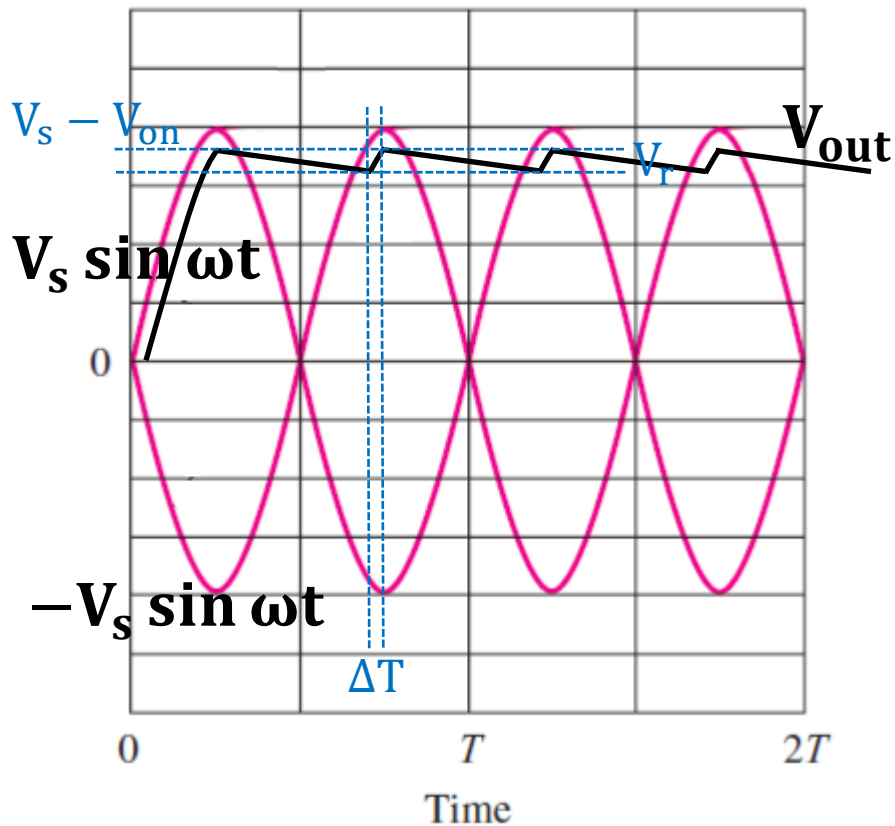
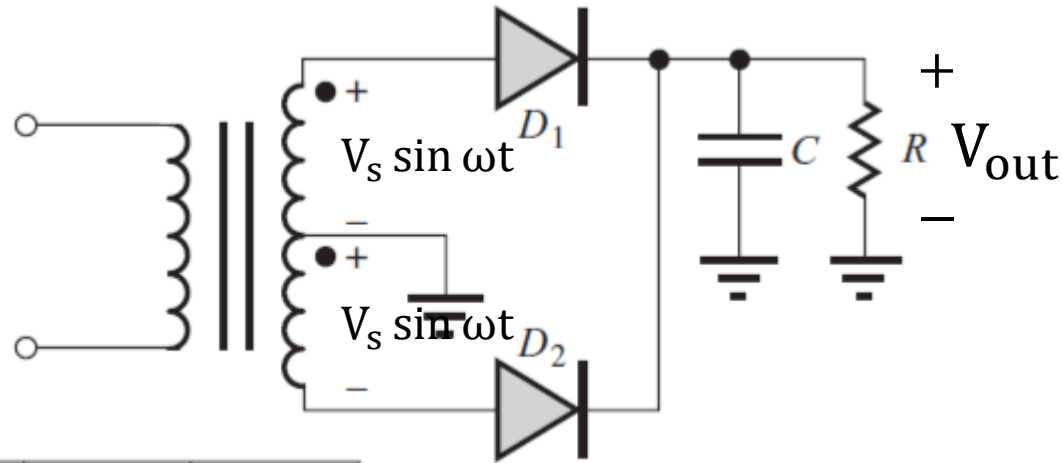
$$\theta_c \cong \sqrt{\frac{2V_r}{V_s}} = \sqrt{\frac{2 \times 0.747}{12.6 \times \sqrt{2}}} = 0.29 \text{ (rad) or } 16.6^\circ$$

$$\Delta T \cong \frac{\theta_c}{\omega} = \frac{0.29}{2\pi \times 60} = 7.69 \times 10^{-4} \text{ (sec)}$$

$$I_{\text{peak}} = \frac{2 \times 1.12 \times \frac{1}{60}}{7.69 \times 10^{-4}} = 48.6 \text{ (A)}$$

- Make sure all assumptions are valid.
- Since  $R$  is small ( $15 \Omega$ ),  $C$  needs to be large ( $25,000 \mu\text{F}$ ) to maintain a low  $V_r$ .
- The diode must be able to handle these repetitively high peak currents.

# Full-Wave Rectifier (I)



$$V_{dc} = V_s - V_{on}$$

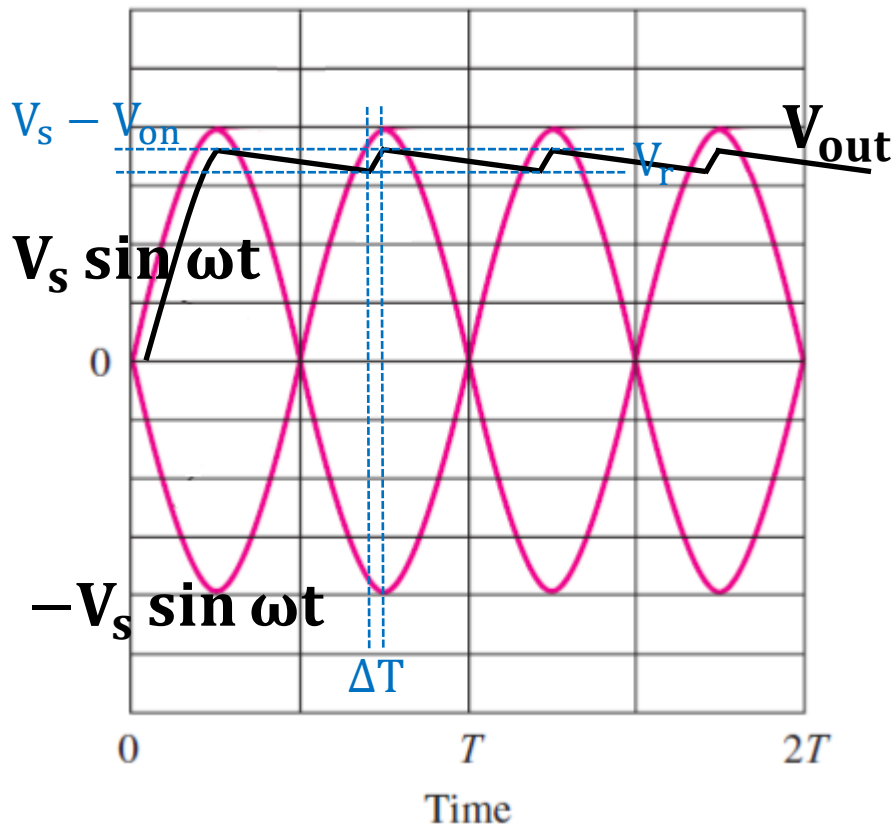
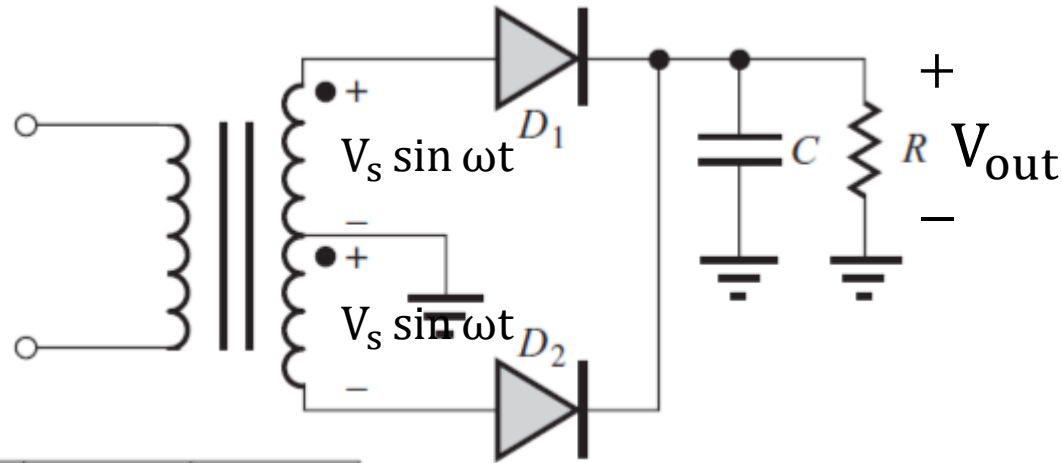
$$I_{dc} = \frac{V_{dc}}{R}$$

$$V_r = (V_s - V_{on}) \left( 1 - e^{-\frac{T/2 - \Delta T}{RC}} \right)$$

$$\cong (V_s - V_{on}) \left( \frac{T/2 - \Delta T}{RC} \right) \text{ if } \left( \frac{T}{2} - \Delta T \right) \ll RC$$

$$\cong (V_s - V_{on}) \left( \frac{T}{2RC} \right) \text{ if } \Delta T \ll \frac{T}{2}$$

# Full-Wave Rectifier (II)



$$-V_s \sin \left[ \omega \left( \frac{3T}{4} - \Delta T \right) \right] - V_{on} = (V_s - V_{on}) - V_r$$

$$-V_s \sin \left( \frac{3\pi}{2} - \theta_c \right) - V_{on} = (V_s - V_{on}) - V_r$$

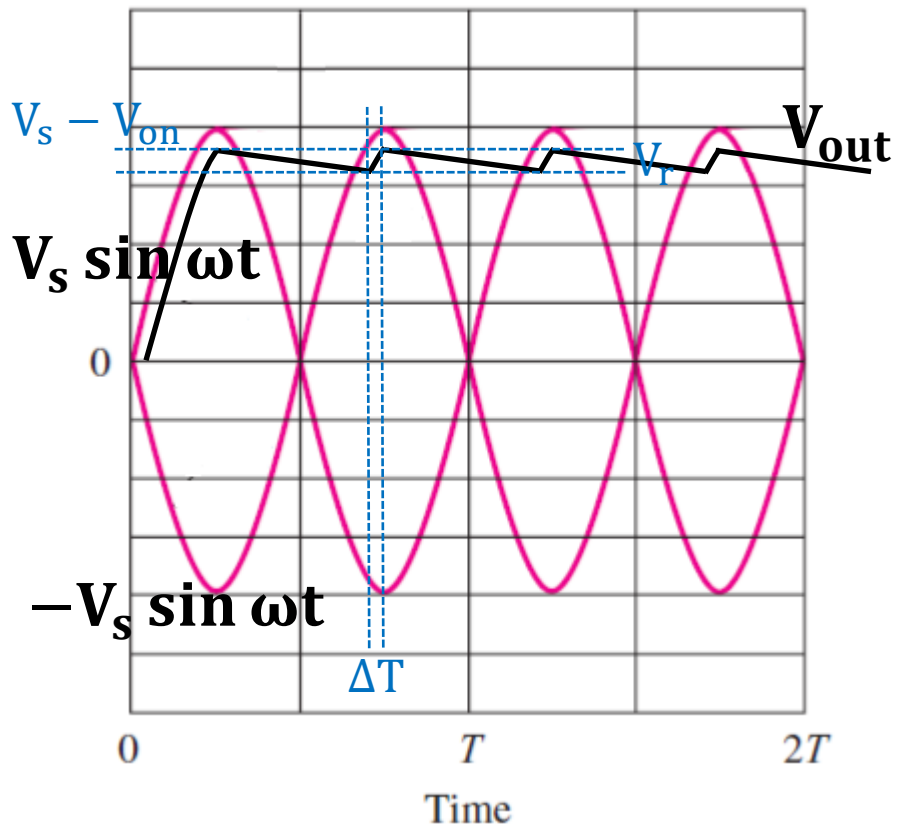
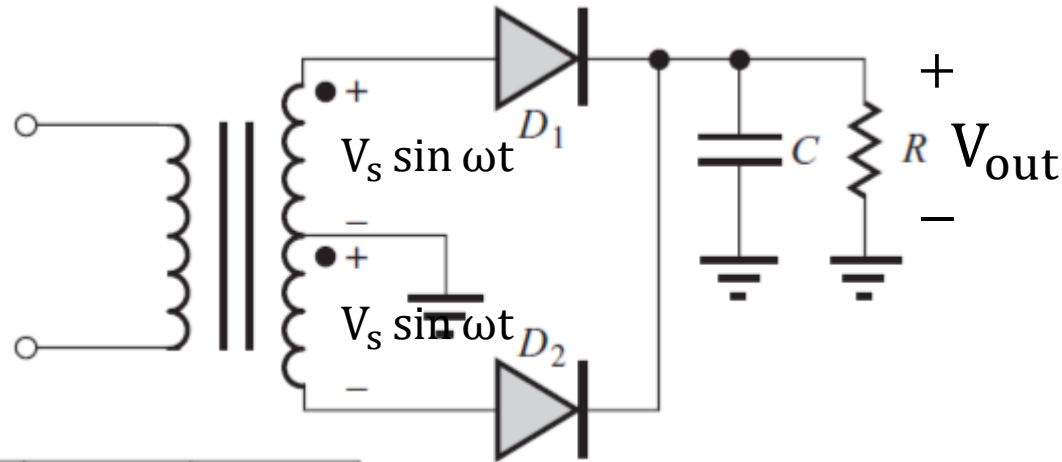
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c^2}{2} \quad \text{if } \theta_c \text{ very small}$$

$$\theta_c = \sqrt{\frac{2V_r}{V_s}}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_s}}$$

# Full-Wave Rectifier (III)



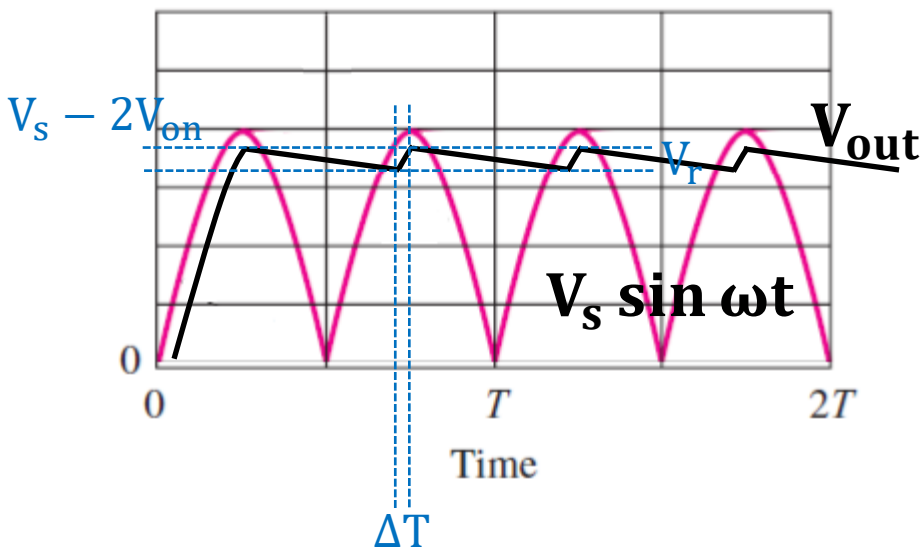
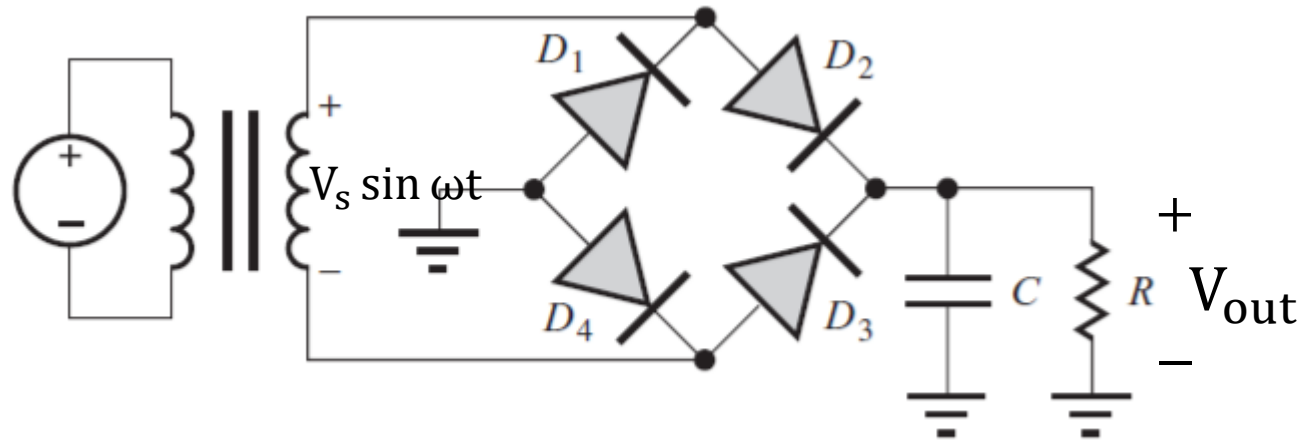
$$Q \cong \frac{I_{peak} \Delta T}{2} = I_{dc} \left( \frac{T}{2} - \Delta T \right) \cong I_{dc} \frac{T}{2}$$

$$I_{peak} = \frac{I_{dc} T}{\Delta T}$$

$$I_{surge} = \omega C V_s$$

$$PIV = 2V_s - V_{on}$$

# Full-Wave Bridge Rectifier (I)



$$V_{dc} = V_s - 2V_{on}$$

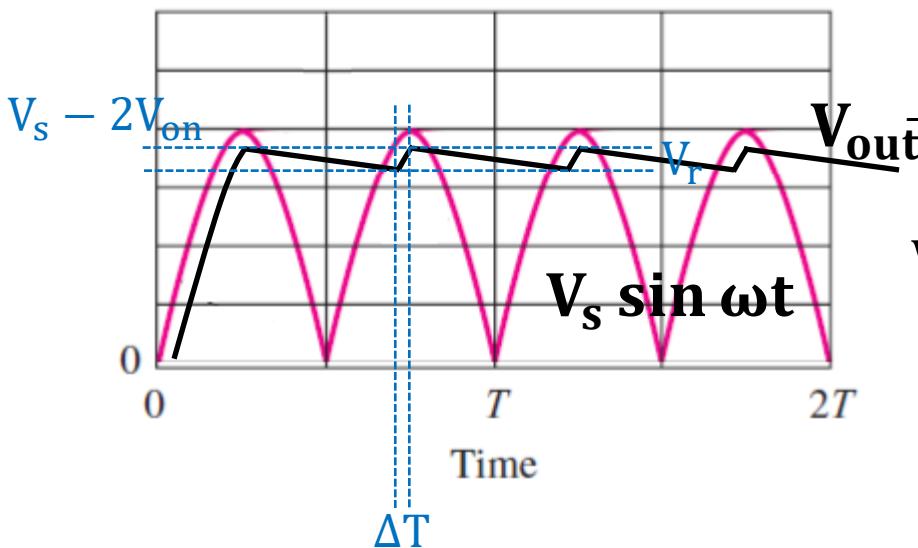
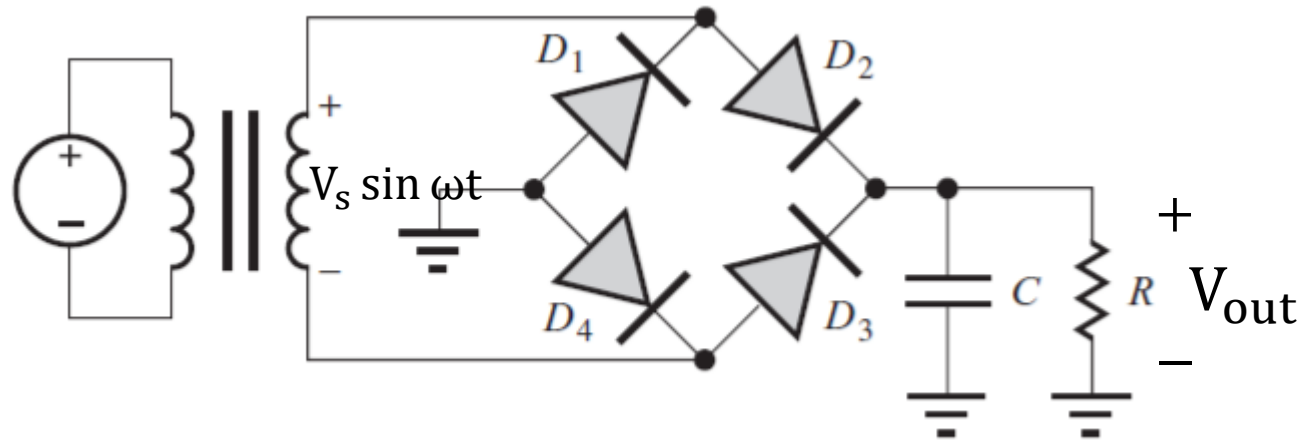
$$I_{dc} = \frac{V_{dc}}{R}$$

$$V_r = (V_s - 2V_{on}) \left( 1 - e^{-\frac{T/2 - \Delta T}{RC}} \right)$$

$$\cong (V_s - 2V_{on}) \left( \frac{T/2 - \Delta T}{RC} \right) \text{ if } \left( \frac{T}{2} - \Delta T \right) \ll RC$$

$$\cong (V_s - 2V_{on}) \left( \frac{T}{2RC} \right) \text{ if } \Delta T \ll \frac{T}{2}$$

# Full-Wave Bridge Rectifier (II)



$$-V_s \sin \left[ \omega \left( \frac{3T}{4} - \Delta T \right) \right] - 2V_{on} = (V_s - 2V_{on}) - V_r$$

$$-V_s \sin \left( \frac{3\pi}{2} - \theta_c \right) - 2V_{on} = (V_s - 2V_{on}) - V_r$$

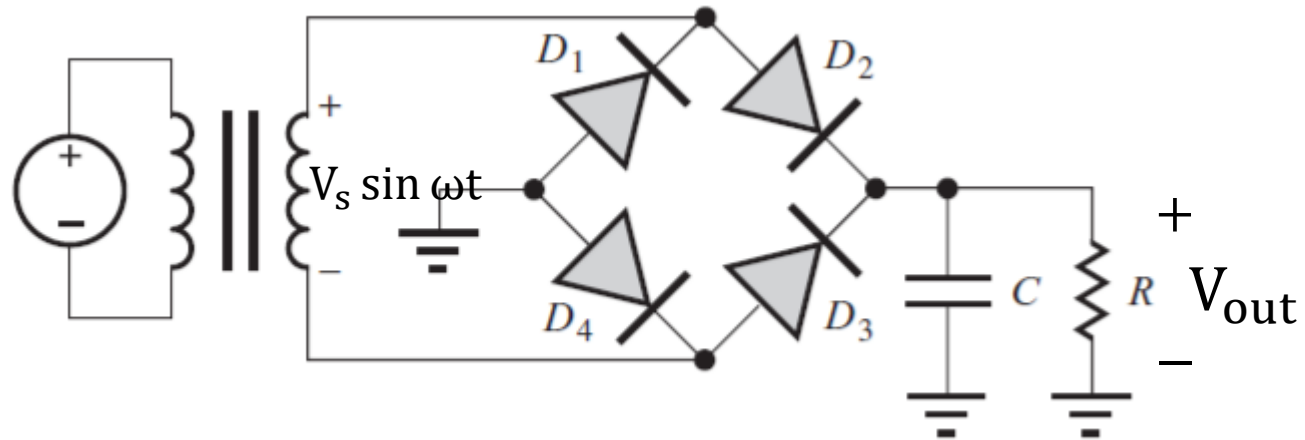
$$V_s \cos \theta_c = V_s - V_r$$

$$\cos \theta_c = \frac{V_s - V_r}{V_s} \cong 1 - \frac{\theta_c^2}{2} \text{ if } \theta_c \text{ very small}$$

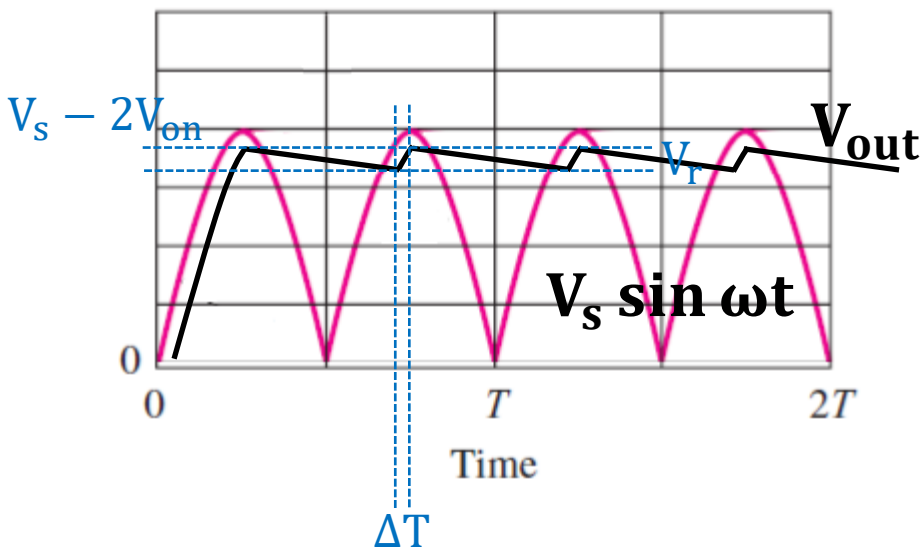
$$\theta_c = \sqrt{\frac{2V_r}{V_s}}$$

$$\Delta T = \frac{\theta_c}{\omega} = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_s}}$$

# Full-Wave Bridge Rectifier (III)



$$Q \cong \frac{I_{peak} \Delta T}{2} = I_{dc} \left( \frac{T}{2} - \Delta T \right) \cong I_{dc} \frac{T}{2}$$



$$I_{peak} = \frac{I_{dc} T}{\Delta T}$$

$$I_{surge} = \omega C V_s$$

$$PIV = V_s - V_{on}$$



# Example

Design a full-wave bridge rectifier to provide a dc output voltage 15 V with no more than 1 percent ripple at a load current of 2A. ( $V_{on} = 1$  V,  $T = 1/60$  sec)

$$V_{dc} = 15 \text{ (V)}$$

$$V_r < 0.15 \text{ (V)}$$

$$I_{dc} = 2 \text{ (A)}$$

$$\text{Load resistance} = 15/2 = 7.5 \text{ (}\Omega\text{)}$$

The required transformer voltage  $V_s = 15 + 2 = \mathbf{17 \text{ (V)}}$  or  $\frac{17}{\sqrt{2}}$  ( $V_{rms}$ )

$$V_r \cong (V_s - 2V_{on}) \left( \frac{T}{2RC} \right) = 15 \left( \frac{1}{2 \times 60 \times 7.5 \times C} \right) = 0.15 \Rightarrow \mathbf{C = 0.111 \text{ (F)}}$$

$$\Delta T = \frac{1}{\omega} \sqrt{\frac{2V_r}{V_s}} = \frac{1}{2\pi \times 60} \sqrt{\frac{2 \times 0.15}{17}} = \mathbf{0.352 \times 10^{-3} \text{ (sec)}}$$

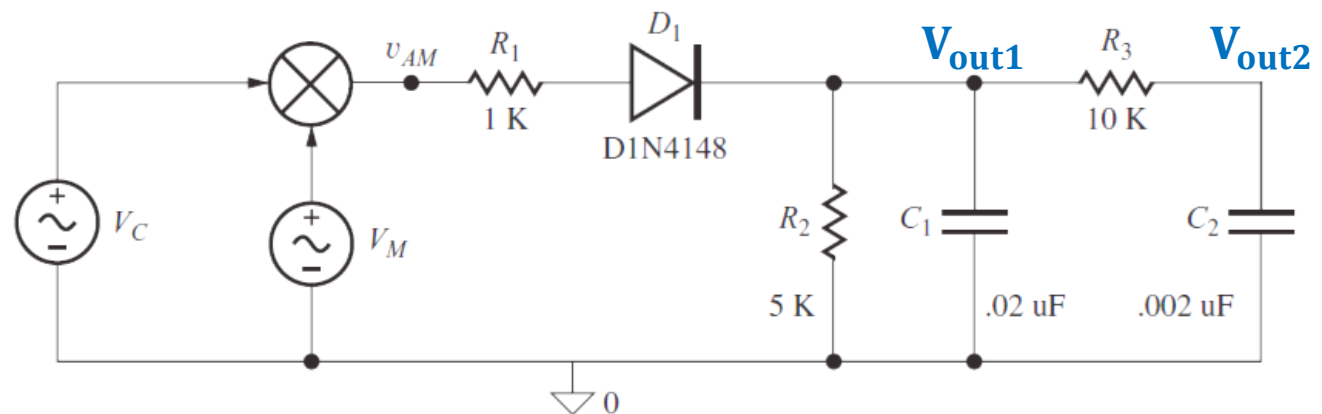
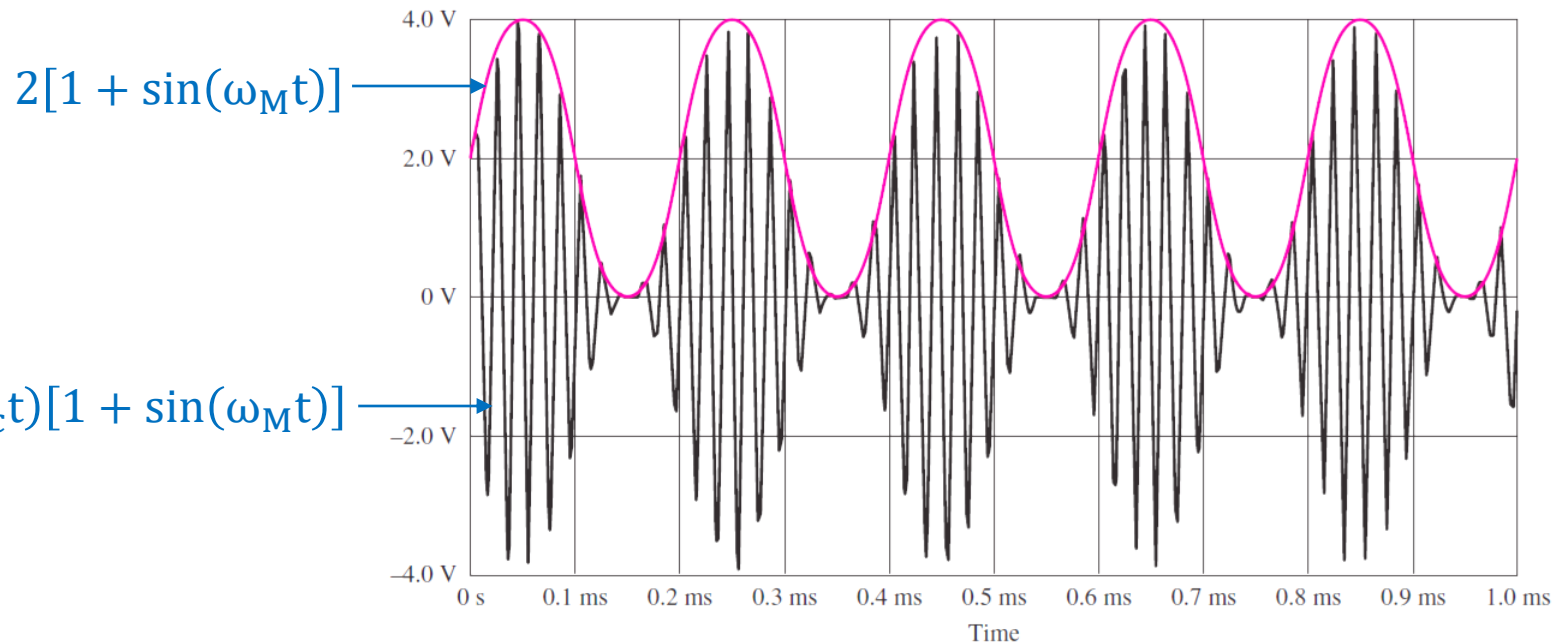
$$I_{peak} = \frac{I_{dc} T}{\Delta T} = \frac{2 \times \frac{1}{60}}{0.352 \times 10^{-3}} = \mathbf{94.7 \text{ (A)}}$$

$$I_{surge} = \omega C V_s = 2\pi \times 60 \times 0.111 \times 17 = \mathbf{711 \text{ (A)}}$$

Make sure the diodes can handle these large currents

# Example

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.



# Example

An amplitude modulated (AM) signal is shown below. The envelope of the AM signal contains the information being transmitted, and the envelope can be recovered using a single half-wave rectifier.

