JOINT INSTITUTE
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Diode

VE311 Electronic Circuits (Fall 2021)

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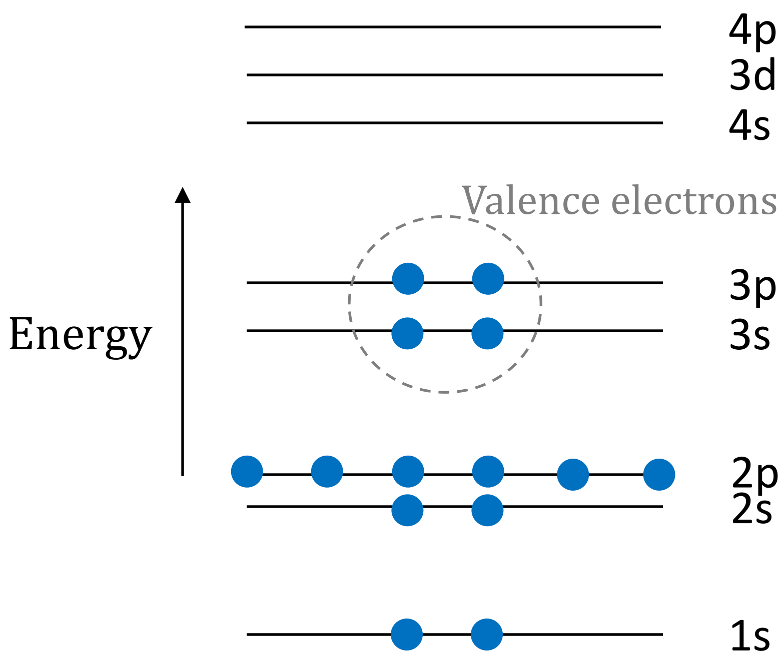
Brief Introduction of Semiconductor Physics

Although not going to be covered in the exams, this part lays a foundation for understanding the working principles of the semiconductor devices.

Silicon Atom

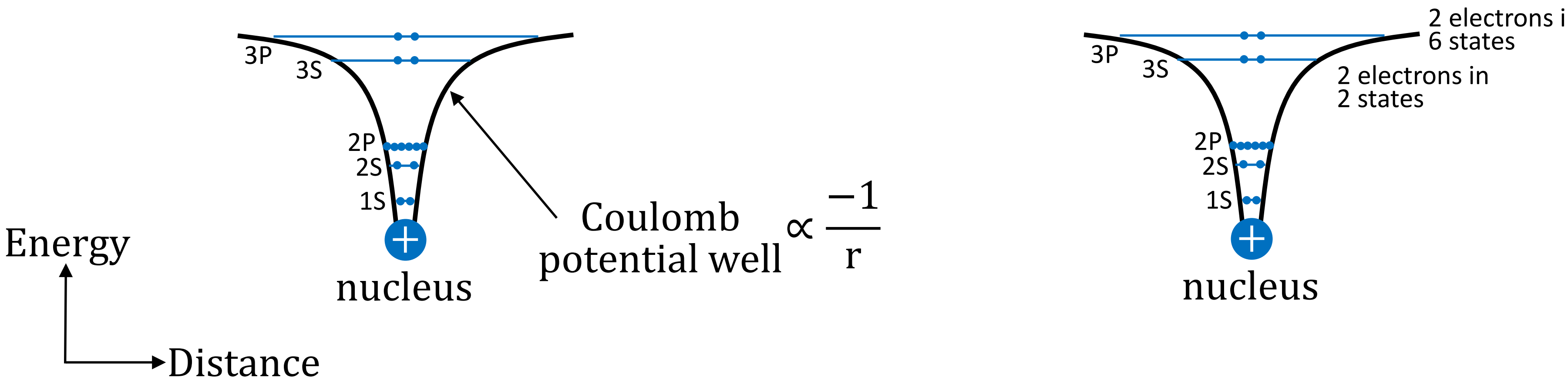
- **Pauli exclusion principle** requires that each electron must have a distinct energy state defined by an unique set of quantum numbers.
- The allowed states and associated wavefunctions can be described by 4 quantum numbers: principle quantum number (n), angular momentum quantum number (l), magnetic quantum number (m) and electron spin (s).



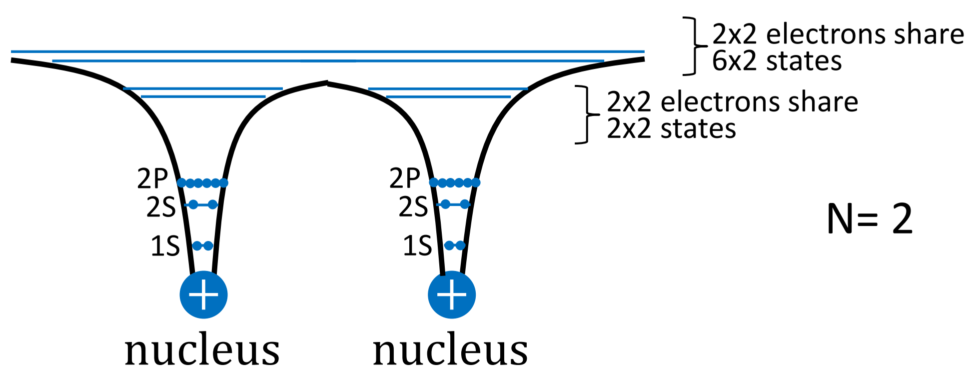
Silicon has its inner shell (1s, 2s and 2p orbitals) totally filled with electrons. It's outer shell has 2 valence electrons in 3s orbital and 2 valence electrons in 3p orbital.

Effect of Lattice (I)

When two silicon atoms are far away from each other, there is no interaction.

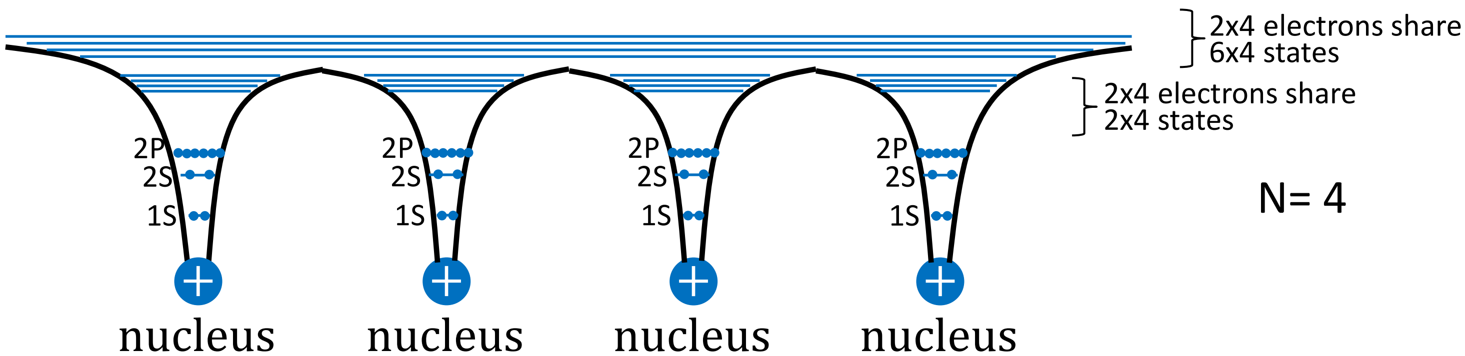


When two silicon atoms are close enough to each other: (1) wavefunctions overlap. (2) Potential wells are influenced by neighboring nucleus. (3) The valence electrons become delocalized (e.g. through tunneling). (4) By Pauli exclusion principle, **each state splits into N substates, where N is number of atoms.**



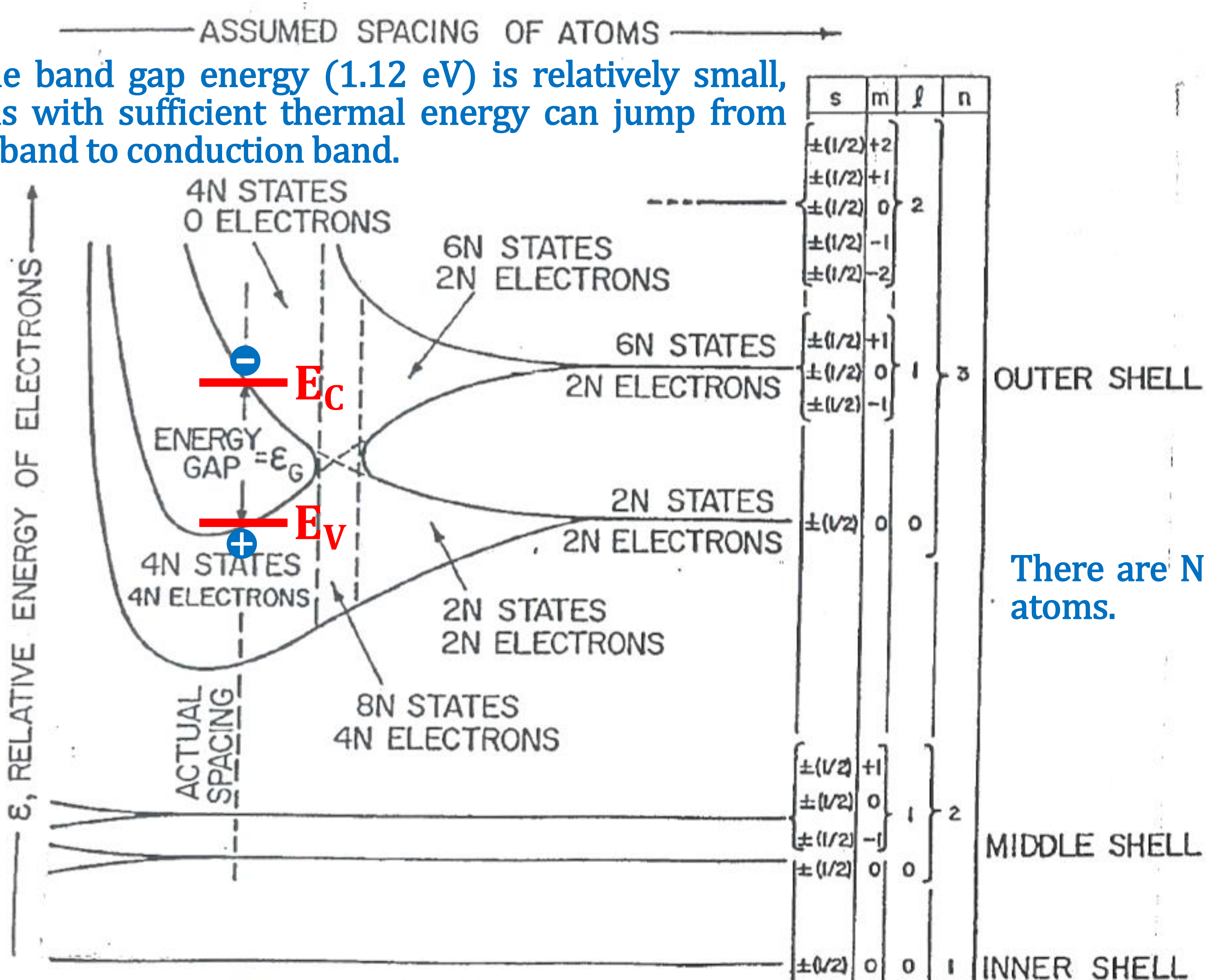
Effect of Lattice (II)

Discrete states grow into bands when N is large.



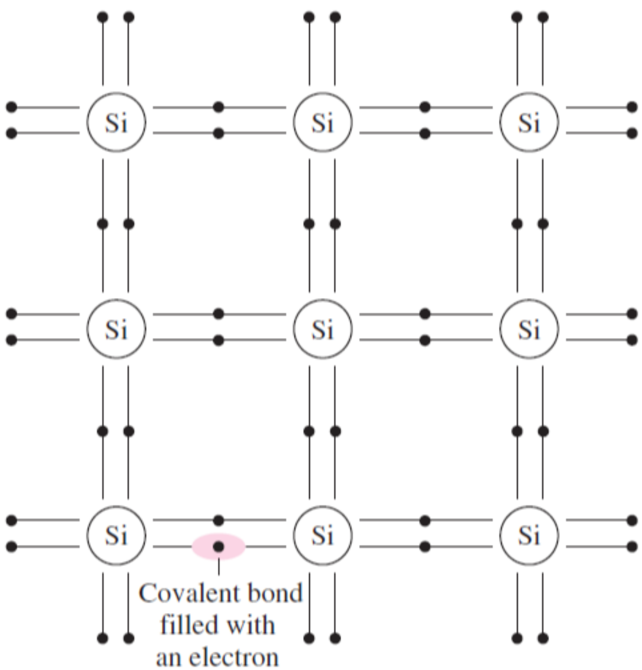
Band Formation for Si

Since the band gap energy (1.12 eV) is relatively small, electrons with sufficient thermal energy can jump from valence band to conduction band.



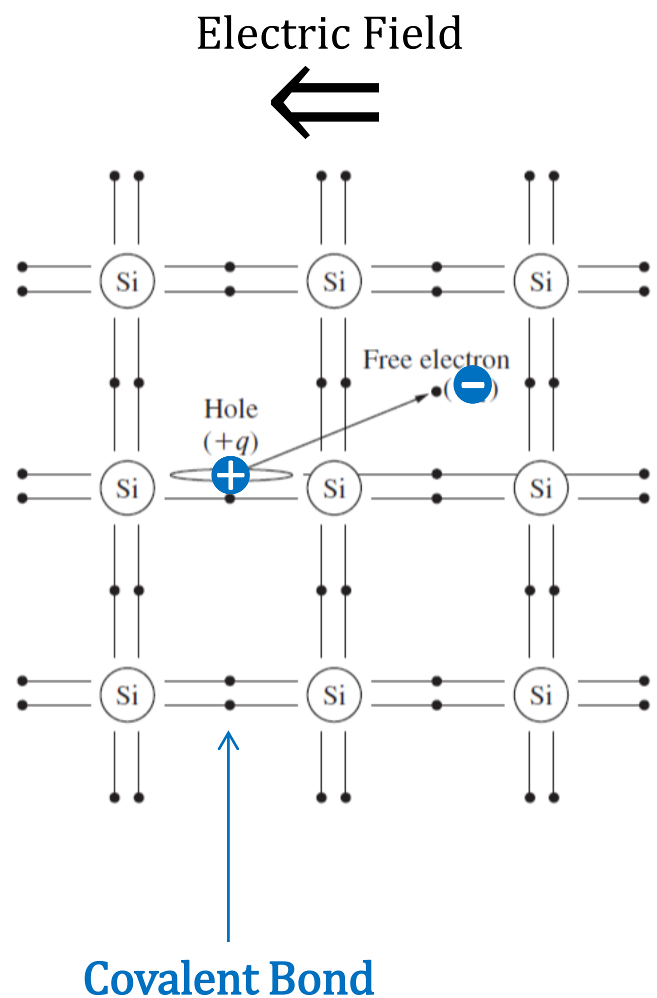
There are N silicon atoms.

Intrinsic (i.e. no impurity) Si (I)



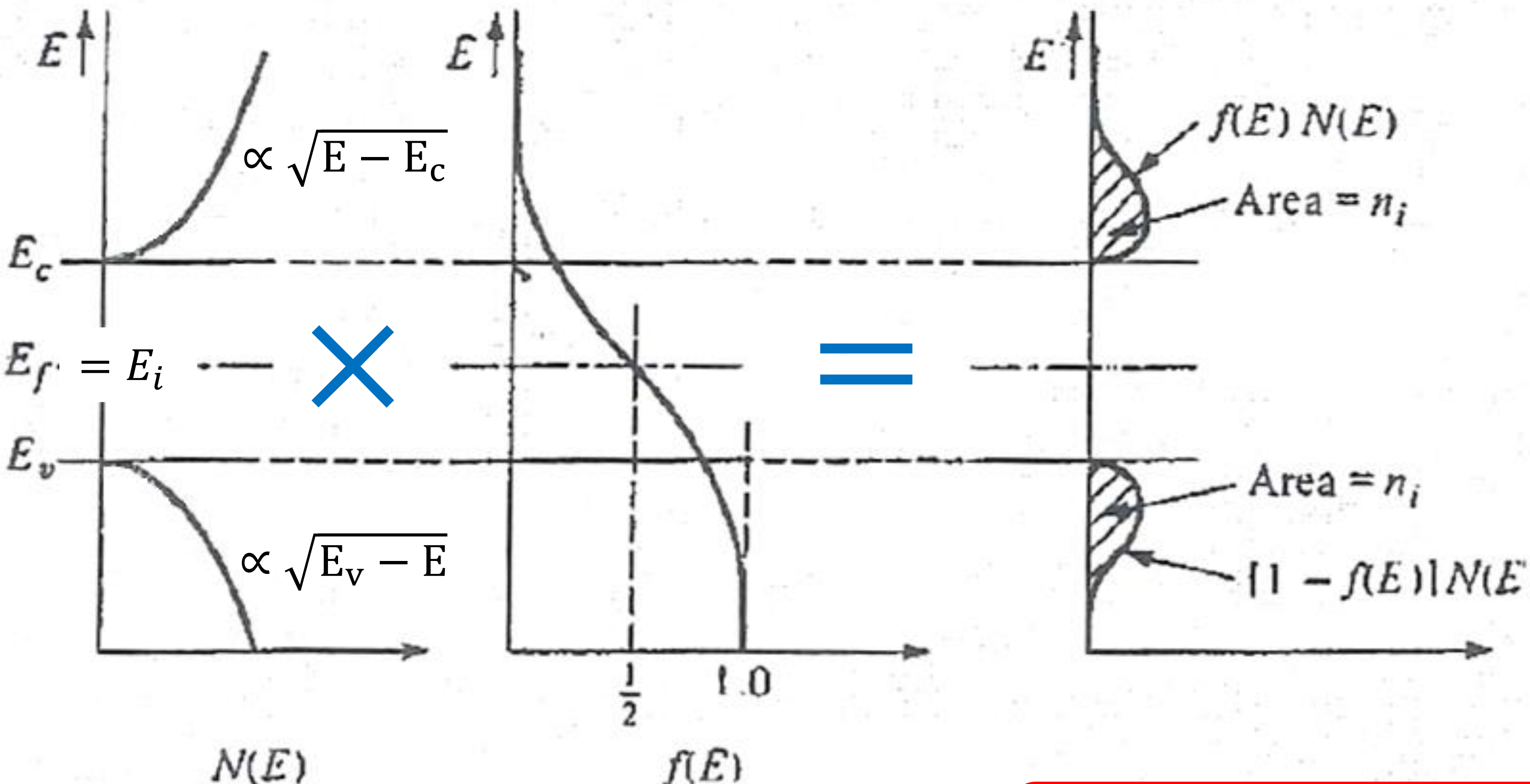
0 K

Electrons with sufficient thermal energy can jump from valence band to conduction band.



300 K

n and p for **Intrinsic Si** (II)



Density of States
\# of states/($cm^3 \cdot J$)

Fermi-Dirac Distribution
(the probability of the state occupied with an electron)

$$\mathbf{f(E) = \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1}}$$

$$\mathbf{n = \int_{E_C}^{\infty} f(E)N(E)dE = n_i}$$

$$\mathbf{p = \int_{-\infty}^{E_V} [1 - f(E)]N(E)dE = n_i}$$

(1 / cm^3)

n and p for **Intrinsic Si** (III)

$$np = n_i^2 = BT^3 \exp\left(-\frac{E_G}{kT}\right) = \text{constant}$$

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

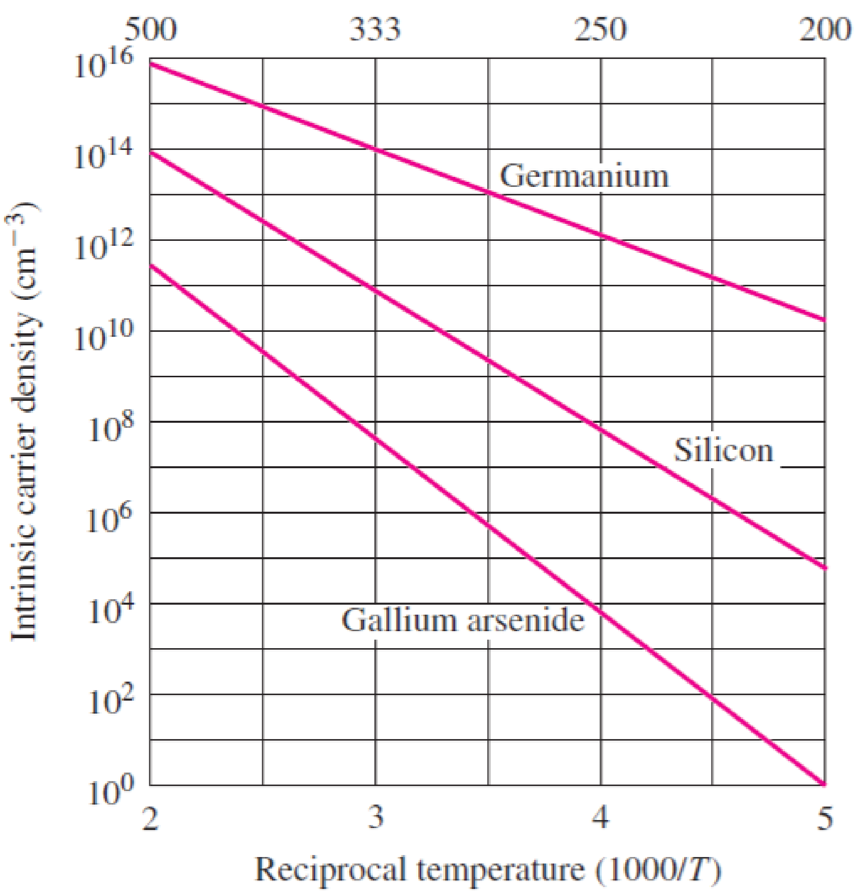


Figure 2.4 Intrinsic carrier density versus temperature from Eq. (2.1).

k (Boltzmann's Constant)
$= 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K

At 300 K:

$$n_i^2 = (1.08 \times 10^{31}) 300^3 e^{\frac{-1.12}{(8.62\times10^{-5})\times 300}}$$
$$= 4.52 \times 10^{19}\ (1/cm^6)$$

$$\boxed{n_i = 6.73 \times 10^9\ (1/cm^3) \cong 10^{10}\ (1/cm^3)}$$

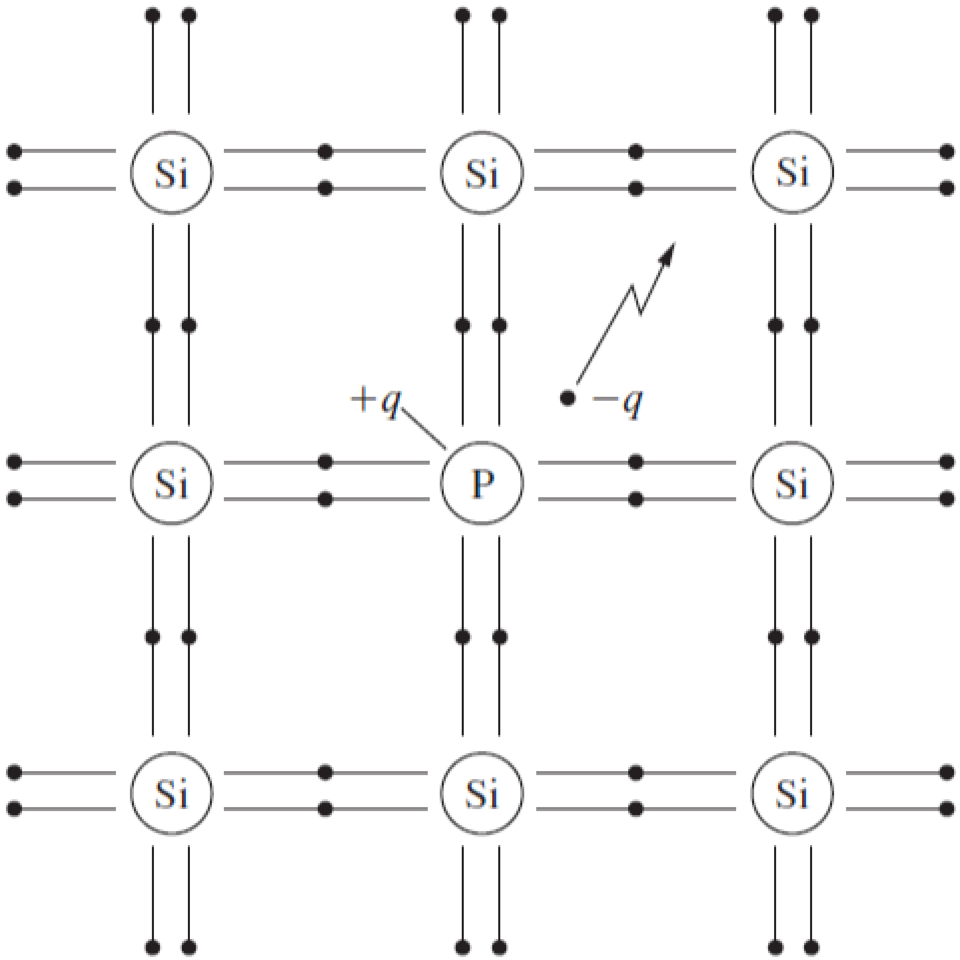
	B ($K^{-3} \cdot cm^{-6}$)	E_G (eV)
Si	1.08×10^{31}	1.12
Ge	2.31×10^{30}	0.66
GaAs	1.27×10^{29}	1.42

n and p for **n-type Si** (I)

300 K

At room temperature, nearly all phosphorus dopants are ionized.

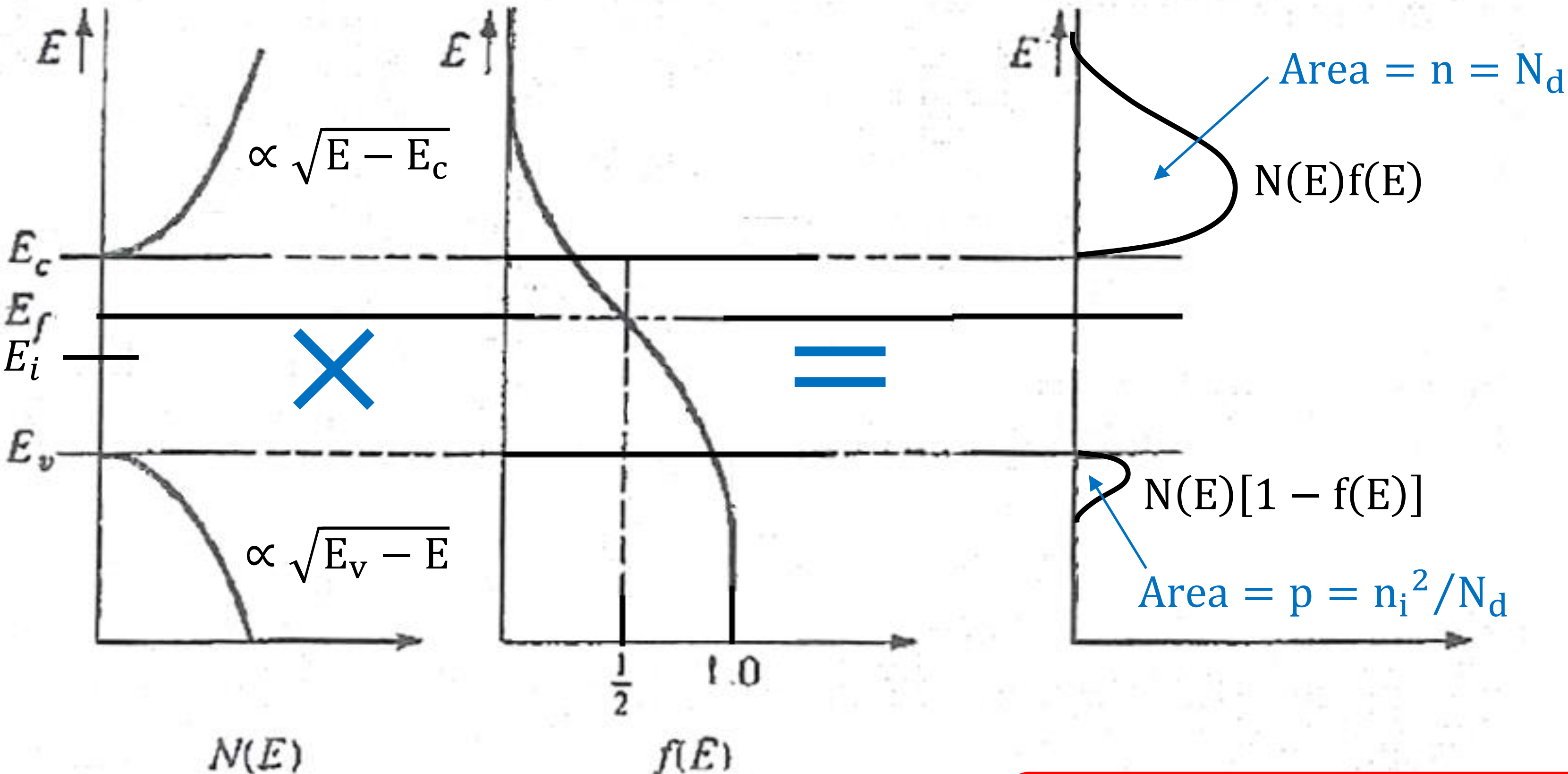
Each dopant donates one electron away, which creates an electron in the conduction band.



If the n-type dopant (e.g. 磷 **phosphorus**) concentration $N_d \gg n_i$,

$n = N_d$ and $p = n_i^2/N_d$ $(1 / cm^3)$

n and p for **n-type Si** (II)



Density of States
$\#$ of states/$(cm^3 \cdot J)$

Fermi-Dirac Distribution
(the probability of the state occupied with an electron)

$$\mathbf{f(E) = \frac{1}{\exp\left(\frac{E - E_f}{kT}\right) + 1}}$$

$$\mathbf{n = \int_{E_C}^{\infty} f(E)N(E)dE = n_i\, e^{\frac{E_f - E_i}{kT}}}$$

$$\mathbf{p = \int_{-\infty}^{E_V} [1 - f(E)]N(E)dE = n_i\, e^{\frac{E_i - E_f}{kT}}}$$

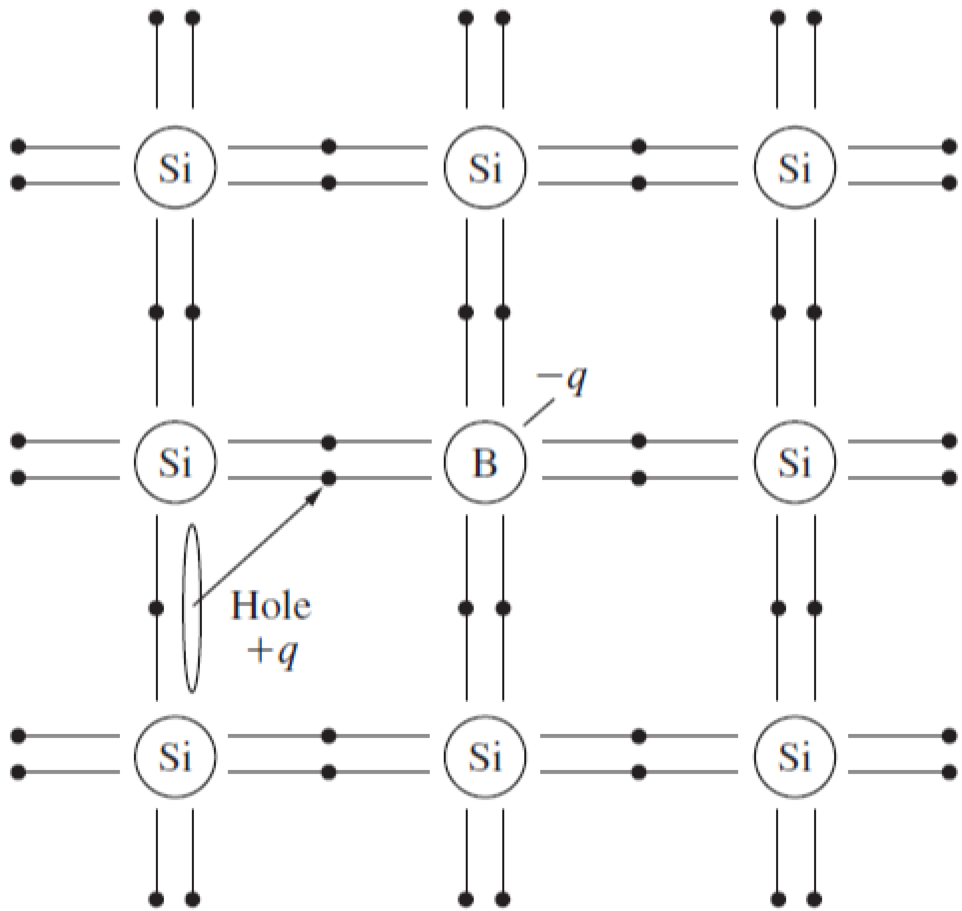
$(1\ /\ cm^3)$

n and p for **p-type Si** (I)

300 K

At room temperature, nearly all boron dopants are ionized.

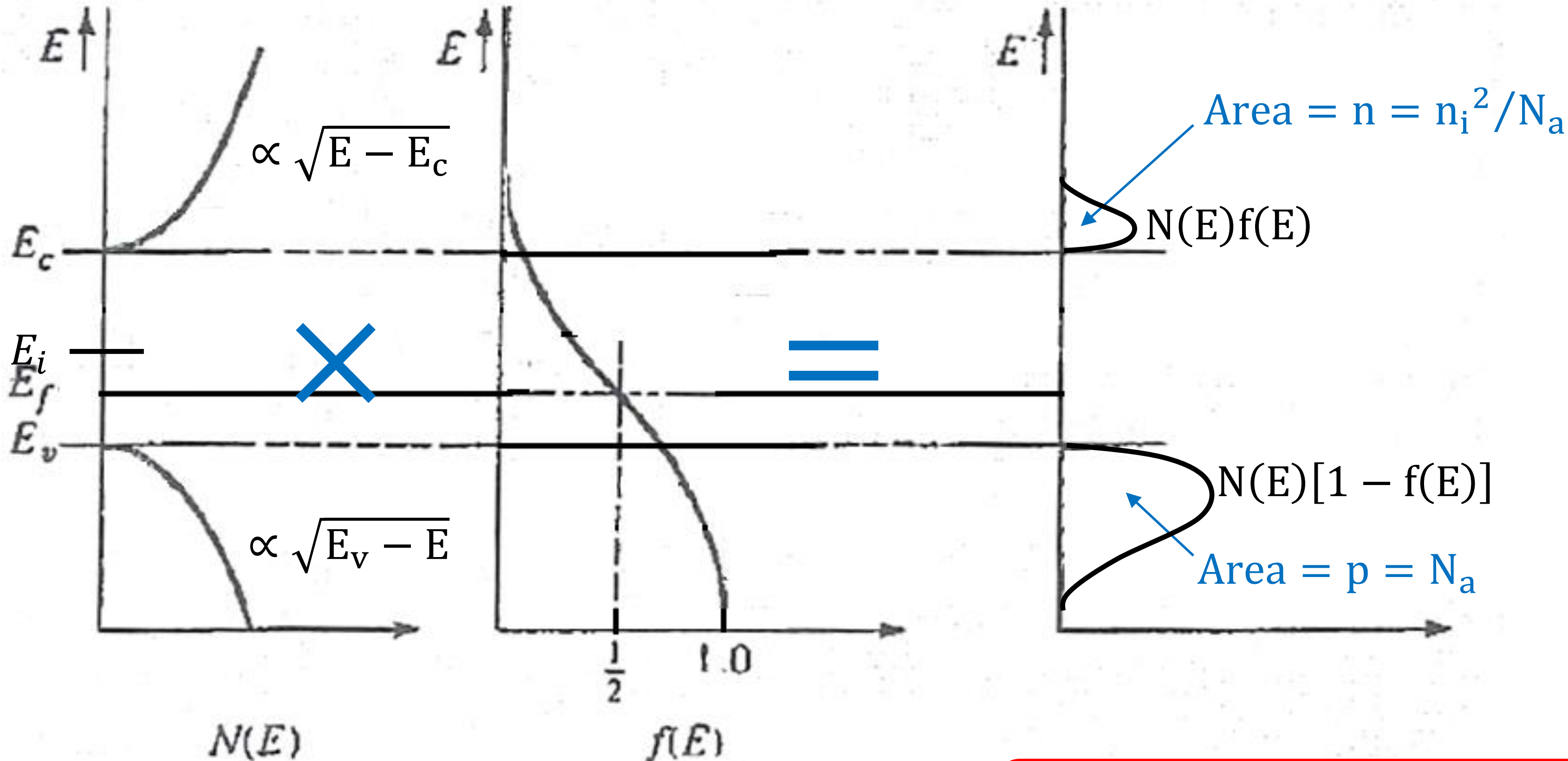
Each dopant takes one electron away from neighboring silicon, which creates a hole in the valence band.



If the p-type dopant (e.g. 硼 **boron**) concentration $N_a \gg n_i$,

$p = N_a$ and $n = n_i^2/N_a$ $(1 / cm^3)$

n and p for **p-type Si** (II)



Density of States
# of states/$(\text{cm}^3 \cdot \text{J})$

Fermi-Dirac Distribution
(the probability of the state occupied with an electron)

$$\mathbf{f(E)} = \frac{\mathbf{1}}{\mathbf{exp}\left(\frac{\mathbf{E - E_f}}{\mathbf{kT}}\right) + \mathbf{1}}$$

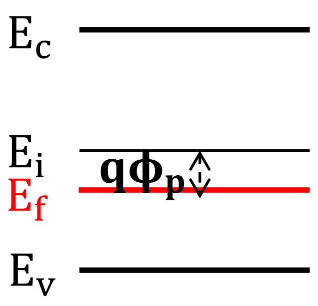
$$\mathbf{n} = \int_{E_C}^{\infty} \mathbf{f(E)N(E)dE} = \mathbf{n_i\, e^{\frac{E_f - E_i}{kT}}}$$

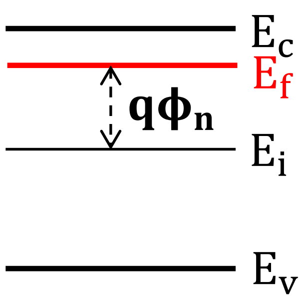
$$\mathbf{p} = \int_{-\infty}^{E_V} \mathbf{[1 - f(E)]N(E)dE} = \mathbf{n_i\, e^{\frac{E_i - E_f}{kT}}}$$

$(1\ /\ \text{cm}^3)$

Summary

p-type	n-type





$$p = N_a = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{q\phi_p}{kT}}$$

$$n = \frac{{n_i}^2}{N_a} = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{-q\phi_p}{kT}}$$

$$np = {n_i}^2$$

$$n = N_d = n_i e^{\frac{E_f - E_i}{kT}} = n_i e^{\frac{q\phi_n}{kT}}$$

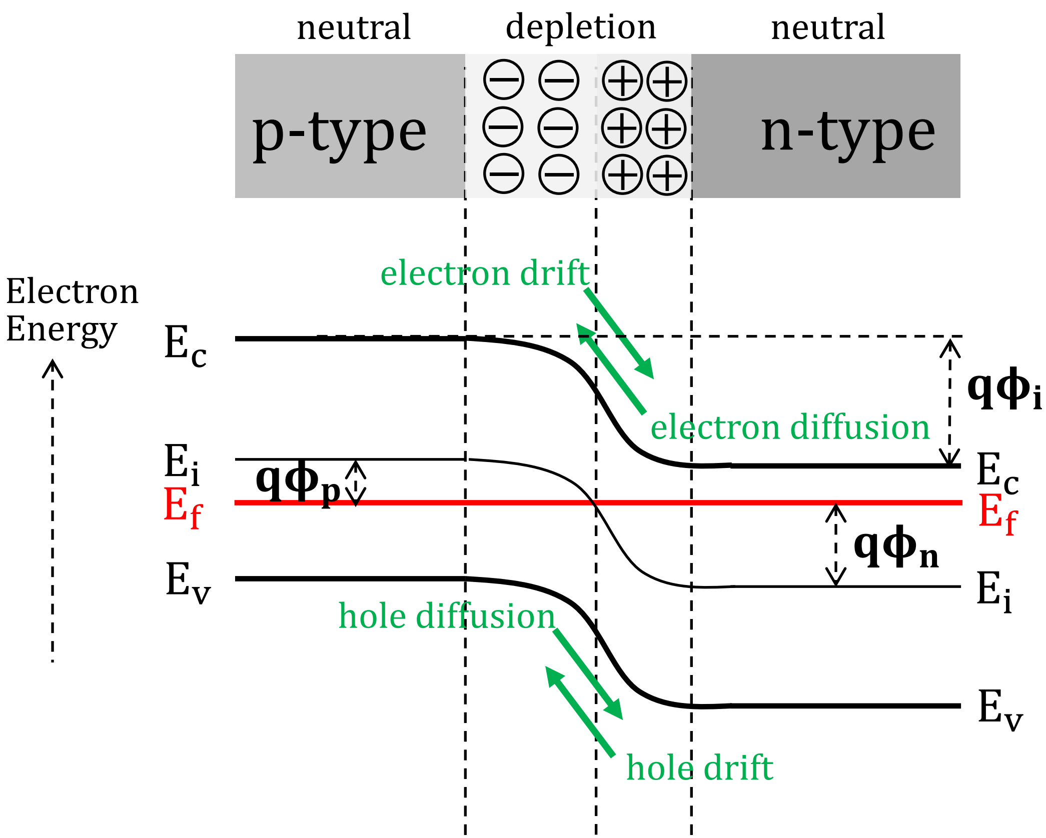
$$p = \frac{{n_i}^2}{N_d} = n_i e^{\frac{E_i - E_f}{kT}} = n_i e^{\frac{-q\phi_n}{kT}}$$

$$np = {n_i}^2$$

Si PN Junction Diode

Qualitative Understanding

Si PN Junction in **Thermal Equilibrium**



Note:

- E_C, E_i and E_V are parallel to each other.
- E_C, E_i and E_V bending means there is electric field.
- E_f bending means there is current.

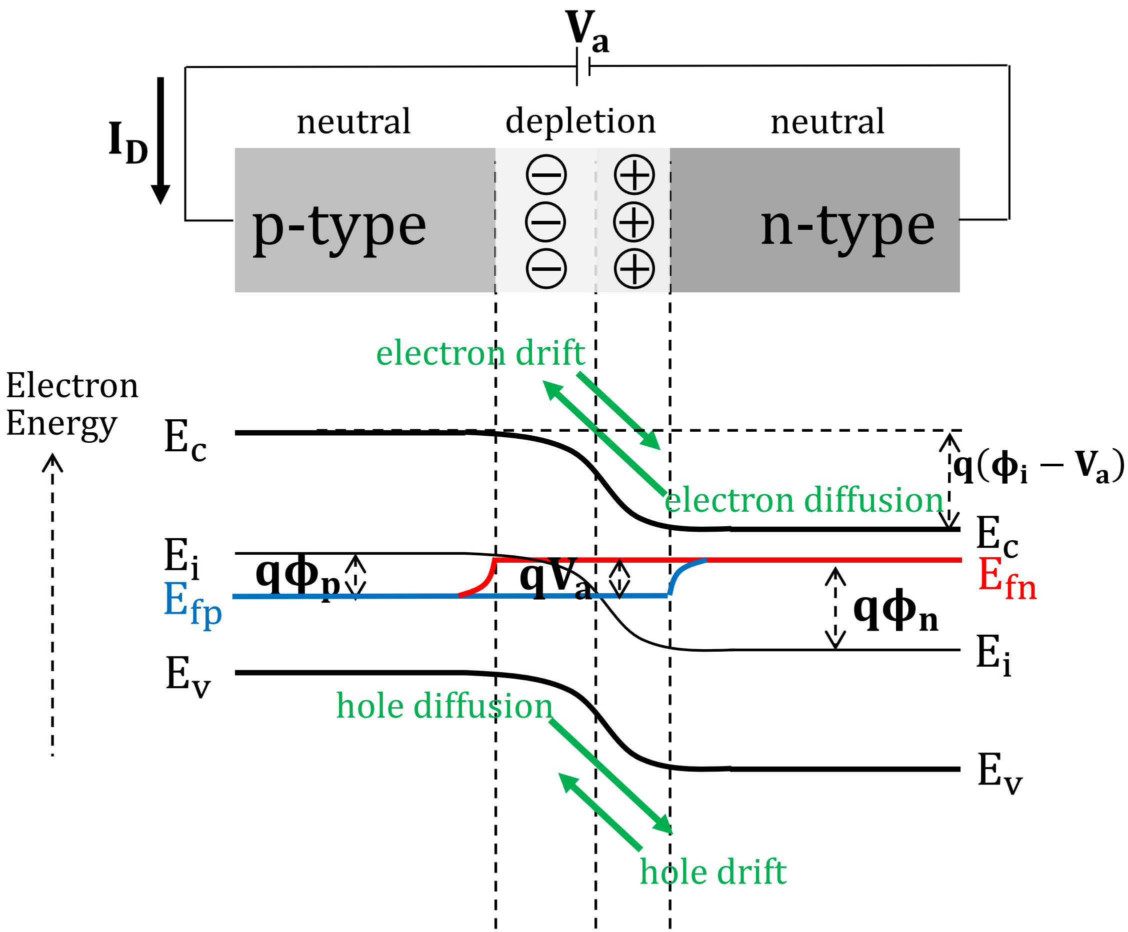
At first

1. Electrons/holes near the junction diffuse to the opposite sides.
2. Ionized dopants, fixed in the lattice, are left behind. → Formation of **built-in electric field** and **energy barrier** (**$q\phi_i$**) for diffusion.

Then

3. Some electrons/holes in the neutral regions with sufficient energy continuously diffuse to the opposite sides. → Formation of **diffusion current.**
4. Some electrons/holes wandering into the in the depletion region get swept by the built-in electric field. → Formation of **drift current.**
5. Diffusion current cancels drift current. No net current flowing.

Si PN Junction in **Forward Bias**



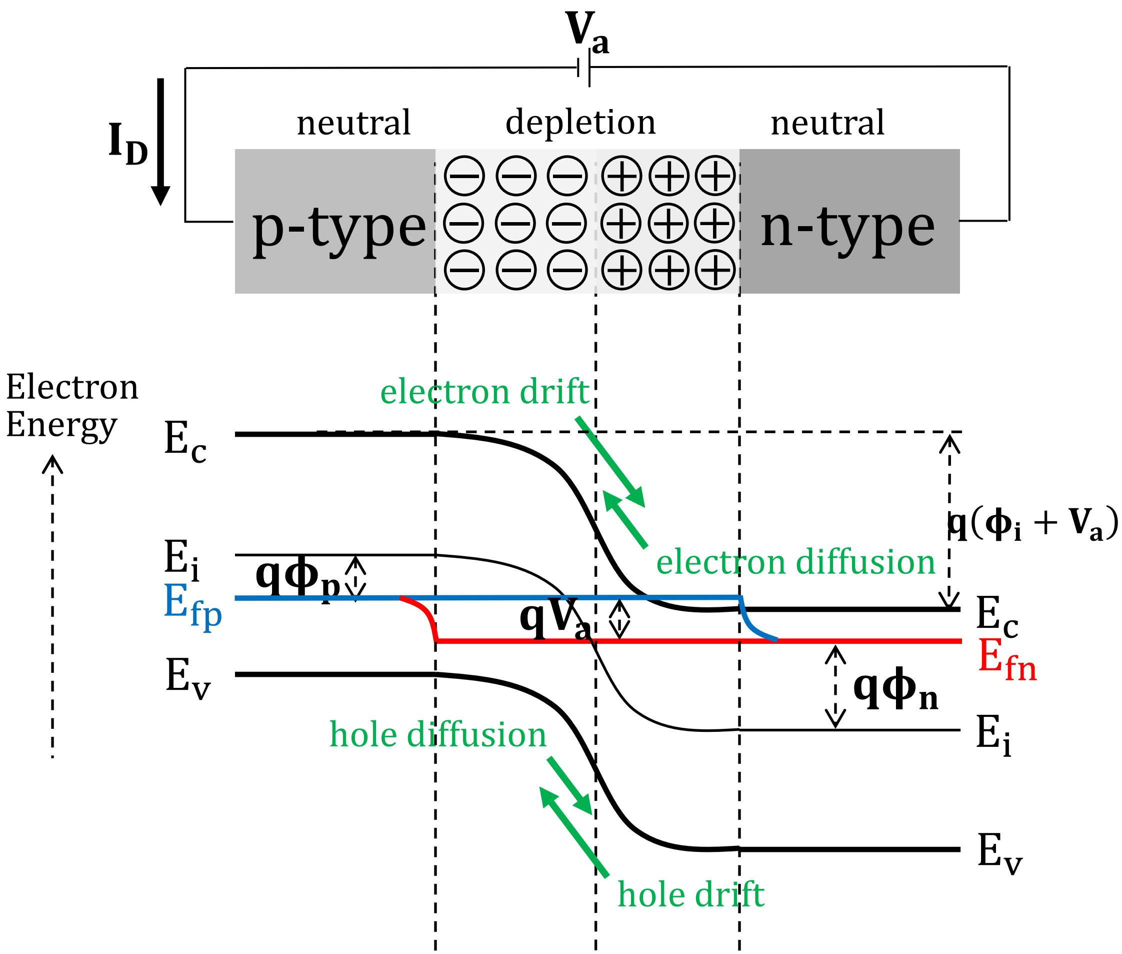
$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) > 0$$

Constant Diffusion Drift

When V_a > 0 applied

1. The energy barrier formed by the built-in electric field becomes smaller, $\mathbf{q(\phi_i - V_a)}$.
2. More electrons/holes diffuse to the opposite sides. → **Diffusion current increases**, while **drift current remains the same**.
3. There is (+) net current flowing.
4. The depletion width becomes narrower.

Si PN Junction in **Reverse Bias**



$$I_D = I_S\left(e^{\frac{-qV_a}{kT}} - 1\right) < 0$$

Constant Diffusion Drift

When V_a < 0 applied

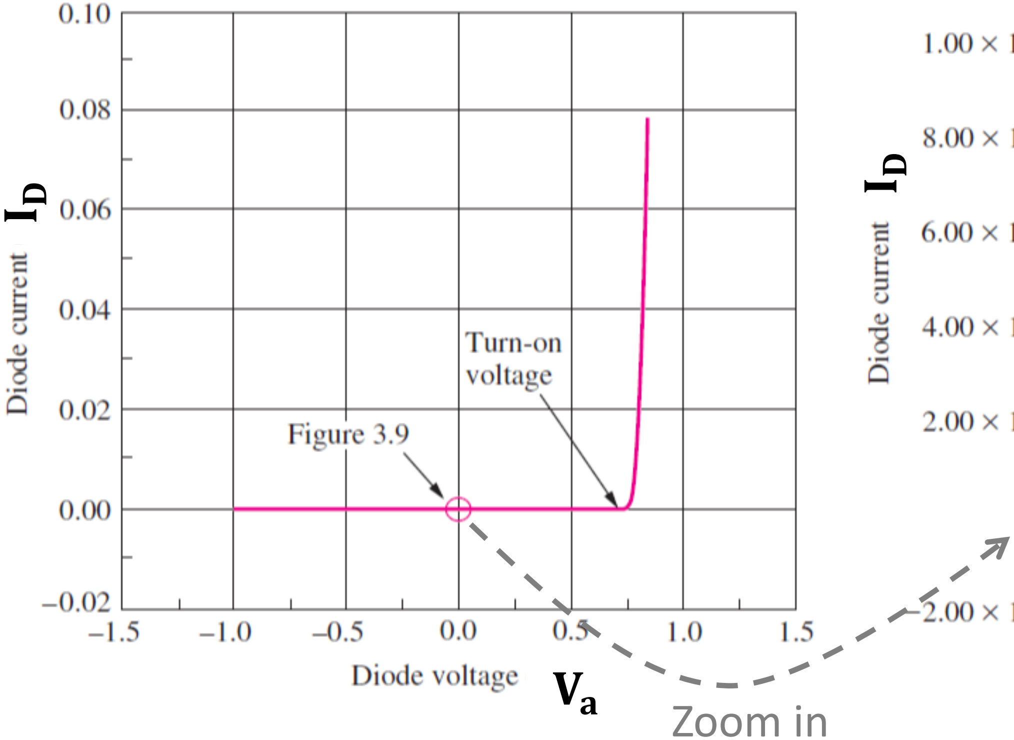
1. The energy barrier formed by the built-in electric field becomes larger, $\mathbf{q(\phi_i + V_a)}$.
2. Less electrons/holes diffuse to the opposite sides. → **Diffusion current decreases**, while **drift current remains the same**.
3. There is (−) net current flowing.
4. The depletion width becomes wider.

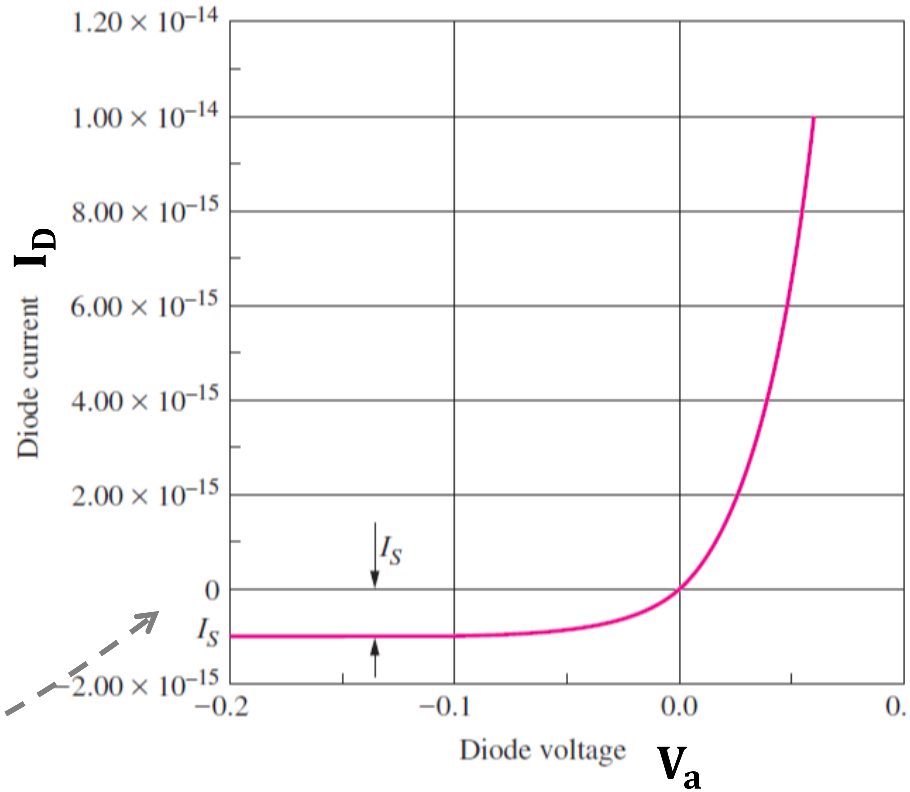
Diode I-V Characteristics

Si Diode I-V Characteristics

$$\mathbf{I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right)}$$

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock





- Turn-on voltage typically 0.5 to 0.7 V
- Saturation current (I_S) typically 10^{-18} to 10^{-9} A
- $kT/q = 0.025875$ V at 300 K

Example

(a) Calculate V_a for a silicon diode with $I_S = 0.1$ fA and I_D increasing from 300 μA to 10 mA at 300 K.

$$300 \times 10^{-6} = (0.1 \times 10^{-15})\left(e^{\frac{V_a}{0.025875}} - 1\right) \qquad V_a = 0.743\ (\mathrm{V})$$

$$10 \times 10^{-3} = (0.1 \times 10^{-15})\left(e^{\frac{V_a}{0.025875}} - 1\right) \qquad V_a = 0.834\ (\mathrm{V})$$

(b) Calculate I_S for a silicon diode with $I_D = 2.5$ mA and $V_a = 0.736$ V at 50°C.

$$2.5 \times 10^{-3} = I_S\left(e^{\frac{(1.6\times10^{-19})\times0.736}{(1.38\times10^{-23})(323)}} - 1\right) \qquad I_S = 8.4 \times 10^{-15}\ (\mathrm{A})$$

Example

Calculate the required V_a for I_D of a silicon diode to increase by a factor 10 at 300 K. Assume $I_D \gg I_S$.

$$\begin{cases} I_{D1} = I_S\left(e^{\frac{qV_{a1}}{kT}} - 1\right) \approx I_S e^{\frac{qV_{a1}}{kT}} \\ I_{D2} = I_S\left(e^{\frac{qV_{a2}}{kT}} - 1\right) \approx I_S e^{\frac{qV_{a2}}{kT}} \end{cases}$$

$$\frac{I_{D2}}{I_{D1}} = 10 = \frac{I_S e^{\frac{qV_{a2}}{kT}}}{I_S e^{\frac{qV_{a1}}{kT}}} = e^{\frac{V_{a2}-V_{a1}}{0.025875}}$$

$$V_{a2} - V_{a1} = 0.025875 \times \ln 10$$
$$= 0.05958\ (\text{V}) \approx 60\ (\text{mV})$$

The diode voltage changes by about 60 mV per decade change in diode current.

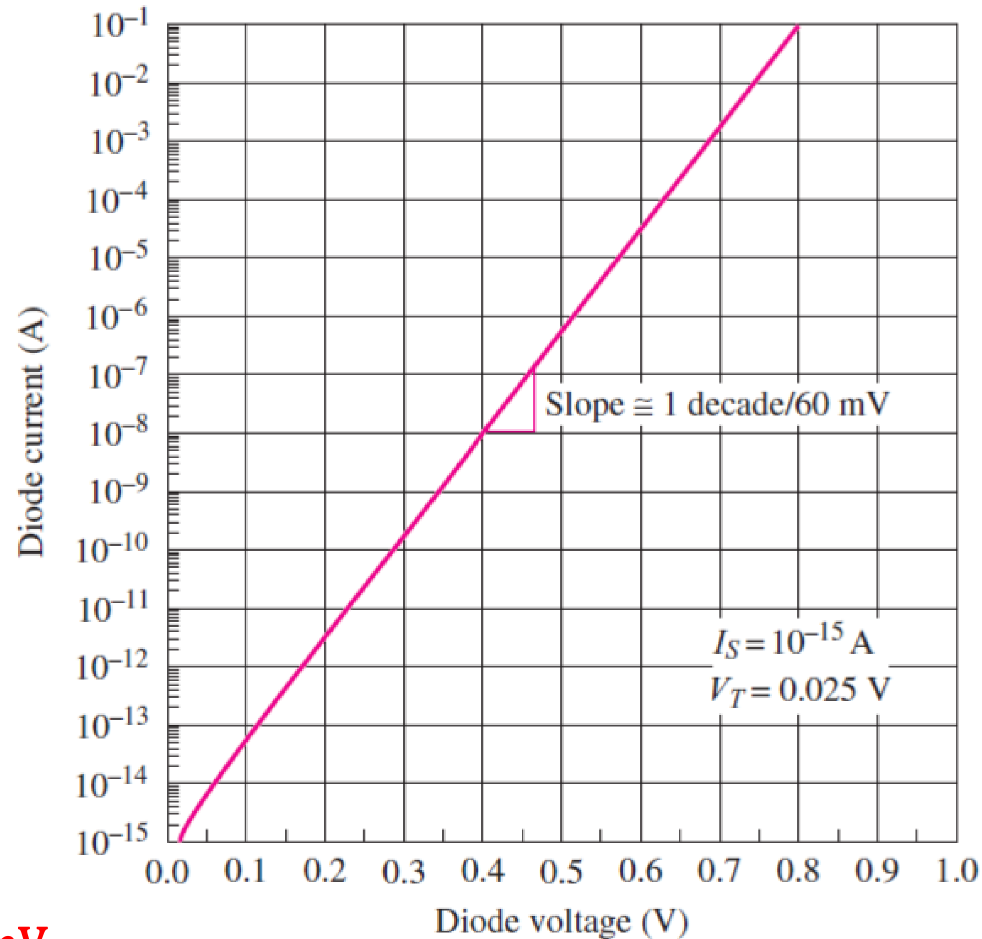
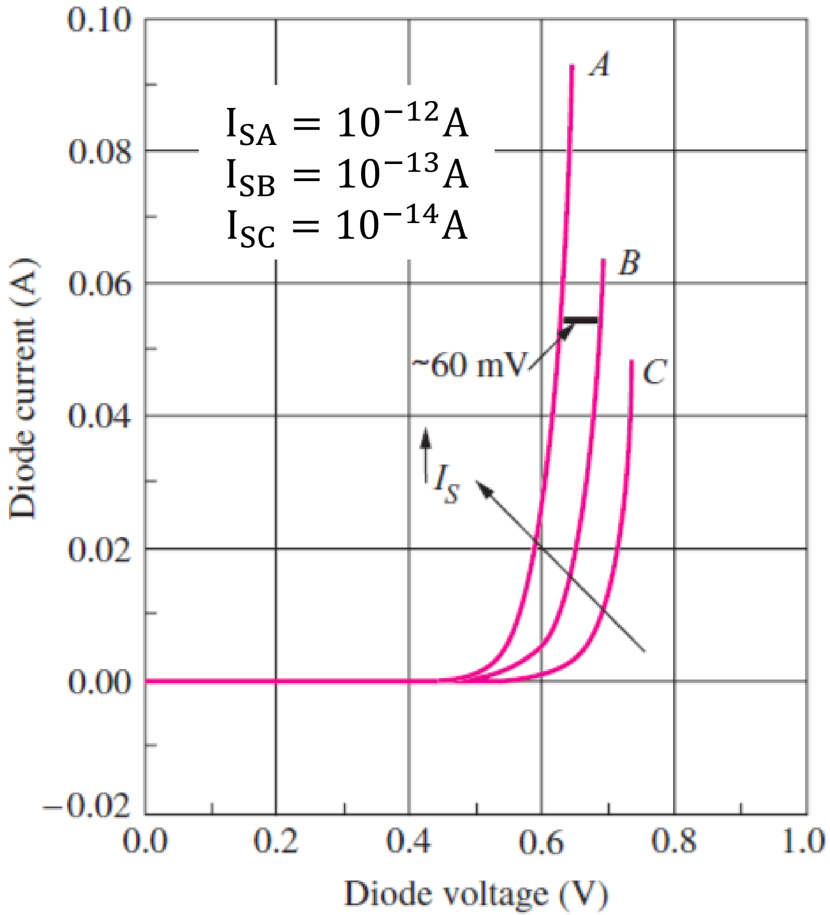


Figure 3.11 Diode *i*-*v* characteristic on semilog scale.

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

I_D and I_S versus V_a



Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

$$\mathbf{I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right)}$$

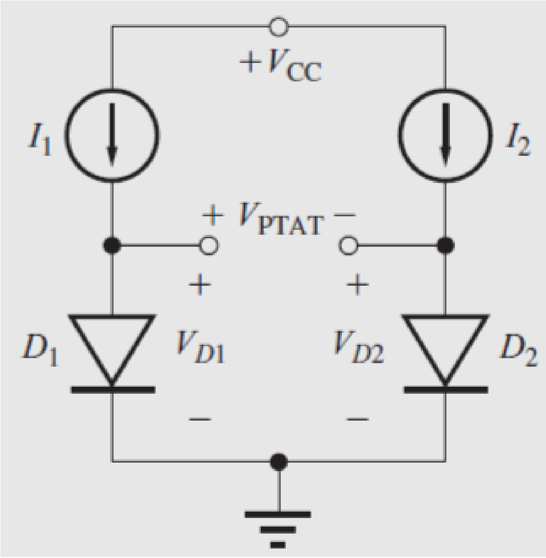
- **At the same I_D, when I_S increases by 10, V_a decreases by 60 mV.**
- **At the same I_S, when I_D increases by 10, V_a increases by 60 mV.**

Diode Temperature Dependence

A Voltage Proportional to Absolute Temperature

- For a fixed $I_D \gg I_S$:

$$I_D = I_S\left(e^{\frac{qV_a}{kT}} - 1\right) \Rightarrow V_a = \frac{kT}{q}\ln\left(\frac{I_D}{I_S} + 1\right) \cong \frac{kT}{q}\ln\frac{I_D}{I_S}$$



Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

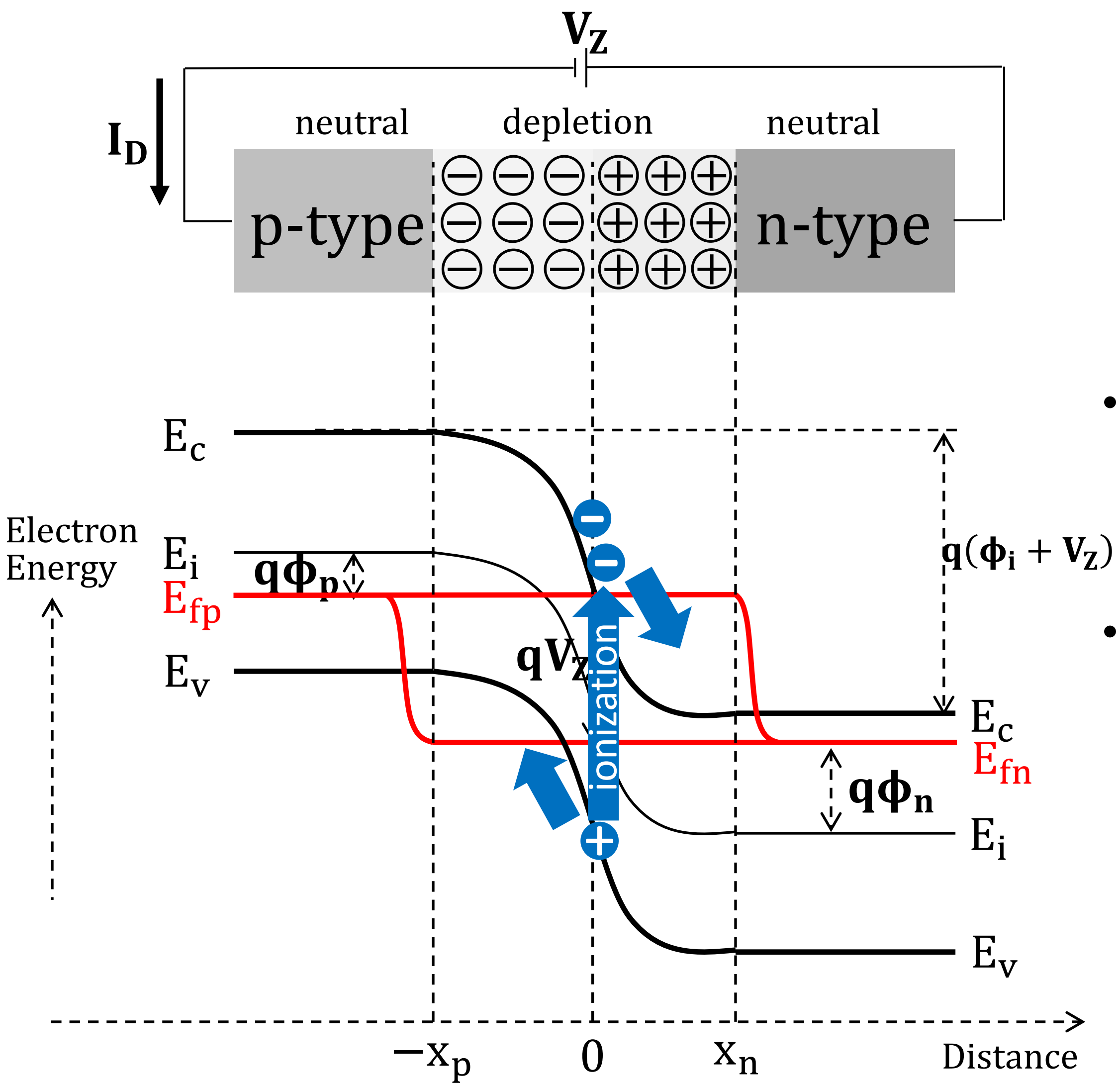
$$\begin{cases} V_{D1} = \dfrac{kT}{q}\ln\dfrac{I_1}{I_S} \\ V_{D2} = \dfrac{kT}{q}\ln\dfrac{I_2}{I_S} \end{cases}$$

I_1 and I_2 are ideal current source.

$$\boxed{V_{PTAT} = V_{D1} - V_{D2} = \frac{kT}{q}\ln\frac{I_1}{I_2} = T \times \text{constant}}$$

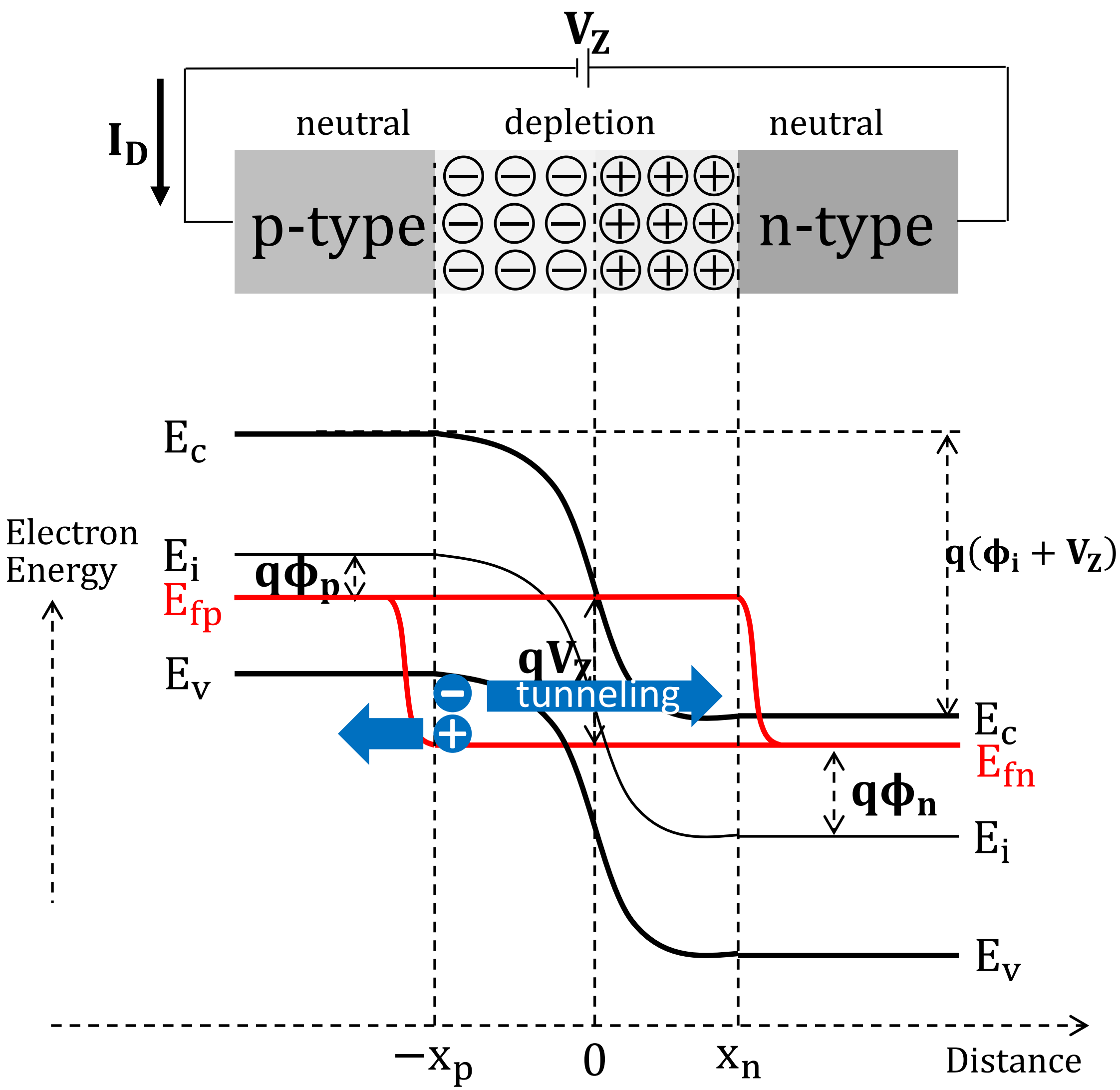
Diode in Reverse Bias

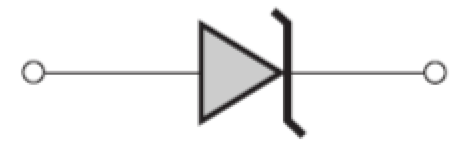
Avalanche Breakdown in Reverse Bias



- Si diode with breakdown voltages greater than about 5.6 V enter breakdown through an avalanche mechanism.
- Carriers accelerated by electric field gain sufficient energy to break covalent bonds upon impact, thereby creating electron-hole pairs.

Zener Breakdown in Reverse Bias





- **Si diode with very heavy doping (i.e. very narrow depletion region)** easily enter into Zener breakdown under reverse bias.
- Electrons tunnel directly between valence and conduction bands.

Diode Spice Model and Layout

Diode Spice Model

$$I_D = \mathbf{IS}\left[\exp\left(\frac{qV_a}{\mathbf{N}kT}\right) - 1\right]$$

$$\boxed{C_D = \mathbf{TT}\frac{I_D}{\mathbf{N}(kT/q)} \qquad C_j = \frac{\mathbf{CJO}}{(1 - \frac{V_a}{\mathbf{VJ}})^{\mathbf{M}}}\mathbf{RAREA}}$$

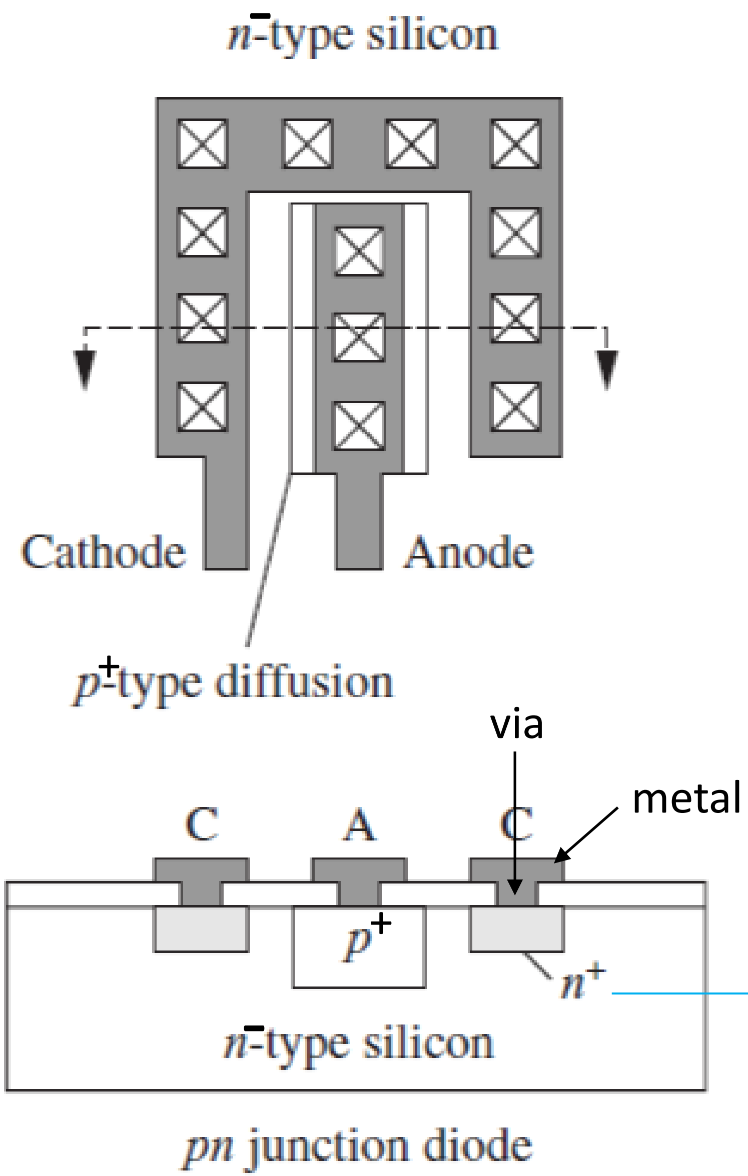
Not covered in Ve311

TABLE 3.1
SPICE Diode Parameter Equivalences

PARAMETER	SPICE	TYPICAL DEFAULT VALUES
Saturation current	IS	10 fA
Ohmic series resistance	RS	0 Ω
Ideality factor or emission coefficient	N	1
Transit time	TT	0 sec
Zero-bias junction capacitance for a unit area diode RAREA = 1	CJO	$0\ F/m^2$
Built-in potential	VJ	1 V
Junction grading coefficient	M	0.5
Relative junction area	RAREA	$1\ m^2$

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

Diode Layout

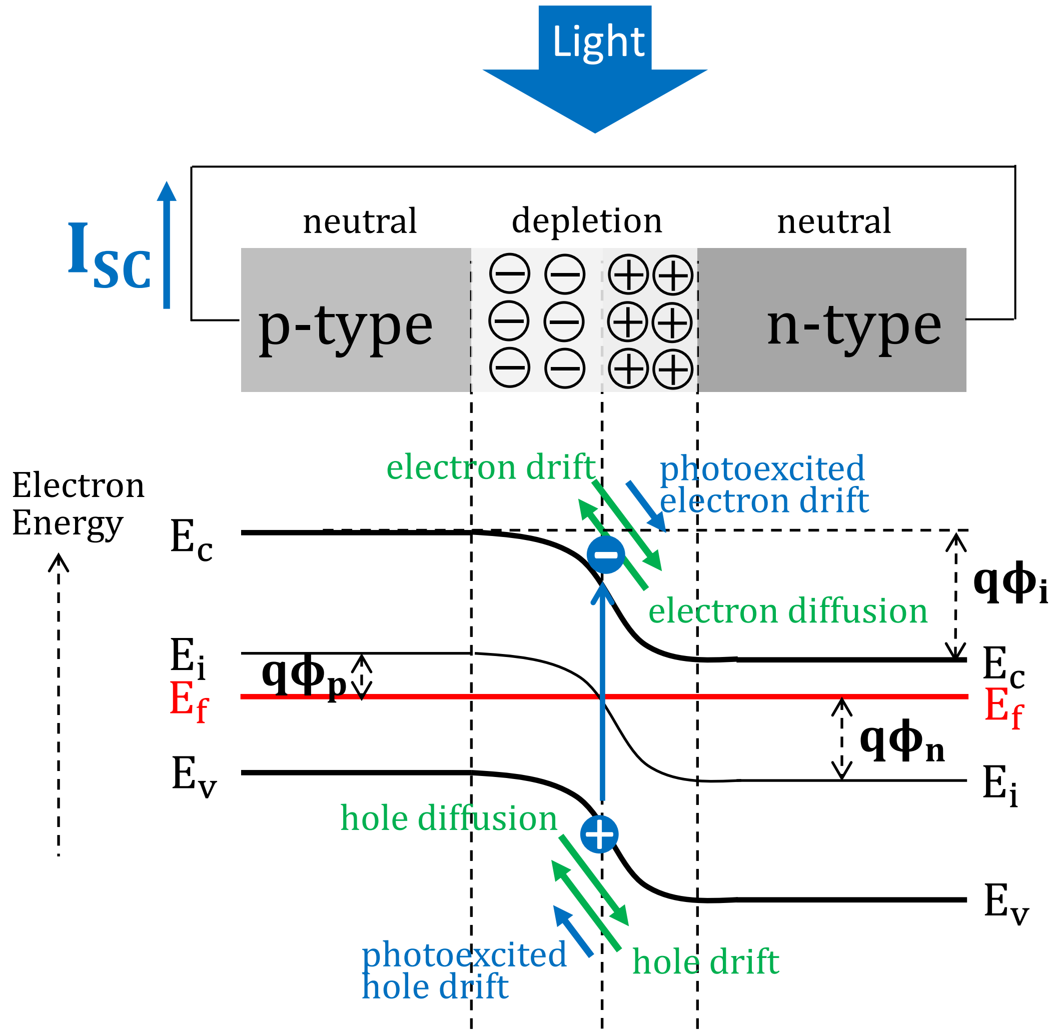


For forming a ohmic contact between metal and n^- type silicon

Source: Microelectronic Circuit Design, 4th Edition, by R. C. Jaeger and T. N. Blalock

Photodiode / Solar Cell

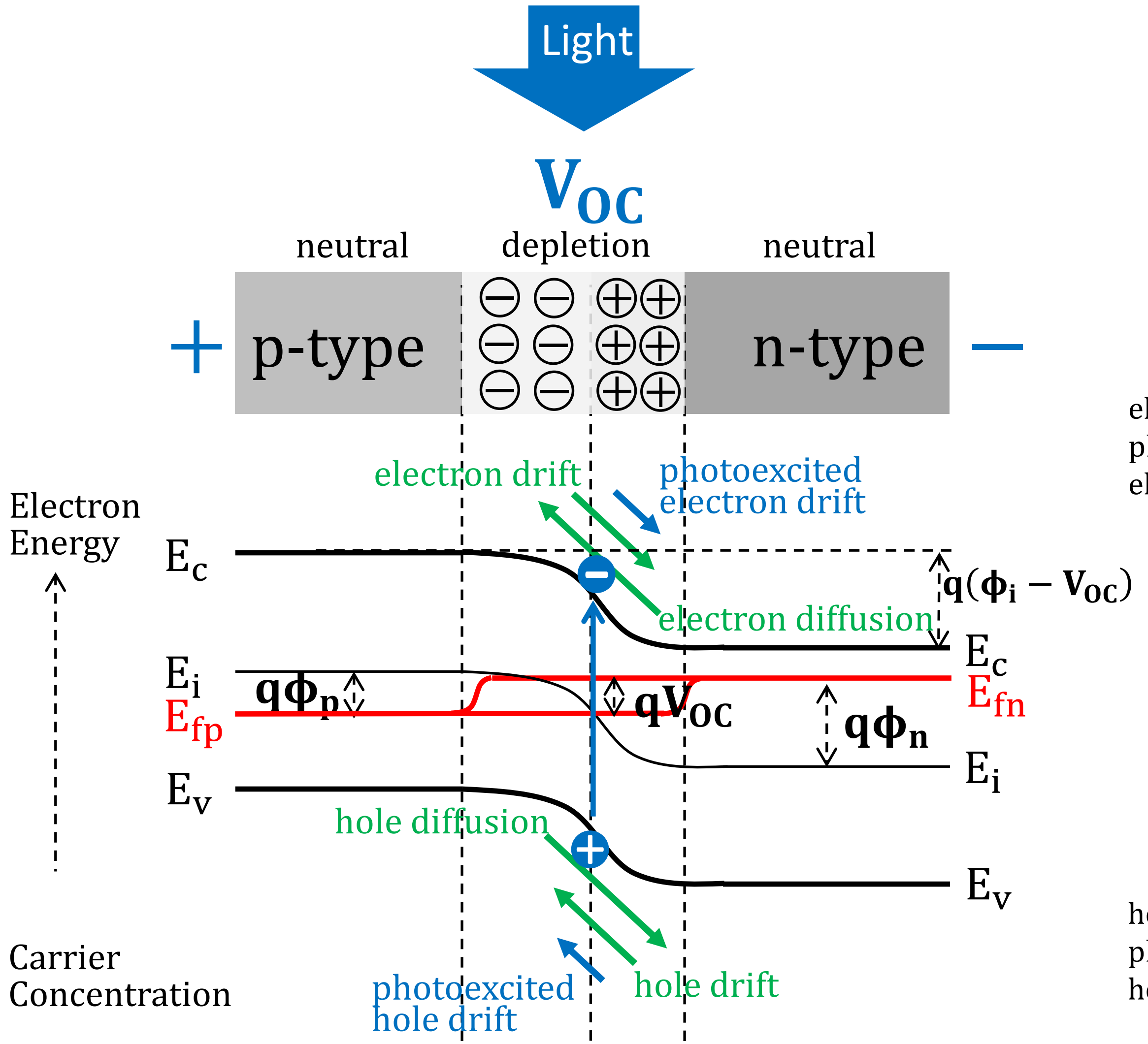
Short Circuit Current



The percentage of incident photons absorbed and converted into electron-hole pairs

$$I_{SC} = \frac{\textbf{Incident Light Power on Depletion Region (W)} \cdot \textbf{Absorption}}{\textbf{Photon Energy (J)}}$$

Open Circuit Voltage

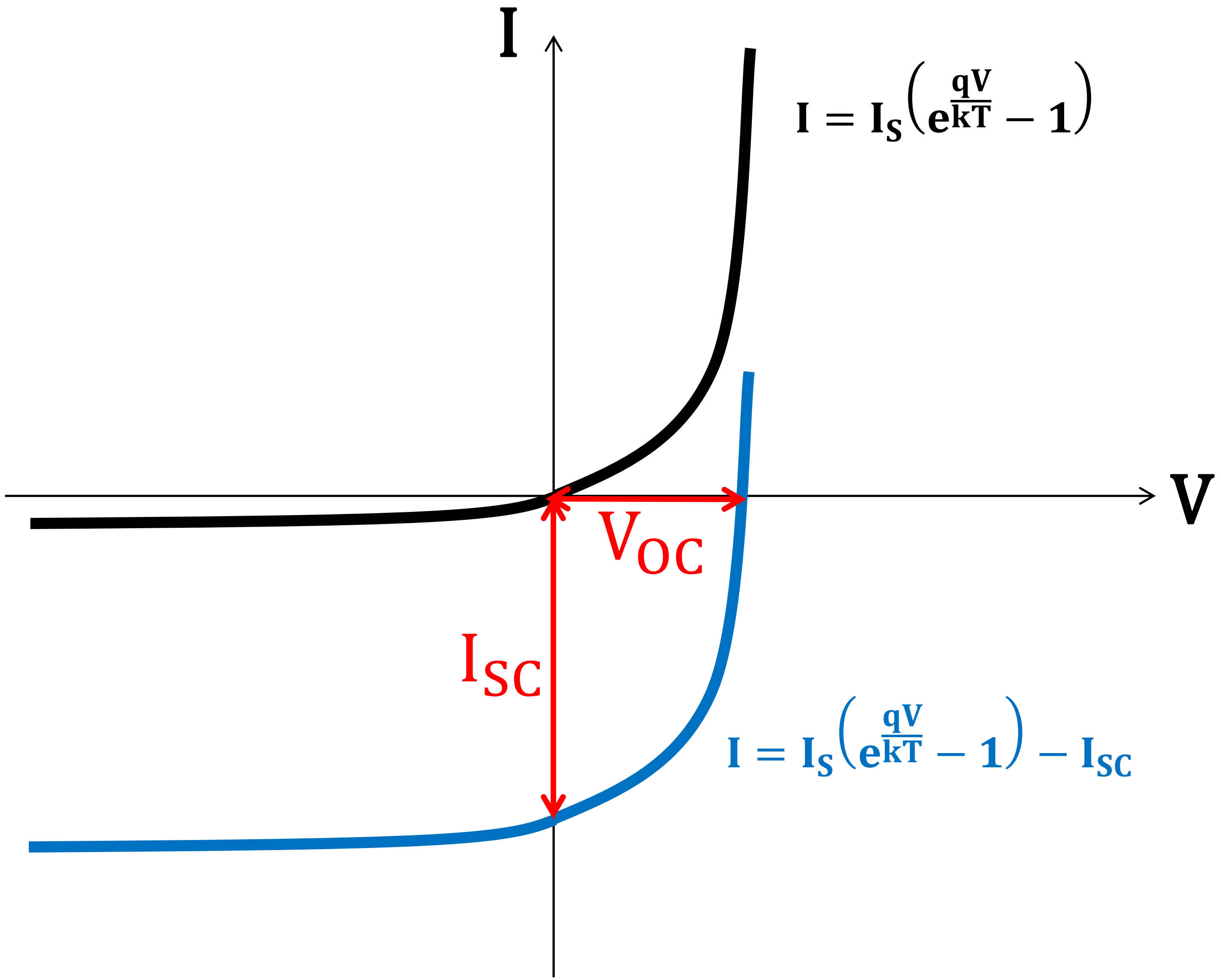


electron drift + photoexcited electron drift = electron diffusion

hole drift + photoexcited hole drift = hole diffusion

$$I_{SC} = I_S\left(e^{\frac{qV_{OC}}{kT}} - 1\right)$$

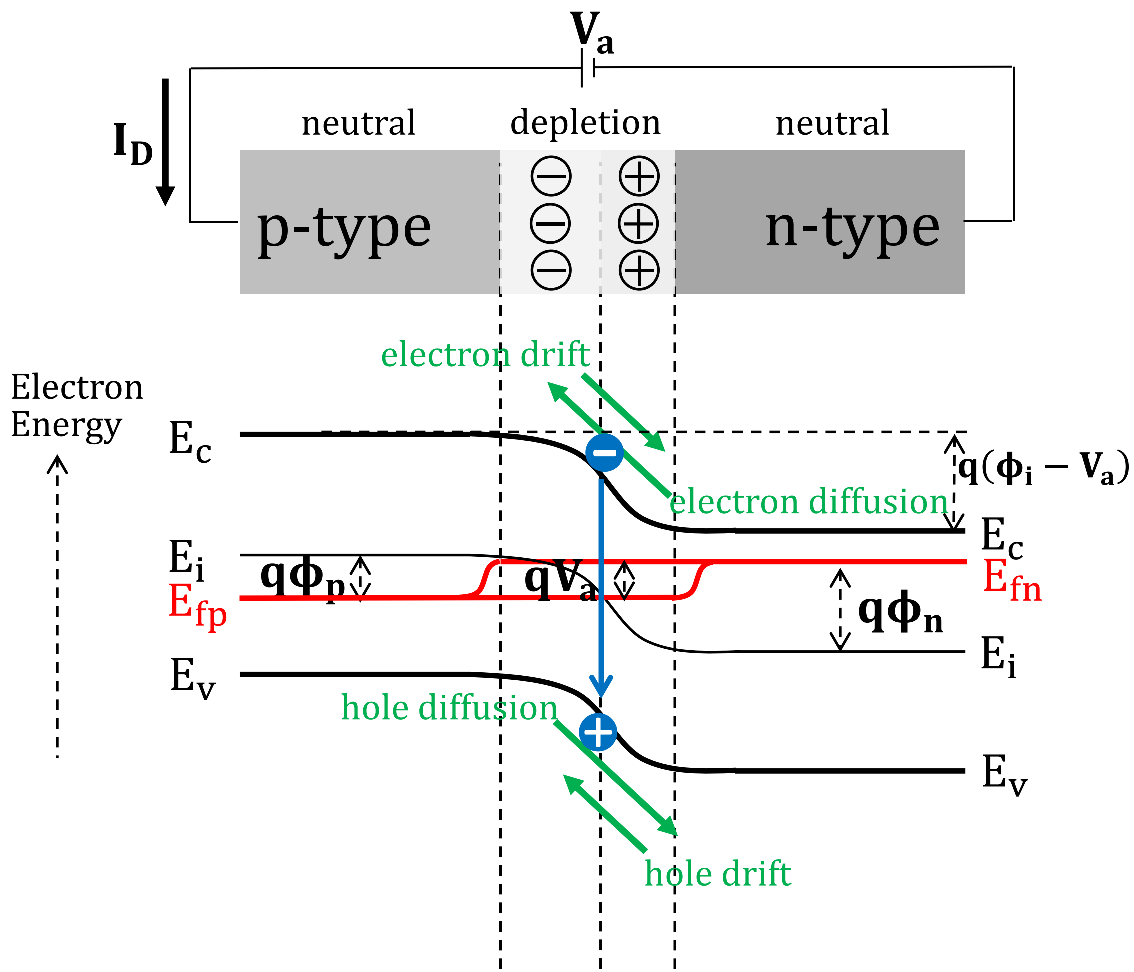
I-V Curve of Photodiode / Solar Cells



What are fill factor and power conversion efficiency?

Light Emitting Diode

Charge Injection



- **Direct bandgap** semiconductor is required. Note that silicon is indirect bandgap semiconductor.
- Under forward bias, charge carriers are **injected and recombined** in the depletion region to emit photons.