VE320 – Summer 2021

Introduction to Semiconductor Devices

Instructor: Yaping Dan （但亚平）
yaping.dan@sjtu.edu.cn

Chapter 3 Introduction to the Quantum Theory of Solids

Outline

- 3.1 Allowed and Forbidden Energy Bands
- 3.2 Electrical Conduction in Solids
- 3.3 Extension to Three Dimensions
- 3.4 Effective Mass
- 3.5 Density of States Function
- 3.6 Statistical Mechanics

Outline

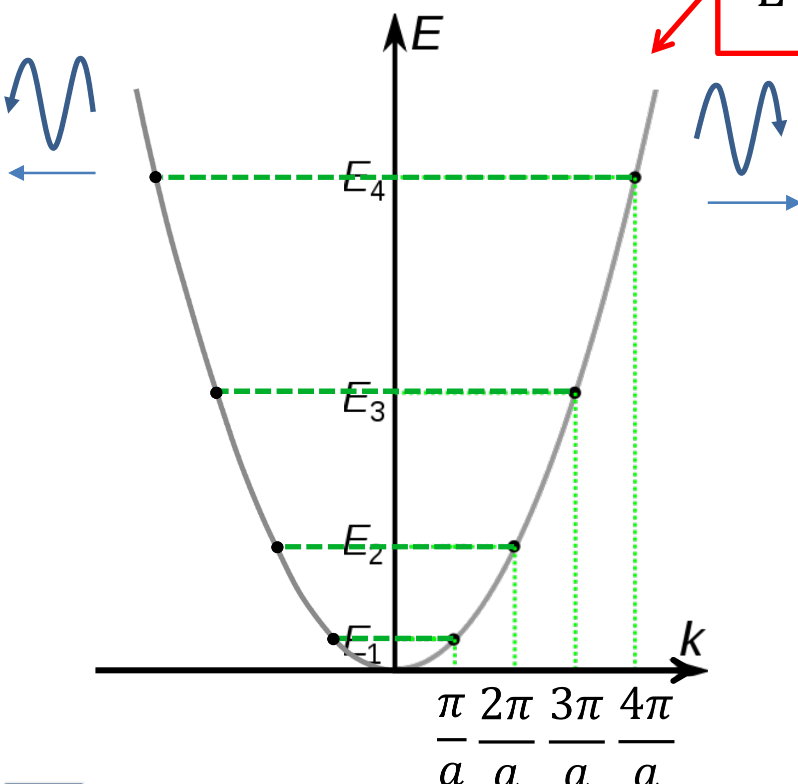
- **3.1 Allowed and Forbidden Energy Bands**
- 3.2 Electrical Conduction in Solids
- 3.3 Extension to Three Dimensions
- 3.4 Effective Mass
- 3.5 Density of States Function
- 3.6 Statistical Mechanics

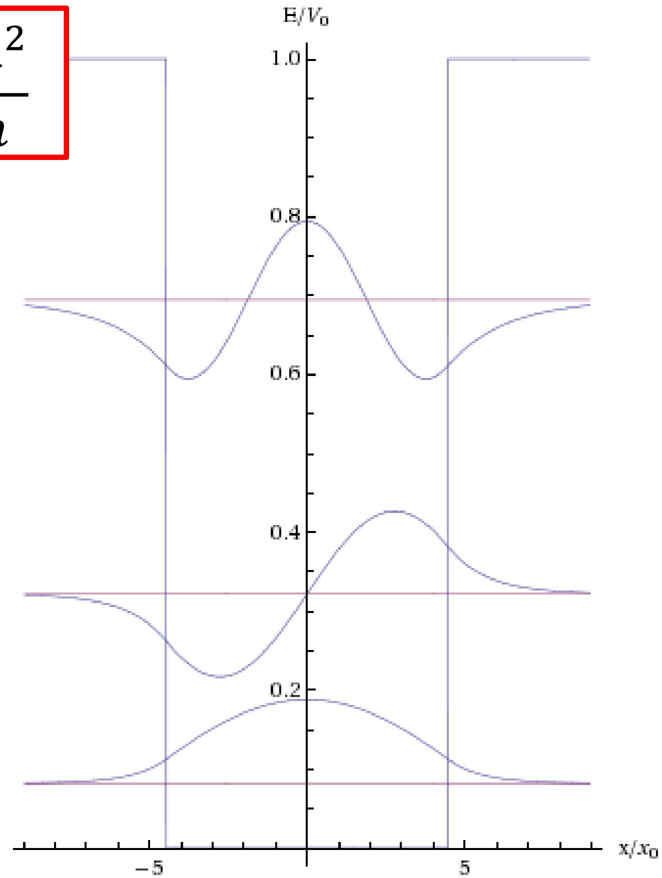
3.1 Allowed and Forbidden Energy Bands

<u>Forming energy bands</u>: analytical

Previously: Electrons in Finite Quantum Well

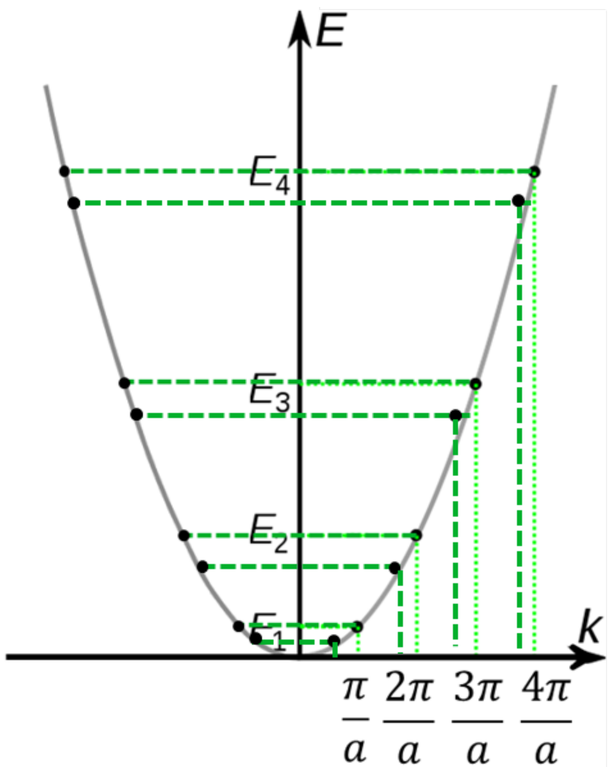
$$\boxed{E = \frac{k^2\hbar^2}{2m}}$$

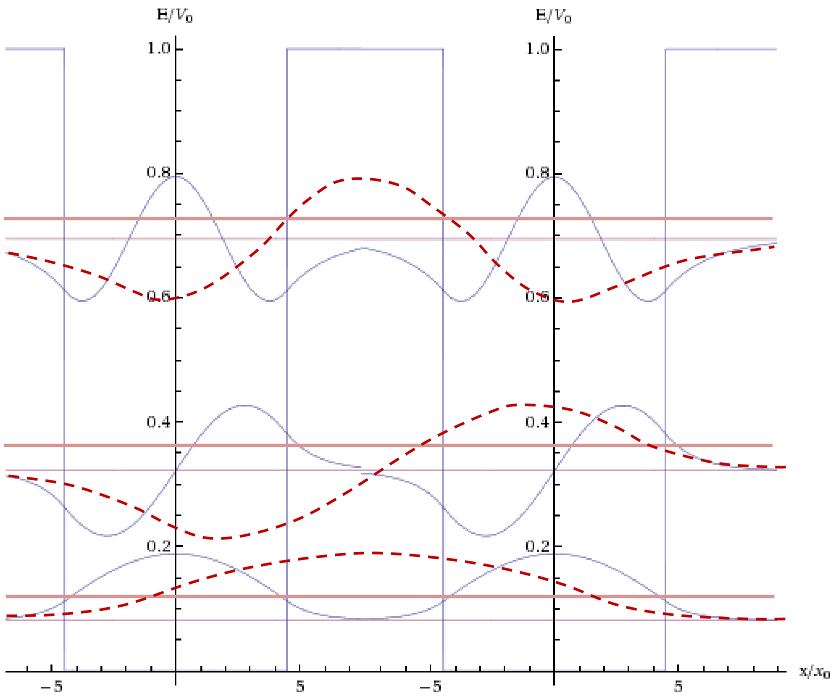




3.1 Allowed and Forbidden Energy Bands

Forming energy bands: analytical

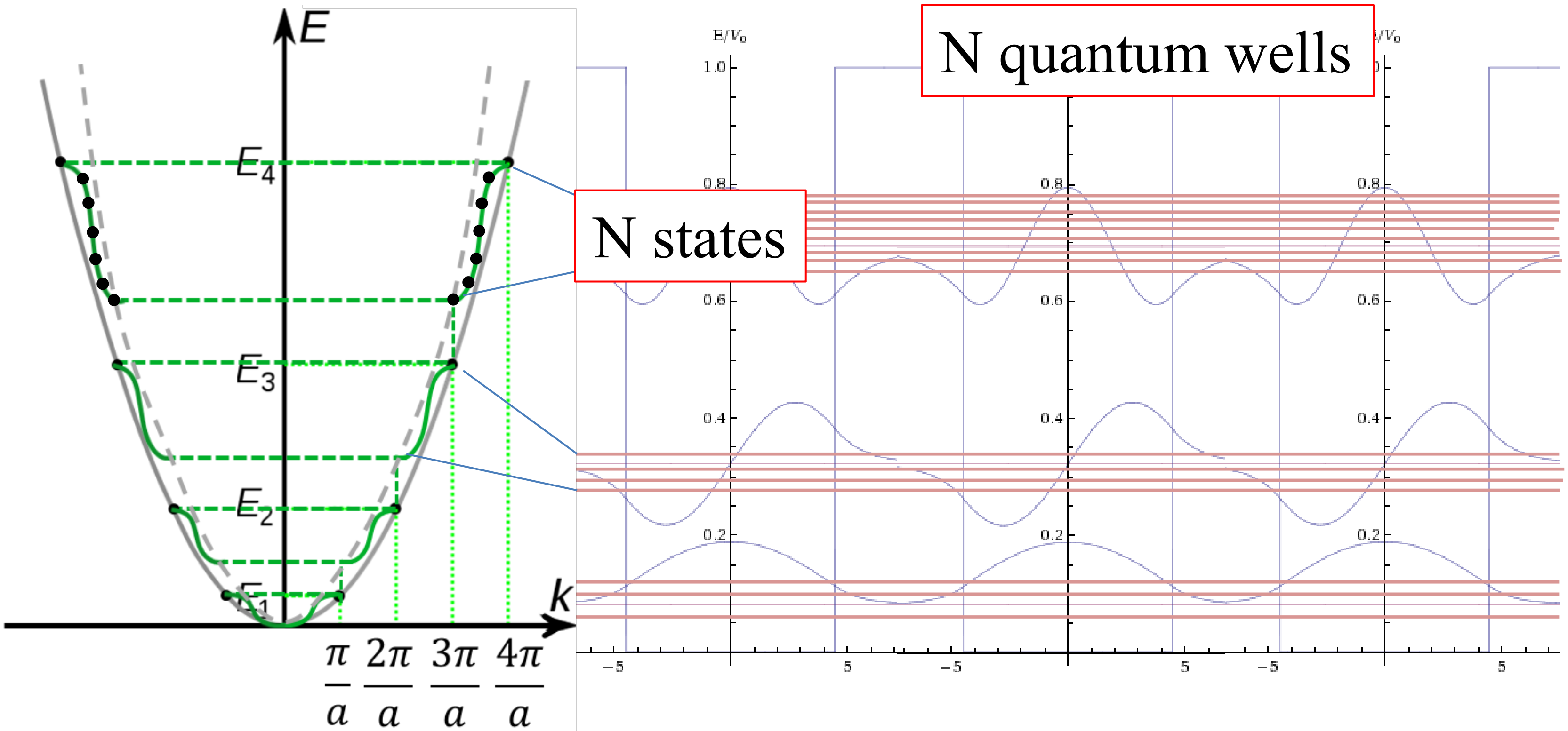




2 quantum wells

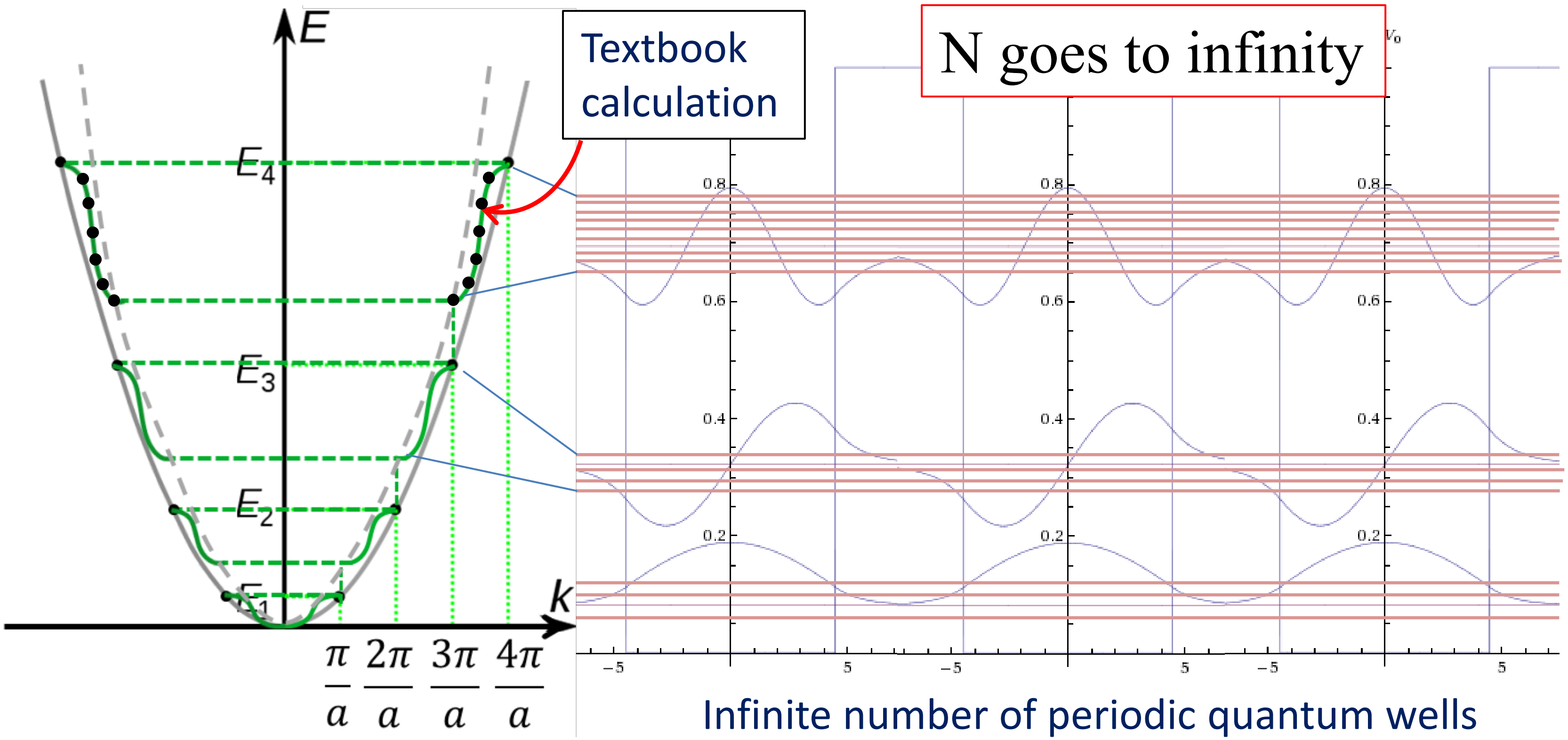
3.1 Allowed and Forbidden Energy Bands

Forming energy bands: analytical



3.1 Allowed and Forbidden Energy Bands

<u>Forming energy bands</u>: analytical

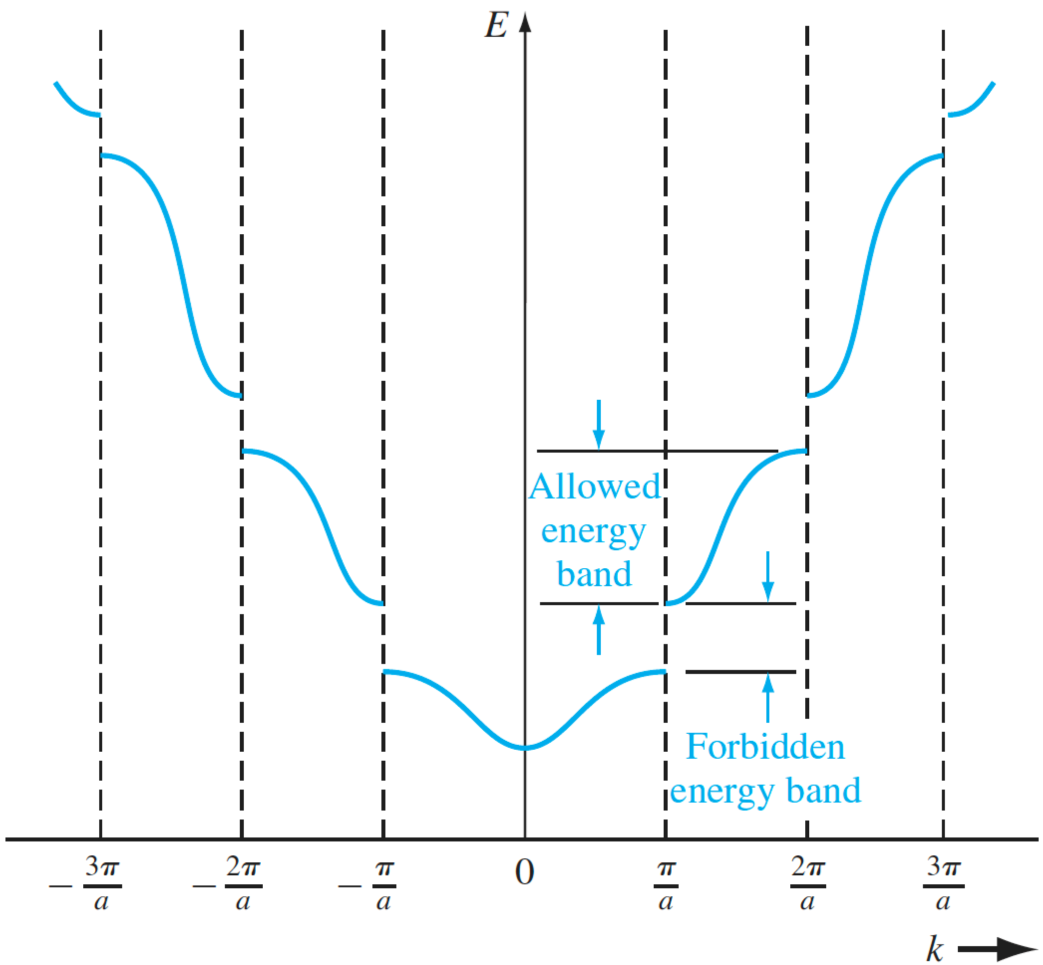


$$\frac{mV_0ba}{\hbar^2}\frac{sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos(ka)$$

On P.67
eq.(3.22)

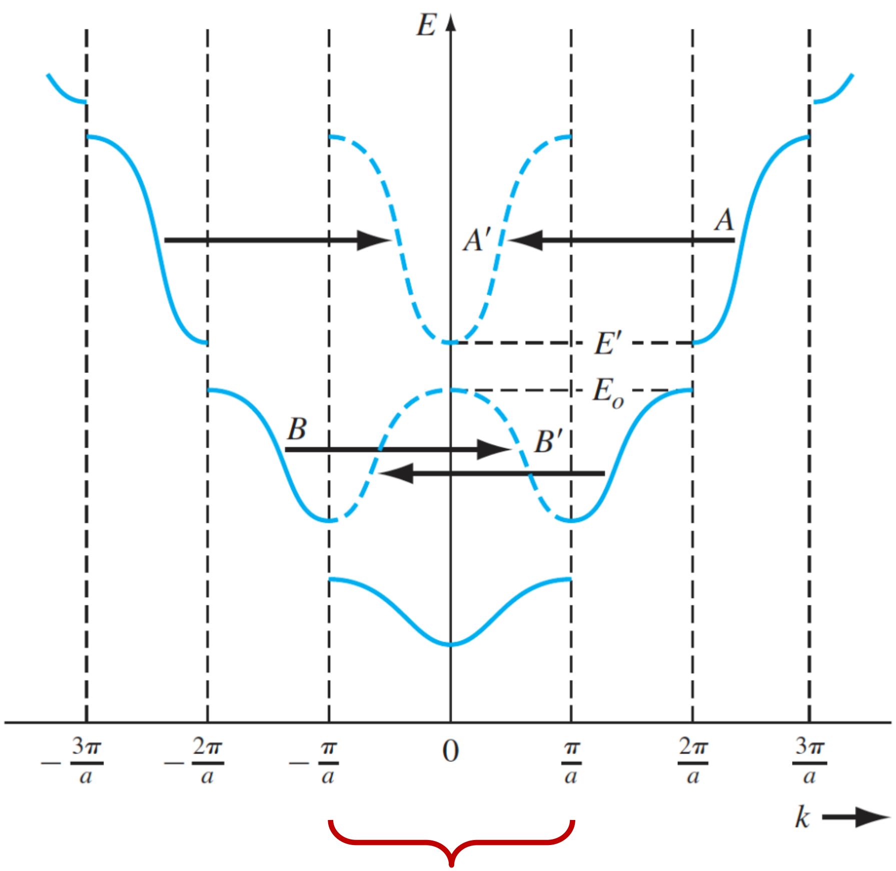
3.1 Allowed and Forbidden Energy Bands

<u>Forming energy bands</u>: analytical

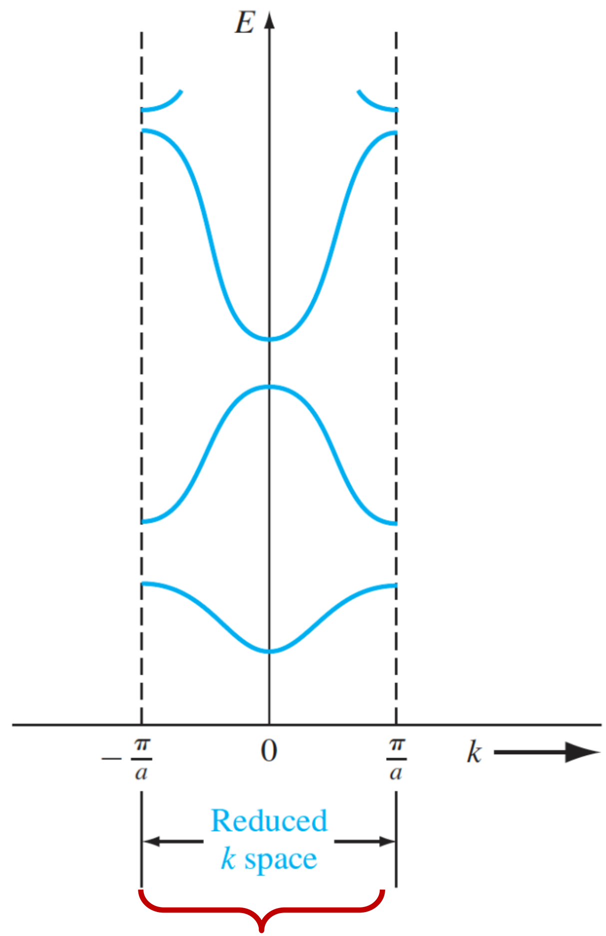


3.1 Allowed and Forbidden Energy Bands

Band structure in physical and k space for 1D periodic quantum wells



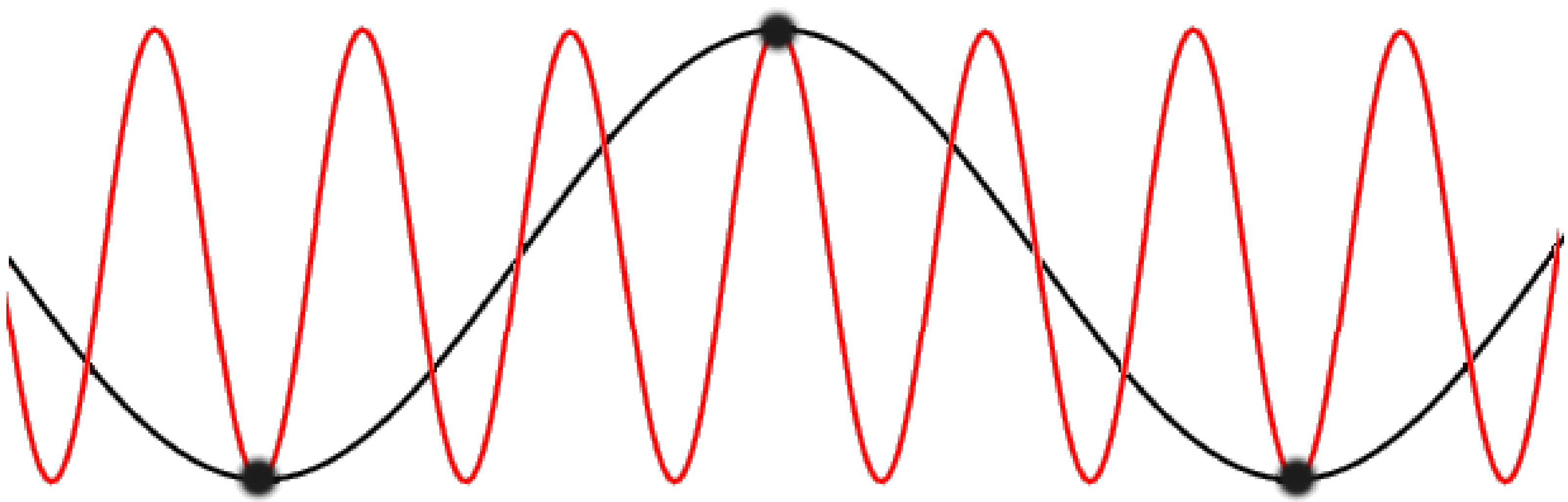
1st Brillouin zone



1st Brillouin zone

3.1 Allowed and Forbidden Energy Bands

- Black wave with a smaller k (longer wavelength) is in the 1st Brillouis zone.
- Red wave with a larger k (short wavelength) is outside of 1st Brillouis zene.
- Both waves have the same frequency (same energy).
- Both waves can describe the exact same information of a particle.



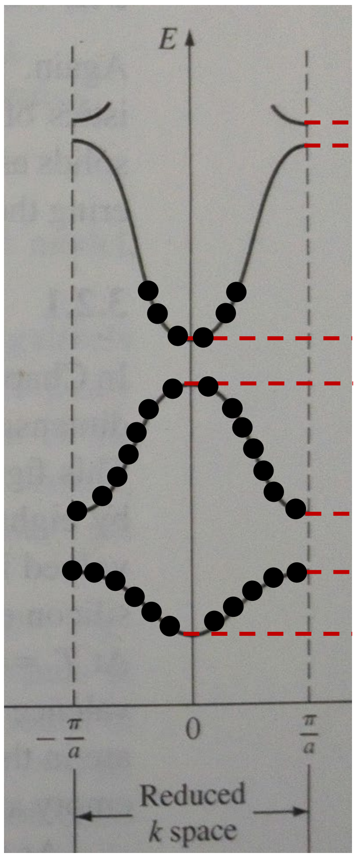
http://en.wikipedia.org/wiki/Phonon#/media/File:Phonon_k_3k.gif

Outline

- 3.1 Allowed and Forbidden Energy Bands
- **3.2 Electrical Conduction in Solids**
- 3.3 Extension to Three Dimensions
- 3.4 Effective Mass
- 3.5 Density of States Function
- 3.6 Statistical Mechanics

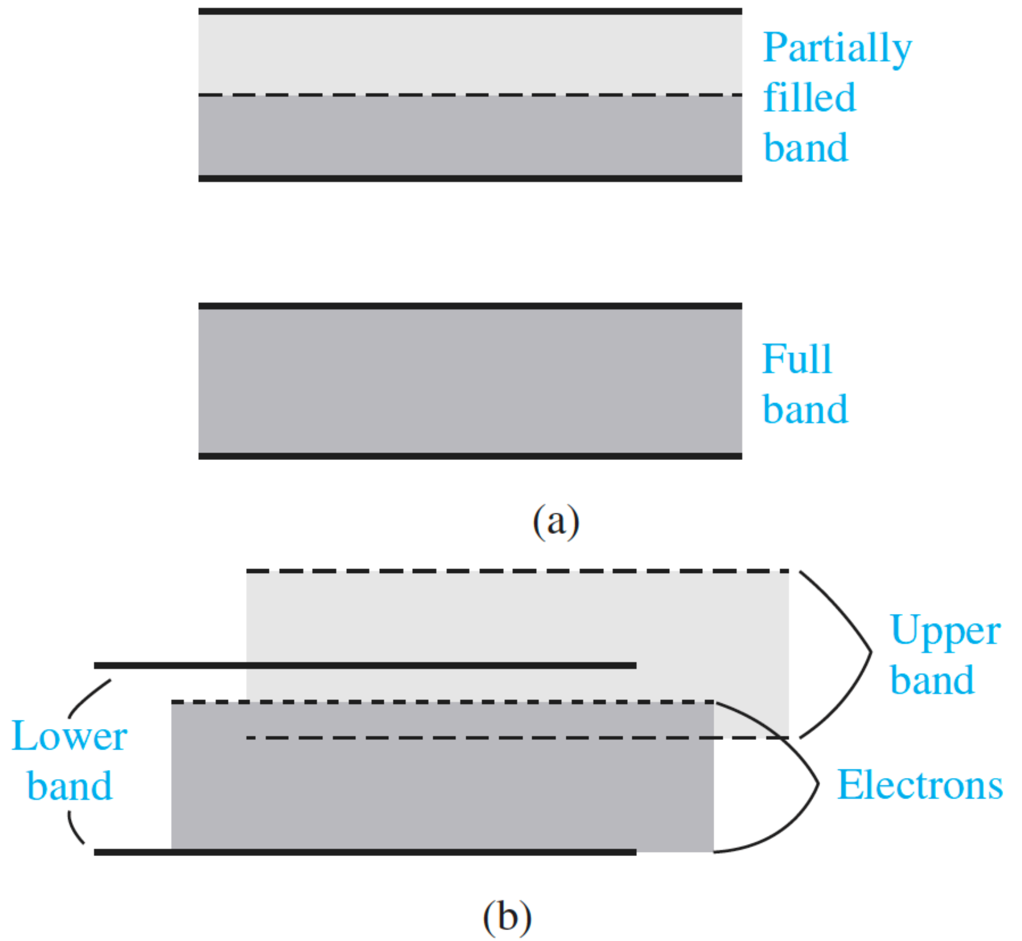
3.2 Electrical Conduction in Solids

Metals, semiconductors and insulators



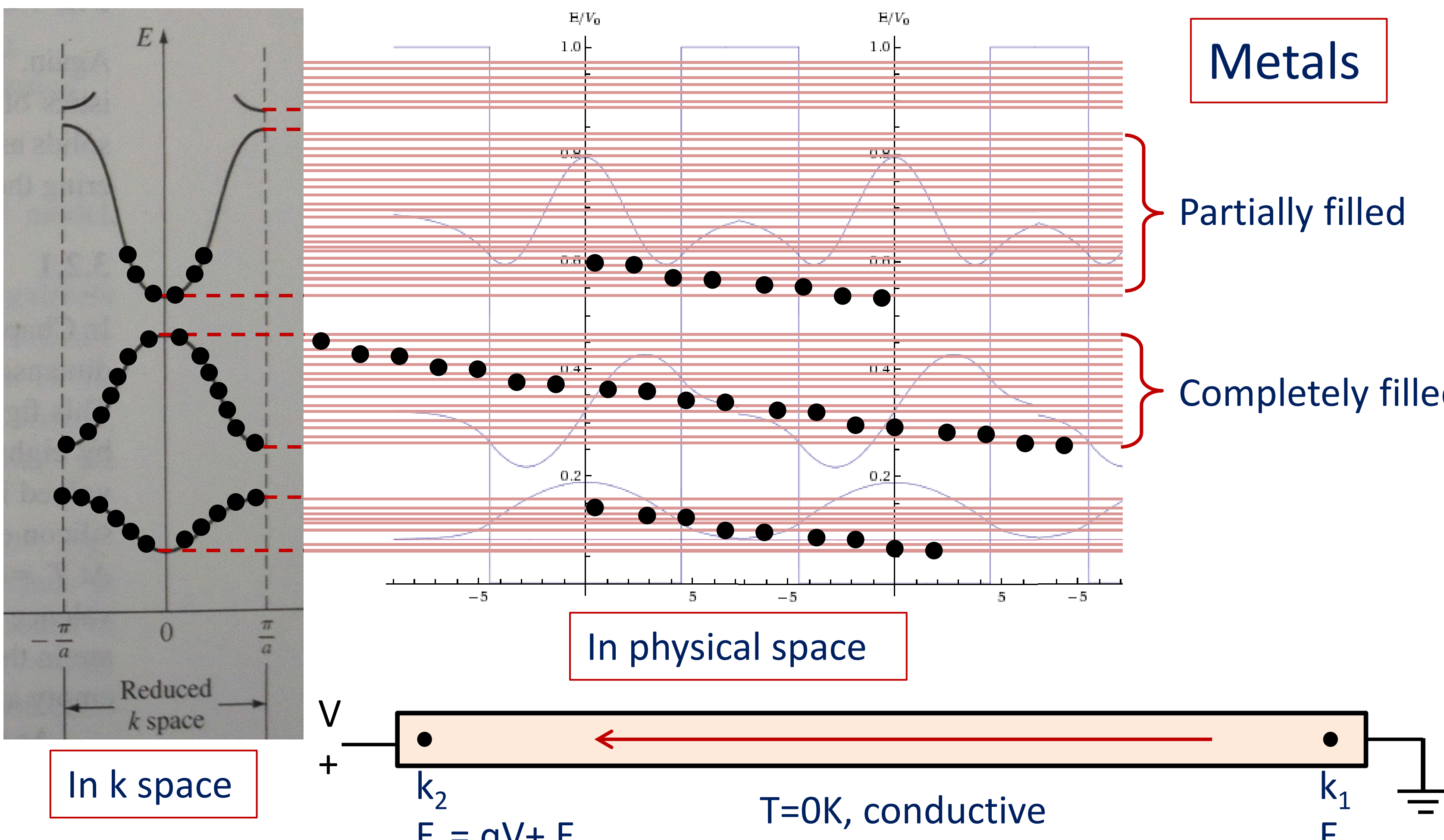
In k space

Forming energy bands is complicated.



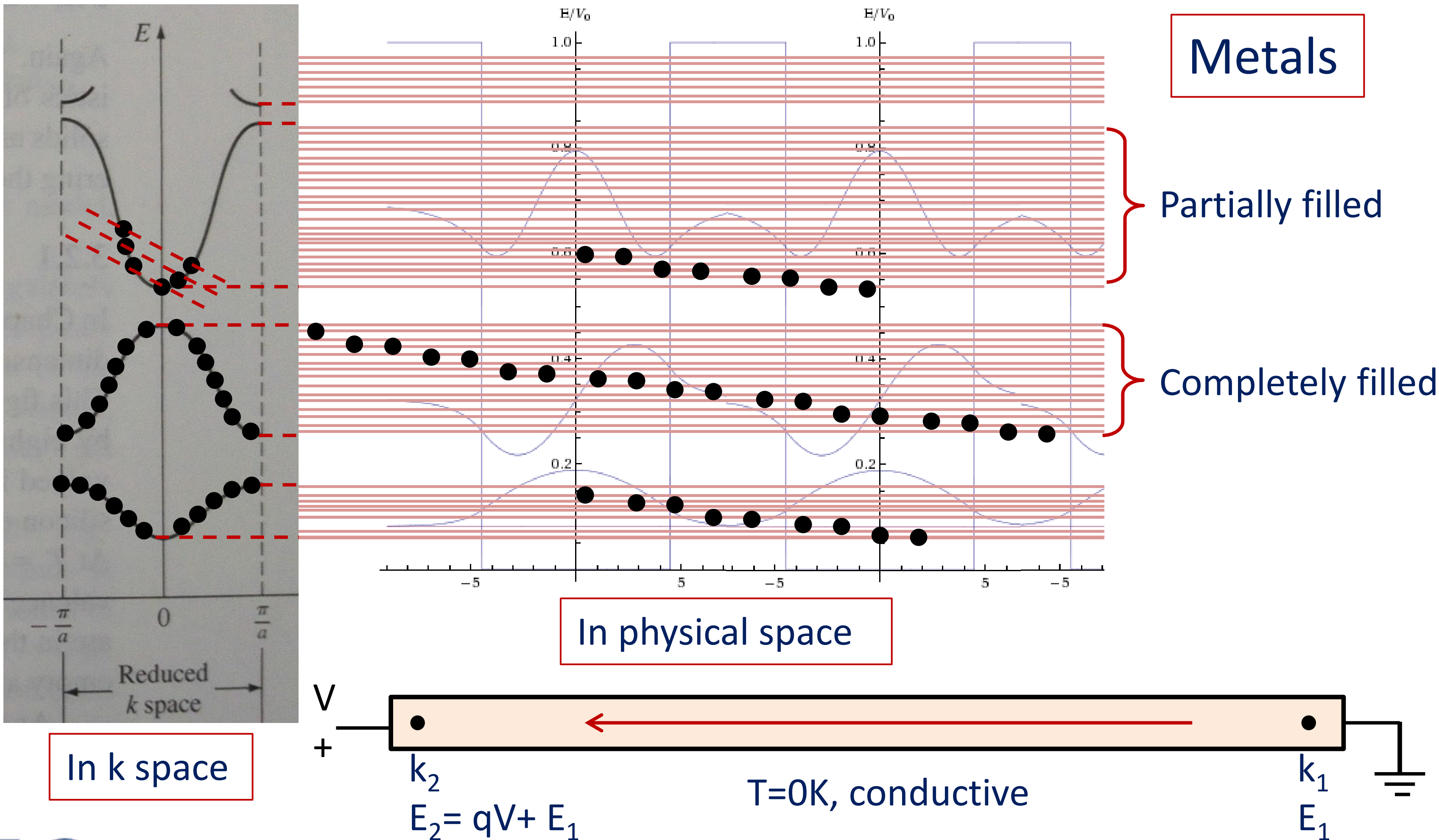
3.2 Electrical Conduction in Solids

Metals, semiconductors and insulators



3.2 Electrical Conduction in Solids

Metals, semiconductors and insulators

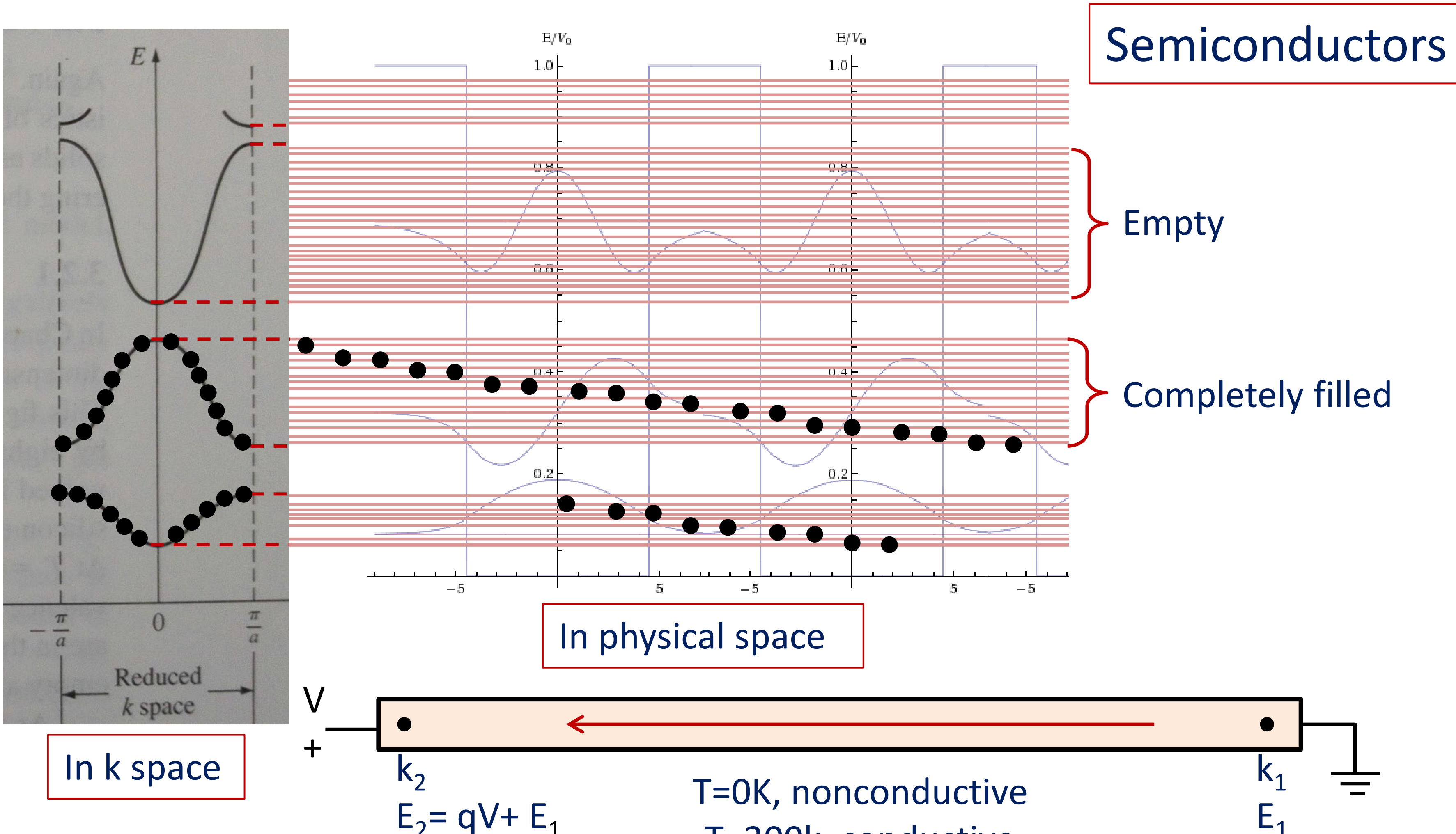


In k space

In physical space

3.2 Electrical Conduction in Solids

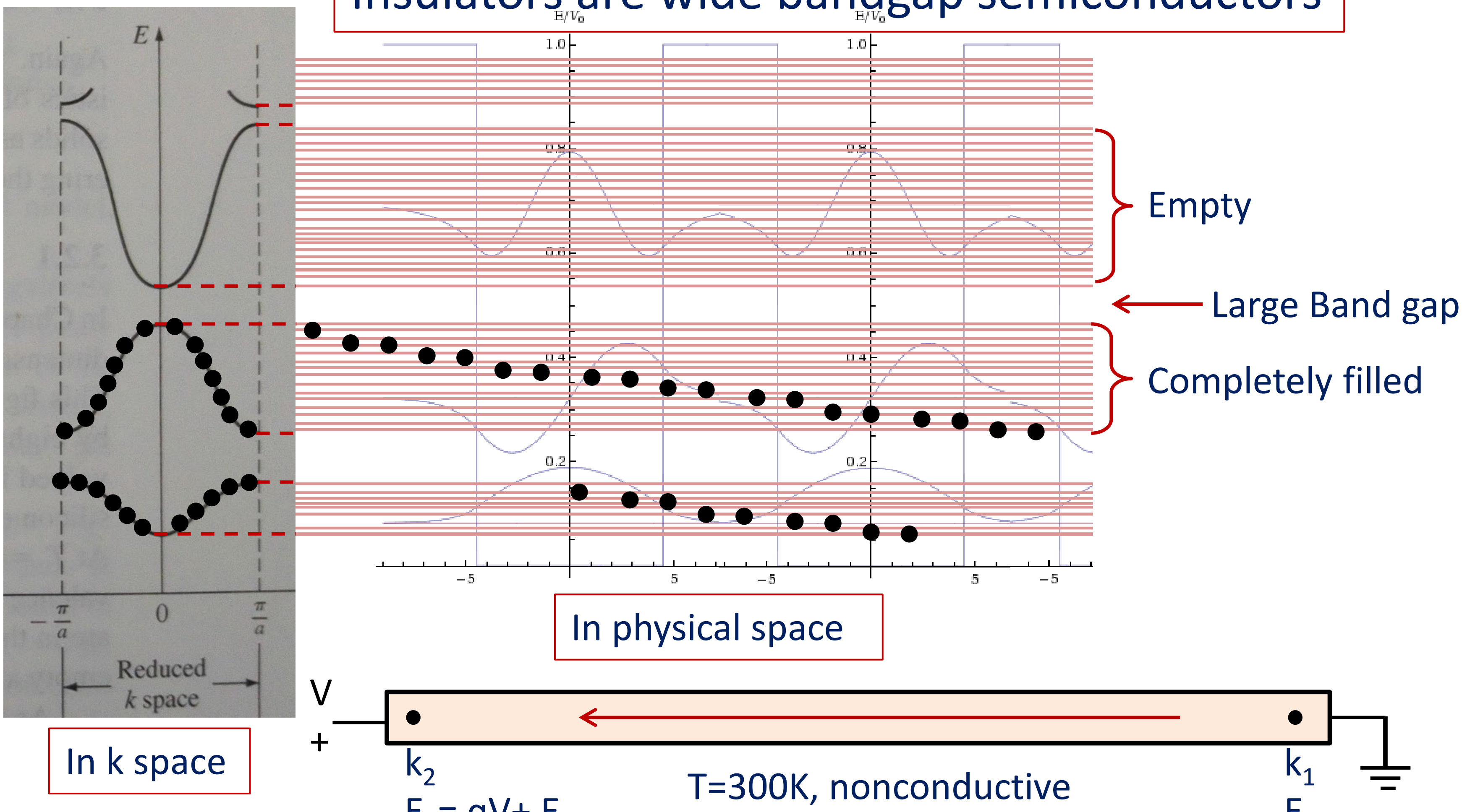
Metals, semiconductors and insulators



3.2 Electrical Conduction in Solids

Metals, semiconductors and insulators

Insulators are wide bandgap semiconductors



3.2 Electrical Conduction in Solids

Doping in semiconductors

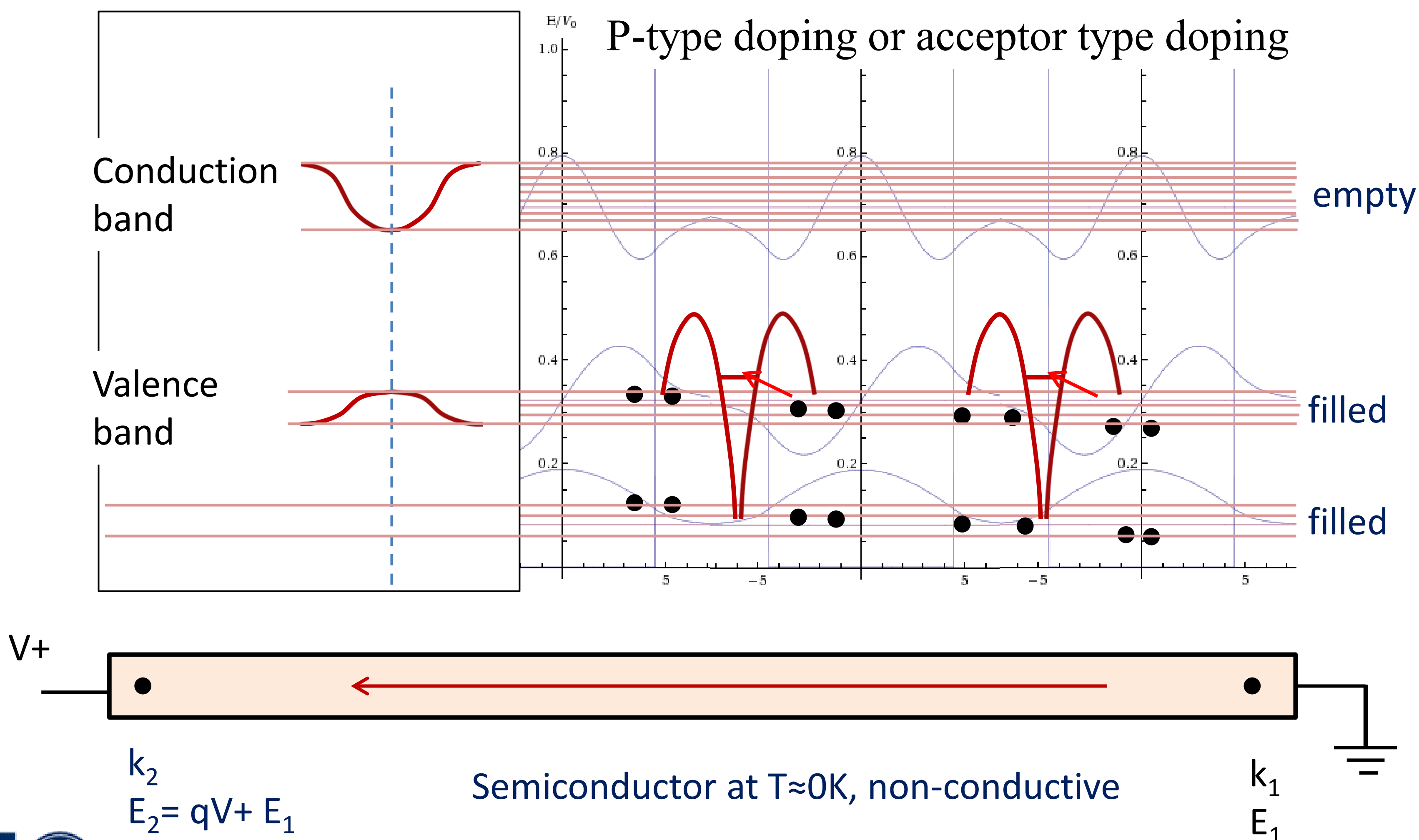
Intrinsic semiconductors:
pure semiconductor, no doping, no defects

n-type semiconductors :
Charge carriers are negative, i.e. electrons
Doped by donor-type of dopants (impurities)

p-type semiconductors:
Charge carriers are positive, i.e. holes
Doped by acceptor-type of dopants (impurities)

3.2 Electrical Conduction in Solids

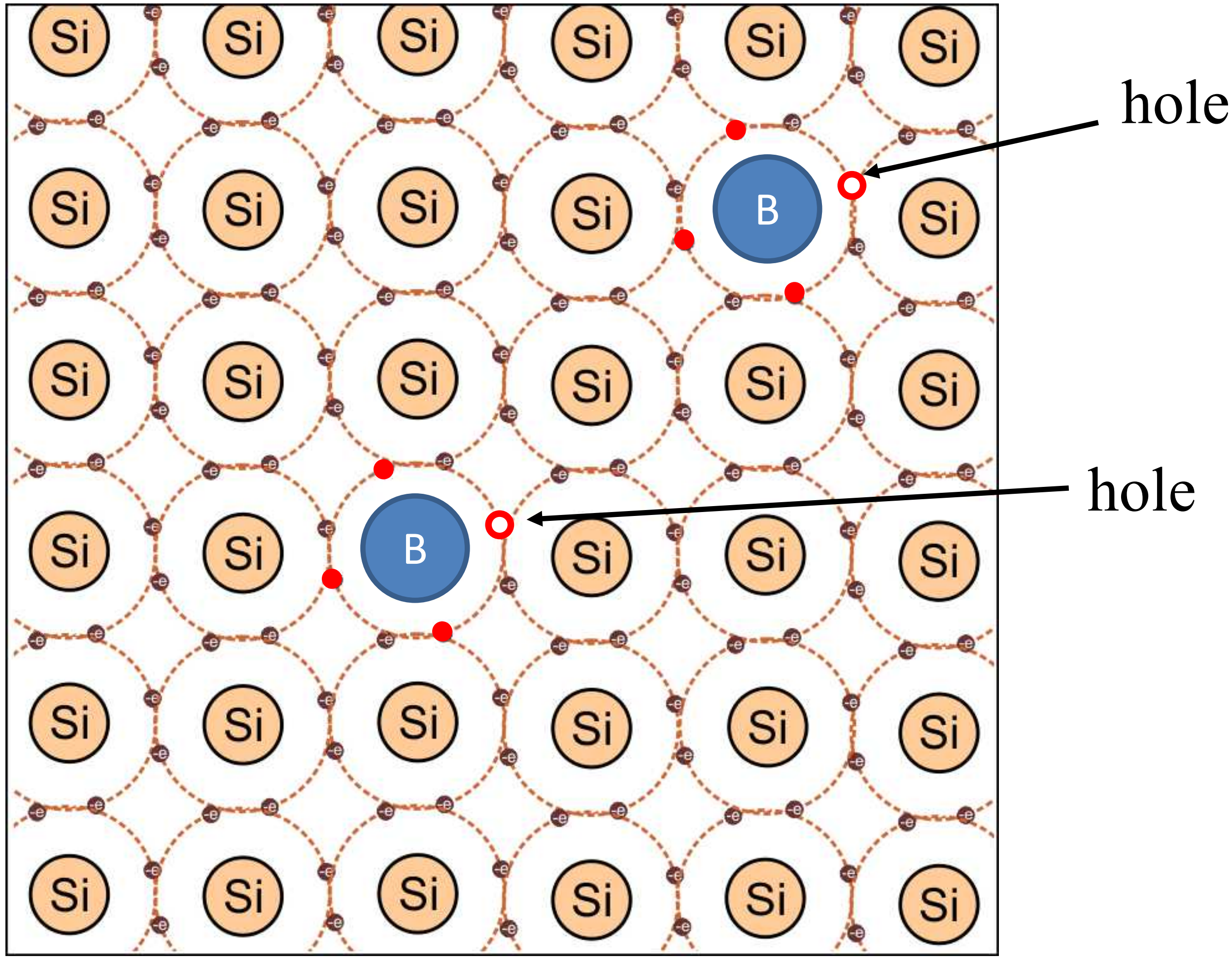
Doping in semiconductors



3.2 Electrical Conduction in Solids

p-type doping

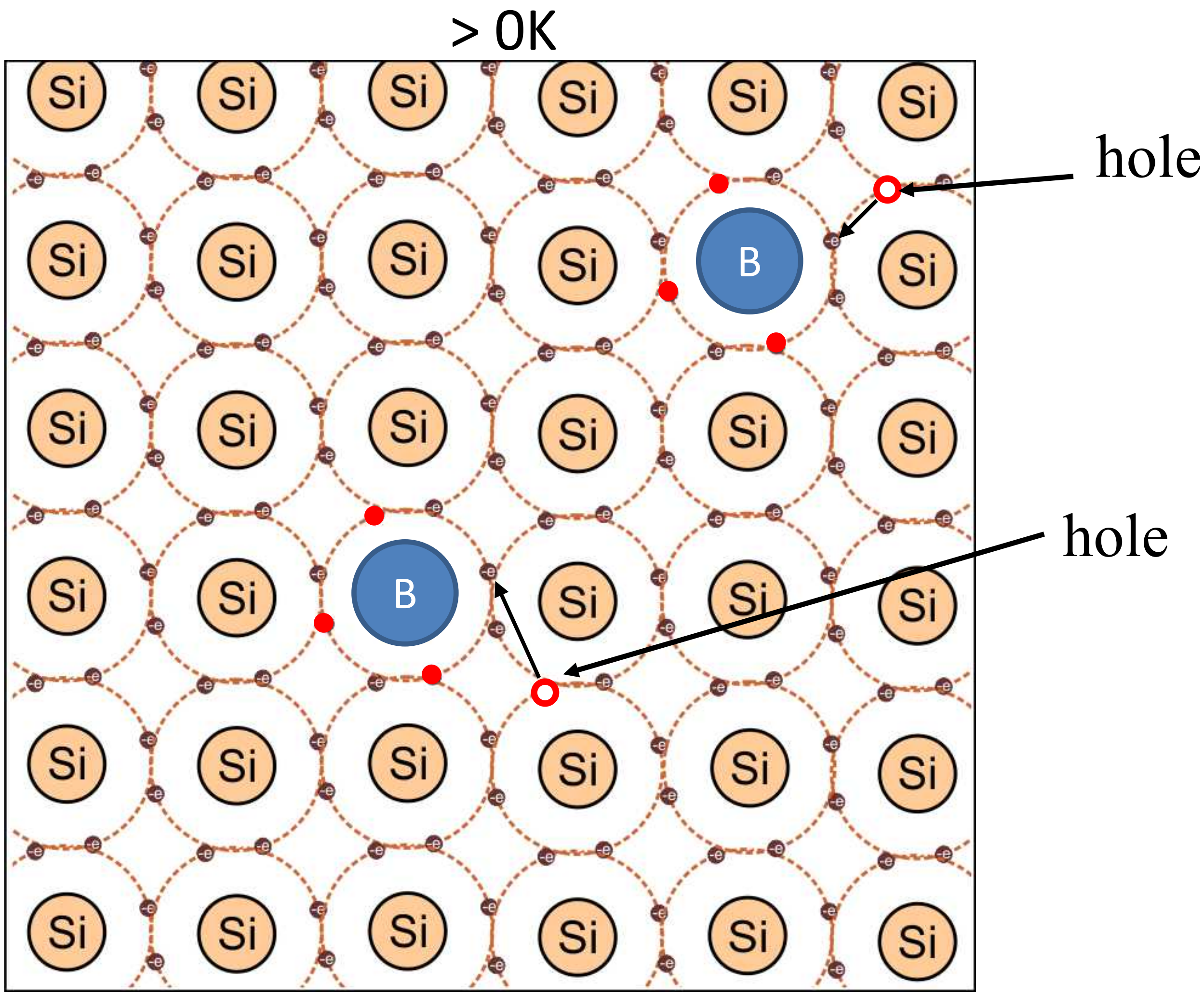
At 0K



Acceptor-type of dopants

3.2 Electrical Conduction in Solids

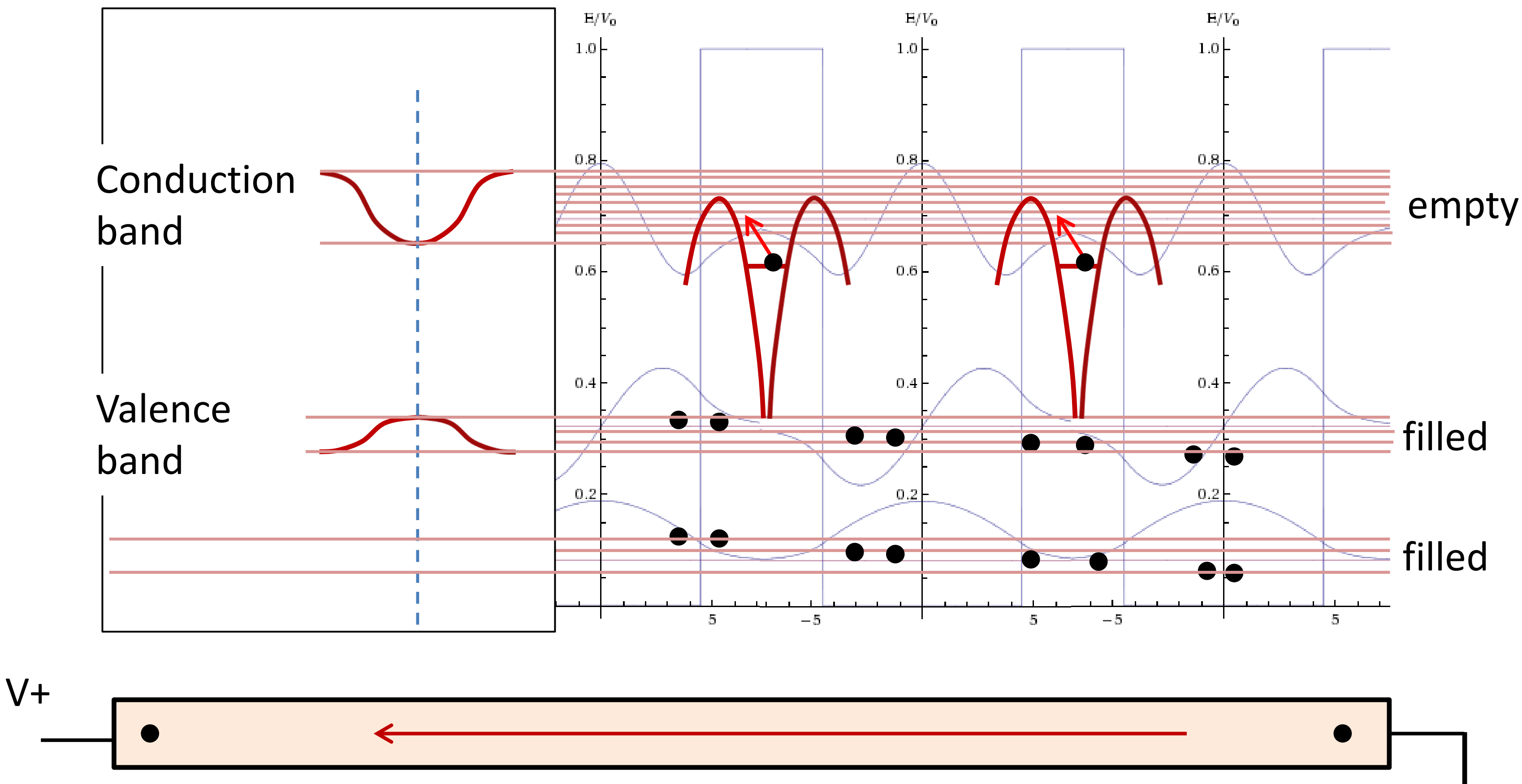
p-type doping



Acceptor-type of dopants

3.2 Electrical Conduction in Solids

n-type doping or donor-type doping



k_2
E_2= qV+ E_1

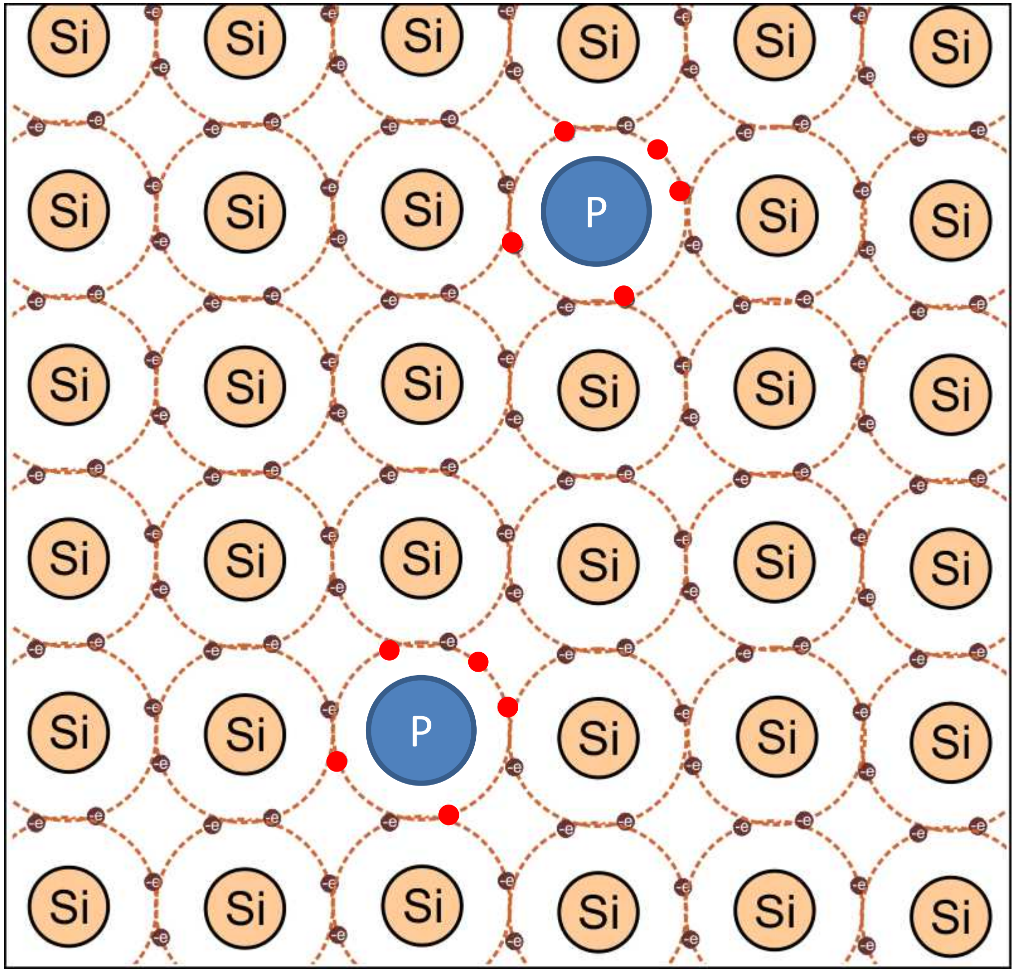
Semiconductor at T≈0K, non-conductive

k_1
E_1

3.2 Electrical Conduction in Solids

n-type doping

At 0K

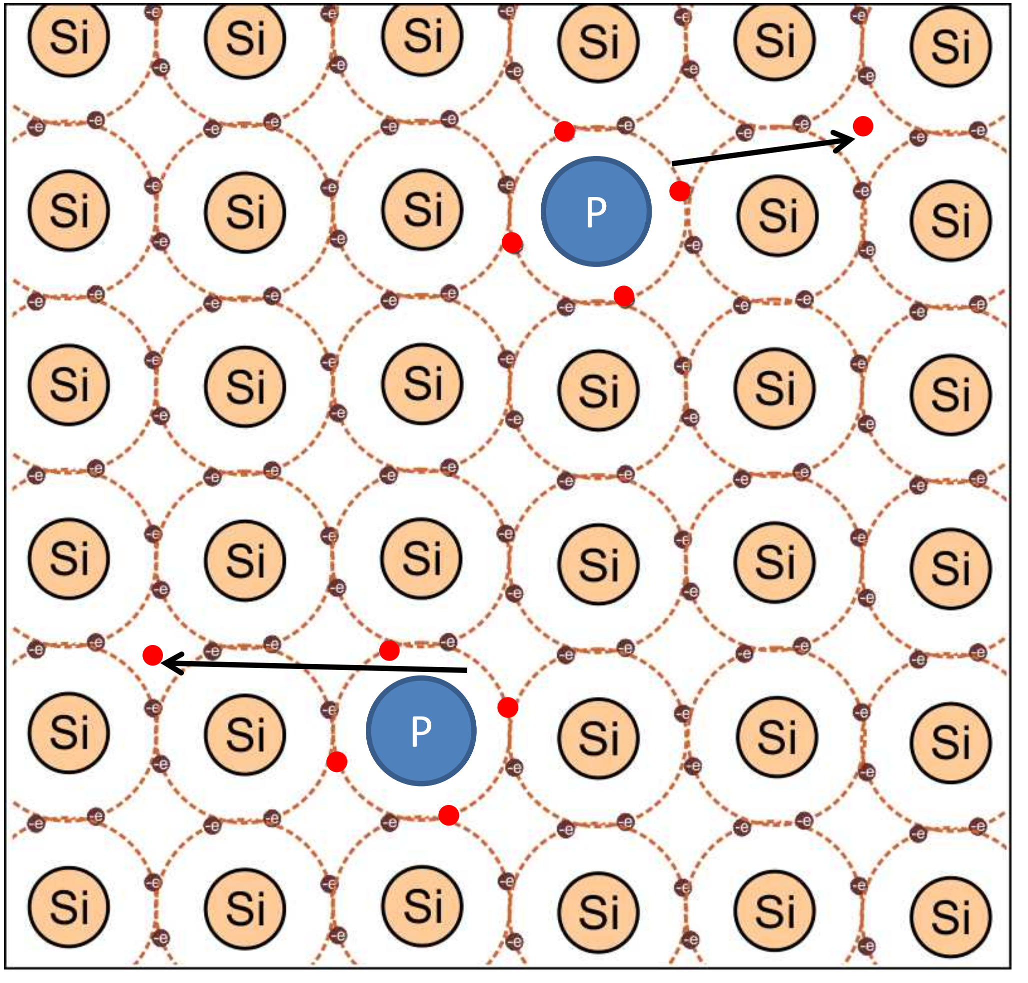


Donor-type of dopants

3.2 Electrical Conduction in Solids

n-type doping

> 0K



Donor-type of dopants

3.2 Electrical Conduction in Solids

Doping in semiconductors

Si atomic concentration: 5×10^{22} cm^{-3}

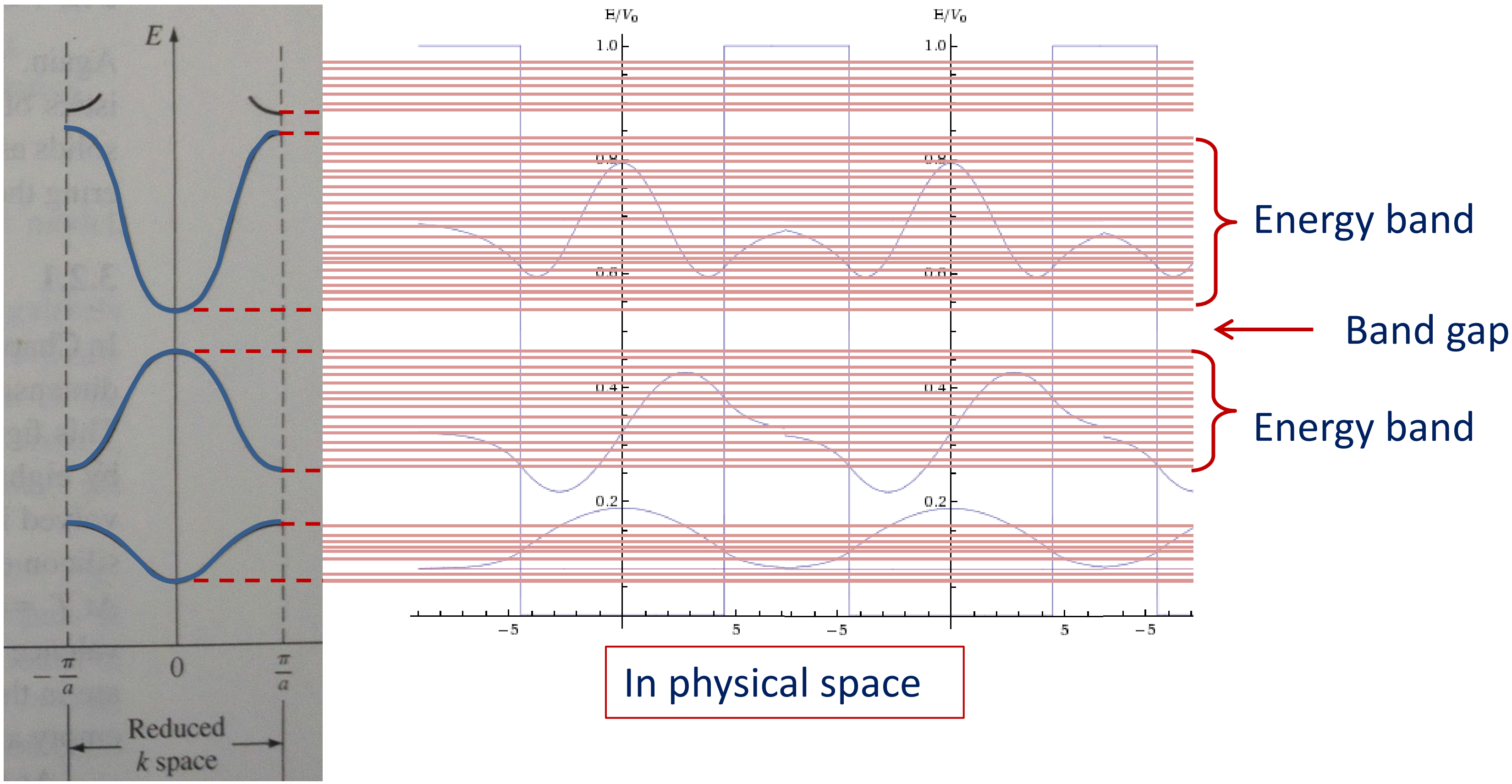
	Low concentration of doping	Medium concentration doping	High concentration of doping
Concentration (cm^{-3})	$< 10^{16}$	10^{16}-10^{18}	10^{18} - 10^{20}
Relative concentration	1ppm	1 -100 ppm	100 ppm – 1%

Outline

- 3.1 Allowed and Forbidden Energy Bands
- 3.2 Electrical Conduction in Solids
- **3.3 Extension to Three Dimensions**
- 3.4 Effective Mass
- 3.5 Density of States Function
- 3.6 Statistical Mechanics

Previously…

Band structure in physical and k space for 1D periodic quantum wells

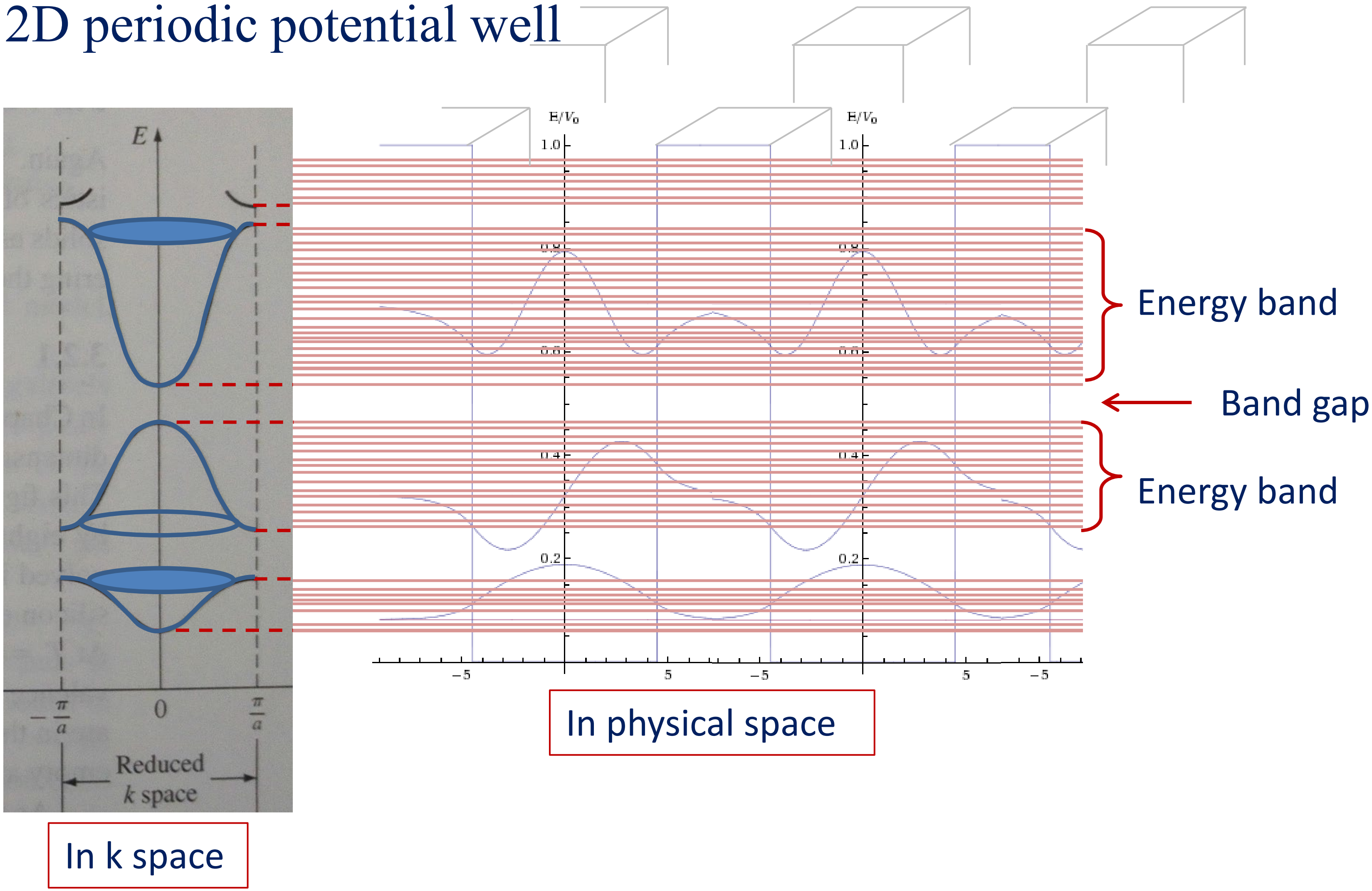


In k space

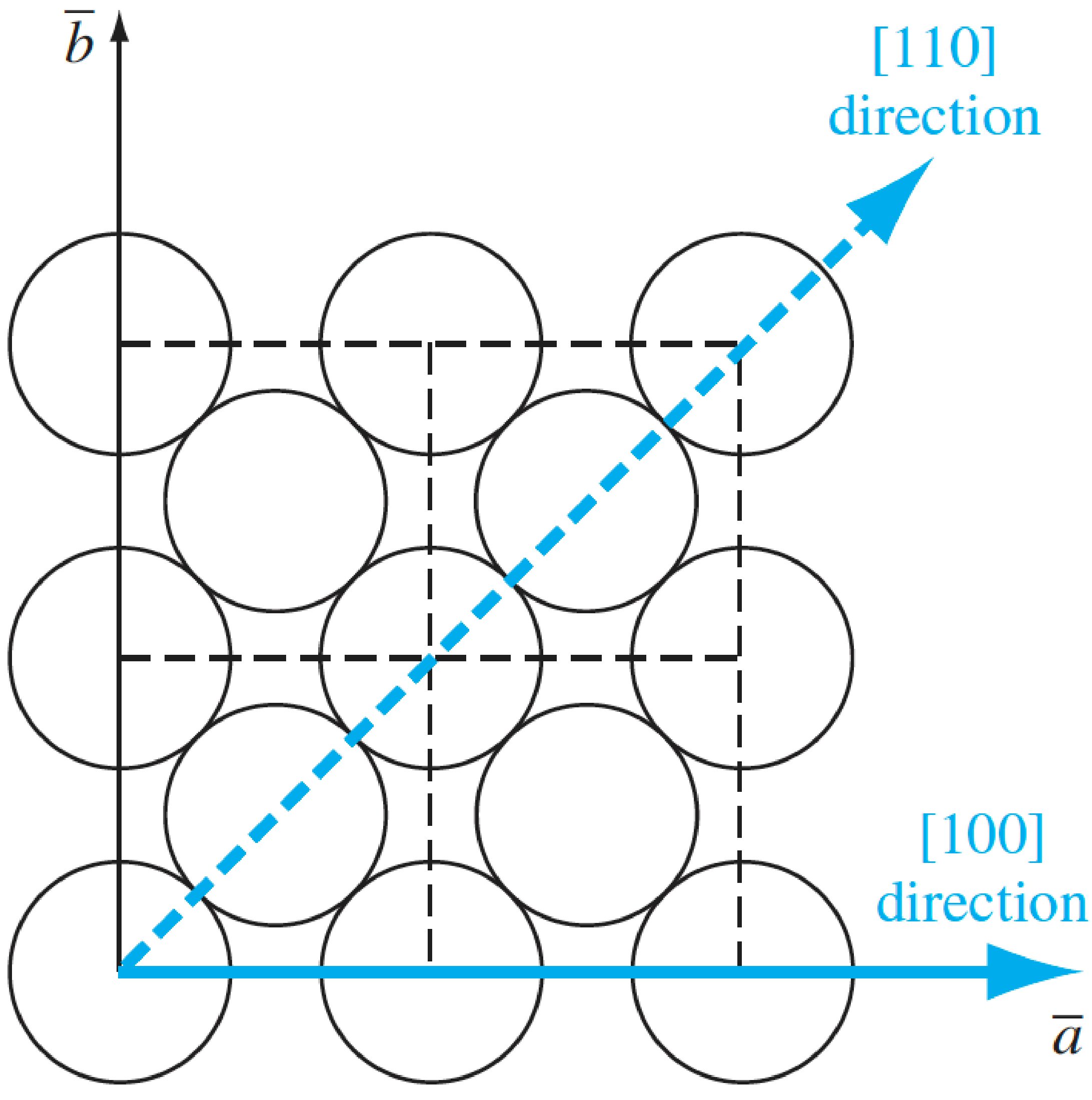
In physical space

3.3 Extension to Three Dimensions

2D periodic potential well

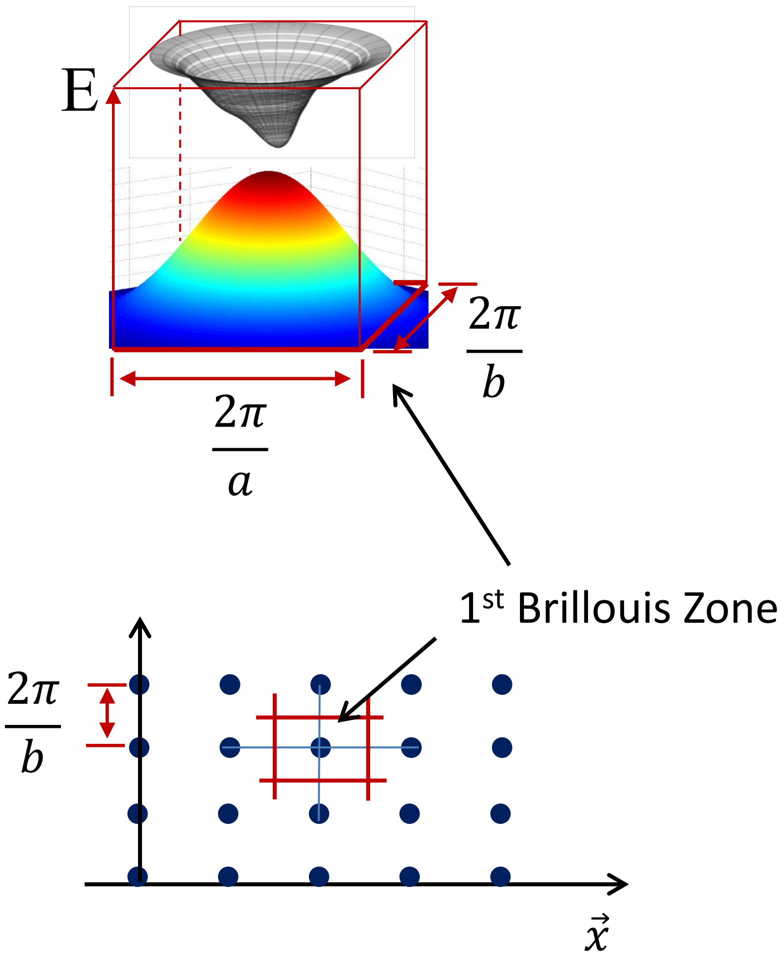


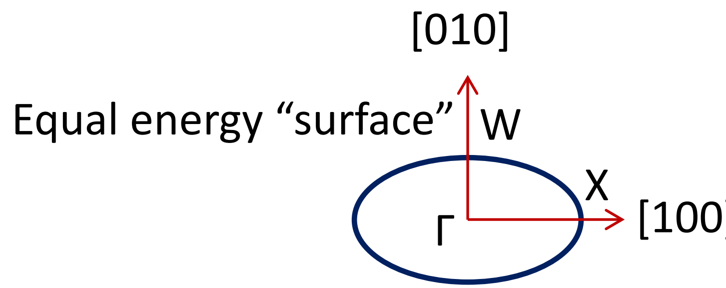
3.3 Extension to Three Dimensions

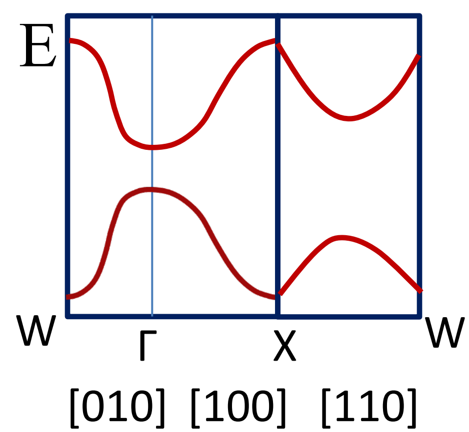


3.3 Extension to Three Dimensions

E in 3rd Dimension

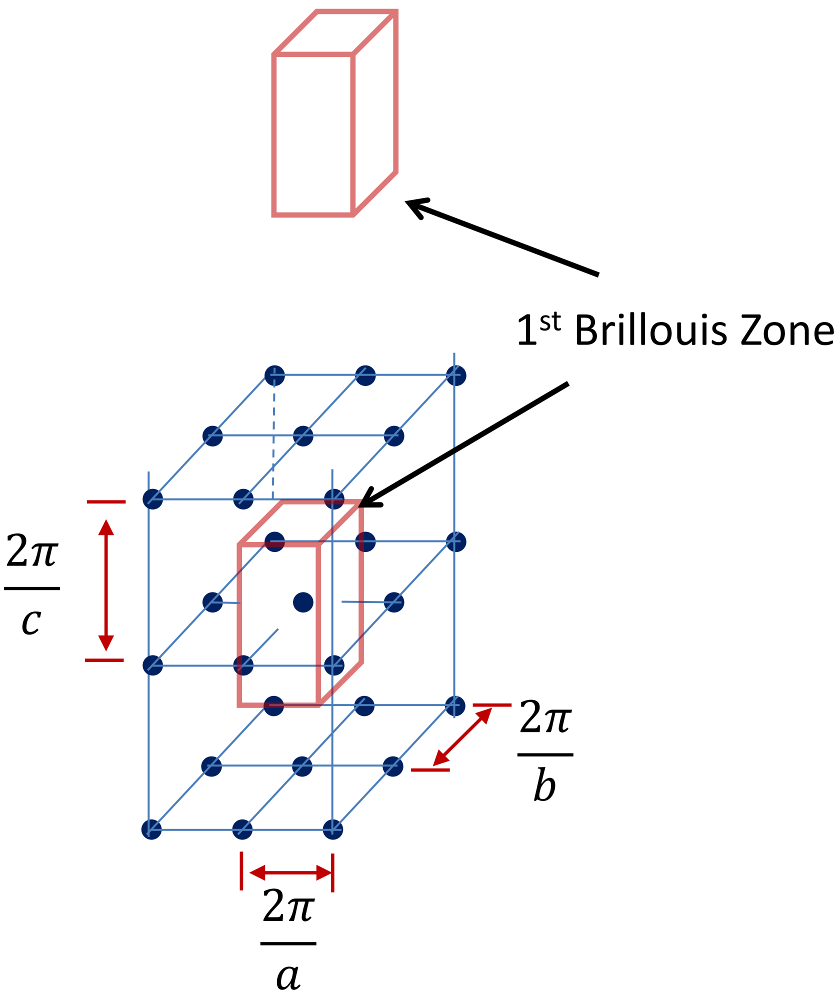




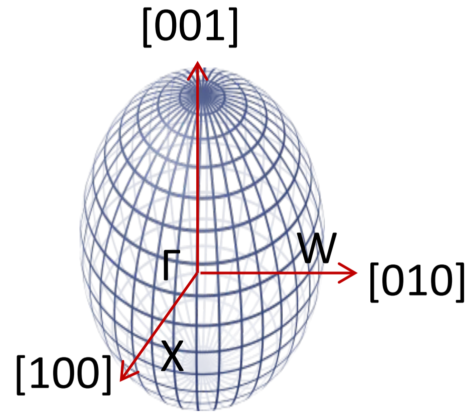


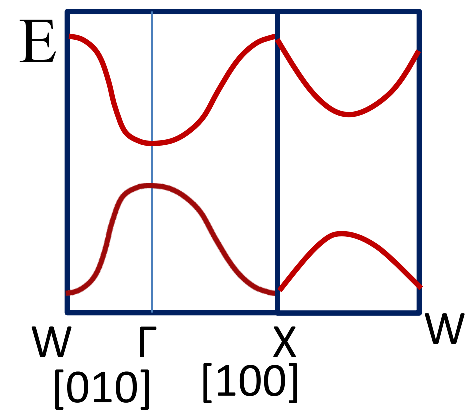
3.3 Extension to Three Dimensions

E in 4^{th} Dimension

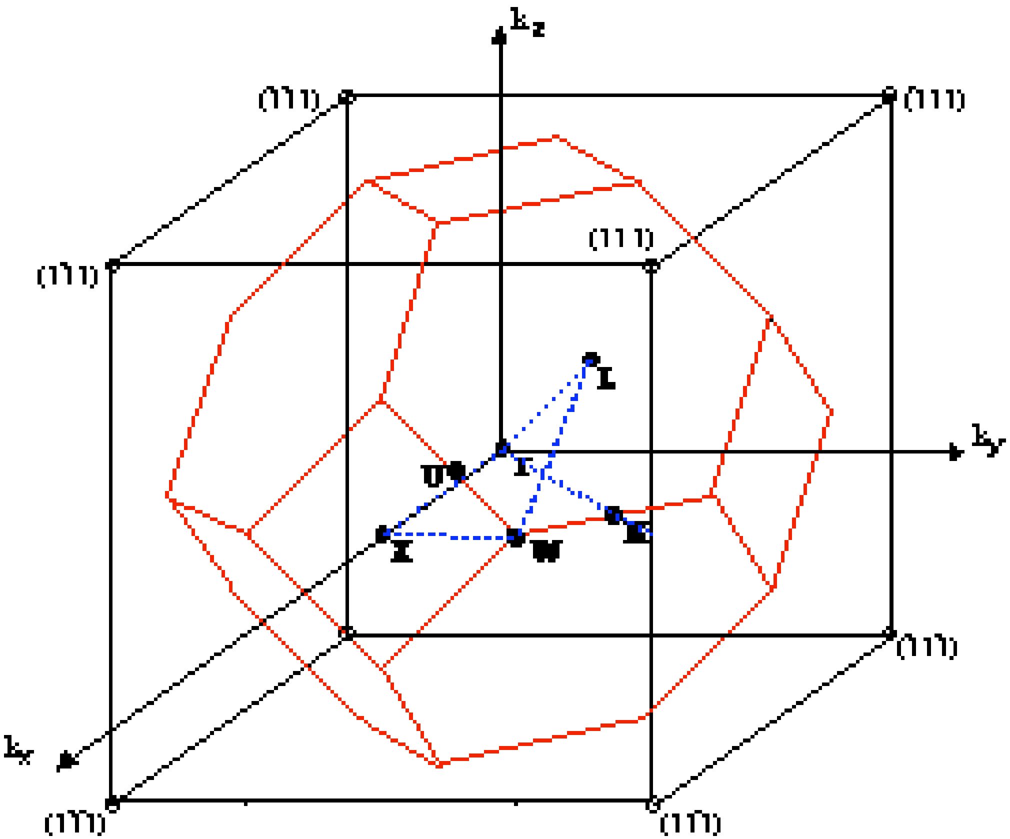


Equal energy "surface"





3.3 Extension to Three Dimensions



Γ - center of the BZ

X - [100] intercept; Γ – X path Δ

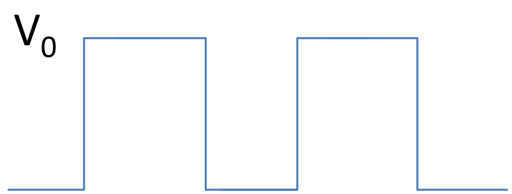
K - [110] intercept; Γ – K path ∑

L - [111] intercept; Γ – L path Λ

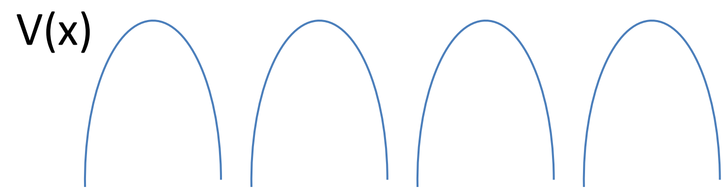
3.3 Extension to Three Dimensions

Ideal

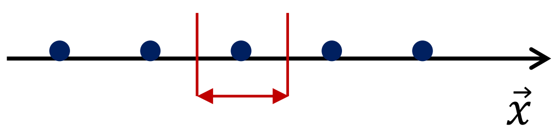
Reality



Constant potential

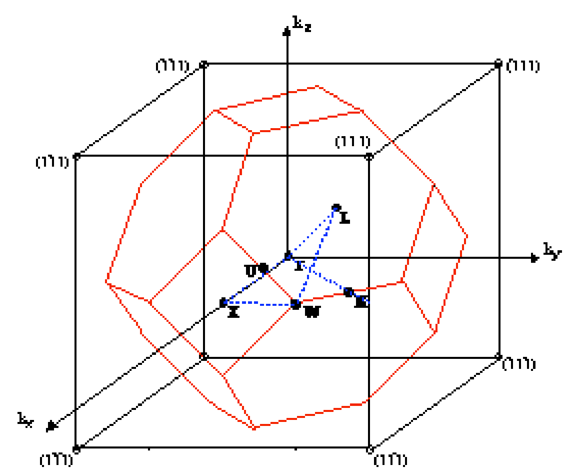


Variable potential



1st Brillouis Zone

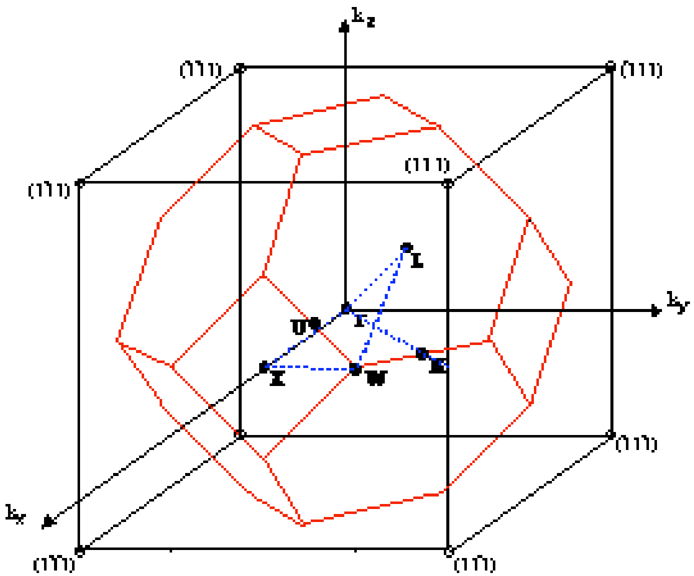
Simple 1D structure

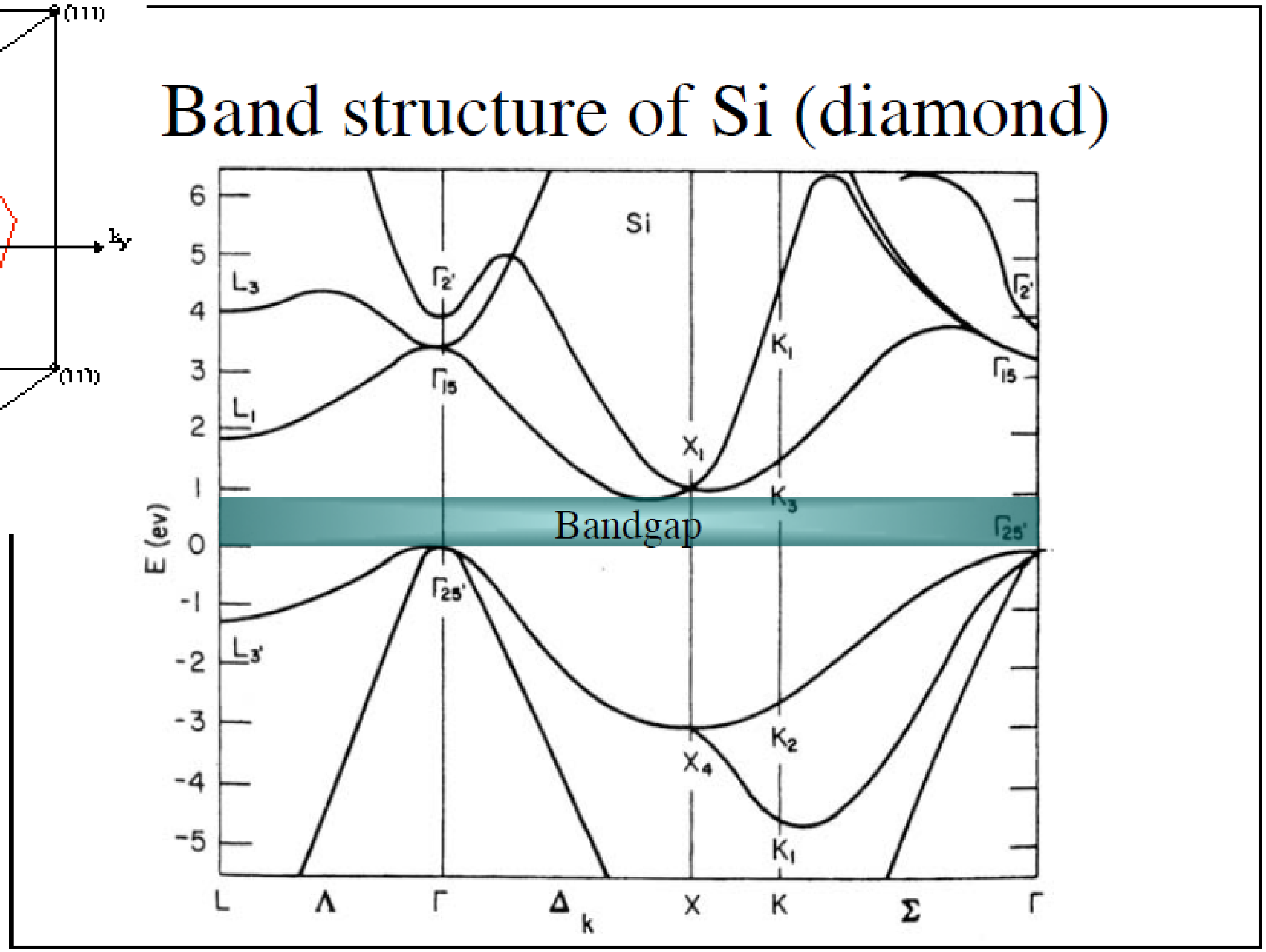


1st Brillouis Zone

Complicated 3D structure

3.3 Extension to Three Dimensions

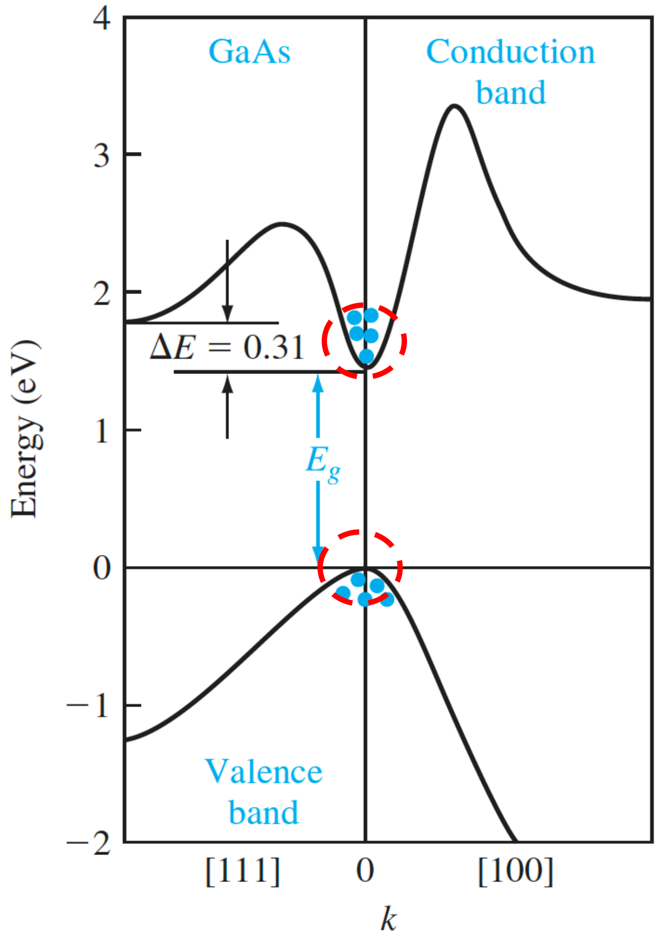




Why is it so complicated?

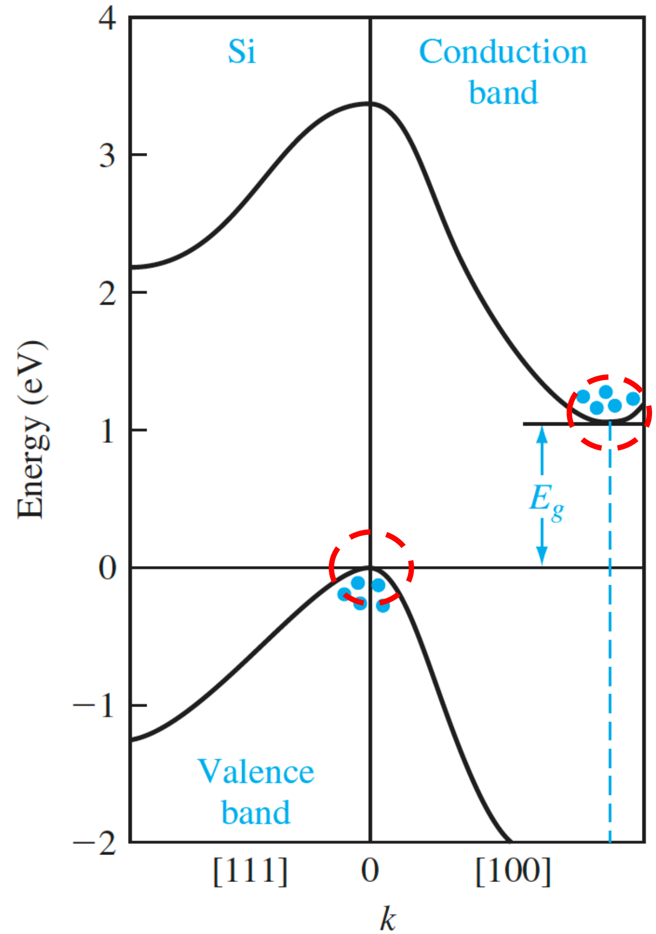
3.3 Extension to Three Dimensions

Direct bandgap



(a)

Indirect bandgap

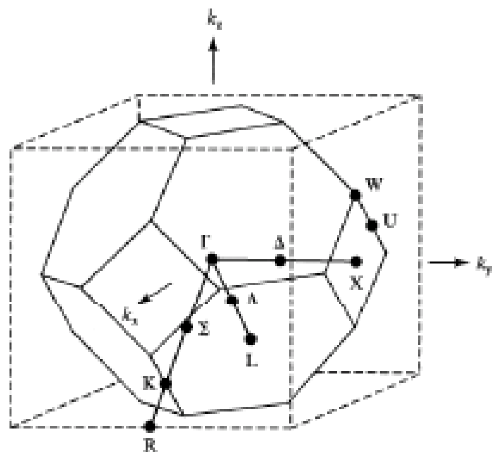


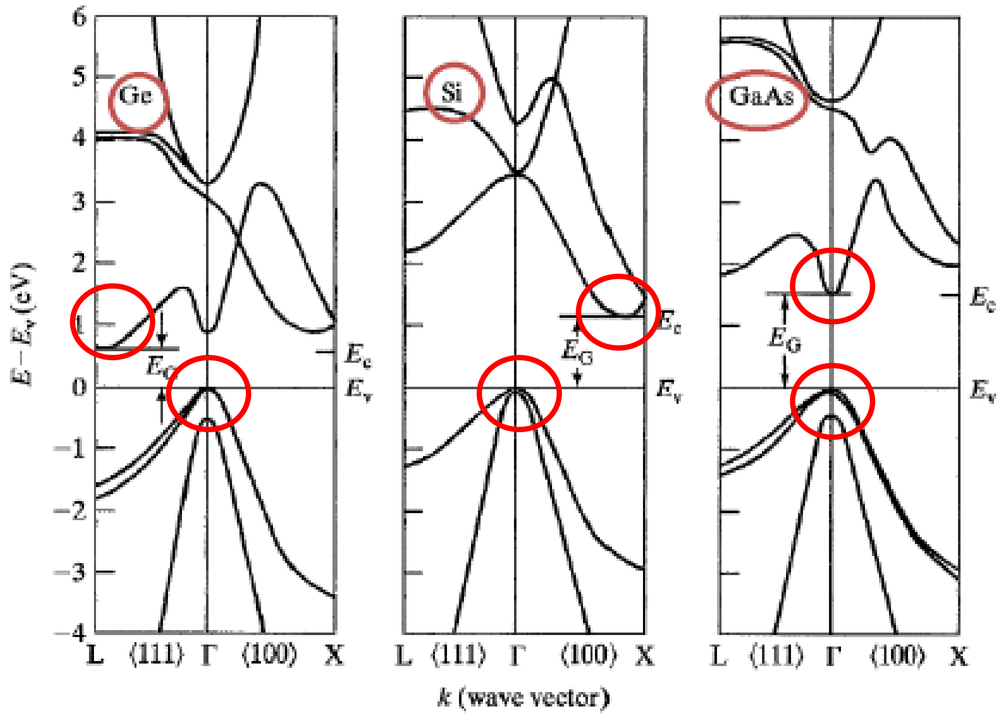
(b)

Outline

- 3.1 Allowed and Forbidden Energy Bands
- 3.2 Electrical Conduction in Solids
- 3.3 Extension to Three Dimensions
- **3.4 Effective Mass**
- 3.5 Density of States Function
- 3.6 Statistical Mechanics

3.4 Effective Mass

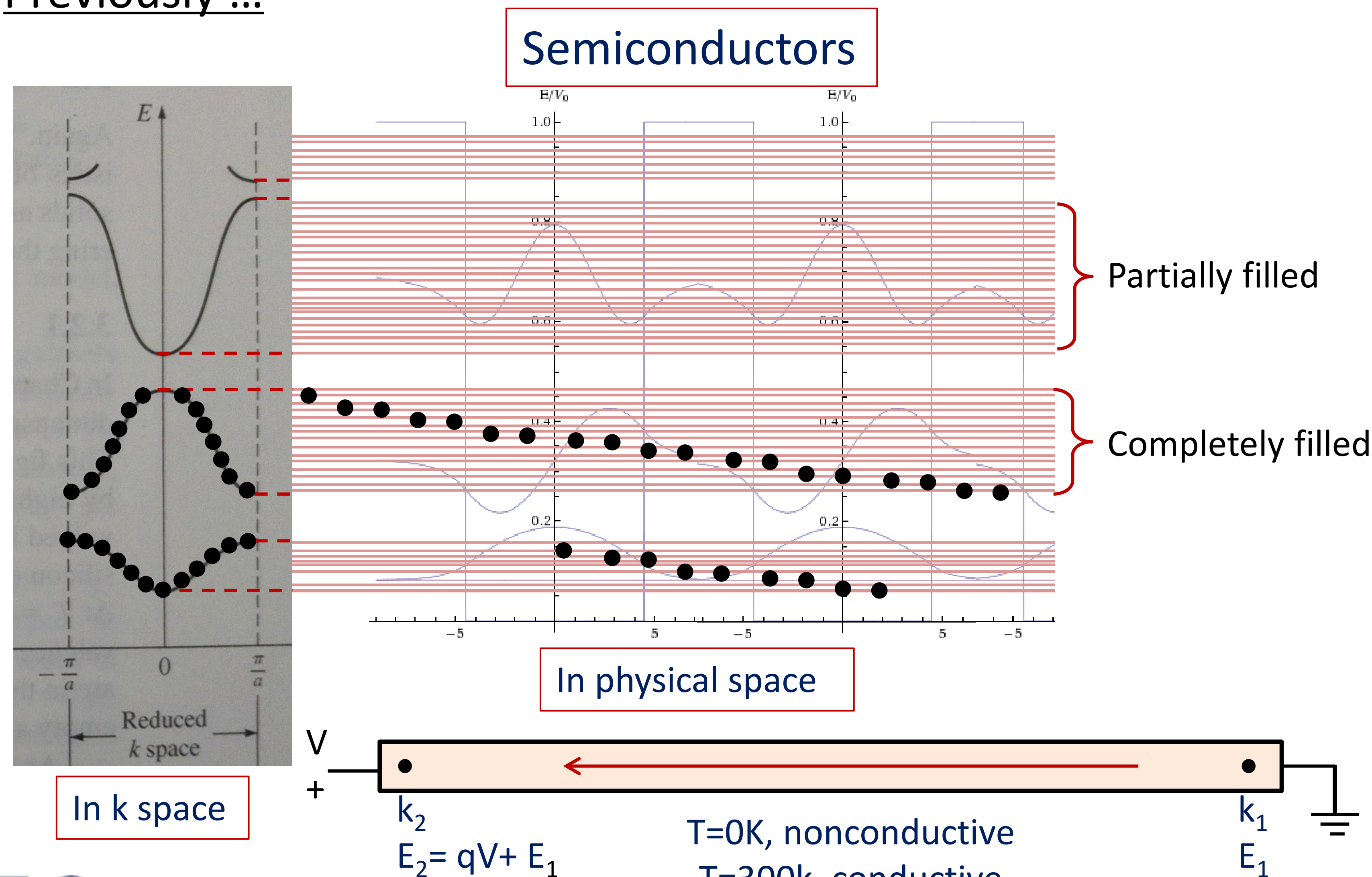




- So far the energy band structure is theoretically calculated.
- How to experimentally find it?

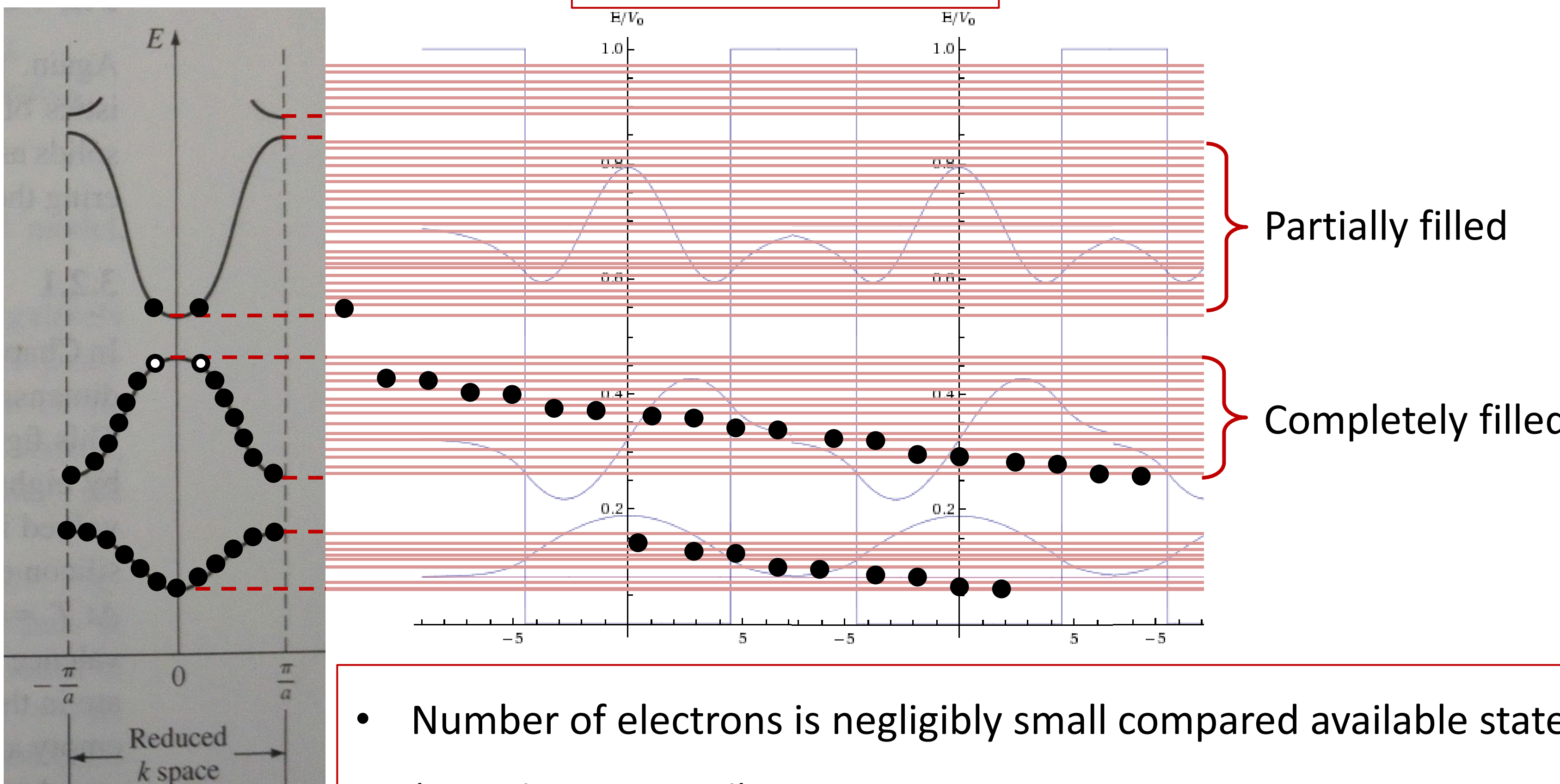
3.4 Effective Mass

Previously …



3.4 Effective Mass

Semiconductors

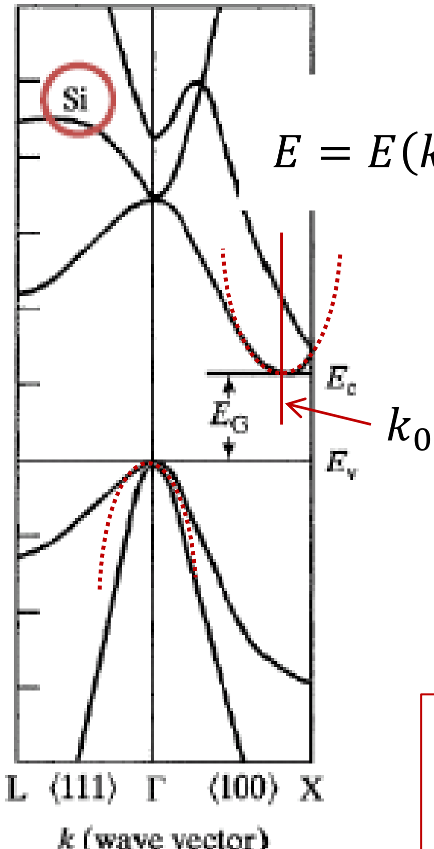


In k space

- Number of electrons is negligibly small compared available states (non-degenerated)
- Electrons mostly located at the bottom of conduction band

3.4 Effective Mass

Semiconductors (1st time approximation)



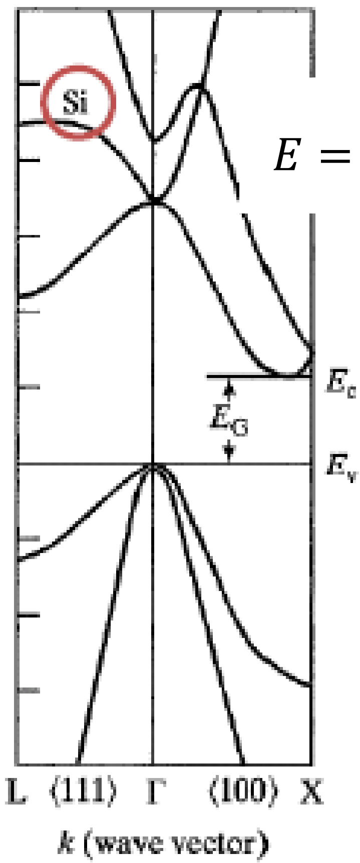
$$E = E(k) = E(k = k_0) + \frac{dE}{dk}|_{k=k_0}(k - k_0) + \frac{d^2E}{2dk^2}|_{k=k_0}(k - k_0)^2 + O((\Delta k)^3)$$

Taylor series

- Number of electrons is negligibly small compared available states (non-degenerated)
- Electrons mostly located at the bottom of conduction band

3.4 Effective Mass

Semiconductors



$$E = E(k) = E(k = k_0) + \frac{dE}{dk}|_{k=k_0}(k - k_0) + \frac{d^2E}{2dk^2}|_{k=k_0}(k - k_0)^2 + O((\Delta k)^3)$$

For electrons in free space:

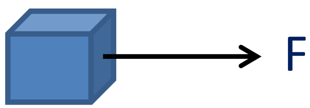
$$E_f = \frac{\hbar^2 k^2}{2m} \Rightarrow \frac{d^2 E_f}{dk^2} = \frac{\hbar^2}{m} \qquad \frac{d^2E}{dk^2}|_{k=0} = \frac{\hbar^2}{m^*}$$

- m* has a unit of mass
- We call it the effective mass of electrons in the crystal

3.4 Effective Mass

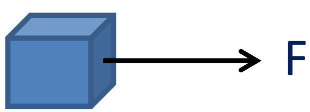
- How to understand effective mass

Example: use Newton's law to find mass of an object



$$m = \frac{F}{a} \qquad a = \frac{d^2x}{dt^2}$$

In the air



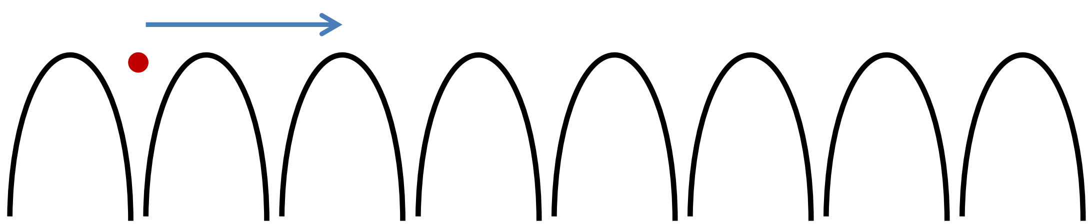
$$m^* = \frac{F}{a} \qquad a = \frac{d^2x}{dt^2}$$

In the water

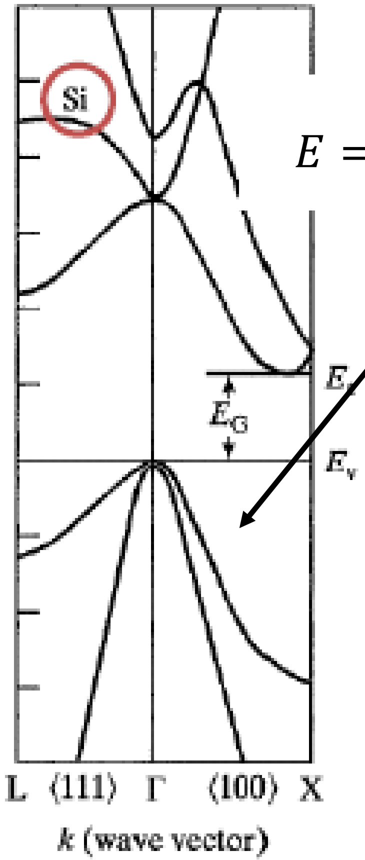
3.4 Effective Mass

- How to understand effective mass

<u>Modulated by Electric potential of ions</u>



3.4 Effective Mass

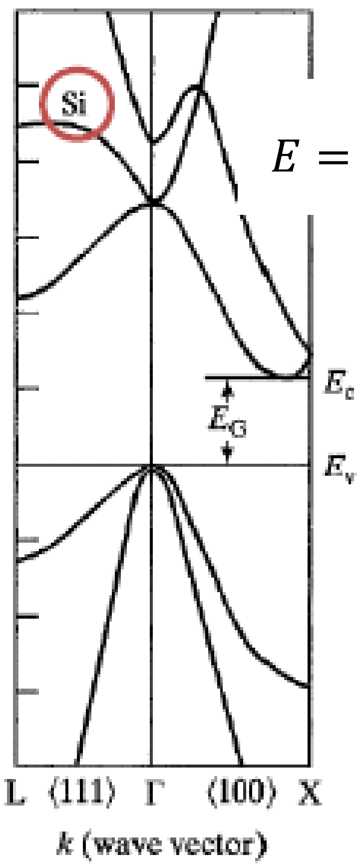


For Electrons in the valence band:

$$E = E(k) = E(k = k_0') + \frac{dE}{dk}|_{k=k_0}(k - k_0') + \frac{d^2E}{2dk^2}|_{k=k_0}(k - k_0')^2 + O((\Delta k)^3)$$

3.4 Effective Mass

Semiconductors



For Electrons in the conduction band:

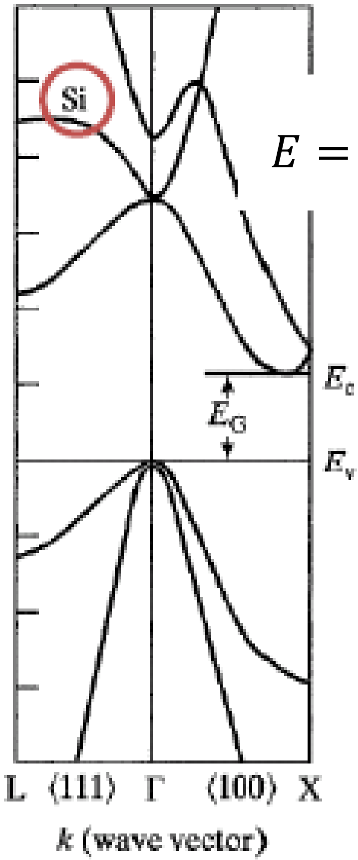
< 0

$$E = E(k) = E(k = k_0') + \frac{dE}{dk}|_{k=k_0}(k - k_0') + \frac{d^2E}{2dk^2}|_{k=k_0}(k - k_0')^2 + O((\Delta k)^3)$$

$$\boxed{E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k - k_0')^2}$$

3.4 Effective Mass

Semiconductors



For Electrons in the conduction band:

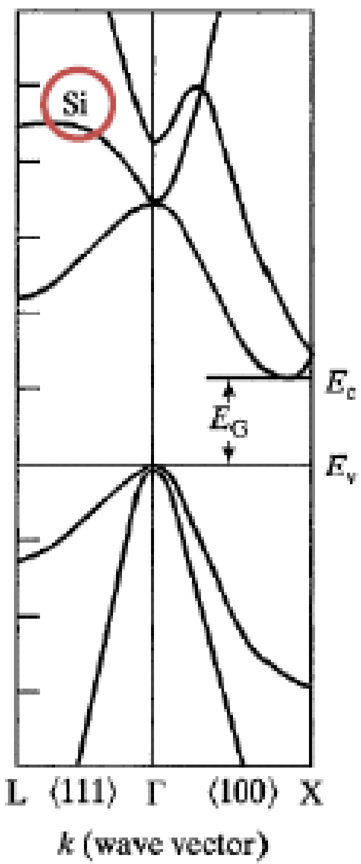
< 0

$$E = E(k) = E(k = k_0') + \frac{dE}{dk}|_{k=k_0}(k - k_0') + \frac{d^2E}{2dk^2}|_{k=k_0}(k - k_0')^2 + O((\Delta k)^3)$$

$$\boxed{E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k - k_0')^2}$$

- Equivalent to a positive charge carrier
- Different effective mass (always larger than electrons)
- Electrons and holes can come from dopants separately

3.4 Effective Mass



Conduction Band:

$$\boxed{E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}(k - k_0)^2}$$

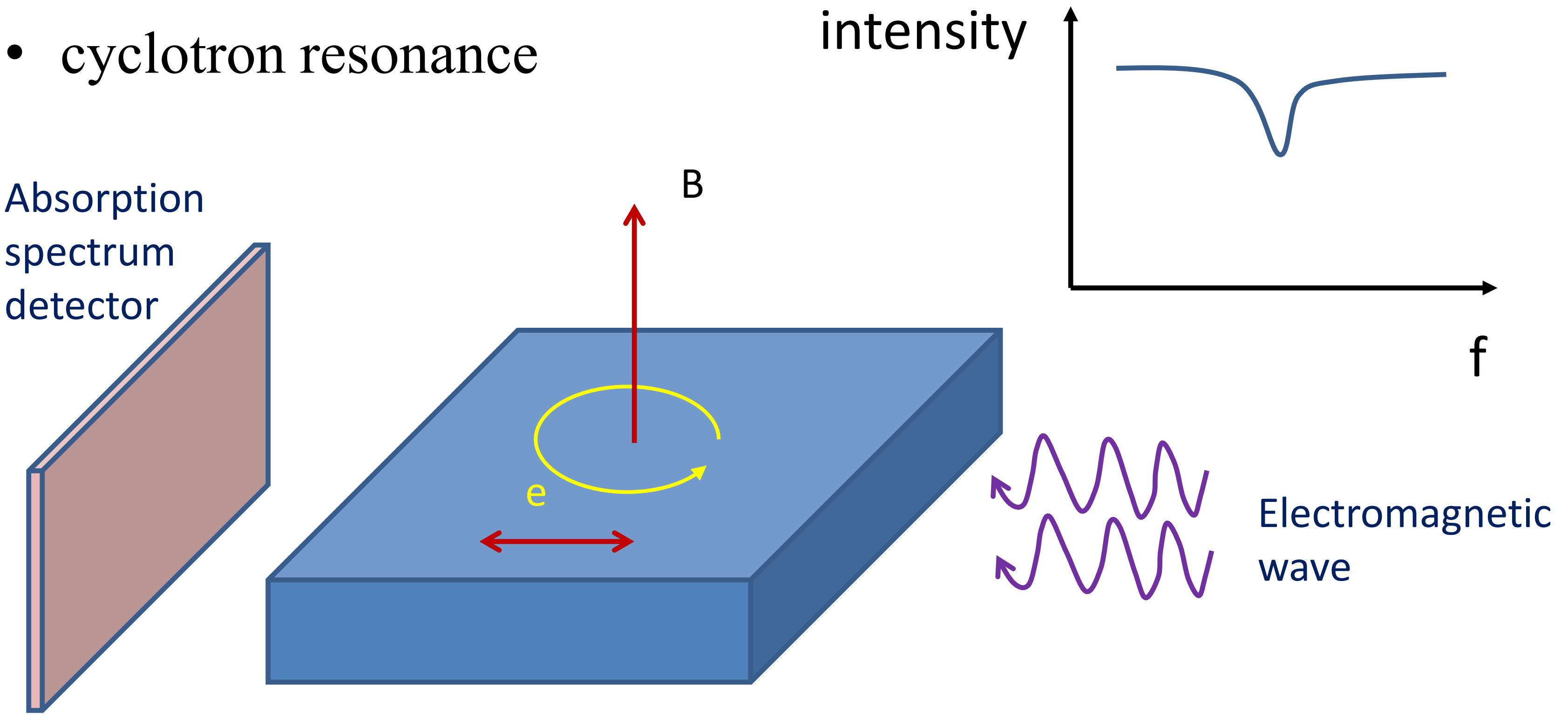
Valence Band:

$$\boxed{E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k - k_0')^2}$$

- If we can experimentally measure the effective mass, we will have found the analytical express of energy band structure for semiconductors.
- How?

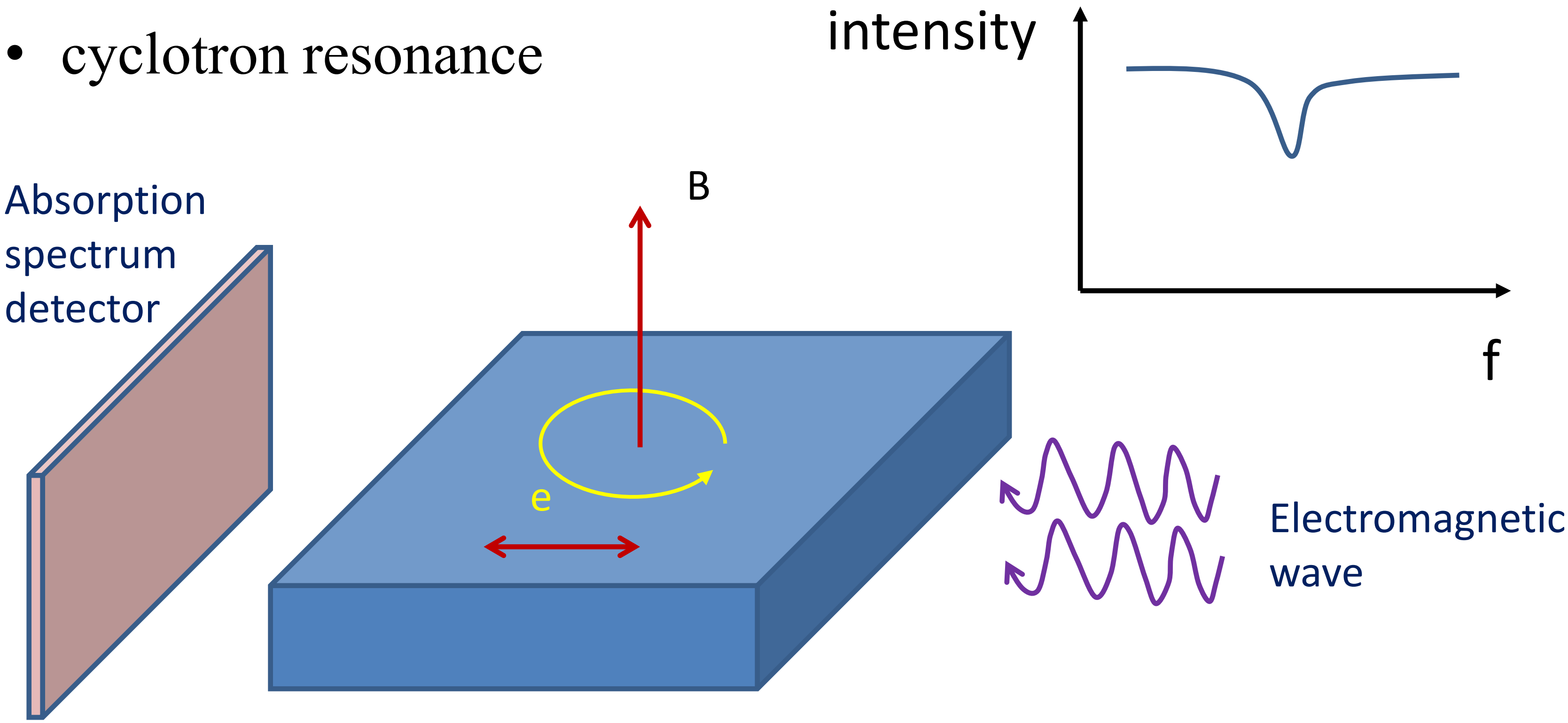
3.4 Effective Mass

- cyclotron resonance



3.4 Effective Mass

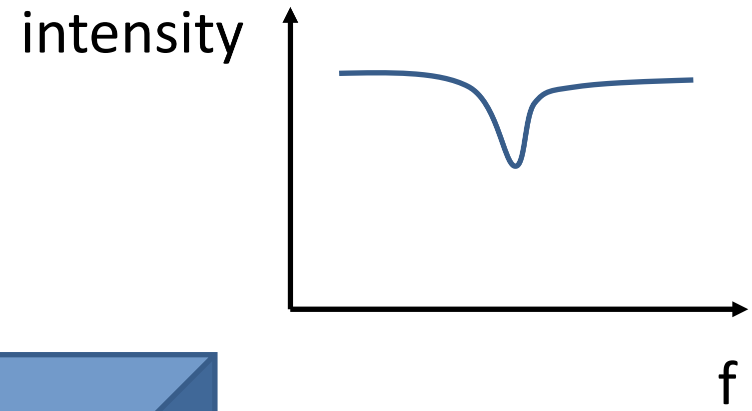
- cyclotron resonance

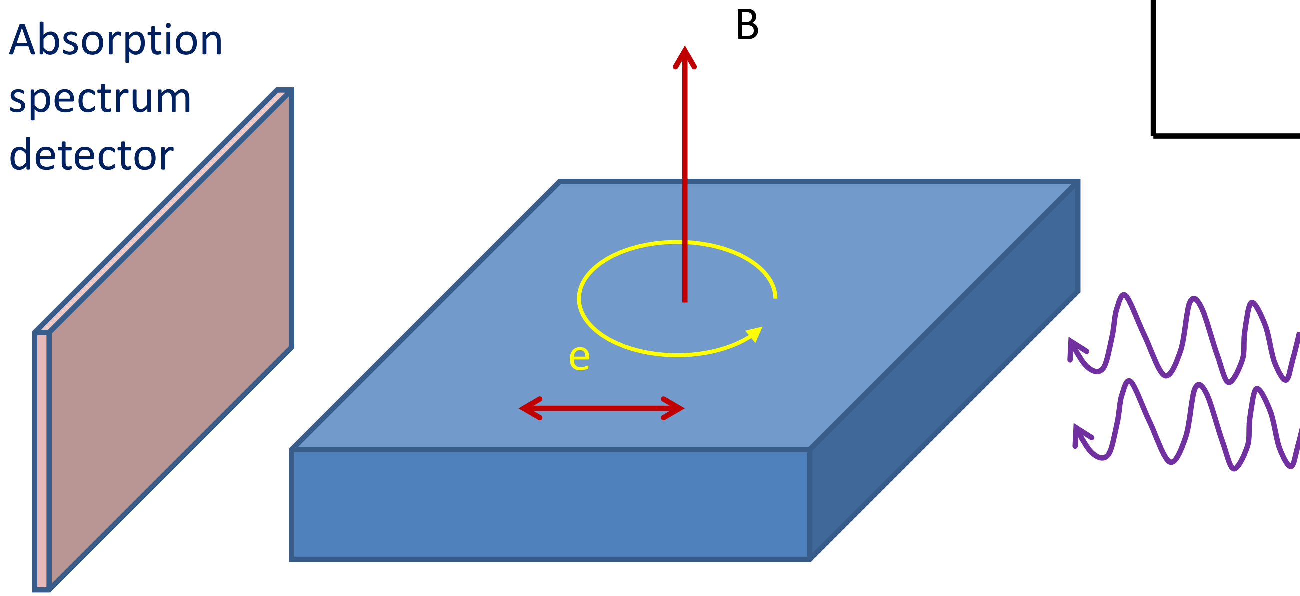


Suppose a intrinsic silicon wafer is placed in a magnetic field B = 1T. We find a dip at λ=5mm in the absorption spectrum, what is the effective mass of electrons? The mass of electrons in free space m_0 = 9.1e-31kg.

3.4 Effective Mass

- cyclotron resonance





Centrifugal force $F = m^* \omega^2 r$

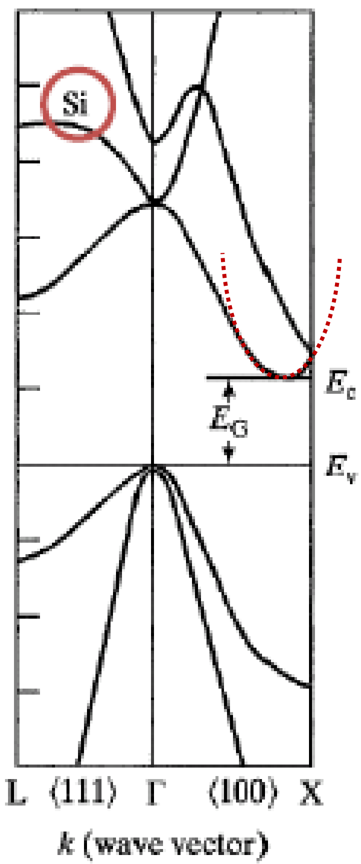
Magnetic force $F_{mag} = e \times v \times B$

$v = \omega r$

$\Rightarrow$ $m^* = eB/\omega \qquad \omega = 2\pi f$

$$\Rightarrow m^* = \frac{eB\lambda}{2\pi c} = \frac{1.6 \times 10^{-19} \times 0.005}{2\pi \times 3 \times 10^{8}} = 0.47 m_0$$

3.4 Effective Mass



- If we can experimentally measure the effective mass, we will have found the analytical express of energy band structure for non-degenerated semiconductors.
- How?

$$\boxed{E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}(k - k_0)^2}$$

For Electrons in the valence band:

$$\boxed{E = E(k) = E_c - \frac{\hbar^2}{2m_p^*}(k - k_0')^2}$$

$$\Rightarrow m^* = \frac{eB\lambda}{2\pi c} = \frac{1.6 \times 10^{-19} \times 0.005}{2\pi \times 3 \times 10^8} = 0.47m_0$$

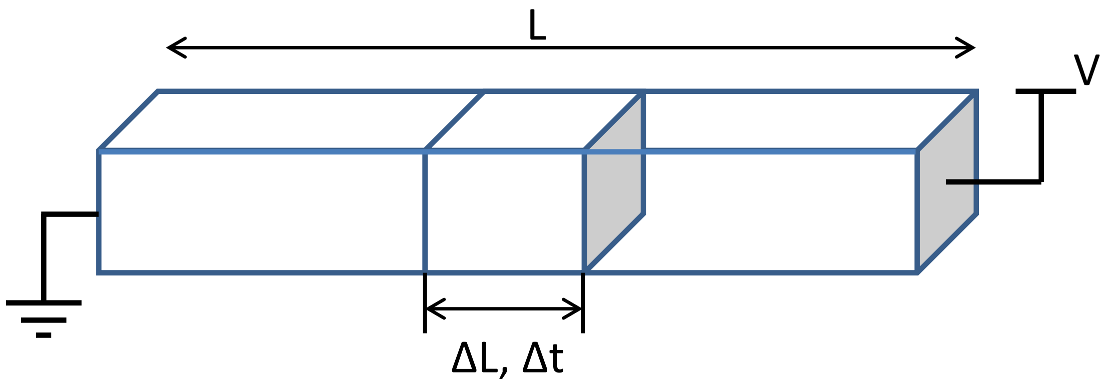
3.4 Effective Mass

	Symbol	Germanium	Silicon	Gallium Arsenide
Bandgap	E_g (eV)	0.66	1.12	1.424
Electrons	m_e^*/m_0	0.067	1.08	0.55
Holes	m_h^*/m_0	0.48	0.56	0.37

Outline

3.5 Density of States Function

$n \ type \ semiconductor$



$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c v$$

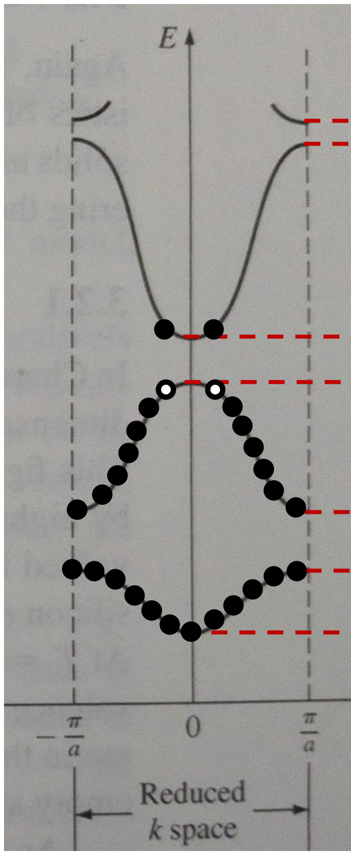
$$v = \mu E = \mu V/L$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c\mu V/L \quad \Rightarrow \quad \sigma = \frac{I}{V} = \frac{nqA_c\mu}{L}$$

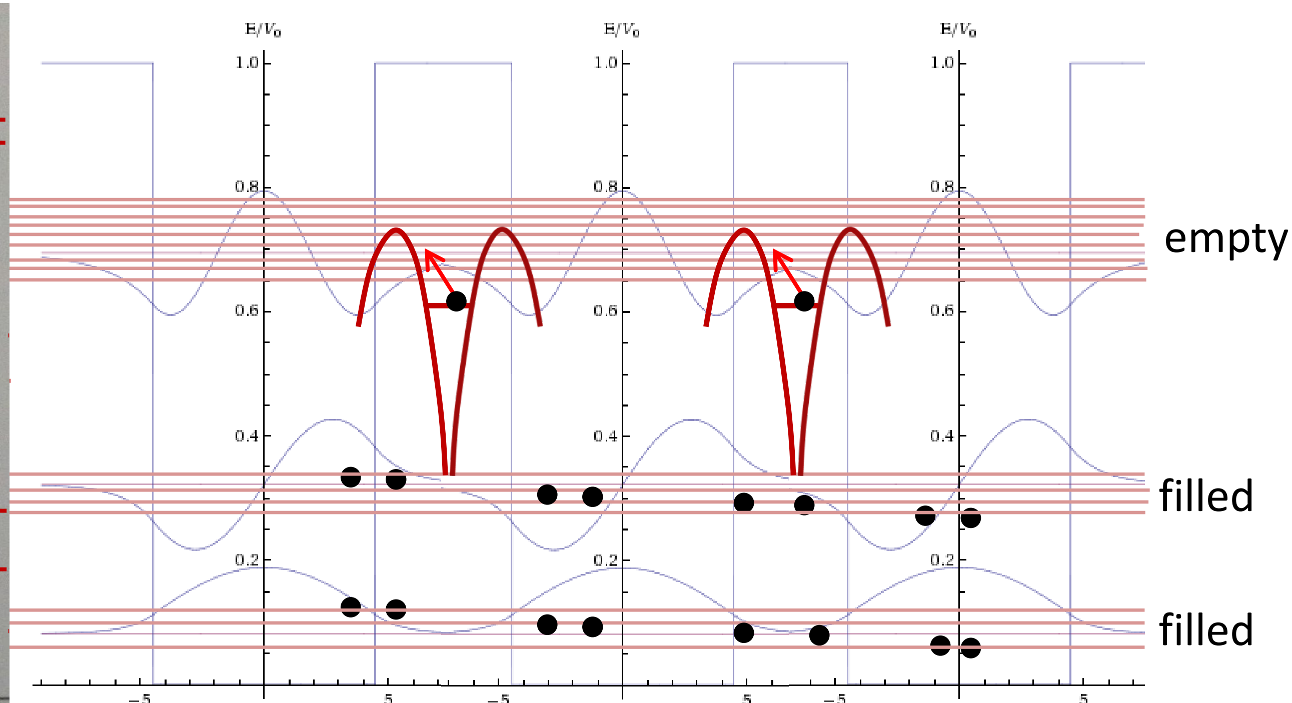
3.5 Density of States Function

Previously…

- Doping concentration: N_D, 100% ionized
- Electrons from the valance are negligible

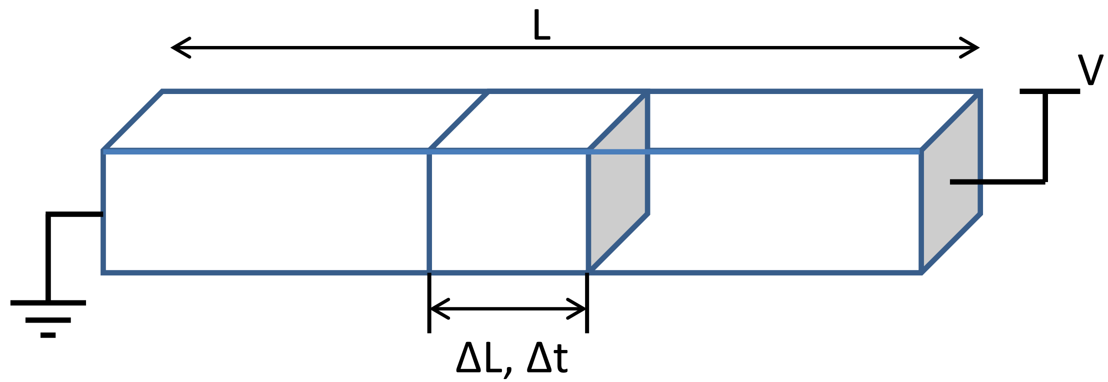


In k space



3.5 Density of States Function

$n\ type\ semiconductor$



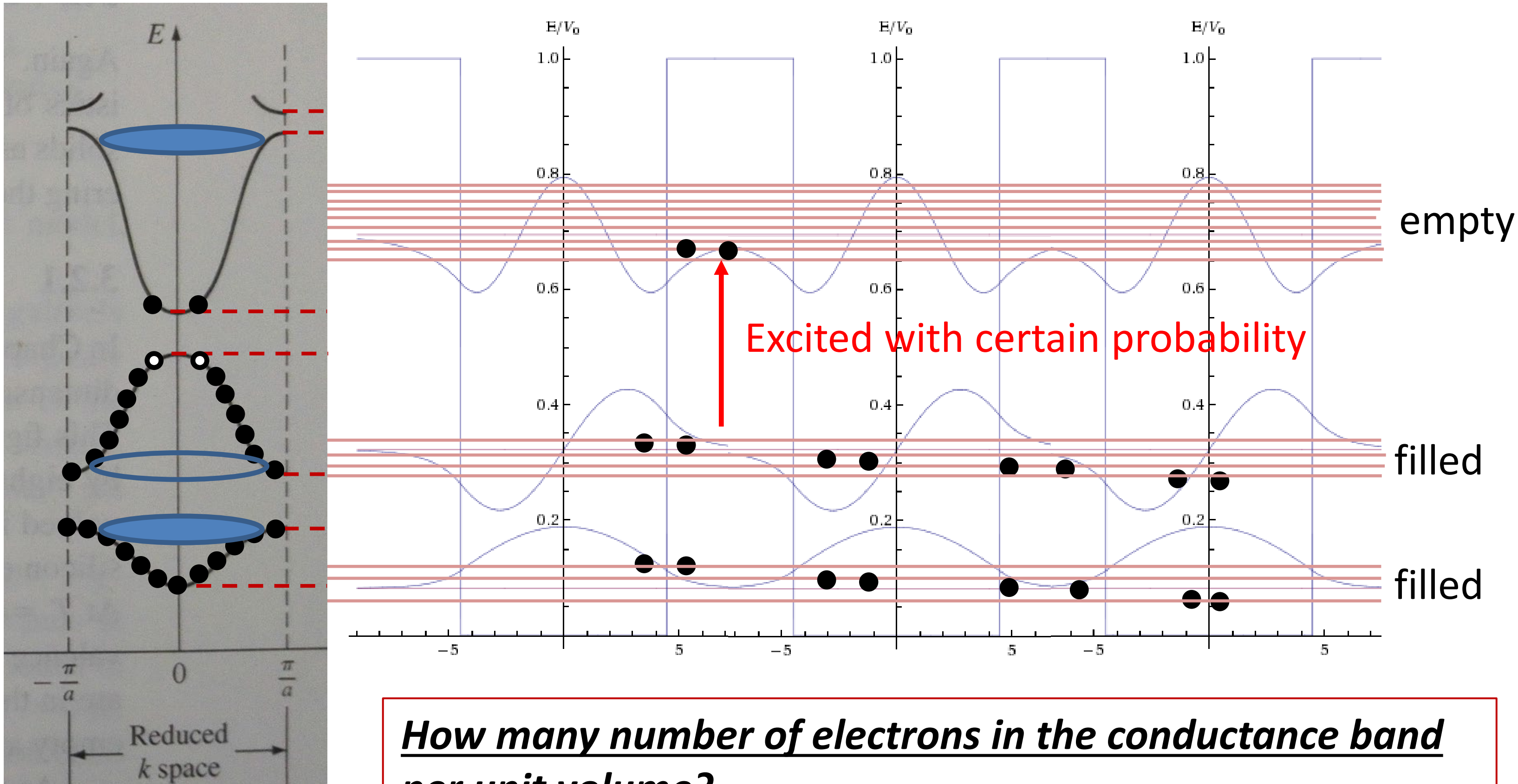
$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c v$$

$$v = \mu E = \mu V/L$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{nqA_c\Delta L}{\Delta t} = nqA_c \mu V/L \quad \Rightarrow \quad \sigma = \frac{I}{V} = \frac{N_D q A_c \mu}{L}$$

3.5 Density of States Function

- If semiconductor is intrinsic



In k space

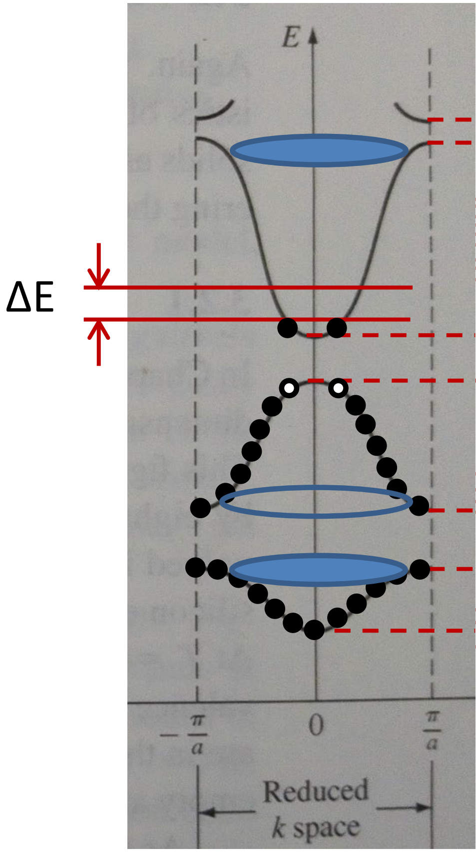
<u>How many number of electrons in the conductance band per unit volume?</u>

3.5 Density of States Function

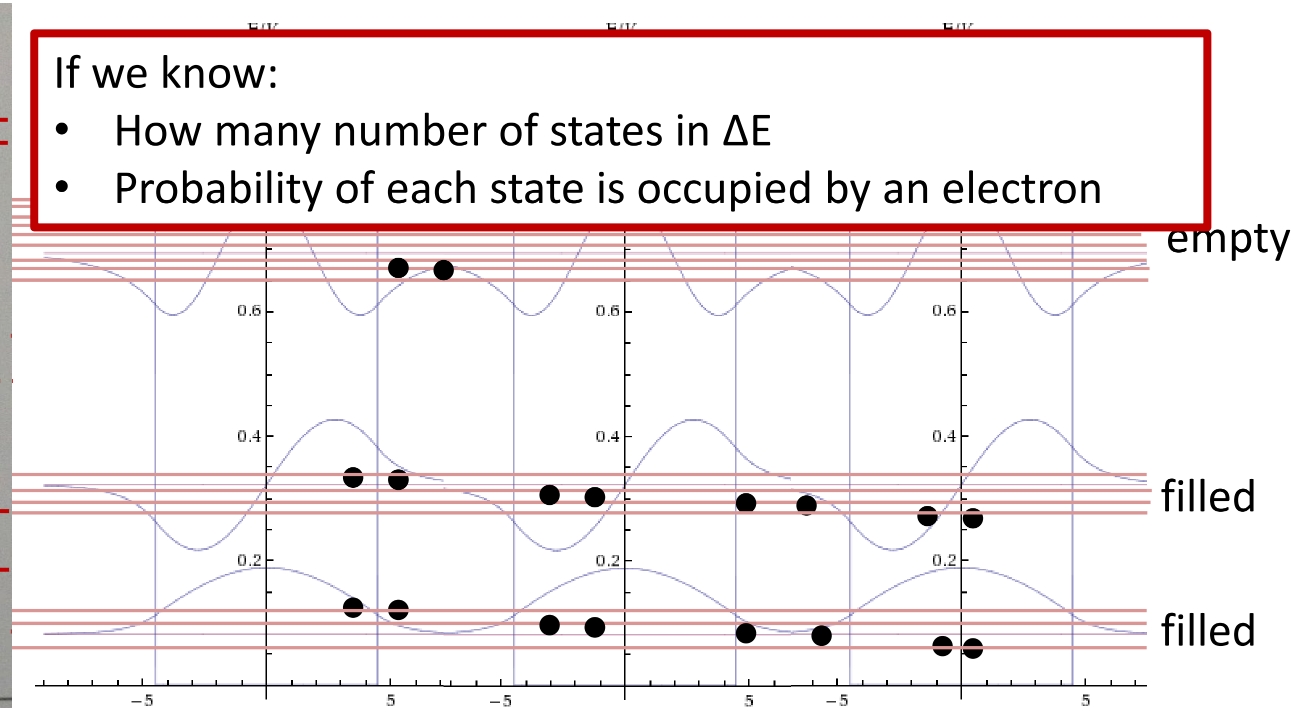
- Intrinsic (undoped)
- Electrons are all from the valance

If we know:

- How many number of states in ΔE
- Probability of each state is occupied by an electron



In k space



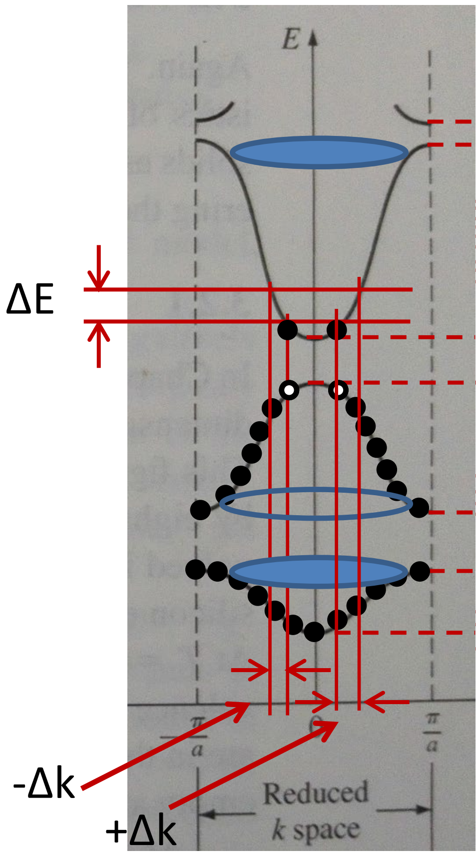
<u>How many number of electrons in the conductance band per unit volume?</u>

3.5 Density of States Function

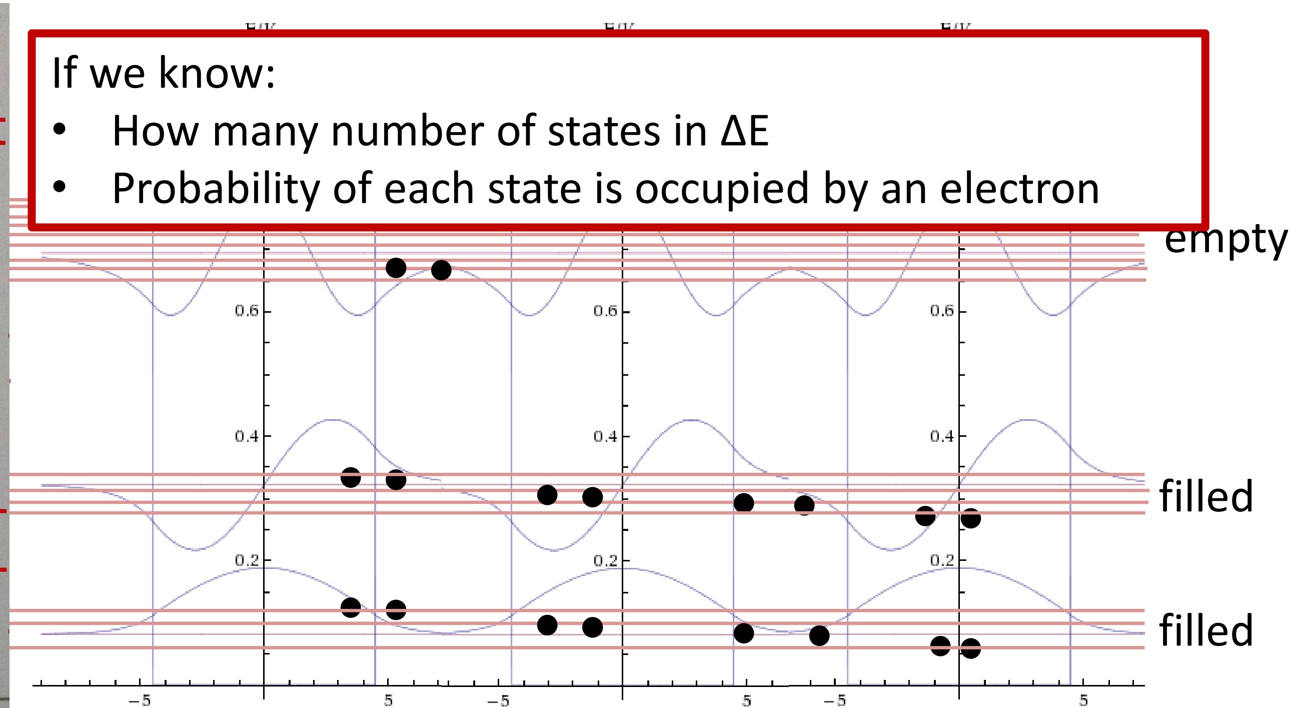
- Intrinsic (undoped)
- Electrons are all from the valance

If we know:

- How many number of states in ΔE
- Probability of each state is occupied by an electron



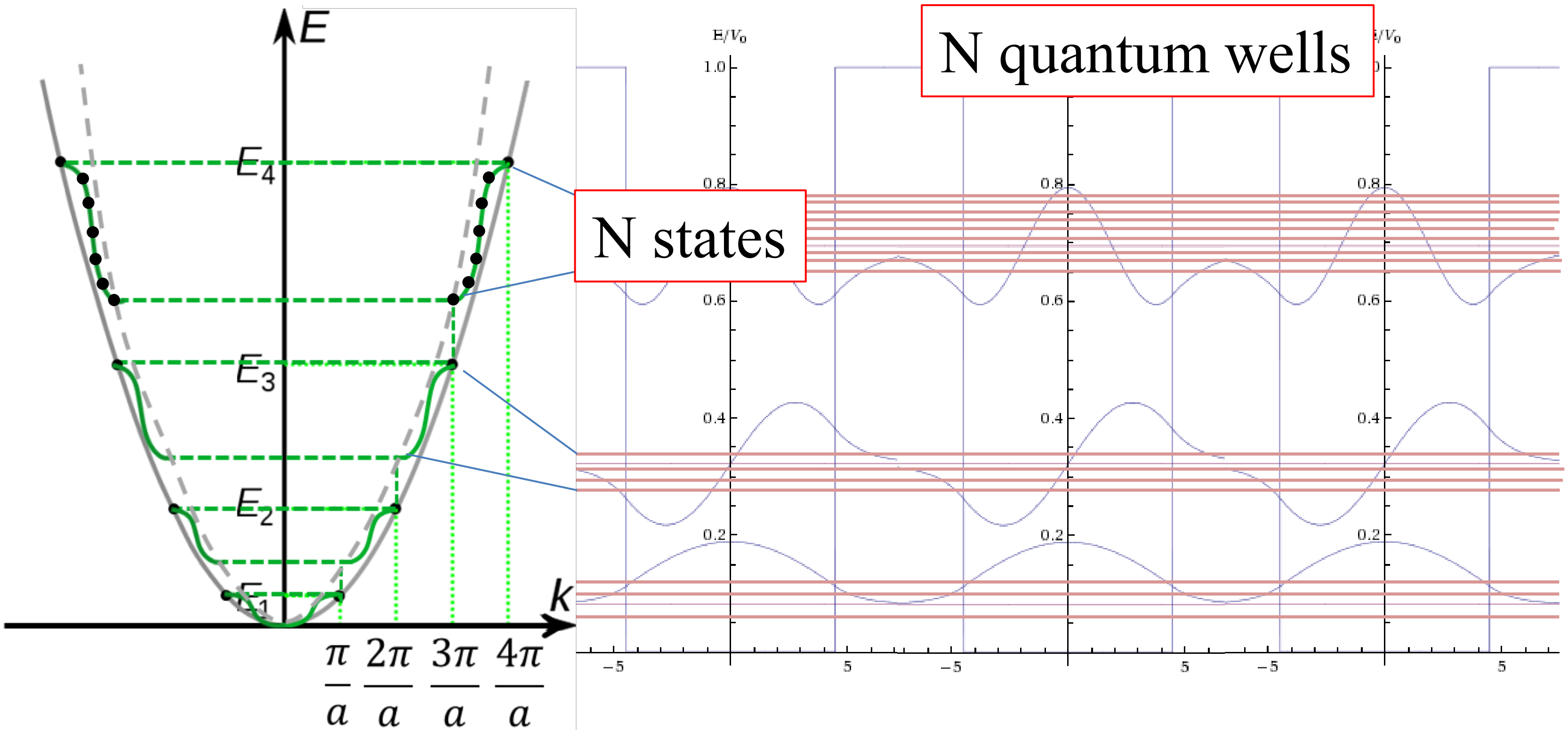
In k space



<u>How many number of electrons in the conductance band per unit volume?</u>

3.1 Allowed and Forbidden Energy Bands

Forming energy bands: analytical



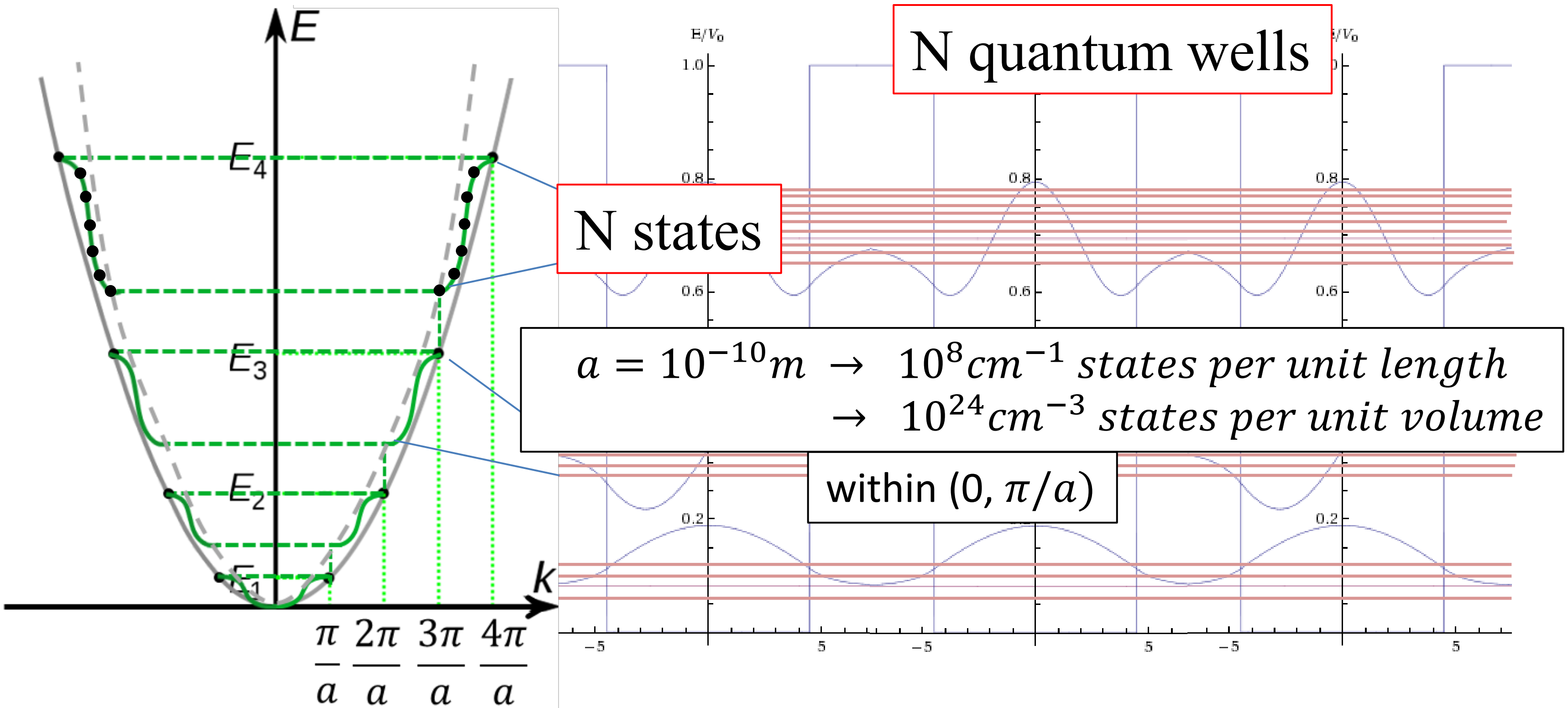
The "density" of states of the whole crystal within $(0, \pi/a)$: $\dfrac{N}{\pi/a}$

The number of states of the whole crystal within Δk: $\dfrac{N}{\pi/a} \times \Delta k$

The number of states per unit volume within Δk: $\dfrac{N}{\pi/a} \times \Delta k \dfrac{1}{Na} = \dfrac{\Delta k}{\pi}$

3.1 Allowed and Forbidden Energy Bands

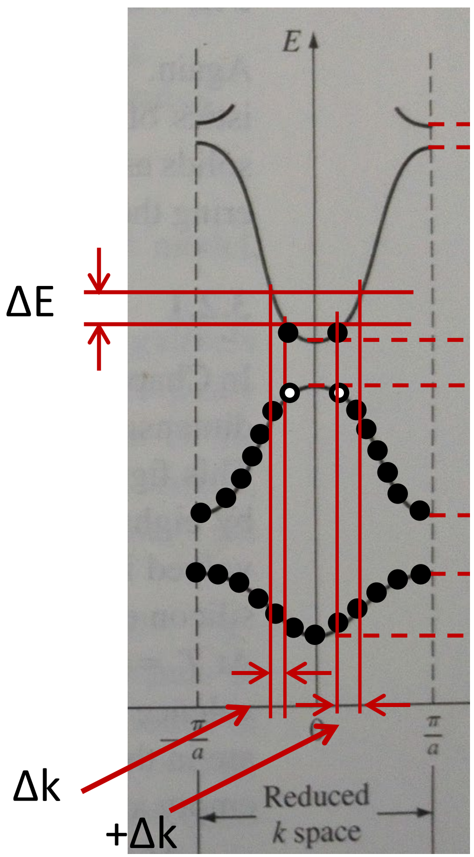
Forming energy bands: analytical



k is wave number. $\frac{k}{\pi}$ *means the number of states per unit volume*

3.5 Density of States Function

- Intrinsic (undoped)
- Electrons are all from the valance

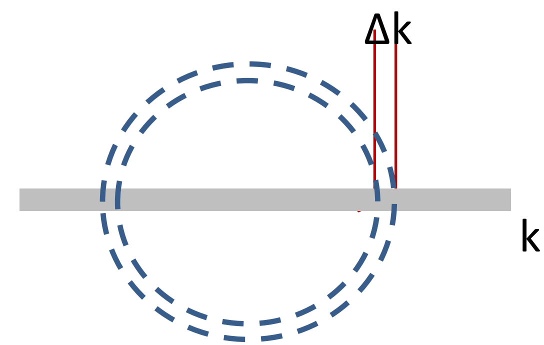


In k space

If we know:

- How many number of states in ΔE
- Probability of each state is occupied by an electron

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$



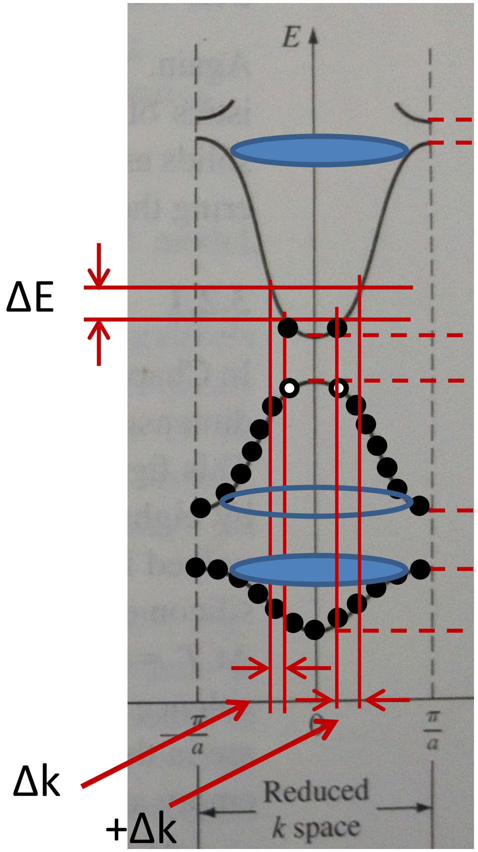
$$k = \mp\frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$

Within ΔE, we have the number of k is $\frac{d(2|k|/\pi)}{dE}\Delta E$

$$g(E) = \frac{1}{2}\frac{d(2|k|/\pi)}{dE}$$

3.5 Density of States Function

- Intrinsic (undoped)
- Electrons are all from the valance

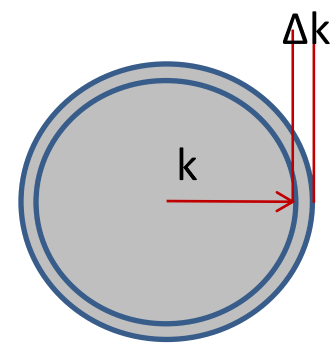


In k space

If we know:
- How many number of states in ΔE
- Probability of each state is occupied by an electron

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$

$$k = \mp\frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$



Within ΔE, we have the number of k is $\frac{d(\pi(k/\pi)^2)}{dE}\Delta E$

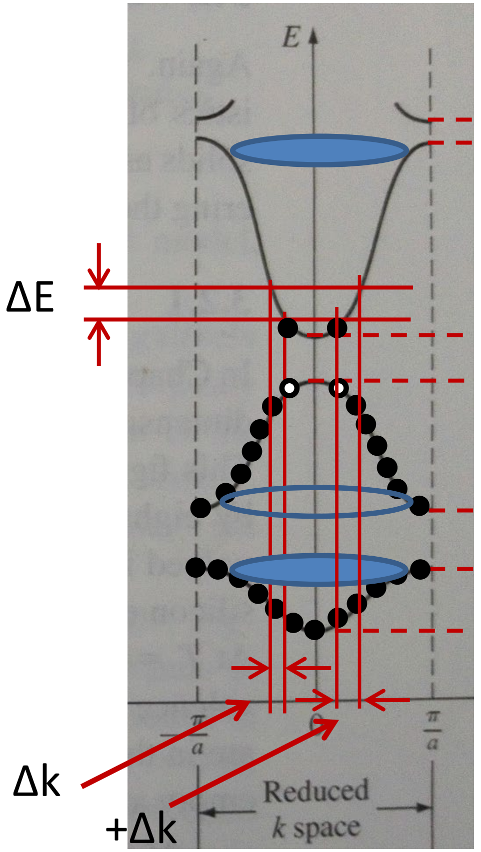
$$g(E) = \frac{1}{4}\frac{d(\pi(k/\pi)^2)}{dE}$$

3.5 Density of States Function

- Intrinsic (undoped)
- Electrons are all from the valance

If we know:

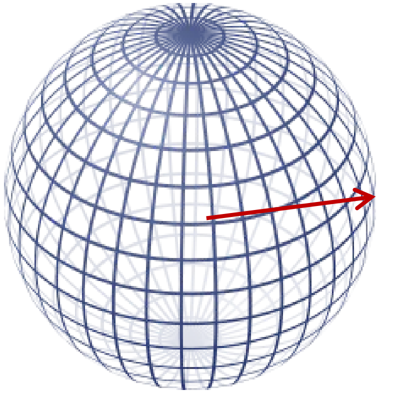
- How many number of states in ΔE
- Probability of each state is occupied by an electron



In k space

$$E = E(k) = E_c + \frac{\hbar^2}{2m_n^*}k^2$$

$$k = \mp\frac{\sqrt{2m_n^*(E - E_c)}}{\hbar}$$

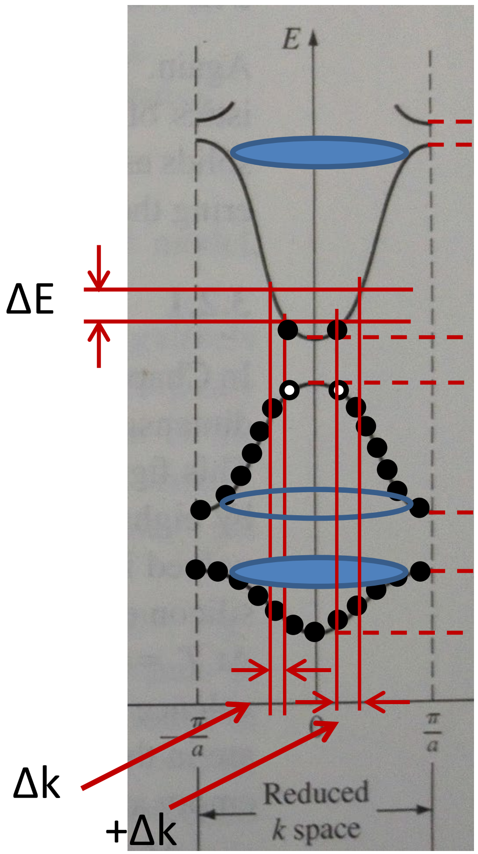


Within ΔE, we have the number of k is $\frac{d(4\pi\left(\frac{k}{\pi}\right)^3/3)}{dE}\Delta E$

$$g(E) = \frac{1}{8}\frac{d(4\pi\left(\frac{k}{\pi}\right)^3/3)}{dE}$$

3.5 Density of States Function

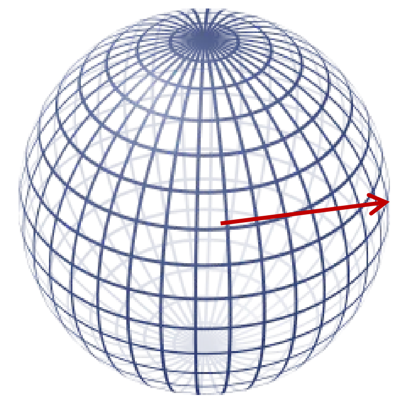
- Intrinsic (undoped)
- Electrons are all from the valance



In k space

If we know:

- How many number of states in ΔE
- Probability of each state is occupied by an electron



spin

$$g(E) = \frac{dV_k}{dE} = 2\,\frac{2\pi(2m^*)^{3/2}}{h^3}\sqrt{E - E_c}$$

3.5 Density of States Function

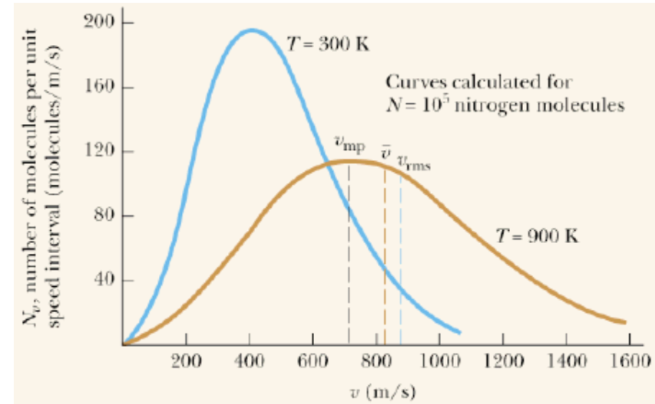
Problem Example #1

Determine the number of quantum states (per unit volume) in silicon between (E_v-kT) and E_v at 300K.

Outline

- 3.1 Allowed and Forbidden Energy Bands
- 3.2 Electrical Conduction in Solids
- 3.3 Extension to Three Dimensions
- 3.4 Effective Mass
- 3.5 Density of States Function
- **3.6 Statistical Mechanics**

3.6 Statistical mechanics



<u>Maxwell-Boltzmann probability function</u>:

- distinguishable
- no limit on the particle number in each state
- Example: gas molecules in a container

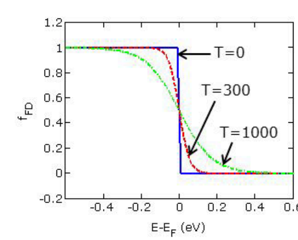
<u>Bose-Einstein probability function</u>:

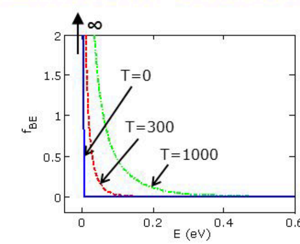
- indistinguishable,
- no limit on the particle number in each state
- Example: photons

<u>Fermi-Dirac probability function</u>:

- indistinguishable
- one particle limit in each state
- Example: electrons in solids

Fermi-Dirac vs. Bose-Einstein Statistics





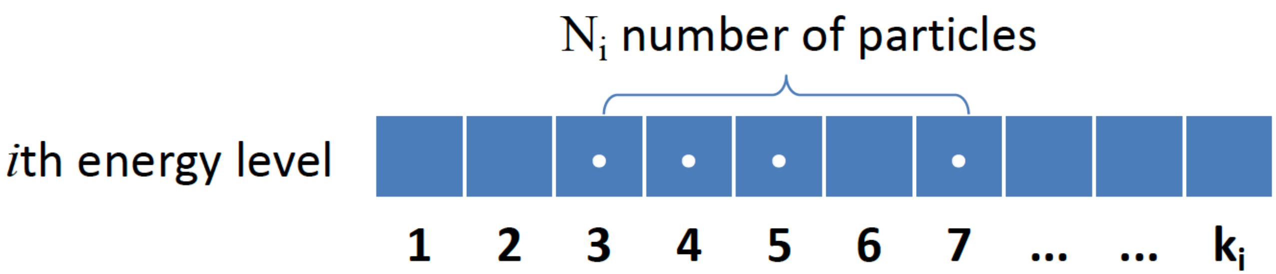
$$f_{FD}(E)=\frac{1}{\exp\left(\frac{E-E_F}{k_BT}\right)+1}$$

$$f_{BE}(E)=\frac{1}{\exp\left(\frac{E}{k_BT}\right)-1}$$

3.6 Statistical mechanics

<u>Fermi-Dirac probability function</u>:

- indistinguishable
- one particle limit in each state
- Example: electrons in solids



The totoal number of ways of arranging N_i particles in each ith energy level

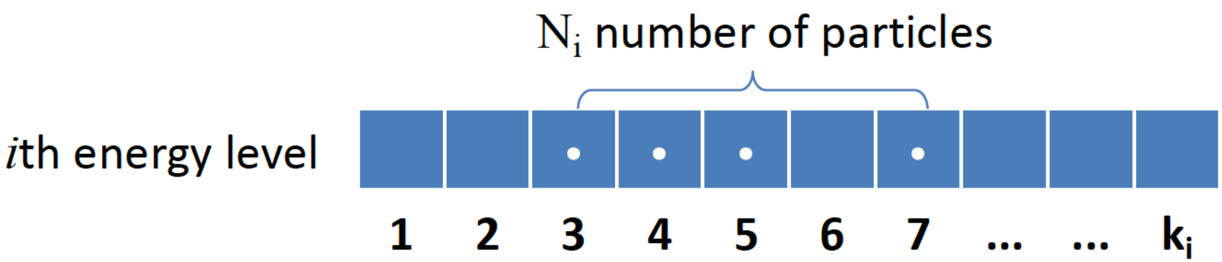
$$k_i(k_i - 1)\cdots(k_i - (N - 1)) = \frac{k_i!}{(k_i - N_i)}$$

(Particles are distinguishable)

3.6 Statistical mechanics

Fermi-Dirac probability function:

- indistinguishable
- one particle limit in each state
- Example: electrons in solids



The totoal number of ways of arranging N_i indistinguishable particles in each ith energy level

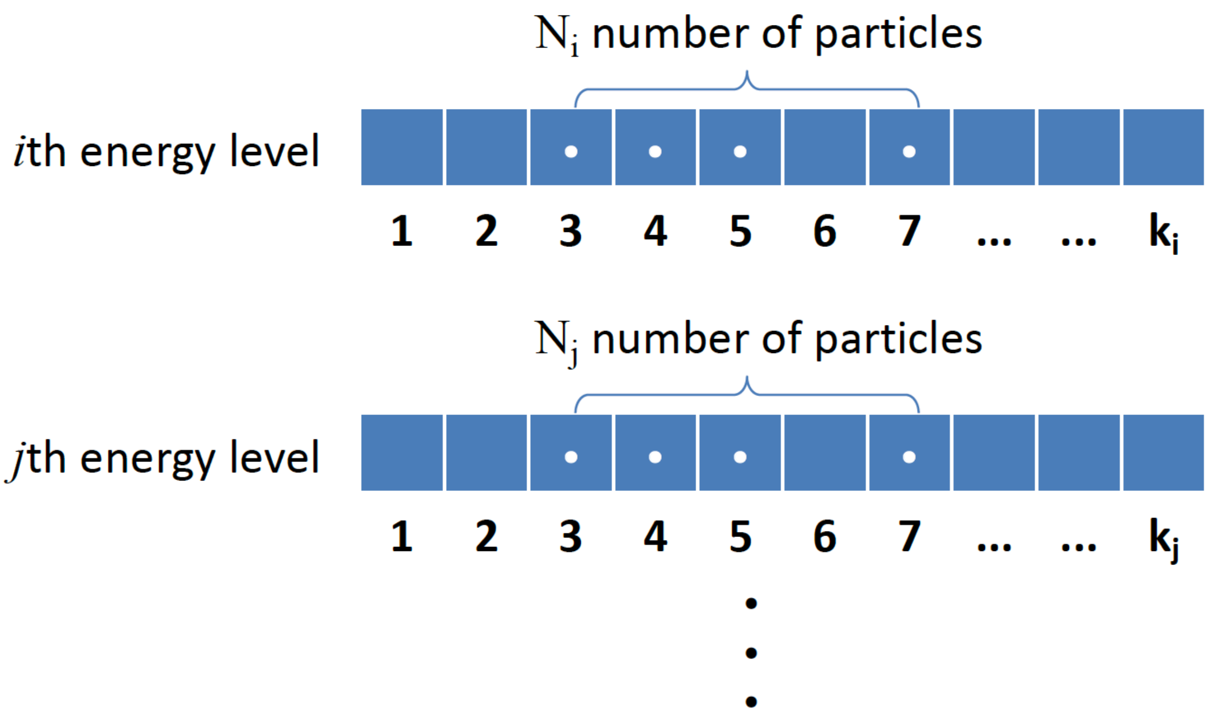
$$W_i = \frac{k_i!}{N_i!(k_i - N_i)!}$$

(Particles are indistinguishable)

3.6 Statistical mechanics

Fermi-Dirac probability function:

- indistinguishable
- one particle limit in each state
- Example: electrons in solids



$$W_i = \frac{k_i!}{N_i!(k_i - N_i)!}$$

$$W_j = \frac{k_j!}{N_j!(k_j - N_j)!}$$

$$\vdots$$

3.6 Statistical mechanics

For a given totoal number (N) of particles, the total number of ways of arranging indistiguishable particles among n energy levels is

$$W = \prod_{i=1}^{n} \frac{k_i!}{N_i!(k_i - N_i)!}$$

$f_F(E)$

The highest probable distribution at following given constraints:

$$N = \sum_{i=1}^{n} N_i \qquad \text{constant}$$

$$E_{total} = \sum_{i=1}^{n} E_i N_i \qquad \text{constant}$$

3.6 Statistical mechanics

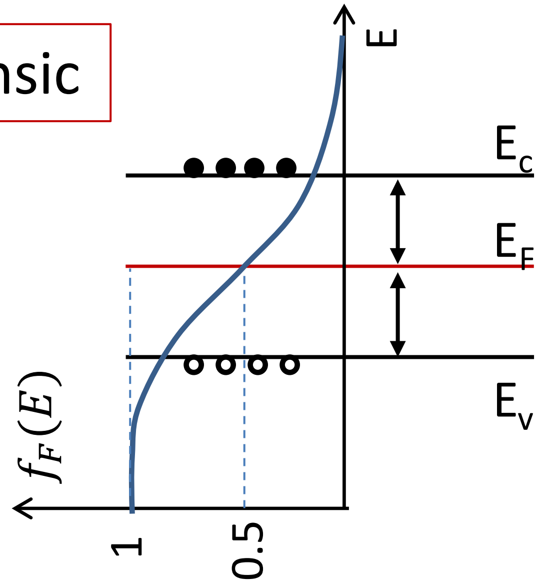
The probability of a state at energy E being occupied by an electron:

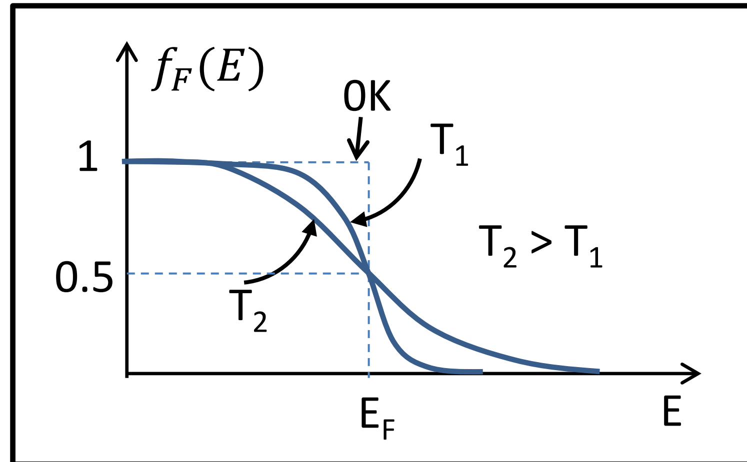
$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

E is the energy level; EF is the Fermi energy level; k is the Boltzmann constant; T is the absolute temperature.

3.6 Fermi distribution and Fermi level

Intrinsic





$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

Probability of a state at E_c occupied

=

Probability of a state at E_v unoccupied

<u>Physical meaning of Fermi energy level:</u>

At equilibrium, when an electron is added the system, the change of the system energy

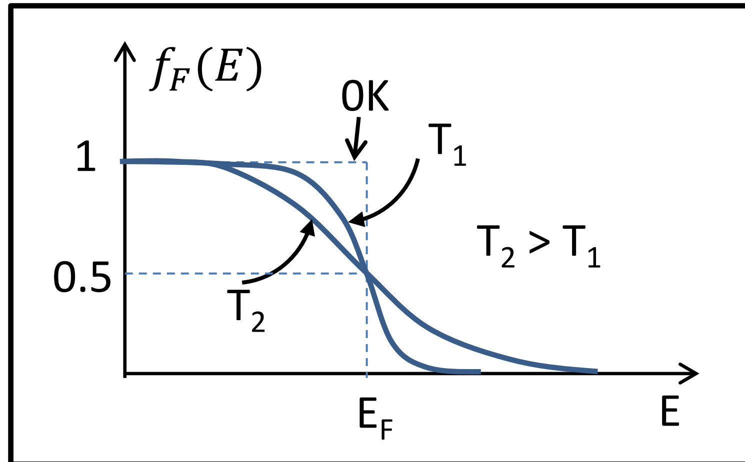
3.6 Boltzmann distribution

$$when\ \exp\left(\frac{E-E_F}{kT}\right) \gg 1 \Rightarrow E - E_F > 2kT$$

$$\boxed{f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}}$$

$$\boxed{f_F(E) \approx \exp(-\frac{E-E_F}{kT})}$$

Boltzmann distribution



$$\boxed{f_F(E) = \frac{1}{1+\exp(\frac{E-E_F}{kT})}}$$

3.6 Boltzmann distribution

Problem Example #2

Assume that the Fermi energy level is 0.35eV above the valence band energy. Let T=300K. Determine the probability of a state being empty of an electron at $E = E_v - kT/2$.