# VE320 – Summer 2021

## Semiconductor Physics

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# Chapter 10 Fundamentals of Metal-Oxide-Semiconductor Field Effect Transistors

# Outline

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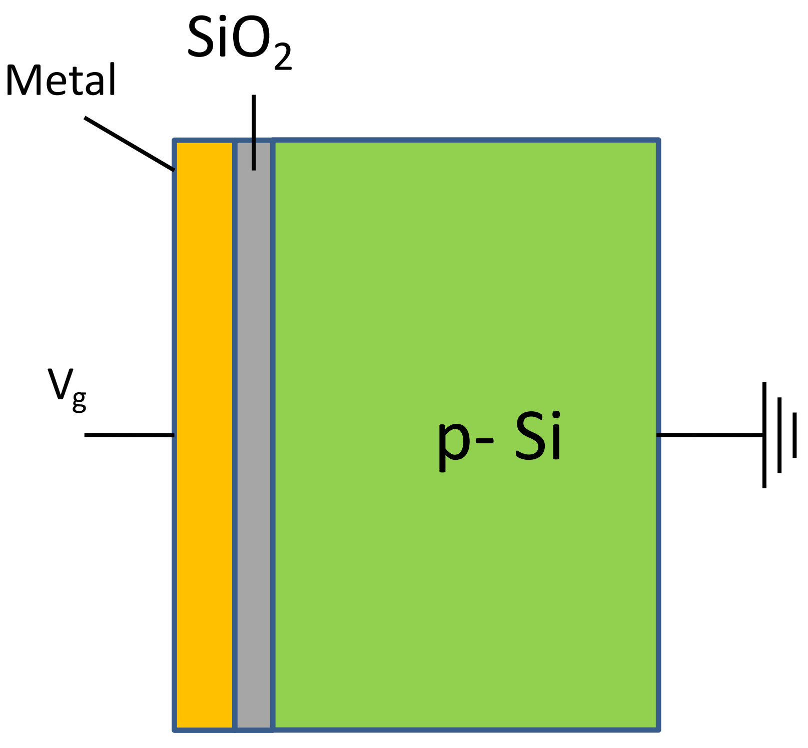
**10.1 The two-terminal MOS structure**

10.2 Capacitance-voltage characteristics

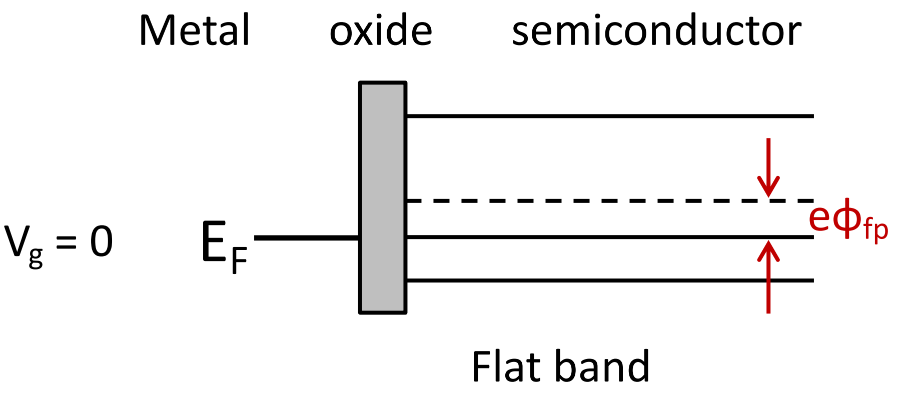
10.3 Non-ideal effects

10.4 The basic MOSFET operation

# 10.1 The two-terminal MOS structure




Metal-insulator-semiconductor (MIS)



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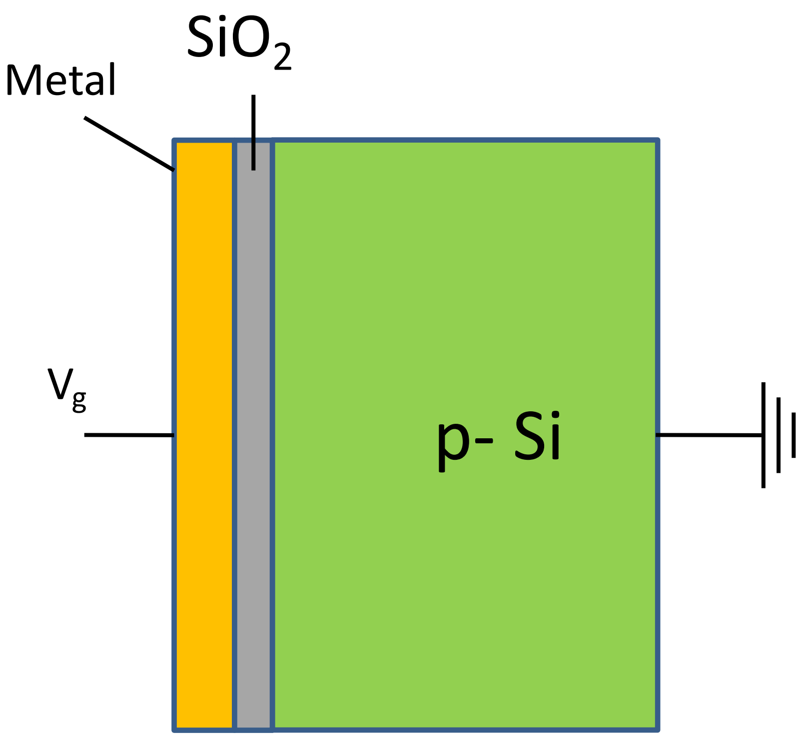


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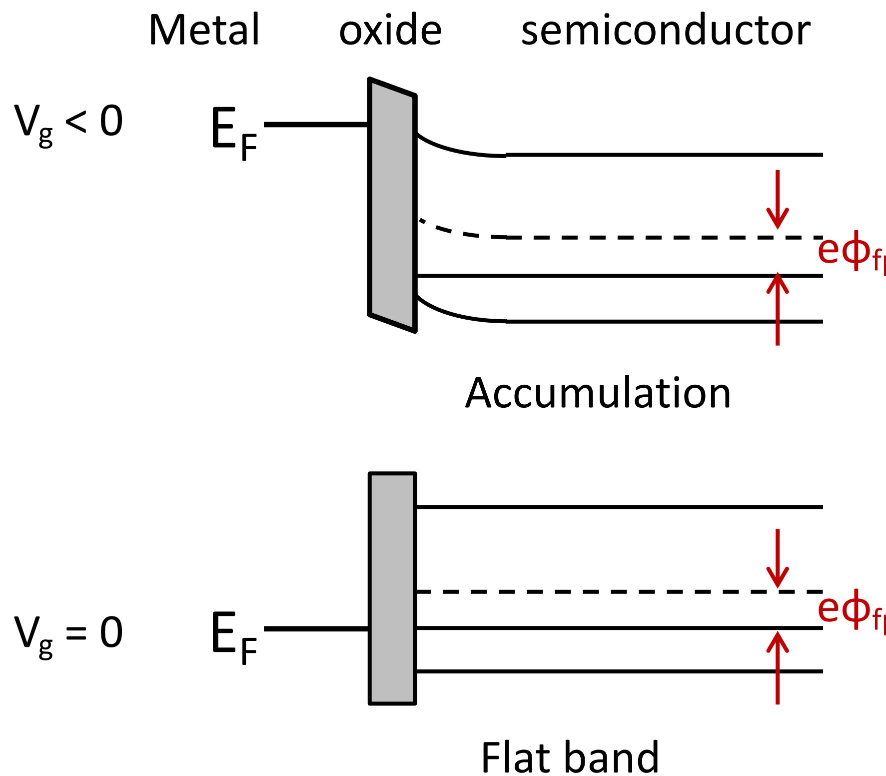

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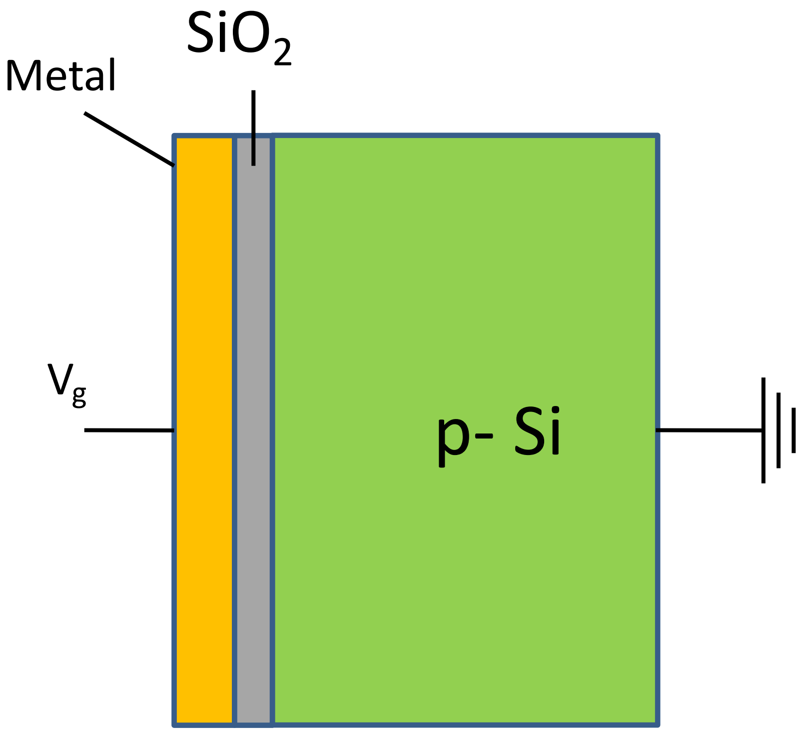


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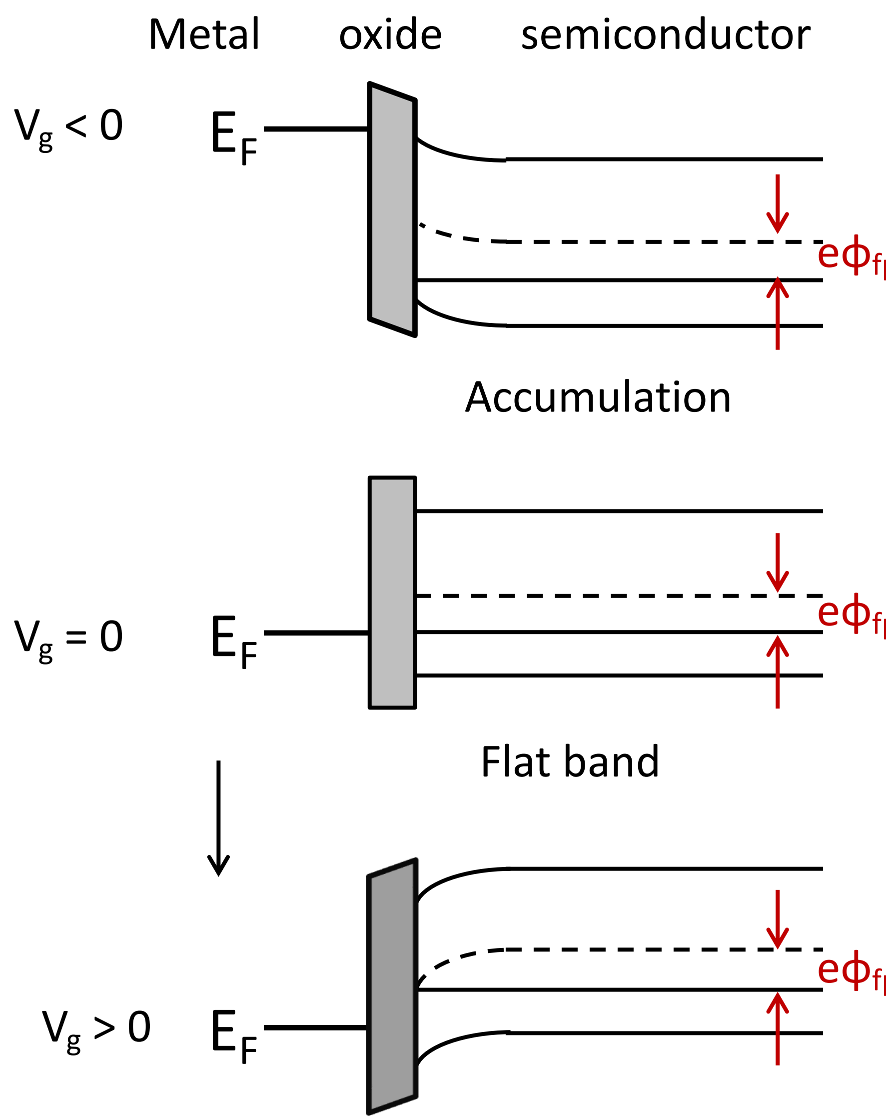

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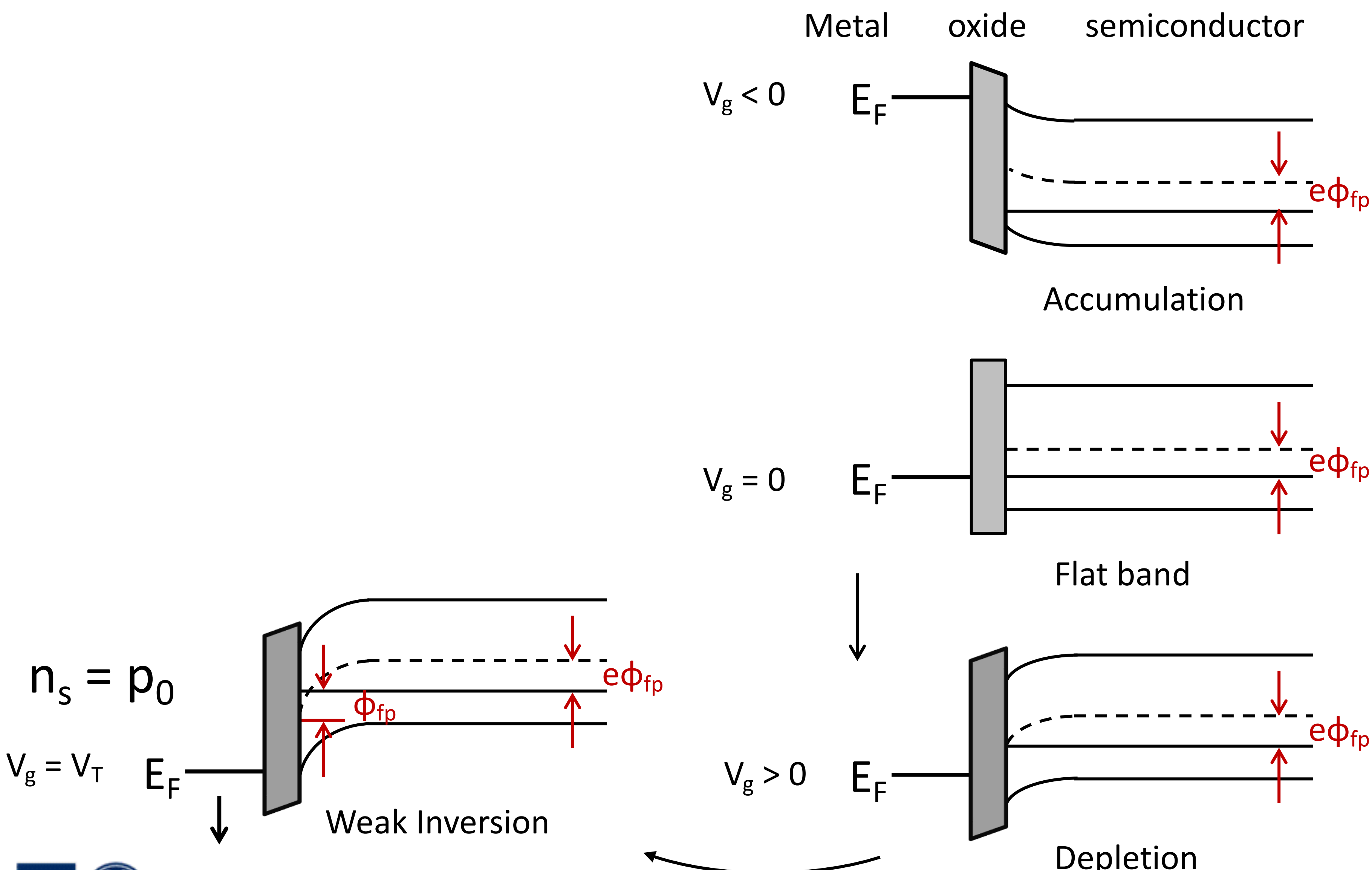

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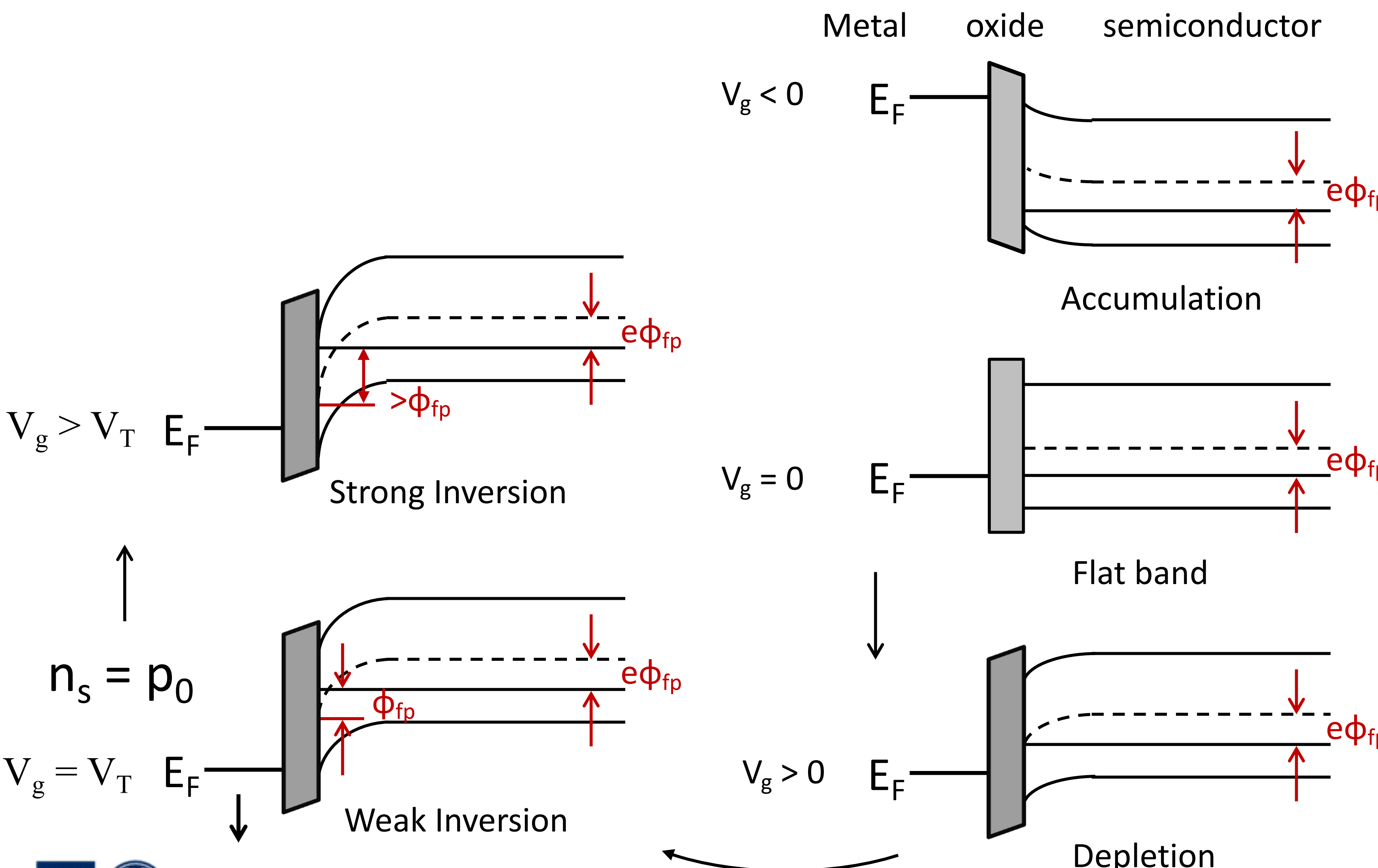

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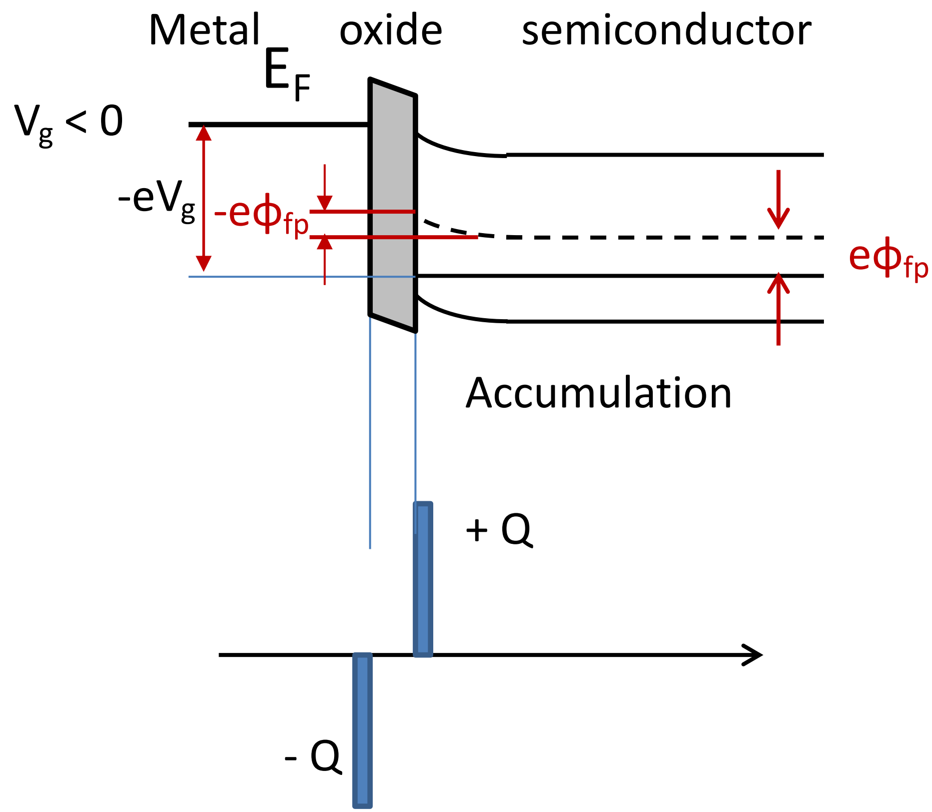
<u>Charge distribution</u>



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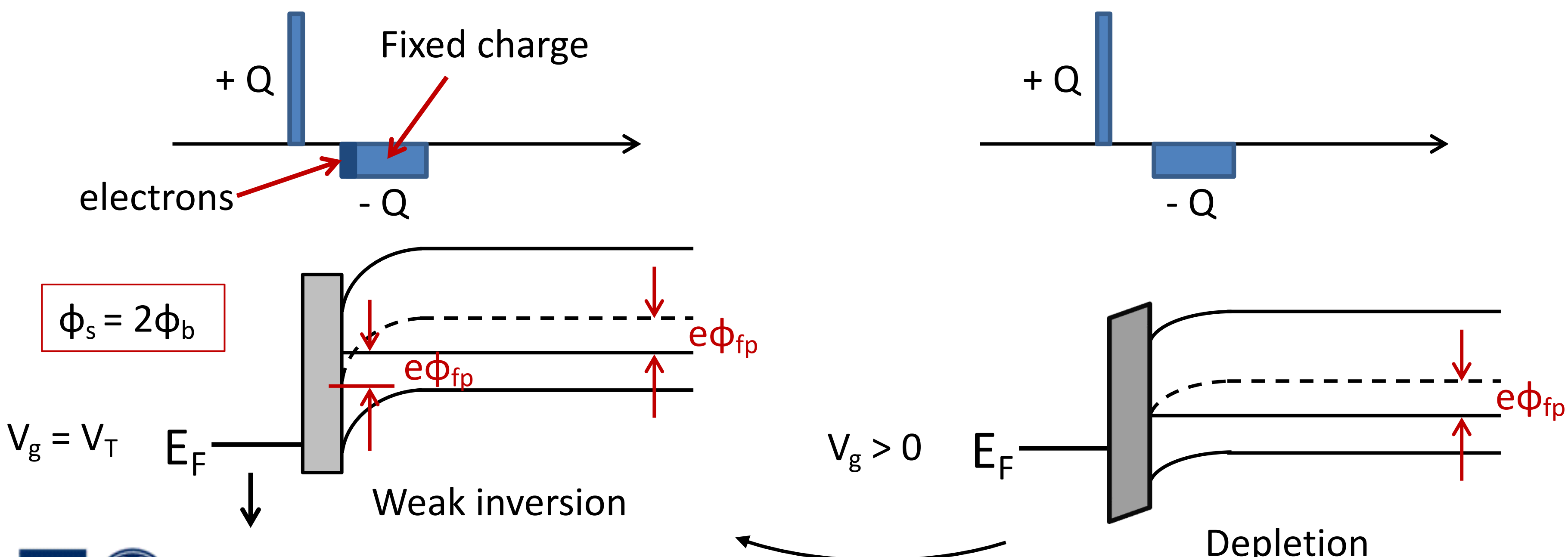
## Charge distribution



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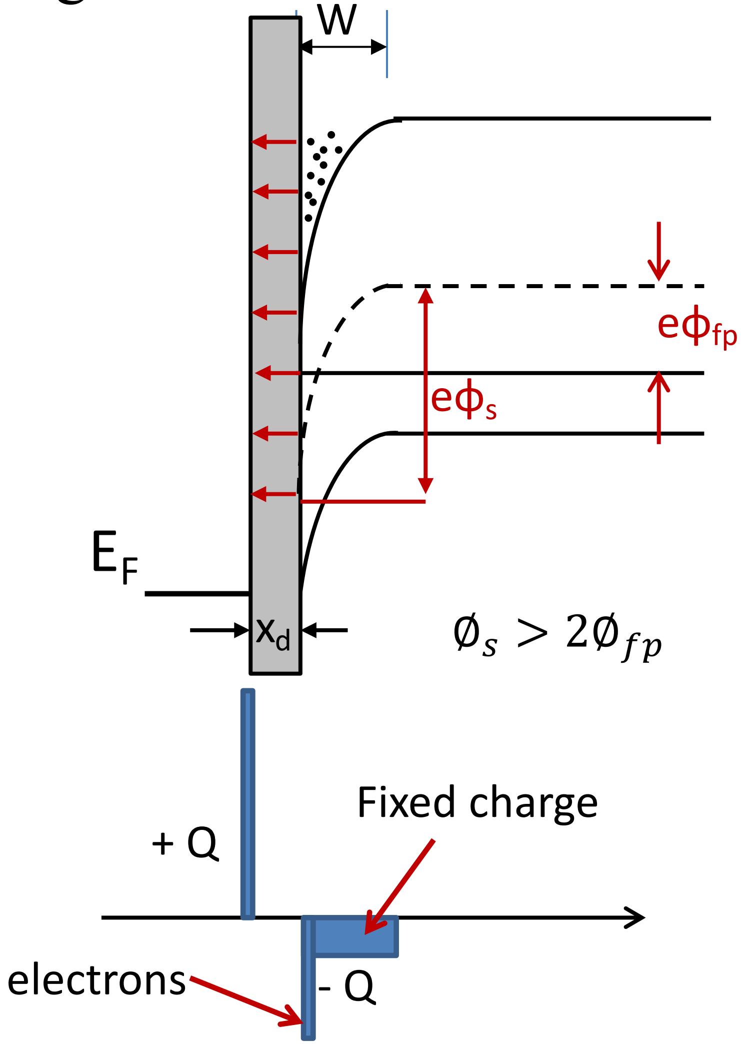
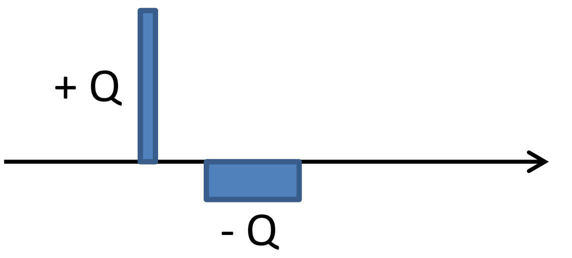
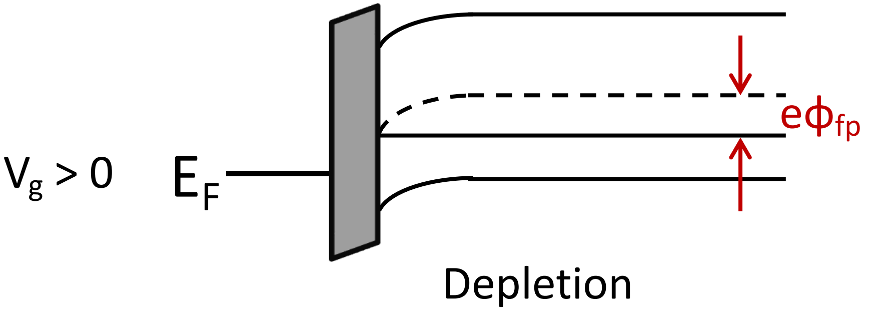


# 10.1 The two-terminal MOS structure

Depletion layer thickness




$$x_d = \left( \frac{2\epsilon_s \phi_s}{eN_a} \right)^{1/2}$$



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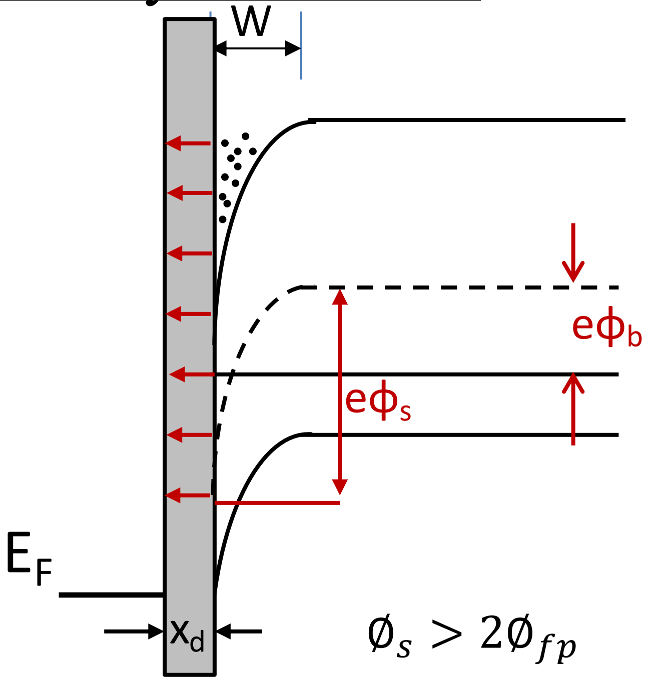
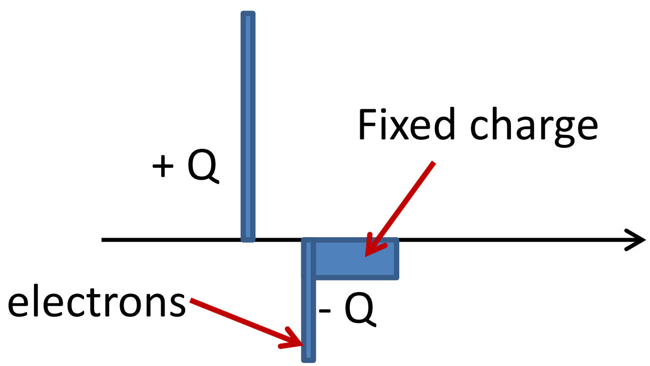
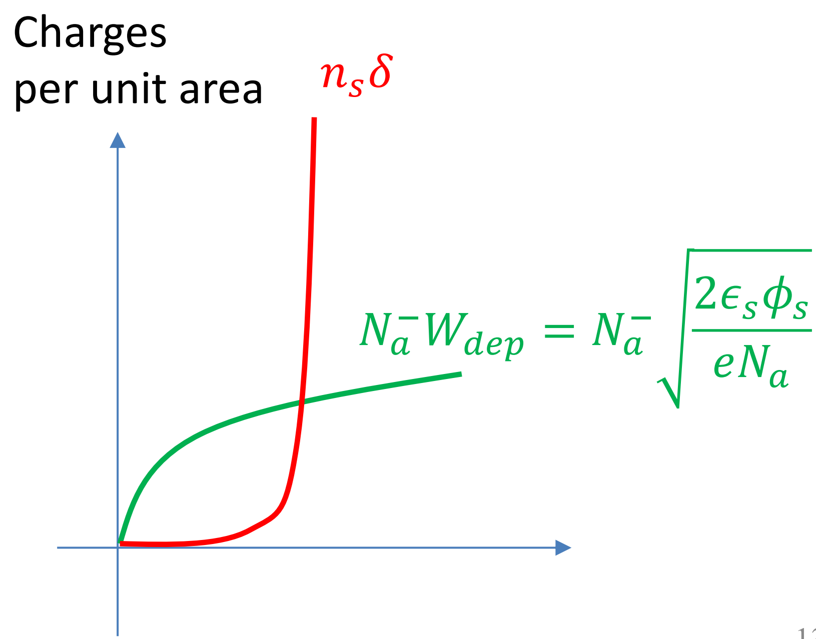



$\emptyset_s > 2\emptyset_{fp}$




Maximum depletion layer

$$\boxed{x_{dT} = \left(\frac{4\epsilon_s \phi_{fp}}{eN_a}\right)^{1/2}}$$



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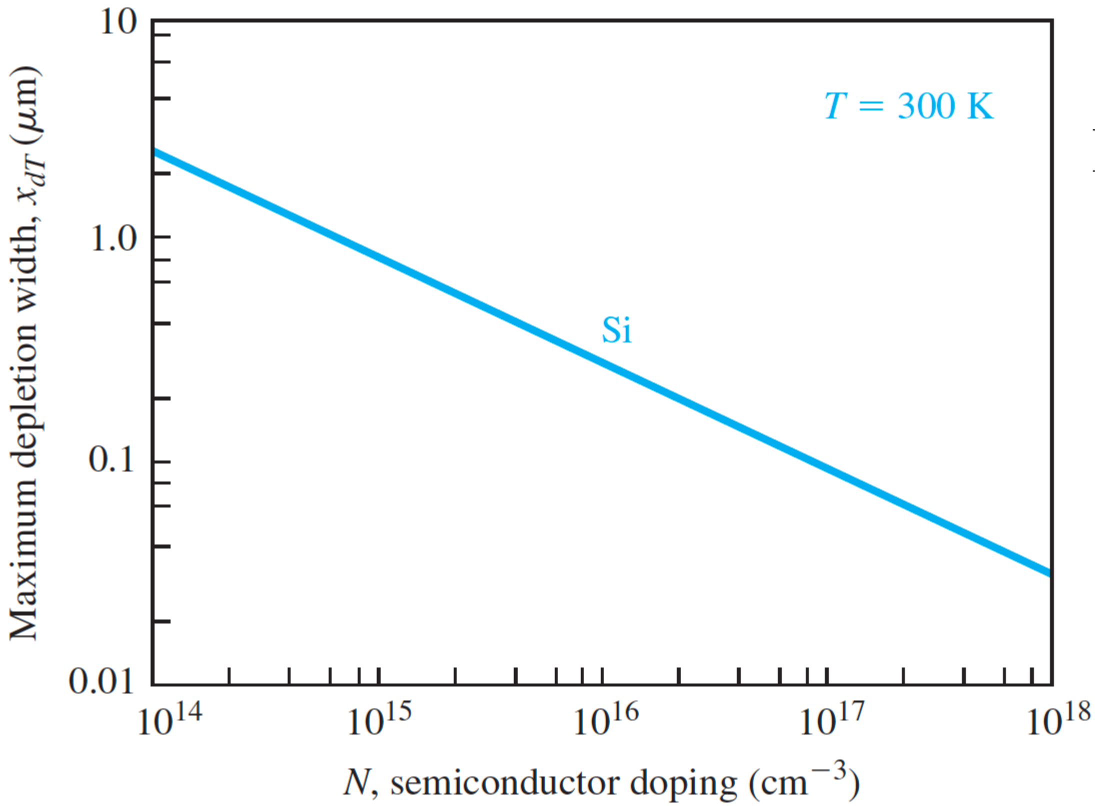


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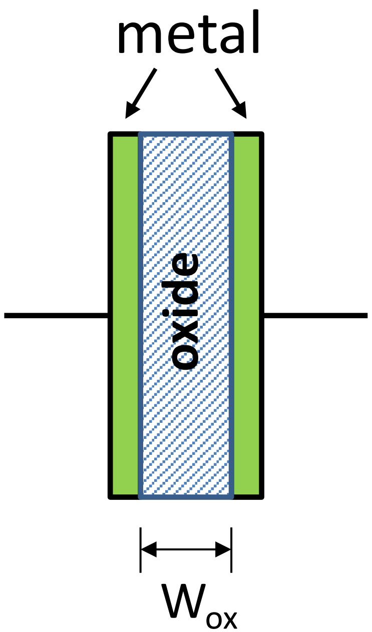
**10.2 Capacitance-voltage characteristics**

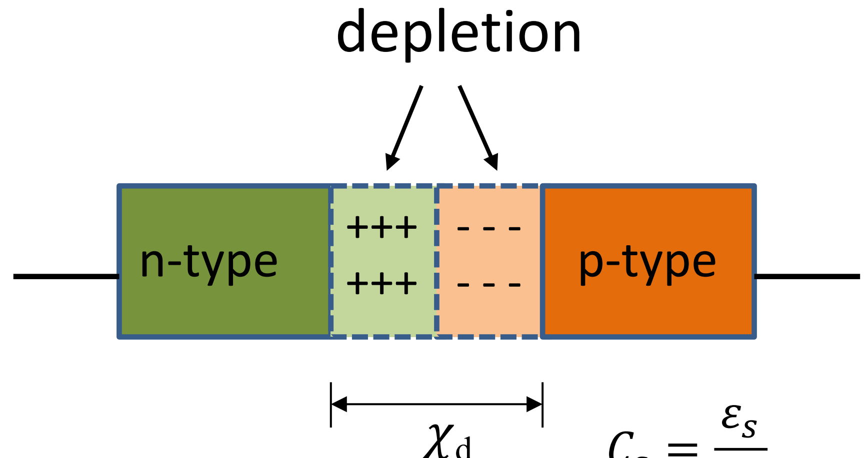
10.3 Non-ideal effects

10.4 The basic MOSFET operation

# 10.2 The capacitance-voltage characteristics



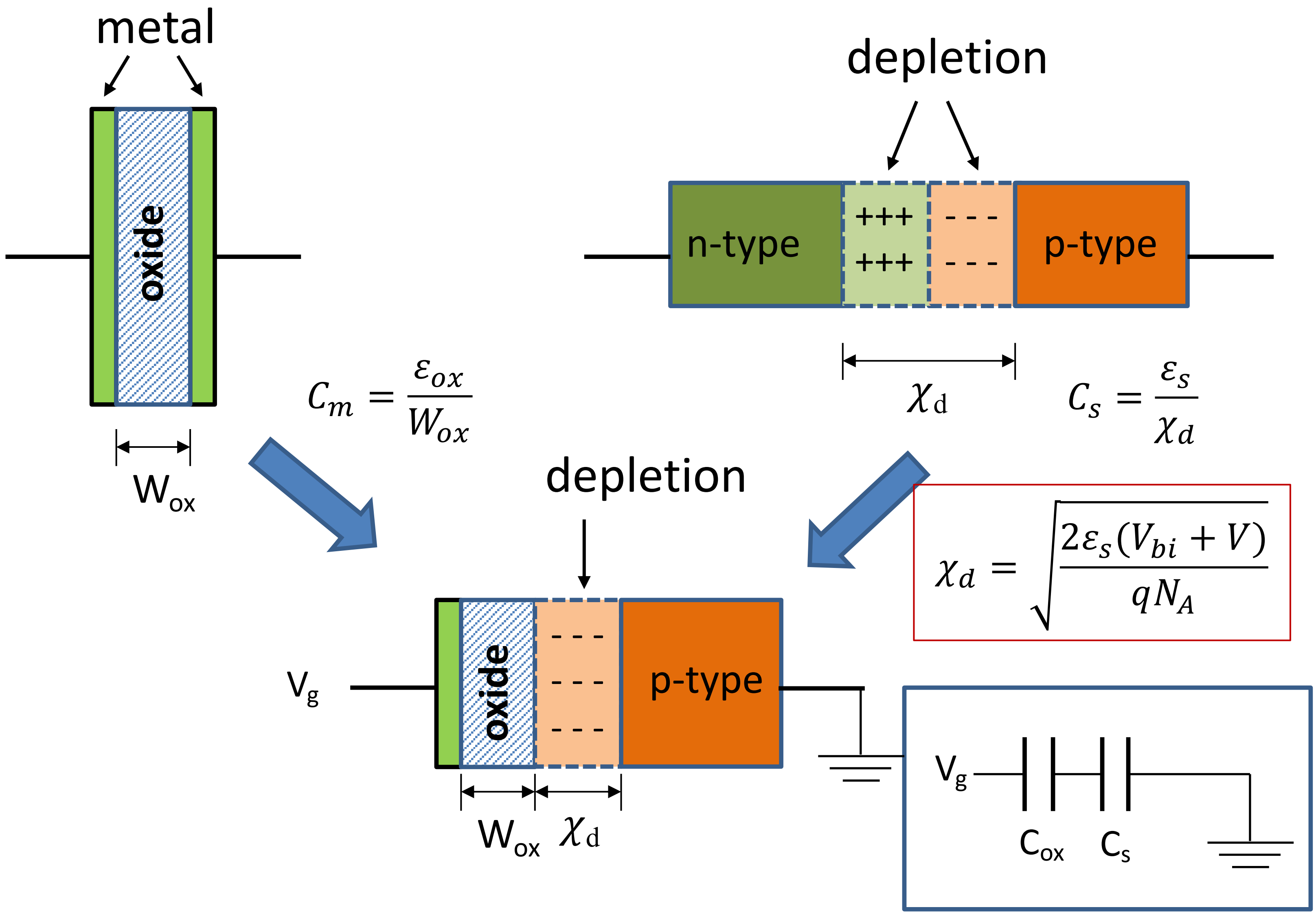

$$C_m = \frac{\varepsilon_{ox}}{W_{ox}}$$




$$C_s = \frac{\varepsilon_s}{\chi_d}$$

$$\boxed{\chi_d = \sqrt{\frac{2\varepsilon_s(V_{bi}+V)}{qN_A}}}$$

# 10.2 The capacitance-voltage characteristics

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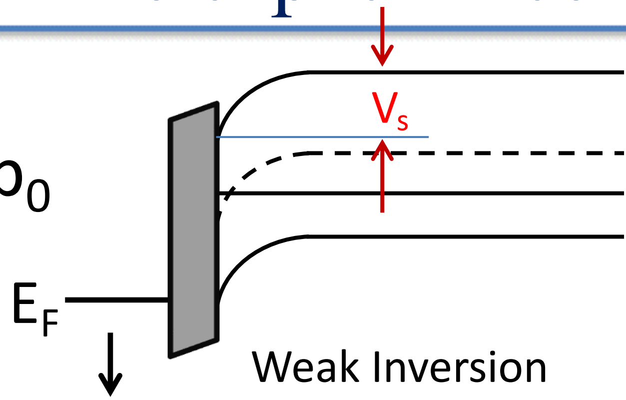
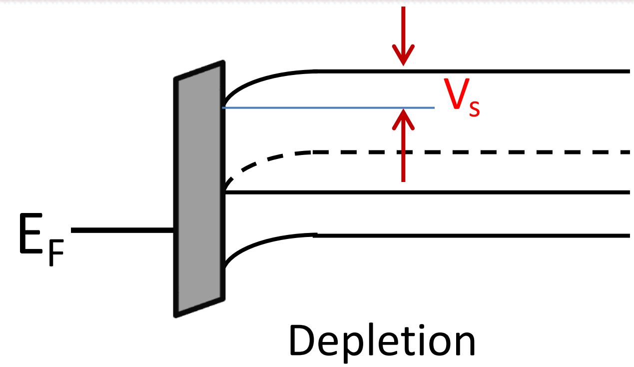
# 10.2 The capacitance-voltage characteristics

$n_s = p_0$

$V_g = V_T$




$V_g > 0$



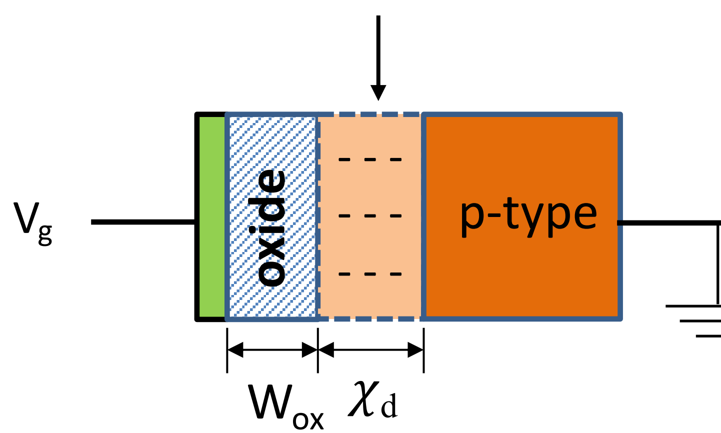

$$C_s = \frac{\varepsilon_s}{\chi_d}$$

Net charge is zero!

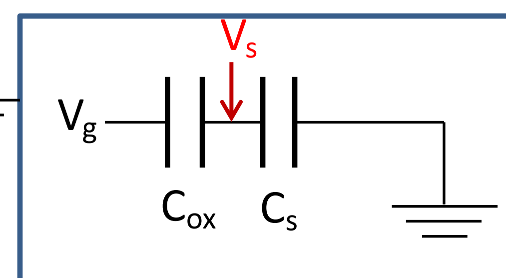
$$(V_g - V_s)C_{ox} = V_s C_s$$

$$V_g C_{ox} = V_s C_{ox} + V_s C_s$$

$$V_s = \frac{V_g C_{ox}}{C_{ox} + C_s}$$




$$\boxed{\chi_d = \sqrt{\frac{2\varepsilon_s(V_{bi} + V)}{qN_A}}}$$



# 10.2 The capacitance-voltage characteristics

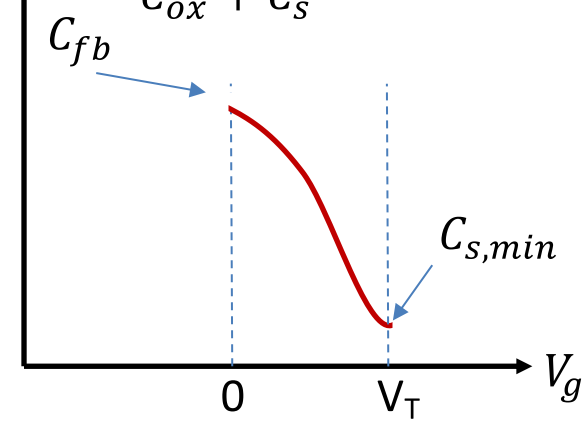
$$\left\{\begin{array}{l} \boxed{V_s = \dfrac{V_g C_{ox}}{C_{ox} + C_s}} \\[2ex] \boxed{C_s = \dfrac{\varepsilon_s}{\sqrt{\dfrac{2\varepsilon_s (V_s)}{qN_A}}}} \end{array}\right.$$

$$C'_{FB} = \frac{\epsilon_{ox}}{t_{ox} + \left(\frac{\epsilon_{ox}}{\epsilon_s}\right)\sqrt{\left(\frac{kT}{e}\right)\left(\frac{\epsilon_s}{eN_a}\right)}}$$

$$C_{min} = \frac{C_{ox} C_{s,min}}{C_{ox} + C_{s,min}}$$

$$C_{s,min} = \frac{\varepsilon_s}{\sqrt{\dfrac{4\varepsilon_s \phi_{fp}}{qN_a}}}$$

$$C = \frac{C_{ox} C_s}{C_{ox} + C_s}$$

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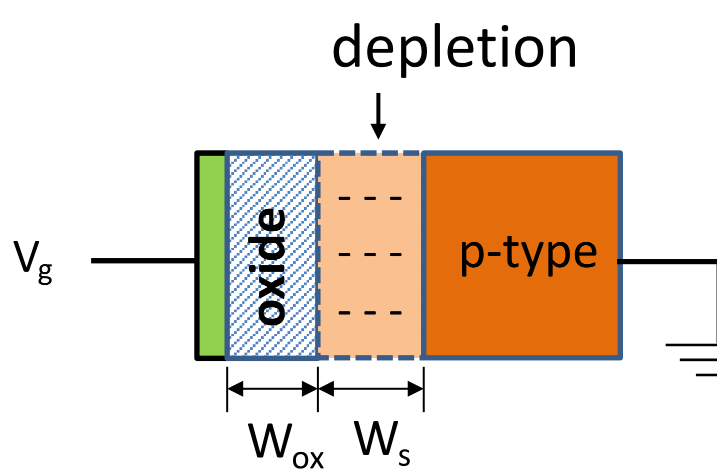
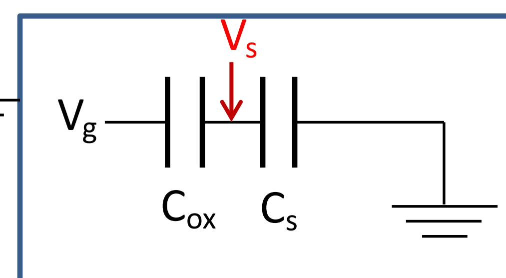
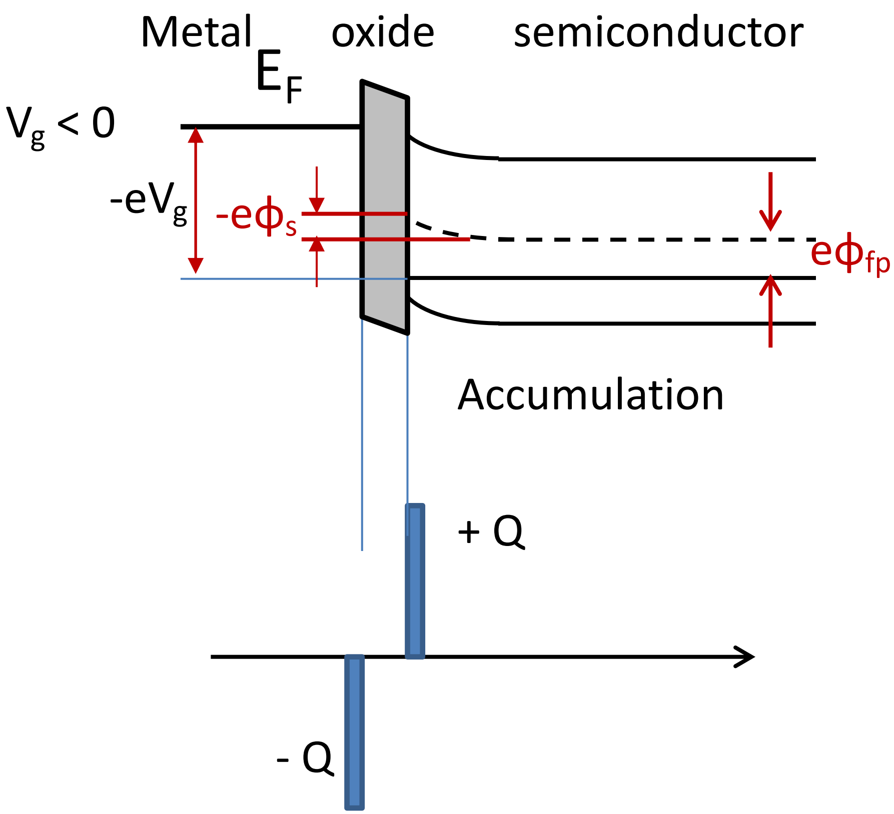



$$\boxed{W_s = \sqrt{\frac{2\varepsilon_s (V_s)}{qN_A}}}$$

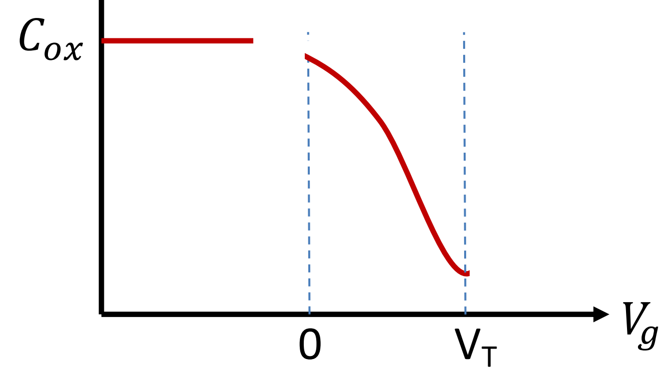


# 10.2 The capacitance-voltage characteristics

Accumulation




$$C = \frac{C_{ox} C_s}{C_{ox} + C_s}$$



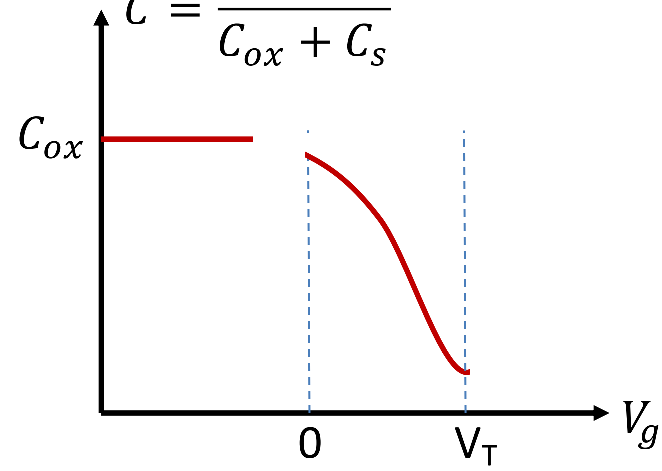

$$C_s \rightarrow \infty$$

$$C = \frac{C_s C_{ox}}{C_s + C_{ox}} \approx C_{ox}$$

# 10.2 The capacitance-voltage characteristics

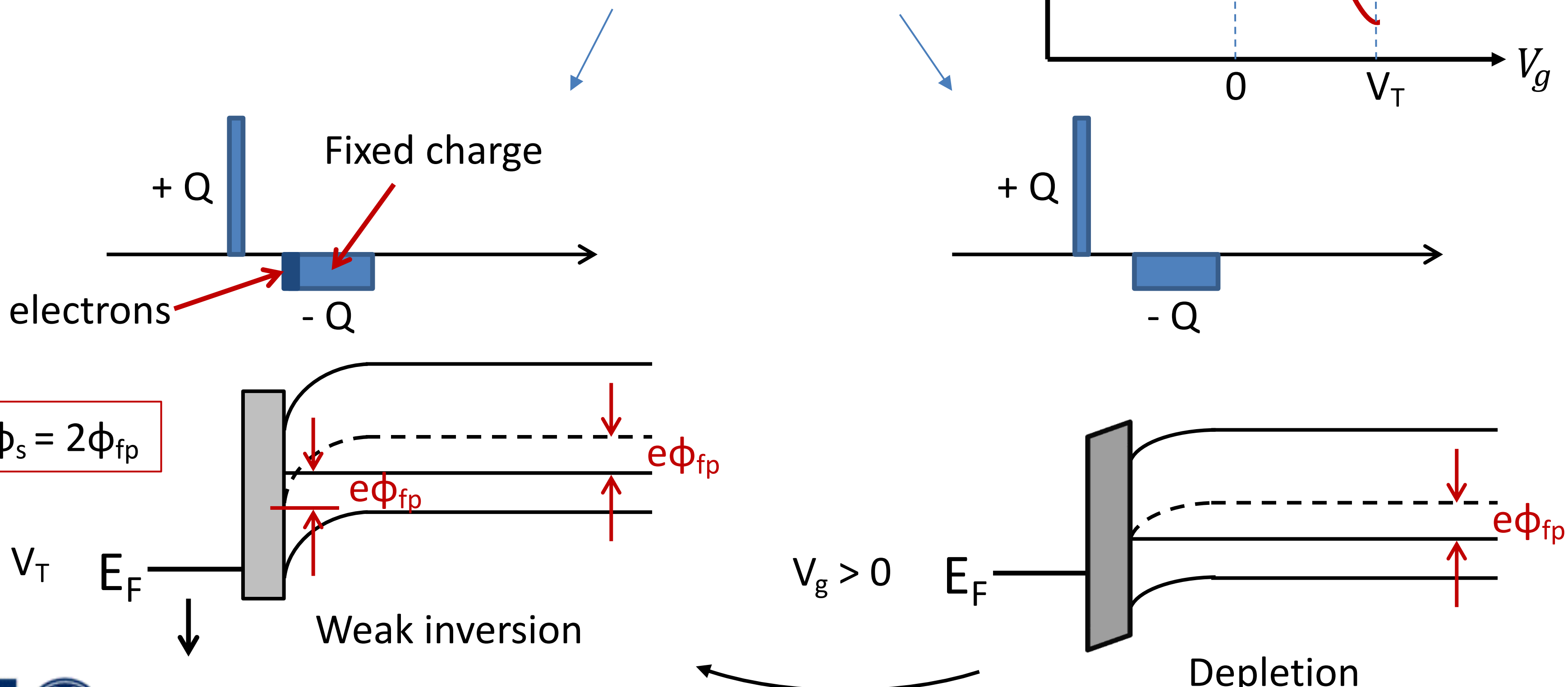
Depletion and weak inversion

$$C = \frac{C_{ox} C_s}{C_{ox} + C_s}$$



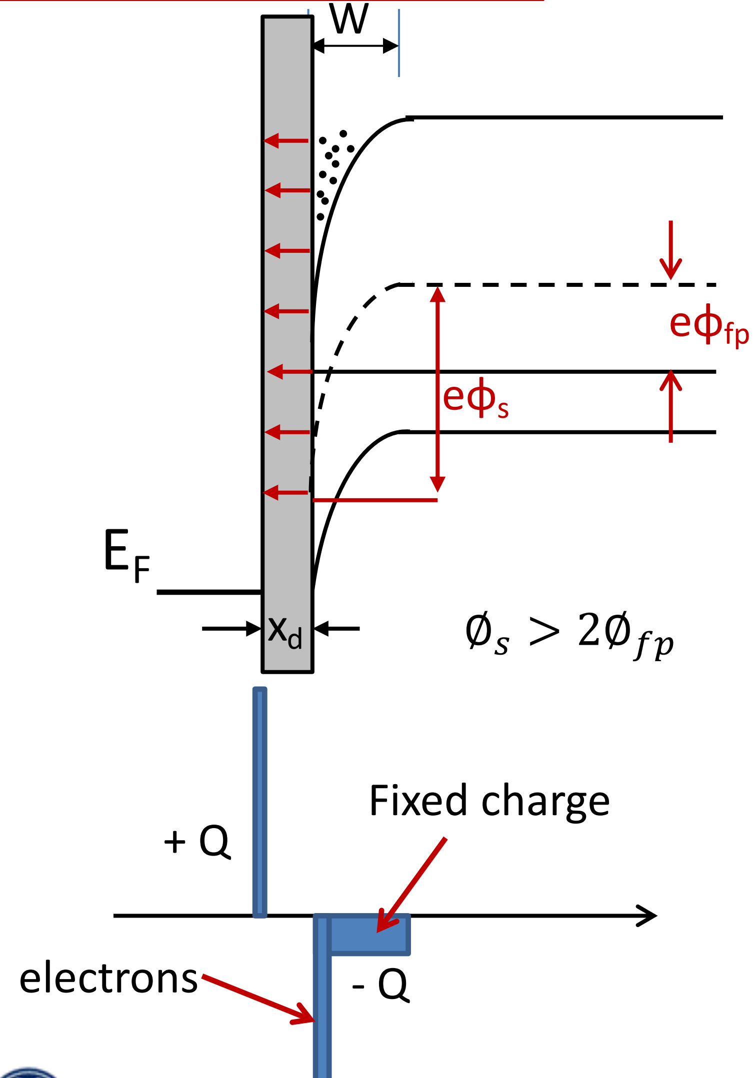

$$\boxed{C_s = \frac{\varepsilon_s}{\sqrt{\frac{2\varepsilon_s (V_s)}{qN_A}}}}$$

$\boxed{\phi_s = 2\phi_{fp}}$

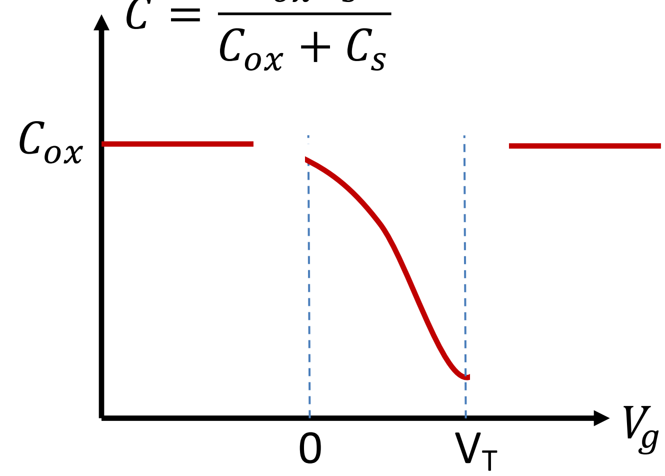


# 10.2 The capacitance-voltage characteristics

Strong inversion




$$\phi_s > 2\phi_{fp}$$




$$C = \frac{C_{ox} C_s}{C_{ox} + C_s}$$

$$C_s \rightarrow \infty$$

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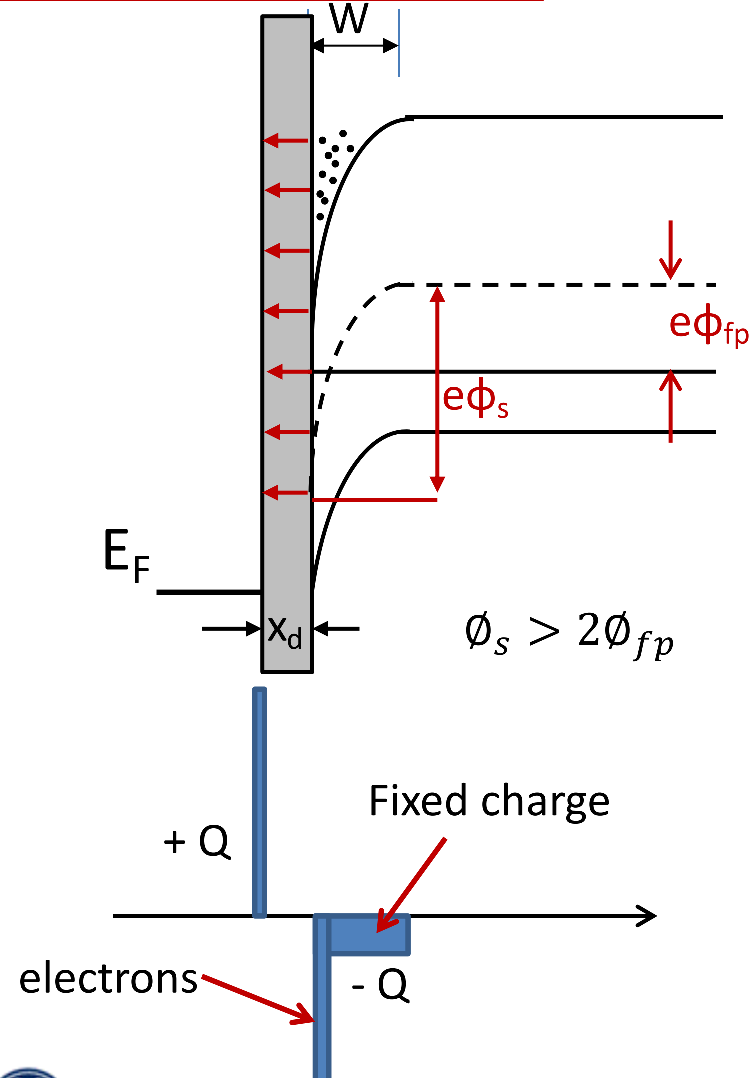
# 10.2 The capacitance-voltage characteristics

Strong inversion

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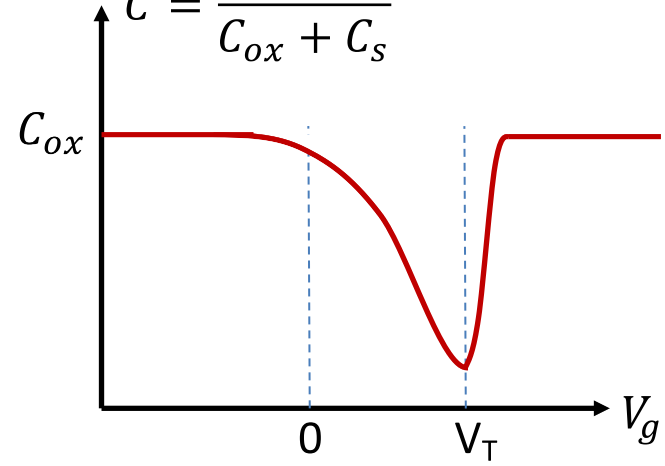


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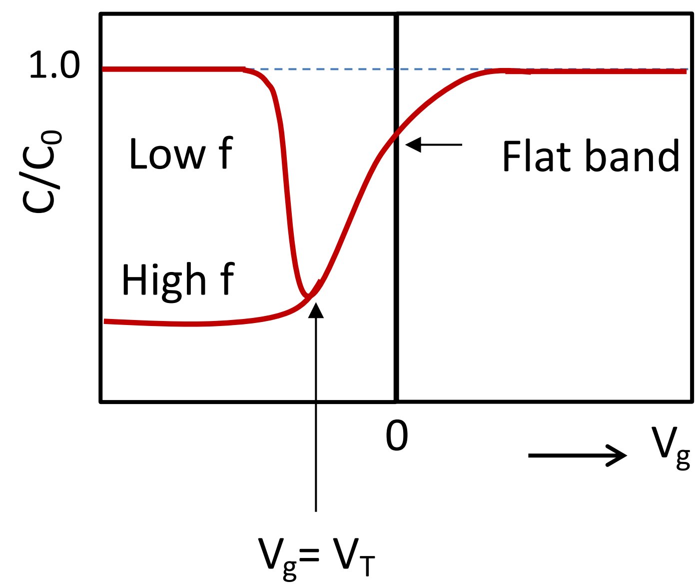
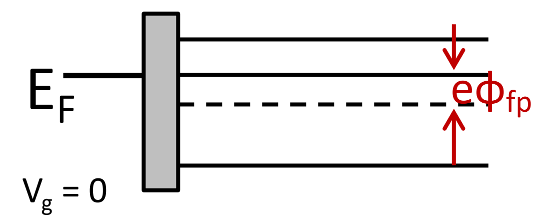
# 10.2 The capacitance-voltage characteristics

n-type semiconductor




n-type substrate

# 10.2 The capacitance-voltage characteristics

Problem Example

Consider a p-type silicon substrate at $T = 300$ K doped to $N_a = 10^{16}$ cm$^{-3}$. The oxide is silicon dioxide with a thickness of $t_{ox} = 18$ nm $= 180$ Å, and the gate is aluminum.

Calculate $C_{ox}$, $C'_{\min}$, and $C'_{FB}$ for a MOS capacitor.

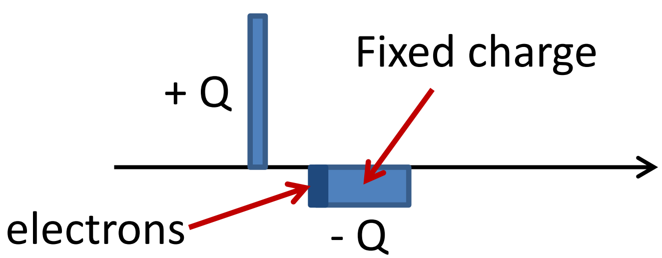
# 10.2 The capacitance-voltage characteristics

## Threshold voltage

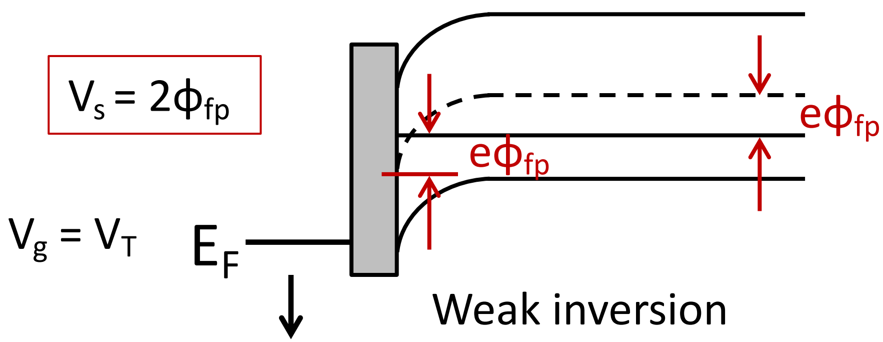
$$C_s = \frac{\varepsilon_s}{\sqrt{\frac{2\varepsilon_s(V_s)}{qN_A}}} = \frac{\varepsilon_s}{\sqrt{\frac{2\varepsilon_s(2\phi_{fp})}{qN_A}}}$$

$$2\phi_{fp} = \frac{V_T C_{ox}}{C_{ox} + C_s} \Rightarrow V_T = 2\phi_{fp} + 2\phi_{fp}\frac{C_s}{C_{ox}}$$

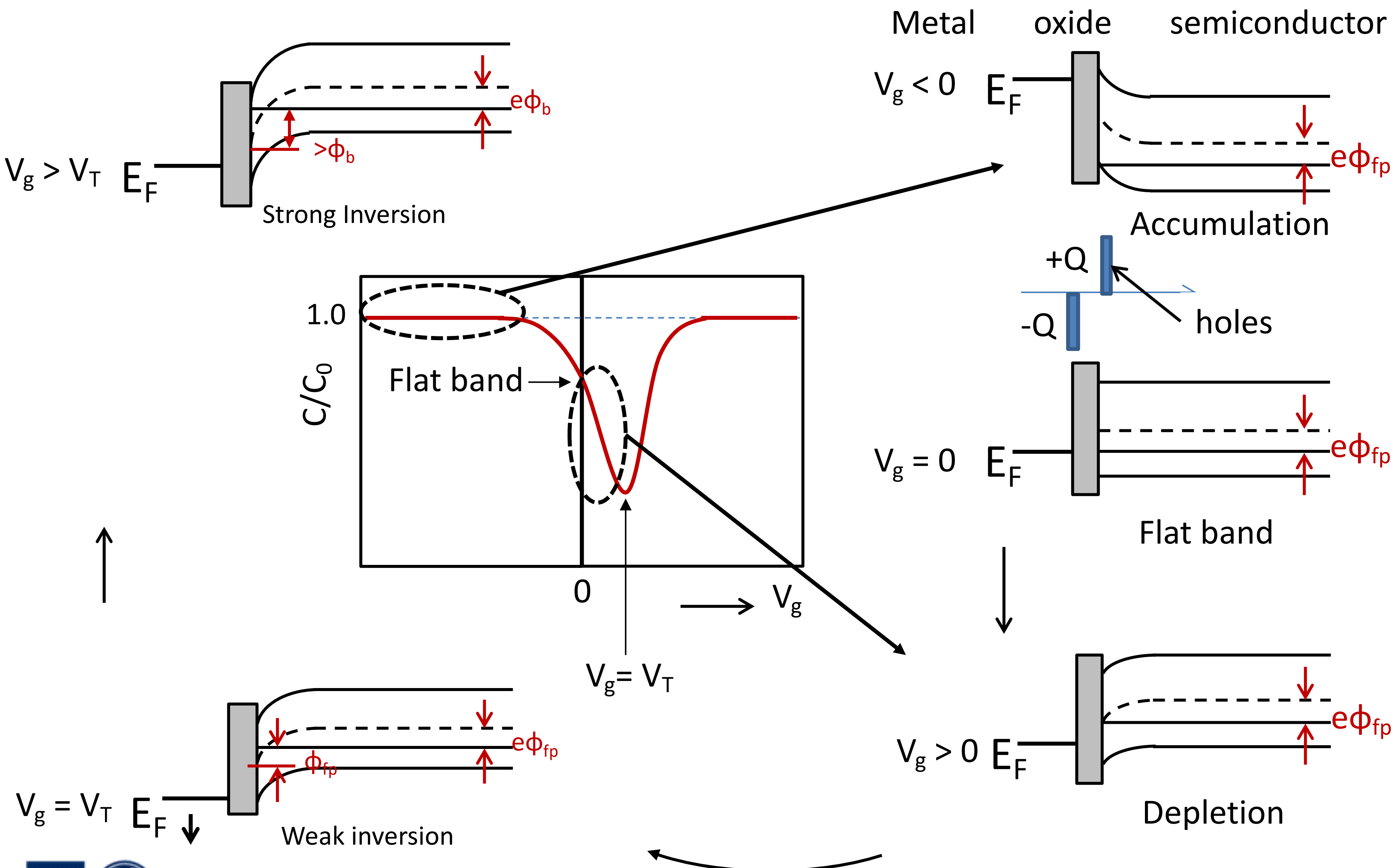
$$V_T = 2\phi_{fp} + \frac{2\sqrt{e\varepsilon_s N_A \phi_{fp}}}{C_{ox}}$$



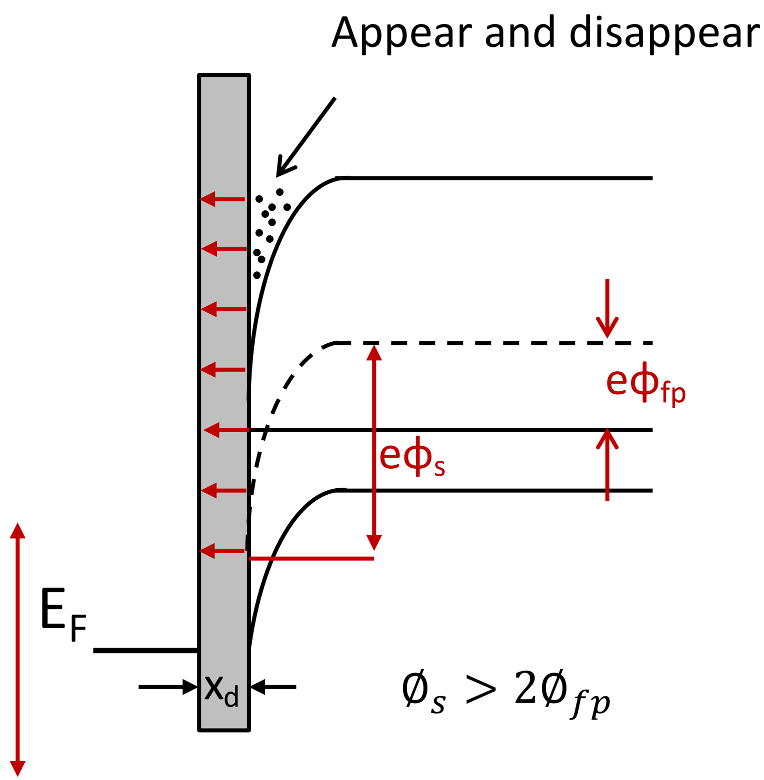

$V_s = 2\phi_{fp}$



# 10.2 The capacitance-voltage characteristics: summary



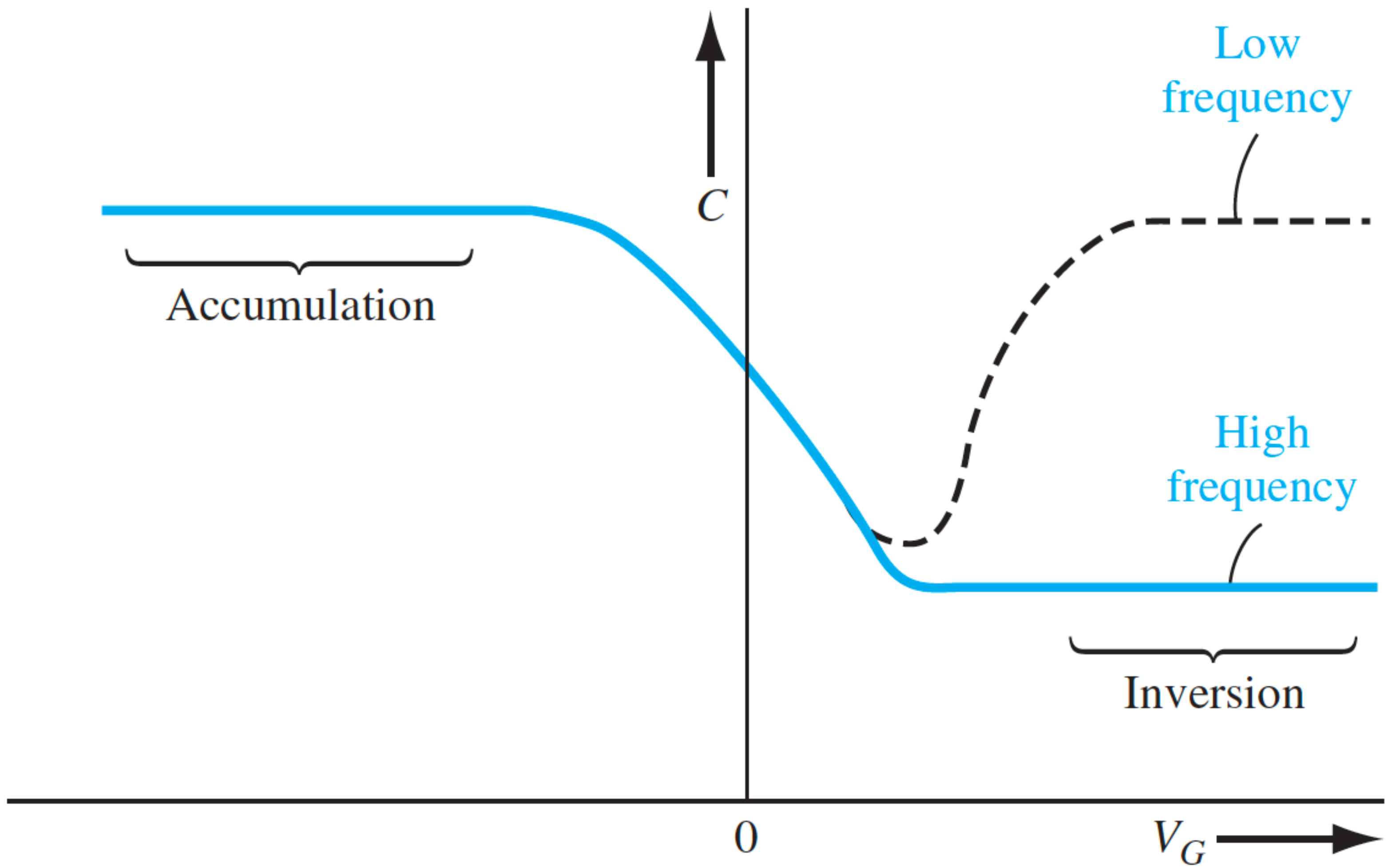
# Question:




Where are the electrons coming from and going to?

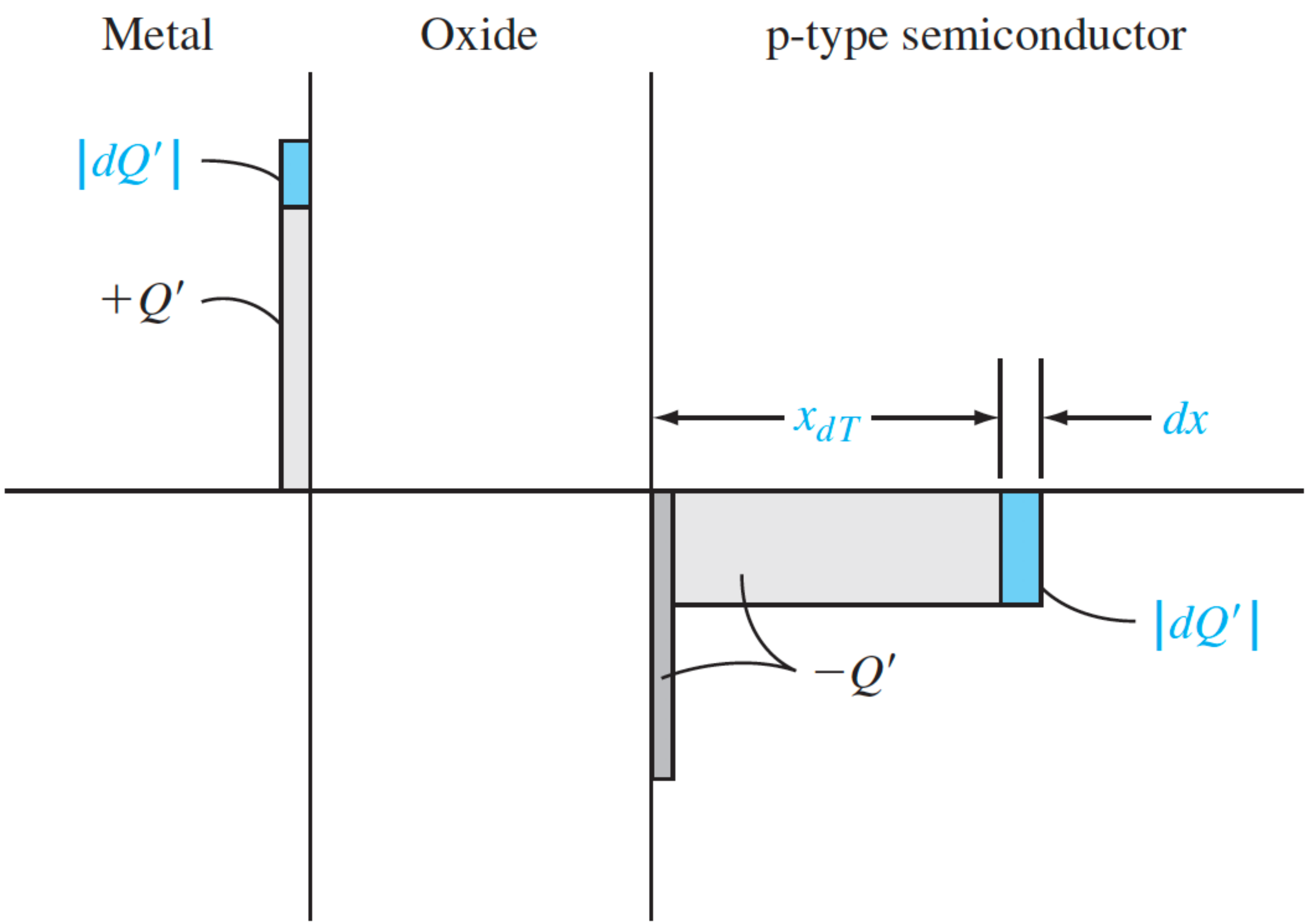
# 10.2 The capacitance-voltage characteristics

## Frequency dependence



# 10.2 The capacitance-voltage characteristics

## Frequency dependence



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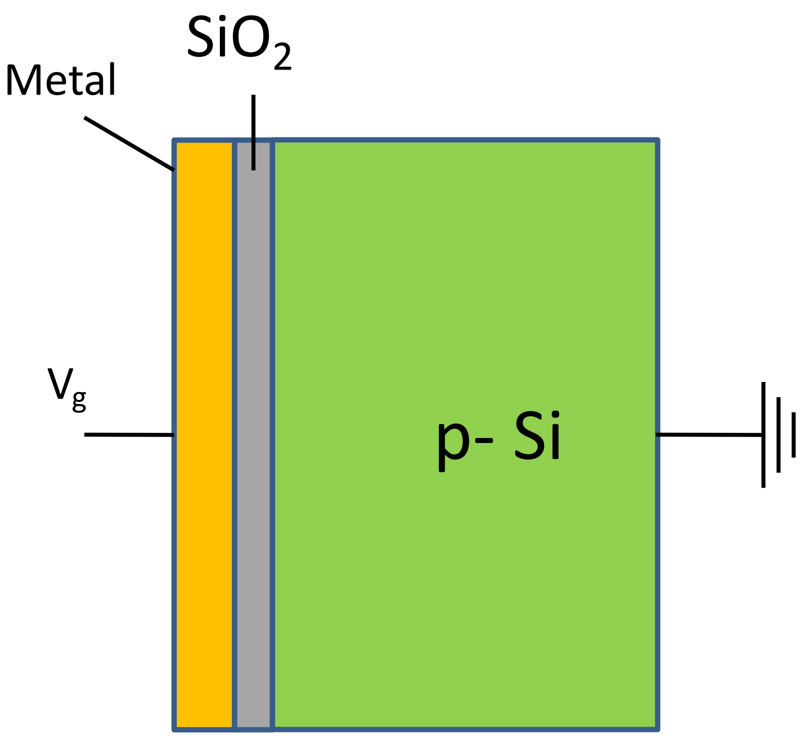
10.2 Capacitance-voltage characteristics

**10.3 Non-ideal effects**

10.4 The basic MOSFET operation

# 10.3 Non-ideal effects

## Work function difference

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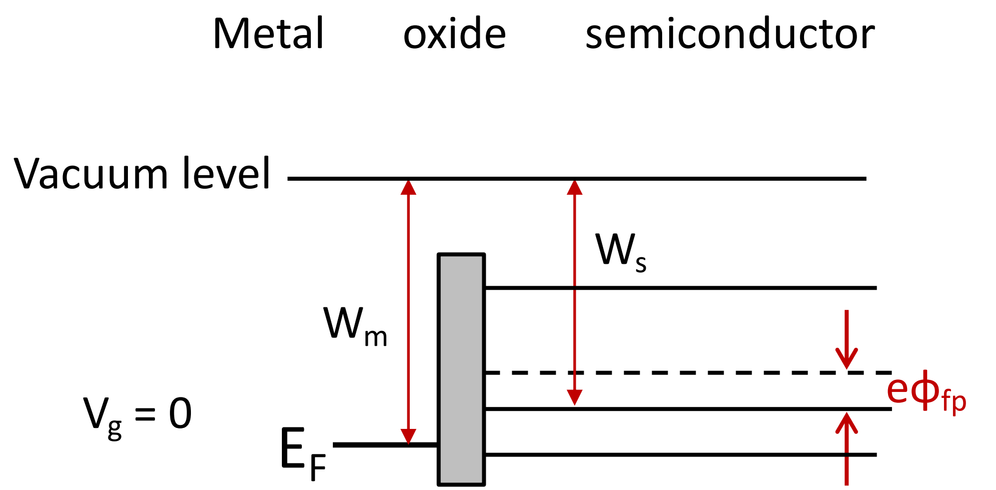
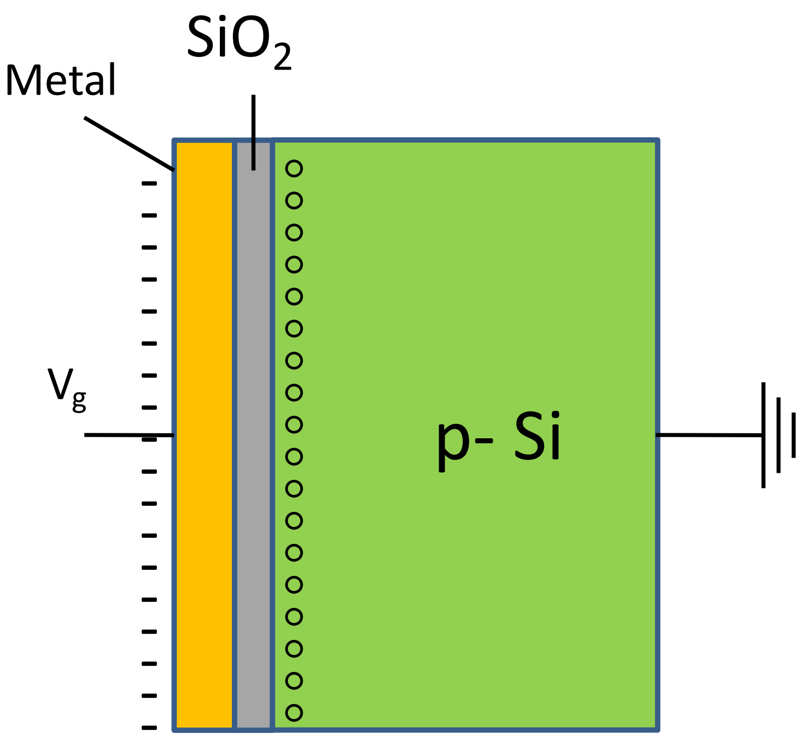



$V_{FB} = 0$

# 10.3 Non-ideal effects

## Work function difference

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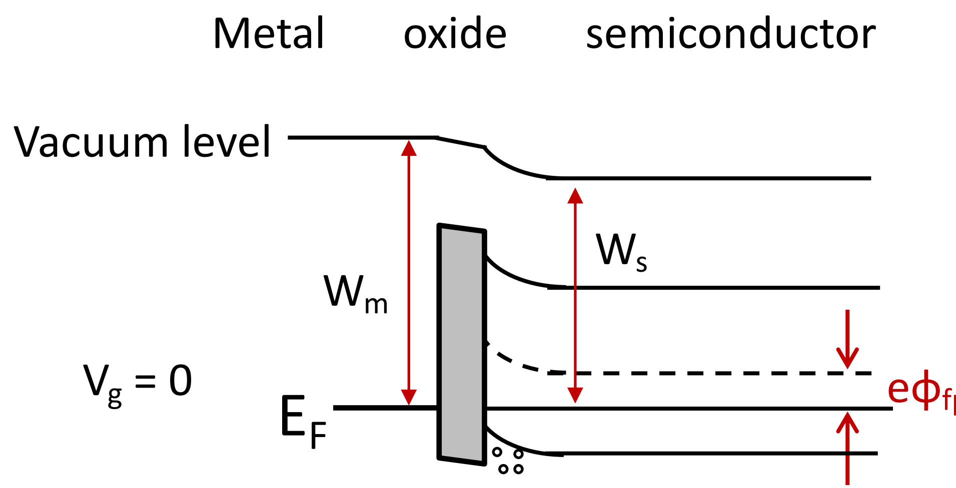

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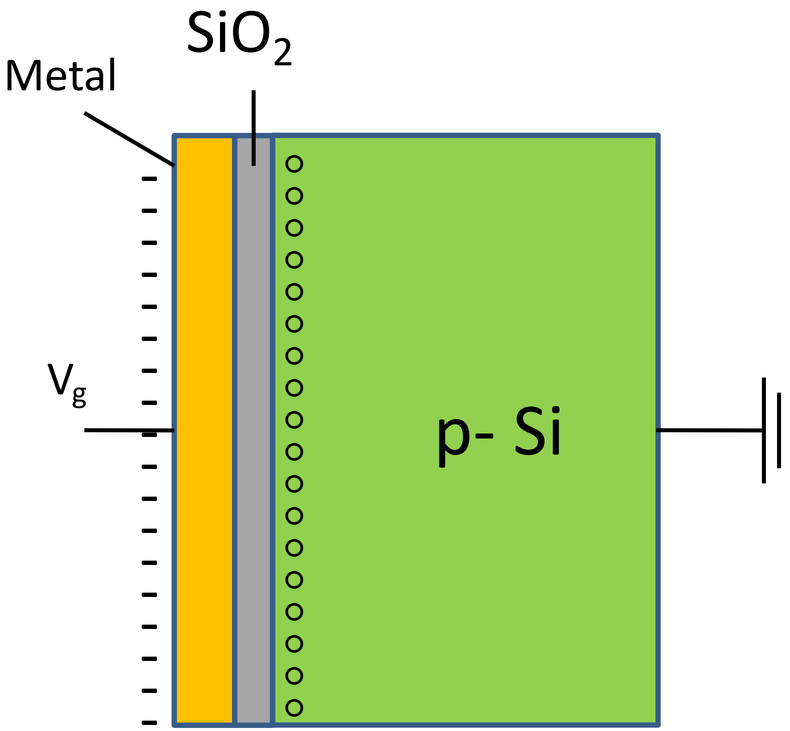


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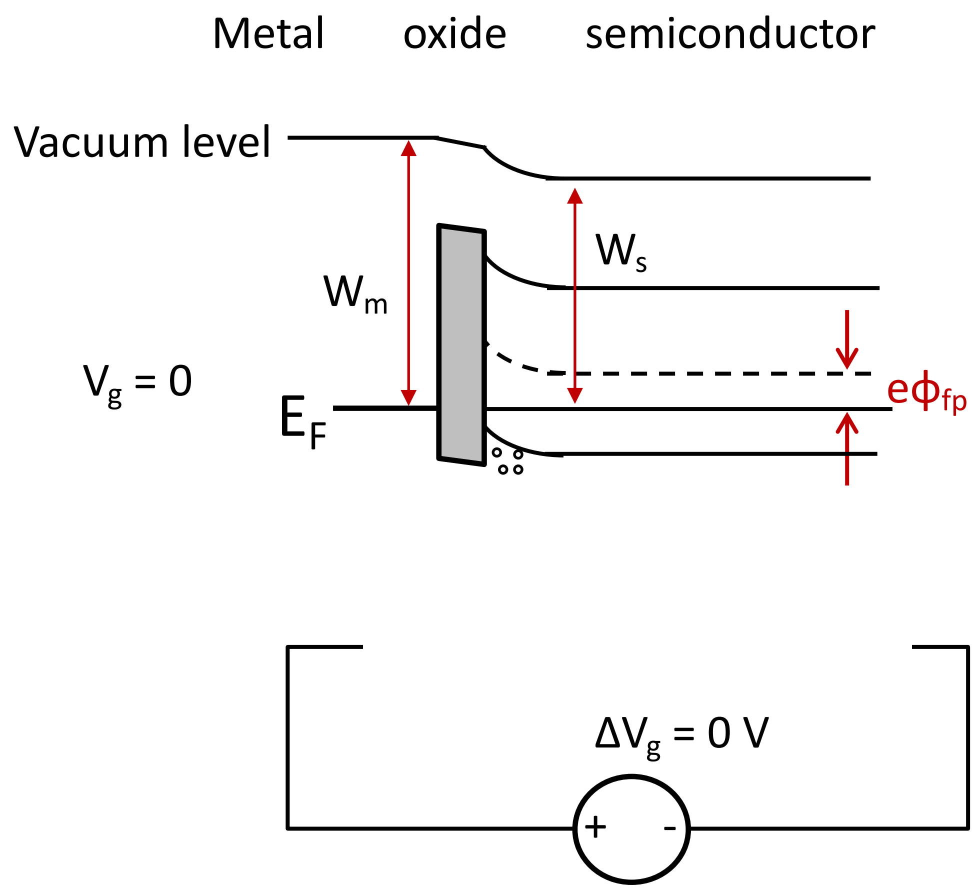

# 10.3 Non-ideal effects

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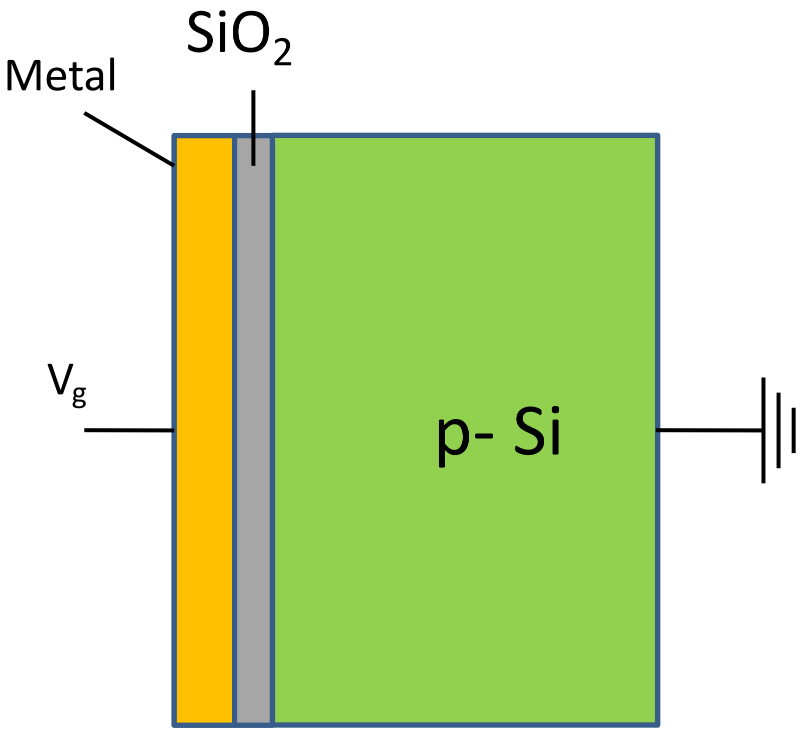


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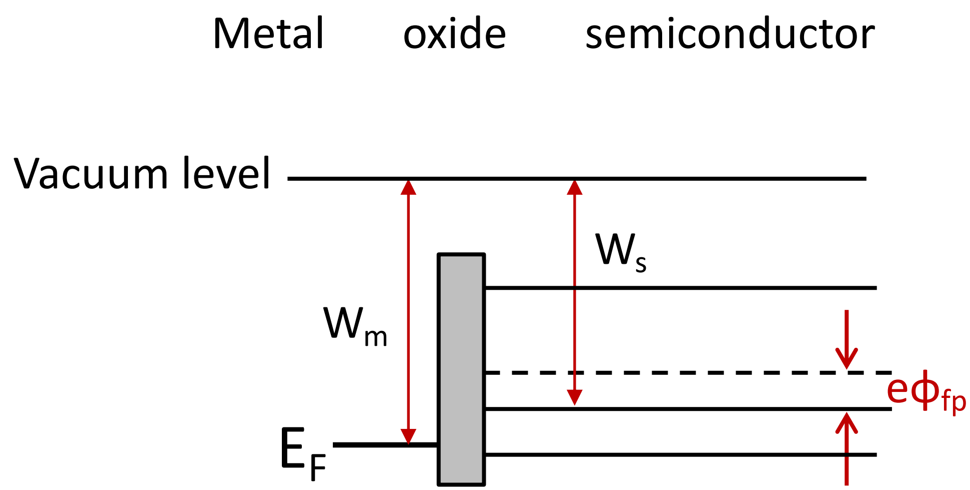
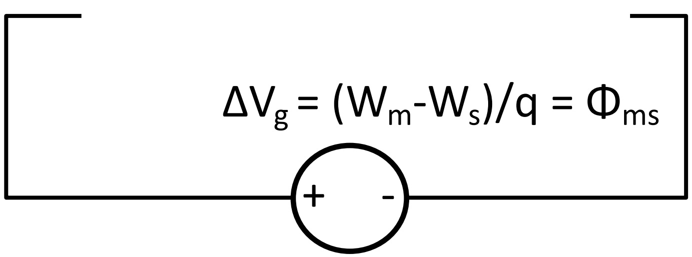



Flat band $V_{FB} = (W_m - W_s)/q$



# 10.3 Non-ideal effects

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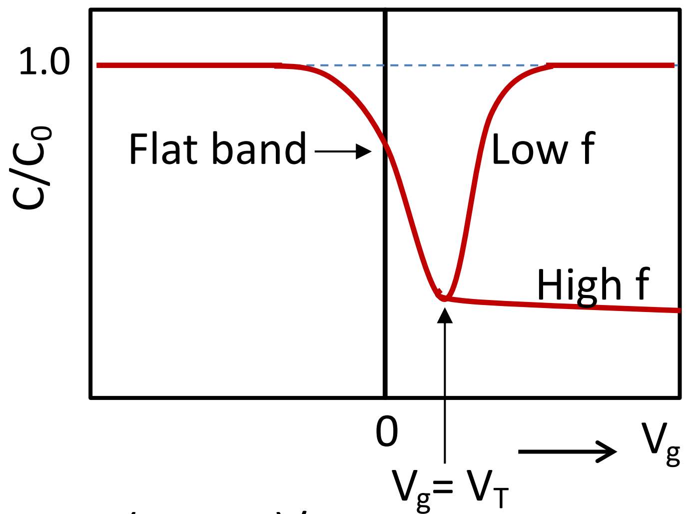


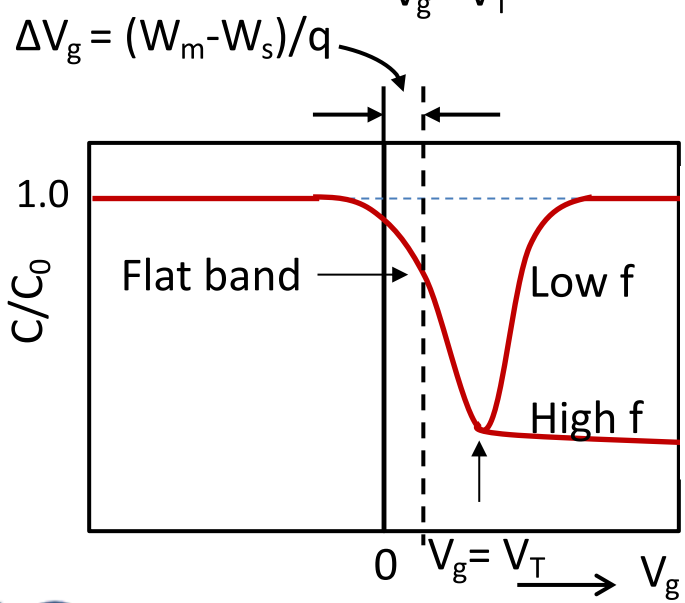


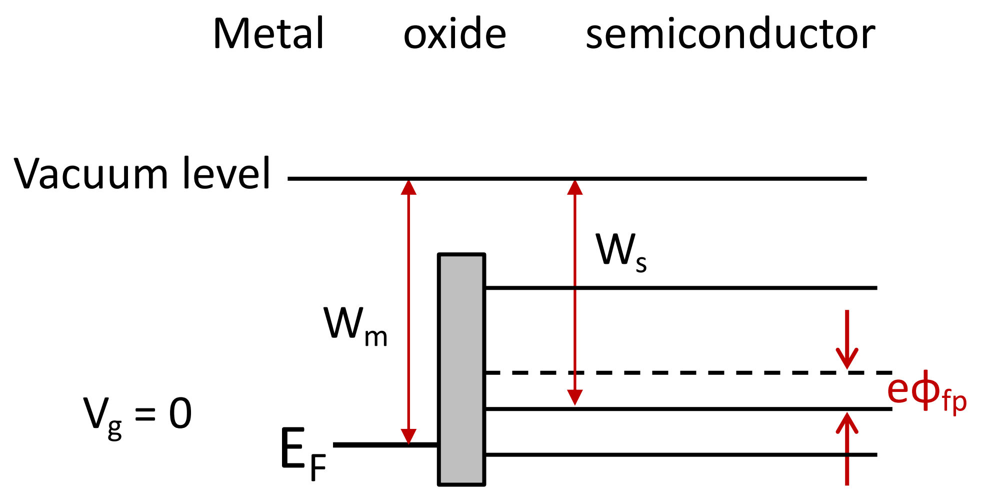


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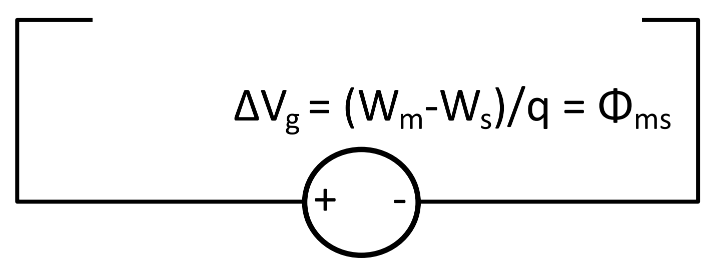
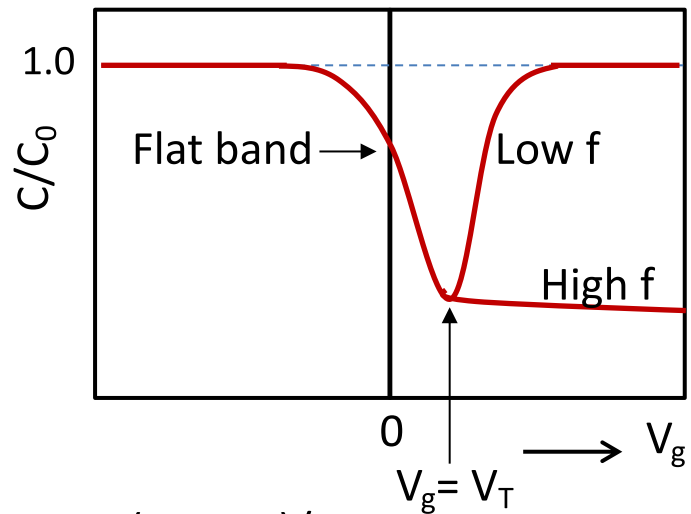
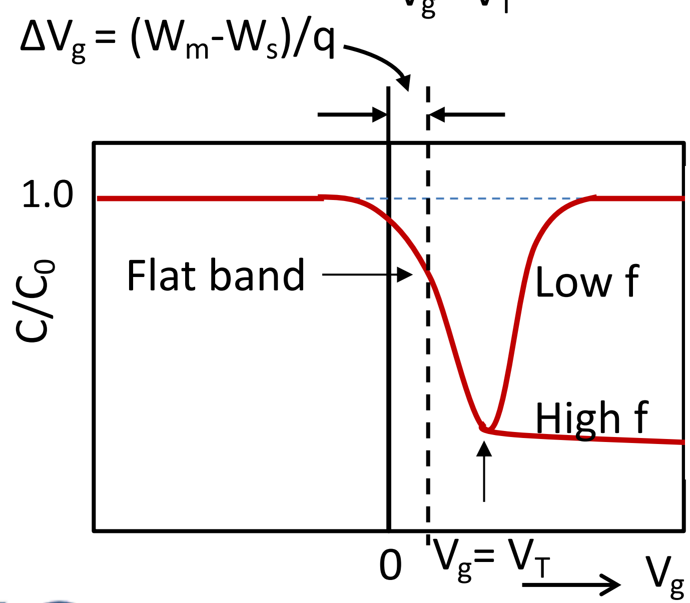


# 10.3 Non-ideal effects



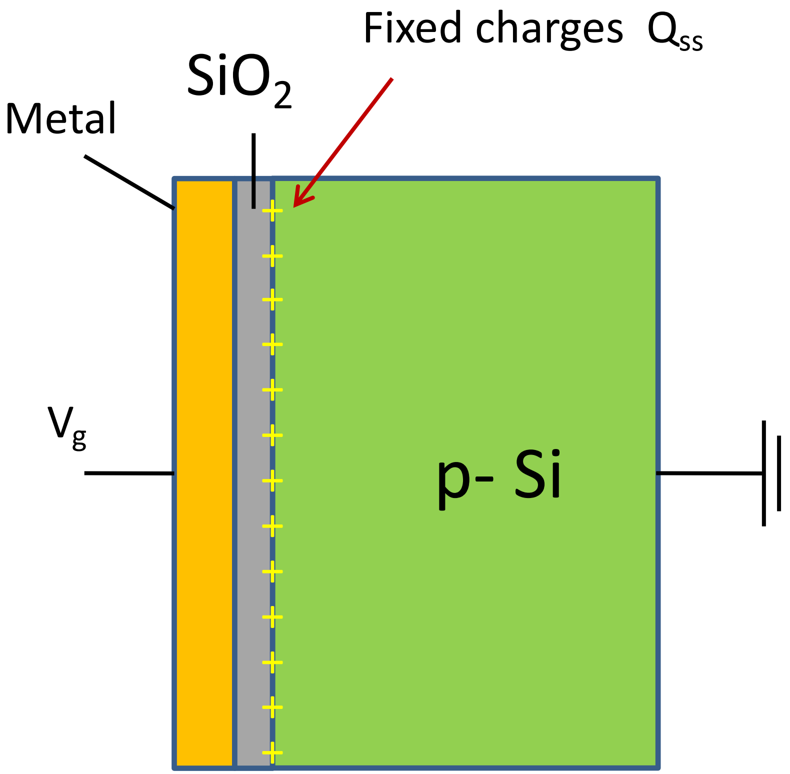

## Work function difference

$$V_T = 2\emptyset_{fp} + t_{ox}\sqrt{\frac{4eN_a\varepsilon_{Si}\emptyset_{fp}}{\varepsilon_{ox}^2}} = 2\emptyset_b + \frac{|Q_{SD}|}{C_{ox}}$$

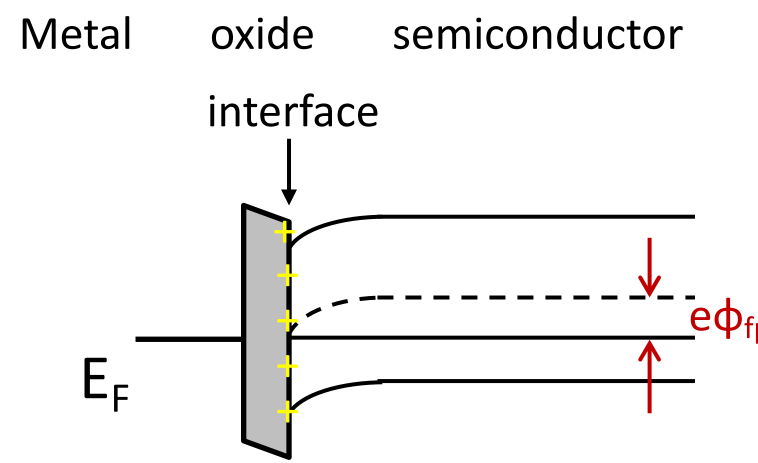
$$V_T = 2\emptyset_{fp} + t_{ox}\sqrt{\frac{4eN_a\varepsilon_{Si}\emptyset_{fp}}{\varepsilon_{ox}^2}} + V_{FB}$$
$$= 2\emptyset_{fp} + \frac{|Q_{SD}|}{C_{ox}} + \emptyset_{ms}$$

# 10.3 Non-ideal effects

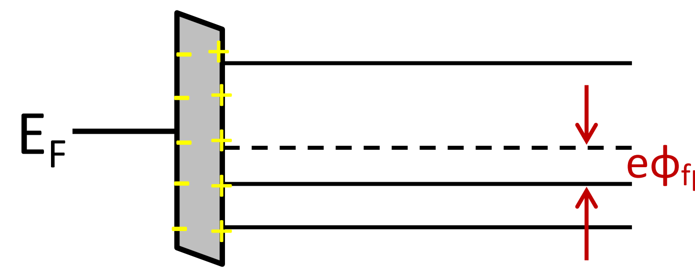
## Fixed charges




Metal-insulator-semiconductor (MIS)




Depletion

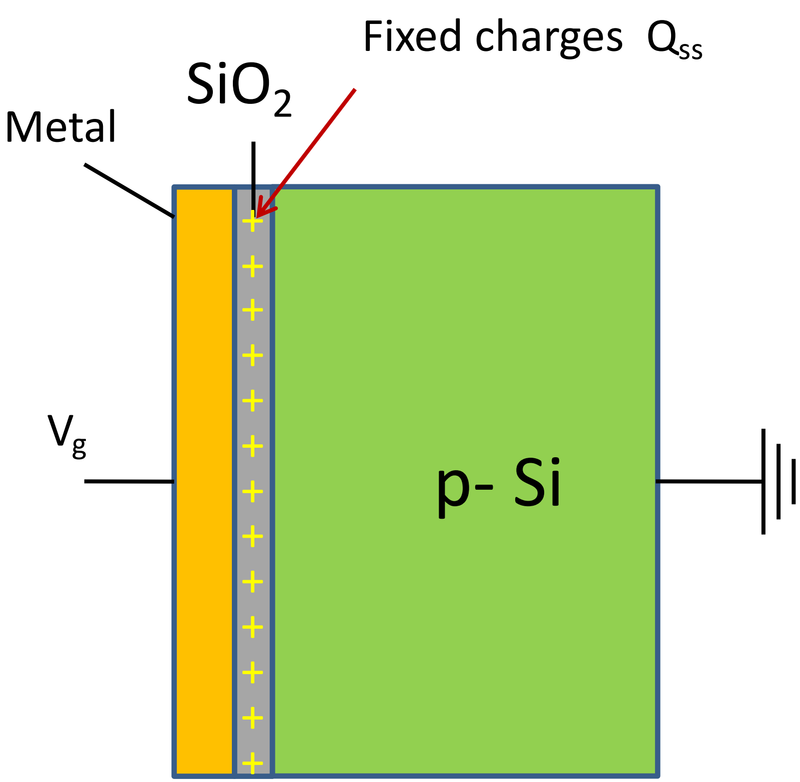



Flat band

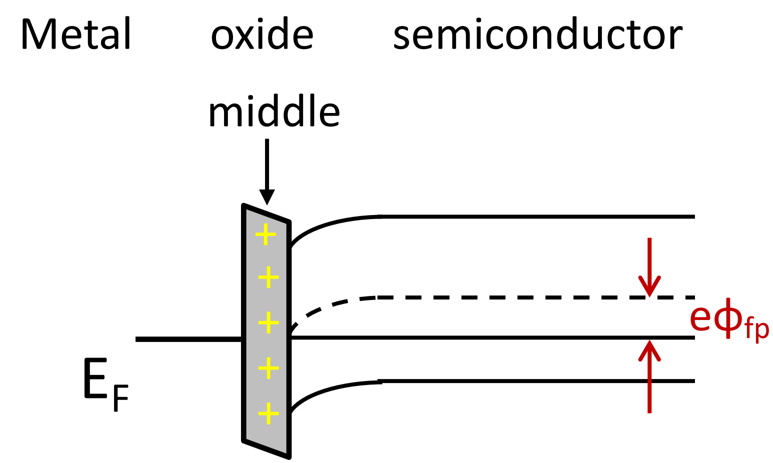
$$V_g = V_{FB} = -Q_{ss}/C$$

# 10.3 Non-ideal effects

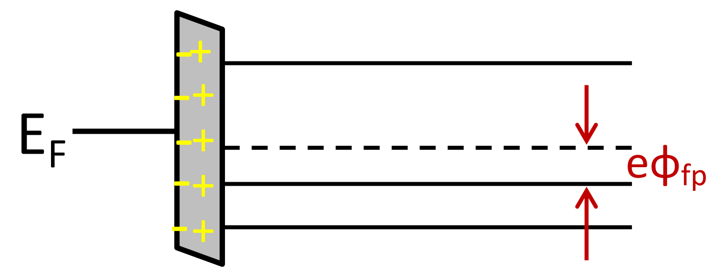
## Fixed charges




Metal-insulator-semiconductor (MIS)




Depletion

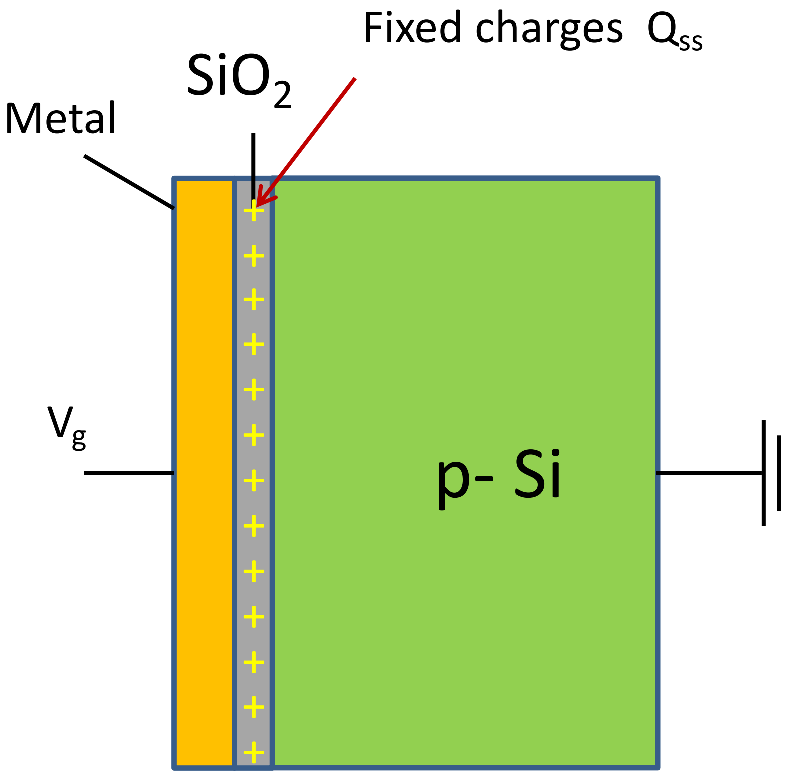



Flat band

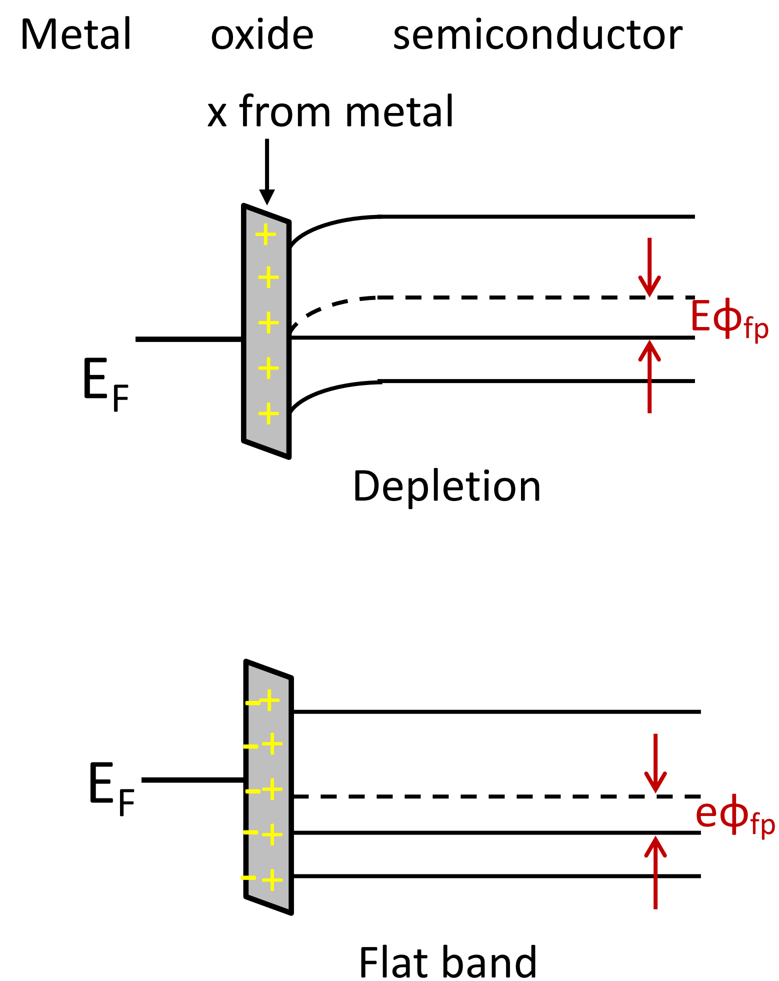
$$V_g = V_{FB} = -Q_{ss}/2C$$

# 10.3 Non-ideal effects

## Fixed charges



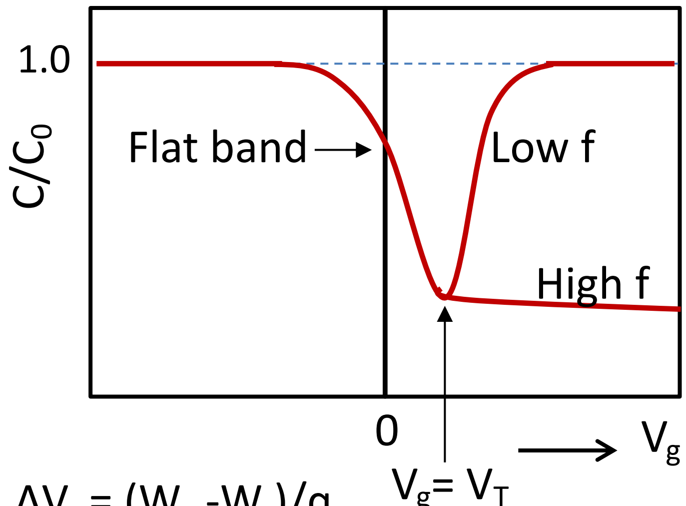

Metal-insulator-semiconductor (MIS)

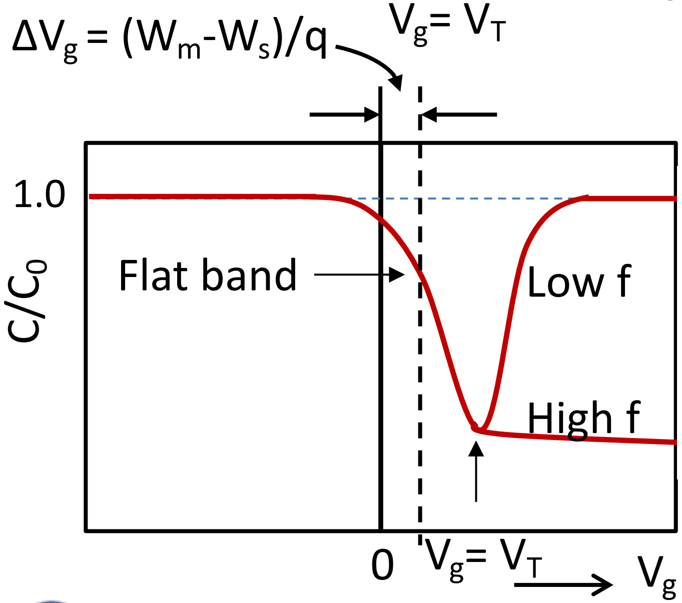


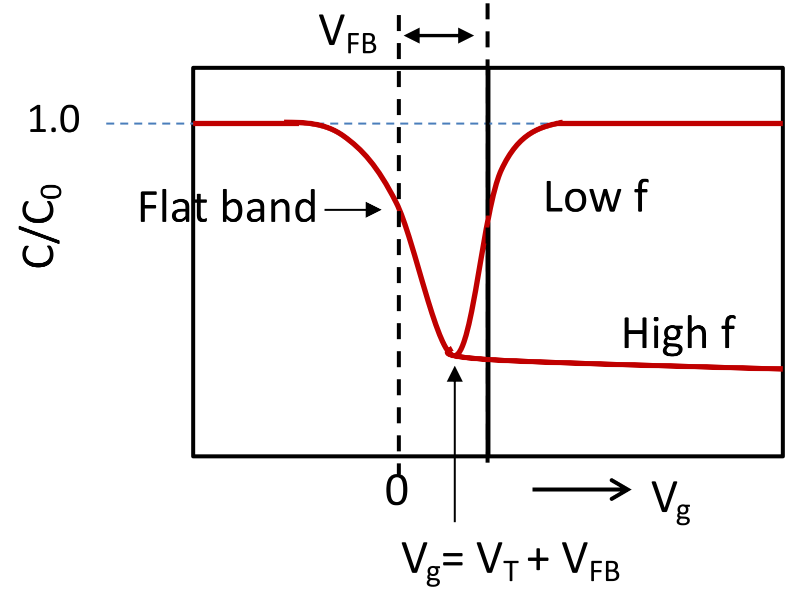

$$V_g = V_{FB} = -\frac{Q_{ss}}{C} \cdot \frac{x}{d}$$

# 10.3 Non-ideal effects

## Fixed charges



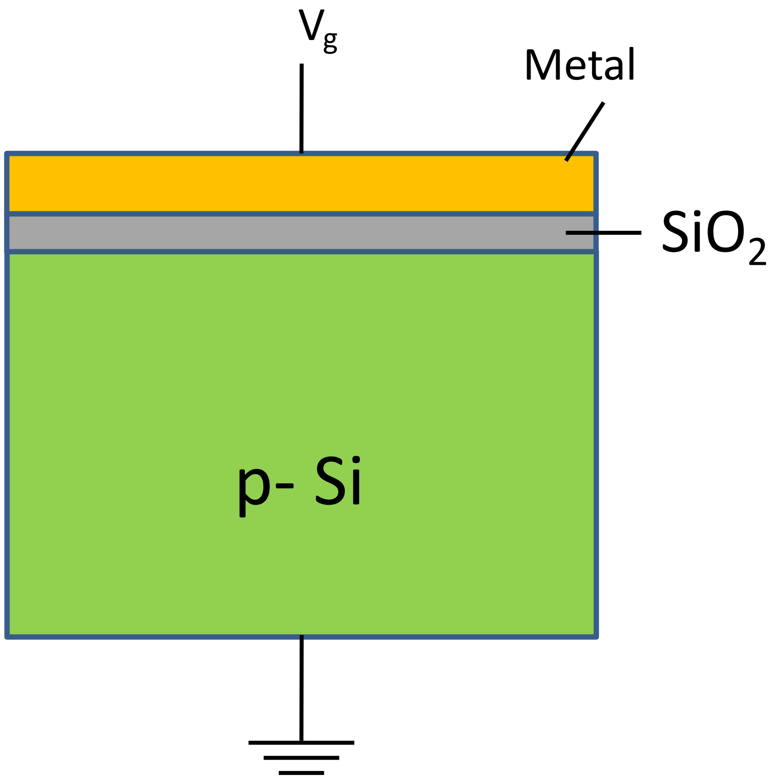

$$V_T = 2\phi_{fp} + t_{ox}\sqrt{\frac{4eN_a\varepsilon_{Si}\phi_{fp}}{\varepsilon_{ox}^2}} + V_{FB}$$

$$= 2\phi_{fp} + \frac{|Q_{SD}|}{C_{ox}} + \phi_{ms} - \frac{Q_{ss}}{C_{ox}}$$

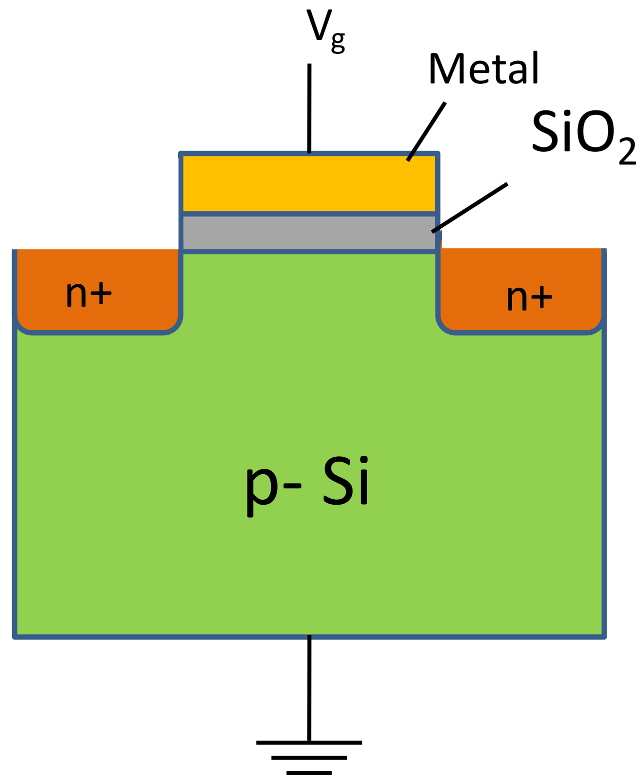
# Outline

# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: MOSFET



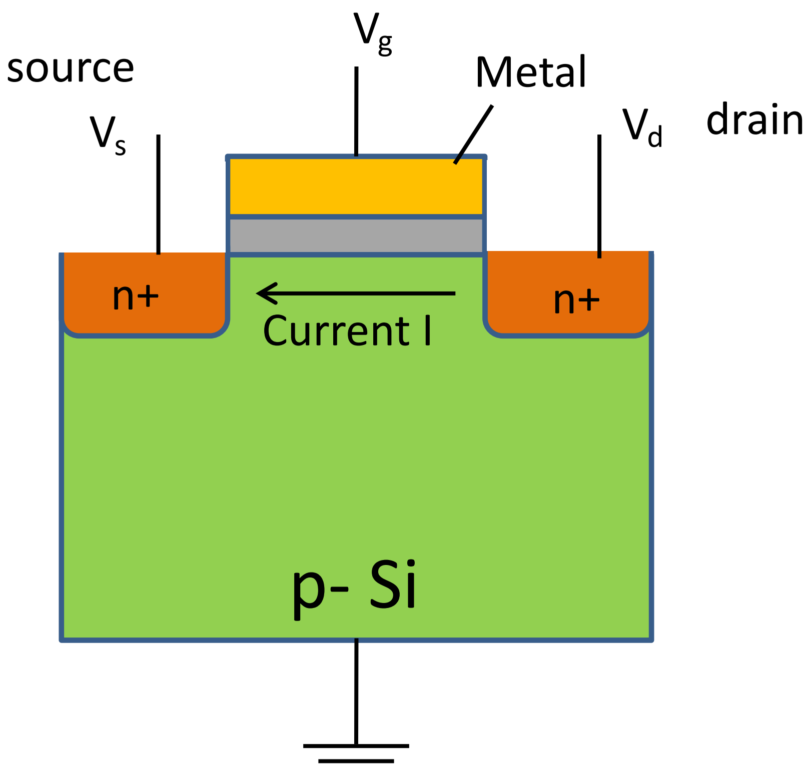

MOS capacitor

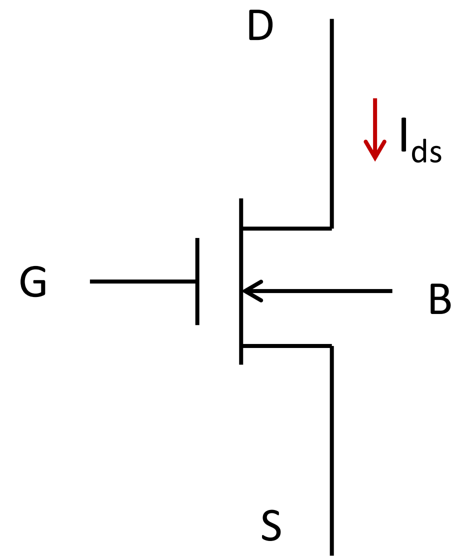



MOSFET

# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: MOSFET

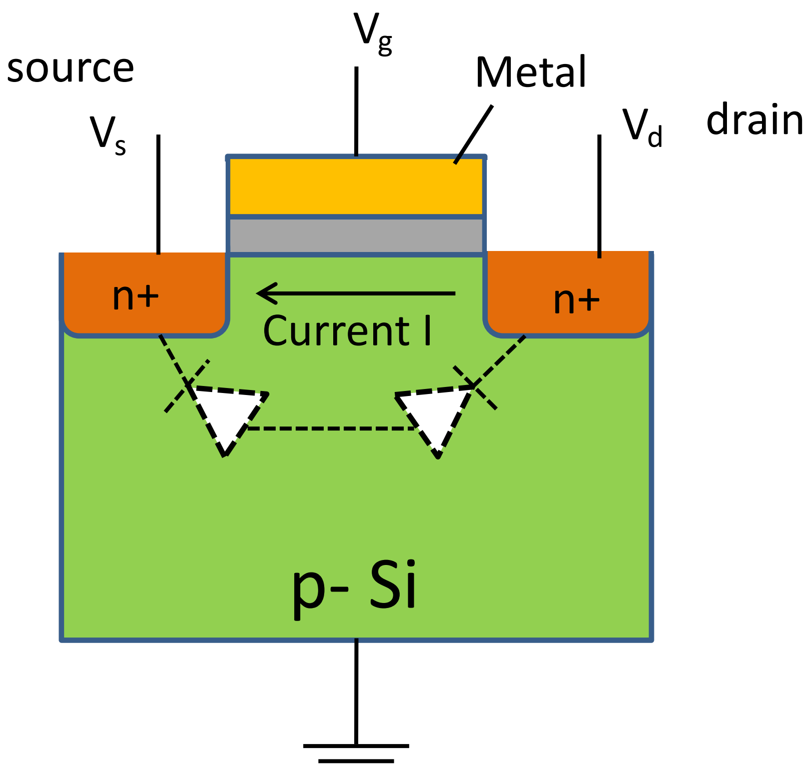


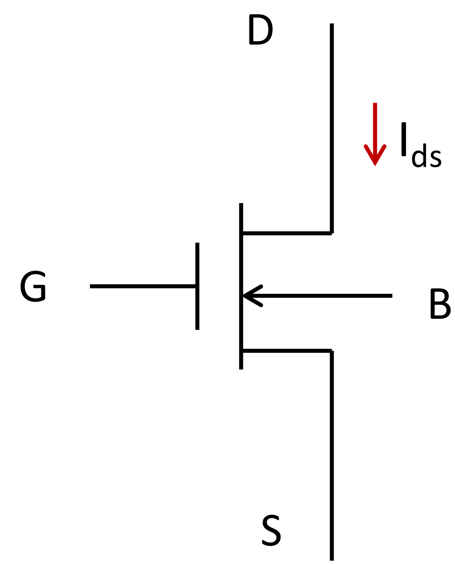

Metal-oxide-semiconductor (MOS)

# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: MOSFET

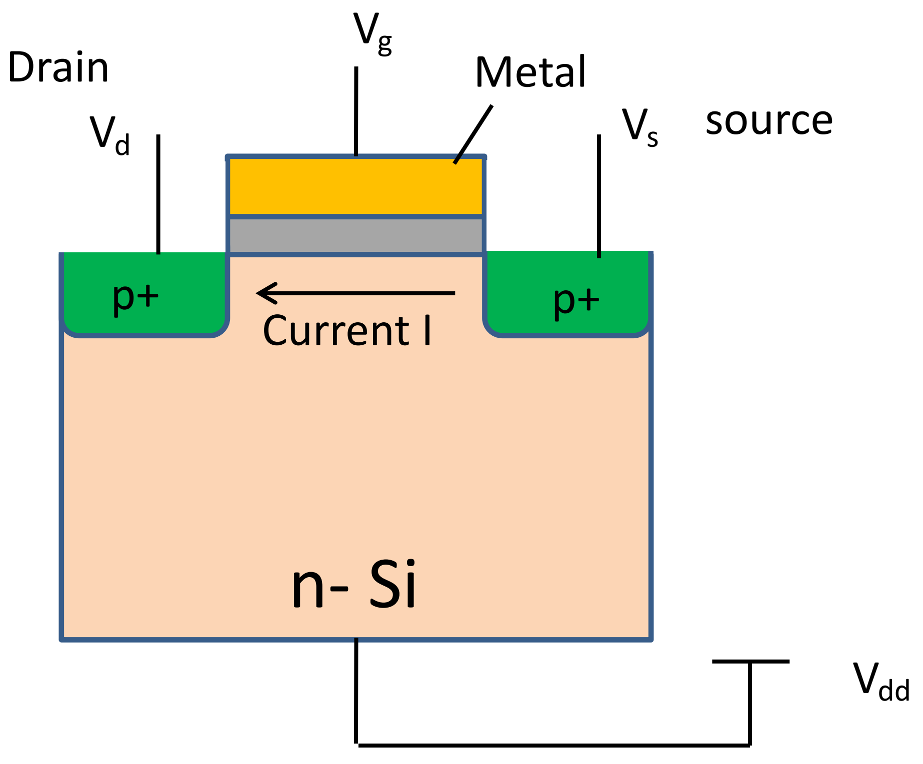


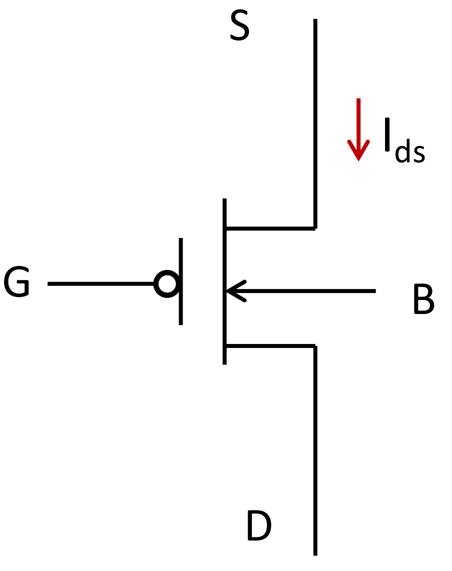

MOSFET



# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: p-type MOSFET

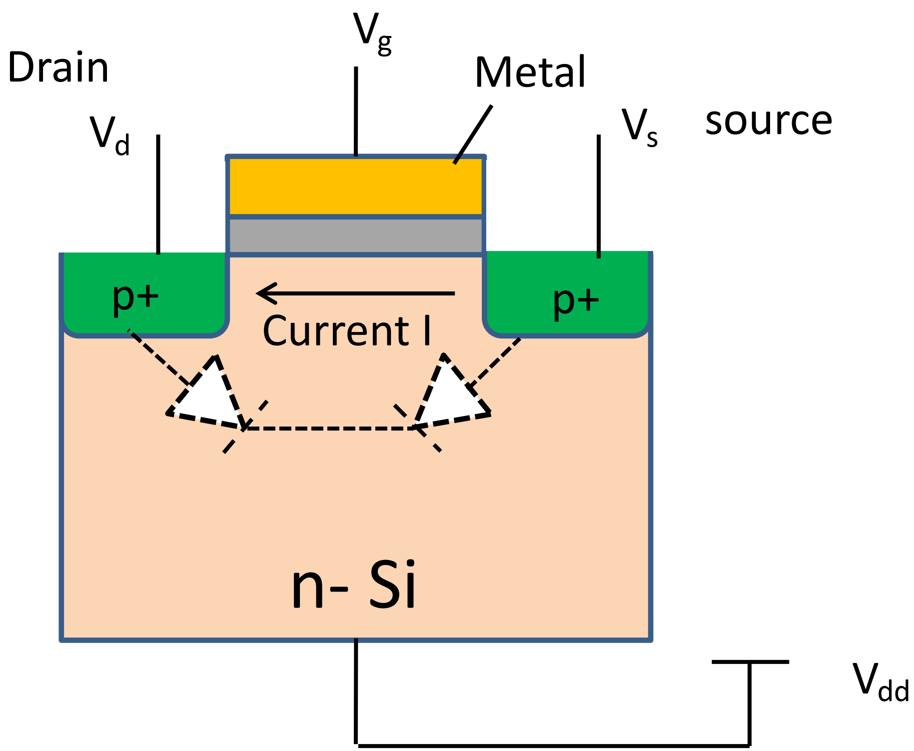


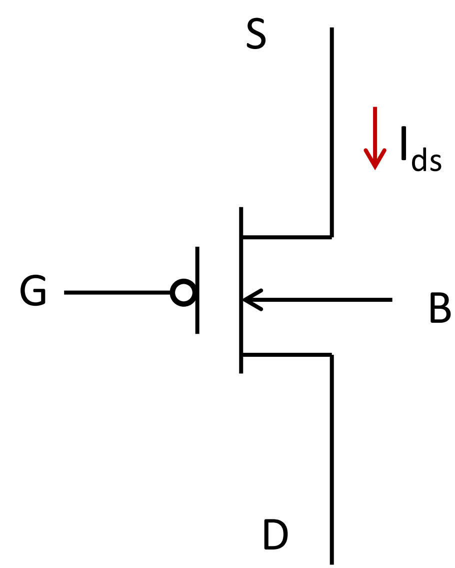




P-type MOSFET

# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: P MOSFET

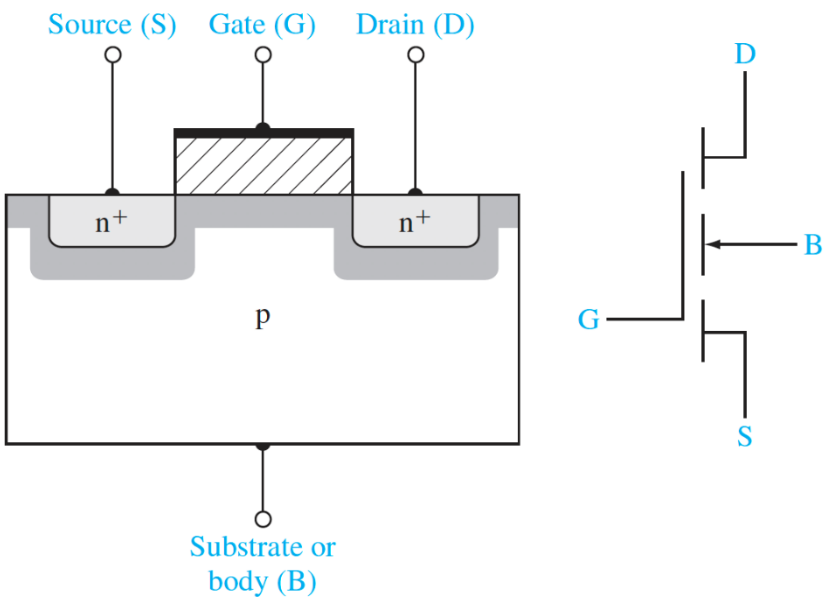


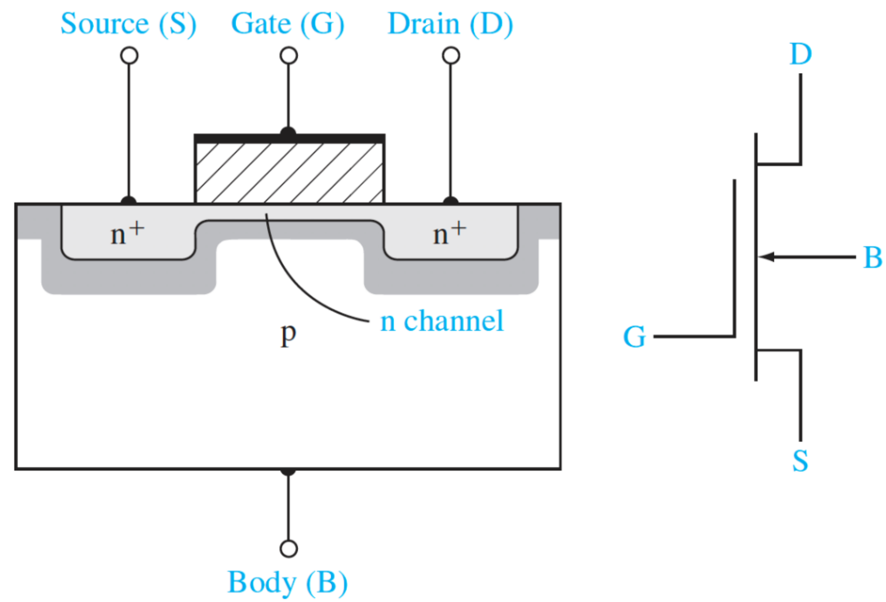

P-type MOSFET

# 10.4 The basic MOSFET operation

## MOSFET structures



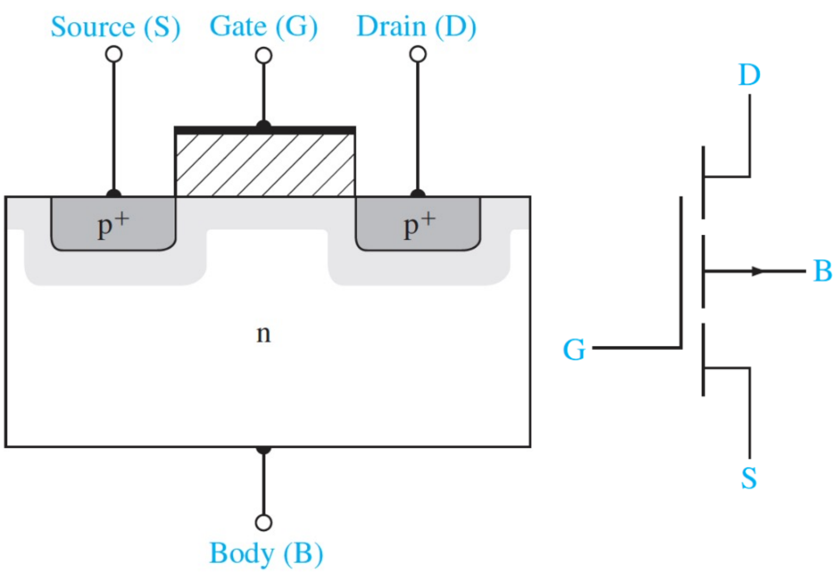

NMOS Enhancement mode

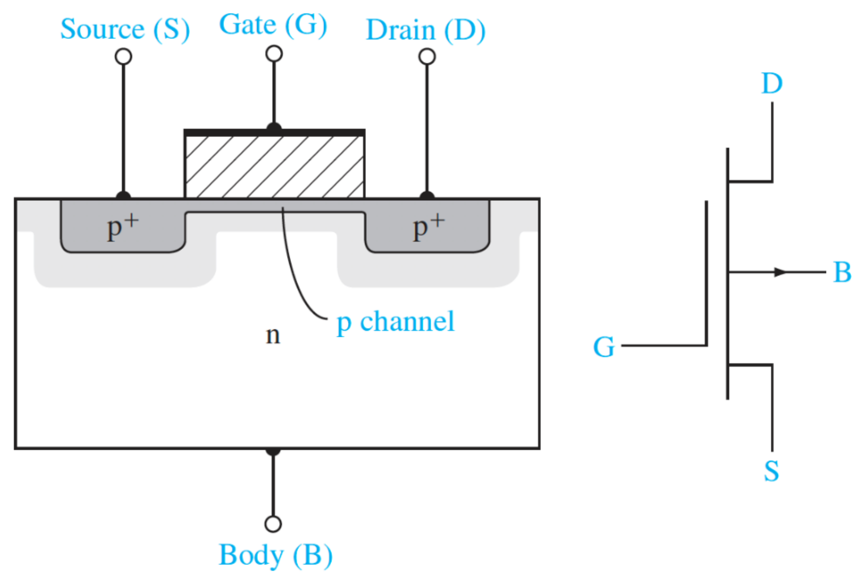



NMOS Depletion mode

# 10.4 The basic MOSFET operation

<u>MOSFET structures</u>



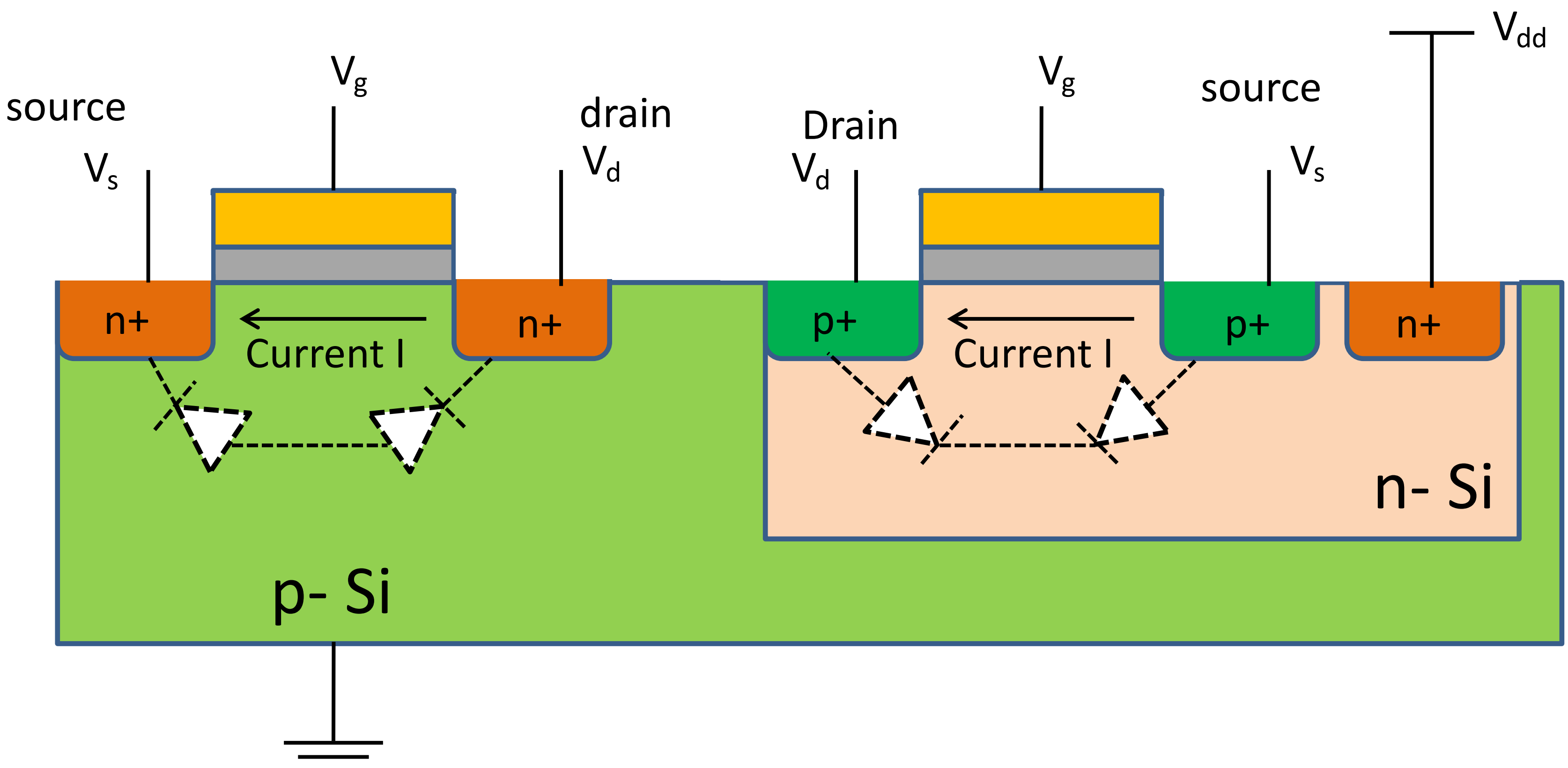




PMOS Enhancement mode

PMOS Depletion mode

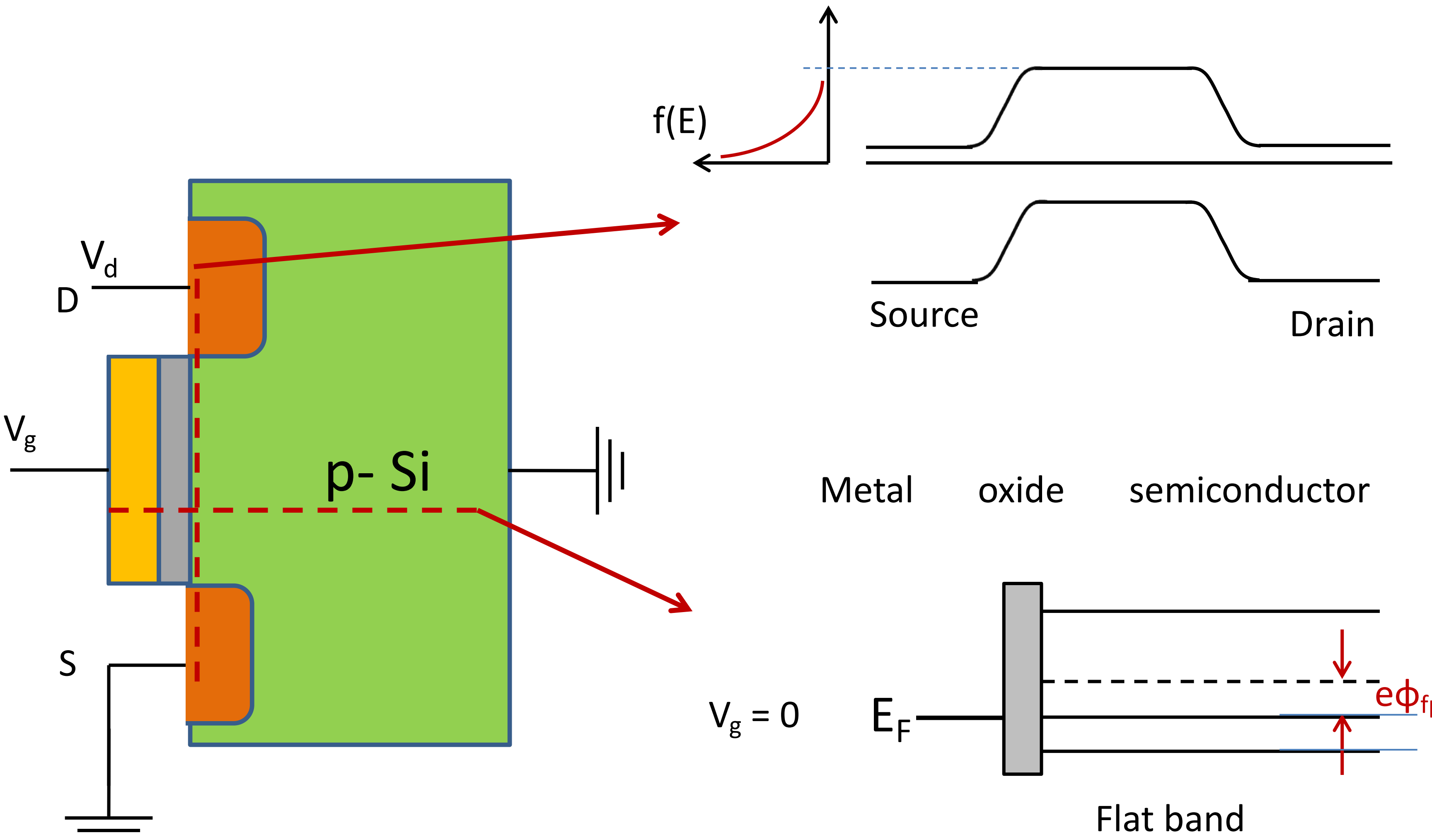
# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: CMOS

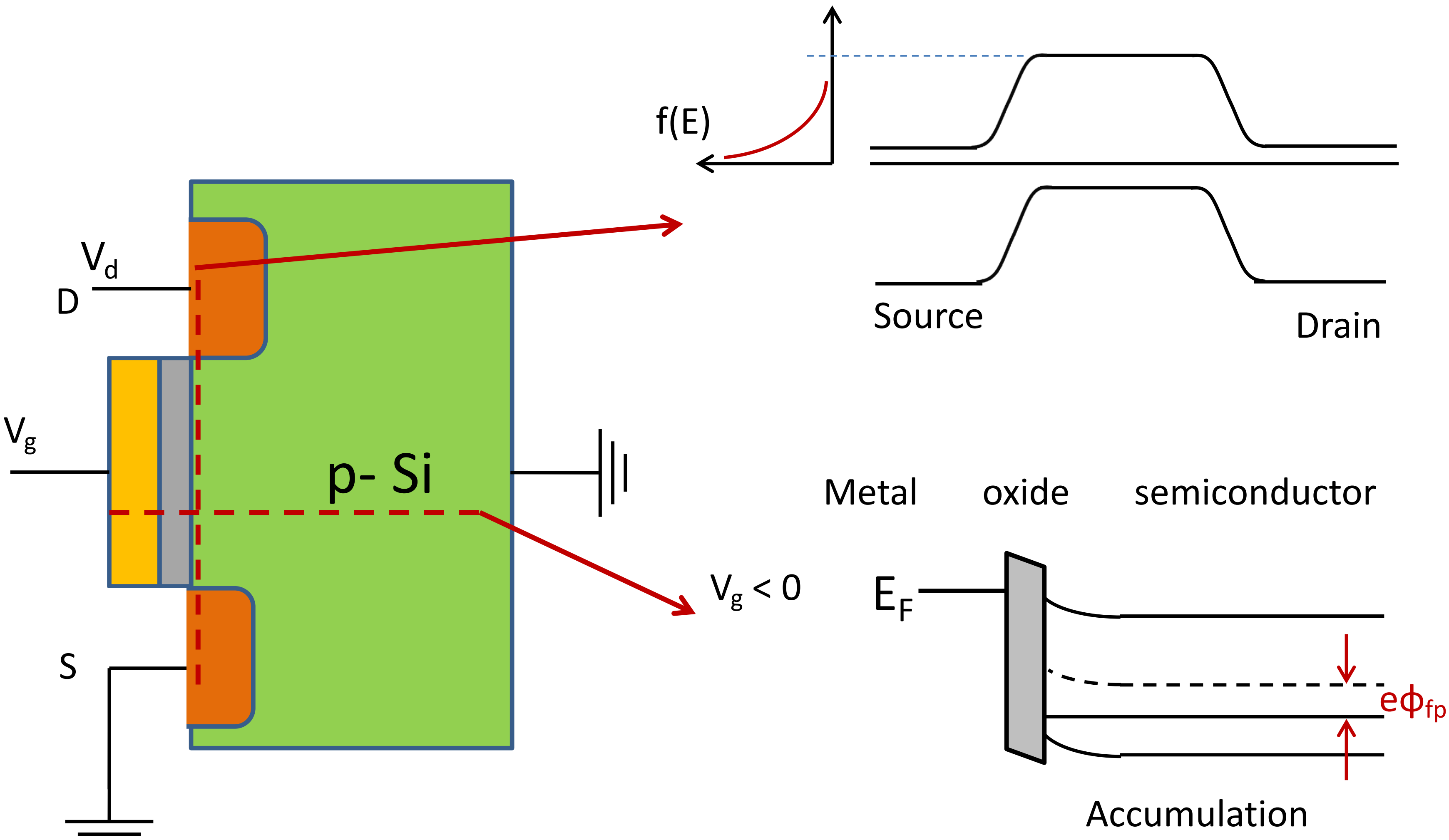



Complementary Metal-oxide-semiconductor (CMOS) field effect transistors

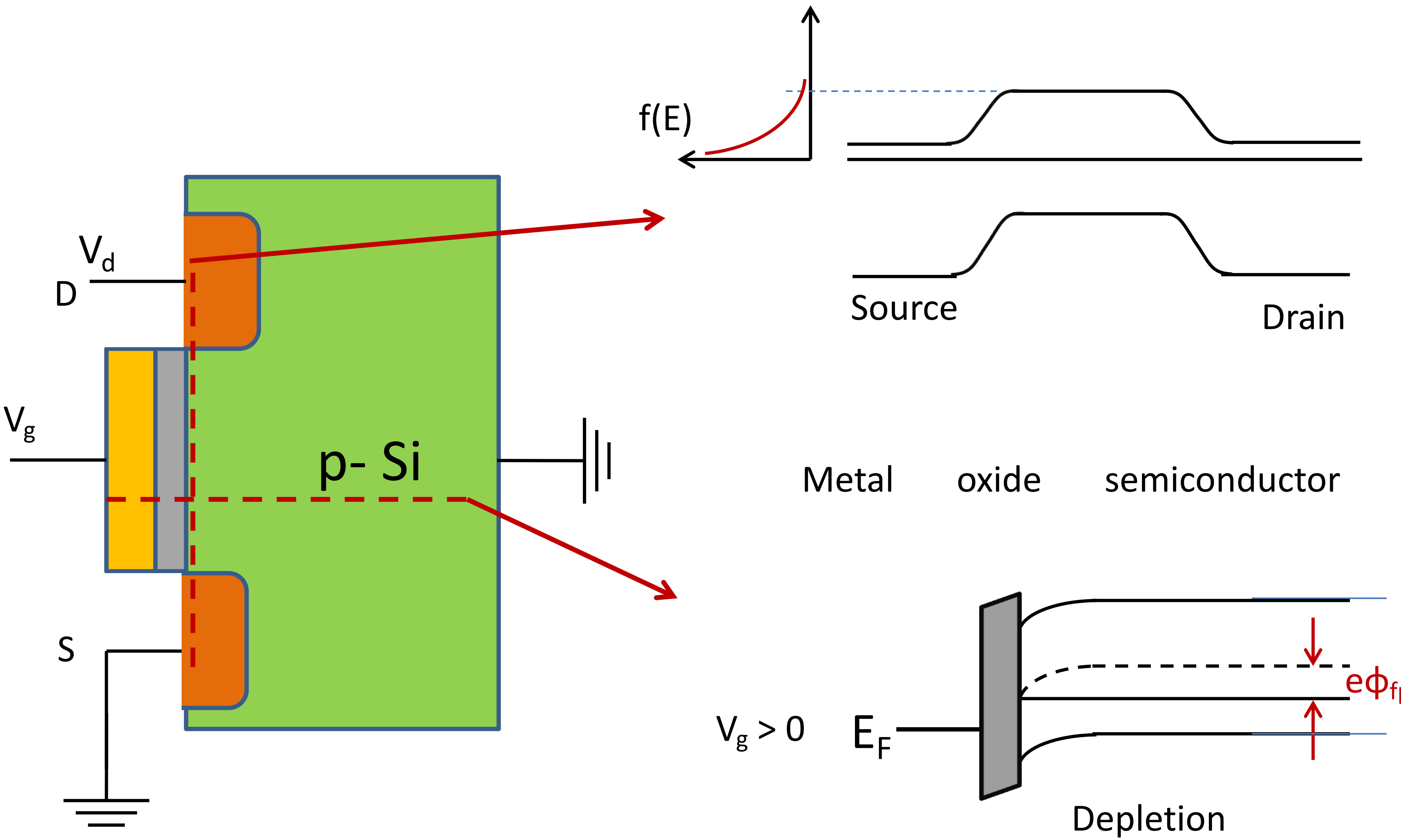
# 10.4 The basic MOSFET operation



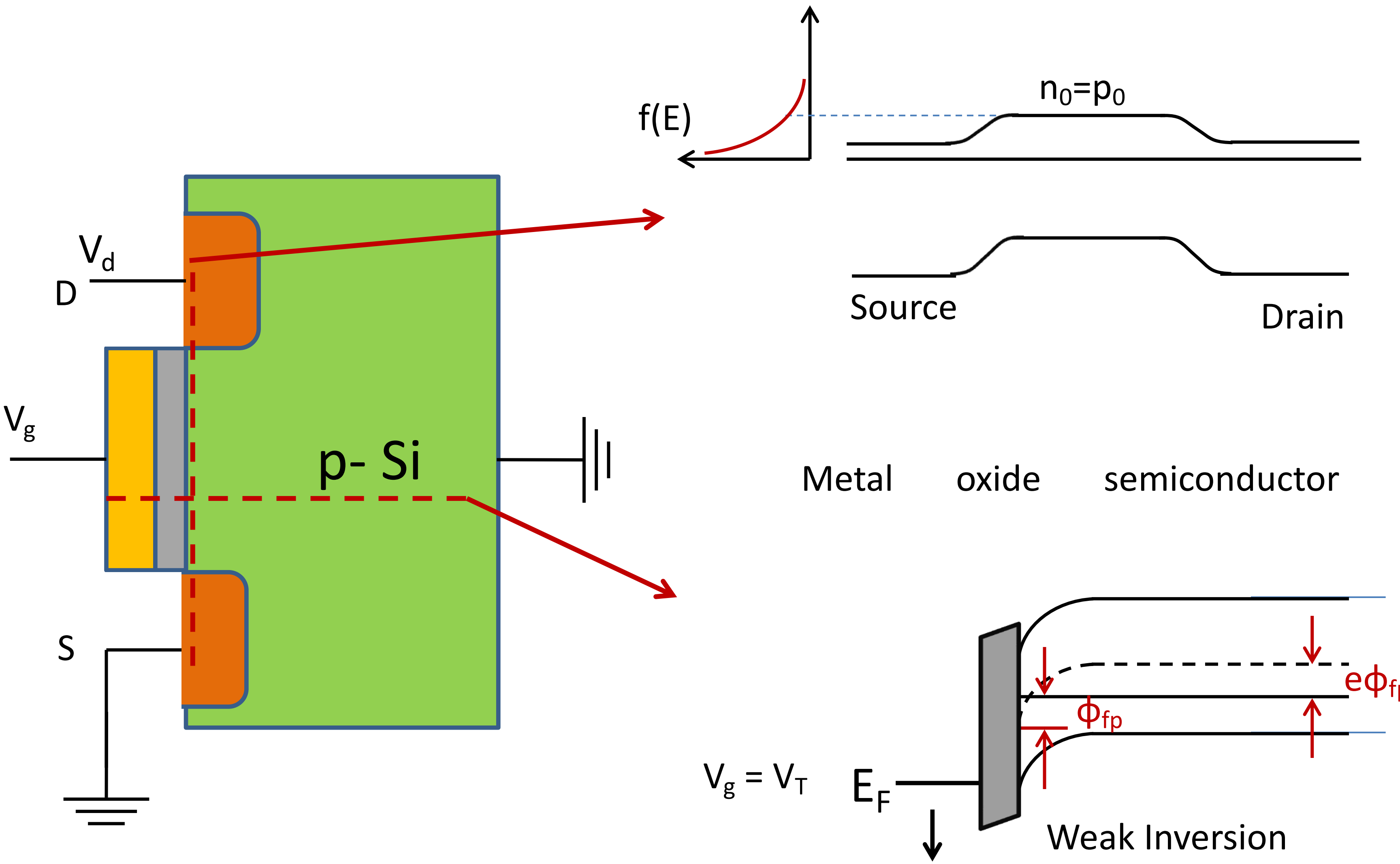
# 10.4 The basic MOSFET operation



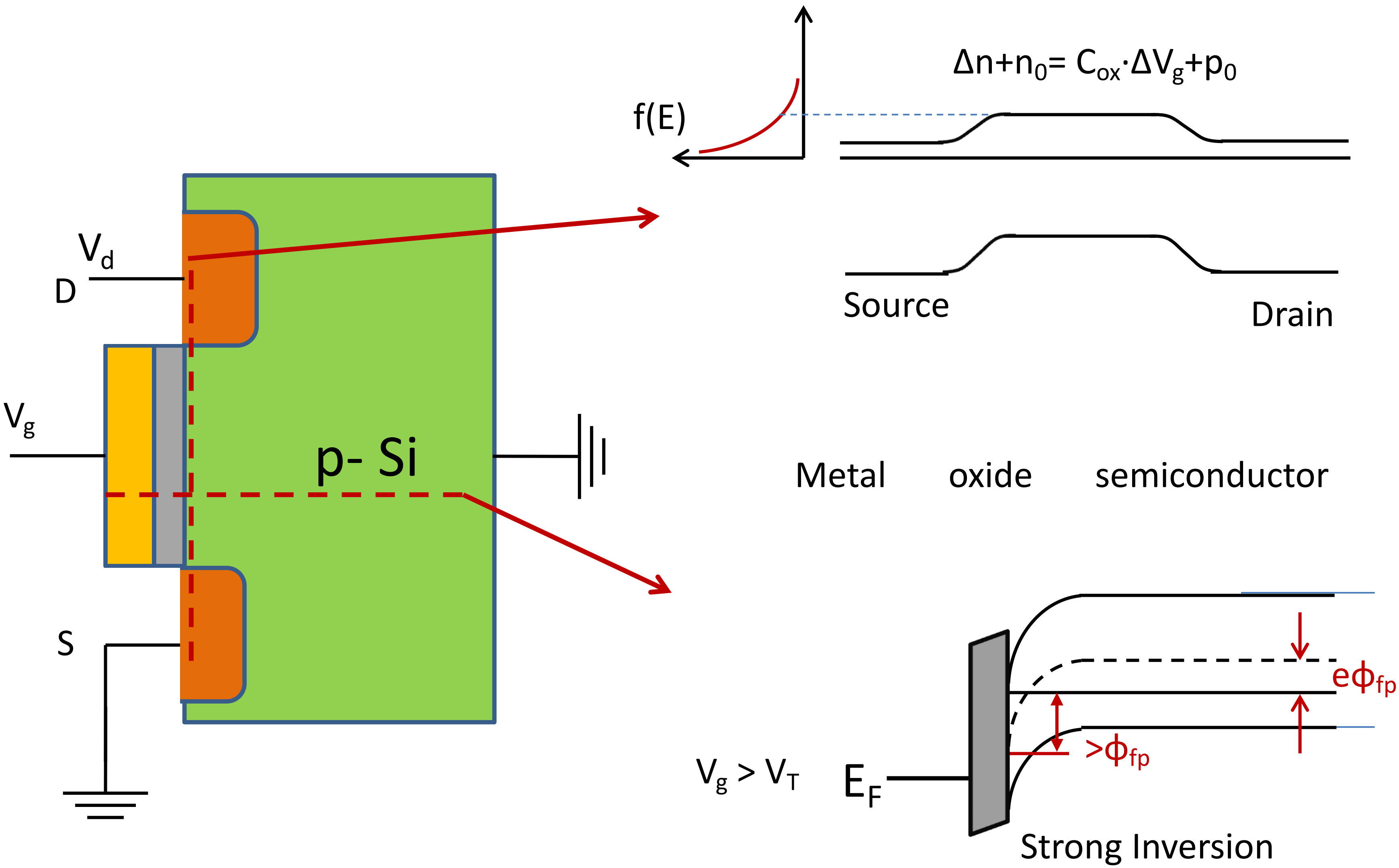
# 10.4 The basic MOSFET operation



# 10.4 The basic MOSFET operation

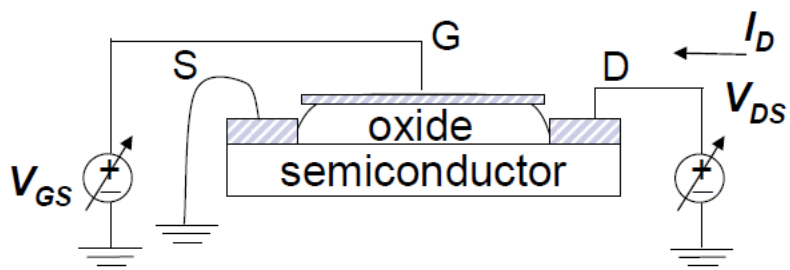


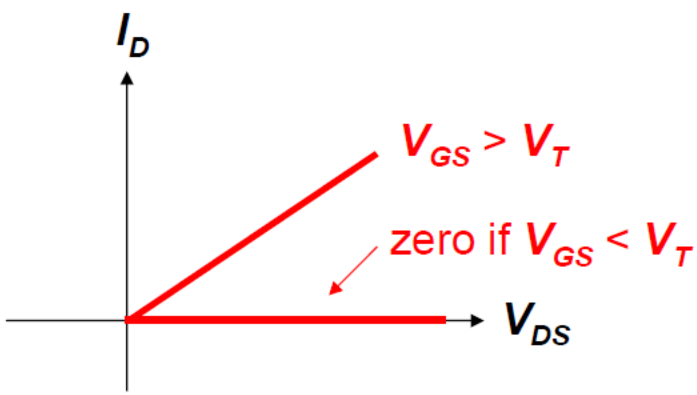
# 10.4 The basic MOSFET operation



## NMOSFET $I_D$ vs. $V_{DS}$ Characteristics

Next consider $I_D$ (flowing into **D**) versus $V_{DS}$, as $V_{GS}$ is varied:




**Above threshold ($V_{GS} > V_T$):**
"inversion layer" of electrons appears, so conduction between **S** and **D** is possible

**Below "threshold" ($V_{GS} < V_T$):**
no charge → no conduction

# 10.4 The basic MOSFET operation

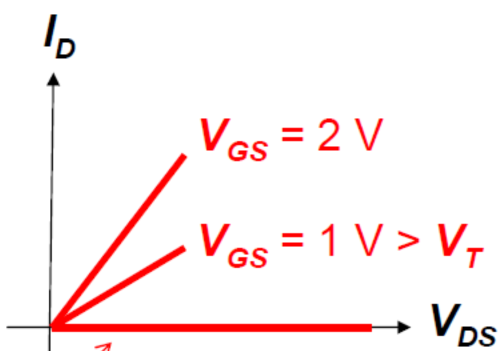
## Current-voltage characteristics

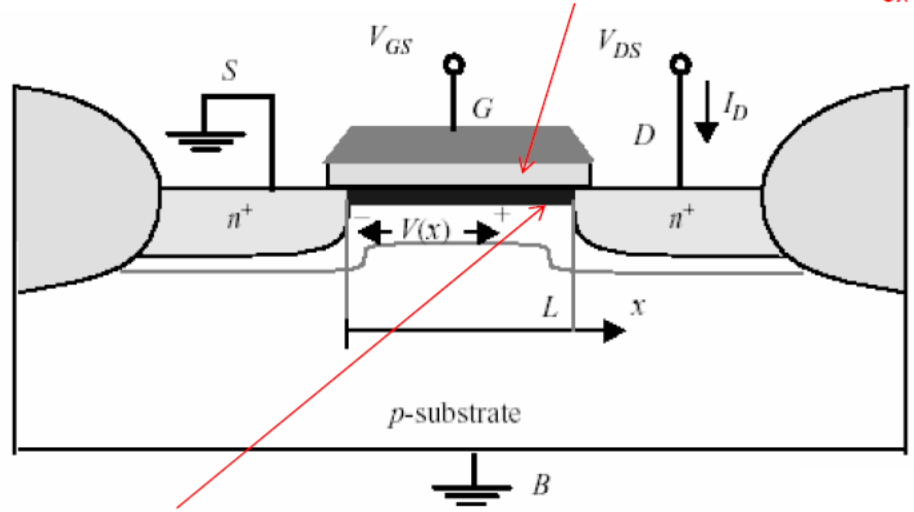
### The MOSFET as a Controlled Resistor

- The MOSFET behaves as a resistor when $V_{DS}$ is low:
  - Drain current $I_D$ increases linearly with $V_{DS}$
  - Resistance $R_{DS}$ between SOURCE & DRAIN depends on $V_{GS}$
    - $R_{DS}$ is lowered as $V_{GS}$ increases above $V_T$

**NMOSFET Example:**

oxide thickness $\equiv t_{ox}$




$I_{DS}$ = 0 if $V_{GS} < V_T$

Inversion charge density $Q_i(x) = -C_{ox}[V_{GS}-V_T-V(x)]$
where $C_{ox} \equiv \varepsilon_{ox} / t_{ox}$

# 10.4 The basic MOSFET operation

$$J_{DS} = qn\mu_n E = qn\mu_n \frac{V_{DS}}{L}$$

$$I_{DS} = J_{DS} A_c = J_{DS} \delta W$$

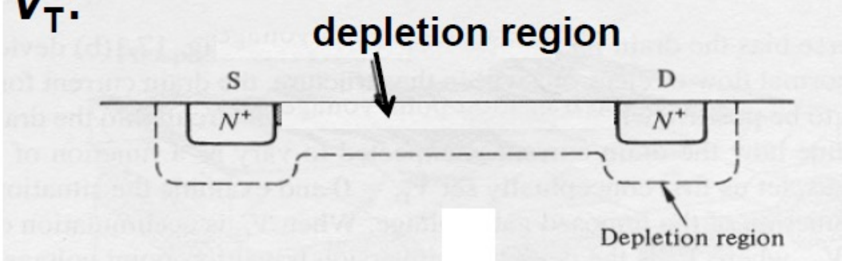
$$I_{DS} = J_{DS} A_c = qn\mu_n \frac{V_{DS}}{L} \delta W = qn\delta \mu_n \frac{W}{L} V_{DS}$$

$$qn\delta = -Q_i(x) = C_{ox}[V_{GS} - V_T - V(x)]$$

$$I_{DS} = C_{ox}[V_{GS} - V_T - \frac{1}{2} V_{DS}] \mu_n \frac{W}{L} V_{DS}$$

## Charge in an N-Channel MOSFET

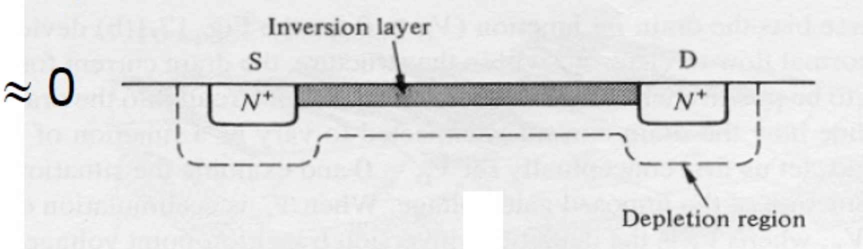
$V_{GS} < V_T$:



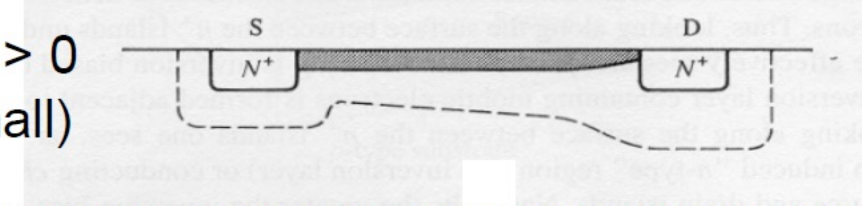

(no inversion layer at surface)

$V_{GS} > V_T$ :

$V_{DS} \approx 0$




$V_{DS} > 0$ (small)




$$I_D = WQ_{inv}v$$
$$= WQ_{inv}\mu_n E$$
$$= WQ_{inv}\mu_n\left(\frac{V_{DS}}{L}\right)$$

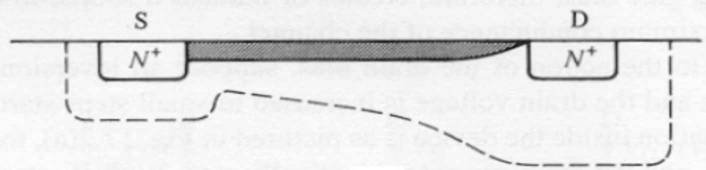
Average electron velocity ***v*** is proportional to lateral electric field ***E***

# 10.4 The basic MOSFET operation

## What Happens at Larger $V_{DS}$?

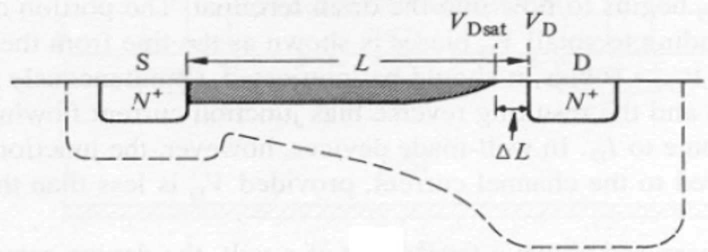
$V_{GS} > V_T$ :

$V_{DS} = V_{GS} - V_T$




Inversion-layer is "pinched-off" at the drain end

$V_{DS} > V_{GS} - V_T$




**As $V_{DS}$ increases above $V_{GS} - V_T \equiv V_{DSAT}$, the length of the "pinch-off" region $\Delta L$ increases:**

- "extra" voltage ($V_{DS} - V_{Dsat}$) is dropped across the distance $\Delta L$
- the voltage dropped across the inversion-layer "resistor" remains $V_{Dsat}$

$\Rightarrow$ the drain current $I_D$ saturates

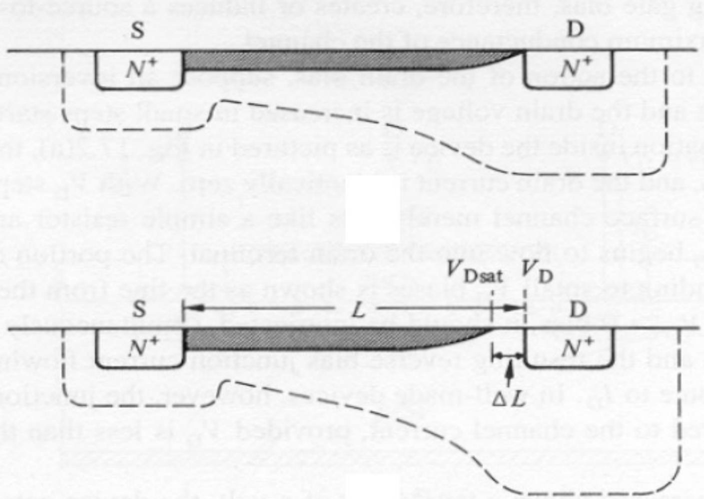
Note: Electrons are swept into the drain by the *E*-field when they enter the pinch-off region.

## What Happens at Larger $V_{DS}$?

$V_{GS} > V_T$ :

$V_{DS} = V_{GS} - V_T$

$V_{DS} > V_{GS} - V_T$




**Inversion-layer is "pinched-off" at the drain end**

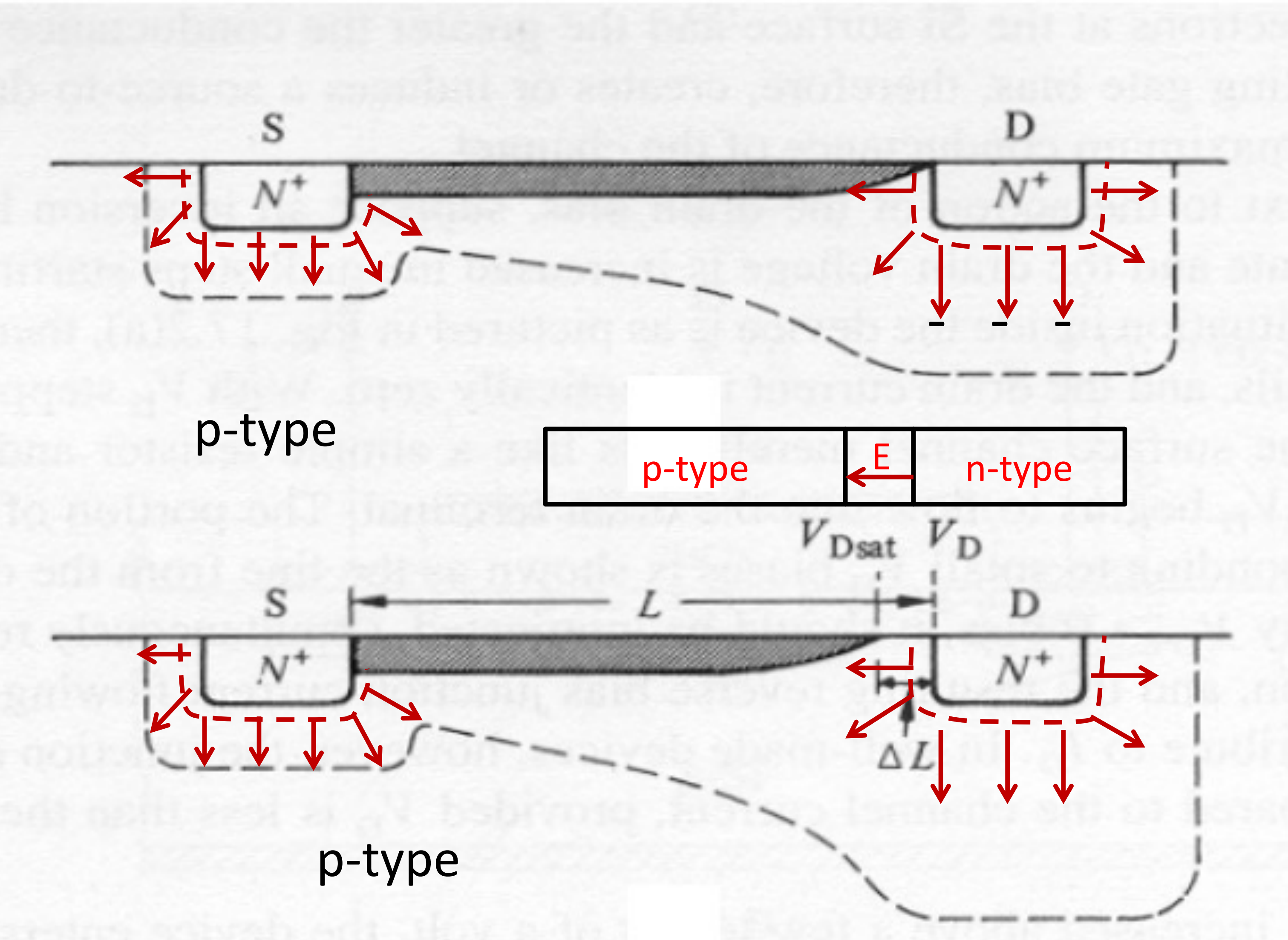
**As $V_{DS}$ increases above $V_{GS} - V_T \equiv V_{DSAT}$,**

$$I_D = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_T - \frac{1}{2}V_{DS}\right)V_{DS}$$

$I_D$ will not increase after $V_{DS} \geq V_{GS} - V_T$

$$I_D = \mu_n C_{ox} \frac{W}{L}\left(V_{GS} - V_T - \frac{1}{2}V_{DS}\right)V_{DS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_T)(V_{GS} - V_T)$$

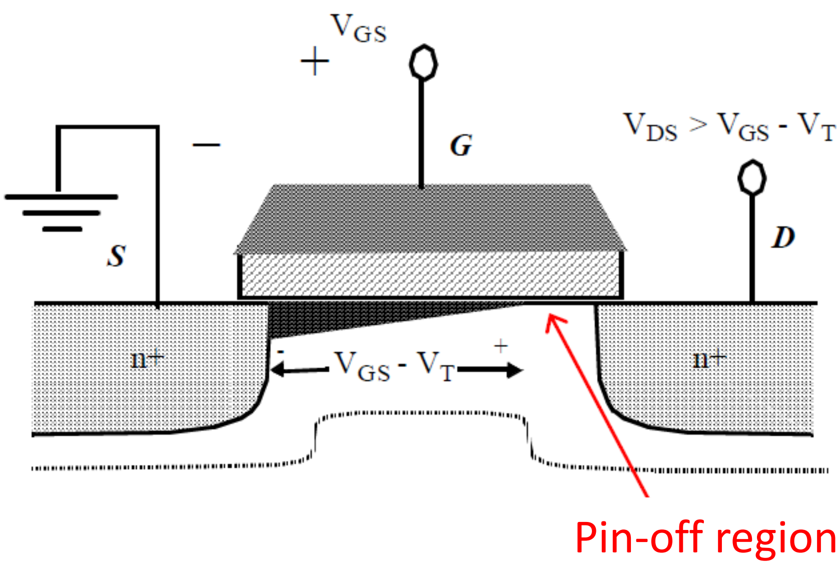
# 10.4 The basic MOSFET operation



# 10.4 The basic MOSFET operation

## Summary of $I_D$ vs. $V_{DS}$

- As $V_{DS}$ increases, the inversion-layer charge density at the drain end of the channel is reduced; therefore, $I_D$ does not increase linearly with $V_{DS}$.
- When $V_{DS}$ reaches $V_{GS} - V_T$, the channel is "pinched off" at the drain end, and $I_D$ saturates (*i.e.* it does not increase with further increases in $V_{DS}$).




$$\boxed{I_{DSAT} = \mu_n C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2}$$

## $I_D$ vs. $V_{DS}$ Characteristics

The MOSFET $I_D$-$V_{DS}$ curve consists of two regions:

**1) Resistive or "Triode" Region: $0 < V_{DS} < V_{GS} - V_T$**

$$I_D = k_n' \frac{W}{L}\left[V_{GS} - V_T - \frac{V_{DS}}{2}\right]V_{DS}$$

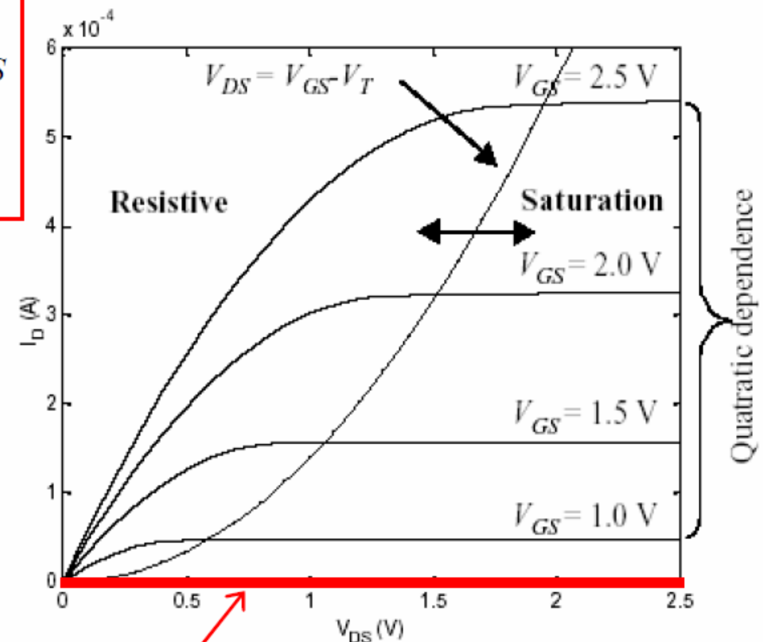
where $k_n' = \mu_n C_{ox}$

process transconductance parameter

**2) Saturation Region: $V_{DS} > V_{GS} - V_T$**

$$I_{DSAT} = \frac{k_n'}{2}\frac{W}{L}(V_{GS} - V_T)^2$$

where $k_n' = \mu_n C_{ox}$

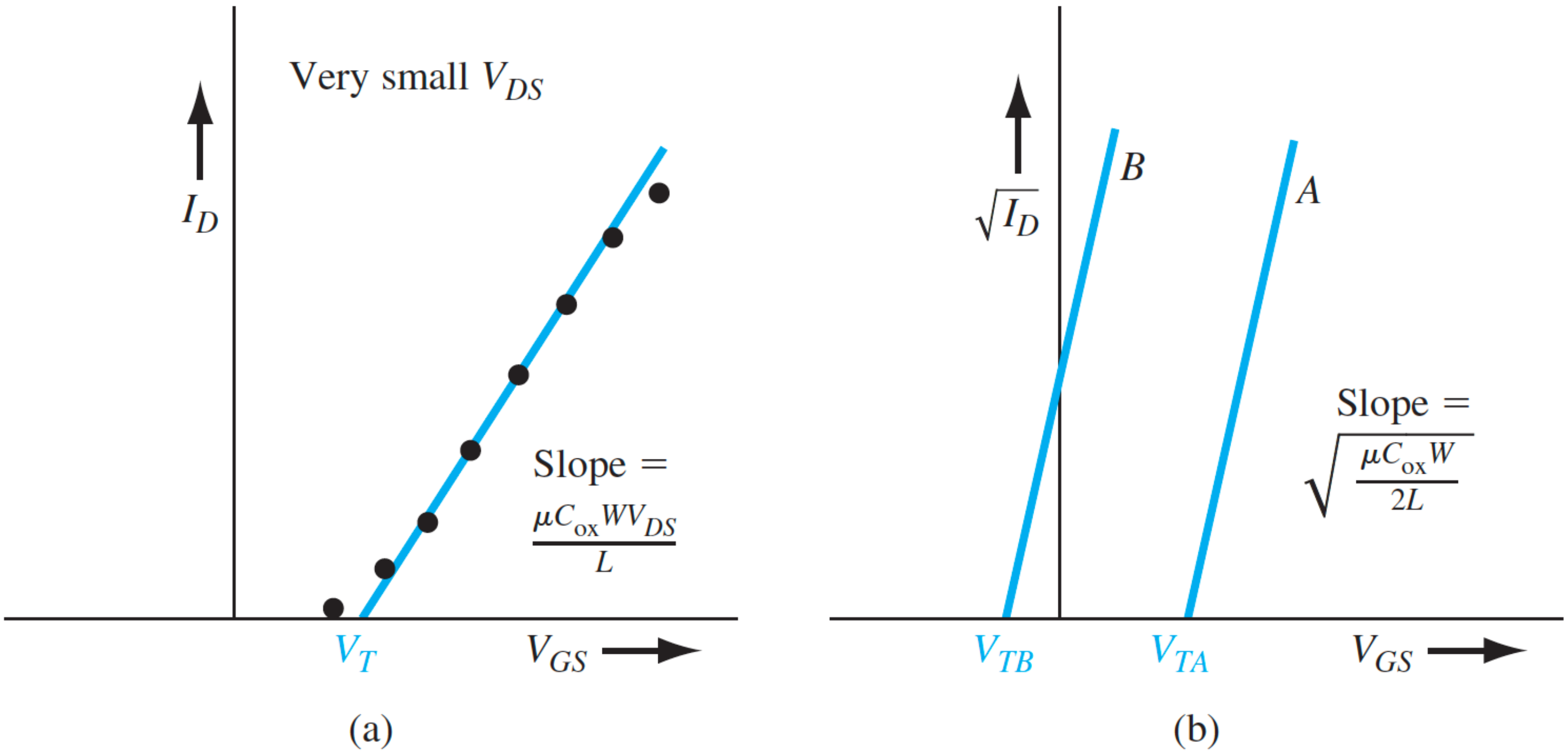



"CUTOFF" region: $V_G < V_T$

# 10.4 The basic MOSFET operation

$$I_D = \frac{W\mu_n C_{ox}}{L}(V_{GS} - V_T)V_{DS}$$

$$\sqrt{I_D(\text{sat})} = \sqrt{\frac{W\mu_n C_{ox}}{2L}}(V_{GS} - V_T)$$



# 10.4 The basic MOSFET operation

Transconductance: $$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

$$I_D = k_n' \frac{W}{L}\left[V_{GS} - V_T - \frac{V_{DS}}{2}\right]V_{DS}$$

$$g_{mL} = \frac{\partial I_D}{\partial V_{GS}} = \frac{W\mu_n C_{\text{ox}}}{L} \cdot V_{DS}$$

**$0 < V_{DS} < V_{GS} - V_T$**

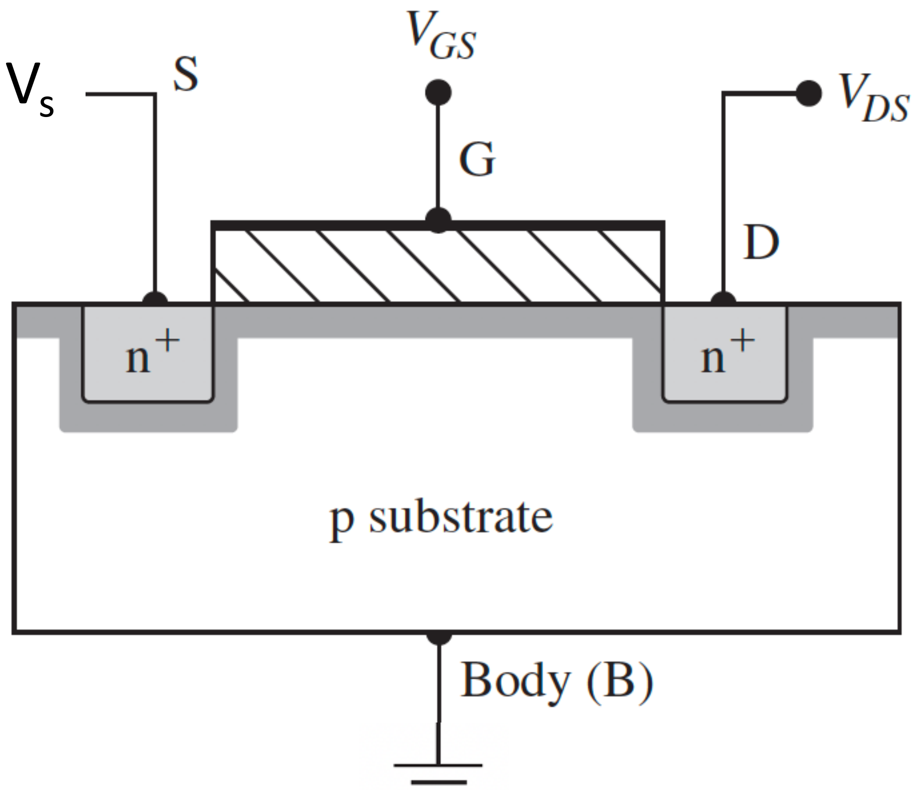
$$I_{DSAT} = \frac{k_n'}{2}\frac{W}{L}\left(V_{GS} - V_T\right)^2$$

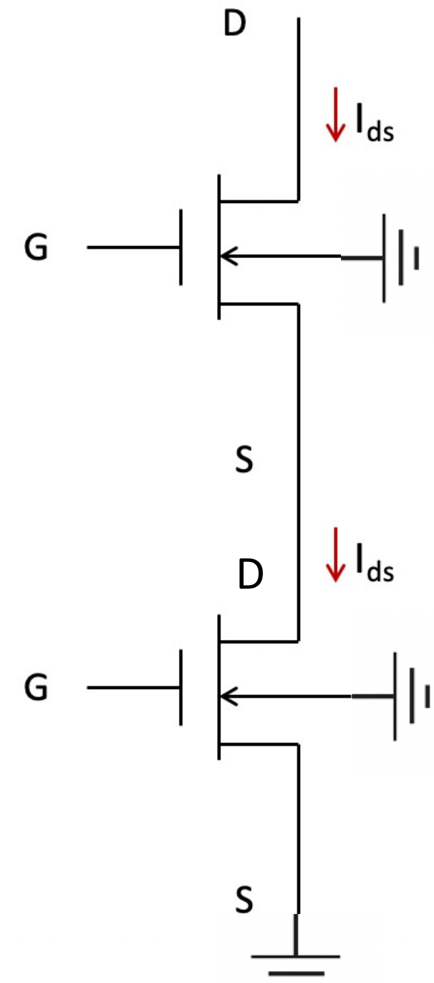
$$g_{ms} = \frac{\partial I_D(\text{sat})}{\partial V_{GS}} = \frac{W\mu_n C_{\text{ox}}}{L}(V_{GS} - V_T)$$

**$V_{DS} > V_{GS} - V_T$**

# 10.4 The basic MOSFET operation

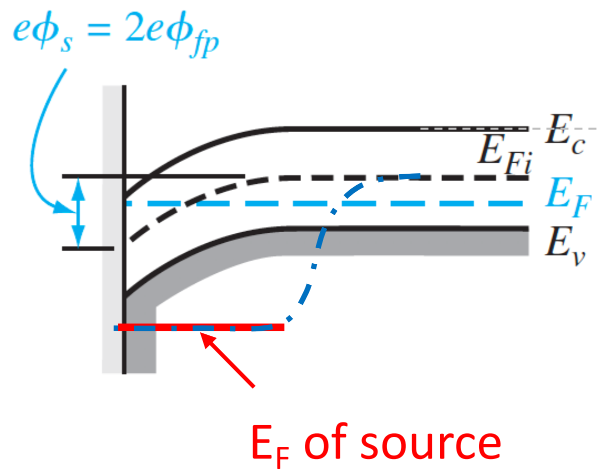
Substrate bias effect

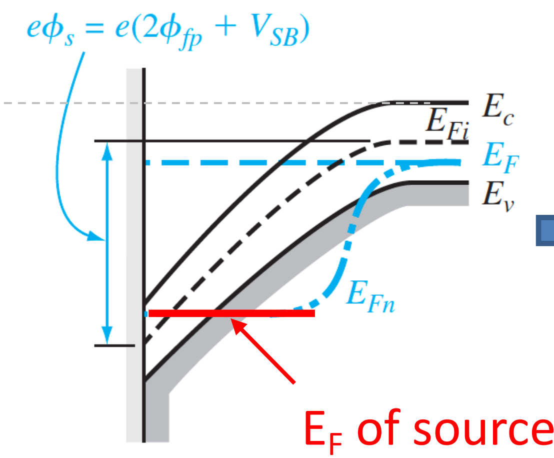


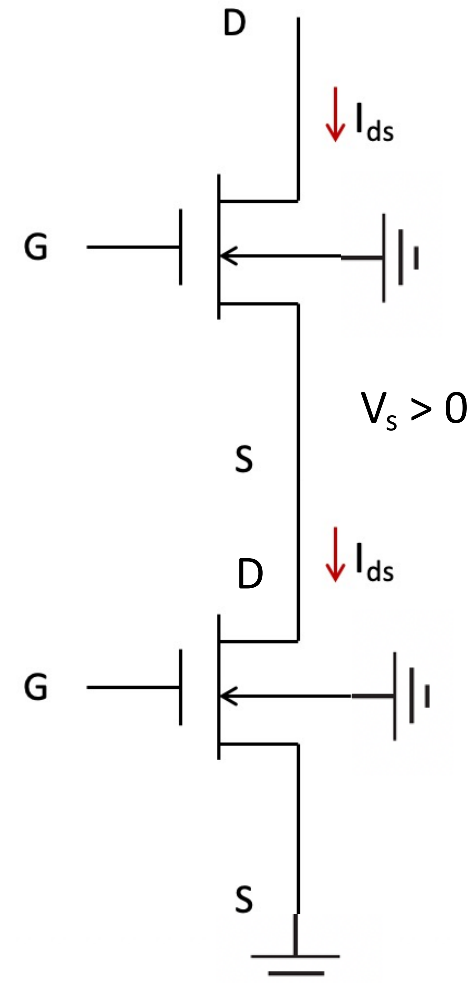



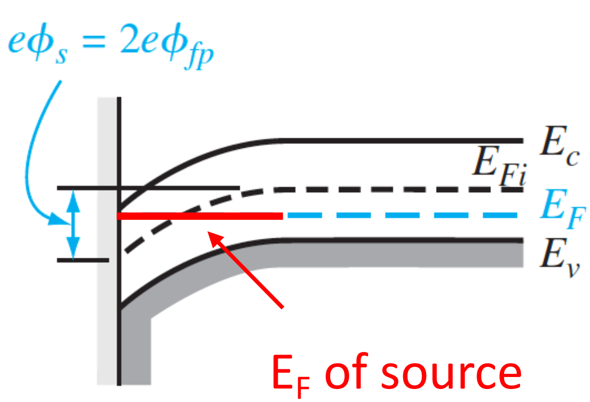
# 10.4 The basic MOSFET operation

## Substrate bias effect



# 10.4 The basic MOSFET operation

Substrate bias effect

When $V_{SB} = 0$, we had

$$Q'_{SD}\,(\max) = -eN_a x_{dT} = -\sqrt{2e\epsilon_s N_a(2\phi_{fp})}$$

When $V_{SB} > 0$, the space charge width increases and we now have

$$Q'_{SD} = -eN_a x_d = -\sqrt{2e\epsilon_s N_a(2\phi_{fp} + V_{SB})}$$

The change in the space charge density is then

$$\Delta Q'_{SD} = -\sqrt{2e\epsilon_s N_a}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$

# 10.4 The basic MOSFET operation

Substrate bias effect

$$\boxed{\Delta V_T = -\frac{\Delta Q'_{SD}}{C_{ox}} = \frac{\sqrt{2e\epsilon_s N_a}}{C_{ox}}\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]}$$

$$\gamma = \frac{\sqrt{2e\epsilon_s N_a}}{C_{ox}} \qquad \Delta V_T = \gamma\left[\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right]$$