VE320 RC Chap 4&5

louyukun@umich.edu

6.10

Outline

- The semiconductor in equilibrium
- Carrier Transport Phenomena
- Graded Impurity Distribution

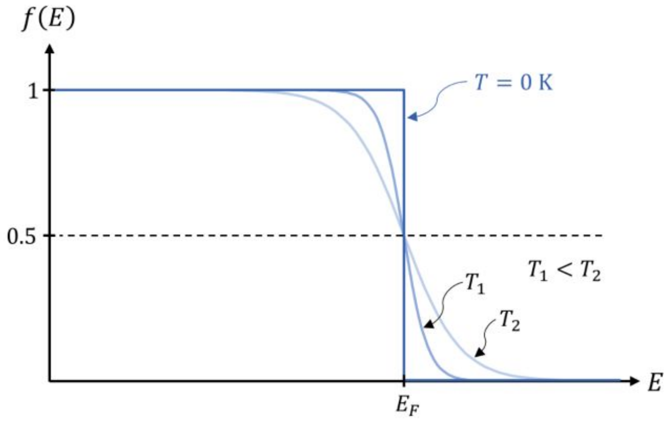
Fermi-Dirac Distribution

Fermi-Dirac Distribution Function $f_E(E)$:

$E - E_F >> kT$

$= e^{-(E-E_F)/kT}$

$$f_E(E) = \frac{1}{1+e^{(E-E_F)/kT}} =$$ Probability of occupancy of a discrete energy state E



$E_F \equiv$ Fermi Energy (or Fermi Level), represents the "filling level"

$k(k_B) \equiv$ Boltzmann constant

$kT = 0.0259$ eV at $T = 300$K

Temperature T determines shape of $f_E(E)$

The semiconductor in equilibrium

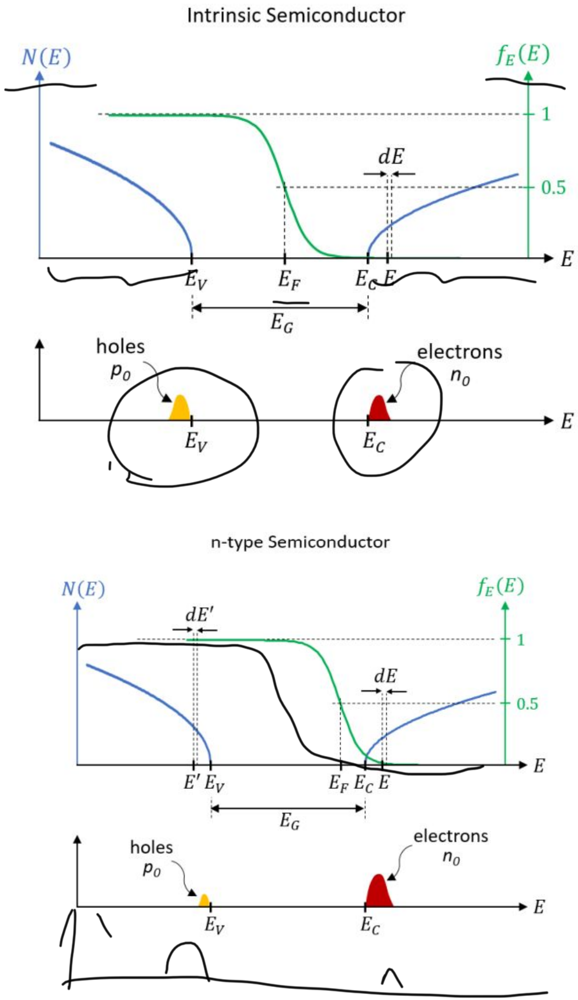
- $n_0 \equiv$ concentration of ***occupied*** states in conduction band
- $p_0 \equiv$ concentration of ***empty*** states in valence band
- g(E)
  Density of states $N(E)$ in conduction and valence bands, respectively, are ***continuous functions***:
  - g(E)
    Conduction band: $N(E) = \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{3/2} \cdot (E - E_C)^{1/2}$
  - g(E)
    Valence band: $N(E) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} \cdot (E_V - E)^{1/2}$

The semiconductor in equilibrium

- $N(E)dE \;\equiv$ number of states per unit volume between E and $E + dE$
- $f_E(E)N(E)dE \;\equiv$ number of occupied states per unit volume between E and $E + dE$

$$n_0 = \int_{E_C}^{\infty} N(E) \cdot f_E(E) \cdot dE$$

$$p_0 = \int_{-\infty}^{E_V} N(E) \cdot \underbrace{[1 - f_E(E)]}_{\text{Probability that state is } empty} \cdot dE$$



The semiconductor in equilibrium

Define $N_C \equiv 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$ = Effective density of states in the conduction band at $E = E_C$

$$\boxed{n_0 = N_C \cdot f_E(E_C) = N_C \cdot e^{-(E_C-E_F)/kT}} \quad [6]$$

$$\boxed{p_0 = N_V \cdot [1 - f_E(E_V)] = N_V \cdot e^{-(E_F-E_V)/kT}} \quad [7]$$

where

$N_V \equiv 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$ = Effective density of states in the valence band at $E = E_V$

$$1 - f_E(E_V) = 1 - \frac{1}{1 + e^{-(E_F-E_V)/kT}} \approx 1 - \left(1 - e^{-(E_F-E_V)/kT}\right)$$

$$= e^{-(E_F-E_V)/kT}$$

The semiconductor in equilibrium

$$n_0 \cdot p_0 = \left(N_C \cdot e^{-(E_C - E_F)/kT}\right)\left(N_V \cdot e^{-(E_F - E_V)/kT}\right)$$

$$= N_C N_V e^{-(E_C - E_V)/kT} = N_C N_V e^{-E_G/kT} \quad \text{(independent of doping)}$$

$$n_0 \cdot p_0 = n_i^2 \quad \rightarrow \quad \boxed{n_i = \sqrt{N_C N_V} e^{-E_G/2kT}} \qquad [8]$$

The semiconductor in equilibrium - Intrinsic

$$n_i = N_C e^{-(E_C - E_i)/kT} = N_V e^{-(E_i - E_V)/kT}$$

$$e^{-(E_C + E_V - 2E_i)/kT} = \left(\frac{N_V}{N_C}\right) \rightarrow \text{Take ln of both sides}$$

$$2E_i - E_C - E_V = kT \cdot \ln\left(\frac{N_V}{N_C}\right)$$

$$\boxed{E_i = \frac{E_C + E_V}{2} + \frac{1}{2}kT \cdot \ln\left(\frac{N_V}{N_C}\right)}$$

For intrinsic semiconductor material:

- $n_0 = p_0 = n_i$
- $n_0 = N_C e^{-(E_C - E_i)/kT} = n_i$ [9a]
- $p_0 = N_V e^{-(E_i - E_V)/kT} = n_i$ [9b]

$$E_{Fi} = \frac{1}{2}(E_c + E_v) + \frac{3}{4}kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

The semiconductor in equilibrium - Extrinsic

e^-

$$n + N_A^- = p + N_D^+$$

$$n.p = n_i^2$$

$$n = n_i \exp\left(\frac{E_f - E_i}{kT}\right)$$

$$n = N_C \exp\left(\frac{E_f - E_C}{kT}\right)$$

$$p = n_i \exp\left(-\frac{E_f - E_i}{kT}\right)$$

$$p = N_V \exp\left(\frac{E_V - E_f}{kT}\right)$$

$$E_f = E_C + kT \ln\left(\frac{n}{N_C}\right)$$

$$E_f = E_V - kT \ln\left(\frac{p}{N_V}\right)$$

$$E_f - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

$$E_i - E_f = kT \ln\left(\frac{p}{n_i}\right)$$

The semiconductor in equilibrium - STATISTICS OF DONORS AND ACCEPTORS

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$n_d = f_d(E) N_d = N_d - N_d^+$$

$$f_a(E) = \frac{1}{1 + \frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)}$$

$$p_a = f_a(E) N_a = N_a - N_a^+$$

$$n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}$$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}$$

The semiconductor in equilibrium - Example 1

- n-type Si ($E_G = 1.1\ \mathrm{eV},\ n_i = 1.5 \times 10^{10}\ \mathrm{cm^{-3}},\ N_C = 2.8 \times 10^{19}\ \mathrm{cm^{-3}},$
  $N_V = 1.8 \times 10^{19}\ \mathrm{cm^{-3}}$)
- $N_D = 10^{17}\ \mathrm{cm^{-3}}$
- $\underline{E_C - E_D = 10\ \mathrm{meV}}$
- $T = 300$ K ($kT = 25.9$ meV)

Shallow donor

$E_D - E_F >> kT$

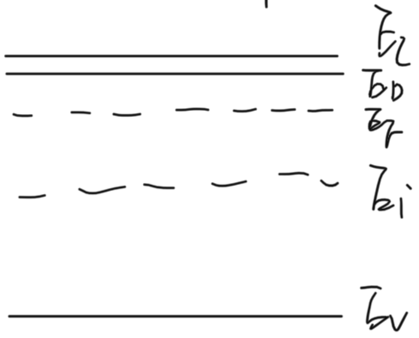
$N_D = N_D^+$

$n_0,\ p_0,\ \underline{E_C - E_F}$

$$\underline{n_0} + \cancel{N_A^-} = \cancel{p_0} + \underline{N_D^+}$$

$$n_0 = N_D^+$$

$kT = 0.0259\ eV \quad E_C - E_D << kT$



$E_D - E_F >> kT$

$$N_D^+ = N_D[1 - f_F(E_D)] = N_D\left[\frac{e^{\underline{(E_D - E_F)}/kT}}{\cancel{1 + e^{(E_D - E_F)/kT}}}\right] \approx 1$$

$$\approx N_D$$

$n_0 = N_D = 10^{17}\ \underline{\mathrm{cm^{-3}}}$

$$p_0 = \frac{n_i^2}{n_0}$$

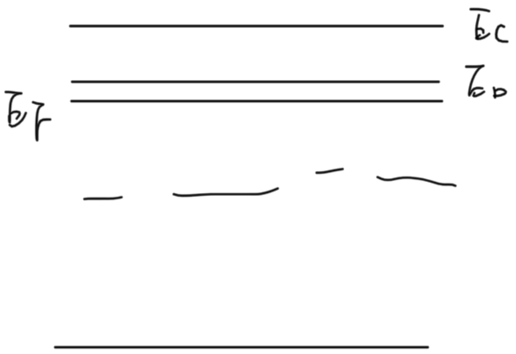
$\underline{E_C - E_F}$.

$$n_0 = N_C \cdot e^{-(E_C - E_F)/kT}$$

$$E_C - E_F = kT \cdot \ln\left(\frac{N_C}{n_0}\right)$$

The semiconductor in equilibrium - Example 2

- *n*-type Si (properties given on last page)
- $N_D = 10^{17}\ \mathrm{cm}^{-3}$
- $E_C - E_D = 155\ \mathrm{meV}$
- $T = 300$ K ($kT = 25.9$ meV)



Deep donor

$$N_D^+ = N_D[1 - f(E_D)]$$

$$n_0 + \cancel{N_A^-} = \cancel{p_0} + N_D^+$$

$$n_0 = N_C \cdot e^{-(E_C - E_F)/kT}$$

$$N_D\left[\frac{e^{(E_D - E_F)/kT}}{1 + e^{(E_D - E_F)/kT}}\right] = N_C \cdot e^{-(E_C - E_F)/kT}$$

$$e^{-(E_C - E_F)/kT} + e^{-(E_C - E_D)/kT} = \frac{N_D}{N_C}\left[e^{(E_D - E_F)/kT}\right]$$

$E_C - E_F$

$= -(E_C - E_D)$

$E_C - E_F$

n_0

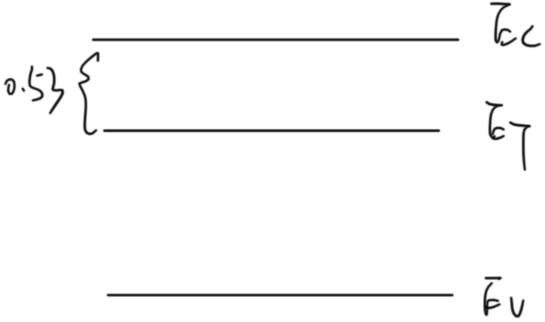
$$p_0 = \frac{n_i^2}{n_0}$$

The semiconductor in equilibrium - Example 3

Prob 1) Manganese makes a donor trap state 0.53 eV below the conduction band edge in silicon. If the silicon is doped with $N_D = 10^{16}\ \text{cm}^{-3}$, what is the ~~probability~~ of occupancy of the trap state? Assume the concentration of Mn is small enough not to affect the overall doping and that the trap state is single-valued (**40 points**).

$$f_E(E_T) = \frac{1}{1+e^{(E_T-E_F)/kT}}$$

$$\underline{E_c - E_F} \leftarrow N_D \qquad \underline{E_T - E_F} = \underline{E_T - E_c} + \underline{E_c - E_F}$$



$$E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) \qquad n_0 \approx N_D$$

$$E_c - E_F = kT \ln\left(\frac{N_c}{N_D}\right)$$

$$f_E(E_T) = \frac{1}{1+e^{(\underline{E_T - E_c} + \underline{E_c - E_F})}}$$

$$\approx 0.999$$

Carrier Transport Phenomena

Drift: Carriers in an E-field move as a result of the force from the E-field

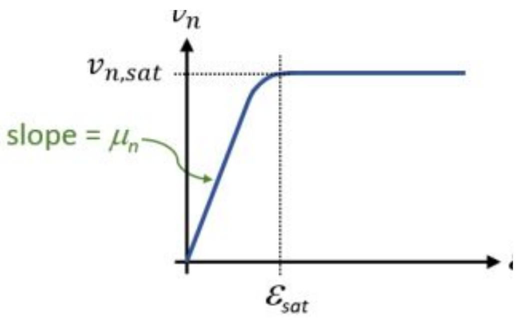
Diffusion: Carriers move from region of high concentration to region of low concentration (statistical process)

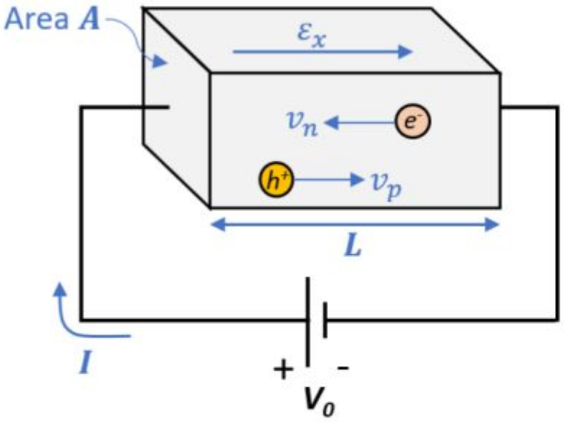
Carrier Transport Phenomena - Drift

- $\varepsilon_x = \dfrac{V_0}{L}$

- At lower voltages V_0 (small ε_x), v_n and v_p increase linearly with V_0

$$\left.\begin{array}{l} \circ\ v_n = \mu_n \varepsilon_x \\ \circ\ v_p = \mu_p \varepsilon_x \end{array}\right\} \varepsilon_x < \varepsilon_{sat} \quad \text{(Ohm's Law)}$$





$\mu_n \equiv$ electron mobility

$\mu_p \equiv$ hole mobility

Carrier Transport Phenomena - Drift

$$I_n = \frac{(\text{charge } q)\cdot(\text{\# of electrons in block})}{(\text{time } t_0 \text{ for electron to transit block})}$$

$$\text{\# of electrons in block} = n_0 \cdot L \cdot A$$

$$t_0 = \frac{L}{v_n} = \frac{L}{\mu_n \varepsilon_x}$$

$$I_n = q \cdot n_0 \cdot L \cdot A \cdot \left(\frac{\mu_n \varepsilon_x}{L}\right) = \boxed{A(q\mu_n n_0)\varepsilon_x}$$

$$J_n = (q\mu_n n_0)\varepsilon_x = \boxed{\sigma_n \varepsilon_x} \rightarrow \sigma_n \equiv \text{electron conductivity} = \boxed{q\mu_n n_0}$$

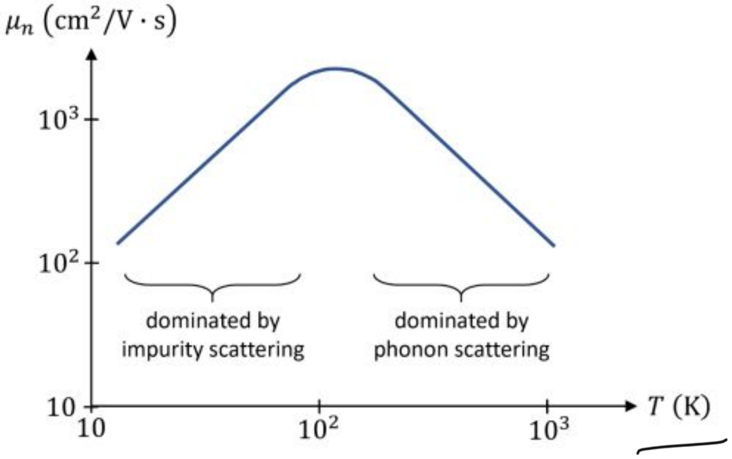
$$I_p = \boxed{A(q\mu_p p_0)\varepsilon_x} \text{ and } J_p = (q\mu_p p_0)\varepsilon_x = \boxed{\sigma_p \varepsilon_x}$$

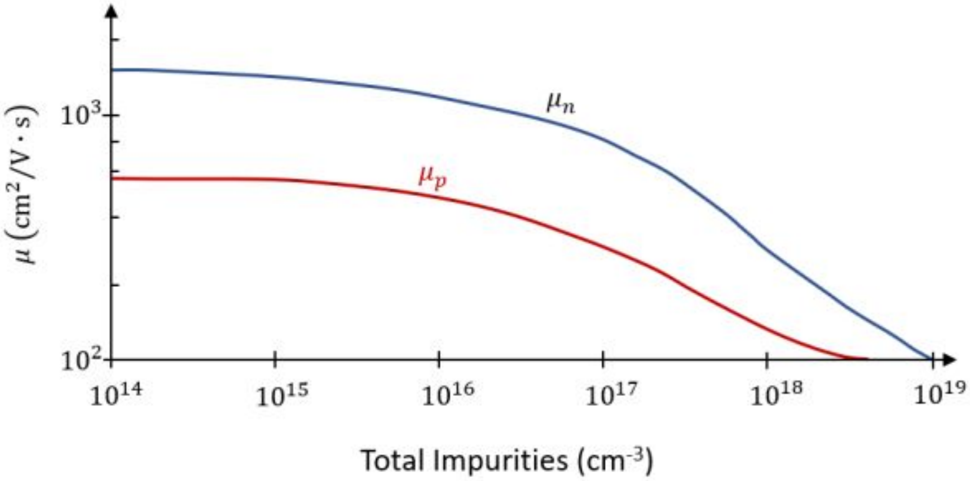
$$J_{tot} = J_n + J_p = \boxed{\sigma \varepsilon_x} \rightarrow \sigma = \sigma_n + \sigma_p = \boxed{q(n_0\mu_n + p_0\mu_p)}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)}$$

ρ : resistivity

Carrier Transport Phenomena - Drift



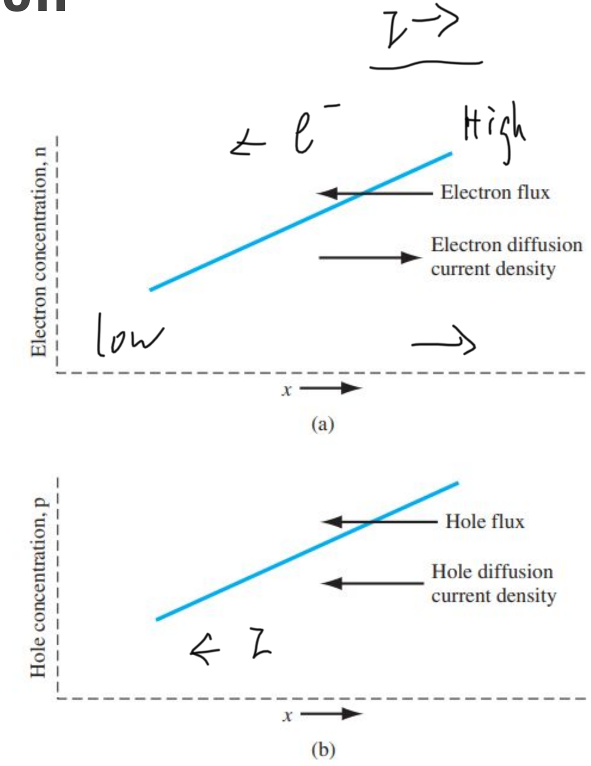


Carrier Transport Phenomena - Diffusion

$$J_{nx|dif} = eD_n \frac{\mathrm{d}n}{\mathrm{d}x}$$

$$J_{px|dif} = -eD_p \frac{\mathrm{d}p}{\mathrm{d}x}$$

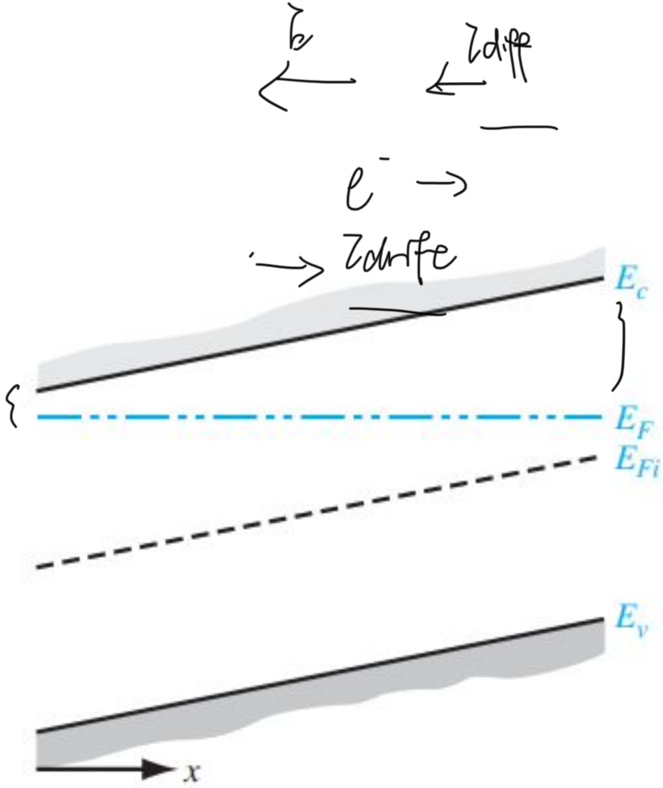
$$\begin{aligned} J &= J_{drf} + J_{dif} \\ &= J_{nx|drf} + J_{px|drf} + J_{nx|dif} + J_{px|dif} \\ &= \boxed{en\mu_n E_x + ep\mu_p E_x + eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}} \end{aligned}$$



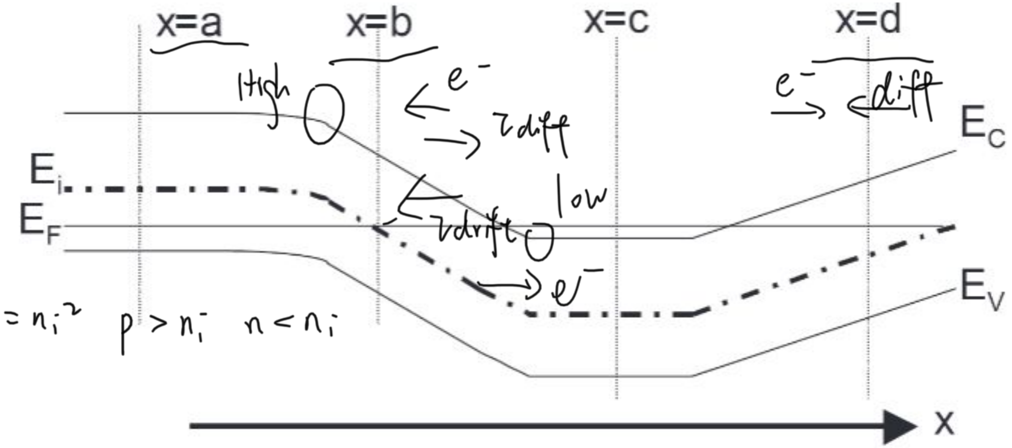
GRADED IMPURITY DISTRIBUTION

$$J_n = 0 = en\mu_n \mathrm{E}_x + eD_n \frac{dn}{dx}$$

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}$$



Exercise 1



a) At x=a, the material is n-type or p-type

b) At x=a, $n < n_i$ or $n=n_i$ or $n > n_i$

$p > n$ $pn = n_i^2$ $p > n_i$ $n < n_i$

c) At x=b, $n \ll p$ or $n \sim p$ or $n \gg p$

d) At x=b, $J_{n,drift} < 0$ or $J_{n,drift} = 0$ or $J_{n,drift} > 0$

e) At x=b, the drift process forces electrons in the +x direction or -x direction

f) At x=c, the semiconductor is degenerate or non-degenerate

g) At x=d, $J_{n,diff} < 0$ or $J_{n,diff} = 0$ or $J_{n,diff} > 0$

h) At x=d, $J_{n,total} < 0$ or $J_{n,total} = 0$ or $J_{n,total} > 0$

Exercise 2

A semiconductor is doped with $N_D = 1x10^{13}$ cm^{-3} and N_A=0. Other parameters of the materials are given below.

Property	Value
E_G (eV)	0.259
m_n^*/m_0	0.30
m_p^*/m_0	0.60
μ_n (cm^2/Vs), doping independent	2,000
μ_p (cm^2/Vs), doping independent	500

a) Determine the intrinsic carrier concentration.

b) Calculate the conductivity of this material. (10 points)

$$n_i = \sqrt{N_c N_v} \cdot e^{-E_G/2kT} = \boxed{4.65 \times 10^{16} cm^{-3}}$$

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

$$n_i >> N_D, N_A$$

$$m_n^* = 0.3 \cdot m_0$$

$$m_p^* = 0.6 \cdot m_0$$

Exercise 2

A semiconductor is doped with $N_D = 1x10^{13}\ cm^{-3}$ and N_A=0. Other parameters of the materials are given below.

a) Determine the intrinsic carrier concentration.

b) Calculate the conductivity of this material. (10 points)

Property	Value
E_G (eV)	0.259
m_n^*/m_0	0.30
m_p^*/m_0	0.60
μ_n (cm^2/Vs), doping independent	2,000
μ_p (cm^2/Vs), doping independent	500

$\sigma = nq\mu_n + pq\mu_p$

$= n_i q\mu_n + n_i q\mu_p$

$N_D \ll n_i \quad N_A \ll n_i$

$p = n = n_i$

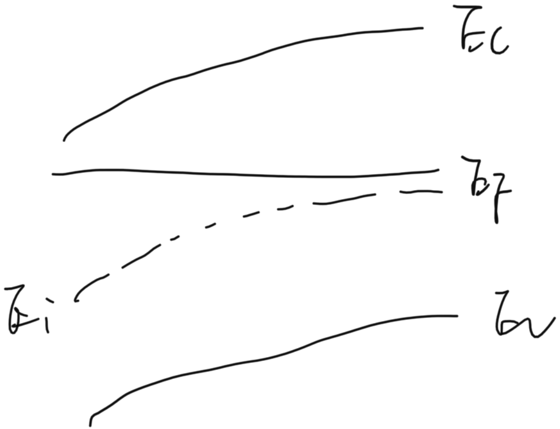
Exercise 3

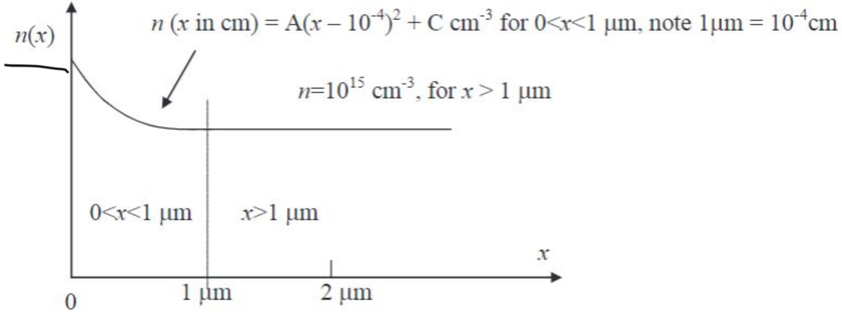
Suppose we have a region of an ***n*-type** semiconductor where we know the electron concentration as shown below. The semiconductor is at 300K, at equilibrium, and has an electron mobility of μ_n=1000 $cm^2/V \cdot s$.

a) Sketch the band diagram showing E_C, E_V, E_F and E_i.

b) Write expressions for the electron drift current density (as a function of position x and electric field) and diffusion current density (as a function of position only) for $0<x<2\mu m$.

c) Find the electric field at x=0.5μm.





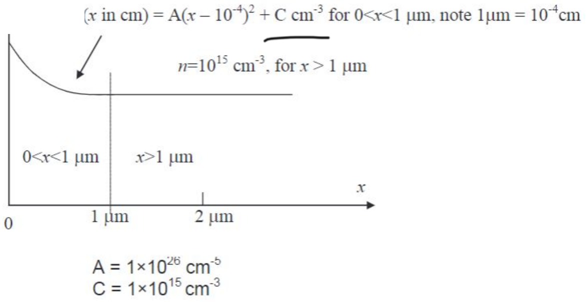
A = 1×10^{26} cm^{-5}
C = 1×10^{15} cm^{-3}

Exercise 3

Suppose we have a region of an ***n*-type** semiconductor where we know the electron concentration as shown below. The semiconductor is at 300K, at equilibrium, and has an electron mobility of μ_n=1000 cm^2/V·s.

b) Write expressions for the electron drift current density (as a function of position x and electric field) and diffusion current density (as a function of position only) for $0<x<2\mu m$.

c) Find the electric field at x=0.5μm.



$$J_{n,drift} = n q \mu_n \mathcal{E} = \begin{cases} [A(x-10^{4})^2 + C] q \mu_n \mathcal{E}, & 0<x<1\mu m \\ 10^{15} q \mu_n \mathcal{E}, & x > 1\mu m \end{cases}$$

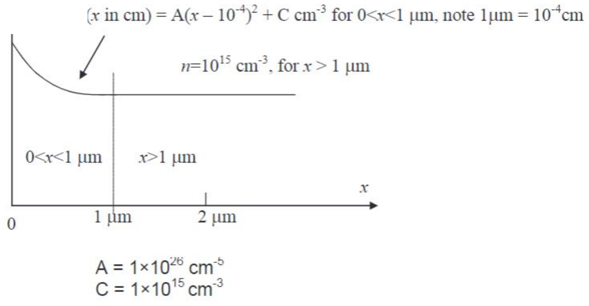
$$J_{n,diff} = q D_n \frac{dn}{dx} \qquad x > 1\mu m, \ \frac{dn}{dx} = 0 \rightarrow J_{n,diff} = 0$$

$$0<x<1\mu m, \ \frac{dn}{dx} = 2A(x-10^{-4}) \rightarrow J_{n,diff} = 2A(x-10^{-4}) q D_n$$

Exercise 3

Suppose we have a region of an ***n*-type** semiconductor where we know the electron concentration as shown below. The semiconductor is at 300K, at equilibrium, and has an electron mobility of μ_n=1000 cm^2/V·s.

c) Find the electric field at x=0.5μm.



$$J_{n,drift}(0.5) = 10^{26}(0.5\times10^{-4} - 10^{-4}) \cdot q \cdot \mu_n \mathcal{E}$$

$$J_{n,diff} = 2\cdot10^{26}(0.5\times10^{-4} - 10^{-4})\, q D_n$$

$$J_{n,drift} = -J_{n,diff}$$

$$\mathcal{E} = \frac{J_{n,drift}}{n q \mu_n} = \frac{-J_{n,diff}}{n q \mu_n}$$

Thanks