

VE320 Intro to Semiconductor Devices

Summer 2022 — Problem Set 1 Solution



May 30, 2022

Exercise 1.1

The lattice constant of a single crystal is 4.73\AA . Calculate the surface density ($\#/\text{cm}^2$) of atoms on the (i) (100), (ii) (110), and (iii) (111) plane for a (a) simple cubic, (b) body-centered cubic, and (c) face-centered cubic lattice.

Answer:

(a) Simple cubic

(i) (100) plane:

$$\begin{aligned}\text{Surface density} &= \frac{1}{a^2} = \frac{1}{(4.73 \times 10^{-8})^2} \\ &= 4.47 \times 10^{14} \text{ cm}^{-2}\end{aligned}$$

(ii) (110) plane:

$$\begin{aligned}\text{Surface density} &= \frac{1}{a^2\sqrt{2}} \\ &= 3.16 \times 10^{14} \text{ cm}^{-2}\end{aligned}$$

(iii) (111) plane:

$$\text{Area of plane} = \frac{1}{2}bh$$

$$\text{where } b = a\sqrt{2} = 6.689\text{\AA}$$

$$\text{Now } h^2 = (a\sqrt{2})^2 - \left(\frac{a\sqrt{2}}{2}\right)^2 = \frac{3}{4}(a\sqrt{2})^2 \text{ So } h = \frac{\sqrt{6}}{2}(4.73) = 5.793\text{\AA}$$

Area of plane

$$\begin{aligned}&= \frac{1}{2} (6.68923 \times 10^{-8}) (5.79304 \times 10^{-8}) \\ &= 19.3755 \times 10^{-16} \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Surface density} &= \frac{3 \times \frac{1}{6}}{19.3755 \times 10^{-16}} \\ &= 2.58 \times 10^{14} \text{ cm}^{-2}\end{aligned}$$

(b) bcc

(i) (100) plane:

$$\text{Surface density} = \frac{1}{a^2} = 4.47 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{2}{a^2\sqrt{2}} = 6.32 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane:

$$\begin{aligned}\text{Surface density} &= \frac{3 \times \frac{1}{6}}{19.3755 \times 10^{-16}} \\ &= 2.58 \times 10^{14} \text{ cm}^{-2}\end{aligned}$$

(c) fcc

(i) (100) plane:

$$\text{Surface density} = \frac{2}{a^2} = 8.94 \times 10^{14} \text{ cm}^{-2}$$

(ii) (110) plane:

$$\text{Surface density} = \frac{2}{a^2\sqrt{2}} = 6.32 \times 10^{14} \text{ cm}^{-2}$$

(iii) (111) plane:

$$\text{Surface density} = \frac{3 \times \frac{1}{6} + 3 \times \frac{1}{2}}{19.3755 \times 10^{-16}} = 1.03 \times 10^{15} \text{ cm}^{-2}$$

Exercise 1.2

The work function of a material refers to the minimum energy required to remove an electron from the material. Assume that the work function of gold is 4.90eV and that of cesium is 1.90eV. Calculate the maximum wavelength of light for the photoelectric emission of electrons for gold and cesium.

Answer:

$$E = hv = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E}$$

$$\text{Gold: } E = 4.90\text{eV} = (4.90) (1.6 \times 10^{-19}) \text{ J}$$

So,

$$\lambda = \frac{(6.625 \times 10^{-34}) (3 \times 10^{10})}{(4.90) (1.6 \times 10^{-19})} = 2.54 \times 10^{-5} \text{ cm}$$

or

$$\lambda = 0.254\mu\text{m}$$

$$\text{Cesium: } E = 1.90\text{eV} = (1.90) (1.6 \times 10^{-19}) \text{ J}$$

So,

$$\lambda = \frac{(6.625 \times 10^{-34}) (3 \times 10^{10})}{(1.90) (1.6 \times 10^{-19})} = 6.54 \times 10^{-5} \text{ cm}$$

or

$$\lambda = 0.654\mu\text{m}$$

Exercise 1.3

According to classical physics, the average energy of an electron in an electron gas at thermal equilibrium is $3kT/2$. Determine, for $T = 300 \text{ K}$, the average electron energy (in eV), average electron momentum, and the de Broglie wavelength.

Answer:

$$E_{avg} = \frac{3}{2}kT = \left(\frac{3}{2}\right) (0.0259) = 0.03885\text{eV}$$

Now

$$\begin{aligned} p_{avg} &= \sqrt{2mE_{avg}} \\ &= \sqrt{2 (9.11 \times 10^{-31}) (0.03885) (1.6 \times 10^{-19})} \end{aligned}$$

or

$$p_{\text{avg}} = 1.064 \times 10^{-25} \text{ kg} \cdot \text{m/s}$$

Now

$$\lambda = \frac{h}{p} = \frac{6.625 \times 10^{-34}}{1.064 \times 10^{-25}} = 6.225 \times 10^{-9} \text{ m}$$

or

$$\lambda = 62.25 \text{ \AA}$$

Exercise 1.4

An electron is described by a wave function given by $\psi(x) = \sqrt{\frac{2}{a}} \cos\left(\frac{\pi x}{a}\right)$ for $-\frac{a}{2} < x < \frac{a}{2}$. The wave function is zero elsewhere. Calculate the probability of finding the electron between (a) $0 < x < \frac{a}{4}$, (b) $\frac{a}{4} < x < \frac{a}{2}$, and (c) $-\frac{a}{2} < x < -\frac{a}{4}$.

Answer:

$$\begin{aligned} \int_{-1}^{+3} A^2 \cos^2\left(\frac{\pi x}{2}\right) dx &= 1 \\ A^2 \left[\frac{x}{2} + \frac{\sin(\pi x)}{2\pi} \right] \Big|_{-1}^{+3} &= 1 \\ A^2 \left[\frac{3}{2} - \left(\frac{-1}{2} \right) \right] &= 1 \end{aligned}$$

so

$$A^2 = \frac{1}{2}$$

or

$$|A| = \frac{1}{\sqrt{2}}$$

Exercise 1.5

Consider the wave function $\Psi(x, t) = A \left(\cos\left(\frac{\pi x}{2}\right) \right) e^{-j\omega t}$ for $-1 \leq x \leq +3$. Determine A so that $\int_{-1}^{+3} |\Psi(x, t)|^2 dx = 1$.

Answer:

$$P = \int |\psi(x)|^2 dx$$

(a)

$$\begin{aligned} & \int_0^{a/4} \left(\frac{2}{a} \right) \cos^2\left(\frac{\pi x}{2}\right) dx \\ &= \left(\frac{2}{a} \right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)} \right]_0^{a/4} \\ &= \left(\frac{2}{a} \right) \left[\frac{\left(\frac{a}{4}\right)}{2} + \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)} \right] \\ &= \left(\frac{2}{a} \right) \left[\frac{a}{8} + \frac{(1)(a)}{4\pi} \right] \\ &\text{or } P = 0.409 \end{aligned}$$

(b)

$$\begin{aligned} P &= \int_{a/4}^{a/2} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{4\left(\frac{\pi}{a}\right)} \right] \Big|_{a/4}^{a/2} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{4} + \frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)} - \frac{a}{8} - \frac{\sin\left(\frac{\pi}{2}\right)}{\left(\frac{4\pi}{a}\right)} \right] \\ &= 2 \left[\frac{1}{4} + 0 - \frac{1}{8} - \frac{1}{4\pi} \right] \\ P &= 0.0908 \end{aligned}$$

(c)

$$\begin{aligned} P &= \int_{-a/2}^{+a/2} \left(\frac{2}{a}\right) \cos^2\left(\frac{\pi x}{a}\right) dx \\ &= \left(\frac{2}{a}\right) \left[\frac{x}{2} + \frac{\sin\left(\frac{2\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)} \right] \Big|_{-a/2}^{+a/2} \\ &= \left(\frac{2}{a}\right) \left[\frac{a}{4} + \frac{\sin(\pi)}{\left(\frac{4\pi}{a}\right)} - \left(\frac{-a}{4}\right) - \frac{\sin(-\pi)}{\left(\frac{4\pi}{a}\right)} \right] \end{aligned}$$

or

$$P = 1$$

Exercise 1.6

An electron is bound in a one-dimensional infinite potential well with a width of 10\AA . (a) Calculate the first three energy levels that the electron may occupy. (b) If the electron drops from the third to the second energy level, what is the wavelength of a photon that might be emitted?

Answer:

$$(a) E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2} = \frac{n^2 (1.054 \times 10^{-34})^2 \pi^2}{2(9.11 \times 10^{-31})(10 \times 10^{-10})^2} = n^2 (6.018 \times 10^{-20}) \text{ J}$$

$$\text{or } E_n = \frac{n^2 (6.018 \times 10^{-20})}{1.6 \times 10^{-19}} = n^2 (0.3761) \text{ eV}$$

Then

$$E_1 = 0.376 \text{ eV}$$

$$E_2 = 1.504 \text{ eV}$$

$$E_3 = 3.385 \text{ eV}$$

(b)

$$\lambda = \frac{hc}{\Delta E}$$

$$\begin{aligned} \Delta E &= (3.385 - 1.504) (1.6 \times 10^{-19}) \\ &= 3.01 \times 10^{-19} \text{ J} \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{(6.625 \times 10^{-34}) (3 \times 10^8)}{3.01 \times 10^{-19}} \\ &= 6.604 \times 10^{-7} \text{ m} \end{aligned}$$

$$\text{or } \lambda = 660.4 \text{ nm}$$

Exercise 1.7

Consider the one-dimensional potential function shown in Figure 1. Assume the total energy of an electron is $E < V_0$. (a) Write the wave solutions that apply in each region. (b) Write the set of equations that result from applying the boundary conditions. (c) Show explicitly why, or why not, the energy levels of the electron are quantized.

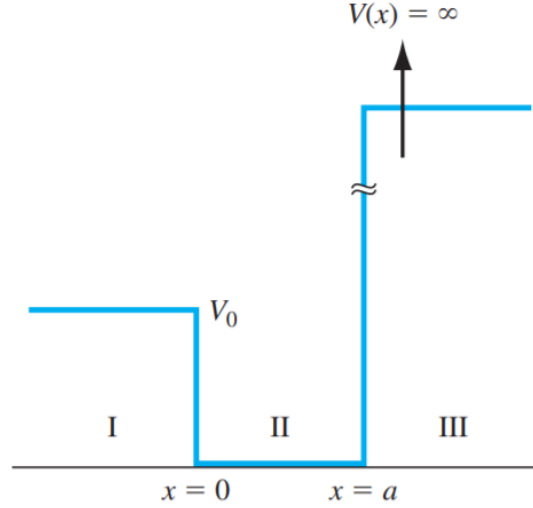


Figure 1: Potential function for Problem 1.7

Answer:

(a) Region I: Since $V_0 > E$, we can write

$$\frac{\partial^2 \psi_1(x)}{\partial x^2} - \frac{2m(V_0 - E)}{\hbar^2} \psi_1(x) = 0$$

Region II: $V = 0$, so

$$\frac{\partial^2 \psi_2(x)}{\partial x^2} + \frac{2mE}{\hbar^2} \psi_2(x) = 0$$

Region III: $V \rightarrow \infty \Rightarrow \psi_3 = 0$

The general solutions can be written, keeping in mind that ψ_1 must remain finite for $x < 0$, as

$$\begin{aligned} \psi_1(x) &= B_1 \exp(k_1 x) \\ \psi_2(x) &= A_2 \sin(k_2 x) + B_2 \cos(k_2 x) \\ \psi_3(x) &= 0 \end{aligned}$$

where

$$k_1 = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} \text{ and } k_2 = \sqrt{\frac{2mE}{\hbar^2}}$$

(b) Boundary conditions

At $x = 0$: $\psi_1 = \psi_2 \Rightarrow B_1 = B_2$

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x} \Rightarrow k_1 B_1 = k_2 A_2$$

At $x = a$: $\psi_2 = \psi_3 \Rightarrow$

$$A_2 \sin(k_2 a) + B_2 \cos(k_2 a) = 0$$

or

$$B_2 = -A_2 \tan(k_2 a)$$

(c)

$$k_1 B_1 = k_2 A_2 \Rightarrow A_2 = \left(\frac{k_1}{k_2}\right) B_1$$

and since $B_1 = B_2$, then

$$A_2 = \left(\frac{k_1}{k_2}\right) B_2$$

From $B_2 = -A_2 \tan(k_2 a)$, we can write

$$B_2 = -\left(\frac{k_1}{k_2}\right) B_2 \tan(k_2 a)$$

or

$$1 = -\left(\frac{k_1}{k_2}\right) \tan(k_2 a)$$

This equation can be written as

$$1 = -\sqrt{\frac{V_O - E}{E}} \cdot \tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

or

$$\sqrt{\frac{E}{V_O - E}} = -\tan\left[\sqrt{\frac{2mE}{\hbar^2}} \cdot a\right]$$

This last equation is valid only for specific values of the total energy E . The energy levels are quantized.

Exercise 1.8

The bandgap energy in a semiconductor is usually a slight function of temperature. In some cases, the bandgap energy versus temperature can be modeled by

$$E_g = E_g(0) - \frac{\alpha T^2}{(\beta + T)}$$

where $E_g(0)$ is the value of the bandgap energy at $T = 0$ K. For silicon, the parameter values are $E_g(0) = 1.170$ eV, $\alpha = 4.73 \times 10^{-4}$ eV/K, and $\beta = 636$ K. Plot E_g versus T over the range $0 \leq T \leq 600$ K. In particular, note the value at $T = 300$ K.

Answer:

For $T = 100$ K,

$$\begin{aligned} E_g &= 1.170 - \frac{(4.73 \times 10^{-4})(100)^2}{636 + 100} \Rightarrow \\ T = 200 \text{ K}, \quad E_g &= 1.164 \text{ eV} \\ T = 300 \text{ K}, \quad E_g &= 1.125 \text{ eV} \\ T = 400 \text{ K}, \quad E_g &= 1.097 \text{ eV} \\ T = 500 \text{ K}, \quad E_g &= 1.066 \text{ eV} \\ T = 600 \text{ K}, \quad E_g &= 1.032 \text{ eV} \end{aligned}$$

You can use MATLAB or other softwares to plot this curve.

Reference

1. Neamen, Donald A. Semiconductor physics and devices: basic principles. McGraw-hill, 2003.