VE320 Intro to Semiconductor Devices Summer 2022 — Problem Set 6

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Constants

[Note: In the following problems, assume $T = 300$ K and the following parameters unless otherwise stated. For silicon pn junctions: $D_n = 25\ \text{cm}^2/\text{s}, D_p = 10\ \text{cm}^2/\text{s}, \tau_{n0} = 5 \times 10^{-7}$ s, $\tau_{p0} = 10^{-7}$ s. For GaAs pn junctions: $D_n = 205\ \text{cm}^2/\text{s}, D_p = 9.8\ \text{cm}^2/\text{s}, \tau_{n0} = 5 \times 10^{-8}$ s, $\tau_{p0} = 10^{-8}$ s.]

Exercise 6.1

(a) Consider an ideal pn junction diode at $T = 300$ K operating in the forward-bias region. Calculate the change in diode voltage that will cause a factor of 10 increase in current.

(b) Repeat part (a) for a factor of 100 increase in current.

Answer:

In forward bias

$$I_f \cong I_S \exp\left(\frac{\text{eV}}{kT}\right)$$

Then

$$\frac{I_{f1}}{I_{f2}} = \frac{I_S \exp\left(\frac{eV_1}{kT}\right)}{I_S \exp\left(\frac{eV_2}{kT}\right)} = \exp\left[\left(\frac{e}{kT}\right)(V_1 - V_2)\right]$$

or

$$V_1 - V_2 = \left(\frac{kT}{e}\right) \ln\left(\frac{I_{f1}}{I_{f2}}\right)$$

(a) For $\frac{I_{f1}}{I_{f2}} = 10$, then

$$V_1 - V_2 = (0.0259) \ln(10)$$

or

$$V_1 - V_2 = 59.6\text{mV} \cong 60\text{mV}$$

(b) For $\frac{I_{f1}}{I_{f2}} = 100$, then

$$V_1 - V_2 = (0.0259) \ln(100)$$

or

$$V_1 - V_2 = 119.3\text{mV} \cong 120\text{mV}$$

Exercise 6.2

Consider a GaAs pn junction diode at $T = 300$ K. The parameters of the device are $N_d = 2 \times 10^{16}\ \text{cm}^{-3}, N_a = 8 \times 10^{15}\ \text{cm}^{-3}, D_n = 210\ \text{cm}^2/\text{s}, D_p = 8\ \text{cm}^2/\text{s}, \tau_{no} = 10^{-7}$ s, and $\tau_{po} = 5 \times 10^{-8}$ s. Determine the ideal reverse-saturation current density.

Answer:

$$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}}+\frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right]$$
$$= \left(1.6\times10^{-19}\right)\left(1.8\times10^{6}\right)^2$$
$$\times\left[\frac{1}{8\times10^{15}}\sqrt{\frac{210}{10^{-7}}}+\frac{1}{2\times10^{16}}\sqrt{\frac{8}{5\times10^{-8}}}\right]$$
$$J_s = 3.30\times10^{-18}\ \mathrm{A/cm^2}$$

Optional: If you need to calculate the total current density

$$L_n = \sqrt{D_n\tau_{no}}$$
$$= \sqrt{(210)\left(10^{-7}\right)} = 4.583\times10^{-3}\ \mathrm{cm}$$
$$L_p = \sqrt{D_p\tau_{po}}$$
$$= \sqrt{(8)\left(5\times10^{-8}\right)} = 6.325\times10^{-4}\ \mathrm{cm}$$

$$n_{po} = \frac{n_i^2}{N_a} = \frac{\left(1.8\times10^{6}\right)^2}{8\times10^{15}}$$
$$= 4.05\times10^{-4}\ \mathrm{cm^{-3}}$$
$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.8\times10^{6}\right)^2}{2\times10^{16}}$$
$$= 1.62\times10^{-4}\ \mathrm{cm^{-3}}$$
$$J_n\left(-x_p\right) = \frac{eD_nn_{po}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right)-1\right]$$
$$= \frac{\left(1.6\times10^{-19}\right)(210)\left(4.05\times10^{-4}\right)}{4.583\times10^{-3}}$$
$$\times\left[\exp\left(\frac{1.05}{0.0259}\right)-1\right]$$
$$J_n\left(-x_p\right) = 1.20\ \mathrm{A/cm^2}$$
$$J_p\left(x_n\right) = \frac{eD_pp_{no}}{L_p}\left[\exp\left[\frac{eV_a}{kT}\right]-1\right]$$
$$= \frac{\left(1.6\times10^{-19}\right)(8)\left(1.62\times10^{-4}\right)}{6.325\times10^{-4}}$$
$$\times\left[\exp\left(\frac{1.05}{0.0259}\right)-1\right]$$
$$J_p\left(x_n\right) = 0.1325\ \mathrm{A/cm^2}$$
$$J_T = J_n\left(-x_p\right)+J_p\left(x_n\right)$$
$$= 1.20+0.1325$$
$$J_T = 1.33\ \mathrm{A/cm^2}$$

Exercise 6.3

Consider an ideal silicon pn junction diode.

(a) What must be the ratio of N_d/N_a so that 90 percent of the current in the depletion region is due to the flow of electrons?

(b) Repeat part (a) if 80 percent of the current in the depletion region is due to the flow of holes.

Answer:
(a)

$$\frac{J_n}{J_n + J_p} = \frac{\frac{eD_n n_{po}}{L_n}}{\frac{eD_n n_{po}}{L_n} + \frac{eD_p p_{no}}{L_p}}$$

$$= \frac{\sqrt{\frac{D_n}{\tau_{no}}} \cdot \frac{n_i^2}{N_a}}{\sqrt{\frac{D_n}{\tau_{no}}} \cdot \frac{n_i^2}{N_a} + \sqrt{\frac{D_p}{\tau_{po}}} \cdot \frac{n_i^2}{N_d}}$$

$$0.90 = \frac{1}{\sqrt{\frac{D_p \tau_{no}}{D_n \tau_{po}}} \cdot \left(\frac{N_a}{N_d}\right) = \frac{1}{0.90} - 1}$$

$$\frac{N_a}{N_d} = \sqrt{\frac{D_n \tau_{po}}{D_p \tau_{no}}} \left(\frac{1}{0.90} - 1\right)$$

$$= \sqrt{\frac{(25)\left(10^{-7}\right)}{(10)\left(5 \times 10^{-7}\right)}} (0.1111)\Big)$$

$$\frac{N_a}{N_d} = 0.07857 \text{ or } \frac{N_d}{N_a} = 12.73$$

(b)
From part (a),

$$\frac{N_a}{N_d} = \sqrt{\frac{D_n \tau_{po}}{D_p \tau_{no}}} \left(\frac{1}{0.20} - 1\right)$$

$$= \sqrt{\frac{(25)\left(10^{-7}\right)}{(10)\left(5 \times 10^{-7}\right)}} (4)$$

$$\frac{N_a}{N_d} = 2.828 \text{ or } \frac{N_d}{N_a} = 0.354$$

Exercise 6.4

Consider a silicon pn junction diode with an applied reverse-biased voltage of $V_R = 5$ V. The doping concentrations are $N_a = N_d = 4 \times 10^{16}$ cm^{-3} and the cross-sectional area is $A = 10^{-4}$ cm^2. Assume minority carrier lifetimes of $\tau_0 = \tau_{n0} = \tau_{p0} = 10^{-7}$ s. Calculate the
(a) ideal reverse-saturation current
(b) reverse-biased generation current
(c) the ratio of the generation current to ideal saturation current.

Answer:

(a)

$$(a)\ I_s = \mathrm{Aen}_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}} \right]$$
$$= \left(10^{-4}\right)\left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2$$
$$\times \left[\frac{1}{4\times10^{16}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{4\times10^{16}}\sqrt{\frac{10}{10^{-7}}} \right]$$
$$I_s = 2.323\times10^{-15}\ \mathrm{A}$$

(b) $I_{gen} = \frac{Aen_iW}{2\tau_0}$ We find

$$V_{bi} = (0.0259)\ln\left[\frac{\left(4\times10^{16}\right)\left(4\times10^{16}\right)}{\left(1.5\times10^{10}\right)^2}\right]$$
$$= 0.7665\ \mathrm{V}$$

and

$$W = \left\{\frac{2\in_s\left(V_{bi}+V_R\right)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right\}^{1/2}$$
$$= \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.7665+5)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left[\frac{4\times10^{16}+4\times10^{16}}{\left(4\times10^{16}\right)\left(4\times10^{16}\right)}\right]\right\}^{1/2}$$

Then

$$W = 6.109\times10^{-5}\ \mathrm{cm}$$

$$I_{gen} = \frac{\left(10^{-4}\right)\left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)\left(6.109\times10^{-5}\right)}{2\left(10^{-7}\right)}$$

$$= 7.331\times10^{-11}\ \mathrm{A}$$

(c) $\frac{I_{gen}}{I_s} = \frac{7.331\times10^{-11}}{2.323\times10^{-15}} = 3.16\times10^4$

Exercise 6.5

Consider, as shown in Figure, a uniformly doped silicon pn junction at $T = 300$ K with impurity doping concentrations of $N_a = N_d = 5\times10^{15}\ \mathrm{cm}^{-3}$ and minority carrier lifetimes of $\tau_{n0} = \tau_{p0} = \tau_0 = 10^{-7}$ s. A reverse-biased voltage of $V_R = 10$ V is applied. A light source is incident only on the space charge region, producing an excess carrier generation rate of $g' = 4\times10^{19}\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1}$. Calculate the generation current density.

Answer:
We have

$$J_{\mathrm{gen}} = \int_0^W eG\,dx$$

In this case, $G = g' = 4\times10^{19}\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1}$ and is a constant through the space charge region. Then

$$J_{gen} = eg'W$$

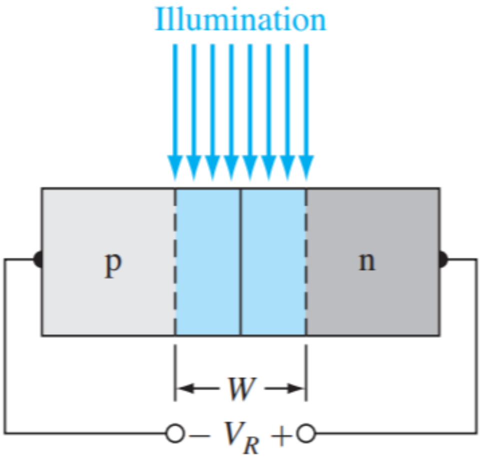


Figure 1: Figure for Problem 6.5

We find

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$
$$= (0.0259)\ln\left[\frac{(5\times10^{15})(5\times10^{15})}{(1.5\times10^{10})^2}\right]$$

or

$$V_{bi} = 0.659 \text{ V}$$

Also

$$W = \left[\frac{2\in_s (V_{bi}+V_R)}{e}\left(\frac{N_a+N_d}{N_aN_d}\right)\right]^{1/2}$$
$$= \left[\frac{2(11.7)(8.85\times10^{-14})(0.659+10)}{1.6\times10^{-19}}\right.$$
$$\left.\times\left(\frac{5\times10^{15}+5\times10^{15}}{(5\times10^{15})(5\times10^{15})}\right)\right]^{1/2}$$

or

$$W = 2.35\times10^{-4} \text{ cm}$$

Then

$$J_{gen} = \left(1.6\times10^{-19}\right)\left(4\times10^{19}\right)\left(2.35\times10^{-4}\right)$$

or

$$J_{gen} = 1.5\times10^{-3} \text{ A/cm}^2$$

Exercise 6.6

A silicon pn junction at $T = 300$ K is reverse biased at $V_{\mathrm{R}} = 8$ V. The doping concentrations are $N_{\mathrm{a}} = 5 \times 10^{16}$ cm^{-3} and $N_{\mathrm{d}} = 5 \times 10^{15}$ cm^{-3}. Draw the band diagram of the pn junction, and determine $x_{\mathrm{n}}, x_{\mathrm{p}}, W$, and $|E_{\max}|$. Please provide the process of derivation.

Answer:

$$V_{bi} = \frac{kT}{e} \ln\left(\frac{N_d N_a}{{n_i}^2}\right) = 0.718 \text{ V}$$

$$W = \left\{\frac{2 \in_s (V_b + V_R)}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{\frac{1}{2}} = 1.576 \times 10^{-4} \text{ cm} = 1.576\mu\text{m}$$

$$|E_{\max}| = \left|\frac{2\,(V_{bi} + V_R)}{w}\right| = 1.11 \times 10^5 \text{V/cm}$$

$$x_n = \left[\frac{2 \in_s (V_{b_i + V_R})}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right]^{\frac{1}{2}} = 1.43 \times 10^{-4} \text{ cm}$$

$$x_p = \left[\frac{2 \in_s (V_{bi} + V_R)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a + N_d}\right]\right]^{\frac{1}{2}} = \begin{array}{r} 1.43 \times 10^{-5} \text{ cm} \\ = 0.143\mu\text{m} \end{array}$$

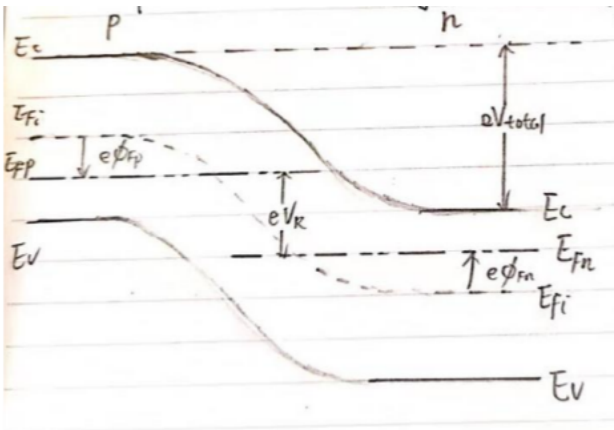


Figure 2: Figure for Problem 6.6

Exercise 6.7

(a) Sketch the energy bands in a zero-biased, reverse-biased, and forward-biased pn junction.

(b) Sketch the steady-state minority carrier concentrations in a forward-biased pn junction.

(c) Sketch the forward-bias I–V characteristics of a pn junction diode showing the effects of recombination and high-level injection.

Answer:

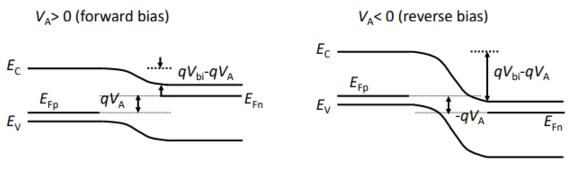
(a) 

Figure 3: Figure for Problem 6.7a

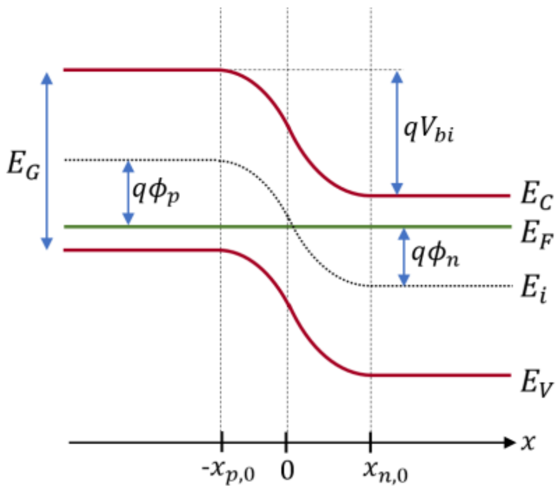


Figure 4: Figure for Problem 6.7a

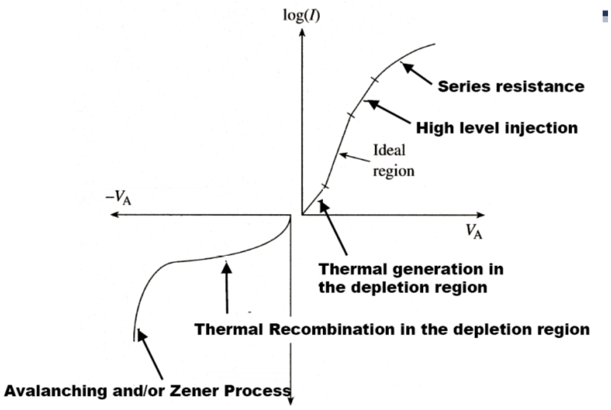
(b) 

Figure 5: Figure for Problem 6.7b

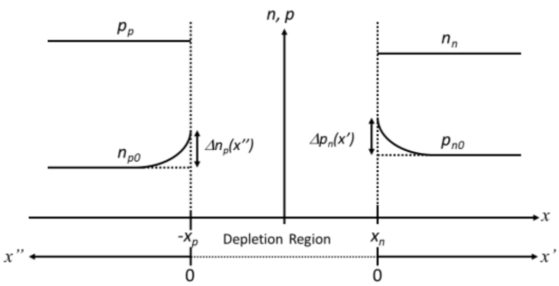
(c) 

Figure 6: Figure for Problem 6.7c

Reference

1. Neamen, Donald A. Semiconductor physics and devices: basic principles. McGraw-hill, 2003.