VE320 – Summer 2022

Introduction to Semiconductor Devices

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Chapter 6 Non-Equilibrium Excess Carriers in Semiconductors

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

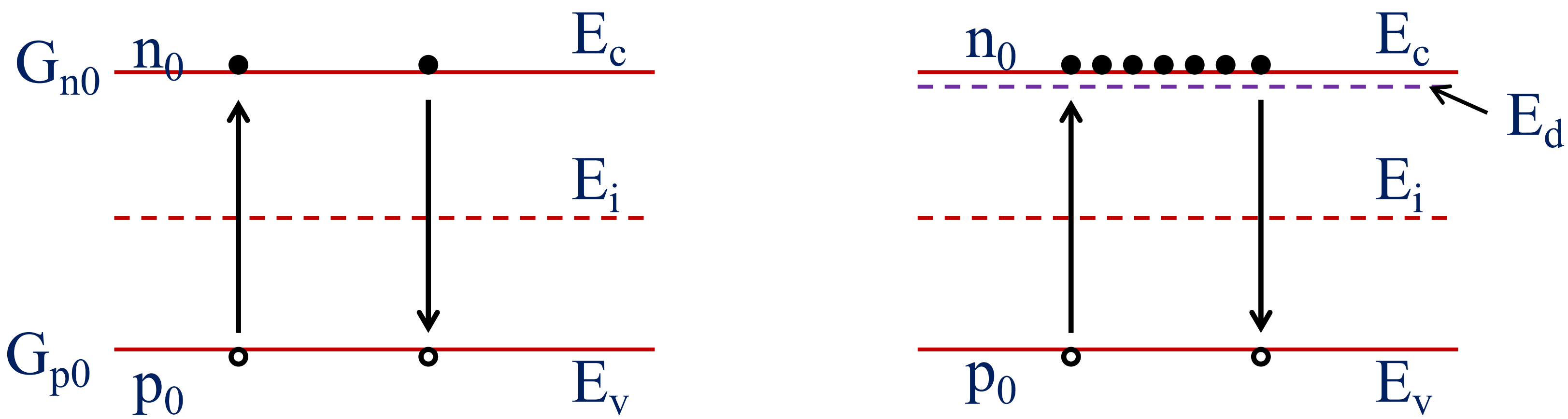
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0=p_0=n_i$

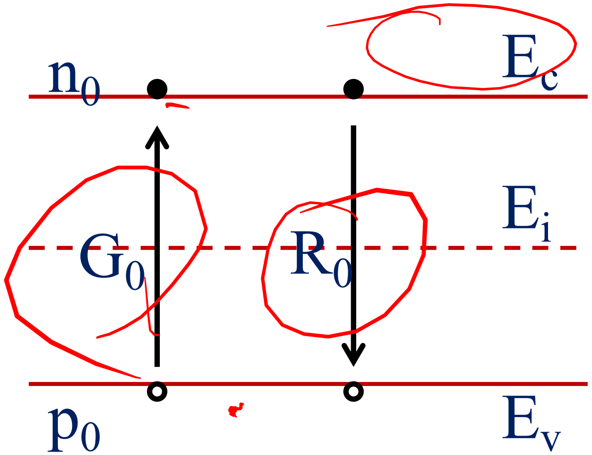
n type : $n_0>>n_i>>p_0$

G_{n0}: the thermal generation rate of electrons
G_{p0}: the thermal generation rate of holes
R_{n0}: the recombination rate of electrons
R_{p0}: the recombination rate of holes

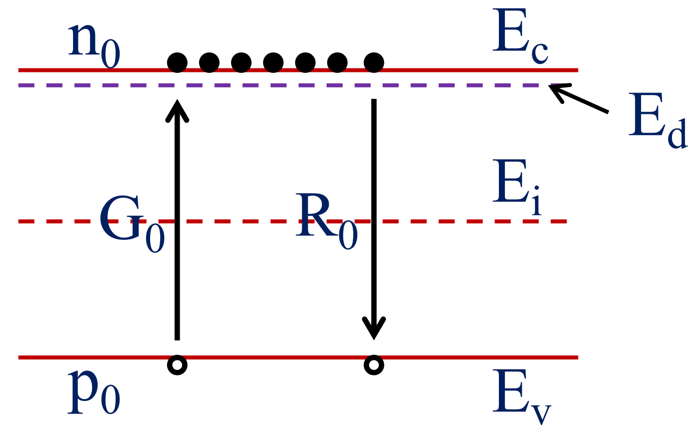
$\boxed{G_{n0} = G_{p0} = R_{n0} = R_{p0}}$ (direct G and R from band to band)

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0=p_0=n_i$



n type : $n_0 >> n_i >> p_0$

$n_0+\Delta n$

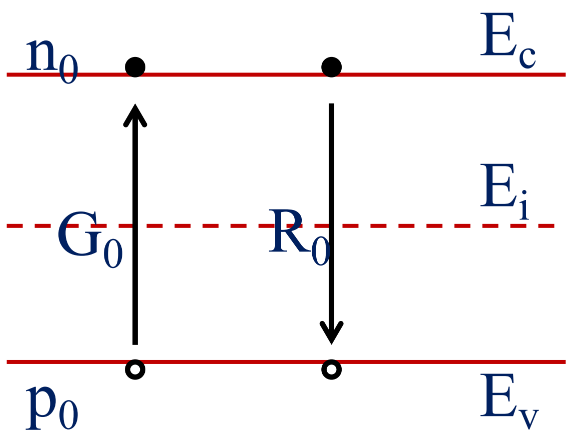
$R_0 \sim n \cdot p = n_0 \cdot p_0$

$G_0 \sim n_0 p_0$

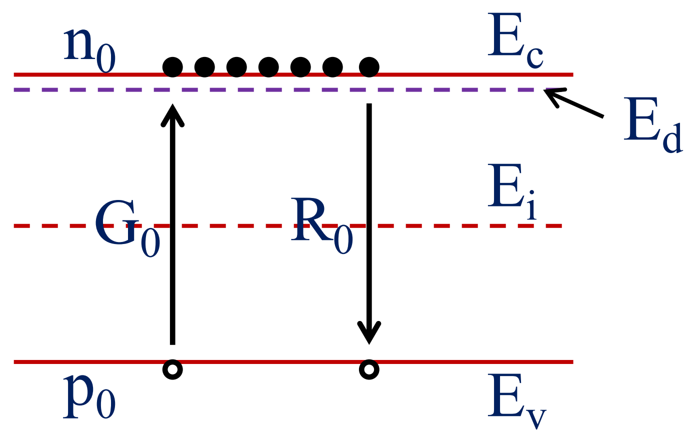
$G_0 = R_0 = n_0 \cdot p_0$

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0 = p_0 = n_i$

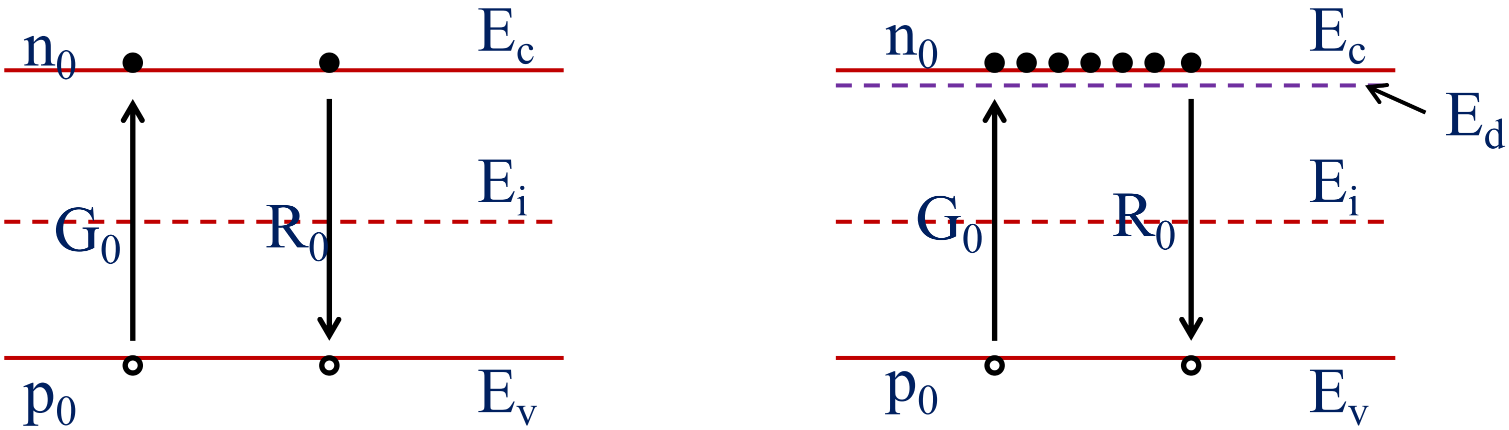


n type : $n_0 >> n_i >> p_0$

$$\boxed{G_0 = R_0 = \alpha_r \cdot n_0 \cdot p_0}$$

6.1 Carrier generation and recombination

The semiconductor in equilibrium

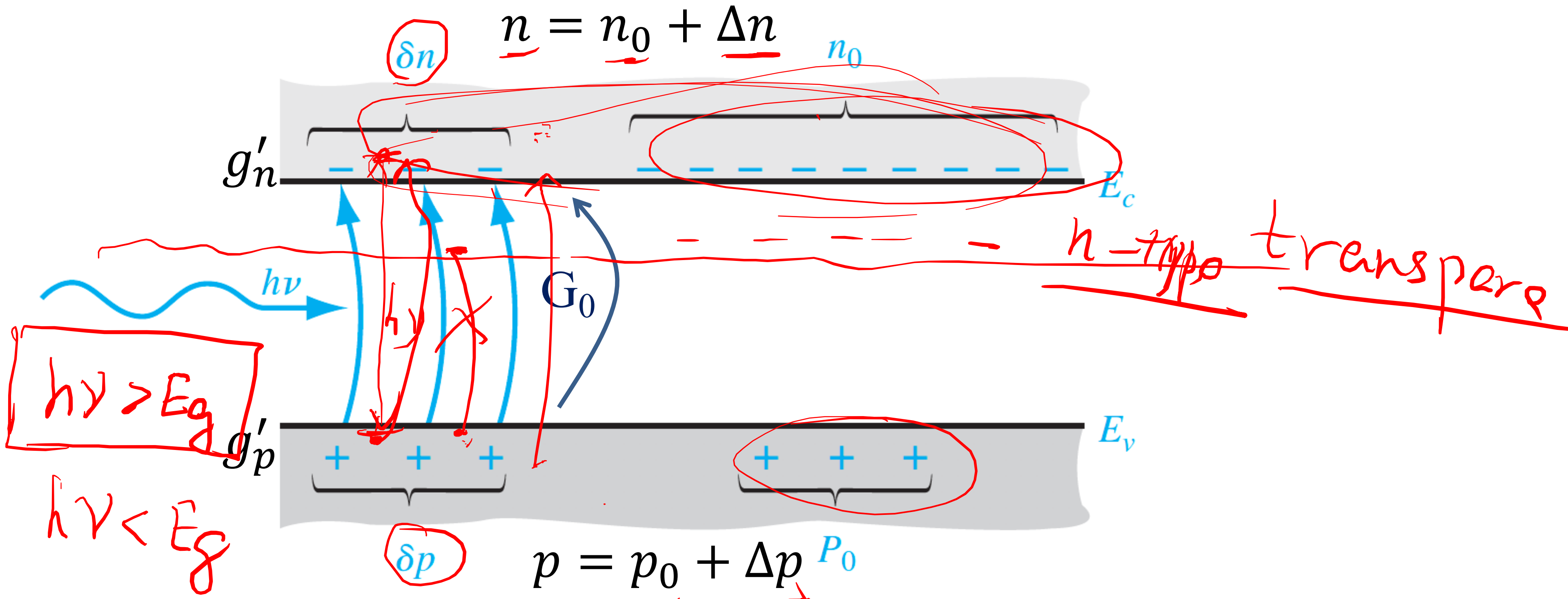


Intrinsic: $n_0 = p_0 = n_i$

n type : $n_0 >> n_i >> p_0$

6.1 Carrier generation and recombination

Excess carrier generation and recombination



$R \sim n \cdot p$

6.1 Carrier generation and recombination

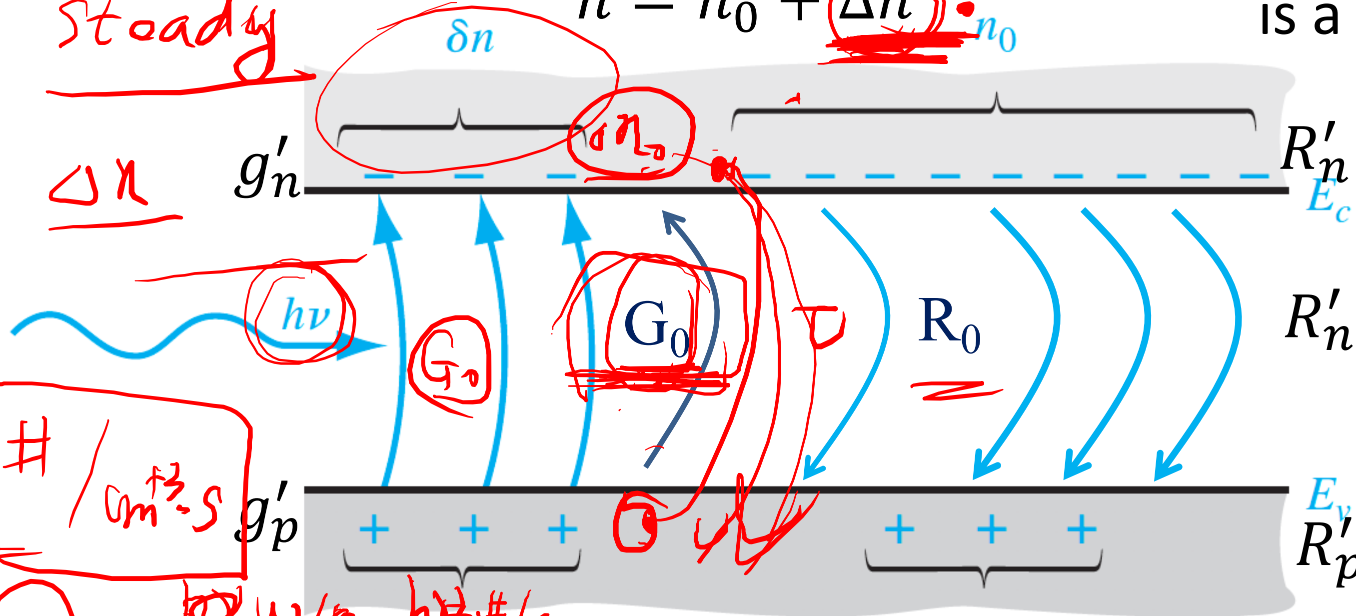
Excess carrier generation and recombination

Recombination process is a function of n and p

steady

$n = n_0 + \Delta n$

Δn



$\#/cm^3 \cdot s$

$R'_n = R'_p = R'$

$\tau = \frac{1}{\alpha_r(n_0+p_0)}$

$I = \frac{W/s}{cm^2} = \frac{h\nu \cdot \#/s}{cm^2}$

$p = p_0 + \Delta p$

$R_0 + R'$ $\quad n_0 + \Delta n$

g’ is not a function of n and p

$\alpha_r \cdot n_0 \cdot p_0 \quad G_0 + G(I_{light}) = R = \alpha_r \cdot n \cdot p \quad = p_0 + \Delta p$

6.1 Carrier generation and recombination

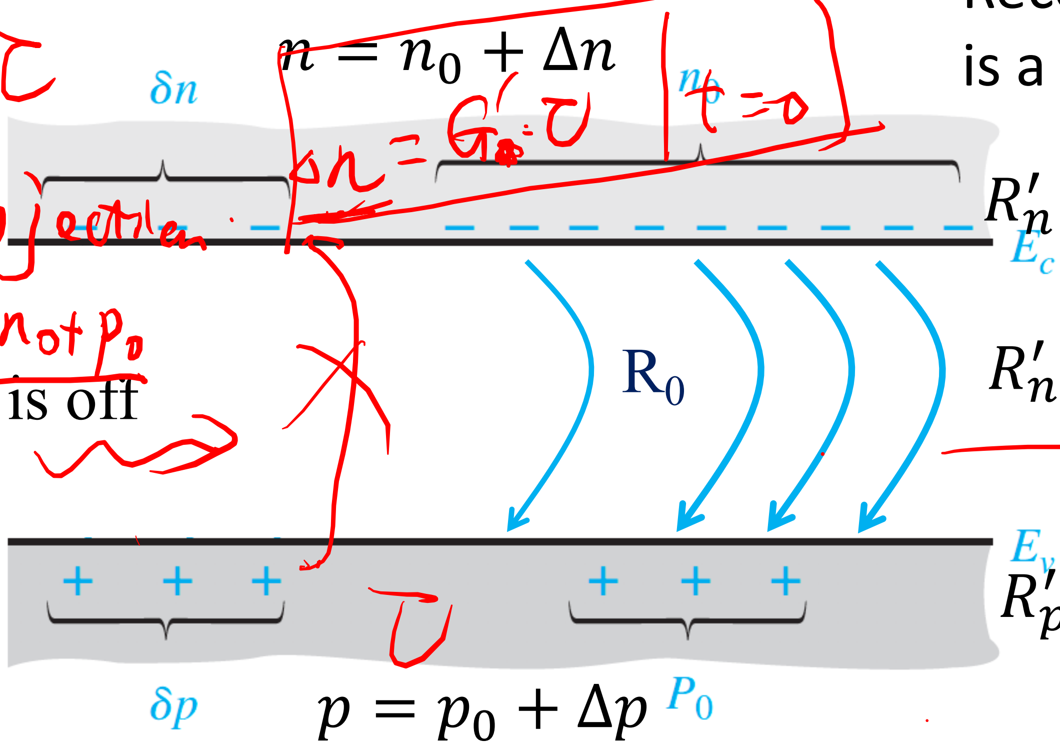
Excess carrier generation and recombination

Recombination process is a function of n and p

lifetime τ

small injection

$\Delta n \ll n_0 + p_0$



$R'_n = R'_p = R'$

$= \alpha_r (n_0 + \Delta n)(p_0 + \Delta p)$

$R = \alpha_r \cdot n_0 \cdot p_0 \quad R_0 = \alpha_r n_0 \cdot p_0$

$R = R' + R_0$

$+ \alpha_r (n_0 + p_0) \Delta n$

$t_0 = 0s$

$G = G_0 + G' = R = \alpha_r \cdot (n_0 + \Delta n) \cdot (p_0 + \Delta p)$

$\frac{d\Delta n(t)}{dt} < 0$

$\frac{d\Delta n(t)}{dt} = -(R' + \cancel{R_0 - G_0}) = \alpha_r (n_0 + p_0) \Delta n$

$= \alpha_r (n_0 \cdot p_0 + n_0 \cdot \Delta p + \Delta n \cdot p_0 + \Delta n \cdot \Delta p)$

$= \alpha_r \, n_0 p_0 + (n_0 + p_0) \cdot \Delta n + \Delta n^2$

6.1 Carrier generation and recombination

Excess carrier generation and recombination

Net recombination rate

$$\cancel{G_0} + G' = R = \cancel{\alpha_r n_0 p_0} + \alpha_r \cdot (n_0+p_0)\Delta n + \alpha_r (\Delta n)^2$$

$$G' = \alpha_r (n_0+p_0)\cdot \Delta n + \alpha_r \cdot (\Delta n)^2$$

$$\Delta n = \frac{G'}{\alpha_r (n_0+p_0+\Delta n)} = G' \cdot \boxed{\frac{1}{\alpha_r (n_0+p_0+\Delta n)}}$$

Small injection condition: if $\Delta n \ll n_0 + p_0$

τ lifetime

6.1 Carrier generation and recombination

10^5

$n_0 = 10^7\,cm^{-3}$

$p_0 = 10^3\,cm^{-3}$

Excess carrier generation and recombination

$R' = R - R_0$ $\Delta n = 10^2\,cm^{-3}$

$= \alpha_r \cdot (n_0 + p_0) \cdot \Delta n$

$$\frac{d\Delta n(t)}{dt} = -\alpha_r (n_0 + p_0) \Delta n(t) = \frac{\Delta n}{\tau_n}$$

For n-type semiconductor, net recombination rate

$\Delta n \ll n_0$

$$\boxed{R_n' = R_p' = \frac{\Delta p(t)}{\tau_{p0}}}$$

$\Delta p \gg p_0$

$$\frac{d\Delta n(t)}{\Delta n(t)} = -\alpha_r (n_0 + p_0) dt$$

$$d(\ln \Delta n(t)) = d(-\alpha_r (n_0 + p_0) t + C)$$

For p-type semiconductor, net recombination rate

$$\boxed{R_n' = R_p' = \frac{\Delta n(t)}{\tau_{n0}}}$$

$\Delta n = G' \cdot \tau$

$$\ln \Delta n(t) = -\alpha_r (n_0 + p_0) t + C$$

$$\Delta n(t) = C' e^{-\alpha_r (n_0 + p_0) t}$$

$$\Delta n(t) = C' e^{-\alpha_r (n_0 + p_0) \cdot t}$$

$$= C' e^{-t/\tau} = G' \cdot \tau$$

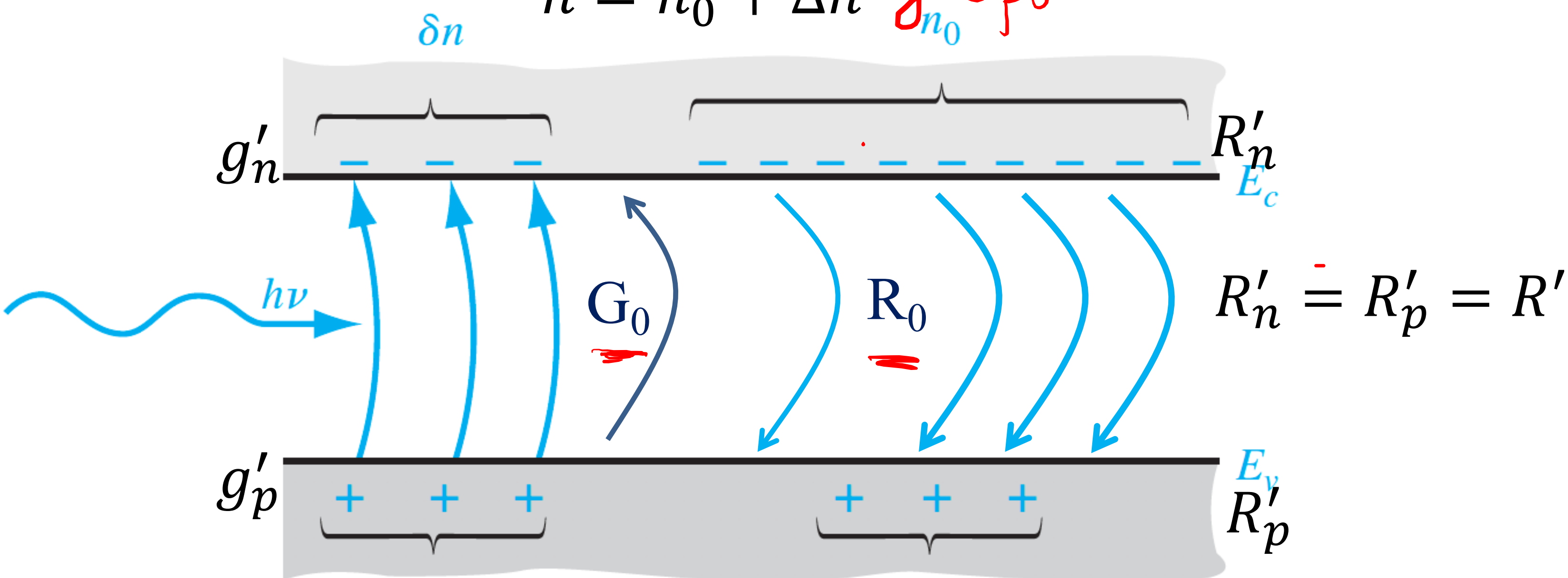
$$\boxed{\Delta n(t) = G' \cdot \tau e^{-t/\tau}}$$

$$\boxed{\tau = \frac{1}{\alpha_r (n_0 + p_0)}}$$ lifetime

6.1 Carrier generation and recombination

Excess carrier generation and recombination

$$n = n_0 + \Delta n = g'\tau_{p0}$$



$$p = p_0 + \Delta p = g' \cdot \tau_{p0}$$

$t=0$

$$\boxed{g' = R' \;\Rightarrow\; \Delta p(t \le 0) = g'\tau_{p0}\ for\ n-type\ semiconductors}$$

$$\boxed{g' = R' \;\Rightarrow\; \Delta n(t \le 0) = g'\tau_{n0}\ for\ p-type\ semiconductors}$$

Chenk your understanding

Problem Example #1

Assume that excess carriers have been generated uniformly in a semiconductor to a concentration of $\Delta n(0) = 10^{15}\ \text{cm}^{-3}$. The generation of the excess carriers turns off at time t=0. Assuming the excess carrier lifetime is $\tau_{n0} = 10^{-6}$ s, calculate the recombination rate of excess carriers for t =4μs.

① $\Delta n = g' \cdot \tau_0 e^{-t/\tau_0}$

② $\Delta n = g' \cdot \tau_0$ minority recomb lifetime

$\approx 1\,\mu s$

$R' = \alpha_r (n_0 + p_0)\Delta n = \frac{\Delta n}{\tau_{n0}}$

$R = \alpha_r \cdot n \cdot p$

$\frac{d\Delta n(t)}{dt} = -R' = -\frac{\Delta n}{\tau_{n0}}$

$R_0 = \alpha_r \cdot n_0 \cdot p_0 = G_0$

$\Delta n(t) = \Delta n(t=0)\, e^{-t/\tau_{n0}}$

$g' \to \Delta n = g' / \alpha_r (n_0 + p_0)$

$= 10^{15} e^{-4/1}$

$\boxed{\Delta n = C' e^{-\alpha_r (n_0 + p_0) t}}$

$\frac{1}{\alpha_r (n_0 + p_0)} = \tau$

$= 10^{15} e^{-4}$

$= \boxed{1.83 \times 10^{13}\ \text{cm}^{-3}}$

$C' e^{-t/\tau}$

$\frac{d\Delta n}{dt} = -(R' + \cancel{R_0 - G_0})$

$\boxed{\tau = \frac{1}{\alpha_r (n_0 + p_0)}}$

$= -\alpha_r (n_0 + p_0 + \Delta n)$

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

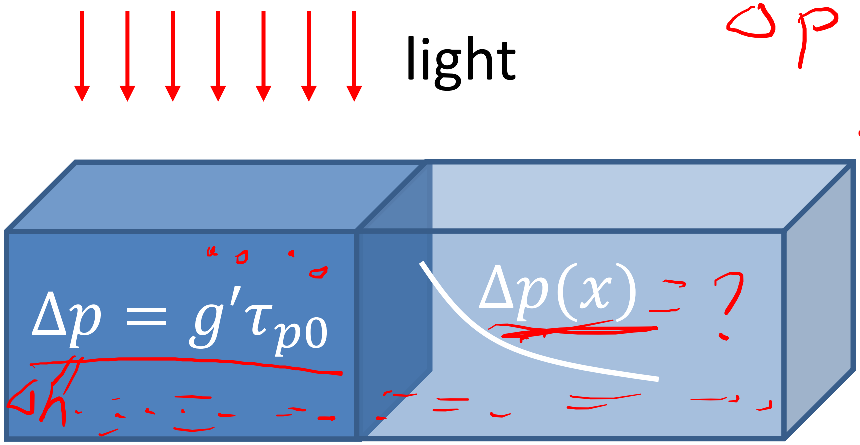
6.2 Characteristics of excess carriers

Continuity equation at steady state

$\Delta p = g' \cdot \tau_{p0}$

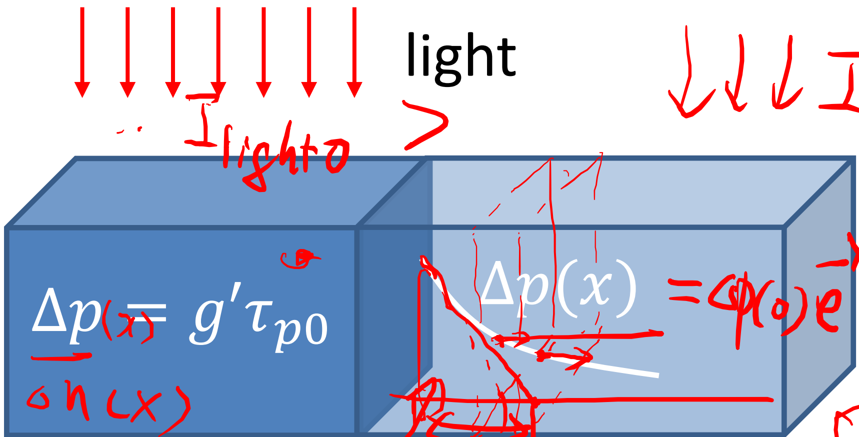
$\Delta p(t) = \Delta p(0) e^{-t/\tau}$

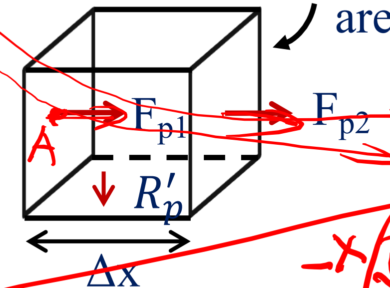
n-type



6.2 Characteristics of excess carriers

Continuity equation at steady state





n_0, p_0

$x \to +\infty$, $\Delta p(x) = 0$

$\Delta p(x) = C \cdot e^{-x/\sqrt{D_p \tau_p}} + B e^{x/\sqrt{D_p \tau_p}}$

$= C e^{-x/L_p}$

$L_p = \sqrt{D_p \tau_p}$ diffusion length

For diffusion current:

carrier (Δn, Δp) into A at a unit time $F_{p1} \cdot A$

of carriers passing (into) the area A at a unit time: $F_{p1} \cdot A$

carrier (Δn, Δp) out A at a unit time

of carriers passing (out) the area A at a unit time: $F_{p2} \cdot A$

carrier recombined in that at a unit of time: $R_p \cdot A \cdot \Delta x$

of carriers recombined in that volume at a unit time: $R'_p \cdot A \cdot \Delta x$

6.2 Characteristics of excess carriers

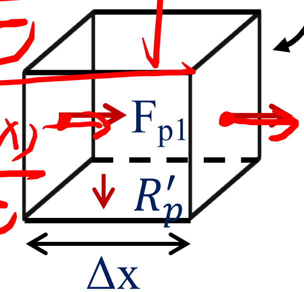
Continuity equation at steady state

diffusion

$$\frac{d^2\Delta p(x)}{dx} = \frac{\Delta p(x)}{\tau}$$

$$\frac{d^2\Delta p(x)}{dx} = \frac{\Delta p(x)}{D_p\tau}$$

$$\Delta p(x) = Ae^{-x/\sqrt{D_p\tau}} + Be^{x/\sqrt{D_p\tau}}$$



For diffusion current:

$$\boxed{F_{p2}\cdot A - F_{p1}\cdot A = -R_p'\cdot A\,\Delta x}$$

$$R_p' = \frac{\Delta n}{\tau} = \alpha_r(n_0+p_0)\cdot\Delta n$$

$$(F_{p2}-F_{p1})\cancel{A} = -\frac{\Delta n}{\tau}\cdot\cancel{A}\cdot\Delta x$$

$$F_p = -D_p\cdot\frac{d\Delta p}{dx}$$

$$\lim_{\Delta x\to 0}\frac{F_{p2}-F_{p1}}{\Delta x} = -\frac{\Delta n}{\tau}$$

$$\frac{dF_p}{dx} = -\frac{\Delta n}{\tau} + g_p'$$

$$p = p_0 + \Delta p(x)$$

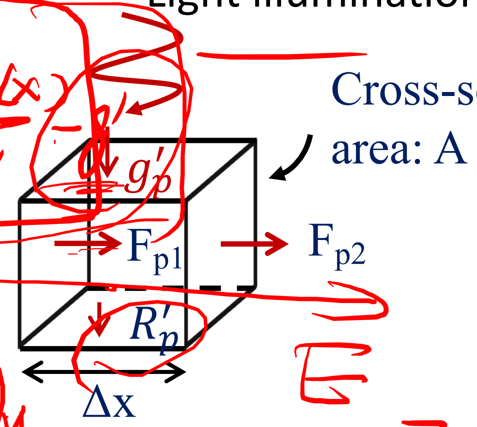
$$= -D_p\cdot\frac{d\Delta p}{dx}$$

6.2 Characteristics of excess carriers

Steady-state continuity equation

diffusion equatin

$$\boxed{\frac{d^2\Delta p(x)}{dx^2} = \frac{\Delta p(x)}{\tau} - g'}$$



$$J_{drf} = q p \cdot \mu_n \cdot E$$

$$F = J/q$$

$$F_{p2}A - F_{p1}A = -R_p' \cdot A \cdot \Delta x + g_p' \cdot A \cdot \Delta x$$

$$F = -D_p \frac{d\Delta p}{dx}$$

$$F = -D_p \frac{dp}{dx}$$

$$\boxed{F = -D_p \frac{dp}{dx} + p\mu_n \cdot E}$$

$$p = p_0 + \Delta p$$

$$\lim_{\Delta x \to 0} \frac{F_{p2} - F_{p1}}{\Delta x} = -R_p' + g_p' = -\frac{\Delta p}{\tau} + g'$$

$$p = p_0 + \Delta p$$

$$\boxed{\frac{dF}{dx} = -\frac{\Delta p}{\tau} + g'}$$

$$\boxed{-D_p \frac{d^2 p}{dx^2} = -\frac{\Delta p}{\tau} + g'}$$

$$-D_p \frac{d^2 p}{dx^2} + \frac{d(p\mu_p E)}{dx} = -D_p \frac{d^2\Delta p}{dx^2} = -D_p \frac{d^2 p(x)}{dx^2}$$

6.2 Characteristics of excess carriers

Steady-state continuity equation

$$-\frac{dF_{drf|p}}{dx}$$

Steady state:

$$\boxed{D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g'_p = 0}$$

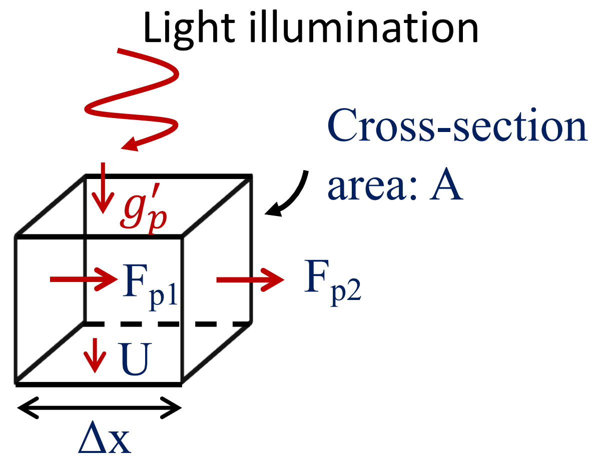
When the n-type semiconductor is uniformly doped,

$$p(x) = p_0 + \Delta p(x)$$

$$\boxed{D_p \frac{d^2 \Delta p}{dx^2} - \mu_p E \frac{d\Delta p}{dx} - \Delta p\mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g'_p = 0}$$

6.2 Characteristics of excess carriers

Time-dependent continuity equation



6.2 Characteristics of excess carriers

Time-dependent continuity equation

For an n-type semiconductor,

$\frac{\Delta p}{\tau_{p0}}$

holes as minority

$$\boxed{\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p\mu_p \frac{dE}{dx} - R_p' + g_p'}$$

$$R_p' = \frac{\Delta p}{\tau_{p0}}$$

(minority carriers)

$\frac{\Delta p}{\tau_{p0}}$

$$\boxed{\frac{d\Delta n}{dt} = D_n \frac{d^2 n}{dx^2} + \mu_n E \frac{dn}{dx} + n\mu_n \frac{dE}{dx} - R_n' + g_n'}$$

$$R_n' = R_p' = \frac{\Delta p}{\tau_{p0}}$$

nonlinear

(majority carriers)

$$g_n' = g_p'$$

Summary

Table 6.2 |

Specification	**Effect**
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate) + no boundary confinement	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E}\frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E}\frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$