VE320 – Summer 2021

Introduction to Semiconductor Devices

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Chapter 8 The pn Junction Diode

Outline

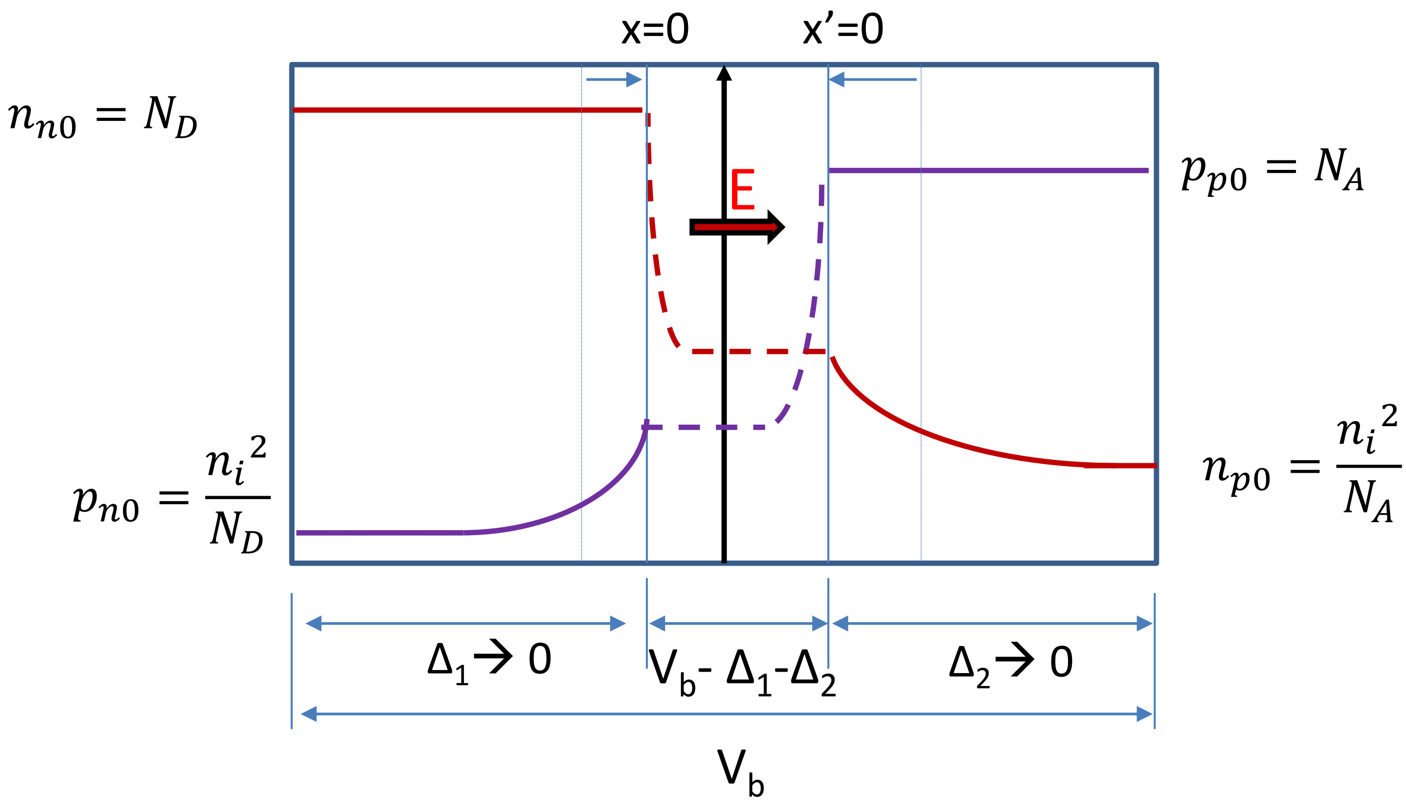
8.1 pn junction current

8.2 Generation-recombination currents

8.3 High-injection levels

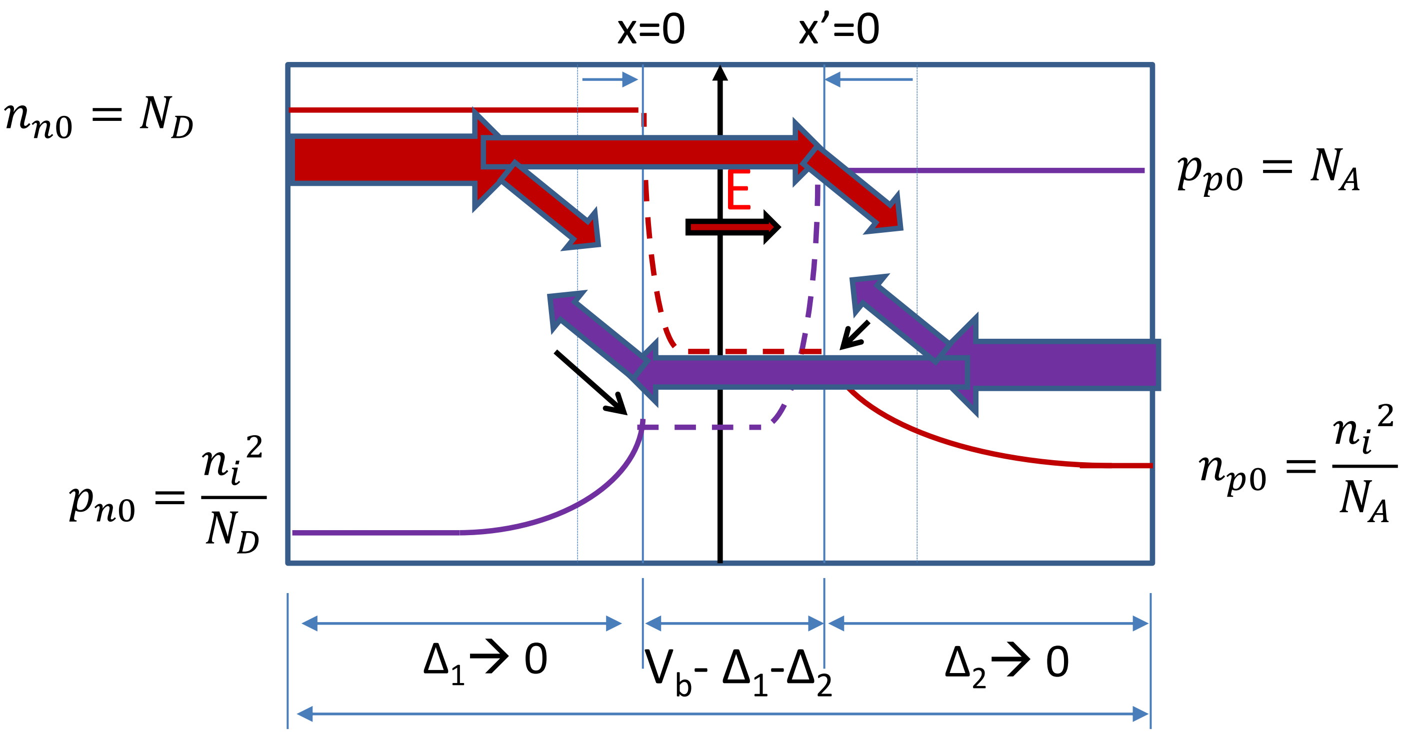
8.4 A few more points on pn junctions (not in the textbook)

8.0 The logic behind the way to derive current



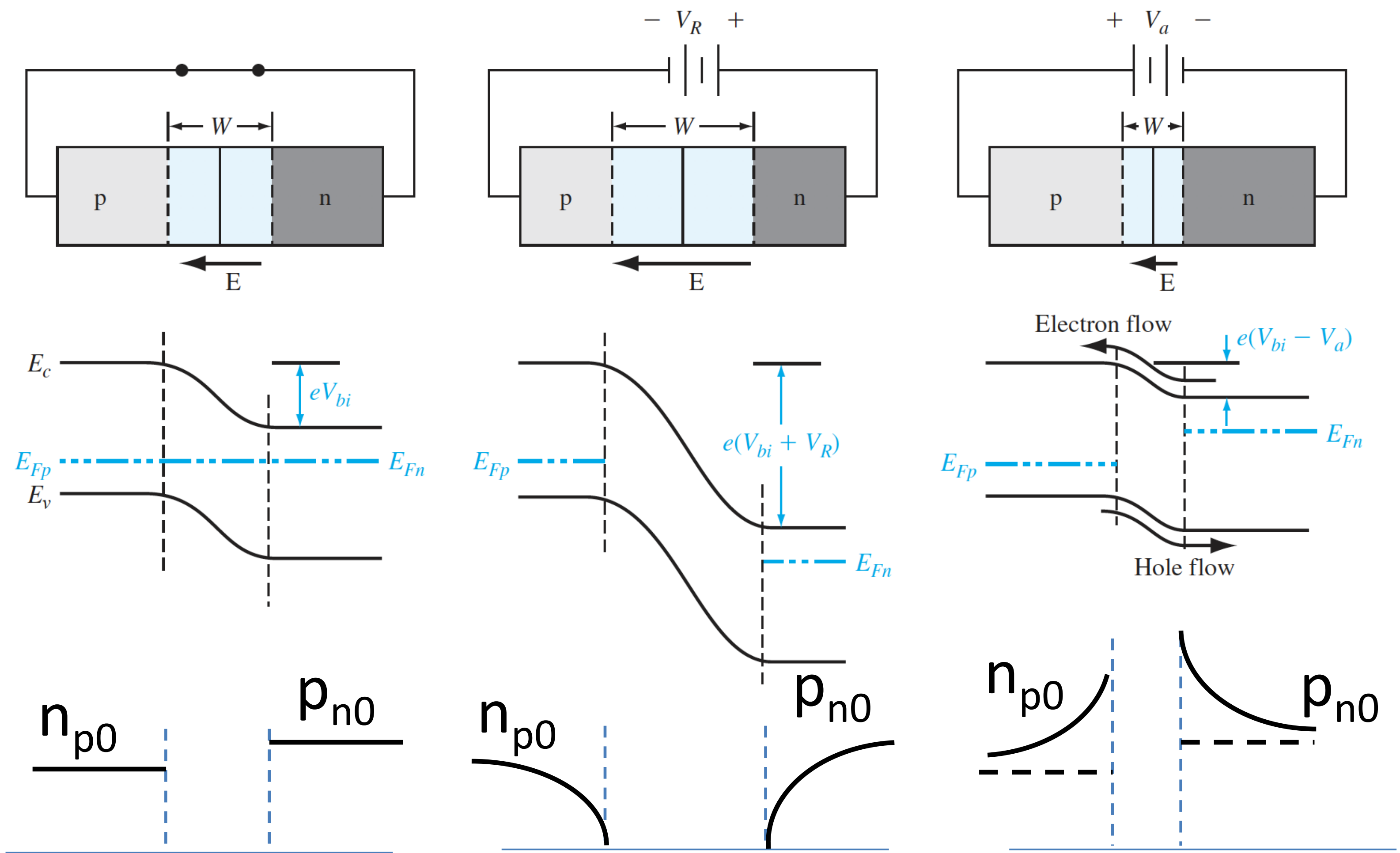
8.0 The logic behind the way to derive current

Total current I_t is uniform at every x → $I_t = I_n\,(x=0) + I_h\,(x=0) = I_n\,(x'=0) + I_h\,(x'=0)$



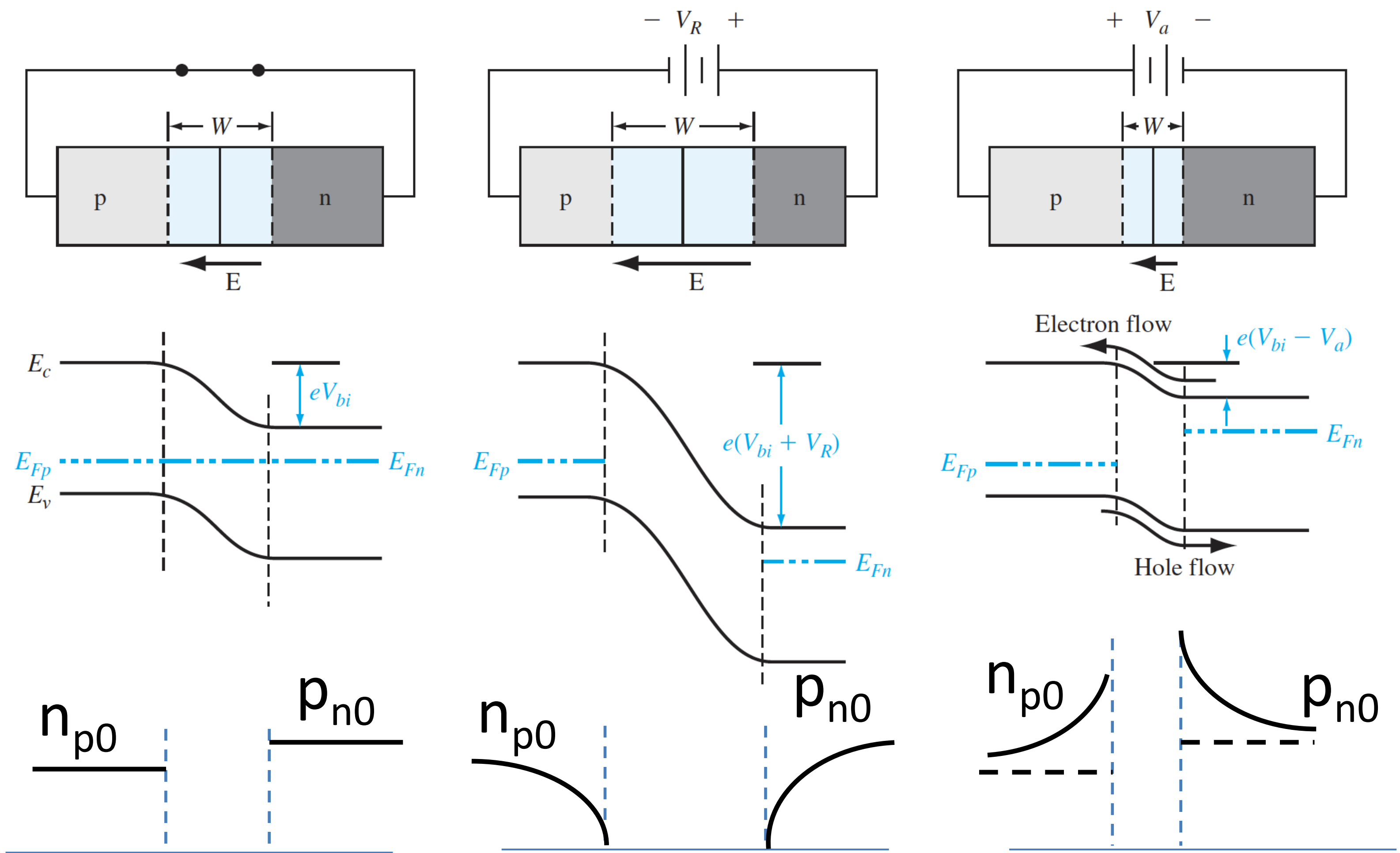
8.1 pn Junction Current

Qualitative Description of Charge Flow in a pn Junction



8.1 pn Junction Current

Qualitative Description of Charge Flow in a pn Junction



8.1 pn Junction Current

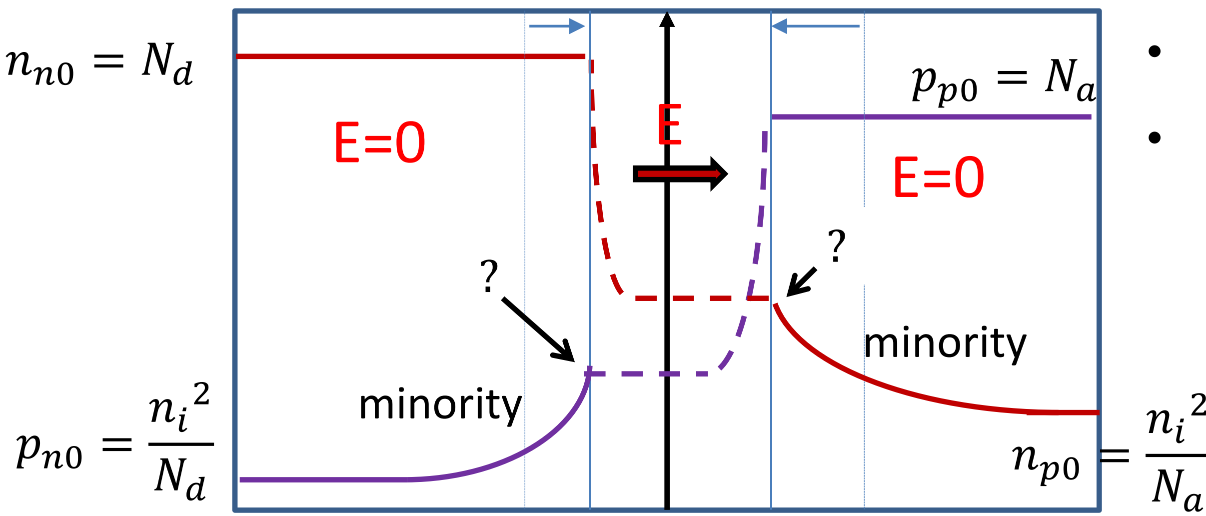
<u>Goal: to find the analytical expression of current</u>

$$\boxed{\frac{\partial n}{\partial t} = D_n \frac{\partial^2 n}{\partial x^2} + \mu_n E \frac{\partial n}{\partial x} + n\mu_n \frac{\partial E}{\partial x} - \frac{\Delta n}{\tau} + G_{ex}}$$

Electrons as minority

$$\boxed{\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p\mu_p \frac{\partial E}{\partial x} - \frac{\Delta p}{\tau} + G_{ex}}$$

holes as minority



- How to simplify?
- Boundary condition?
- how to get total current from minority currents?

8.1 pn Junction Current

Assumptions of an ideal PN junction

1. The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.

2. The Maxwell–Boltzmann approximation applies to carrier statistics.

3. The concepts of low injection and complete ionization apply.

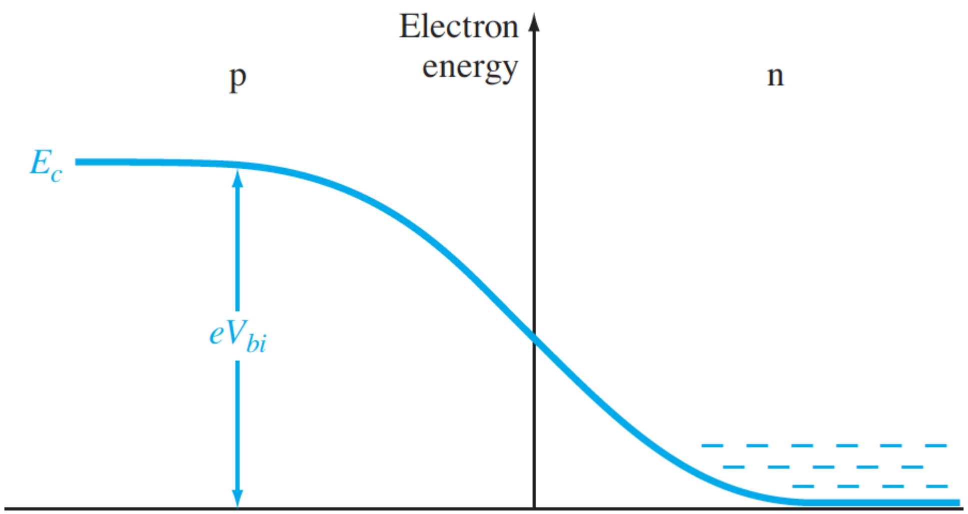
4a. The total current is a constant throughout the entire pn structure.

4b. The individual electron and hole currents are continuous functions through the pn structure.

4c. The individual electron and hole currents are constant throughout the depletion region.

8.1 pn Junction Current

Boundary condition



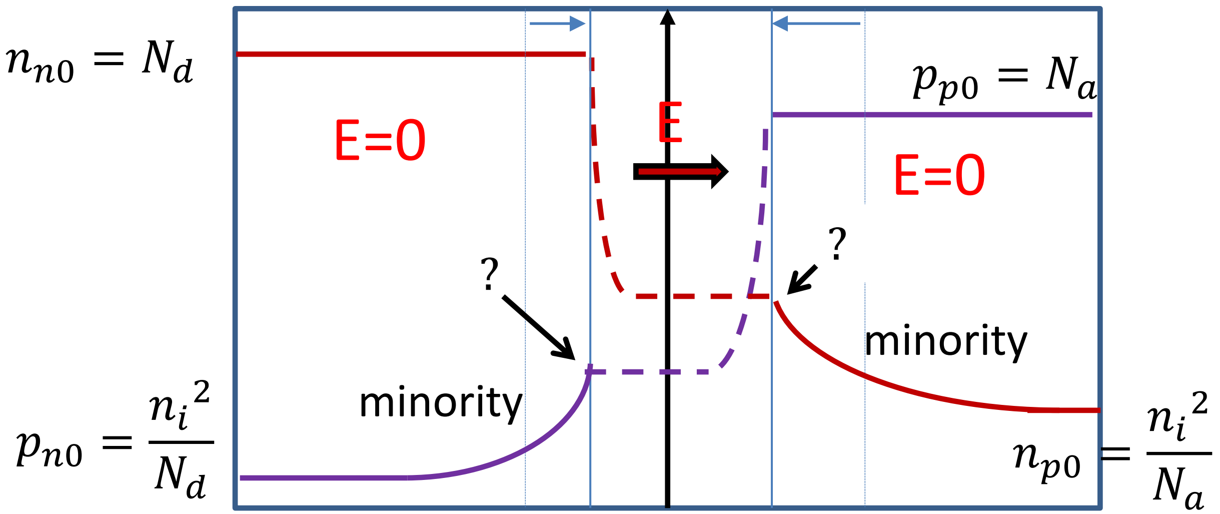
$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

8.1 pn Junction Current

Boundary condition

$$p_{n0} = p_{p0} exp\left(\frac{-eV_{bi}}{kT}\right) \qquad n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

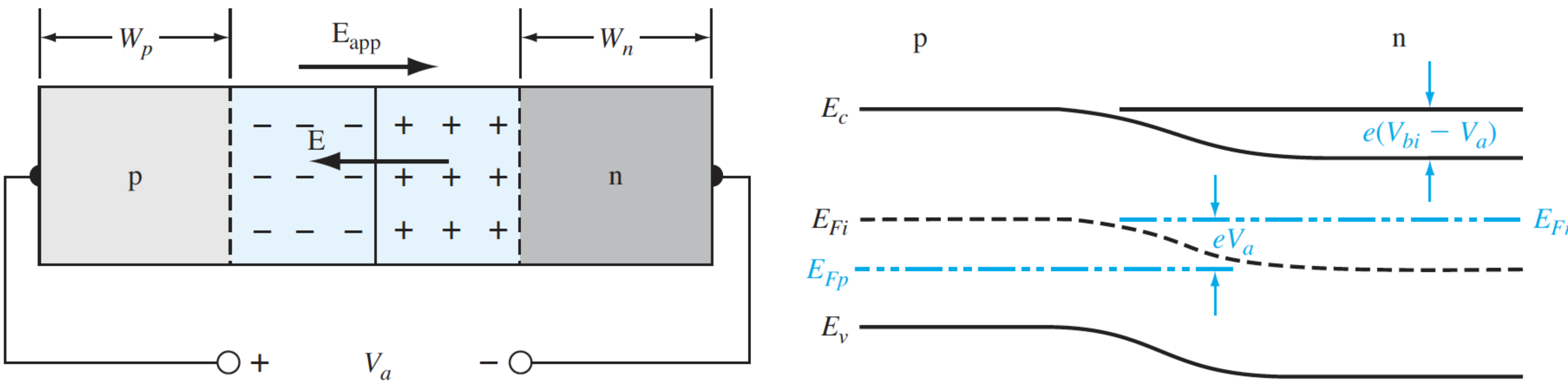


8.1 pn Junction Current

Boundary condition

$$p_{n0} = p_{p0} exp\left(\frac{-eV_{bi}}{kT}\right) \qquad n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

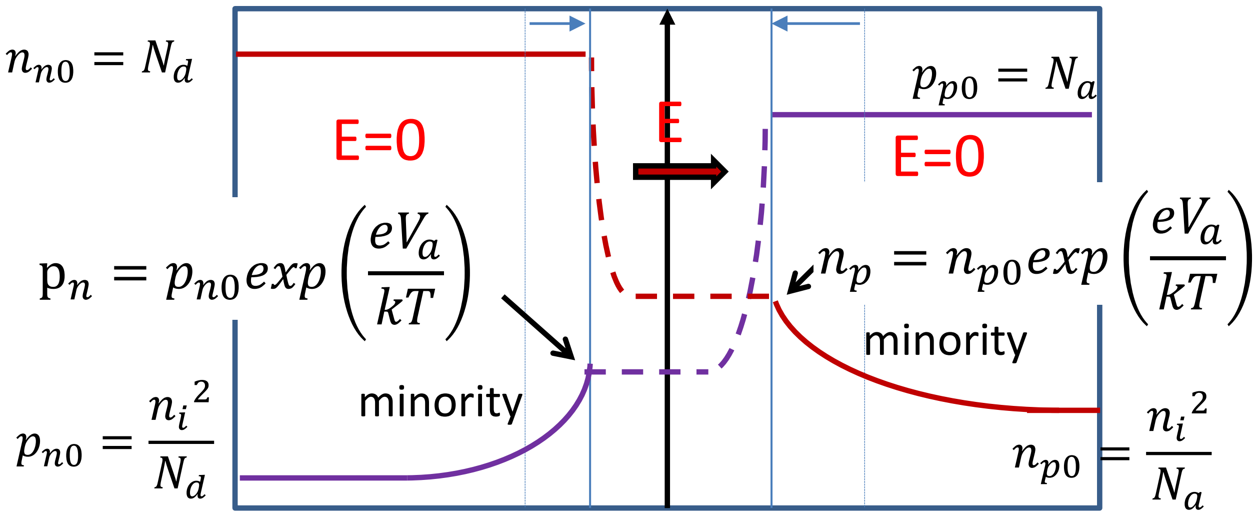
$$p_n = p_{p0} exp\left(\frac{-e(V_{bi}-V_a)}{kT}\right) = p_{n0} exp\left(\frac{eV_a}{kT}\right) \qquad n_p = n_{p0} exp\left(\frac{eV_a}{kT}\right)$$



(a) (b)

8.1 pn Junction Current

Boundary condition



Check your understanding

Problem Example #1

Consider a silicon pn junction at T = 300K. Assume the doping concentration in the n region is $N_d = 10^{16}$ cm^{-3} and the doping concentration in the p region is $N_a = 6 \times 10^{15}$ cm^{-3}. Assume a forward bias of 0.6V is applied to the pn junction. Calculate the minority concentration at the edge of the depletion region.

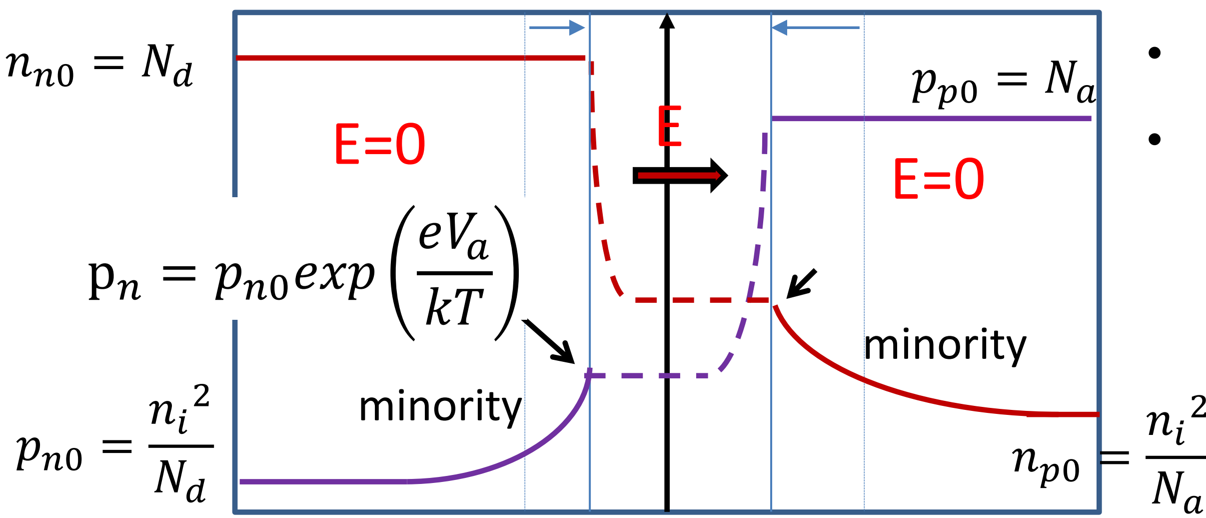
8.1 pn Junction Current

Minority carrier distribution

$$\cancelto{0}{\frac{\partial p}{\partial t}} = D_p \frac{\partial^2 \cancelto{\Delta p}{p}}{\partial x^2} - \mu_p \cancelto{0}{E} \frac{\partial p}{\partial x} - p\mu_p \cancelto{0}{\frac{\partial E}{\partial x}} - \frac{\Delta p}{\tau} + \cancelto{0}{G_{ex}}$$

holes as minority

$$\boxed{D_p \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\Delta p}{\tau} = 0}$$



- How to simplify?
- Boundary condition?
- how to get total current from minority currents?

8.1 pn Junction Current

Minority carrier distribution

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \quad (x \le -x_p)$$

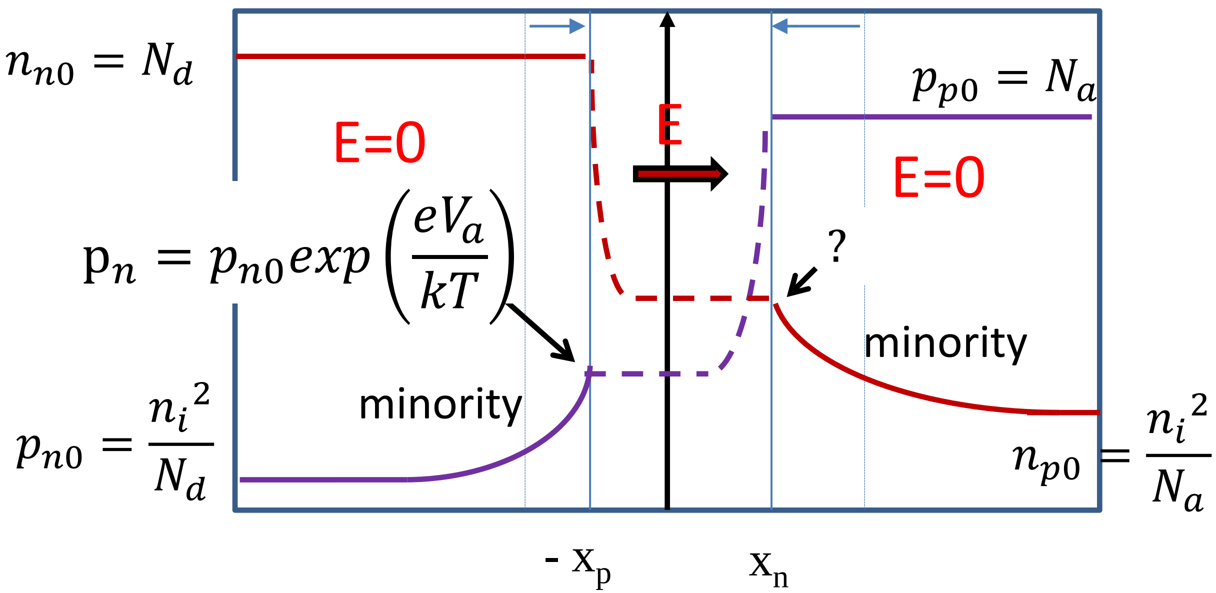
$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \quad (x \ge x_n)$$

$$p_n(x_n) = p_{n0}\exp\left(\frac{eV_a}{kT}\right)$$

$$n_p(-x_p) = n_{p0}\exp\left(\frac{eV_a}{kT}\right)$$

$$p_n(x \to +\infty) = p_{n0}$$

$$n_p(x \to -\infty) = n_{p0}$$

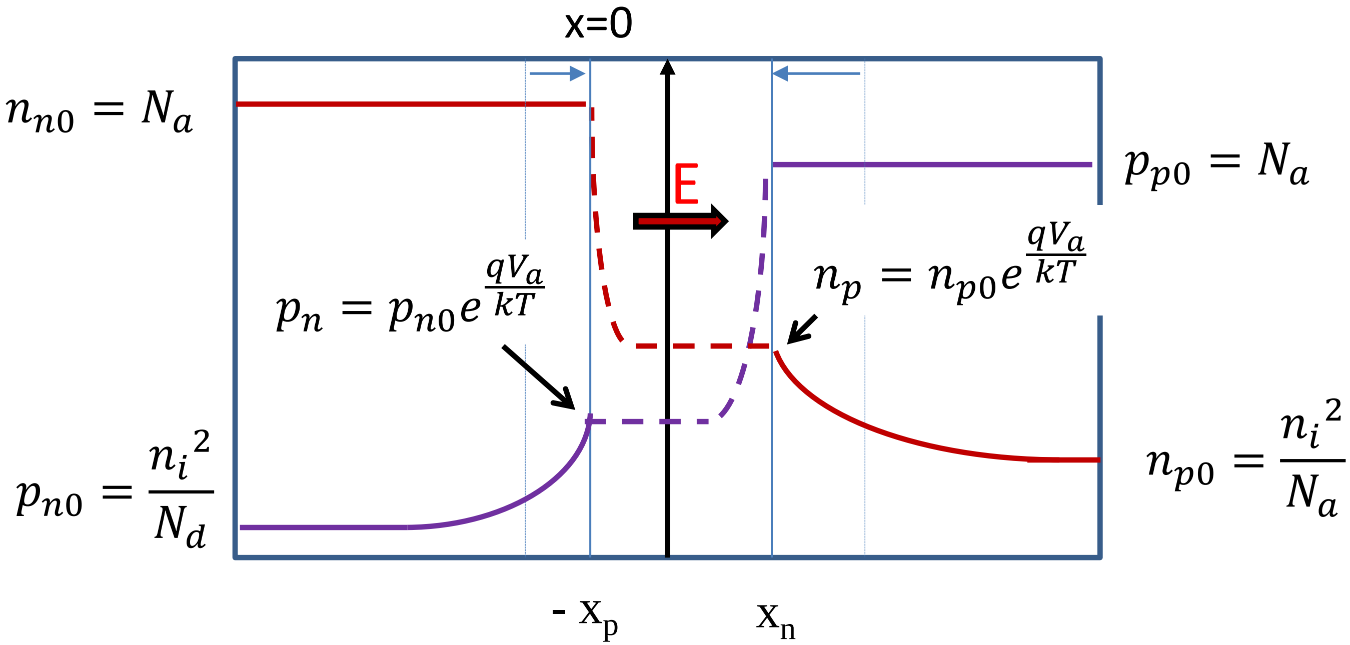


8.1 pn Junction Current

Minority carrier distribution

$$\Rightarrow \Delta p = p_n(x) - p_{n0} = p_{n0}(e^{\frac{qV_a}{kT}} - 1)e^{(x+\mathrm{x_p})/L_p}$$

$$\Rightarrow \Delta n = n_p(x) - n_{p0} = n_{p0}(e^{\frac{qV_a}{kT}} - 1)e^{(x_n-\mathrm{x})/L_n}$$

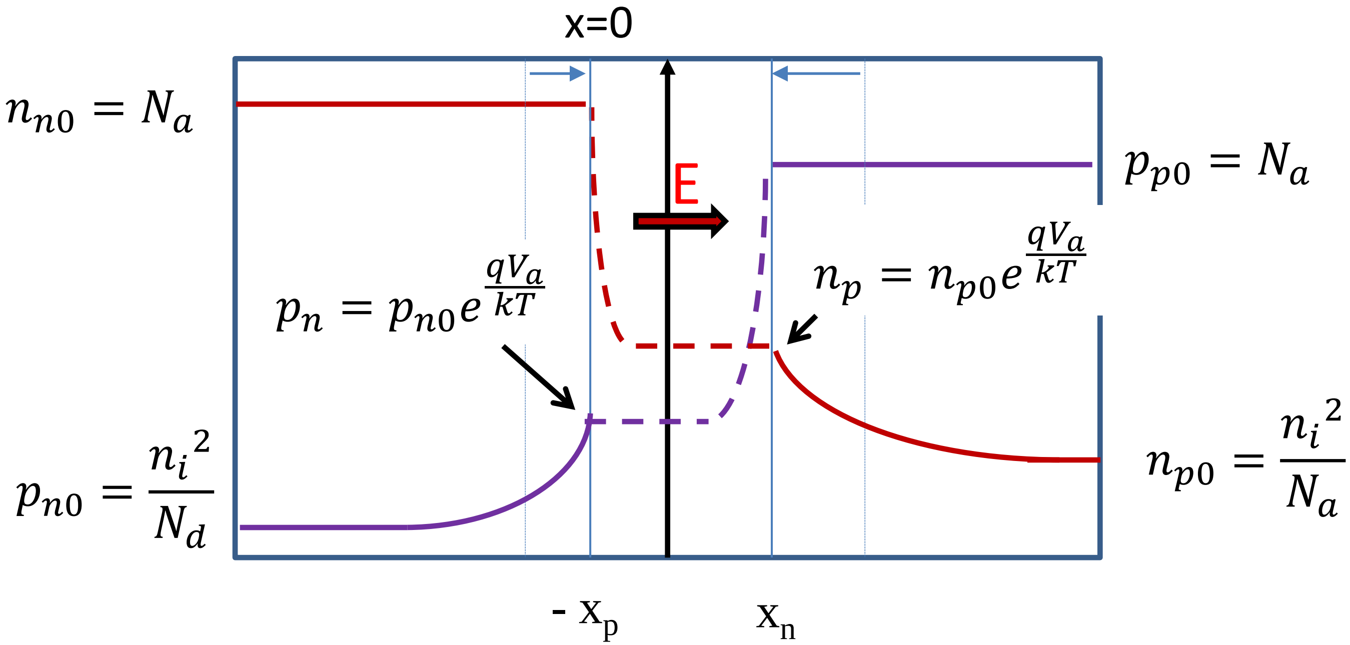


8.1 pn Junction Current

Minority carrier distribution

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

$$n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

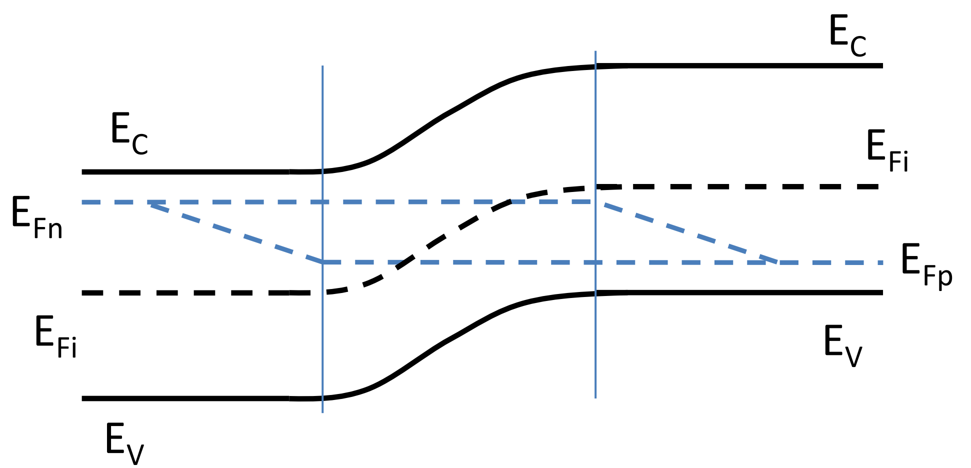


8.1 pn Junction Current

Minority carrier distribution

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$



8.1 pn Junction Current

<u>Minority carrier distribution</u>

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

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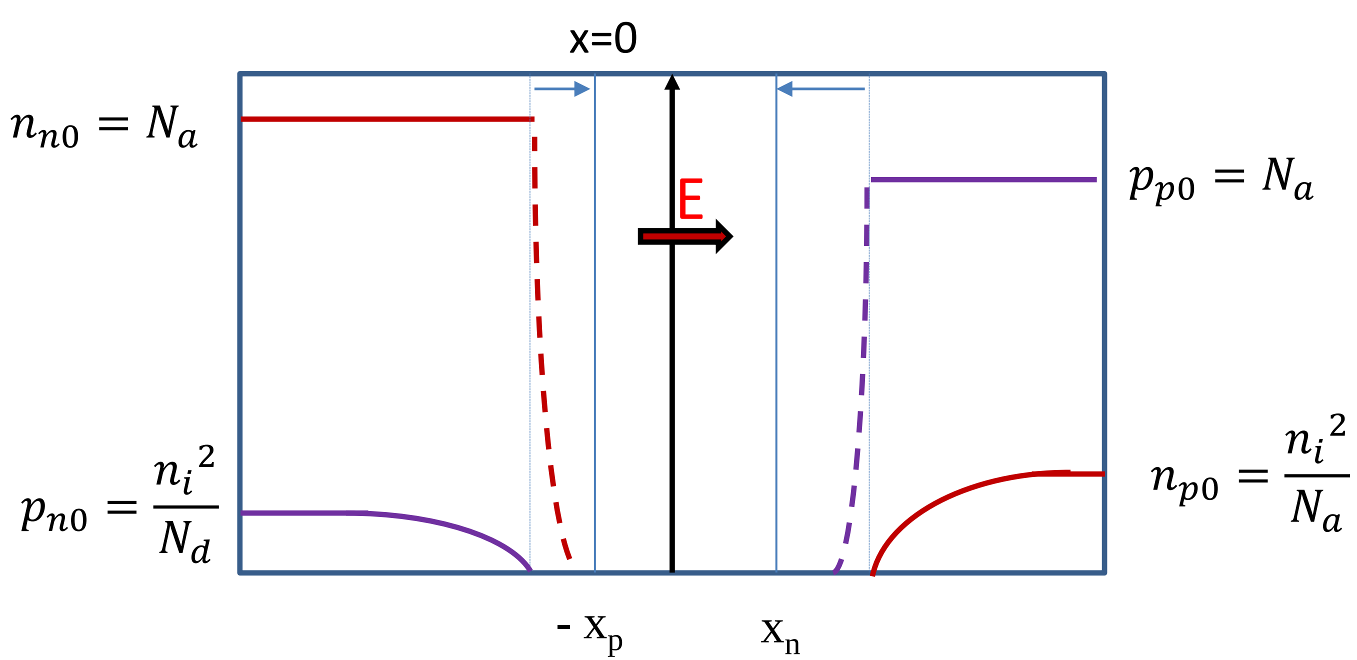

8.1 pn Junction Current

Minority carrier distribution

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \qquad n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

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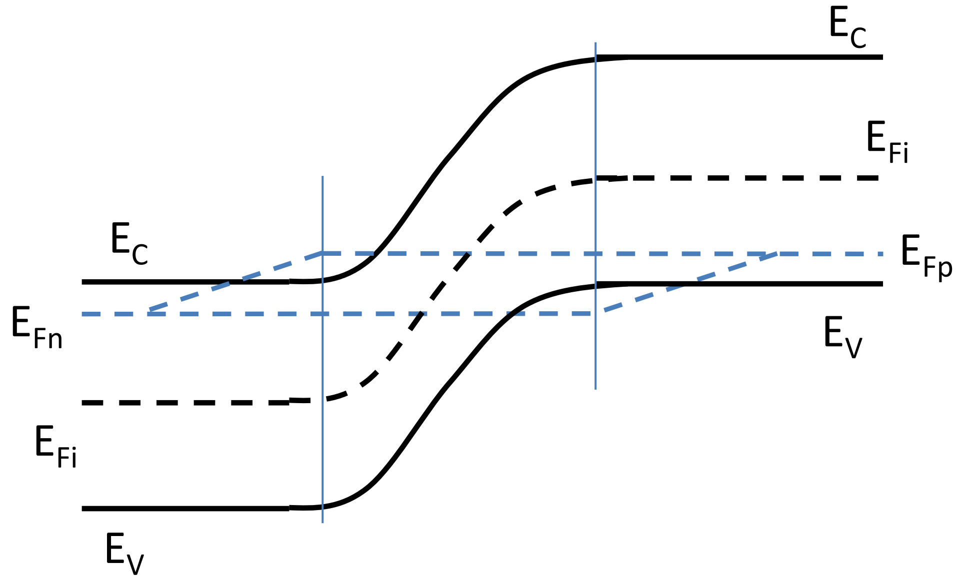
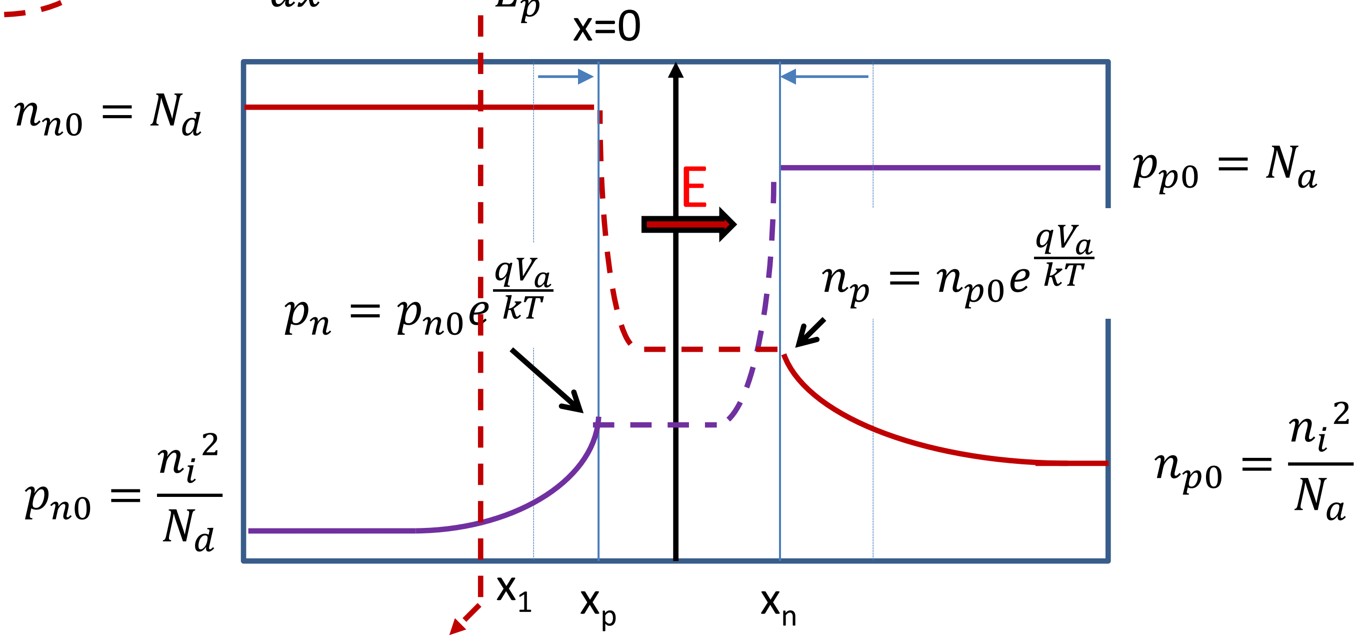


8.1 pn Junction Current

- charge carrier transport: <u>forward bias</u>

$$J_{n,diff} = qD_n \frac{dn_p}{dx} = -\frac{qD_n n_{p0}}{L_n}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{\frac{x_n - x}{L_n}}$$

$$J_{p,diff} = -qD_p \frac{dp_n}{dx} = -\frac{qD_p p_{n0}}{L_p}\left(e^{\frac{qV_a}{kT}} - 1\right)e^{\frac{x+xp}{L_p}}$$



8.1 pn Junction Current

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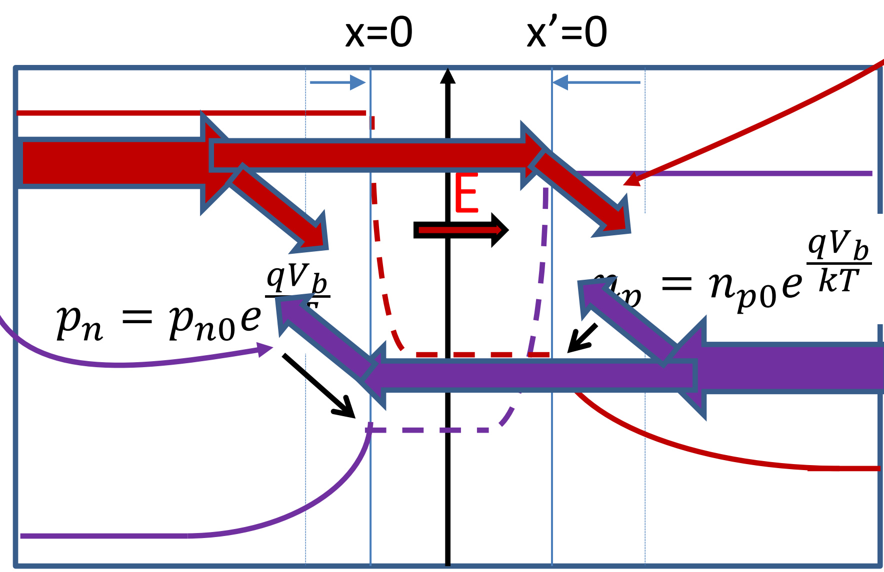
$$\boxed{J = J_n|_{x'=0} + J_n|_{x=0}}$$



$n_{n0} = N_D$

$p_{p0} = N_A$

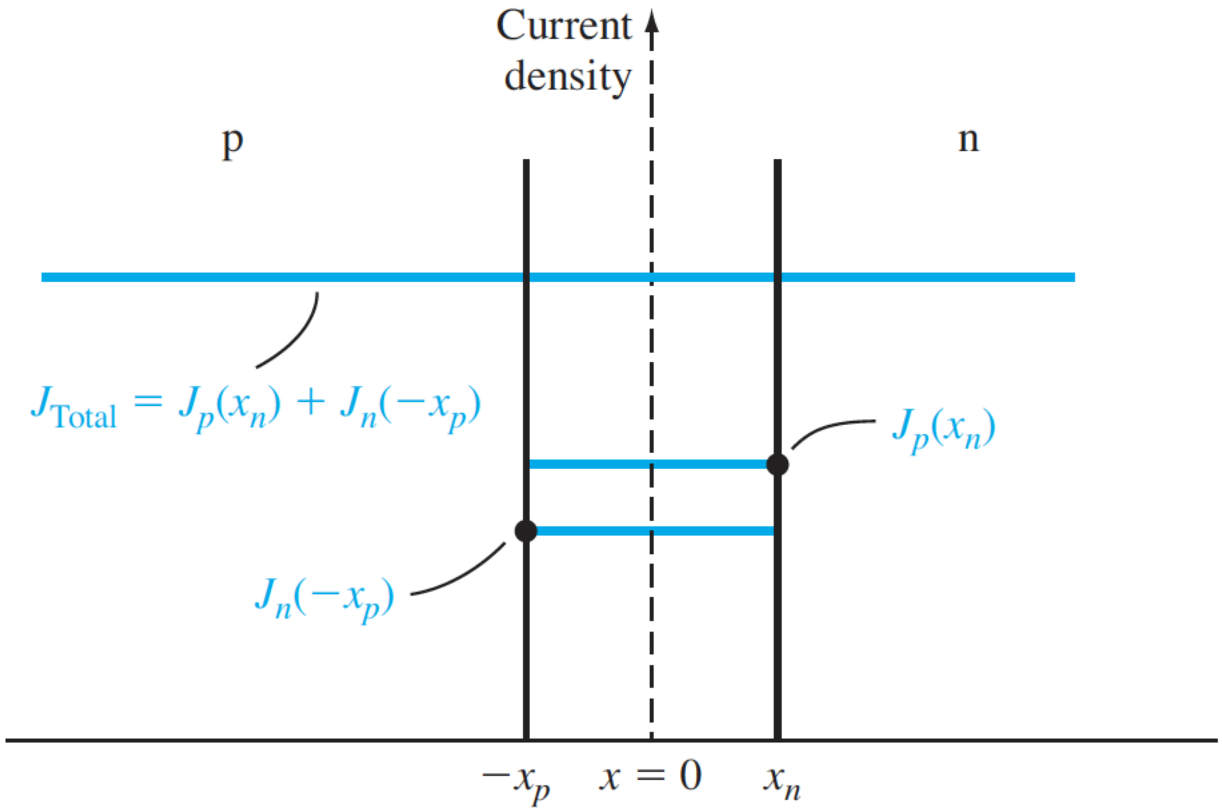
$p_{n0} = \frac{{n_i}^2}{N_D}$

$n_{p0} = \frac{{n_i}^2}{N_A}$

Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- Ideal pn junction current



Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- Ideal pn junction current

$$\boxed{J = J_n|_{x'=0} + J_n|_{x=0}}$$

$$\boxed{J = J_s\left(e^{\frac{qV_b}{kT}} - 1\right)}$$

$$\boxed{J_s = \frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}}$$

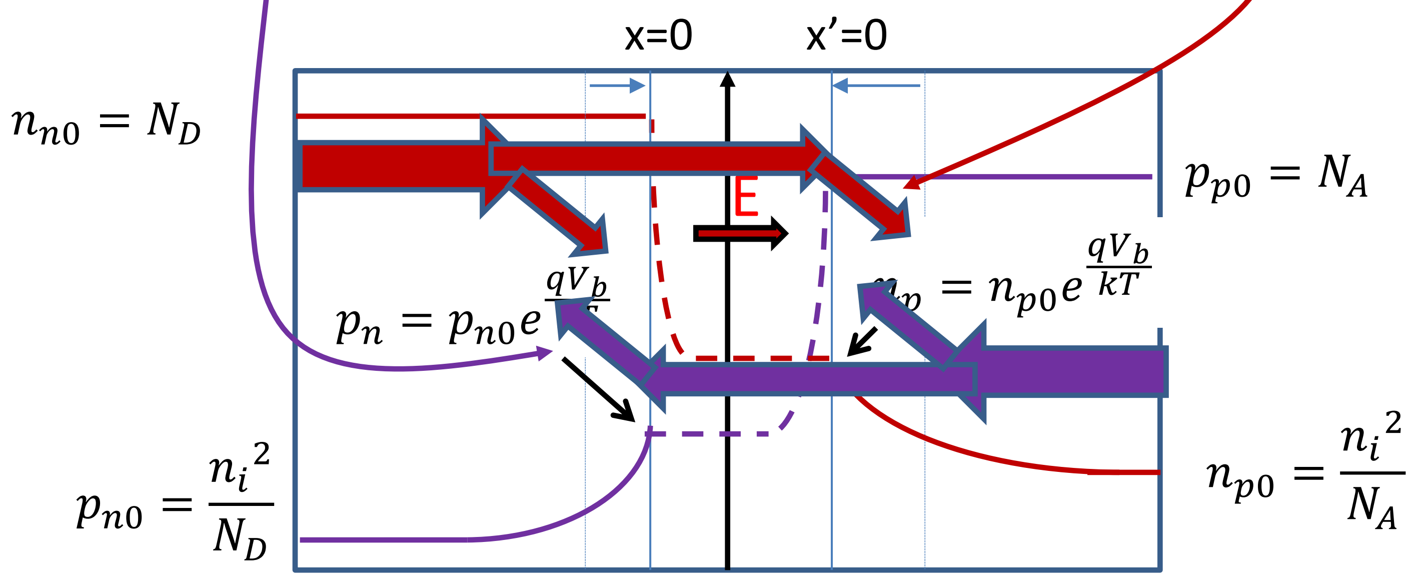
8.1 pn Junction Current

- charge carrier transport: forward bias: current ratio

$$J_n = qD_n \frac{dn_p}{dx} = -\frac{qD_n n_{p0}}{L_n}(e^{\frac{qV_b}{kT}} - 1)$$

$$J_p = -qD_p \frac{dp_n}{dx} = -\frac{qD_p p_{n0}}{L_p}(e^{\frac{qV_b}{kT}} - 1)$$

$$\boxed{\frac{J_n}{J_p} = \frac{D_n n_{p0}/L_n}{D_p p_{n0}/L_p}}$$



Assumption: No recombination-generation in depletion region.

8.1 pn Junction Current

- charge carrier transport: reverse bias

$$J_n = qD_n \frac{dn_p}{dx} = \frac{qD_n n_{p0}}{L_n}$$

$$J_p = -qD_p \frac{dp_n}{dx} = \frac{qD_p p_{n0}}{L_p}$$

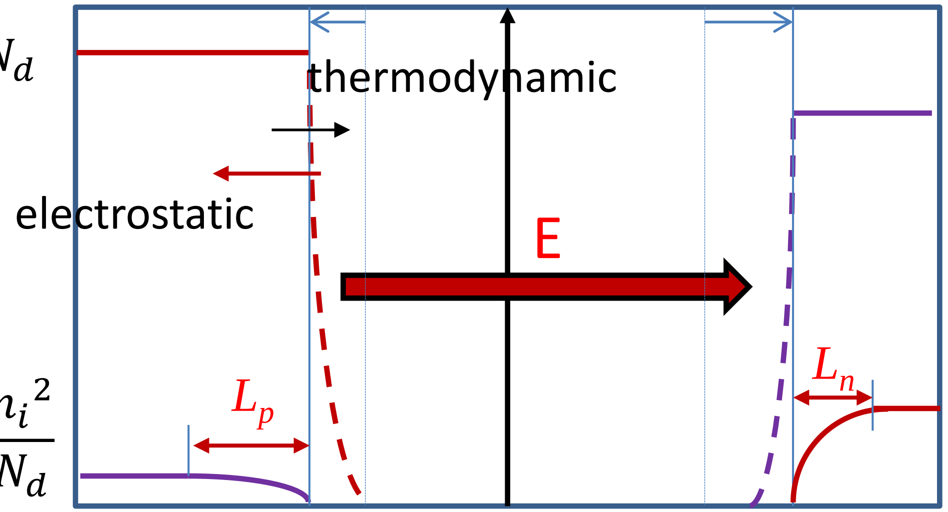
$$\boxed{J = J_n|_{x'=0} + J_n|_{x=0}}$$

$$\boxed{J = J_s\left(e^{\frac{qV_b}{kT}} - 1\right) = -J_s}$$

$$\boxed{J_s = \frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}}$$

$n_{n0} = N_d$

$p_{n0} = \frac{{n_i}^2}{N_d}$



$p_{p0} = N_a$

$n_{p0} = \frac{{n_i}^2}{N_a}$

Assumption: No recombination-generation in depletion region.

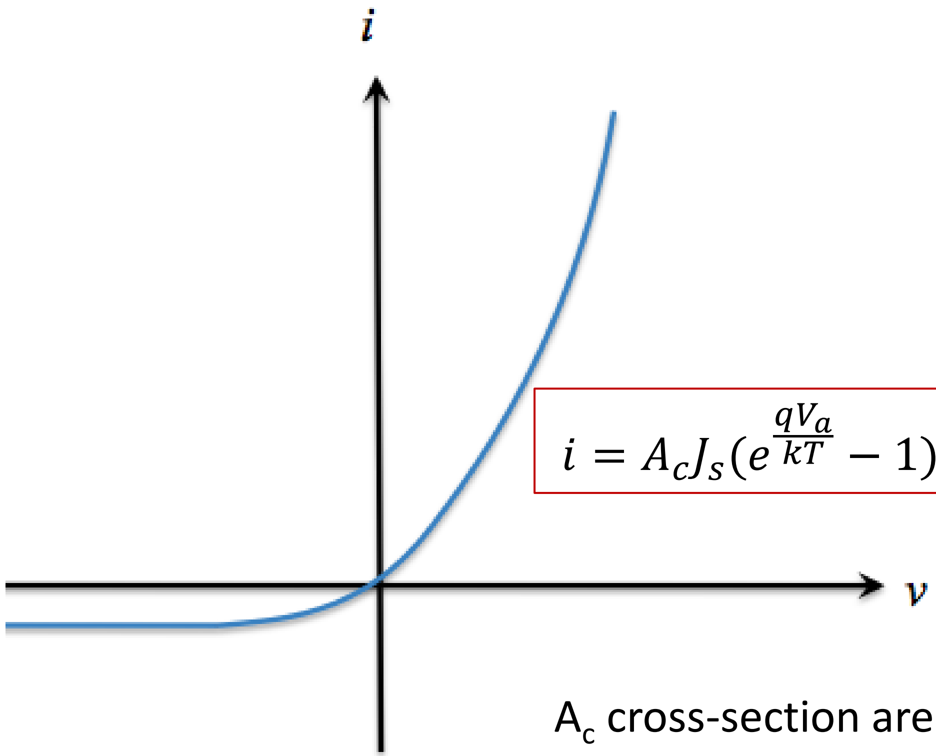
8.1 pn Junction Current

- charge carrier transport: forward bias

$$J = J_n|_{x'=0} + J_n|_{x=0}$$

$$J = J_s(e^{\frac{qV_b}{kT}} - 1)$$

$$J_s = \frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}$$



$$i = A_c J_s(e^{\frac{qV_a}{kT}} - 1)$$

A_c cross-section area

Check your understanding

Problem Example #2

Given the following parameters in a silicon pn junction, determine the ideal reverse-saturation current density of this pn junction at 300K.

$$N_a = N_d = 10^{16}\ \text{cm}^{-3} \qquad n_i = 1.5 \times 10^{10}\ \text{cm}^{-3}$$

$$D_n = 25\ \text{cm}^2/\text{s} \qquad \tau_{p0} = \tau_{n0} = 5 \times 10^{-7}\ \text{s}$$

$$D_p = 10\ \text{cm}^2/\text{s} \qquad \epsilon_r = 11.7$$

Outline

8.1 pn junction current

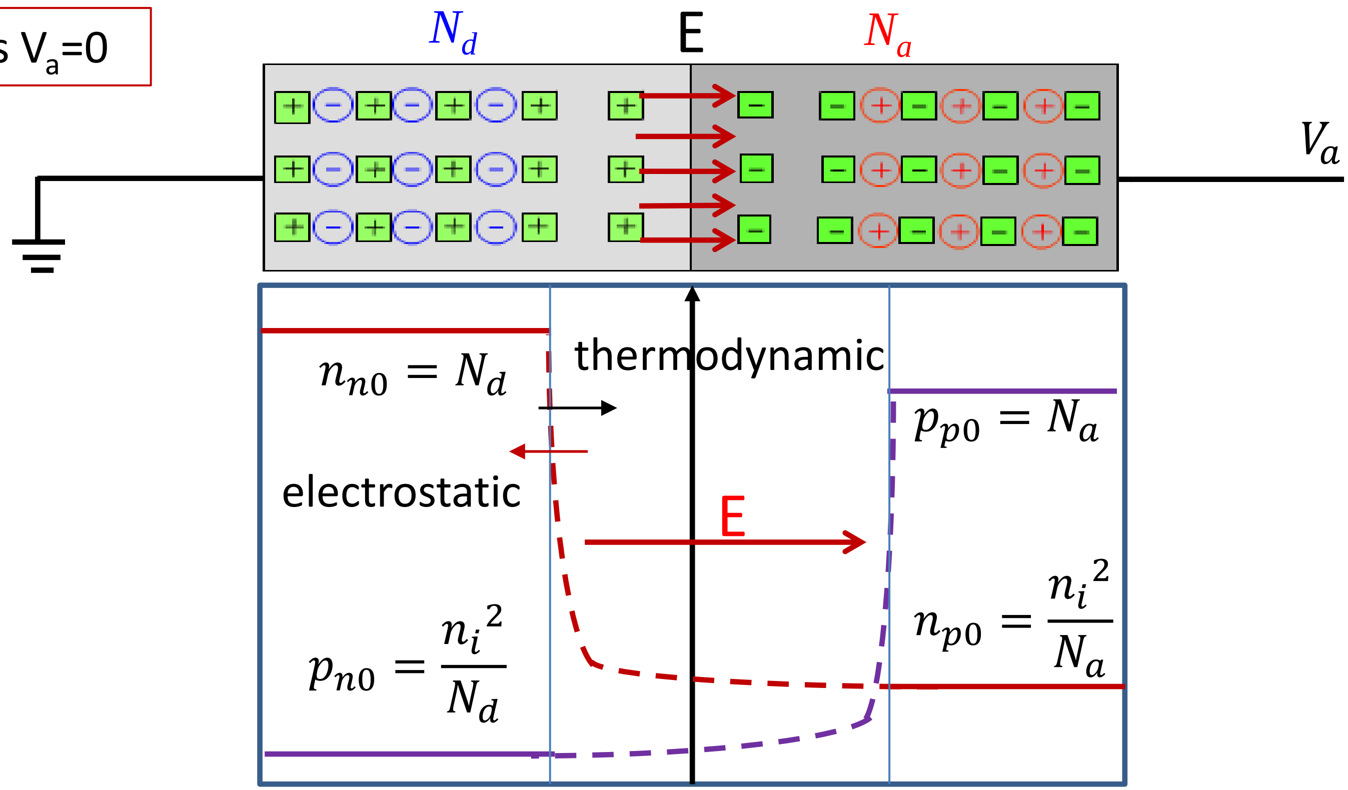
8.2 Generation-recombination currents

8.3 High-injection levels

8.4 A few more points on pn junctions (not in the textbook)

8.2 Generation-recombination currents

Zero Bias V_a=0

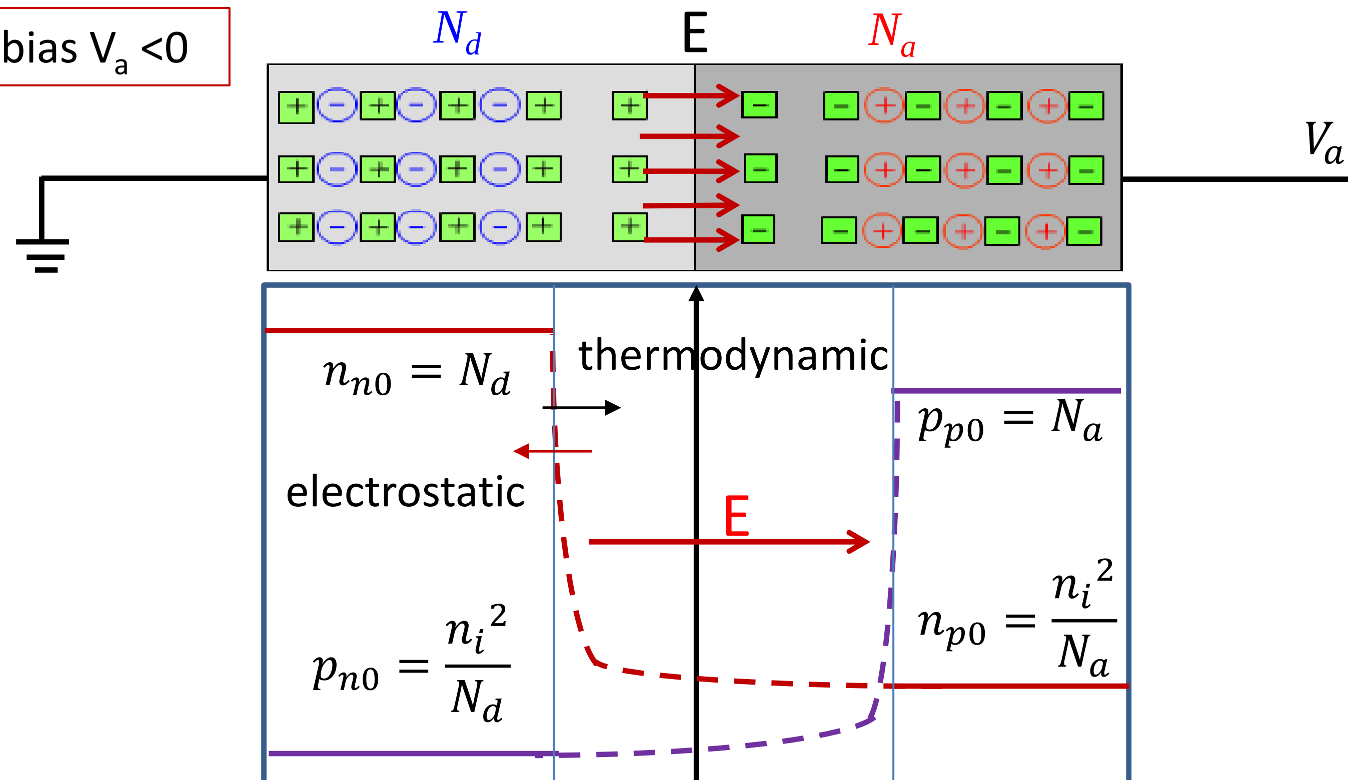


$$R_n = \frac{(np - n_i^2)}{\tau_p \left[n + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n \left[p + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

$$In\ depletion\ region:\ np = n_i^2 \exp\left(\frac{qV_a}{kT}\right)$$

8.2 Generation-recombination currents

Reverse bias V_a <0



$$R_n = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p\left[\cancelto{0}{n} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[\cancelto{0}{p} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}$$

8.2 Generation-recombination currents

Reverse bias $V_a < 0$

To simplify the calculation, we assume

$$\boxed{E_t = E_i, \tau_n = \tau_p = \tau}$$

$$\boxed{R_n = \frac{-n_i}{2\tau} = -G_0}$$

$$\boxed{R_n = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p\left[\cancelto{0}{n} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[\cancelto{0}{p} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}}$$

8.2 Generation-recombination currents

Reverse bias V_a <0

To simplify the calculation, we assume

$$\boxed{E_t = E_i, \tau_n = \tau_p = \tau}$$

$$\boxed{R_n = \frac{-n_i}{2\tau} = -G_0}$$

Current density from G-R in the depletion region:

$$J_r = \int_0^W qG_0 dx = \frac{qWn_i}{2\tau}$$

$$\boxed{R_n = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p\left[\cancelto{0}{n} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n\left[\cancelto{0}{p} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}}$$

8.2 Generation-recombination currents

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Current density from G-R in the depletion region:

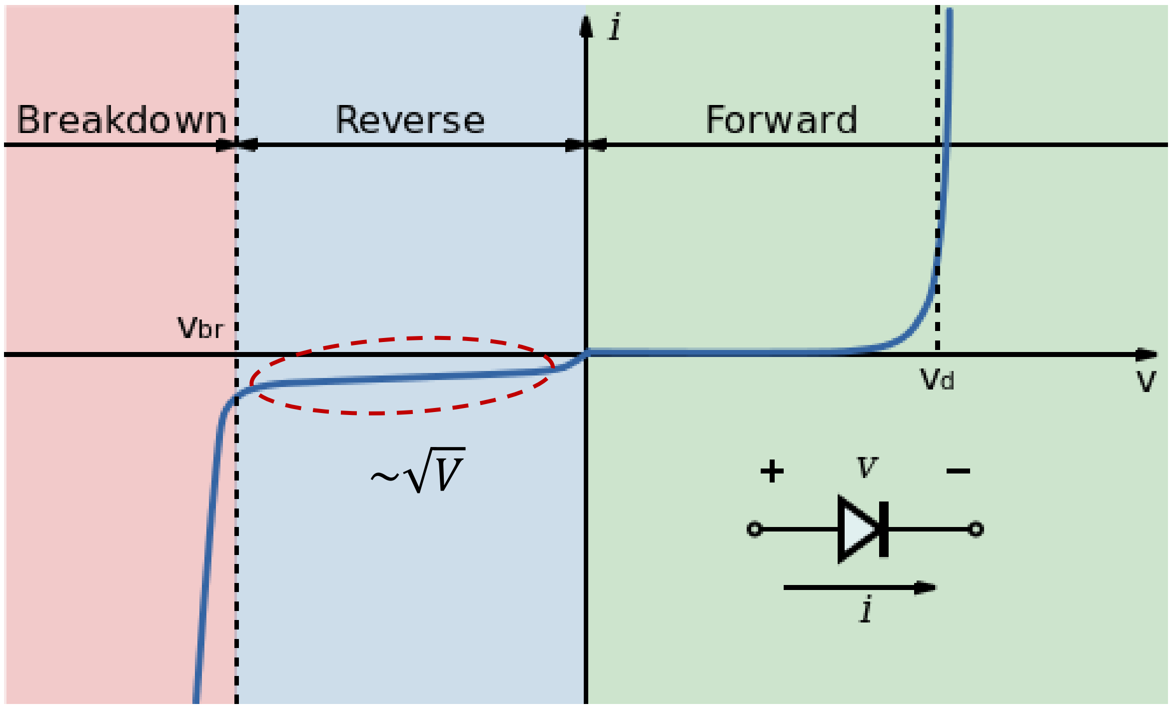
$$J_r = \int_0^W qG_0 dx = \frac{qWn_i}{2\tau}$$

$$\boxed{W = a + b = \sqrt{\frac{2\varepsilon(V_{bi} - V_a)}{q} \frac{N_d + N_a}{N_a N_d}}}$$

$$\boxed{R_n = \frac{(\cancelto{0}{np} - n_i^2)}{\tau_p \left[\cancelto{0}{n} + n_i \exp\left(\frac{E_t - E_i}{kT}\right)\right] + \tau_n \left[\cancelto{0}{p} + n_i \exp\left(\frac{E_i - E_t}{kT}\right)\right]}}$$

8.2 8.2 Generation-recombination currents

Reverse bias V_a <0

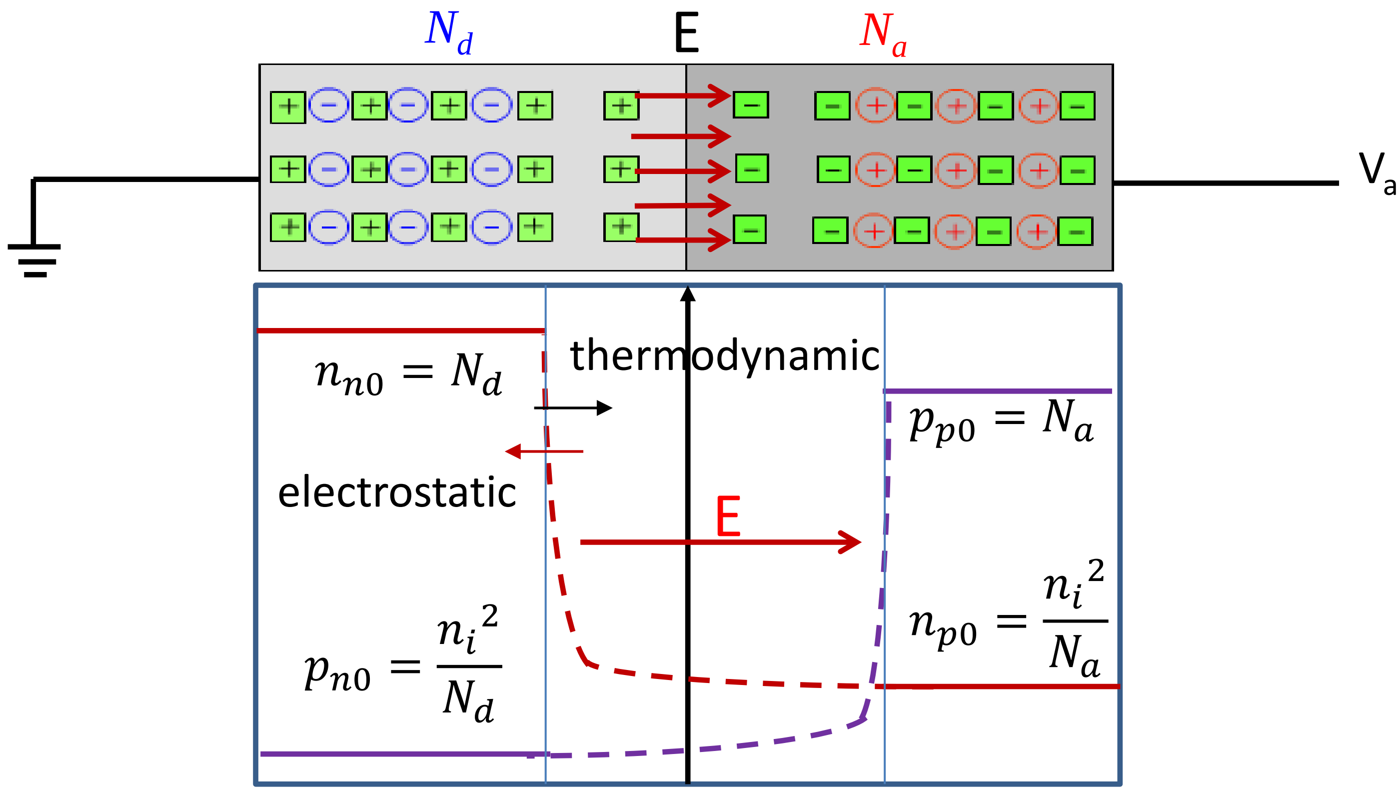


Current density from G-R in the depletion region:

$$J_r = \int_0^W qG dx = \frac{qWn_i}{2\tau}$$

$$\boxed{W = a + b = \sqrt{\frac{2\varepsilon(V_{bi}-V_a)}{q}\frac{N_d + N_a}{N_a N_d}}}$$

8.2 Generation-recombination currents



$$In\ depletion\ region:\ np = n_i^2 \exp\left(\frac{qV_a}{kT}\right)$$

8.2 Generation-recombination currents

To simplify the calculation, we assume

$$E_t = E_i, \tau_n = \tau_p = \tau$$

$$R_n = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$

When n=p, U reaches its max value.

$$R_{n,max} = \frac{np - n_i^2}{\tau\left[n_i \exp(\frac{qV_a}{2kT}) + n_i \exp(\frac{qV_a}{2kT}) + 2n_i\right]} = \frac{n_i\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right]}{2\tau\left[\exp\left(\frac{qV_a}{2kT}\right) + 1\right]}$$

$$= \frac{n_i\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]}{2\tau}$$

$$In\ depletion\ region{:}\ np = n_i^2 \exp(\frac{qV_a}{kT})$$

8.2 Generation-recombination currents

$$\boxed{R_{n,max} = \frac{n_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]}$$

Current density from G-R in the depletion region:

$$J_r = \int_0^W qR_{n,max}dx = \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

For a non-ideal pn junction, the total current density:

$$J = J_F + J_r = J_s\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

8.2 Generation-recombination currents

$$J = J_F + J_r = J_s\left[\exp\left(\frac{qV_a}{kT}\right) - 1\right] + \frac{qWn_i}{2\tau}\left[\exp\left(\frac{qV_a}{2kT}\right) - 1\right]$$

Forward bias V> 3kT/q=0.078V:

$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau}\exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp\left(\frac{qV_a}{nkT}\right)$$

the ideality factor

8.2 Generation-recombination currents

$$J = J_F + J_r = J_s \left[\exp\left(\frac{qV_a}{kT}\right) - 1 \right] + \frac{qWn_i}{2\tau} \left[\exp\left(\frac{qV_a}{2kT}\right) - 1 \right]$$

Forward bias V> 3kT/q=0.078V:

$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau} \exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp\left(\frac{qV_a}{nkT}\right)$$

the ideality factor

Reverse bias:

$$J_0 = -J_s - \frac{qWn_i}{2\tau} = -\left(\frac{qD_n n_{p0}}{L_n} + \frac{qD_p p_{n0}}{L_p}\right) - \frac{qWn_i}{2\tau}$$

Check your understanding

Problem Example #3

A PN junction consisting an n-type semiconductor in contact with another p-type semicondcutor (to be covered later) has a depletion region in which n_0 and p_0 are nearly zero. Suppose a silicon PN junction has defects located at the middle of the semiconductor. The defect concentration is 10^{16} cm^{-3} and the capture rate C_n and C_p for electrons and holes are 10^{-10} cm^{-3}/s. Find the leakage current of the Si PN junction.

Depletion region

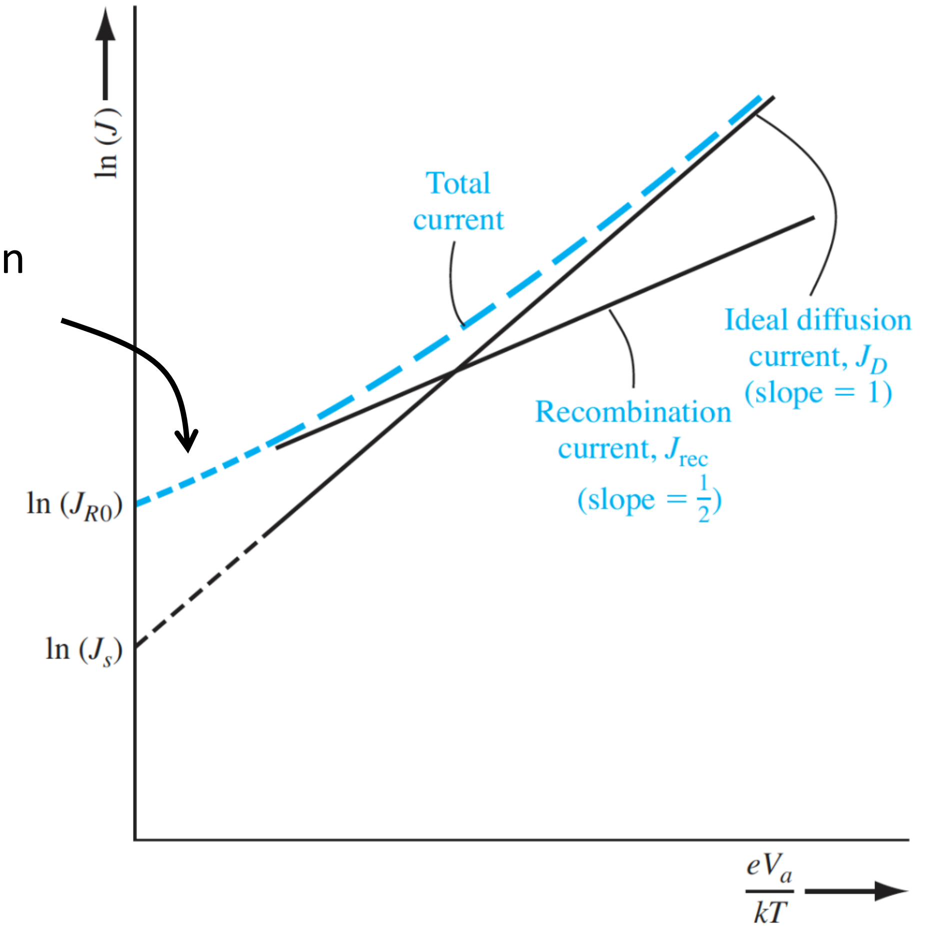
$N_t = 10^{16}$ cm^{-3}
$C_n=C_p=10^{-10}$ cm^{-3}/s

Outline

8.3 High inject level

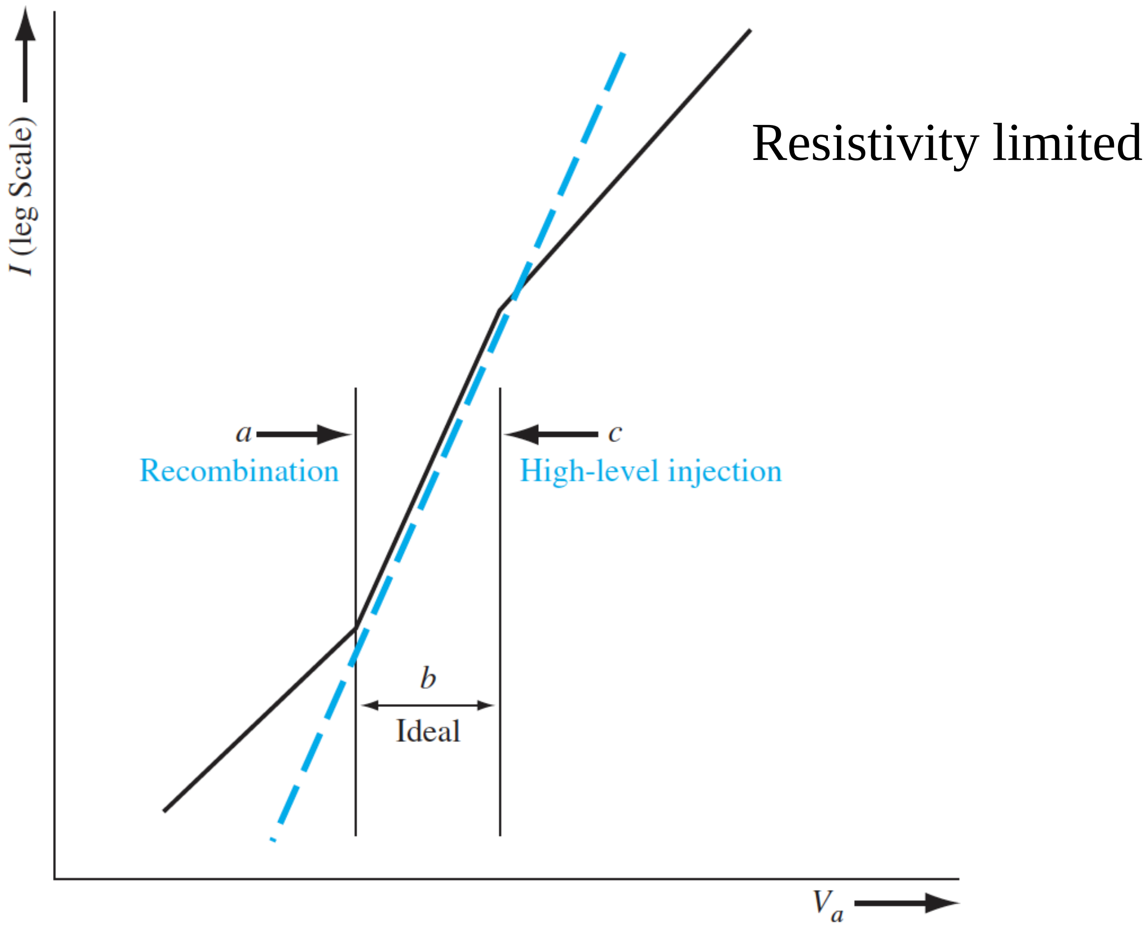
$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau} \exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp\left(\frac{qV_a}{nkT}\right)$$

e-h pairs recombine in the depletion region



8.3 High inject level

$$J = J_F + J_r = J_s \exp\left(\frac{qV_a}{kT}\right) + \frac{qWn_i}{2\tau}\exp\left(\frac{qV_a}{2kT}\right) = J_0 \exp(\frac{qV_a}{nkT})$$



Outline

8.1 pn junction current

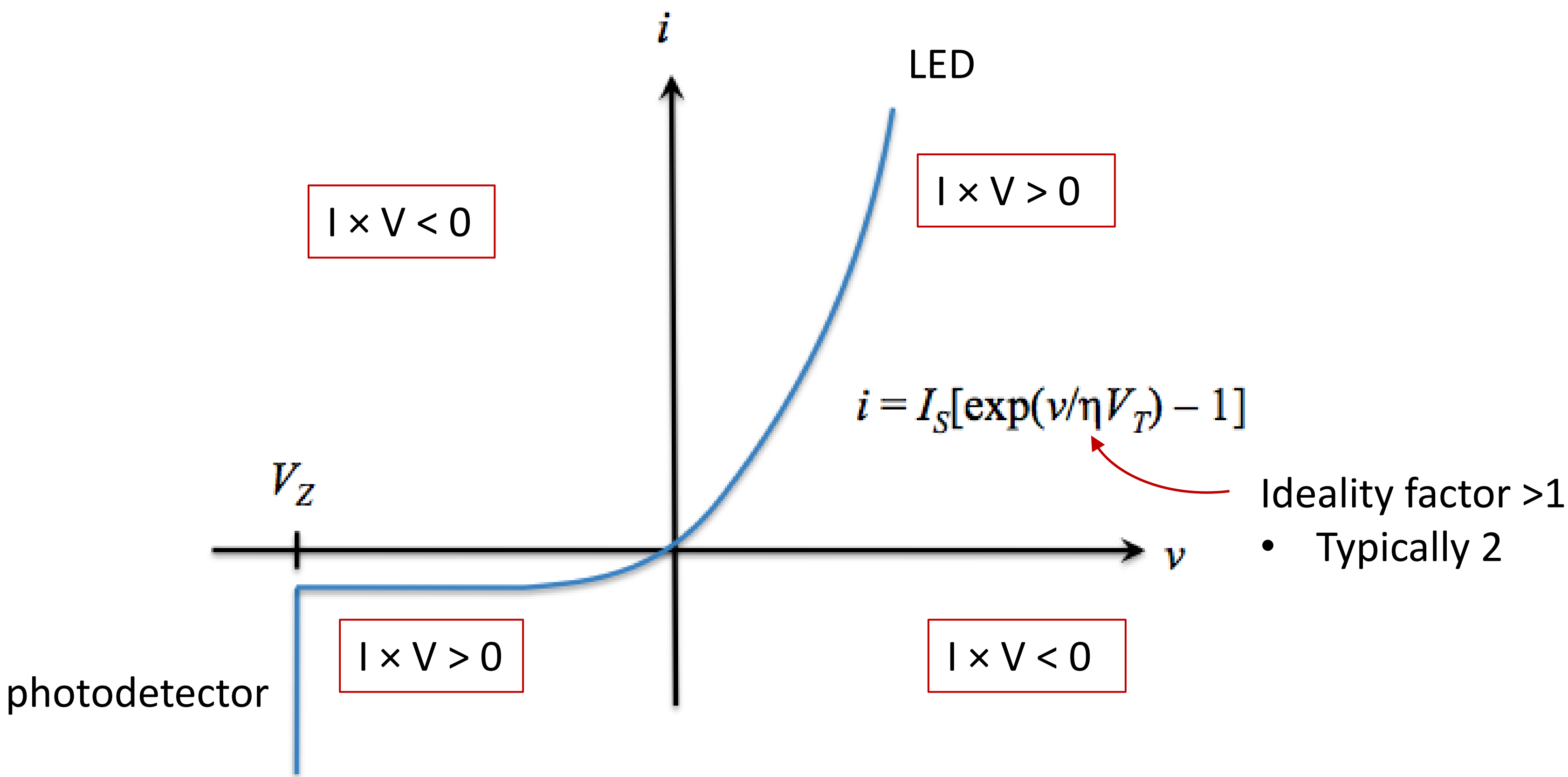
8.2 Generation-recombination currents

8.3 High-injection levels

8.4 A few more points on pn junctions (not in the textbook)

8.4 A few points about pn junction

- Energy consumption:



8.4 A few points about pn junction

- Energy consumption: