VE320 Introduction to Semiconductor Physics and Devices

Recitation Class 3

VE320 Teaching Group SU2022

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Joint Institute

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Contents

Thermal Equilibrium

- Equilibrium or thermal equilibrium means no external forces such as:
    - Electric fields
    - Magnetic fields
    - Temperature variations
- Additionally we are looking at:
    - The inside of a very large piece of uniformly-doped semiconductor
- We study the properties that are independent of time

Motivation and Starting Point

- Motivation
    - We want to know the current
    - We need to know the conductivity
    - We need to know the density of electrons in conduction band and holes in valance band
- Staring point
    - Density of states
    - Fermi distribution

n_0 and p_0

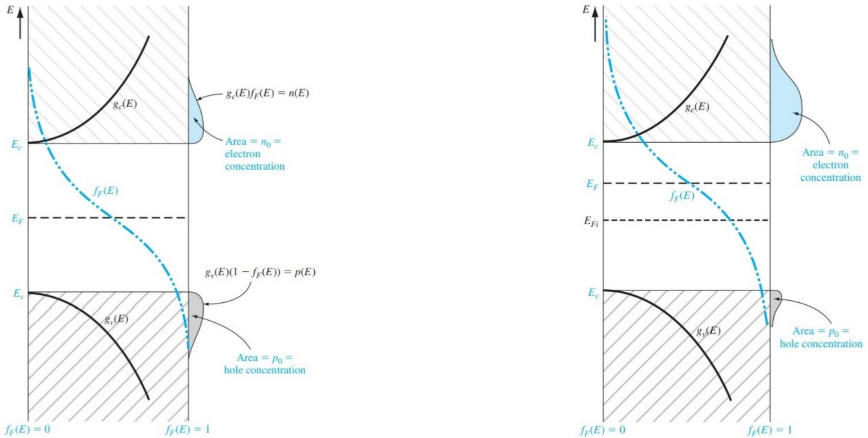

Figure: Intrinsic (left) and n-type (right) semiconductors' density of states, Fermi distribution and carrier density.

n_0 and p_0, Calculation

$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) \mathrm{d}E$$

$$\Rightarrow n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right], \quad N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)\left(1 - f_F(E)\right) \mathrm{d}E$$

$$\Rightarrow p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right], \quad N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

- Note: derivation is done under Boltzmann approximation $E - E_{Fi} > 3kT$, so that the semiconductor cannot be very heavily doped
- The equations apply for both intrinsic and doped semiconductors
- What is the unit of n_0 and p_0?
- Verify that $n_i^2 = n_0 p_0$

Example

Calculate the thermal-equilibrium hole concentration in silicon at $T = 400\ \mathrm{K}$. Assume that the Fermi energy is $0.27\mathrm{eV}$ above the valence-band energy. The value of N_v for silicon at $T = 300K$ is $N_v = 1.04 \times 10^{19}\ \mathrm{cm}^{-3}$.

Example, Solution

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$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453\mathrm{eV}$$

$$N_v = \left(1.04 \times 10^{19}\right)\left(\frac{400}{300}\right)^{3/2} = 1.60 \times 10^{19}\ \mathrm{cm}^{-3}$$

The hole concentration is then

$$\begin{aligned} p_0 = N_v \exp\left[\frac{-\left(E_F - E_v\right)}{kT}\right] &= \left(2.60 \times 10^{19}\right)\exp\left(\frac{-0.27}{0.03453}\right) \\ &= 6.43 \times 10^{15}\ \mathrm{cm}^{-3} \end{aligned}$$

Example 2

The magnitude of the product $g_C(E)f_F(E)$ in the conduction band is a function of energy. Assume the Boltzmann approximation is valid. Determine the energy with respect to E_c at which the maximum occurs.

Example 2, Solution

The magnitude of the product $g_C(E)f_F(E)$ in the conduction band is a function of energy. Assume the Boltzmann approximation is valid. Determine the energy with respect to E_c at which the maximum occurs.

$$g_c f_F \propto \sqrt{E-E_c}\exp\left[\frac{-(E-E_F)}{kT}\right] \propto \sqrt{E-E_c}\exp[\frac{-(E-E_c)}{kT}\right] \times \exp\left[\frac{-(E_c-E_F)}{kT}\right]$$

Let $E-E_c=x$ To find the maximum value:

$$\frac{d\left(g_c f_F\right)}{dx} \propto \frac{1}{2}x^{-1/2}\exp\left(\frac{-x}{kT}\right) \Rightarrow -\frac{1}{kT}\cdot x^{1/2}\exp\left(\frac{-x}{kT}\right)=0 \Rightarrow x=\frac{kT}{2}$$

The maximum value occurs at $E=E_c+\frac{kT}{2}$

Intrinsic Semiconductors

- Intrinsic Fermi energy level:
  - Approximately at the middle of E_c and E_v
  - More accurately: $E_{Fi} - E_{\text{midgap}} = \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right) = \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right)$
    - Derived with $n_0 = p_0$
- $n_0 \times p_0 = n_i^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$

Example and Solution

The carrier effective masses in a semiconductor are $m_n^* = 1.21 m_0$ and $m_p^* = 0.70 m_0$. Determine the position of the intrinsic Fermi level with respect to the center of the bandgap at $T = 300\ \mathrm{K}$.

$$
\begin{aligned}
E_{Fi} - E_{\text{midgap}} &= \frac{3}{4} kT \ln\left(\frac{m_p^*}{m_n^*}\right) \\
&= \frac{3}{4}(0.0259) \ln\left(\frac{0.70}{1.21}\right) \\
&\Rightarrow -10.63 \mathrm{meV}
\end{aligned}
$$

Intrinsic Semiconductors, Two Approaches

For Si at $300K$:

$$E_g = 1.12\mathrm{eV}$$
$$N_c = 2.8 \times 10^{19}\ \mathrm{cm}^{-3}$$
$$N_v = 1.04 \times 10^{19}\ \mathrm{cm}^{-3}$$
$$kT = 0.0259\mathrm{eV}$$

so that

$$n_i^2 = 4.82936 \times 10^{19}\ \mathrm{cm}^{-6}$$

But experimental data show:

$$n_i = 1.5 \times 10^{10}\ \mathrm{cm}^{-3}$$

so that

$$n_i^2 = 2.25 \times 10^{20}\ \mathrm{cm}^{-6}$$

- Both are acceptable in this course

Intrinsic Semiconductors, Two Approaches

For n-doped Silicon semiconductor at $300K$, the Fermi level is $E_F = E_c - 0.3\mathrm{eV}$. Calculate p_0.
- Approach I (n_0 first):

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = 2.61 \times 10^{14}\ \mathrm{cm}^{-3}$$
$$p_0 = \frac{n_i^2}{n_0} = 8.62 \times 10^5\ \mathrm{cm}^{-3}$$

- Approach II (p_0 directly):

$$E_F - E_v = E_g - (E_c - E_F) = 0.82\mathrm{eV}$$
$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = 1.85 \times 10^5\ \mathrm{cm}^{-3}$$

p-Type Doping and Energy Levels

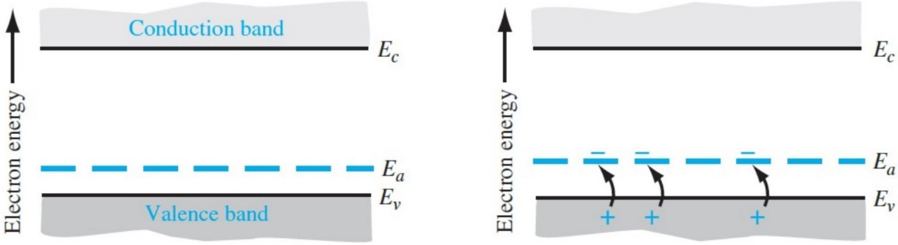

Figure: The acceptor energy state and electrons in the valance band jump into this energy state.

n-Type Doping and Energy Levels

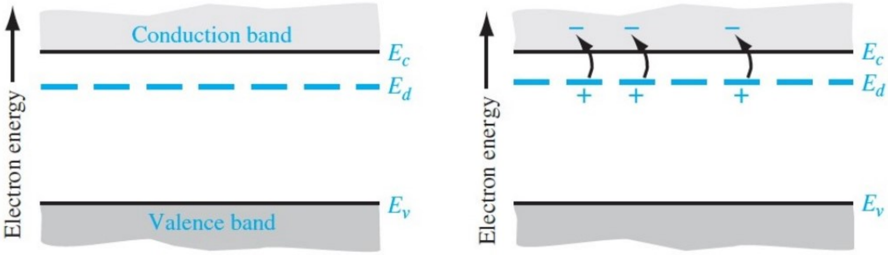

Figure: The donor energy state and electrons in this energy band jump into the conduction band.

Extrinsic Semiconductor, Equations

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \quad p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$n_0 = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \quad p_0 = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$$n_i^2 = n_0 p_0$$

Example

Silicon at $T = 300$ K is doped with arsenic atoms such that the concentration of electrons is $n_0 = 7 \times 10^{15}\ \mathrm{cm}^{-3}$. (a) Find $E_c - E_F$. (b) Determine $E_F - E_v$. (c) Calculate p_0. (d) Find $E_F - E_{Fi}$.

Example, Solution

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(a)

$$E_c - E_F = kT \ln\left(\frac{N_c}{n_o}\right)$$

$$= (0.0259) \ln\left(\frac{2.8 \times 10^{19}}{7 \times 10^{15}}\right)$$

$$= 0.2148\mathrm{eV}$$

(b)

$$E_F - E_v = E_g - (E_c - E_F)$$

$$= 1.12 - 0.2148 = 0.90518\mathrm{eV}$$

Example, Solution

Silicon at $T = 300\ \mathrm{K}$ is doped with arsenic atoms such that the concentration of electrons is $n_0 = 7 \times 10^{15}\ \mathrm{cm}^{-3}$. (a) Find $E_c - E_F$. (b) Determine $E_F - E_v$. (c) Calculate p_0. (d) Find $E_F - E_{Fi}$.

(c)

$$\begin{aligned} p_o &= N_v \exp\left[\frac{-\left(E_F - E_v\right)}{kT}\right] \\ &= \left(1.04 \times 10^{19}\right) \exp\left[\frac{-0.90518}{0.0259}\right] \\ &= 6.90 \times 10^{3}\ \mathrm{cm}^{-3} \end{aligned}$$

(d)

$$\begin{aligned} E_F - E_{Fi} &= kT \ln\left(\frac{n_o}{n_i}\right) \\ &= 0.338\mathrm{eV} \end{aligned}$$

Degenerate Semiconductors

Impurity concentration increases
$\Rightarrow$ distance between impurity atoms decreases
$\Rightarrow$ donor electrons start to interact with each other
$\Rightarrow$ single discrete donor energy level splits into a band
$\Rightarrow$ overlaps with conduction band.

Degenerate Semiconductors

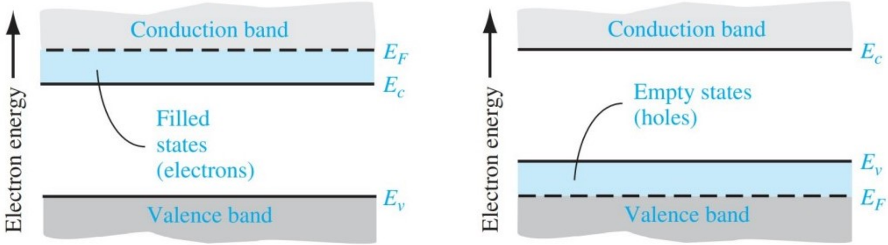


Figure: Energy-band diagrams for degenerate semiconductors: n-type (left) and p-type (right).

Statistics of Donors and Acceptors

How many conduction band electrons are provided by n-type semiconductor with doping concentration N_d?

$$f_d(E) = \frac{1}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

$$n_d = f_d(E)N_d = N_d - N_d^+$$

where N_d^+ is the concentration of ionized donors, and the concentration of electrons combined with these donors is n_d.

Statistics of Donors and Acceptors

How many holes are provided by p-type semiconductor with doping concentration N_a?

$$f_a(E) = \frac{1}{1 + \frac{1}{g}\exp\left(\frac{E_F - E_a}{kT}\right)}$$

g is the degeneracy factor, normally taken as 4 for acceptor level in silicon and gallium arsenide (because of band structure).

$$p_a = f_a(E)N_a = N_a - N_a^-$$

where N_a^- is the concentration of accepters that receive an electron, and the concentration of holes remained with these acceptors is p_a.

Example

We calculate the relative number of electrons in the donor state compared with the total number of electrons. Assume $(E_d - E_F) \gg kT$ so that $n_d = 2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)$.

$$\frac{n_d}{n_d + n_0} = \frac{1}{1 + \frac{N_c}{2N_d} \exp\left[\frac{-(E_c - E_d)}{kT}\right]}$$

Determine the fraction of total electrons still in the donor states at $T = 300K$. Consider phosphorus doping in silicon, for $T = 300K$, at a concentration of $N_d = 10^{16}\ \mathrm{cm}^{-3}$.

- Answer: 0.41%.
- Note: in many cases, you can assume complete ionization.

Charge Neutrality

- $n_0 + N_a^- = N_d^+ + p_0$
- When complete ionization (most cases): $n_0 = \frac{N_d - N_a + \sqrt{(N_d - N_a)^2 + 4n_i^2}}{2}$
- When the temperature is very high so that it is like intrinsic: $n_0 = p_0 = n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$
- When the temperature is not very high (most cases): $n_0 = N_d^+ - N_a^-$
  - Assuming only donors: $n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c} \exp\left(\frac{E_a}{kT}\right)}}{4 \exp\left(\frac{E_a}{kT}\right)}$

Charge Neutrality Derivation 1

$$n_0 = \frac{N_d}{2} + \sqrt{\frac{N_d^2}{2} + n_i^2}$$

Charge neutrality:

$$n_0 = p_0 + N_d^+$$

Complete ionization:

$$n_0 = \frac{n_i^2}{n_0} + N_d \Rightarrow \quad n_0^2 - N_d n_0 - n_i^2 = 0$$

Charge Neutrality Derivation 2

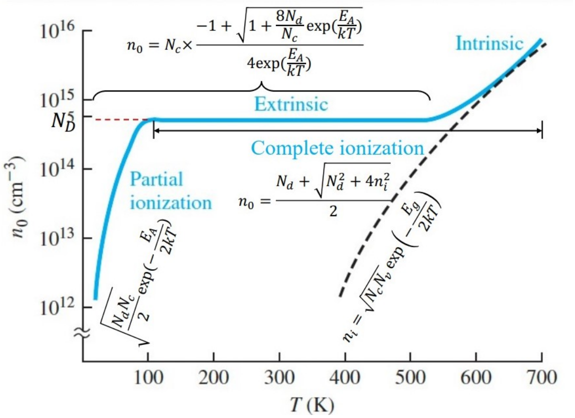
$$n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c} \exp\left(\frac{E_a}{kT}\right)}}{4 \exp\left(\frac{E_a}{kT}\right)}$$

$$n_0 = N_d^+ \quad \text{when } T \text{ is not high}$$

$$n_0 = \frac{N_d}{1 + 2\exp\left(\frac{E_F - E_d}{kT}\right)} = \frac{N_d}{1 + 2\exp\left(\frac{E_c - E_d}{kT}\right)\exp\left(\frac{E_F - E_c}{kT}\right)} = \frac{N_d}{1 + 2\exp\left(\frac{E_a}{kT}\right)\frac{n_0}{N_c}}$$

$$\Rightarrow \quad 2\exp\left(\frac{E_a}{kT}\right) n_0^2 + N_c n_0 - N_d N_c = 0$$

Charge Neutrality Summary



- Note: in most cases, we first get majority carrier concentration with complete ionization equation and use $n_0 p_0 = n_i^2$ to get the other.

Example

Determine the equilibrium electron and hole concentrations in silicon for the following conditions:

(a) $T = 300\ \mathrm{K}, N_d = 10^{15}\ \mathrm{cm}^{-3}, N_a = 4 \times 10^{15}\ \mathrm{cm}^{-3}$

(b) $T = 300\ \mathrm{K}, N_d = 3 \times 10^{16}\ \mathrm{cm}^{-3}, N_a = 0$

(c) $T = 450\ \mathrm{K}, N_d = 10^{14}\ \mathrm{cm}^{-3}, N_a = 0$

Example, Solution

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(c) $T = 450\ \mathrm{K}, N_d = 10^{14}\ \mathrm{cm}^{-3}, N_a = 0$

(a) $p_o = 4 \times 15 - 10^{15} = 3 \times 10^{15}\ \mathrm{cm}^{-3} \Rightarrow n_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{15}} = 7.5 \times 10^4\ \mathrm{cm}^{-3}$

(b) $n_o = N_d = 3 \times 10^{16}\ \mathrm{cm}^{-3} \Rightarrow p_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}} = 7.5 \times 10^3\ \mathrm{cm}^{-3}$

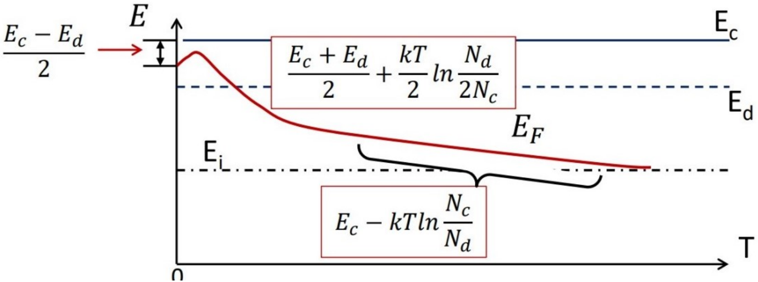
(c) $n_i^2 = \left(2.8 \times 10^{19}\right)\left(1.04 \times 10^{19}\right)\left(\frac{450}{300}\right)^3 \times \exp\left[\frac{-(1.12)(300)}{(0.0259)(450)}\right] \Rightarrow n_i = 1.722 \times 10^{13}\ \mathrm{cm}^{-3}$

Fermi Energy Level Position

$$E_F = E_c + kT \ln \left(\frac{\sqrt{1 + \frac{8N_d}{N_c} \exp\left(\frac{E_a}{kT}\right)} - 1}{4 \exp\left(\frac{E_a}{kT}\right)} \right)$$

$$= \begin{cases} \frac{E_c+E_d}{2} + \frac{kT}{2} \ln \frac{N_d}{2N_c}, & T \text{ small} \\ E_c - kT \ln \frac{N_c}{N_d}, & T \text{ big} \end{cases}$$

Fermi Energy Level Position



Questions?