VE320
Intro to Semiconductor Devices
RC Week6

Yucheng Huang

University of Michigan
Shanghai Jiao Tong University
Joint Institute

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Carrier Generation and Recombination

Let G_{n0} and G_{p0} be the thermal-generation rates of electrons and holes, respectively, given in units of $\#/\mathrm{cm}^3 \cdot \mathrm{s}$. For the direct band-to-band generation, the electrons and holes are created in pairs, so we must have that

$$G_{n0} = G_{p0}$$

Let R_{n0} and R_{p0} be the recombination rates of electrons and holes, respectively, for a semiconductor in thermal equilibrium, again given in units of $\#/\mathrm{cm}^3 \cdot \mathrm{s}$. In direct band-to-band recombination, electrons and holes recombine in pairs, so that

$$R_{n0} = R_{p0}$$

In thermal equilibrium, the concentrations of electrons and holes are independent of time; therefore, the generation and recombination rates are equal, so we have

$$G_{n0} = G_{p0} = R_{n0} = R_{p0}$$

Excess Carrier Generation and Recombination

Electrons in the valence band may be excited into the conduction band when, for example, high-energy photons are incident on a semiconductor. When this happens, not only is an electron created in the conduction band, but a hole is created in the valence band; thus, an electron-hole pair is generated. The additional electrons and holes created are called excess electrons and excess holes. The excess electrons and holes are generated by an external force at a particular rate. Let g_n' be the generation rate of excess electrons and g_p' be that of excess holes. These generation rates also have units of $\#/\mathrm{cm}^3 \cdot \mathrm{s}$. For the direct band-to band generation, the excess electrons and holes are also created in pairs, so we must have

$$g_n' = g_p'$$

Relevant notation used in Chapter 6

Symbol	Definition
n_0, p_0	Thermal-equilibrium electron and hole concentrations (independent of time and also usually position)
n, p	Total electron and hole concentrations (may be functions of time and/or position)
$\delta n = n - n_0$ $\delta p = p - p_0$	Excess electron and hole concentrations (may be functions of time and/or position)
g'_n, g'_p	Excess electron and hole generation rates
R'_n, R'_p	Excess electron and hole recombination rates
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes

Excess Carrier Generation and Recombination

In a nonequilibrium condition,

$$np \neq n_0 p_0 = n_i^2$$

Now, we have

$$n(t) = n_0 + \delta n(t)$$

and

$$p(t) = p_0 + \delta p(t)$$

Excess Carrier Generation and Recombination

The recombination rate-which is defined as a positive quantity-of excess minority carrier electrons can be written, as

$$R'_n = \frac{-d(\delta n(t))}{dt} = +\alpha_r p_0 \delta n(t) = \frac{\delta n(t)}{\tau_{n0}}$$

For the direct band-to-band recombination, the excess majority carrier holes recombine at the same rate, so that for the p-type material

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{n0}}$$

Excess Carrier Generation and Recombination

In the case of an n-type material ($n_0 \gg p_0$) under low-level injection ($\delta n(t) \ll n_0$), the decay of minority carrier holes occurs with a time constant $\tau_{p0} = (\alpha_r n_0)^{-1}$, where τ_{p0} is also referred to as the excess minority carrier lifetime. The recombination rate of the majority carrier electrons will be the same as that of the minority carrier holes, so we have

$$R'_n = R'_p = \frac{\delta n(t)}{\tau_{p0}}$$

The generation rates of excess carriers are not functions of electron or hole concentrations. In general, the generation and recombination rates may be functions of the space coordinates and time.

1. Nonequilibrium Excess Carriers in Semiconductors
   - Carrier Generation and Recombination
   - Characteristics of Excess Carriers
   - Quasi-Fermi Energy Levels
   - Surface Effects

Time-Dependent Diffusion Equations

For p-type: $D_n \frac{\mathrm{d}^2 n}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x}\right) + g_n - \frac{n}{\tau_{n0}} = \frac{\mathrm{d}n}{\mathrm{d}t}$

For n-type: $D_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x}\right) + g_p - \frac{p}{\tau_{p0}} = \frac{\mathrm{d}p}{\mathrm{d}t}$

where g_n and g_p are the total generation rates. For homogeneous semiconductor, $n(x) = n_0 + \delta n(x)$, the equation can be simplified to

$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x}\right) + g'_n - \frac{\delta n}{\tau_{nt}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

$$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x}\right) + g'_p - \frac{\delta p}{\tau_{pt}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

Common equation simplifications

Specification	Effect	
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0,$	$\frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0,$	$D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E} \frac{\partial(\delta n)}{\partial x} = 0,$	$\mathrm{E} \frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$	
No excess carrier recombination	$\frac{\delta n}{\tau_{n0}} = 0,$	$\frac{\delta p}{\tau_{p0}} = 0$

low injection

For a p-type semiconductor under low injection then becomes

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$

The parameter δn is the excess minority carrier electron concentration, the parameter τ_{n0} is the minority carrier lifetime under low injection, and the other parameters are the usual minority carrier electron parameters.

Similarly, for an extrinsic n-type semiconductor under low injection, the ambipolar transport equation becomes

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t}$$

The parameter δp is the excess minority carrier hole concentration, the parameter τ_{p0} is the minority carrier hole lifetime under low injection, and the other parameters are the usual minority carrier hole parameters.

Example 1

Consider an infinitely large, homogeneous n-type semiconductor with zero applied electric field. Assume that at time $t = 0$, a uniform concentration of excess carriers exists in the crystal, but assume that $g' = 0$ for $t > 0$. If we assume that the concentration of excess carriers is much smaller than the thermal-equilibrium electron concentration, then the lowinjection condition applies. Calculate the excess carrier concentration as a function of time for $t \geq 0$.

Example 1 Solution

For the n-type semiconductor, we need to consider the ambipolar transport equation for the minority carrier holes. The equation is

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial(\delta p)}{\partial x} + g' - \frac{\delta p}{\tau_{p0}} = \frac{\partial(\delta p)}{\partial t}$$

We are assuming a uniform concentration of excess holes so that $\partial^2(\delta p)/\partial x^2 = \partial(\delta p)/\partial x = 0$. For $t > 0$, we are also assuming that $g' = 0$. Equation reduces to

$$\frac{d(\delta p)}{dt} = -\frac{\delta p}{\tau_{p0}}$$

Since there is no spatial variation, the total time derivative may be used. At low injection, the minority carrier hole lifetime, τ_{p0}, is a constant. The left-side of Equation is the time rate of change of δp and the right-side of the equation is the recombination rate. The solution to Equation is

Example 1 Solution

$$\delta p(t) = \delta p(0)e^{-t/\tau_{p0}}$$

where $\delta p(0)$ is the uniform concentration of excess carriers that exists at time $t = 0$. The concentration of excess holes decays exponentially with time, with a time constant equal to the minority carrier hole lifetime.

From the charge-neutrality condition, we have that $\delta n = \delta p$, so the excess electron concentration is given by

$$\delta n(t) = \delta p(t) = \delta p(0)e^{-t/\tau_{p0}}$$

Example 2

Again consider an infinitely large, homogeneous n-type semiconductor with a zero applied electric field. Assume that, for $t < 0$, the semiconductor is in thermal equilibrium and that, for $t \geq 0$, a uniform generation rate exists in the crystal. Calculate the excess carrier concentration as a function of time assuming the condition of low injection.

Example 2 Solution

The condition of a uniform generation rate and a homogeneous semiconductor again implies that $\partial^2(\delta p)/\partial x^2 = \partial(\delta p)/\partial x = 0$ in last Equation. The equation, for this case, reduces to

$$g' - \frac{\delta p}{\tau_{p0}} = \frac{d(\delta p)}{dt}$$

The solution to this differential equation is

$$\delta p(t) = g' \tau_{p0} \left(1 - e^{-t/\tau_{p0}}\right)$$

Example 3

Consider a p-type semiconductor that is homogeneous and infinite in extent. Assume a zero applied electric field. For a one-dimensional crystal, assume that excess carriers are being generated at $x = 0$ only, as indicated in Figure. The excess carriers being generated at $x = 0$ will begin diffusing in both the $+x$ and $-x$ directions. Calculate the steady-state excess carrier concentration as a function of x.

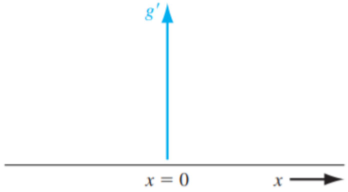

Figure: Steady-state generation rate

Example 3 Solution

The equation for excess minority carrier electrons is written as

$$D_n \frac{\partial^2(\delta n)}{\partial x^2} + \mu_n \mathrm{E} \frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$

From our assumptions, we have $\mathrm{E} = 0, g' = 0$ for $x \neq 0$, and $\partial(\delta n)/\partial t = 0$ for steady state. Assuming a one-dimensional crystal, Equation reduces to

$$D_n \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{\tau_{n0}} = 0$$

Dividing by the diffusion coefficient, Equation may be written as

$$\frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{D_n \tau_{n0}} = \frac{d^2(\delta n)}{dx^2} - \frac{\delta n}{L_n^2} = 0$$

Example 3 Solution

where we have defined $L_n^2 = D_n \tau_{n0}$. The parameter L_n has the unit of length and is called the minority carrier electron diffusion length. The general solution to Equation is

$$\delta n(x) = Ae^{-x/L_n} + Be^{x/L_n}$$

As the minority carrier electrons diffuse away from $x = 0$, they will recombine with the majority carrier holes. The minority carrier electron concentration will then decay toward zero at both $x = +\infty$ and $x = -\infty$. The solution to Equation may then be written as

$$\delta n(x) = \delta n(0)e^{-x/L_n} \quad x \geq 0$$

and

$$\delta n(x) = \delta n(0)e^{+x/L_n} \quad x \leq 0$$

where $\delta n(0)$ is the value of the excess electron concentration at $x = 0$. The steady-state excess electron concentration decays exponentially with distance away from the source at $x = 0$.

Quasi-Fermi Energy Levels

If excess carriers are created in a semiconductor, we are no longer in thermal equilibrium and the Fermi energy is strictly no longer defined. However, we may define a quasi-Fermi level for electrons and a quasi-Fermi level for holes that apply for nonequilibrium. If δn and δp are the excess electron and hole concentrations, respectively, we may write

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

and

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

where E_{Fn} and E_{Fp} are the quasi-Fermi energy levels for electrons and holes, respectively. The total electron concentration and the total hole concentration are functions of the quasi-Fermi levels.

Example 4

Consider an n-type semiconductor at $T = 300\ \mathrm{K}$ with carrier concentrations of $n_0 = 10^{15}\ \mathrm{cm}^{-3}$, $n_i = 10^{10}\ \mathrm{cm}^{-3}$, and $p_0 = 10^5\ \mathrm{cm}^{-3}$. In nonequilibrium, assume that the excess carrier concentrations are $\delta n = \delta p = 10^{13}\ \mathrm{cm}^{-3}$.

Example 4 Solution

The Fermi level for thermal equilibrium can be determined from Equation. We have

$$E_F - E_{Fi} = kT \ln\left(\frac{n_0}{n_i}\right) = 0.2982\mathrm{eV}$$

We can use Equation to determine the quasi-Fermi level for electrons in nonequilibrium. We can write

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{n_0 + \delta n}{n_i}\right) = 0.2984\mathrm{eV}$$

Equation can be used to calculate the quasi-Fermi level for holes in nonequilibrium. We can write

$$E_{Fi} - E_{Fp} = kT \ln\left(\frac{p_0 + \delta p}{n_i}\right) = 0.179\mathrm{eV}$$

We may note that the quasi-Fermi level for electrons is above E_{Fi} while the quasi-Fermi level for holes is below E_{Fi}.

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Surface Effects

In all previous discussions, we have implicitly assumed the semiconductors were infinite in extent; thus, we were not concerned with any boundary conditions at a semiconductor surface. In any real application of semiconductors, the material is not infinitely large and therefore surfaces do exist between the semiconductor and an adjacent medium.

Surface Effects

The recombination rate of excess carriers in the bulk, given by

$$R = \frac{\delta p}{\tau_{p0}} \equiv \frac{\delta p_B}{\tau_{p0}}$$

where δp_B is the concentration of excess minority carrier holes in the bulk material. We may write a similar expression for the recombination rate of excess carriers at the surface as

$$R_s = \frac{\delta p_s}{\tau_{p0s}}$$

where δp_s is the excess minority carrier hole concentration at the surface and τ_{p0s} is the excess minority carrier hole lifetime at the surface.

Surface Effects

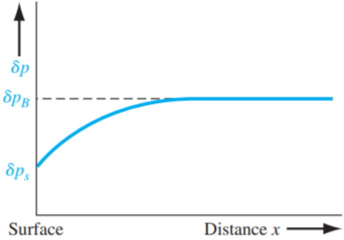

Figure: Steady-state excess hole concentration versus distance from a semiconductor surface

END

Thanks