VE320 – Summer 2022

Introduction to Semiconductor Devices

Instructor: Yaping Dan （但亚平）
yaping.dan@sjtu.edu.cn

Chapter 4 The Semiconductor in Equilibrium

Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

① ΔE # states

② ΔE $f_F(E)$

③ ΔE # state $\times f_F(E)$

$$\Delta N_e = g(E) \cdot f(E) \cdot \Delta E$$

$$N_e = \int_{E_c}^{+\infty} g(E) f(E) dE$$

Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

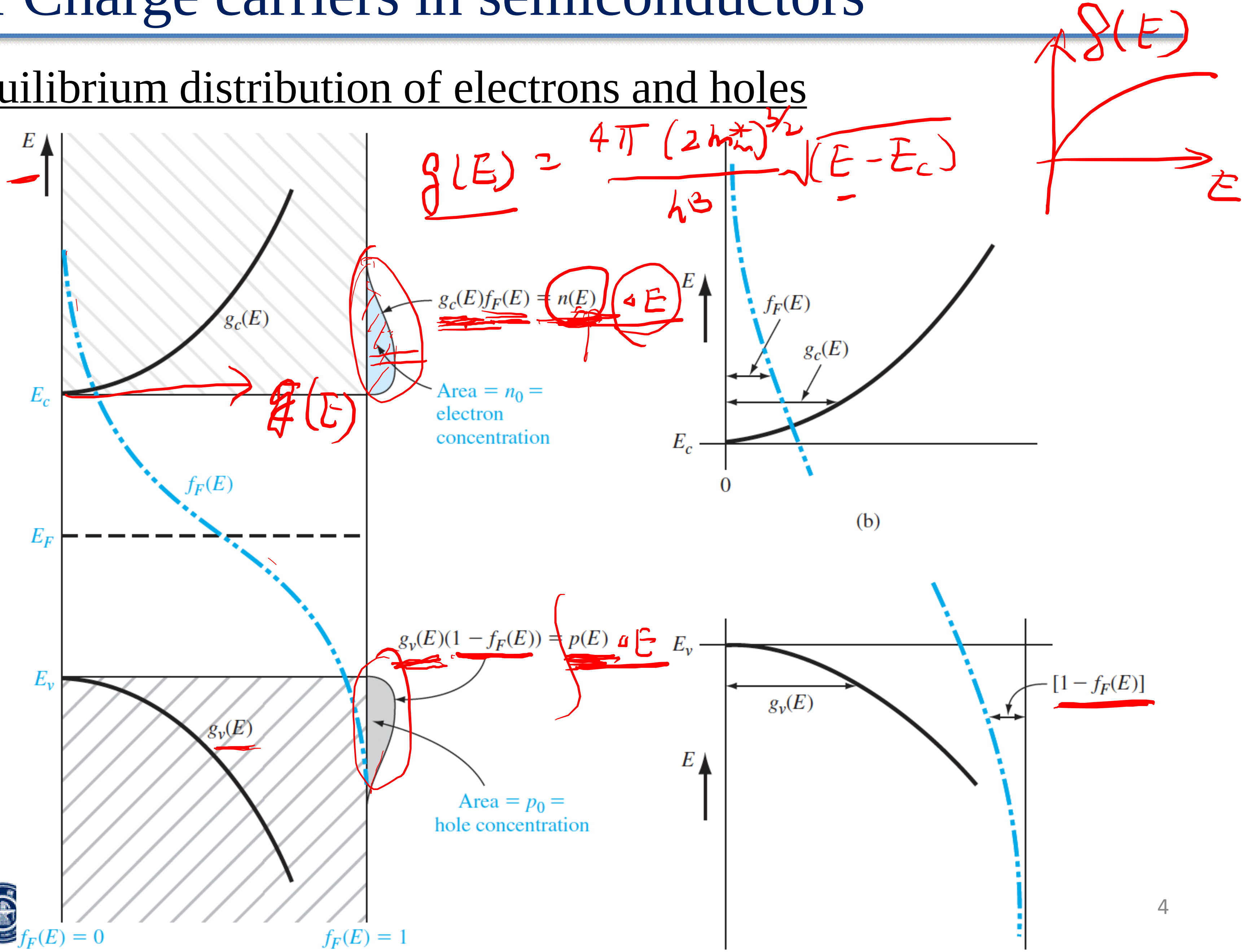
4.5 Charge neutrality

4.6 Position of Fermi energy level

4.1 Charge carriers in semiconductors

Equilibrium distribution of electrons and holes

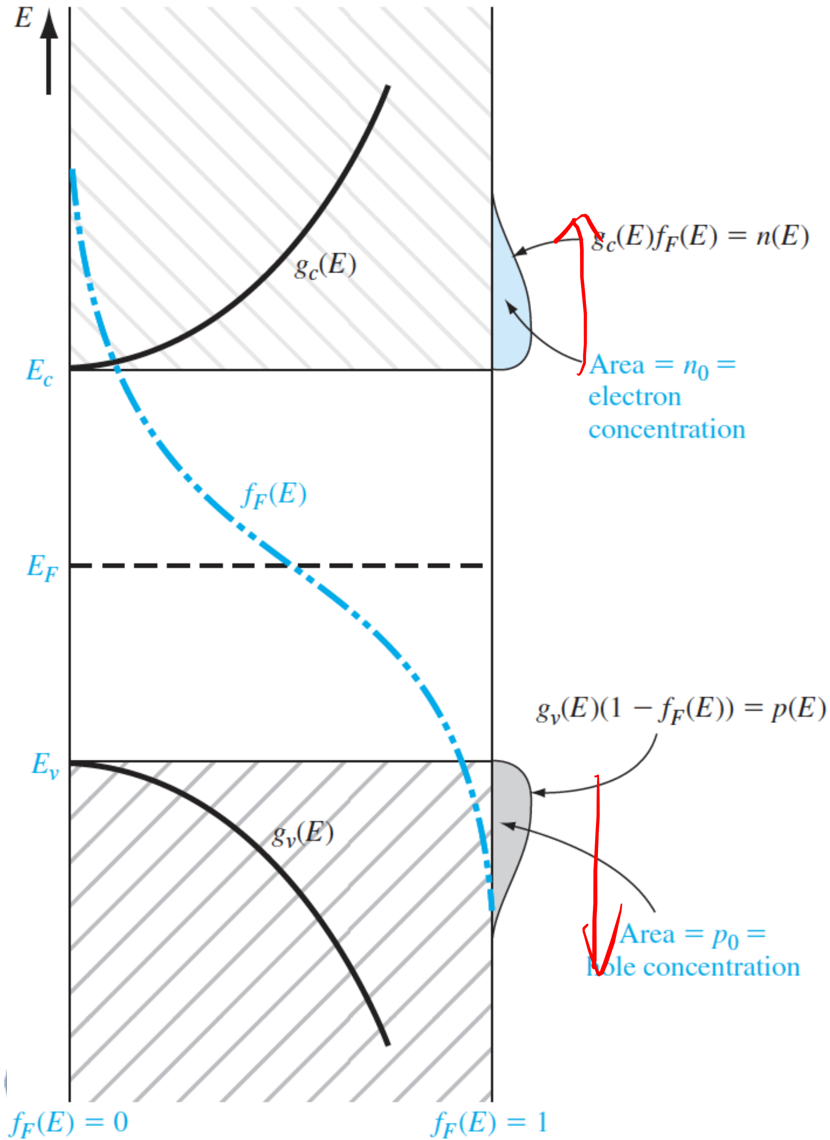
$$g(E) = \frac{4\pi (2m_n^*)^{3/2}}{h^3}\sqrt{(E-E_c)}$$



4.1 Charge carriers in semiconductors

The n_0 and p_0 equations

$\int g(E) \cdot f_F(E) \cdot dE$



$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1 - f_F(E)] dE$$

4.1 Charge carriers in semiconductors

The n_0 and p_0 equations

4.1 Charge carriers in semiconductors

<u>The n_0 and p_0 equations</u>

$$\frac{E-E_F}{kT} > 3$$

$$n_0 = \int_{E_c}^{+\infty} g_c(E)\, f_F(E)\, dE$$

$$= \int_{E_c}^{+\infty} 4\pi \frac{(2m_n^*)^{3/2}}{h^3} (E-E_c)^{1/2} \cdot \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$d\eta = dE/kT$$

$$E - E_F = E - E_c + E_c - E_F = kT\eta - kT\eta_F$$

$$\eta = \frac{E-E_c}{kT} \qquad = 4\pi \frac{(2m_n^*)^{3/2}}{h^3} \int_0^{+\infty} \frac{\eta^{1/2}}{1+\exp(\eta-\eta_F)} d\eta$$

$$\exp(\eta-\eta_F) \gg 10$$

$$E - E_F > 3kT$$

$$\eta_F = \frac{E_F - E_c}{kT} \qquad = 4\pi \frac{(2m_n^*)^{3/2}}{h^3} \int_0^{+\infty} \eta^{1/2} \exp(\eta_F - \eta)\, d\eta$$

4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$ (2nd time approximation)

$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

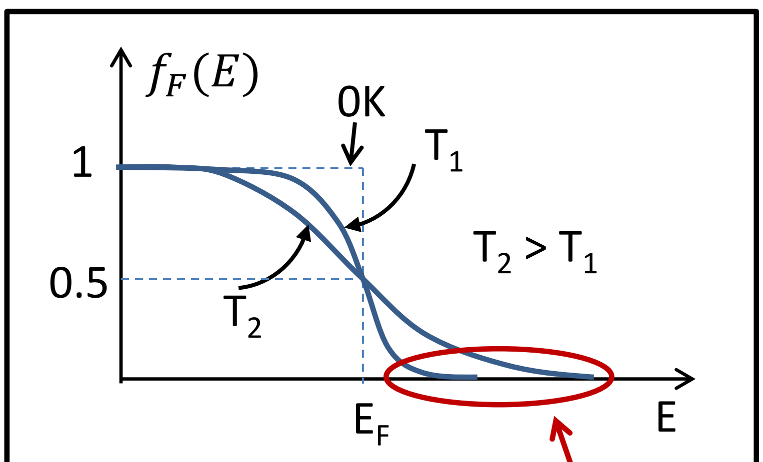
Fermi-Dirac Distribution

$$\Rightarrow$$

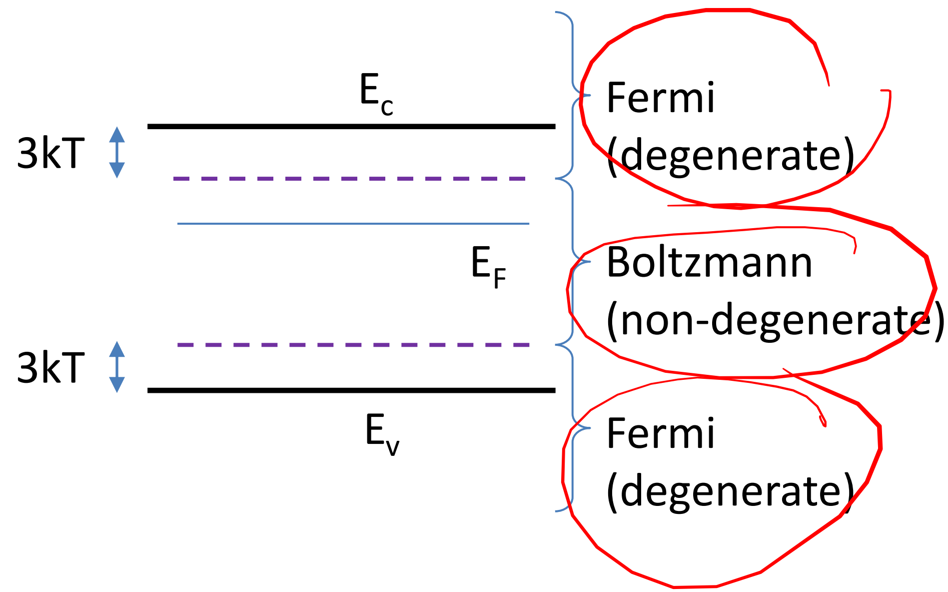
$$f_F(E) = \exp(\frac{E_F - E}{kT})$$

Boltzmann Distribution

Fermi-Dirac



Boltzmann



4.1 Charge carriers in semiconductors

$$\boxed{if\ \ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}}$$

Fermi-Dirac Distribution

$$\boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Boltzmann Distribution

4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}}$$

Fermi-Dirac Distribution

$$\Rightarrow$$

$$\boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Boltzmann Distribution

4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}}$$

Fermi-Dirac Distribution

$$\Rightarrow$$

$$\boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

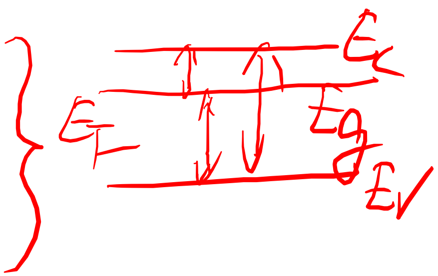
Boltzmann Distribution

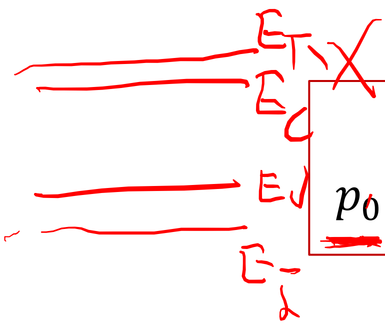
4.1 Charge carriers in semiconductors

$$if\ \ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow \boxed{E - E_F > 3kT}$$

electron concentration in conduction band

$$n_0 = \frac{2(2\pi m_n^* kT)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_F - E_c}{kT}\right) = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$



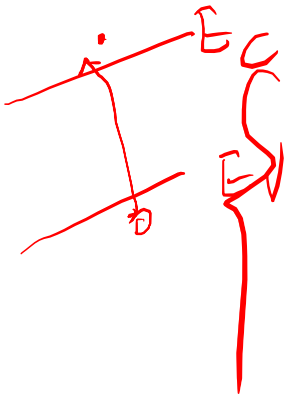


$$p_0 = \frac{2(2\pi m_p^* kT)^{\frac{3}{2}}}{h^3} \exp\left(\frac{E_v - E_F}{kT}\right) = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

4.1 Charge carriers in semiconductors

The intrinsic carrier concentration

$n_0 p_0 = N_c N_v \exp[-\frac{E_g}{kT}]$



$$n_0 \times p_0 = {n_i}^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v} \exp(-\frac{E_g}{2kT})$$

$$n_0 = N_c \exp(\frac{E_F - E_c}{kT}) \qquad p_0 = N_v \exp(\frac{E_v - E_F}{kT})$$

$N_c \sim 10^{19} cm^{-3}$ 300k

$N_v \sim 10^{19} cm^{-3}$ 300k

for Si

- The equations are universal for doped and undoped semiconductors

Check your understanding

Problem Example #1

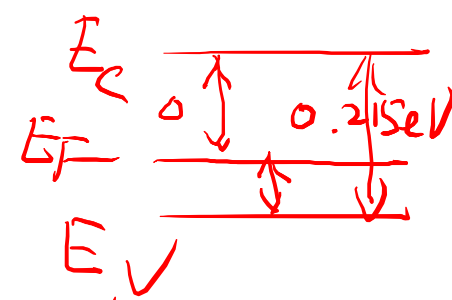
$KT(T=300K) = 0.0259\,eV$

$KT(T=200K) = 0.0259 \times \frac{200}{300}$

Determine the thermal-equilibrium concentrations of electrons and holes in silicon at T = 300K if the Fermi energy level E_F is 0.215eV above the valence band energy E_V. $N_C = 2.8 \times 10^{19}$ cm^{-3} and $N_V = 1.04 \times 10^{19}$ cm^{-3}. $E_g = 1.12$ eV for Si.

$\boxed{E_c - E_V = 1.12\,eV}$

$E_c - E_F = 1.12\,eV - 0.215\,eV = \cancel{0.97}\ \ 0.905\,eV$



$$n_0 = N_c \exp\left(\frac{E_F - E_c}{KT}\right) = 2.8\times10^{19} \exp\left(\frac{-0.905\,eV}{0.0259\,eV}\right) = 18707\ cm^{-3}$$

$$p_0 = N_v \exp\left(\frac{-0.215\,eV}{KT}\right) = 1.04\times10^{19} \exp\left(\frac{-0.215}{0.0259}\right) = 2.58\times10^{15}\ cm^{-3}$$

4.1 Charge carriers in semiconductors

The intrinsic carrier concentration

300K

Table 4.1 | Effective density of states function and density of states effective mass values

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_p^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

μ

Table 4.2 | Commonly accepted values of n_i at $T = 300$ K

Silicon	$n_i = 1.5 \times 10^{10}$ cm^{-3}
Gallium arsenide	$n_i = 1.8 \times 10^{6}$ cm^{-3}
Germanium	$n_i = 2.4 \times 10^{13}$ cm^{-3}

4.1 Charge carriers in semiconductors

The intrinsic carrier concentration

$$n_i = \sqrt{N_c \cdot N_v}\, \exp\left(-\frac{E_g}{2kT}\right) \qquad (kT)^{3/2}$$

$$N_c \sim (kT)^{3/2}$$

$$N_v \sim (kT)^{3/2}$$

$$\ln n_i = \ln\sqrt{N_c N_v} - \frac{E_g \times 1000}{2kT \; 1000}$$

$$= \ln\sqrt{N_c N_v} - \frac{E_g}{2k\times 1000} \times \frac{1000}{T}$$

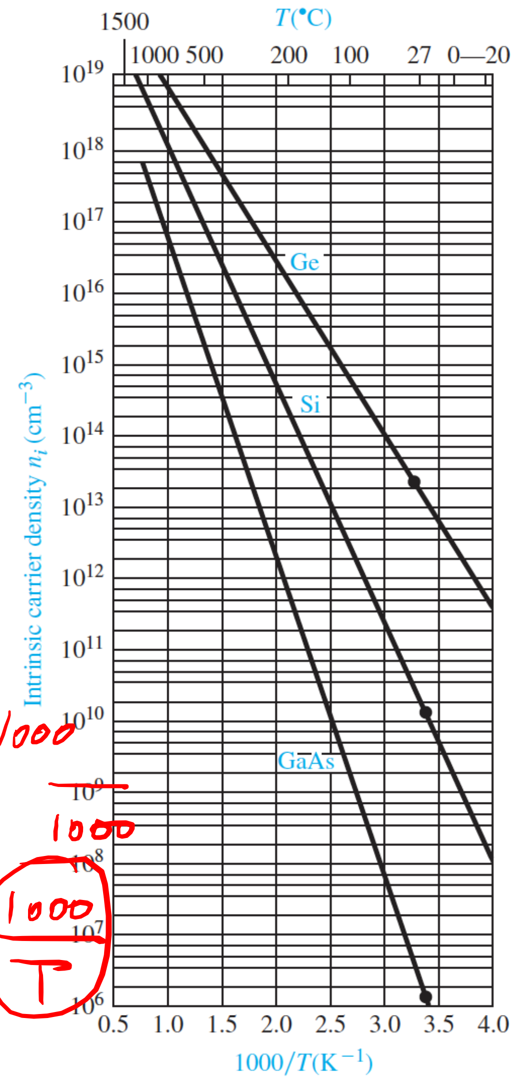


Figure 4.2 | The intrinsic carrier concentration of Ge, Si, and GaAs as a function of temperature.
(From Sze [14].)

Check your understanding

Problem Example #2

Calculate the intrinsic carrier concentration in silicon at T=250K and at 400K.

$$kT\Big|_{T=250K} = 0.0259 \times \frac{250}{300} = 0.0259 \times \frac{5}{6} = 0.0215$$

0.0043×5

215

$$kT\Big|_{T=400k} = 0.0259 \times \frac{400}{300} = 0.00863 \times 4 = 0.0345$$

0.03452

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = 4.90 \times 10^{5}\ cm^{-3} \quad @\ 250k$$

E_g: 1.12

$$N_c = 2.8 \times 10^{19} \times (kT)^{3/2} \times \frac{(250)^{3/2}}{(300)^{3/2}}$$

$$N_v = 1.04 \times 10^{19} \times \frac{(250)^{3/2}}{(300)^{3/2}}$$

4.1 Charge carriers in semiconductors

The intrinsic Fermi-level position

$n_0 = p_0 = n_i$

$\boxed{\text{undoped}}$

$N_c = 4\pi \left(\frac{2m_n^* kT}{h^2}\right)^{3/2}$

$N_v = 4\pi \left(\frac{2m_p^* kT}{h^2}\right)^{3/2}$

$n_0 = N_c \exp\left(\frac{E_{Fi} - E_c}{kT}\right) = N_v \exp\left(\frac{E_v - E_{Fi}}{kT}\right)$

$\frac{N_v}{N_c} = \left(\frac{m_p^*}{m_n^*}\right)^{3/2}$

$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$

$\ln N_c + \frac{E_{Fi} - E_c}{kT} = \ln N_v + \frac{E_v - E_{Fi}}{kT}$

$\frac{2E_{Fi}}{kT} = \ln N_v - \ln N_c + \frac{E_v + E_c}{kT}$

$E_{Fi} = \frac{1}{2} kT \ln \frac{N_v}{N_c} + \frac{E_v + E_c}{2}$

Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

4.2 Dopant atoms and energy levels

Qualitative description

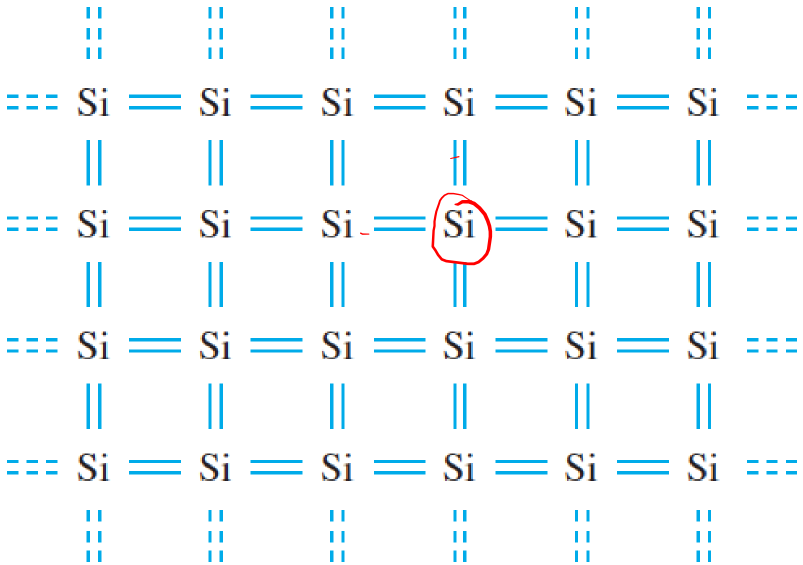


Figure 4.3 | Two-dimensional representation of the intrinsic silicon lattice.

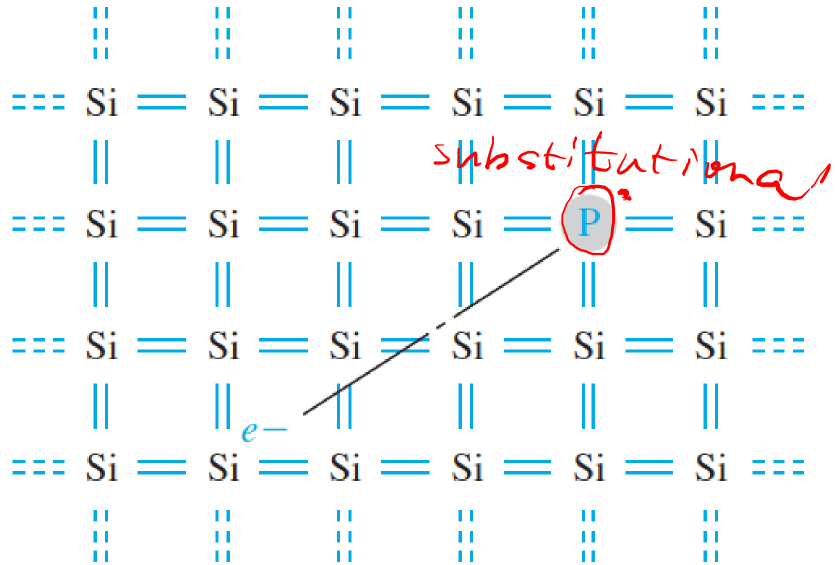
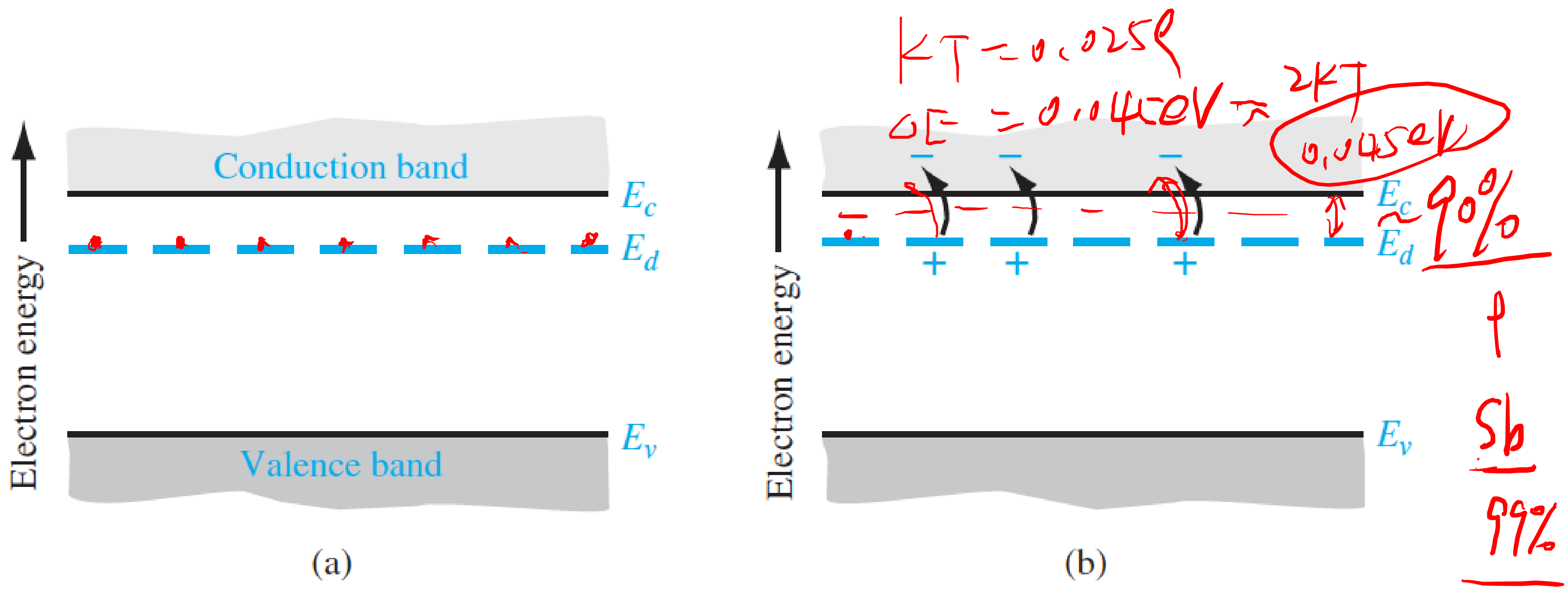


Figure 4.4 | Two-dimensional representation of the silicon lattice doped with a phosphorus atom.

4.2 Dopant atoms and energy levels

Qualitative description



4.2 Dopant atoms and energy levels

Qualitative description

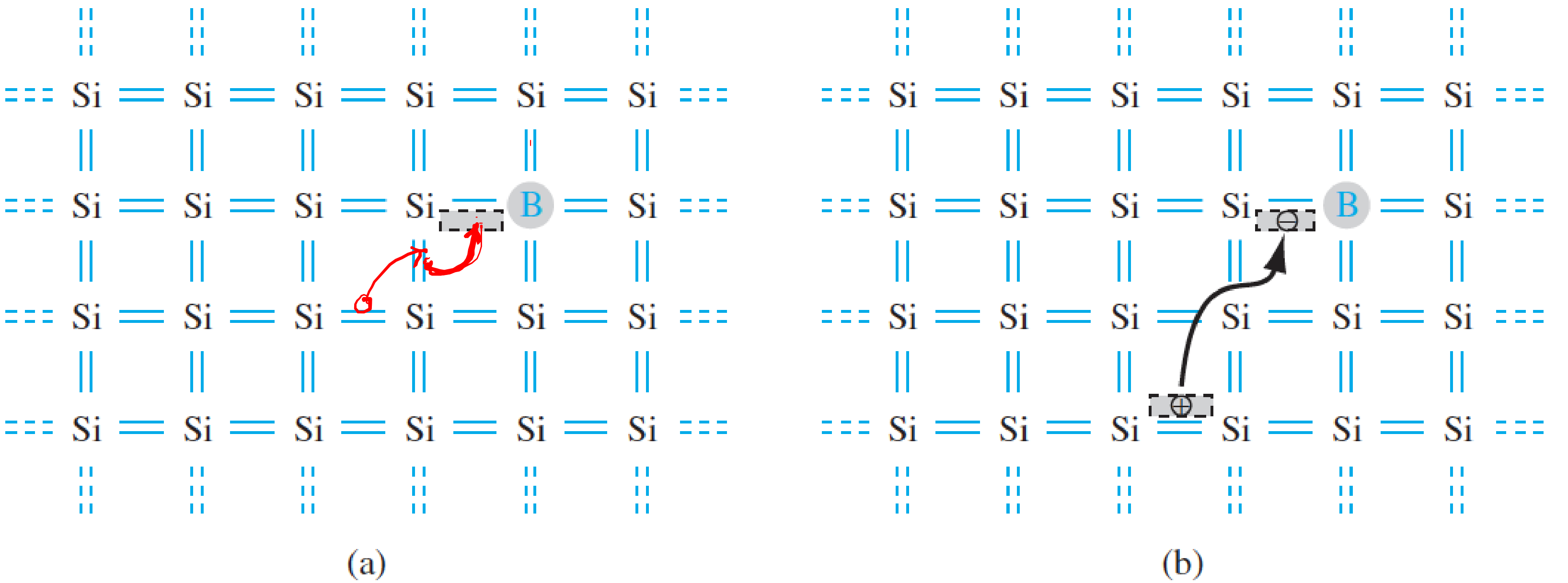


Figure 4.6 | Two-dimensional representation of a silicon lattice (a) doped with a boron atom and (b) showing the ionization of the boron atom resulting in a hole.

4.2 Dopant atoms and energy levels

Qualitative description

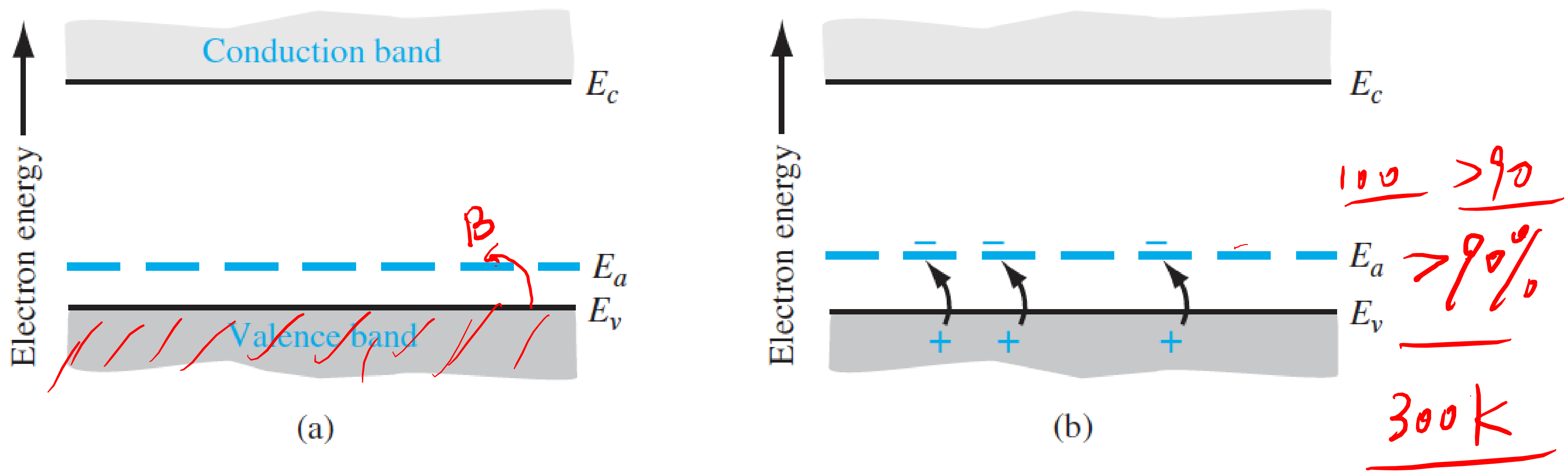
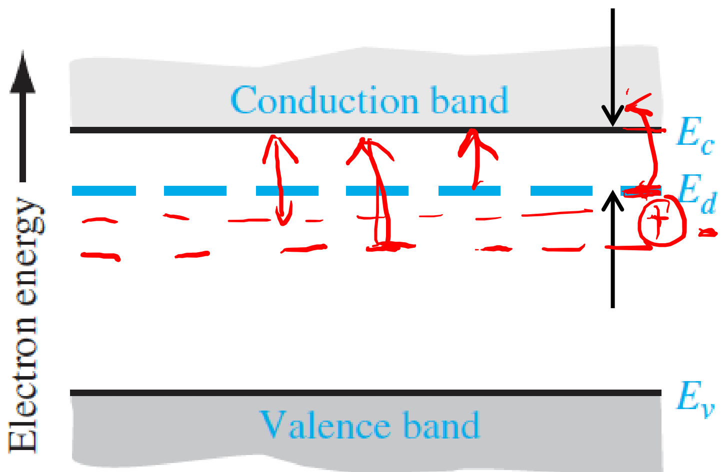


Figure 4.7 | The energy-band diagram showing (a) the discrete acceptor energy state and (b) the effect of an acceptor state being ionized.

4.2 Dopant atoms and energy levels

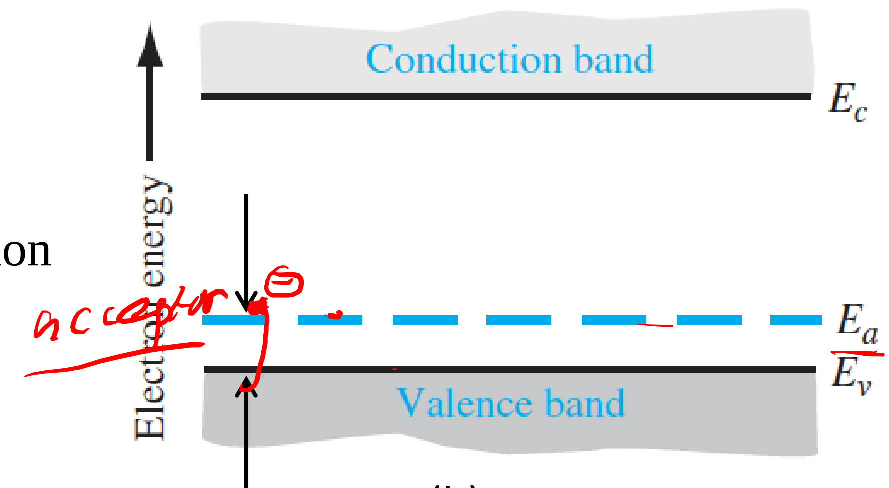
Ionization energy

activation energy based phenomena



(a)

$E_{ionization}$



(b)

$$E_{ionization} = E_c - E_d$$

$$E_{ionization} = E_a - E_v$$

4.2 Dopant atoms and energy levels

Ionization energy

Table 4.3 | Impurity ionization energies in silicon and germanium

	Ionization energy (eV)	
Impurity	**Si**	**Ge**
Donors		
Phosphorus	0.045	0.012 → 100%
Arsenic	0.05 ~2kT	0.0127 → 100%
Acceptors		
Boron	0.045	0.0104
Aluminum	0.06 ~2kT	0.0102

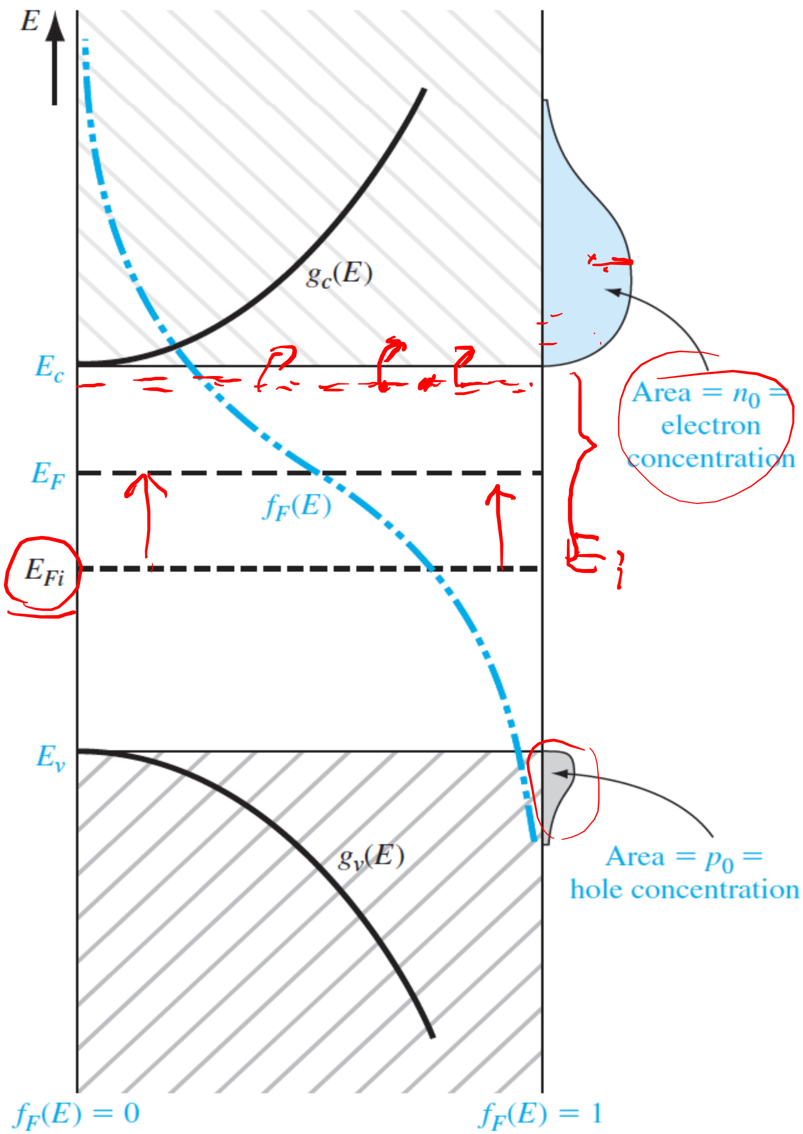
E_c

E_v

Outline

4.3 The extrinsic semiconductor

Equilibrium distribution of electrons and holes



n-type

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

n-type E_F in upper half bandgap

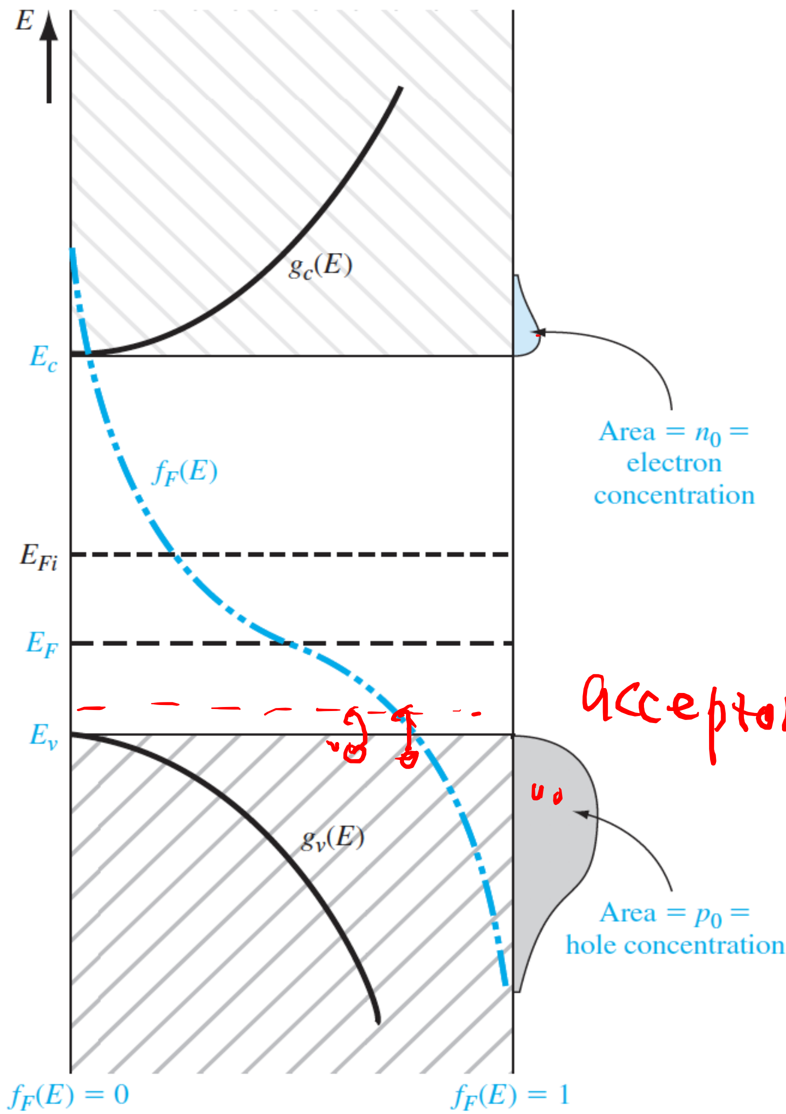
electrons are majority carriers

holes are minority carriers

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

4.3 The extrinsic semiconductor

Equilibrium distribution of electrons and holes



$$n_0 = N_c \exp(\frac{E_F - E_c}{kT})$$

p-type holes are majority
electrons are minority
E_F is lower half bandgap

$$p_0 = N_v \exp(\frac{E_v - E_F}{kT})$$

4.3 The extrinsic semiconductor

<u>The $n_0 p_0$ product</u>

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \qquad p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$\underline{n_0} \cdot \underline{p_0} = N_c \cdot N_v \exp\left(\frac{E_v - E_c}{kT}\right) = \underline{\text{constant}(T)}$$

$$\underline{\text{doping}}$$

$$\underline{n_0 = p_0} \Rightarrow n_0 \cdot p_0 = n_i^2$$

$$\boxed{n_0 \cdot p_0 (n_0 \neq p_0) = n_0 \cdot p_0 (n_0 = p_0) = n_i^2(T)}$$

$$\boxed{n_0 \cdot p_0 = n_i^2} = (1.5 \times 10^{10})^2$$

4.3 The extrinsic semiconductor

E_i

The n_0p_0 product

E_{Fi}

$n_0 = n_i = N_c \exp\left(\frac{E_{Fi} - E_c}{kT}\right)$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \qquad p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$= N_c \exp\left(\frac{E_F - E_i + E_i - E_c}{kT}\right) = N_c \exp\left(\frac{E_F - E_i}{kT}\right) \exp\left(\frac{E_i - E_c}{kT}\right) = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

✓ $$n_0 = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \qquad p_0 = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

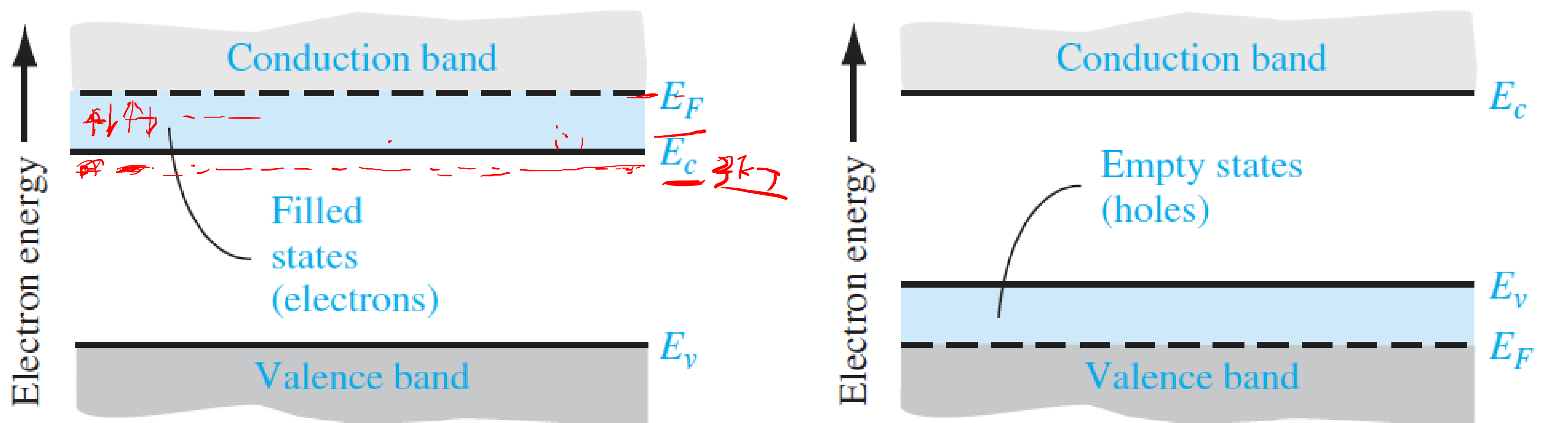
✓ $n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$

$$\boxed{n_i^2 = n_0 p_0}$$

$p_0 = \frac{n_i^2}{n_0}$

4.3 The extrinsic semiconductor

<u>Degenerate and nondegenerate semiconductors</u>



(a) (b)

Degenerate semiconductors:

- Extremely high doping concentration
- Fermi level in the band
- Electron cloud in dopants overlap,
- dopant energy level splitting

Check your understanding

Problem Example #3

Determine the thermal-equilibrium concentrations of electrons and holes in silicon at $T = 300K$ if the Fermi energy level E_F is 0.215eV above the valence band energy E_V. $N_V = 1.04 \times 10^{19}\ cm^{-3}$, $n_i = 1.5 \times 10^{10}\ cm^{-3}$.

Outline

4.1 Charge carriers in semiconductors

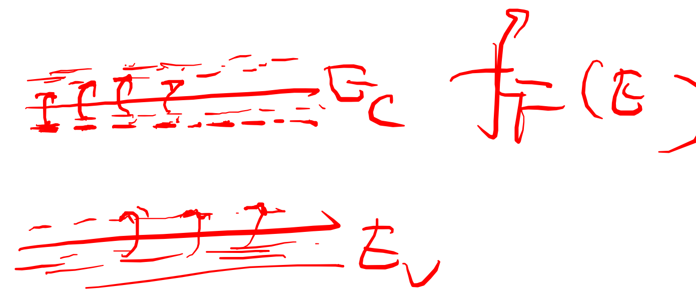
4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

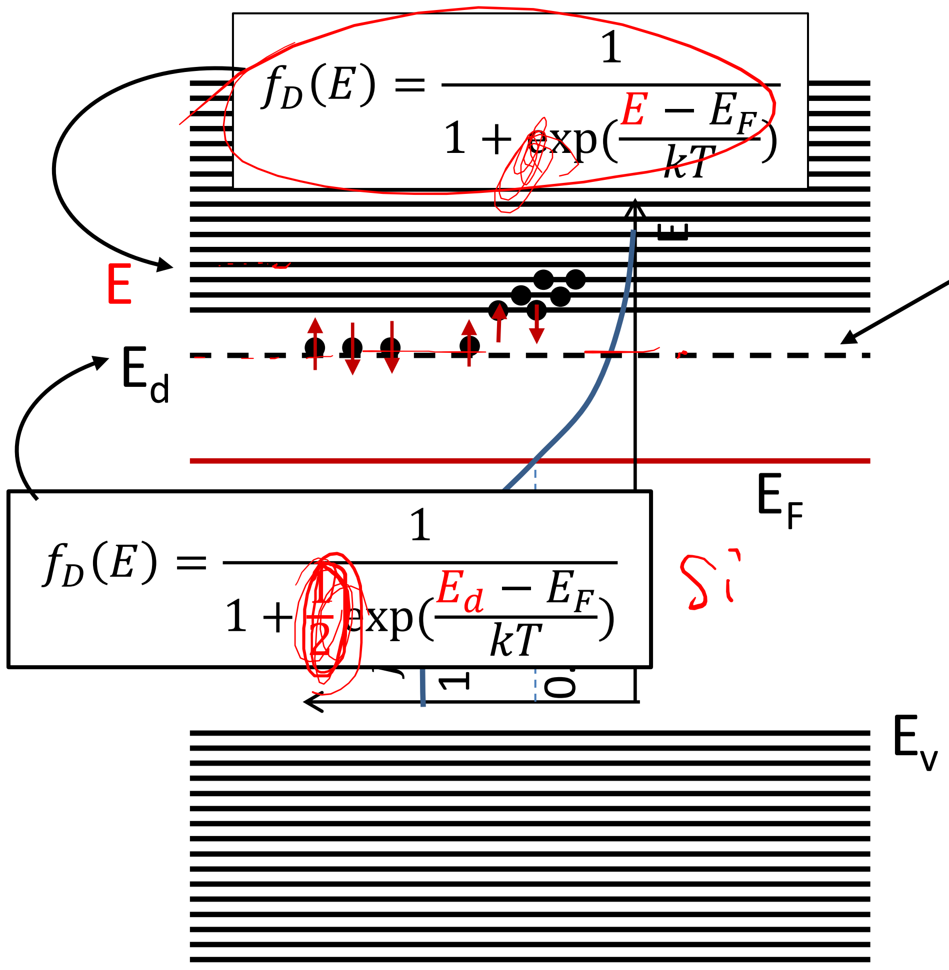
4.5 Charge neutrality

4.6 Position of Fermi energy level



4.4 Statistics of donors and acceptors

Probability function



$$f_D(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

$$f_D(E) = \frac{1}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

Given the concentration of donors is N_d

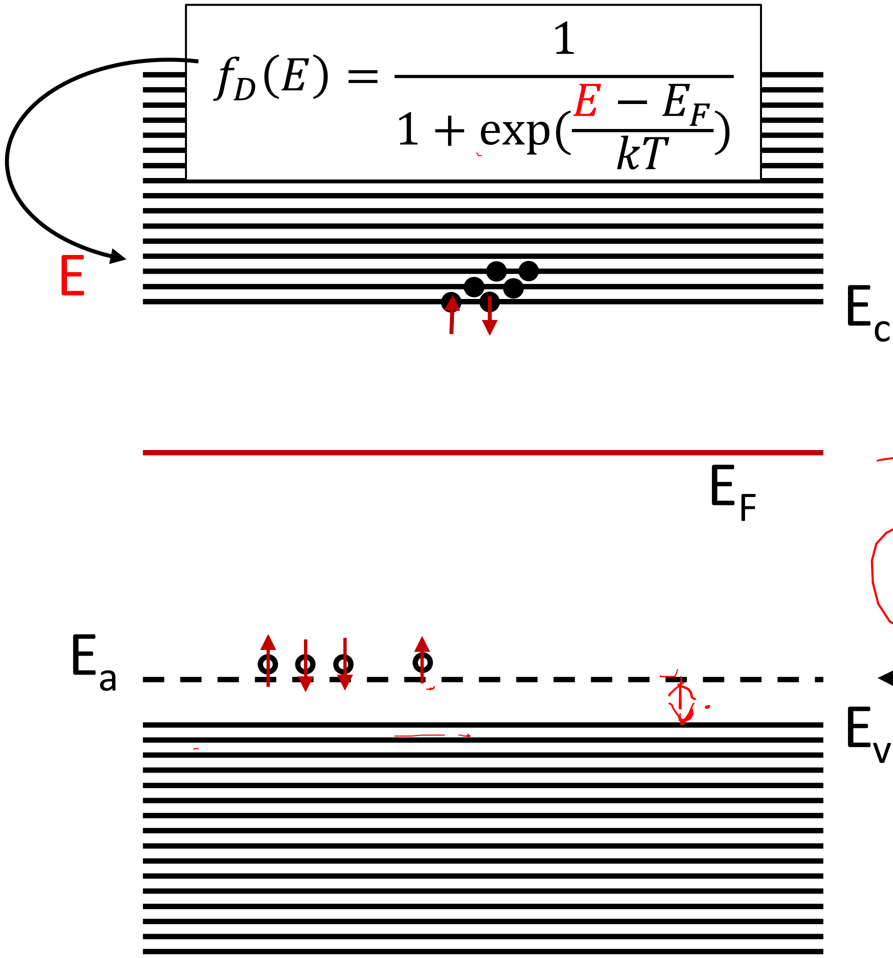
The concentration of electrons on these donors is n_d

$$n_d = N_d - N_d^+$$

$$= \frac{N_d}{1 + \frac{1}{2}\exp(\frac{E_d - E_F}{kT})}$$

4.4 Statistics of donors and acceptors

<u>Probability function</u>



$$f_D(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

The concentration of holes on these acceptors is n_d

$$p_a = N_a - N_a^-$$

$$= \frac{N_d}{1 + \frac{1}{g}\exp(\frac{E_d - E_F}{kT})}$$

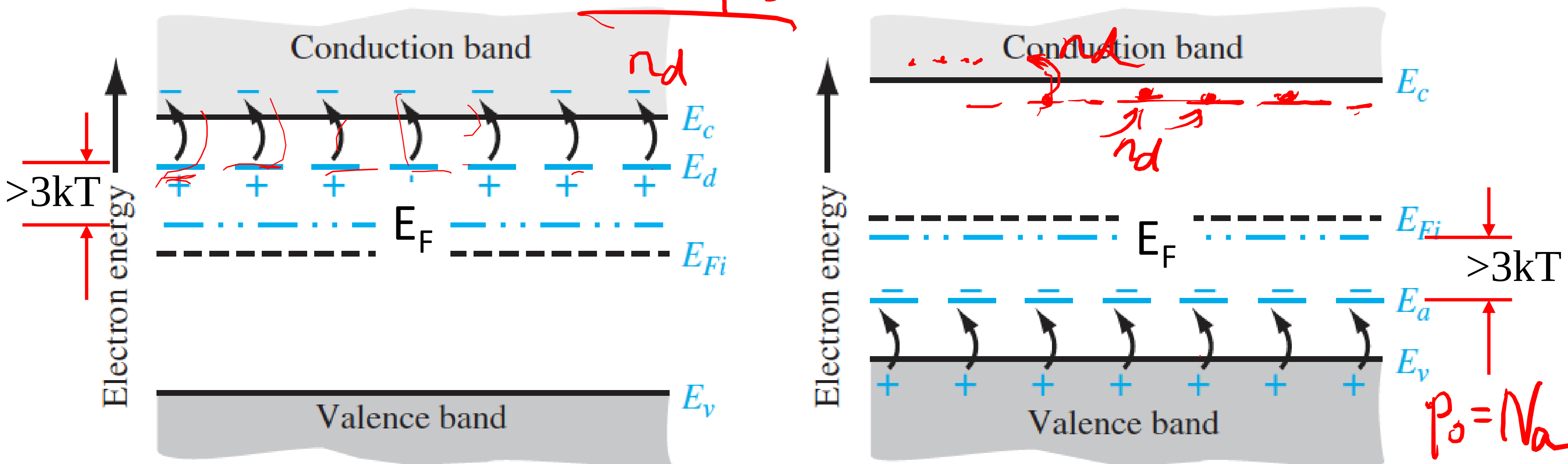
(g=4 for Si, GaAs …)

Given the concentration of acceptors is N_a

4.4 Statistics of donors and acceptors

Complete ionization

$n_0 = N_d$



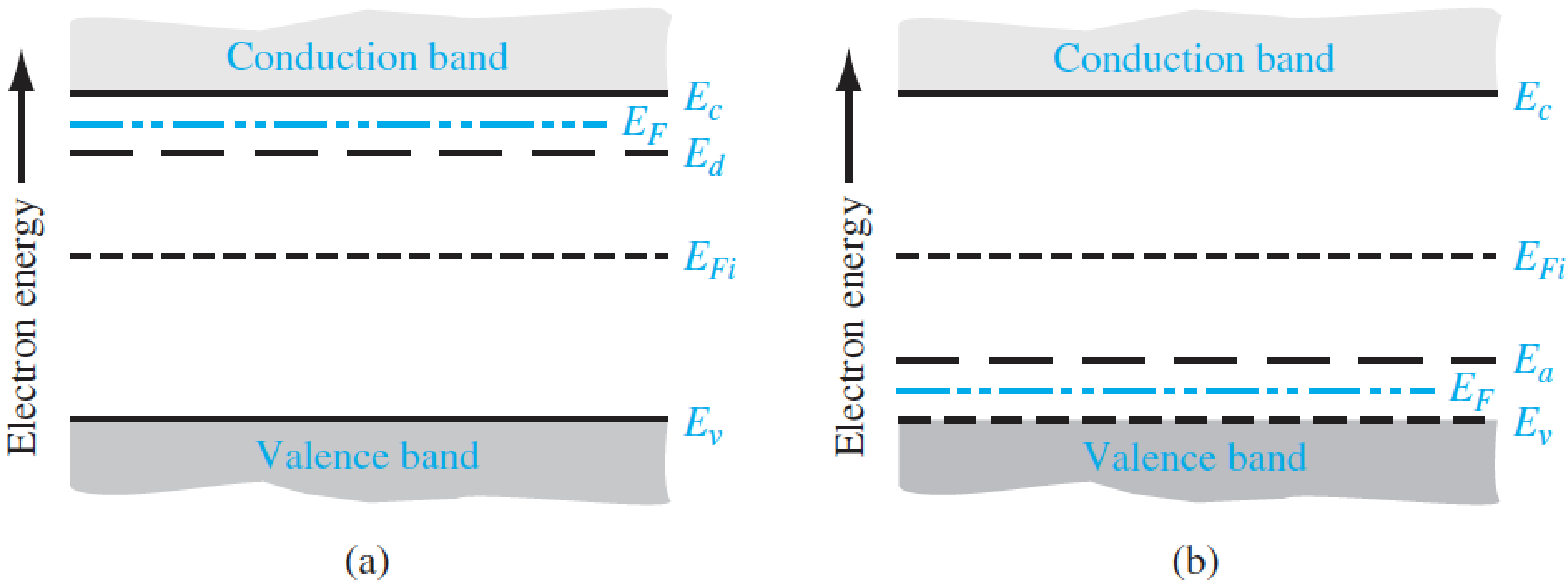
(a) (b)

$$n_d = N_d - N_d^+ = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} = 2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)$$

>10

4.4 Statistics of donors and acceptors

Complete freeze-out



$$n_d = N_d - N_d^+ = \frac{N_d}{1 + \underbrace{\frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)}_{\rightarrow 0}} \approx N_d$$

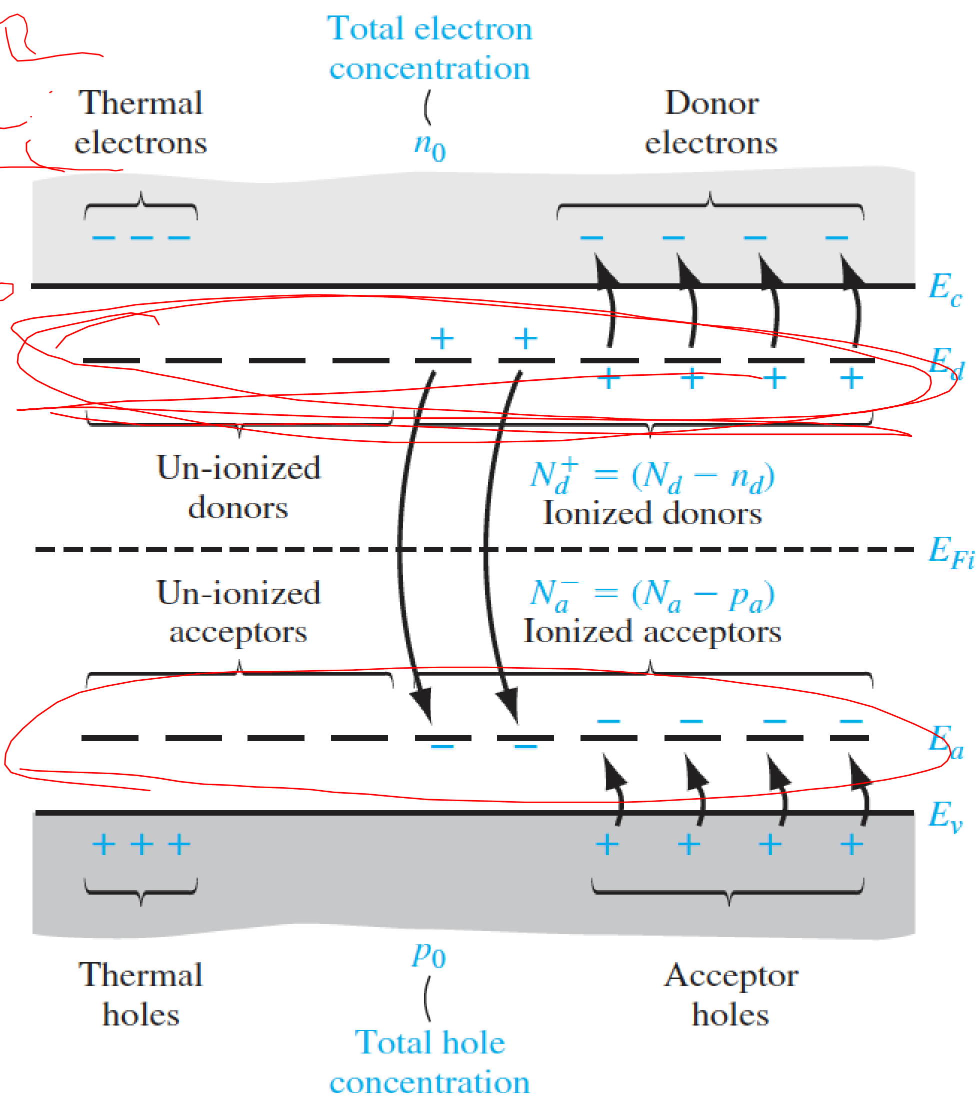
Outline

4.5 Charge neutrality

Compensated semiconductor

$n_0 = \frac{n_i^2}{p_0}$

- $N_d > N_a$:
  n-type compensated $(N_d - N_a)$
- $N_a > N_d$:
  p-type compensated $(N_a - N_d)$
- $N_d = N_a$:
  completely compensated,
  like intrinsic semiconductors



4.5 Charge neutrality

Equilibrium electron and hole concentration

E_c

E_v

Charge neutrality:

$$\boxed{n_0 + N_a^- = N_d^+ + p_0}$$

Or

$$\boxed{n_0 = N_d^+ - N_a^- + p_0}$$

Complete ionization

$$\boxed{n_0 = N_d - N_a + p_0}$$

$$n_0 p_0 = n_i^2$$

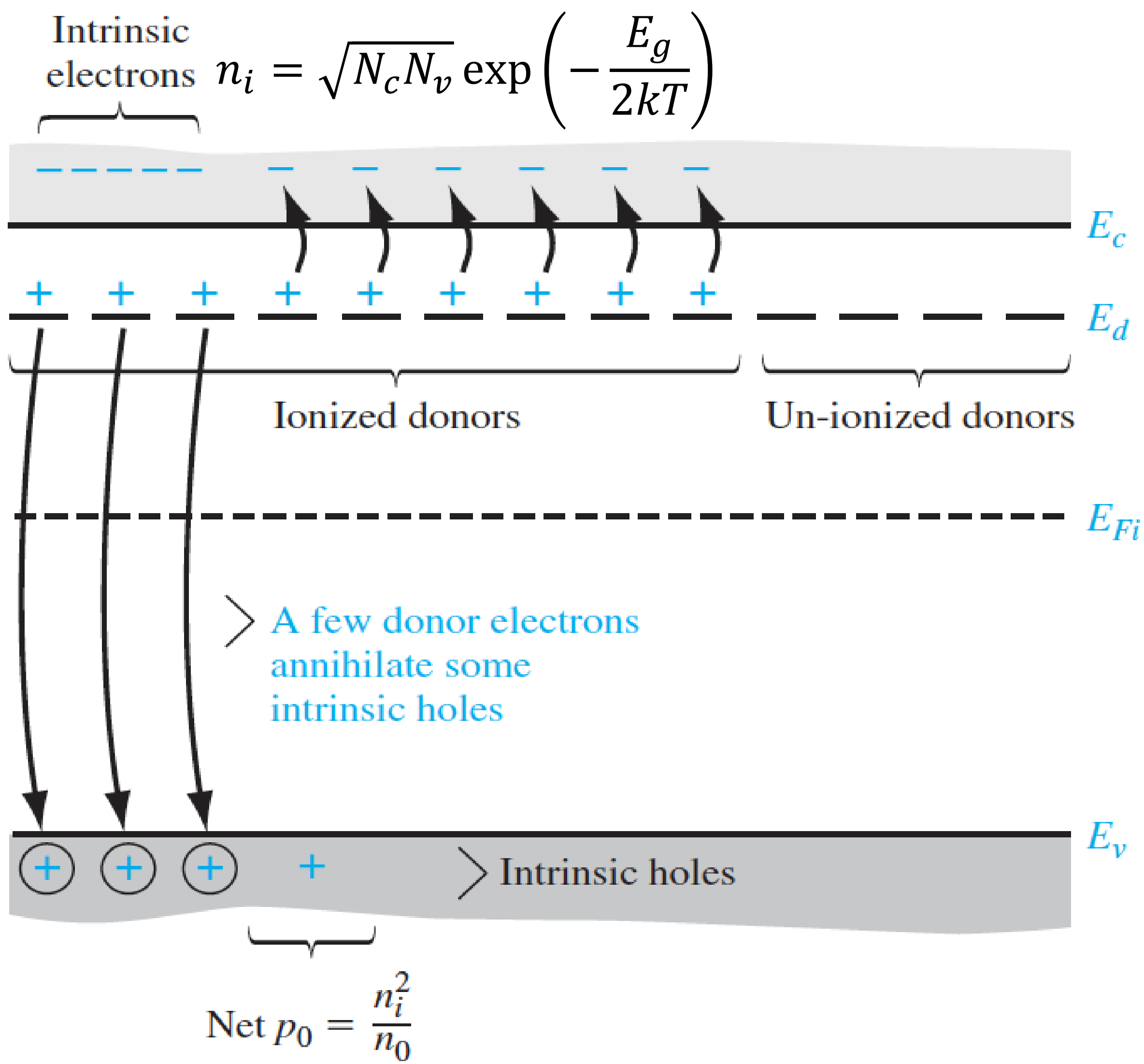
$$p_0 = \frac{n_i^2}{n_0}$$

$$N_d^+ = N_d$$

$$N_a^- = N_a$$

4.5 Charge neutrality

Equilibrium electron and hole concentration



4.5 Charge neutrality

Equilibrium electron and hole concentration

$$n_0 = \frac{N_d^+ + \sqrt{(N_d^+)^2 + 4n_i^2}}{2} \ (but\ N_d^+ \text{ unknown})$$

① $n_i >> N_d^+ \Rightarrow T$ *very high*

$$\boxed{n_0 = p_0 = n_i = \sqrt{N_c N_v}\exp(-\frac{E_g}{2kT})}$$

Check your understanding

Problem Example #4

Determine the thermal-equilibrium electron and hole concentrations in silicon at T = 300K for given doping concentrations on the assumption of 100% ioniztion of dopants. (a) Let $N_d = 10^{11}\text{cm}^{-3}$ and $N_a=0$. (b) Let $N_d = 10^{12}\text{cm}^{-3}$ and $N_a=0$. (c) Let $N_d = 5 \times 10^{15}\ \text{cm}^{-3}$ and $Na = 2 \times 10^{15}\ \text{cm}^{-3}$.

4.5 Charge neutrality

Equilibrium electron and hole concentration

$$n_0 = \frac{N_d^+ + \sqrt{(N_d^+)^2 + 4n_i^2}}{2} \ (but\ N_d^+ \text{ unknown})$$

① $n_i >> N_d^+ \Rightarrow T\ very\ high$

$$\boxed{n_0 = p_0 = n_i = \sqrt{N_c N_v} \exp(-\frac{E_g}{2kT})}$$

② $n_i << N_d^+ \Rightarrow T\ not\ very\ high$ (meaning charge carriers mostly come from dopants, which is often true for a doped semiconductor)

$$\boxed{n_0 = N_d^+}$$

4.5 Charge neutrality

Equilibrium electron and hole concentration

4.5 Charge neutrality

<u>Equilibrium electron and hole concentration</u>

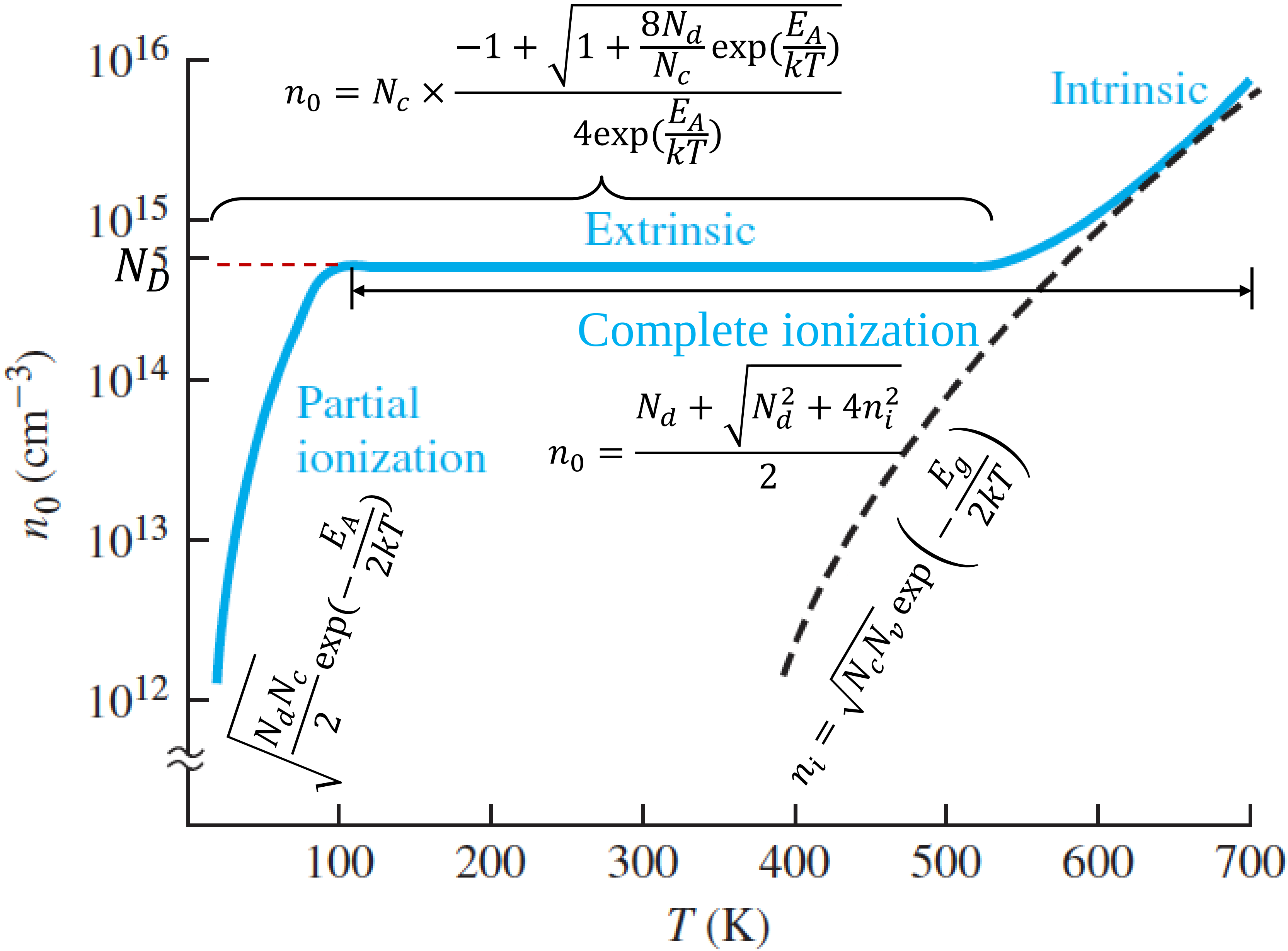
4.5 Charge neutrality

Equilibrium electron and hole concentration

$$\boxed{2\exp\left(\frac{E_A}{kT}\right) n_0^2 + N_c n_0 - N_d N_c = 0}$$

$$\boxed{n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}}$$

Ionization of dopants



Outline

4.6 Position of Fermi energy level

Mathematical Derivation

$$\boxed{n_0 = N_c \times \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}}$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \Rightarrow \ \exp\left(\frac{E_F - E_c}{kT}\right) = \frac{-1 + \sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}}{4\exp(\frac{E_A}{kT})}$$

$$E_F = E_c + kTln(\frac{\sqrt{1 + \frac{8N_d}{N_c}\exp(\frac{E_A}{kT})} - 1}{4\exp(\frac{E_A}{kT})})$$

4.6 Position of Fermi energy level

Mathematical Derivation

$$E_F = E_c + kTln\left(\frac{\sqrt{1+\frac{8N_d}{N_c}\exp(\frac{E_A}{kT})}-1}{4\exp(\frac{E_A}{kT})}\right) = \begin{cases} \frac{E_c+E_D}{2}+\frac{kT}{2}ln\frac{N_d}{2N_c} & T\ small \\ E_c - kTln\frac{N_c}{N_d} & T\ big \end{cases}$$