VE320 Introduction to Semiconductor Physics and Devices

Recitation Class for Midterm 2, Part 1

VE320 Teaching Group SU2022

University of Michigan-Shanghai Jiaotong University
Joint Institute

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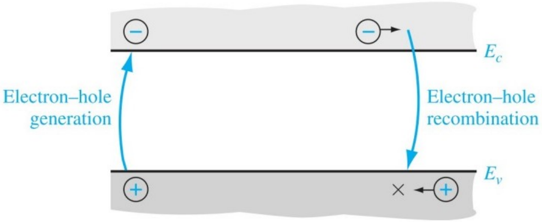
Contents

Notation

Symbol	Definition
n_0, p_0	Thermal-equilibrium electron and hole concentrations (independent of time and also usually position)
n, p	Total electron and hole concentrations
$\delta n = n - n_0$	Excess electron and hole concentrations (may
$\delta p = p - p_0$	be functions of time and/or position)
g'_n, g'_p	Excess electron and hole generation rates
R'_n, R'_p	Excess electron and hole recombination rates
τ_{n0}, τ_{p0}	Excess minority carrier electron and hole lifetimes

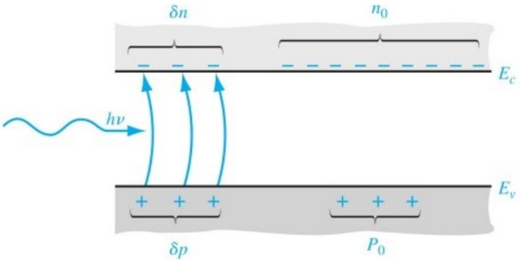
Thermal Equilibrium

- Thermal equilibrium
  - The net carrier concentrations are independent of time.
  - The generation and recombination of electrons and holes are equal.
  - Generation rate = Recombination rate: $G_{n0} = G_{p0} = R_{n0} = R_{p0}$. Unit: $(cm^3 \cdot s)^{-1}$



Non-equilibrium

- Non-equilibrium
  - The semiconductor is affected by time-varying factors like light/current.
  - A higher generation rate: total generation rate $= G_{n0} + g'_n = G_{p0} + g'_p$.
  - A higher amount of n and p: $n = n_0 + \delta n$, $p = p_0 + \delta p$.
  - In normal cases (direct generation), $\delta n = \delta p$.
  - Note $np \neq n_0 p_0 = n_i^2$.
  - Generation rate is only decided by temperature and light/current but not by n or p.
  - Recombination rate is decided by n and p: $R_n = R_p = \alpha_r np$



Net Recombination Rate

n-type:

$$R'_n = R'_p = \frac{\delta p}{\tau_{p0}}$$

p-type:

$$R'_n = R'_p = \frac{\delta n}{\tau_{n0}}$$

Continuity Equation

$$\text{For p-type: } D_n \frac{\mathrm{d}^2 n}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}n}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{n0}} = \frac{\mathrm{d}n}{\mathrm{d}t}$$

$$\text{For n-type: } D_p \frac{\mathrm{d}^2 p}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}p}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{p0}} = \frac{\mathrm{d}p}{\mathrm{d}t}$$

where g_n and g_p are the total generation rates. For homogeneous semiconductor, $n(x) = n_0 + \delta n(x)$, the equation can be simplified to

$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g'_n - \frac{\delta n}{\tau_{n0}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

$$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g'_p - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

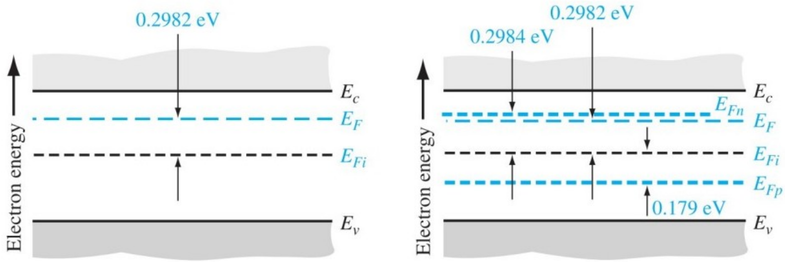
Solving Continuity Equation

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$\mathrm{E} \frac{\partial(\delta n)}{\partial x} = 0, \quad \mathrm{E} \frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

Quasi-Fermi Energy Level

$$n_0 + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

$$p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$



With excess carriers, quasi-Fermi energy level for minority carriers may vary much.

Excess Carrier Lifetime

$$R_n = R_p = \frac{C_n C_p N_t \left(np - n_i^2\right)}{C_n \left(n + n'\right) + C_p \left(p + p'\right)}$$
$$= \frac{\left(np - n_i^2\right)}{\tau_{p0} \left(n + n'\right) + \tau_{n0} \left(p + p'\right)}$$

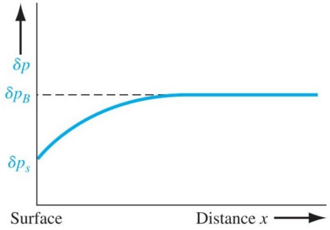
where

$$n' = N_c \exp\left[-\frac{E_c - E_t}{kT}\right], p' = N_v \exp\left[-\frac{E_t - E_v}{kT}\right], \text{ and } \tau_{n0} = \frac{1}{C_n N_t}$$

Surface Effect

- Periodic potential wells break on the surface.
- Allowed energy levels appear inside forbidden band.
- A lot of traps in the middle of forbidden band.
- Higher recombination rate.
- Lower excess carrier concentration on the surface.

Surface Effect



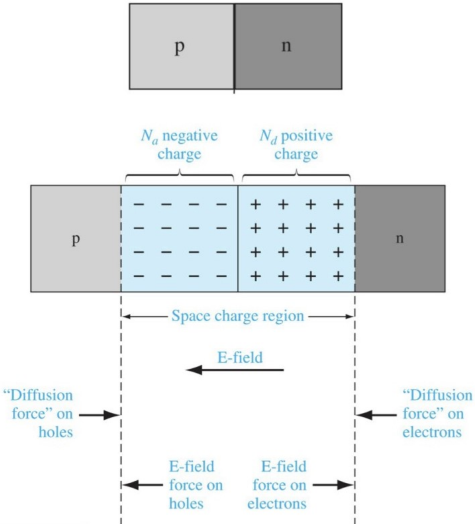
$$-D_p\left[\hat{n}\cdot\frac{\mathrm{d}(\delta p)}{\mathrm{d}x}\right]\Bigg|_{\text{surf}}=s\delta p|_{\text{surf}}$$

where s is surface recombination velocity.

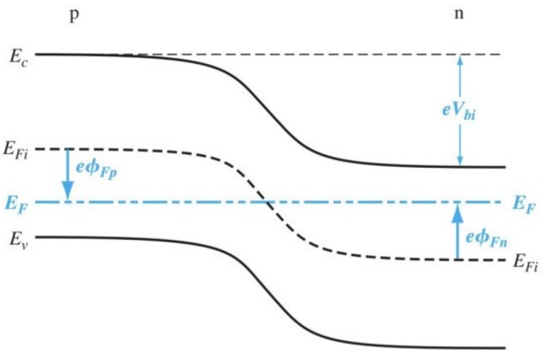
If we have a semiconductor that has a surface at one end and an even excess carrier generation rate without electric field,

$$\delta p(x)=g'\tau_{p0}\left(1-\frac{sL_p\mathrm{e}^{-x/L_p}}{D_p+sL_p}\right)$$

Basic Structure of pn Junction



Built-in Potential Barrier



$$
\begin{aligned}
V_{bi} &= |\phi_{Fn}| + |\phi_{Fp}| \\
&= \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)
\end{aligned}
$$

V_{bi} is the built-in potential barrier. $V_t = kT/e$ is the thermal voltage.

Electric Field

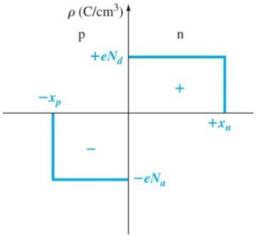


Figure: space charge density

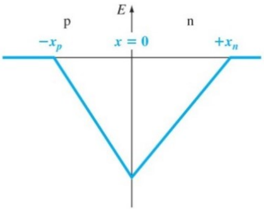


Figure: electric field

$$\rho(x) = \begin{cases} -eN_a, & -x_p \le x \le 0 \\ +eN_d, & 0 \le x \le x_n \end{cases} \quad \frac{d^2\Phi(x)}{dx^2} = -\frac{\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx}$$ where ϵ_s is the dielectric constant.

Electric Field

$$E(x) = \begin{cases} -\frac{eN_a}{\epsilon_s}(x + x_p), & -x_p \le x \le 0 \\ -\frac{eN_d}{\epsilon_s}(x_n - x), & 0 \le x \le x_n \end{cases}$$

At $x = 0$, E is continuous.

$$N_a x_p = N_d x_n$$

Maximum electric field intensity at $x = 0$ is then

$$E_{\max} = -\frac{eN_a x_p}{\epsilon_s} = -\frac{eN_d x_n}{\epsilon_s}$$

Potential

Suppose $\Phi(x) = 0$ when $x = -x_p$.

$$\Phi(x) = \begin{cases} \frac{eN_a}{2\epsilon_s}\left(x + x_p\right)^2, & -x_p \leq x \leq 0 \\ \frac{eN_d}{\epsilon_s}\left(x_n x - \frac{x^2}{2}\right) + \frac{eN_a}{2\epsilon_s} x_p^2, & 0 \leq x \leq x_n \end{cases}$$

When $x = x_n$, the potential is the same as the built-in potential barrier.

$$V_{bi} = \left|\Phi\left(x = x_n\right)\right| = \frac{e}{2\epsilon_s}\left(N_d x_n^2 + N_a x_p^2\right)$$

Space Charge Width

$$\begin{cases} x_n = \left[\frac{2\epsilon_s(V_{bi})}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2} \\ x_p = \left[\frac{2\epsilon_s(V_{bi})}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2} \end{cases}$$

The total depletion region is then

$$\begin{aligned} W &= x_n + x_p \\ &= \left[\frac{2\epsilon_s\left(V_{bi}\right)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} \end{aligned}$$

Example

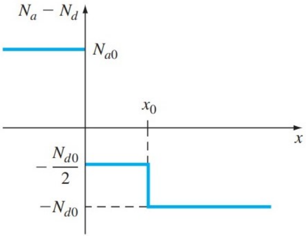
A pn junction has the doping profile shown in Figure. Assume that $x_n > x_0$ for all reverse-biased voltages.

(a) For the abrupt junction approximation, sketch the charge density through the junction.

(b) What is the built-in potential across the junction?

(c) Derive the expression for the electric field through the space charge region, in terms of x_p and x_n.

(d) Find out x_p and x_n.



Example Solution

(b)

$$V_{bi} = V_t \ln \left(\frac{N_{a0} N_{d0}}{n_i^2} \right)$$

(c) p-region

$$\frac{d\mathrm{E}}{dx} = \frac{\rho(x)}{\epsilon_s} = -\frac{eN_{a0}}{\epsilon_s}$$

or

$$\mathrm{E} = -\frac{eN_{a0}x}{\epsilon_s} + C_1$$

We have

$$\mathrm{E} = 0 \text{ at } x = -x_p \Rightarrow C_1 = -\frac{eN_{a0}x_p}{\epsilon_s}$$

Then for $-x_p < x < 0$

$$\mathrm{E} = -\frac{eN_{a0}}{\epsilon_s}(x + x_p)$$

n-region, $0 < x < x_0$

$$\frac{d\mathrm{E}_1}{dx} = \frac{\rho(x)}{\epsilon_s} = \frac{eN_{d0}}{2\epsilon_s}$$

or

$$\mathrm{E}_1 = \frac{eN_{d0}x}{2\epsilon_s} + C_2$$

n-region, $x_0 < x < x_n$

$$\frac{d\mathrm{E}_2}{dx} = \frac{\rho(x)}{\epsilon_s} = \frac{eN_{d0}}{\epsilon_s}$$

or

$$\mathrm{E}_2 = \frac{eN_{d0}x}{\epsilon_s} + C_3$$

We have $\mathrm{E}_2 = 0$ at $x = x_n$

$$\Rightarrow C_3 = -\frac{eN_{d0}x_n}{\epsilon_s}$$

so that for $x_0 < x < x_n$, we have

$$\mathrm{E}_2 = -\frac{eN_{d0}}{\epsilon_s}(x_n - x)$$

We also have $\mathrm{E}_2 = \mathrm{E}_1$ at $x = x_0$ Then

$$\frac{eN_{d0}x_0}{2\epsilon_s} + C_2 = -\frac{eN_{d0}}{\epsilon_s}(x_n - x_0)$$

which gives

$$C_2 = -\frac{eN_{d0}}{\epsilon_s}\left(x_n - \frac{x_0}{2}\right)$$

(d)

$$V_{bi} = -\int_{-x_p}^{x_n} \mathrm{E}dx, \quad N_{a0}x_p = \frac{N_{d0}x_0}{2} + (x_n - x_0)N_{d0}$$

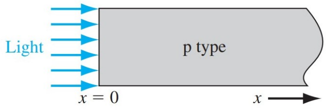
Sample Exam Question 2

Consider a bar of p-type silicon that is uniformly doped with a value of $\mathrm{N_a} = 2 \times 10^{16}\ \mathrm{cm}^{-3}$ at $\mathrm{T} = 300\ \mathrm{K}$. A light source is incident on the end of the semiconductor. The steady-state concentration of excess carriers generated at $\mathrm{x} = 0$ is $\delta \mathrm{p}(0) = \delta \mathrm{n}(0) = 2 \times 10^{14}\ \mathrm{cm}^{-3}$. $\mu_\mathrm{n} = 1200\ \mathrm{cm}^2/\mathrm{Vs}, \mu_\mathrm{p} = 400\ \mathrm{cm}^2/\mathrm{Vs}, \tau_{n0} = 1\mu s, \tau_{p0} = 0.5\mu s$. Neglecting surface effects.

(a) Determine the steady-state excess electron and hole concentration as a function of distance into the semiconductor, when the applied electric field is zero.

(b) Calculate the steady-state electron and hole diffusion current densities as a function of distance into the semiconductor, when the applied electric field is zero.

(c) Assume a constant electric field of $5\ \mathrm{V/cm}$ is applied in the $+\mathrm{x}$ direction. Derive the expression for the steady-state excess electron concentration as a function of distance into the semiconductor.



Example Solution

Minority carriers are electrons.

$$\begin{aligned} D_n &= \left(\frac{kT}{e}\right)\mu_n = (0.0259)(1200) \\ &= 31.08\ \mathrm{cm}^2/\mathrm{s} \\ L_n &= \sqrt{D_n\tau_{n0}} = \left[(31.08)\left(10^{-6}\right)\right]^{1/2} \\ &= 5.575\times 10^{-3}\ \mathrm{cm} \end{aligned}$$

(a) $\delta n(x) = \delta p(x) = 2\times 10^{14} e^{-x/L_n}\ \mathrm{cm}^{-3}$

(b)

$$
\begin{aligned}
J_n &= eD_n \frac{d(\delta n)}{dx} = eD_n \frac{d}{dx}\left[2 \times 10^{14} e^{-x/L_n}\right] \\
&= \frac{-eD_n}{L_n}\left(2 \times 10^{14}\right) e^{-x/L_n} \\
&= \frac{-\left(1.6 \times 10^{-19}\right)(31.08)\left(2 \times 10^{14}\right)}{\left(5.575 \times 10^{-3}\right)} e^{-x/L_n} \\
J_n &= -0.1784 e^{-x/L_n} \ \mathrm{A/cm^2}
\end{aligned}
$$

Holes diffuse at same rate as minority carrier electrons, so

$$J_p = +0.1784 e^{-x/L_n} \ \mathrm{A/cm^2}$$

(c) Write out continuity equation for p-type semiconductor.

$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n' - \frac{\delta n}{\tau_{n0}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

Simplify the equation to get

$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} - \frac{\delta n}{\tau_{n0}} = 0$$

Boundary condition

$$\delta n(\infty) = 0$$

Use the solution model and do some simplifications to get

$$\delta n(x) = \delta n(0) \exp(\lambda x)$$

where $\lambda = \frac{-\tau_{n0}\mu_n E - \sqrt{(\tau_{n0}\mu_n E)^2 + 4\tau_{n0} D_n}}{2\tau D_n}$.

Supplementary Materials

Case Study 1: Removing the Light

We consider a case where a light is on the semiconductor for a long time so that n and p becomes constant.

$$G_n = R_n = \alpha_r np$$

After removing the light:

$$\begin{aligned} R'_{n/p} &= -\tfrac{d\delta n}{dt} \\ &= -(G_n - R_n) = -((G_{n0} + g'_n) - \alpha_r np) \\ &= -\alpha_r(n_i^2 - (n_0 + \delta n)(p_0 + \delta p)) \end{aligned}$$

where g'_n is now 0.
We assume the low-injection condition: the maximum of n_0 and p_0 is much greater than excess carriers. But the excess carriers can be much more than the minimum of n_0 and p_0.

$$R'_{n/p} = \alpha_r \max(n_0, p_0)\delta n$$

Case Study 1, Continued

$$\delta n = \delta p = \delta n(0)e^{-t/\tau}$$

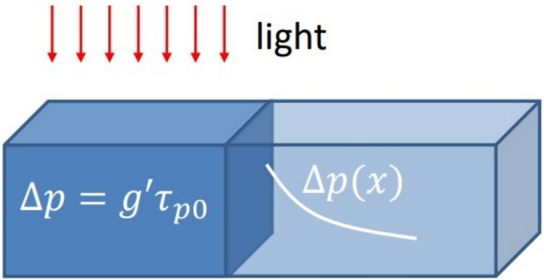
where $\tau = 1/\max(n_0, p_0)$. We call it excess minority carrier lifetime.

- Note
  - In p-type, we only care about δn since δp is small compared with p_0. And we use $\tau_{n0} = 1/\alpha_r p_0$.
  - In n-type, we only care about δp since δn is small compared with n_0. And we use $\tau_{p0} = 1/\alpha_r n_0$.
  - Excess carrier recombination rate: $R'_n = R'_p = \delta n/\tau = \delta p/\tau$.

  Summary of case study: when light applied suddenly decreased, excess carriers decrease exponentially with respect to time.

Case Study 2: Diffusion

N-type semiconductor. On the one end, there is non-zero excess carrier generation rate, and on the other end, there is no excess carrier generation.



Case Study 2: Diffusion

$$\frac{d\text{flux}}{dx} = -R'_p = -\frac{\delta p}{\tau_{p0}}$$

$$D_p \frac{d^2 \delta p}{dx^2} = \frac{\delta p}{\tau_{p0}}$$

Therefore,

$$\delta p(x) = Ae^{-x/L_p} + Be^{x/L_p}$$

where $L_p = \sqrt{D_p \tau_{p0}}$.
Summary of case study: excess carriers change exponentially with respect to space.

Solution Model: Time

$$\frac{\mathrm{d}(\delta p)}{\mathrm{d}t} = -\frac{\delta p}{\tau_{p0}}$$

solution:

$$\delta p(t) = \delta p(0)\mathrm{e}^{-t/\tau_{p0}}$$

$$g' - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

solution:

$$\delta p(t) = g'\tau_{p0}\left(1 - \mathrm{e}^{-t/\tau_{p0}}\right)$$

Solution Model: Distance

$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} - \frac{\delta n}{\tau_{n0}} = 0$$

solution:

$$\delta n(x) = A\mathrm{e}^{-x/L_n} + B\mathrm{e}^{x/L_n}, \quad L_n = \sqrt{D_n \tau_{n0}}$$

special:

$$\delta n(x) = \begin{cases} \delta n(0)\mathrm{e}^{-x/L_n}, x \geq 0 \\ \delta n(0)\mathrm{e}^{+x/L_n}, x \leq 0 \end{cases}$$

$$D_p \frac{\mathrm{d}^2 \delta p}{\mathrm{d}x^2} - \frac{\delta p}{\tau} + g = 0$$

solution:

$$\delta p(x) = A \exp(\lambda x) + B \exp(-\lambda x) + g\tau, \quad \lambda = \frac{1}{\sqrt{D_p \tau}}$$

Solution Model: Electrical Field

$$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p E_0 \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} - \frac{\delta p}{\tau_{p0}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

solution:

$$\delta p(x, t) = \frac{\mathrm{e}^{-t/\tau_{p0}}}{(4\pi D_p t)^{1/2}} \exp\left[\frac{-(x - \mu_p E_0 t)^2}{4 D_p t}\right]$$

Solution Model: Electrical Field

$$D_p \frac{\mathrm{d}^2 \delta p}{\mathrm{d}x^2} - \mu_p E \frac{\mathrm{d}\delta p}{\mathrm{d}x} - \frac{\delta p}{\tau} = 0$$

solution:

$$\delta p(x) = A \exp(\lambda x) + C$$

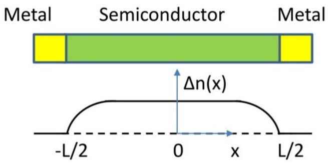
$$\lambda = \frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2}, \quad L_p = \sqrt{\tau D_p}, \quad L_p(E) = \tau \mu_p E$$

special:

$$\delta p(x) = \delta p(0) \exp\left[\frac{L_p(E) \pm \sqrt{L_p^2(E) + 4L_p^2}}{2L_p^2} x\right] = \begin{cases} \delta p(0) \exp\left(-\frac{x}{L_p}\right), & \text{if } L_p(E) \ll L_p \\ \delta p(0) \exp\left(-\frac{x}{L_p(E)}\right), & \text{if } L_p(E) \gg L_p \end{cases}$$

Example 1

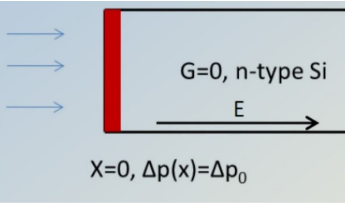
Given a piece of p-type uniformly doped semiconductor in contact with two metal electrodes separated by a length of L, forming a photoconductor device. The light illumination will create electron-hole pairs at a generation rate of g'. The minority carrier recombination lifetime is τ_0. Find the analytical distribution of the excess minority electrons at zero external bias. Note that light illumination will not create excess carriers in metals.



$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

Example 2

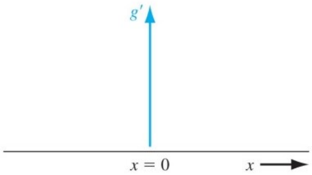
A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface ($x = 0$). The wafer is placed in a constant electric field with a known intensity E. We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface ($x = 0$). Small injection condition is always maintained and the wafer is uniformly doped as N_d.



$$D_p \frac{\mathrm{d}^2(\delta p)}{\mathrm{d}x^2} - \mu_p \left(E \frac{\mathrm{d}(\delta p)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_p - \frac{p}{\tau_{pt}} = \frac{\mathrm{d}(\delta p)}{\mathrm{d}t}$$

Example 3

Consider a p-type semiconductor that is homogeneous and infinite in extent. Assume a zero applied electric field. For a one-dimensional crystal, assume that excess carriers are being generated at $x = 0$ only, as indicated in Figure below. The excess carriers being generated at $x = 0$ will begin diffusing in both the $+x$ and $-x$ directions. Calculate the steady-state excess carrier concentration as a function of x.



$$D_n \frac{\mathrm{d}^2(\delta n)}{\mathrm{d}x^2} + \mu_n \left(E \frac{\mathrm{d}(\delta n)}{\mathrm{d}x} + n \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g_n - \frac{n}{\tau_{nt}} = \frac{\mathrm{d}(\delta n)}{\mathrm{d}t}$$

Good Luck!