# VE320 – Summer 2022

# Introduction to Semiconductor Devices

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## Chapter 4 The Semiconductor in Equilibrium

# Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

① $\Delta E$ # states

② $\Delta E$ $f_F(E)$

③ $\Delta E$ # state × $f_F(E)$

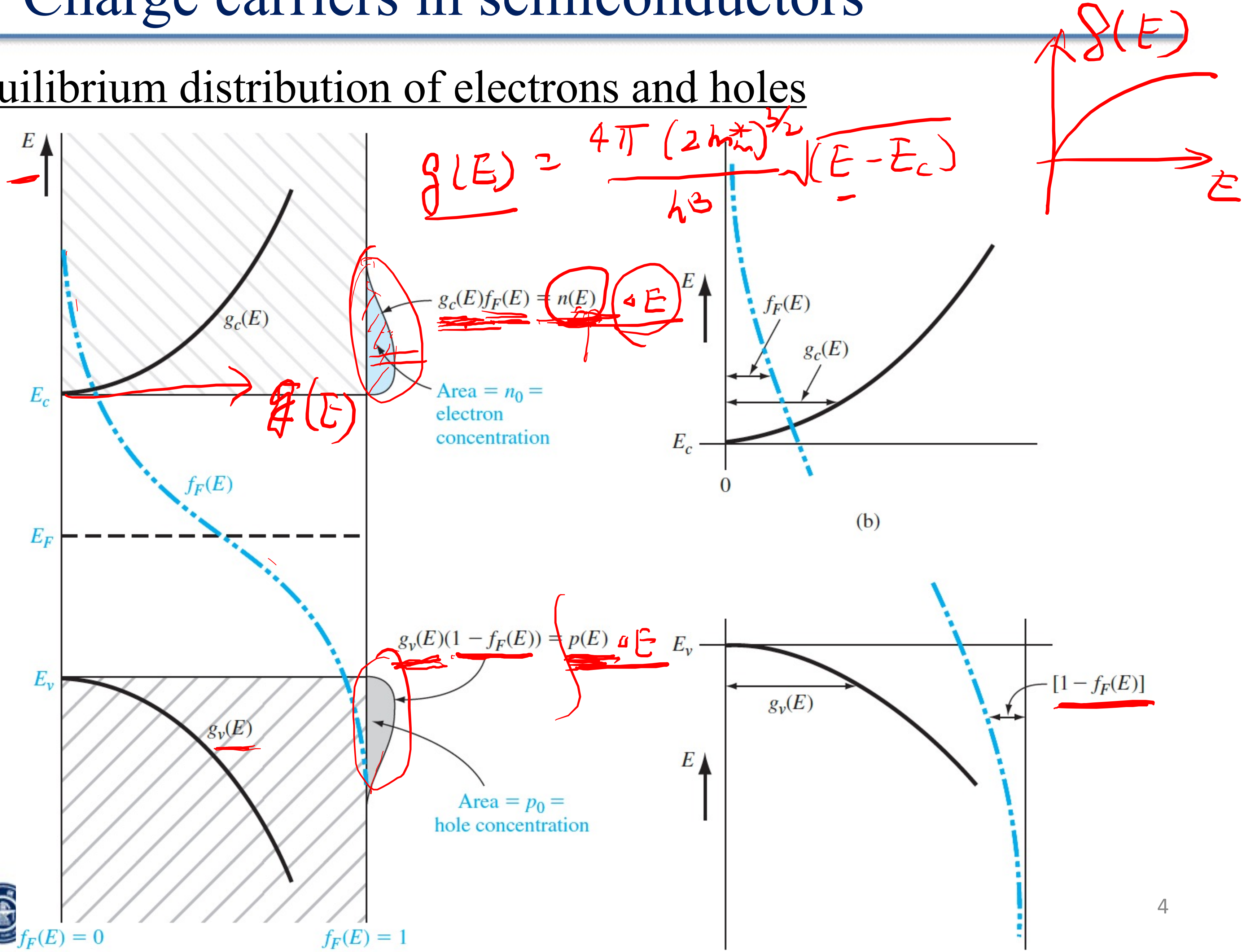
$$\Delta N_e = g(E) \cdot f(E) \cdot \Delta E$$

$$N_e = \int_{E_c}^{+\infty} g(E) f(E) dE$$

# Outline

# 4.1 Charge carriers in semiconductors

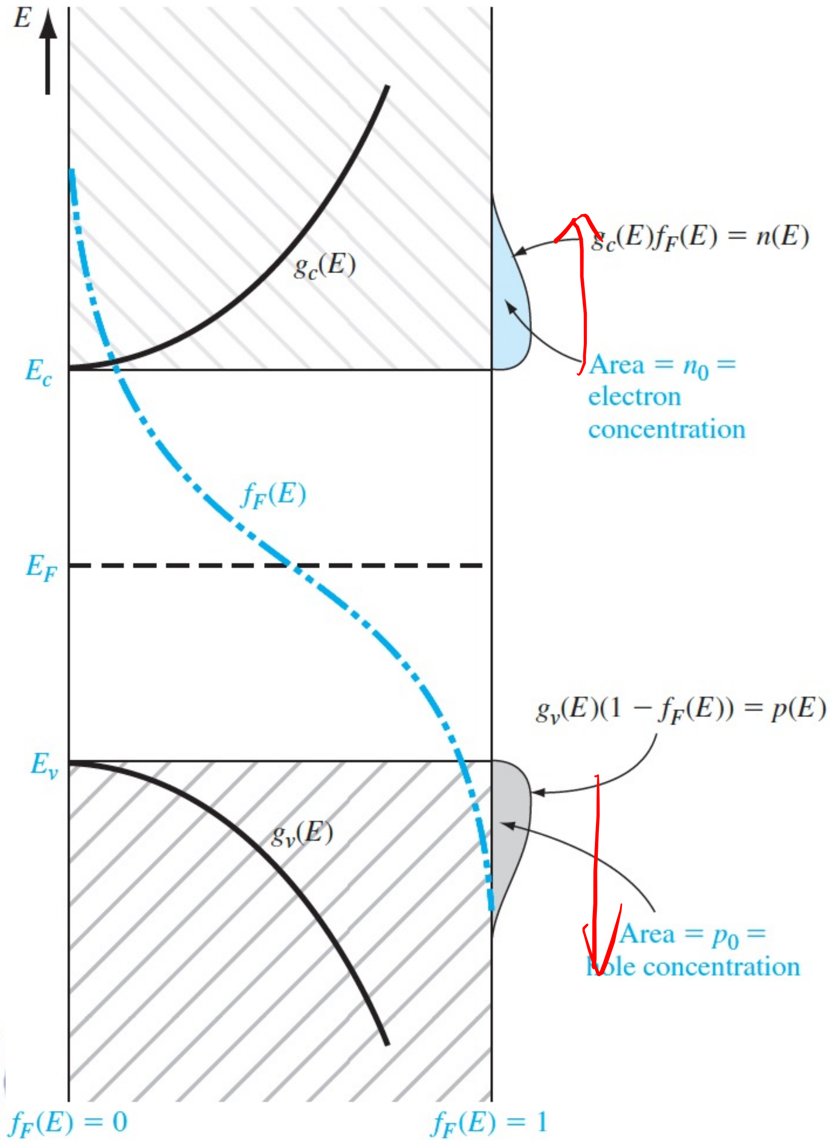
## Equilibrium distribution of electrons and holes



# 4.1 Charge carriers in semiconductors

## The $n_0$ and $p_0$ equations

$\int g(E) \cdot f_F(E) \cdot dE$




$$n_0 = \int_{E_c}^{\infty} g_c(E) f_F(E) dE$$

$$p_0 = \int_{-\infty}^{E_v} g_v(E)[1 - f_F(E)] dE$$

# 4.1 Charge carriers in semiconductors

The $n_0$ and $p_0$ equations

# 4.1 Charge carriers in semiconductors

The $n_0$ and $p_0$ equations

$$n_0 = \int_{E_c}^{+\infty} g_c(E)\, f_F(E)\, dE \qquad \frac{E-E_F}{kT} > 3$$

$$= \int_{E_c}^{+\infty} 4\pi \frac{(2m_n^*)^{3/2}}{h^3} (E-E_c)^{1/2} \cdot \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)} dE$$

$$d\eta = dE/kT$$

$$E-E_F = E-E_c+E_c-E_F = kT\eta - kT\eta_F$$

$$\eta = \frac{E-E_c}{kT} \qquad = 4\pi \frac{(2m_n^*)^{3/2}}{h^3} \int_0^{+\infty} \frac{\eta^{1/2}}{1+\exp(\eta-\eta_F)} d\eta$$

$$\exp(\eta-\eta_F) \gg 10 \qquad E-E_F > 3kT$$

$$\eta_F = \frac{E_c-E_F}{kT} \qquad = 4\pi \frac{(2m_n^*)^{3/2}}{h^3} \int_0^{+\infty} \eta^{1/2} \exp(\eta_F-\eta)\, d\eta$$

# 4.1 Charge carriers in semiconductors

$$\boxed{if \;\; \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$ (2nd time approximation)

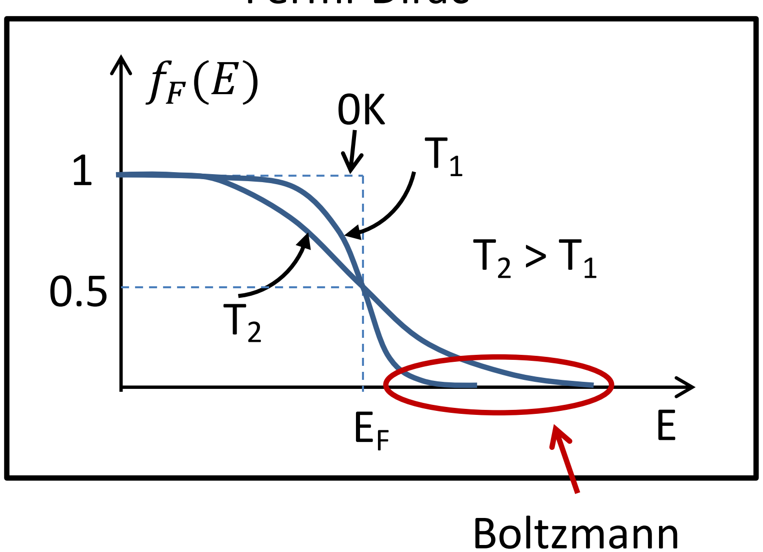
$$f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}$$

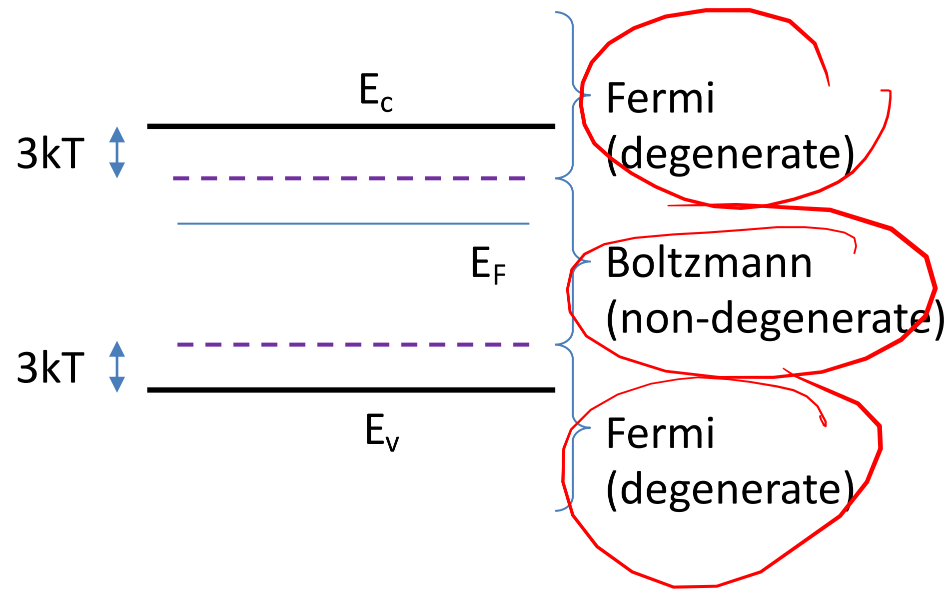
Fermi-Dirac Distribution

$$f_F(E) = \exp(\frac{E_F - E}{kT})$$

Boltzmann Distribution

Fermi-Dirac



# 4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}}$$

Fermi-Dirac Distribution

$$\Rightarrow$$

$$\boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Boltzmann Distribution

# 4.1 Charge carriers in semiconductors

$$\boxed{if\ \exp(\eta-\eta_F)>10 \Leftrightarrow \frac{E-E_F}{kT}>3 \Leftrightarrow E-E_F>3kT}$$

$$\boxed{f_F(E)=\frac{1}{1+\exp(\frac{E-E_F}{kT})}} \Rightarrow \boxed{f_F(E)=\exp(\frac{E_F-E}{kT})}$$

Fermi-Dirac Distribution

Boltzmann Distribution

# 4.1 Charge carriers in semiconductors

$$\boxed{if\ \ \exp(\eta - \eta_F) > 10 \Leftrightarrow \frac{E - E_F}{kT} > 3 \Leftrightarrow E - E_F > 3kT}$$

$$\boxed{f_F(E) = \frac{1}{1 + \exp(\frac{E - E_F}{kT})}} \Rightarrow \boxed{f_F(E) = \exp(\frac{E_F - E}{kT})}$$

Fermi-Dirac Distribution

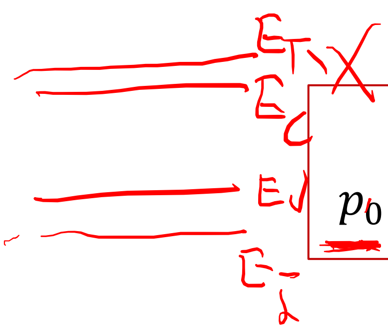
Boltzmann Distribution

# 4.1 Charge carriers in semiconductors

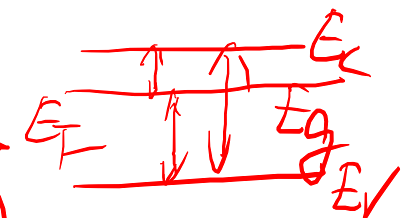
$$if\ \ \exp(\eta-\eta_F) > 10 \Leftrightarrow \frac{E-E_F}{kT} > 3 \Leftrightarrow \boxed{E-E_F > 3kT}$$

electron concentration in conduction band

$$n_0 = \frac{2(2\pi m_n{}^* kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{E_F-E_c}{kT}\right) = N_c \exp\left(\frac{E_F-E_c}{kT}\right)$$



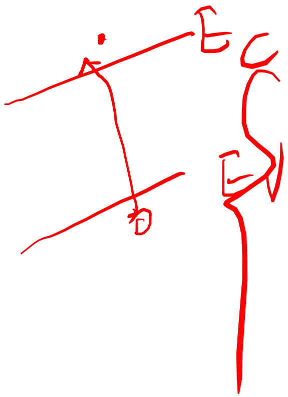

$$p_0 = \frac{2(2\pi m_p{}^* kT)^{\frac{3}{2}}}{h^3}\exp\left(\frac{E_v-E_F}{kT}\right) = N_v \exp\left(\frac{E_v-E_F}{kT}\right)$$



# 4.1 Charge carriers in semiconductors

## The intrinsic carrier concentration

$n_0 p_0 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$




$$n_0 \times p_0 = n_i^2 = N_c N_v \exp\left(\frac{E_v - E_c}{kT}\right) \Rightarrow n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \qquad p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$N_c \sim 10^{19} cm^{-3}$ 300K

$N_v \sim 10^{19} cm^{-3}$ 300K

for Si

- The equations are universal for doped and undoped semiconductors

# Check your understanding

Problem Example #1

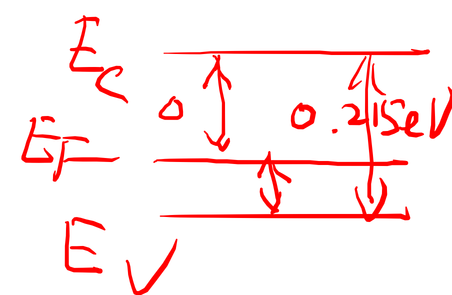
$KT(T=300k) = 0.0259eV$

$KT(T=200k) = 0.0259 \times \frac{200}{300}$

Determine the thermal-equilibrium concentrations of electrons and holes in silicon at T = 300K if the Fermi energy level $E_F$ is 0.215eV above the valence band energy $E_V$. $N_C = 2.8 \times 10^{19}$ cm$^{-3}$ and $N_V = 1.04 \times 10^{19}$ cm$^{-3}$. $E_g = 1.12$ eV for Si.

$\boxed{E_c - E_V = 1.12eV = E_g}$

$$E_c - E_F = 1.12eV - 0.215eV = \cancel{0.97}\ 0.905eV$$




$$n_0 = N_c \exp\left(\frac{E_F - E_c}{KT}\right) = 2.8\times10^{19} \exp\left(\frac{-0.905eV}{0.0259eV}\right) = 18707\ cm^{-3}$$

$$p_0 = N_V \exp\left(\frac{-0.215eV}{KT}\right) = 1.04\times10^{19} \exp\left(\frac{-0.215}{0.0259}\right) = 2.58\times10^{15}\ cm^{-3}$$

# 4.1 Charge carriers in semiconductors

## The intrinsic carrier concentration

300k

**Table 4.1** | Effective density of states function and density of states effective mass values

| | $N_c$ (cm$^{-3}$) | $N_v$ (cm$^{-3}$) | $m_n^*/m_0$ | $m_p^*/m_0$ |
|---|---|---|---|---|
| Silicon | $2.8 \times 10^{19}$ | $1.04 \times 10^{19}$ | 1.08 | 0.56 |
| Gallium arsenide | $4.7 \times 10^{17}$ | $7.0 \times 10^{18}$ | 0.067 | 0.48 |
| Germanium | $1.04 \times 10^{19}$ | $6.0 \times 10^{18}$ | 0.55 | 0.37 |

M

**Table 4.2** | Commonly accepted values of $n_i$ at $T = 300$ K

| | |
|---|---|
| Silicon | $n_i = 1.5 \times 10^{10}$ cm$^{-3}$ |
| Gallium arsenide | $n_i = 1.8 \times 10^{6}$ cm$^{-3}$ |
| Germanium | $n_i = 2.4 \times 10^{13}$ cm$^{-3}$ |

# 4.1 Charge carriers in semiconductors

## The intrinsic carrier concentration

$$n_i = \sqrt{N_c \cdot N_v}\,\exp\left(-\frac{E_g}{2kT}\right) \qquad (kT)^{3/2}$$

$$N_c \sim (kT)^{3/2}$$

$$N_c \sim (kT)^{3/2}$$

$$\ln n_i = \ln\sqrt{N_c N_c} - \frac{E_g \times 1000}{2kT \cdot 1000}$$

$$= \ln\sqrt{N_c N_c} - \frac{E_g}{2k \times 1000} \times \frac{1000}{T}$$

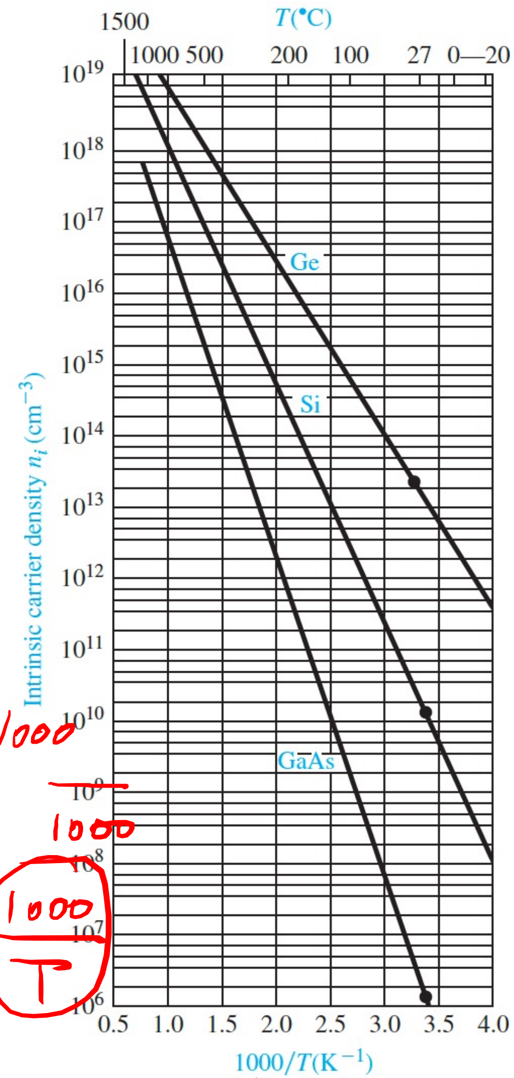



**Figure 4.2** | The intrinsic carrier concentration of Ge, Si, and GaAs as a function of temperature. *(From Sze [14].)*

# Check your understanding

Problem Example #2

Calculate the intrinsic carrier concentration in silicon at T=250K and at 400K.

$$kT\Big|_{T=250K} = 0.0259 \times \frac{250}{300} = 0.0259 \times \frac{5}{6} = 0.0215$$

$0.0043 \times 5$

$215$

$$kT\Big|_{T=400K} = 0.0259 \times \frac{400}{300} = 0.00863 \times 4 = 0.0345$$

$0.03452$

$1.12$

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = 4.90 \times 10^5\ cm^{-3} \quad @\ 250K$$

$$N_c = 2.8 \times 10^{19} \times (kT)^{3/2} \times \frac{(250)^{3/2}}{(300)^{3/2}}$$

$$N_v = 1.04 \times 10^{19} \times \frac{(250)^{3/2}}{(300)^{3/2}}$$

# 4.1 Charge carriers in semiconductors

The intrinsic Fermi-level position

$$N_c = 4\pi \frac{(2m_n^* kT)^{3/2}}{h^3}$$

$$N_v = 4\pi \frac{(2m_p^* kT)^{3/2}}{h^3}$$

$$\frac{N_v}{N_c} = \left(\frac{m_p^*}{m_n^*}\right)^{3/2}$$

$\boxed{\text{undoped}}$

$$n_0 = p_0 = n_i$$

$$n_0 = N_c \exp\left(\frac{E_{Fi} - E_c}{kT}\right) = N_v \exp\left(\frac{E_v - E_{Fi}}{kT}\right)$$

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$\ln N_c + \frac{E_{Fi} - E_c}{kT} = \ln N_v + \frac{E_v - E_{Fi}}{kT}$$

$$\frac{2E_{Fi}}{kT} = \ln N_v - \ln N_c + \frac{E_v + E_c}{kT}$$

$$E_{Fi} = \frac{1}{2} kT \ln \frac{N_v}{N_c} + \frac{E_v + E_c}{2}$$

# Outline

4.1 Charge carriers in semiconductors

**4.2 Dopant atoms and energy levels**

4.3 The extrinsic semiconductor

4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

# 4.2 Dopant atoms and energy levels

## Qualitative description

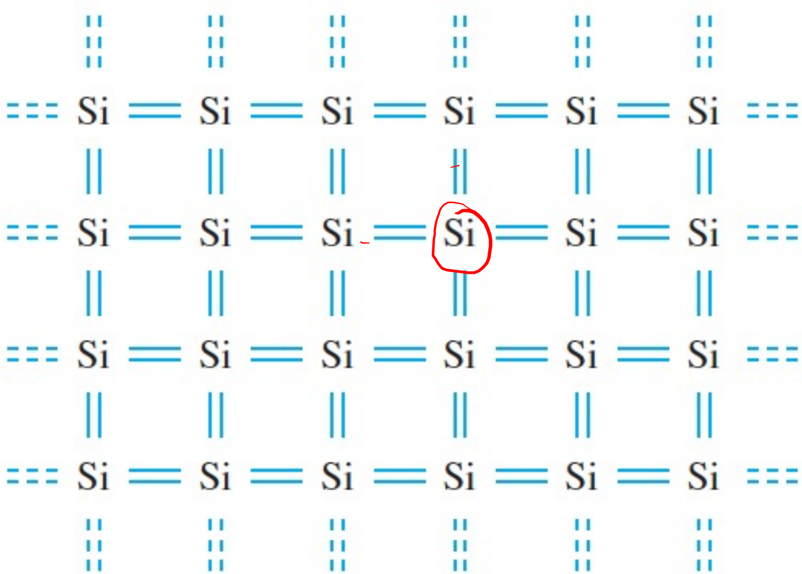



**Figure 4.3** | Two-dimensional representation of the intrinsic silicon lattice.

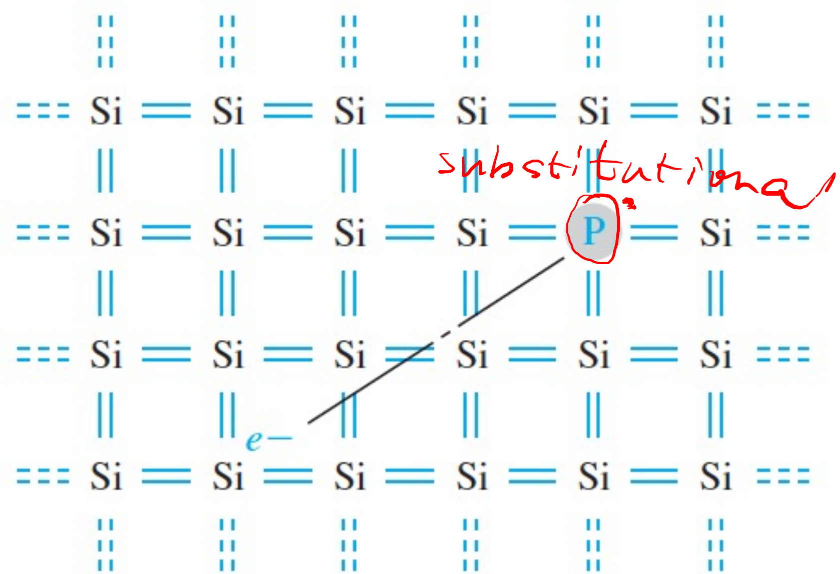
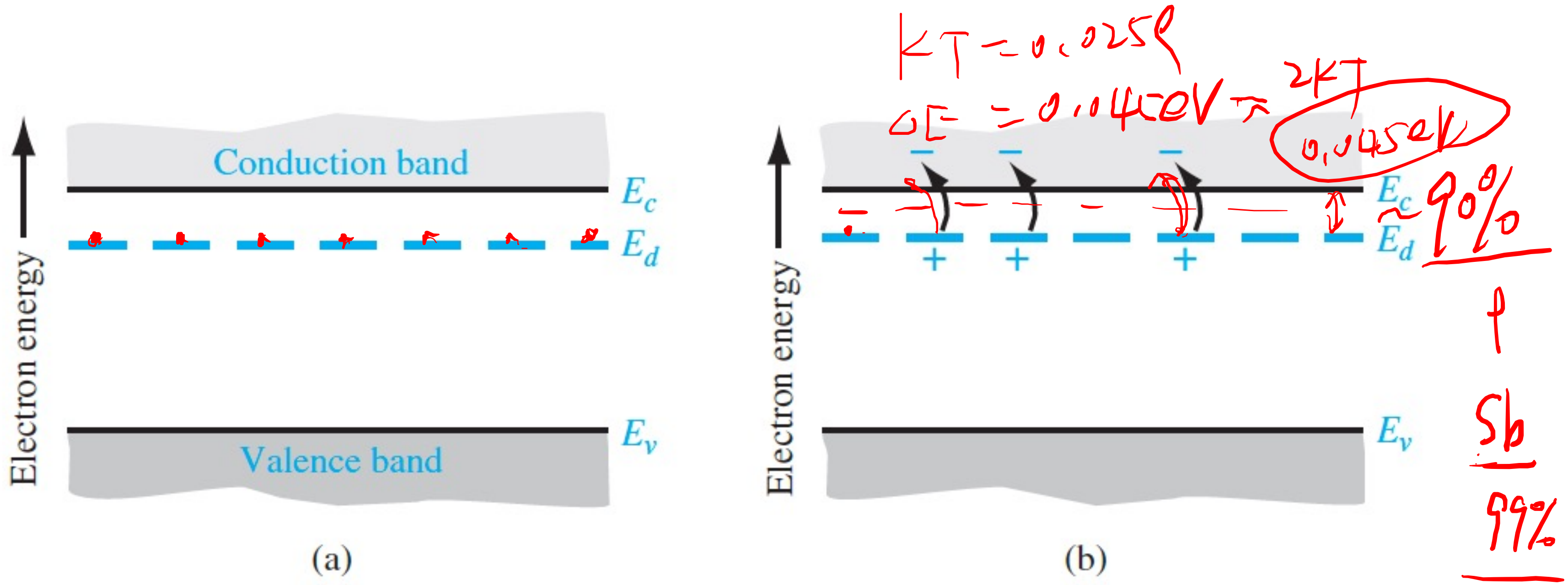



**Figure 4.4** | Two-dimensional representation of the silicon lattice doped with a phosphorus atom.

# 4.2 Dopant atoms and energy levels

## Qualitative description



# 4.2 Dopant atoms and energy levels

## Qualitative description

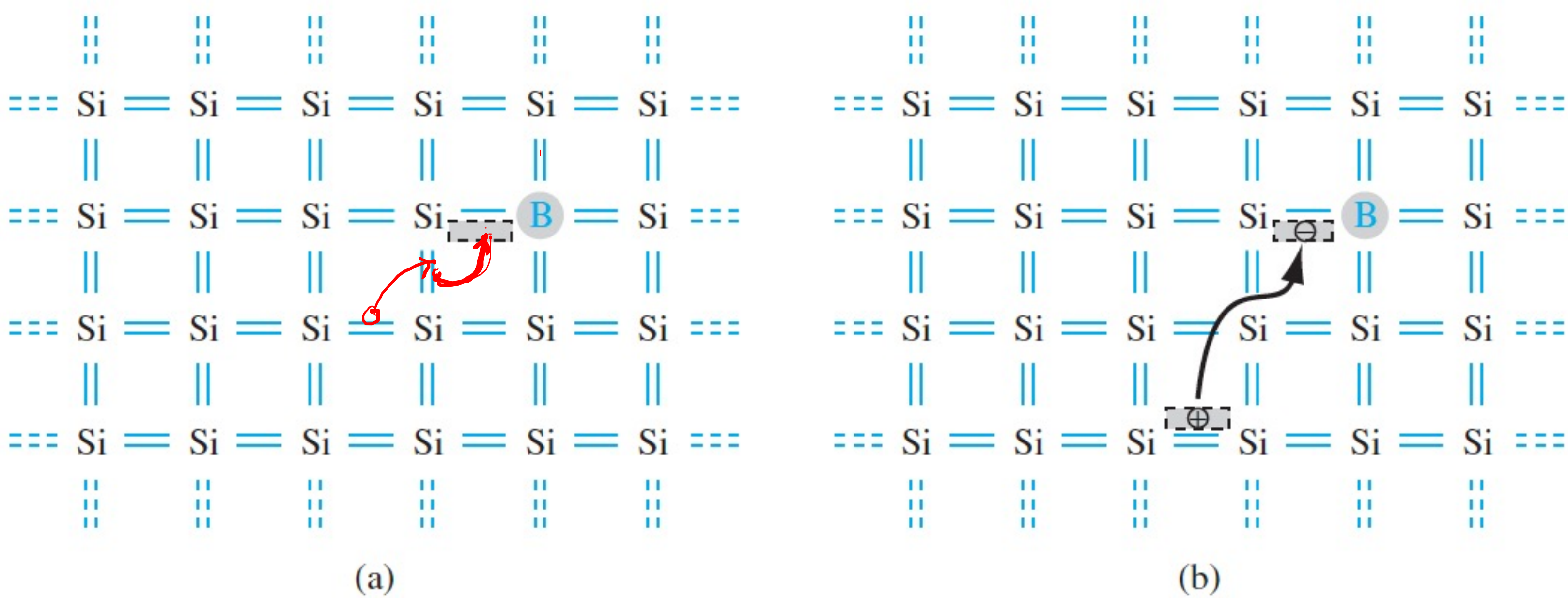



**Figure 4.6** | Two-dimensional representation of a silicon lattice (a) doped with a boron atom and (b) showing the ionization of the boron atom resulting in a hole.

# 4.2 Dopant atoms and energy levels

## Qualitative description

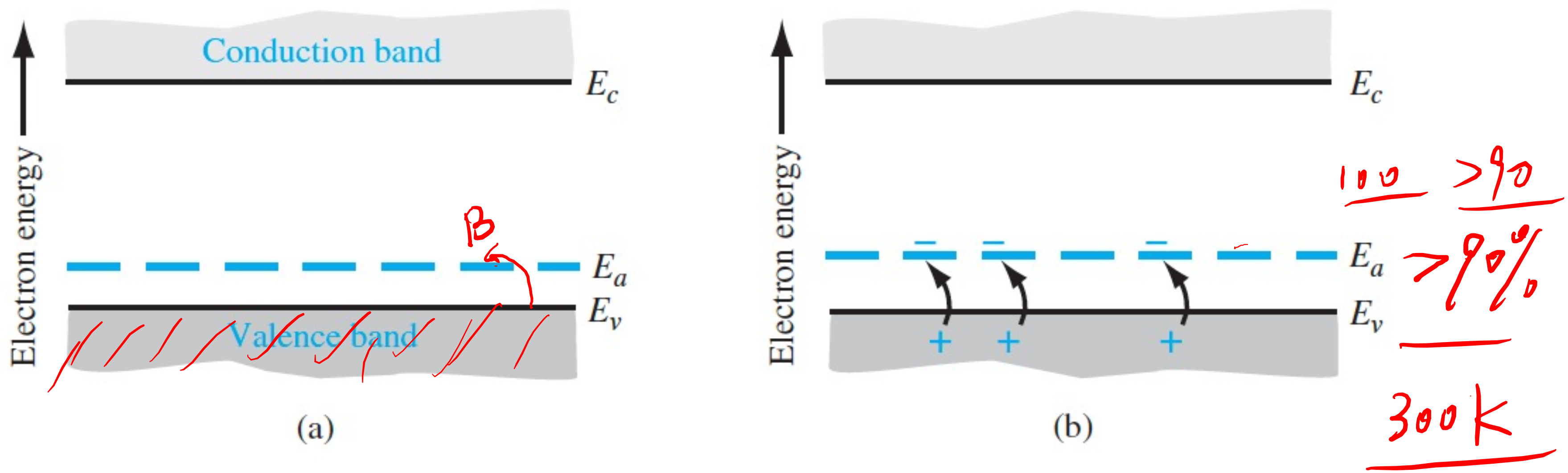
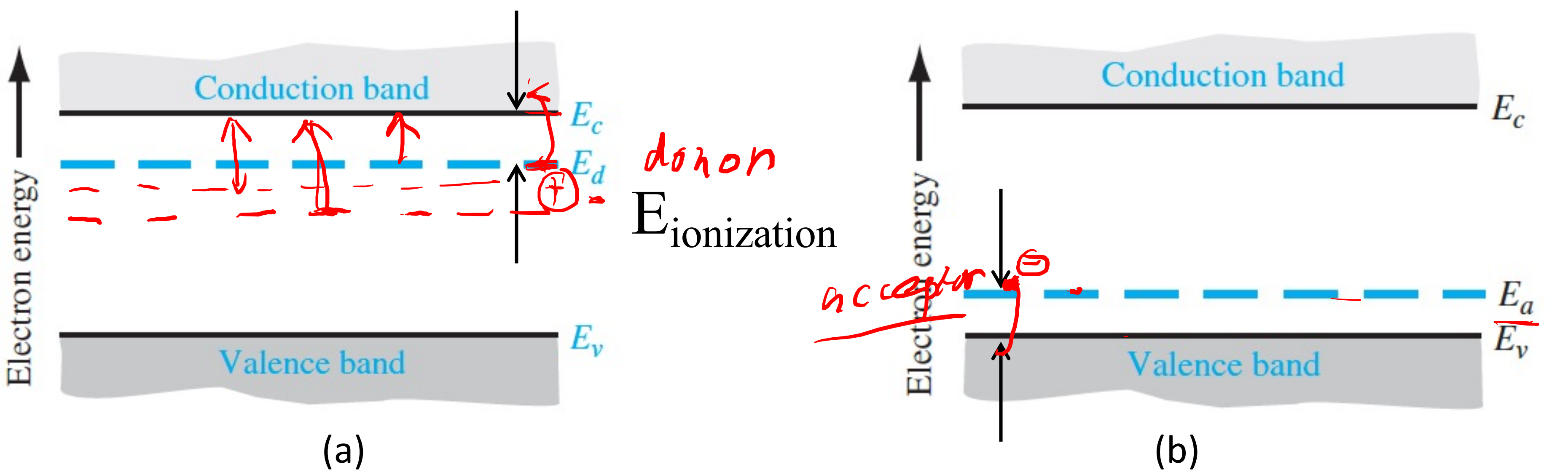



**Figure 4.7** | The energy-band diagram showing (a) the discrete acceptor energy state and (b) the effect of an acceptor state being ionized.

# 4.2 Dopant atoms and energy levels

Ionization energy

activation energy based phenomena




$$E_{ionization} = E_c - E_d$$

$$E_{ionization} = E_a - E_v$$

# 4.2 Dopant atoms and energy levels

## Ionization energy

**Table 4.3 |** Impurity ionization energies in silicon and germanium

| | Ionization energy (eV) | |
|---|---|---|
| **Impurity** | **Si** | **Ge** |
| *Donors* | | |
| Phosphorus | 0.045 | 0.012 → 100% |
| Arsenic | 0.05 ~2kT | 0.0127 → 100% |
| *Acceptors* | | |
| Boron | 0.045 | 0.0104 |
| Aluminum | 0.06 ~2kT | 0.0102 |

$E_c$

$E_v$

# Outline

4.1 Charge carriers in semiconductors

4.2 Dopant atoms and energy levels

undoped

doped

**4.3 The extrinsic semiconductor** ↔ intrinsic seconduc

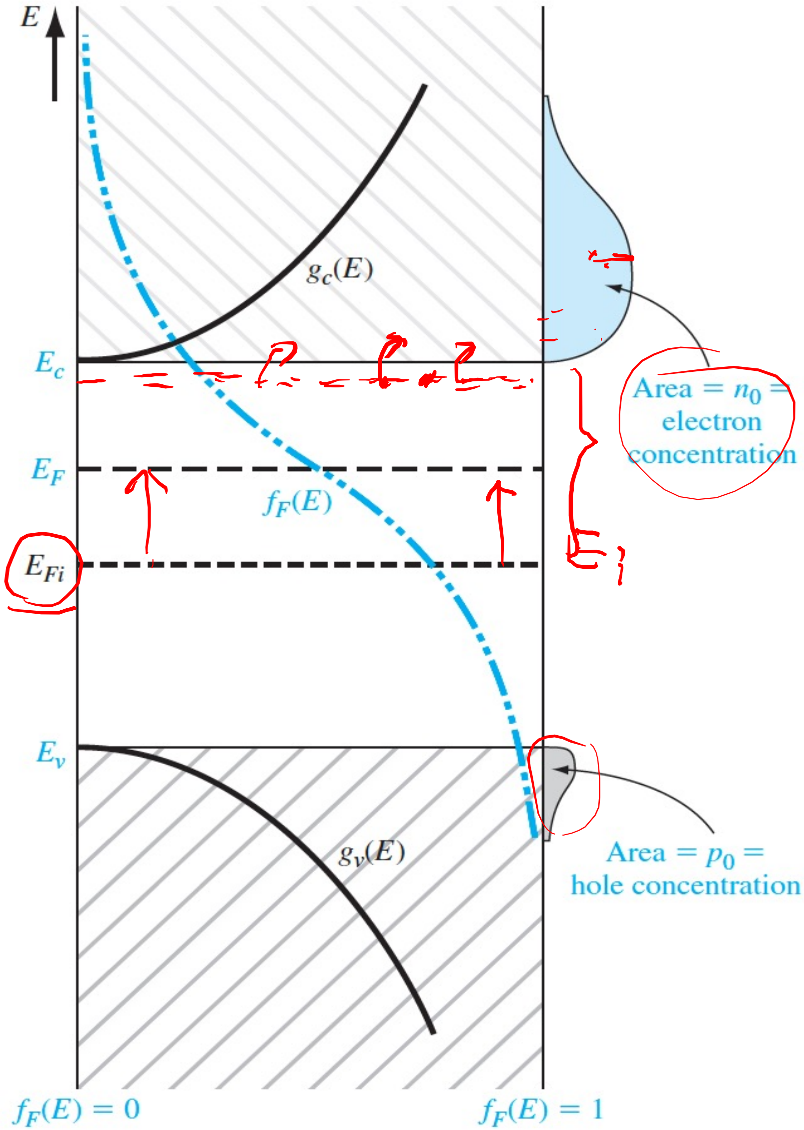
4.4 Statistics of donors and acceptors

4.5 Charge neutrality

4.6 Position of Fermi energy level

# 4.3 The extrinsic semiconductor

Equilibrium distribution of electrons and holes




n-type

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

n-type $E_F$ in upper half bandgap

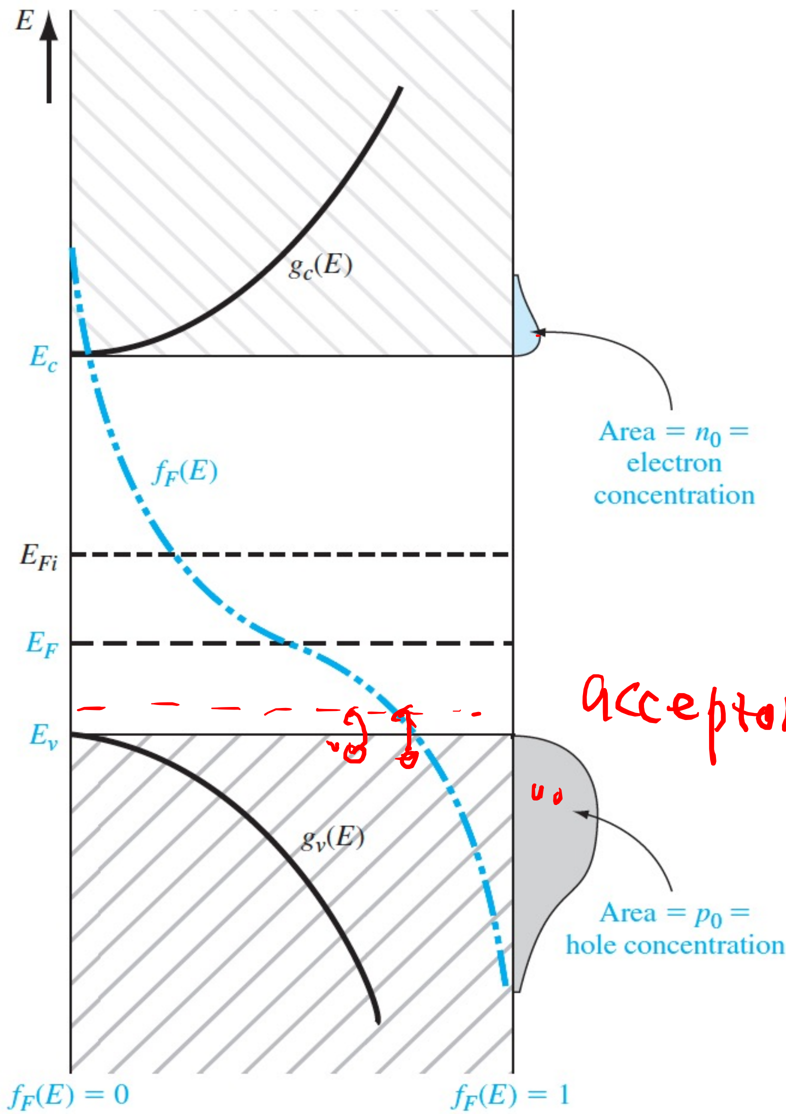
electrons are majority carriers

holes are minority carriers

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

# 4.3 The extrinsic semiconductor

Equilibrium distribution of electrons and holes




$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

p-type holes are majority
electrons are minority
$E_F$ is lower half bandgap

$$p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

# 4.3 The extrinsic semiconductor

The $n_0p_0$ product

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \qquad p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$\underline{n_0} \cdot \underline{p_0} = N_c \cdot N_v \exp\left(\frac{E_v - E_c}{kT}\right) = \underline{\text{constant}\,(T)}$$

$\underline{\text{doping}}$

$\underline{n_0 = p_0} \Rightarrow n_0 \cdot p_0 = n_i^2$

$$\boxed{n_0 \cdot p_0\,(n_0 \neq p_0) = n_0 \cdot p_0\,(n_0 = p_0) = n_i^2(T)}$$

$$\boxed{n_0 \cdot p_0 = n_i^2} = (1.5 \times 10^{10})^2$$

# 4.3 The extrinsic semiconductor

The $n_0p_0$ product

$$n_0 = n_i = N_c \exp\left(\frac{E_{Fi}-E_c}{kT}\right)$$

$$n_0 = N_c \exp\left(\frac{E_F - E_c}{kT}\right) \qquad p_0 = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$= N_c \exp\left(\frac{E_F - E_i + E_i - E_c}{kT}\right) = N_c \exp\left(\frac{E_F - E_i}{kT}\right) \exp\left(\frac{E_i - E_c}{kT}\right) = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

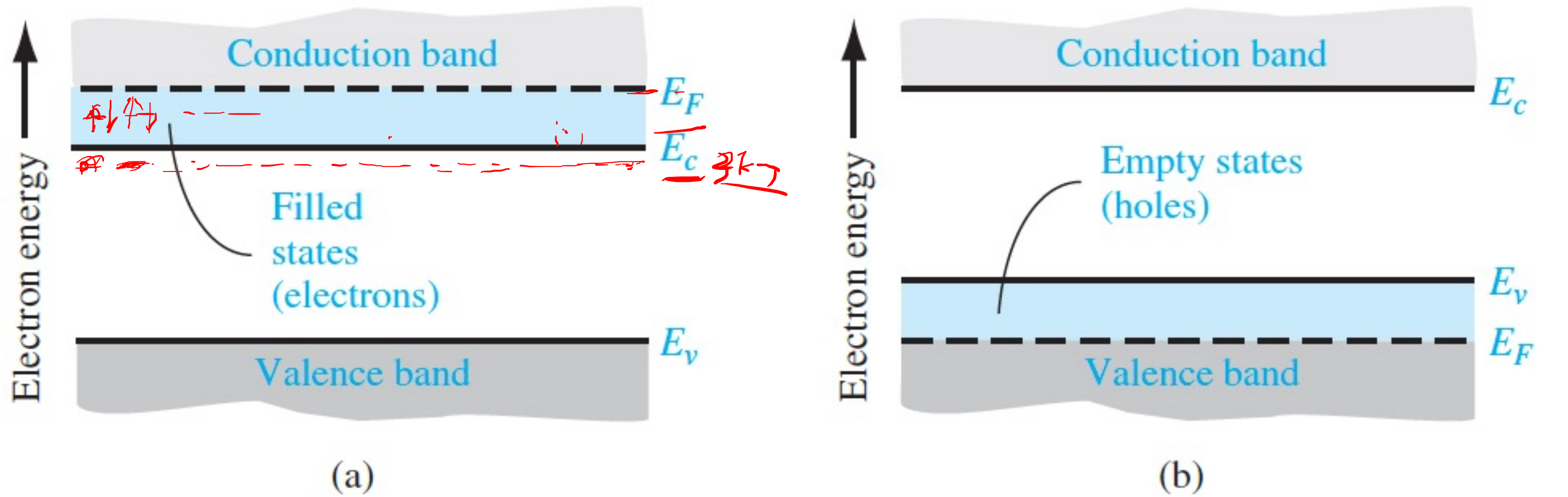
$$✓\ n_0 = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \qquad p_0 = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$$✓\ n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right)$$

$$\boxed{n_i^2 = n_0 p_0} \qquad p_0 = \frac{n_i^2}{n_0}$$

# 4.3 The extrinsic semiconductor

Degenerate and nondegenerate semiconductors




Degenerate semiconductors:

- Extremely high doping concentration
- Fermi level in the band
- Electron cloud in dopants overlap,
- dopant energy level splitting

# Check your understanding

Problem Example #3

Determine the thermal-equilibrium concentrations of electrons and holes in silicon at T = 300K if the Fermi energy level $E_F$ is 0.215eV above the valence band energy $E_V$. $N_V = 1.04 \times 10^{19}$ cm$^{-3}$, $n_i = 1.5 \times 10^{10}$ cm$^{-3}$.

# Outline

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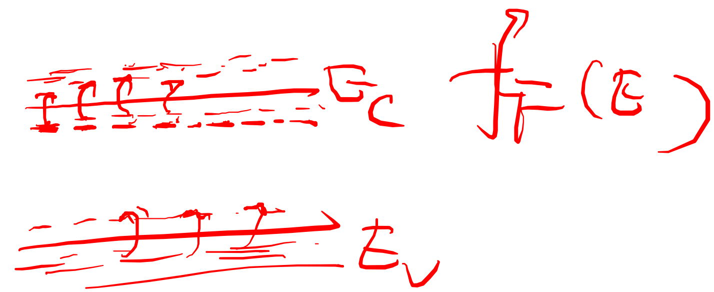
**4.4 Statistics of donors and acceptors**

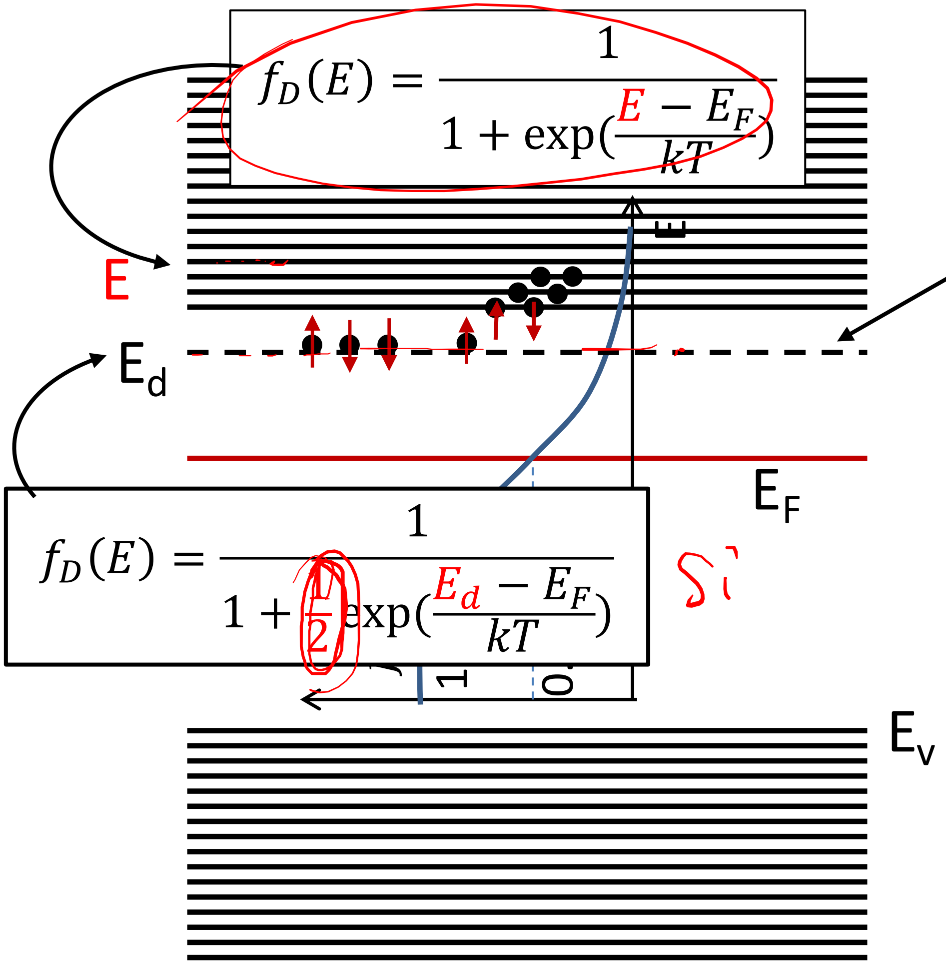
4.5 Charge neutrality

4.6 Position of Fermi energy level



# 4.4 Statistics of donors and acceptors

## Probability function




Given the concentration of donors is $N_d$

The concentration of electrons on these donors is $n_d$

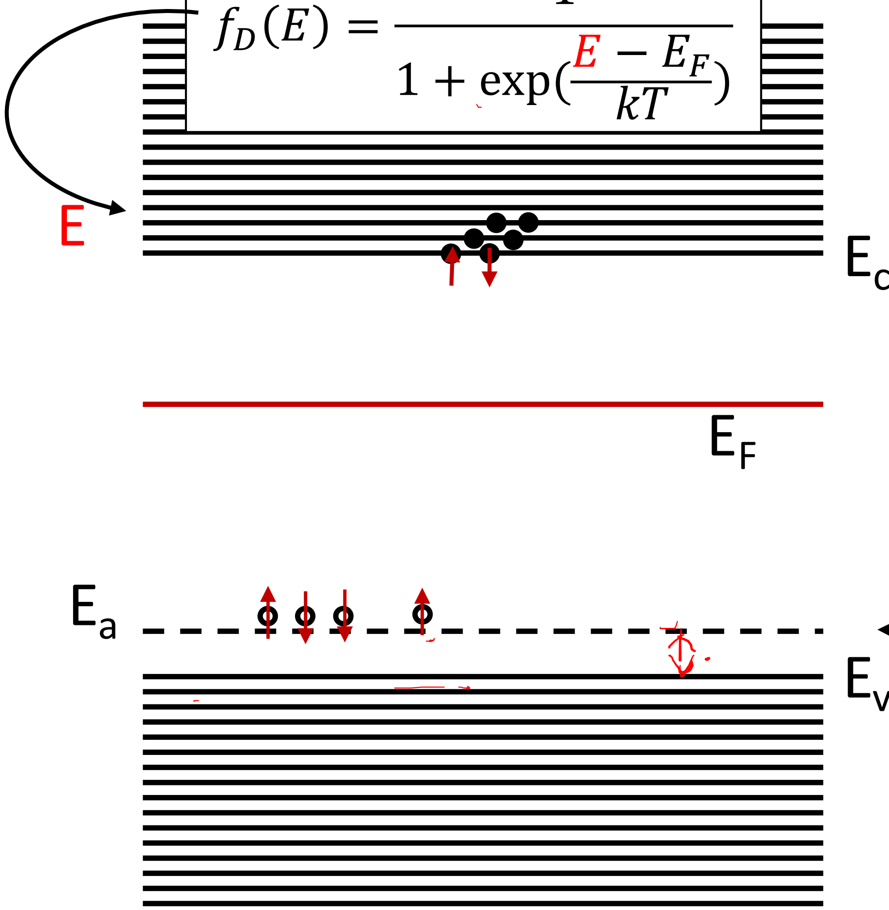
$$n_d = N_d - N_d^+$$

$$= \frac{N_d}{1+\frac{1}{2}\exp(\frac{E_d-E_F}{kT})}$$

# 4.4 Statistics of donors and acceptors

<u>Probability function</u>

$$f_D(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$




The concentration of holes on these acceptors is $n_d$

$$p_a = N_a - N_a^-$$

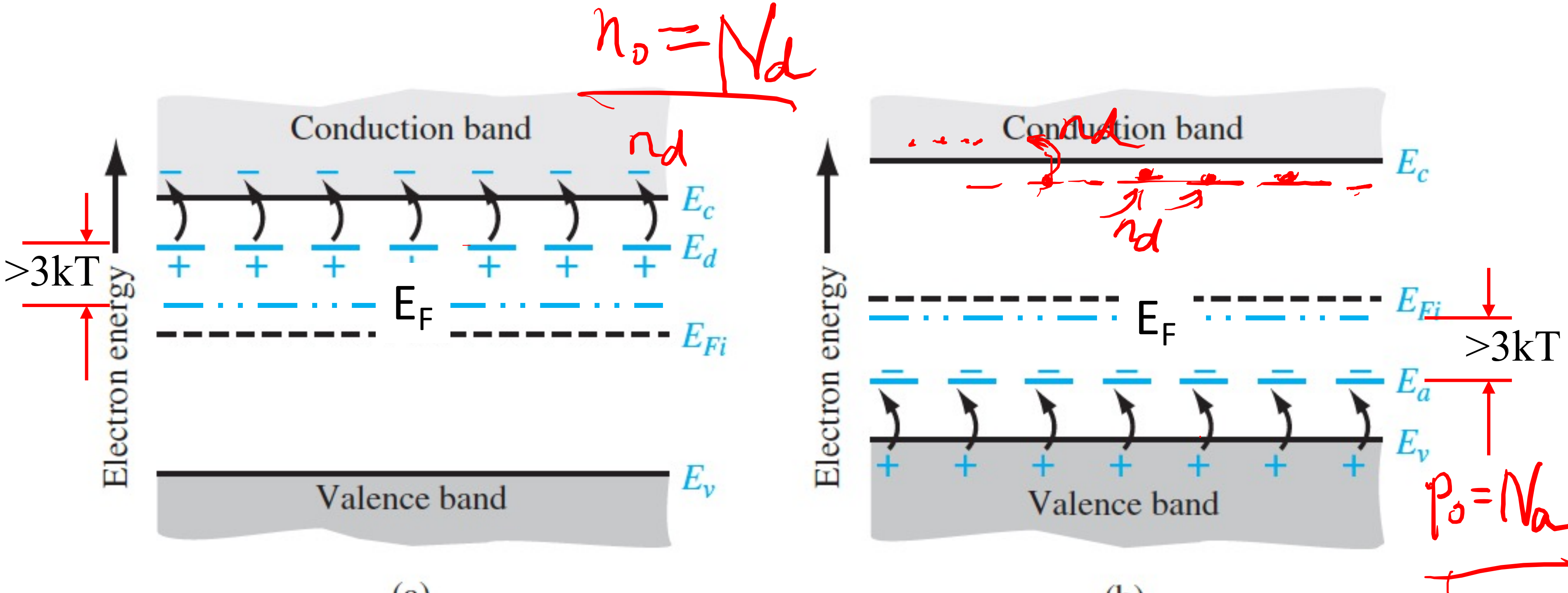
$$= \frac{N_d}{1 + \frac{1}{g}\exp\left(\frac{E_d - E_F}{kT}\right)}$$

(g=4 for Si, GaAs …)

Given the concentration of acceptors is $N_a$

# 4.4 Statistics of donors and acceptors

## Complete ionization



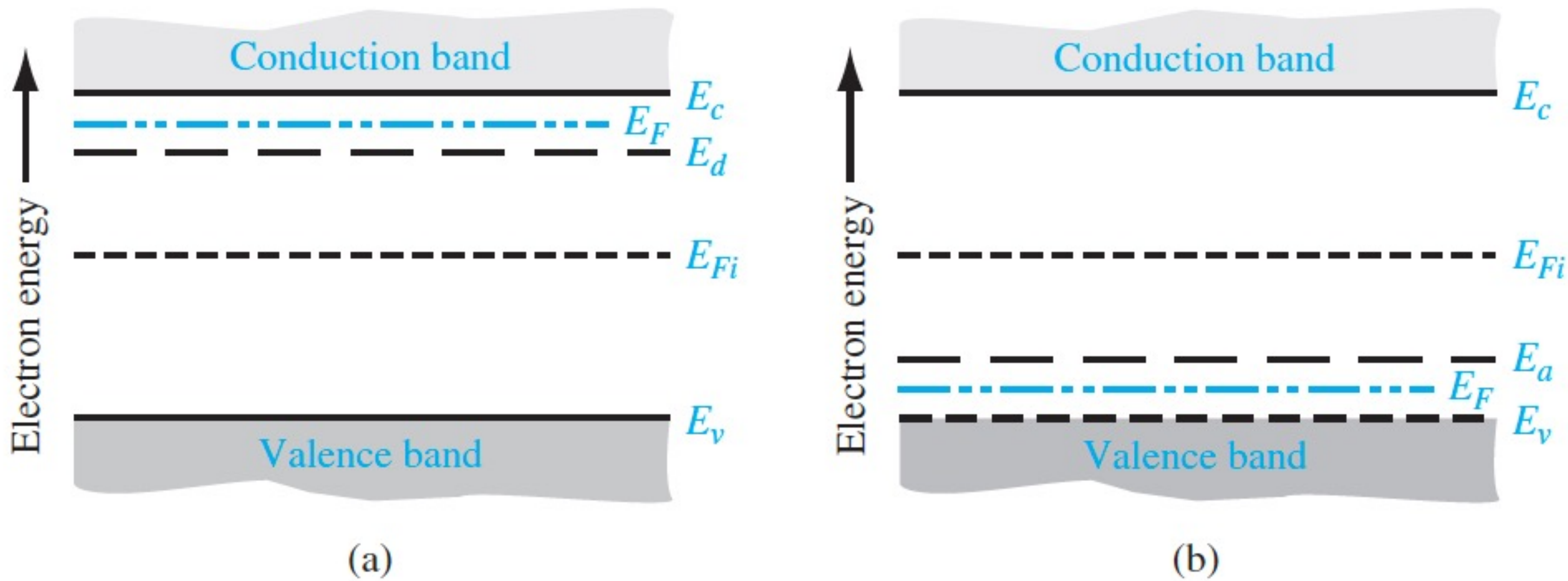

(a) (b)

$$\underline{n_d} = \underline{N_d} - \underline{N_d^+} = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} = 2N_d \exp\left(-\frac{E_d - E_F}{kT}\right)$$

>10

# 4.4 Statistics of donors and acceptors

Complete freeze-out




$$n_d = N_d - N_d^+ = \frac{N_d}{1 + \frac{1}{2}\exp\left(\frac{E_d - E_F}{kT}\right)} \approx N_d$$

$\rightarrow 0$