# VE320 Introduction to Semiconductor Physics and Devices

Recitation Class

VE320 Teaching Group SU2022

University of Michigan-Shanghai Jiaotong University
Joint Institute

June 28, 2022

# Contents

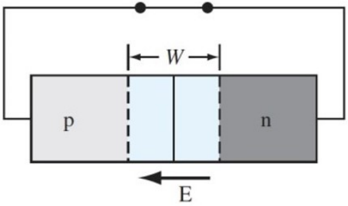
# Notations

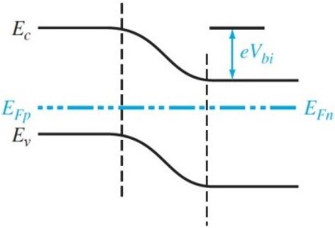
| | |
|---|---|
| $N_a$ | Acceptor concentration in the p region of the pn junction |
| $N_d$ | Donor concentration in the $n$ region of the pn junction |
| $n_{n0} = N_d$ | Thermal-equilibrium majority carrier electron concentration in the n region |
| $p_{p0} = N_a$ | Thermal-equilibrium majority carrier hole concentration in the p region |
| $n_{p0} = n_i^2 / N_a$ | Thermal-equilibrium minority carrier electron concentration in the p region |
| $p_{n0} = n_i^2 / N_d$ | Thermal-equilibrium minority carrier hole concentration in the n region |
| $n_p$ | Total minority carrier electron concentration in the p region |
| $p_n$ | Total minority carrier hole concentration in the n region |
| $n_p(-x_p)$ | Minority carrier electron concentration in the p region at space charge edge |
| $p_n(x_n)$ | Minority carrier hole concentration in the n region at space charge edge |
| $\delta n_p = n_p - n_{p0}$ | Excess minority carrier electron concentration in the p region |
| $\delta p_n = p_n - p_{n0}$ | Excess minority carrier hole concentration in the n region |

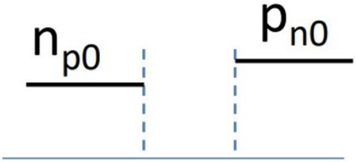
# Ideal Assumptions

- The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.
- The Maxwell-Boltzmann approximation applies to carrier statistics.
- The concepts of low injection and complete ionization apply.
- The total current is a constant throughout the entire pn structure.
- The individual electron and hole currents are continuous functions through the pn structure.
- The individual electron and hole currents are constant throughout the depletion region.

# Concentration Relation on Two Sides



# Concentration Relation on Two Sides

$$V_{bi} = \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

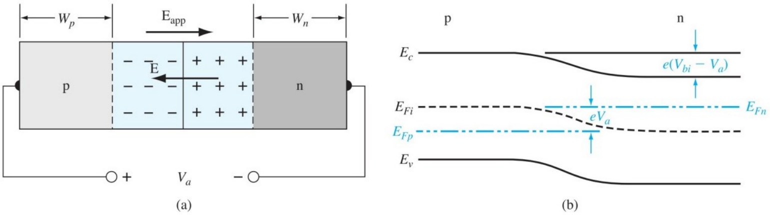
$$n_{n0} = N_d$$

$$n_{p0} = \frac{n_i^2}{N_a}$$

$$\Rightarrow \quad n_p(-x_p) = n_{p0} = n_{n0} \exp\left(-\frac{eV_{bi}}{kT}\right)$$

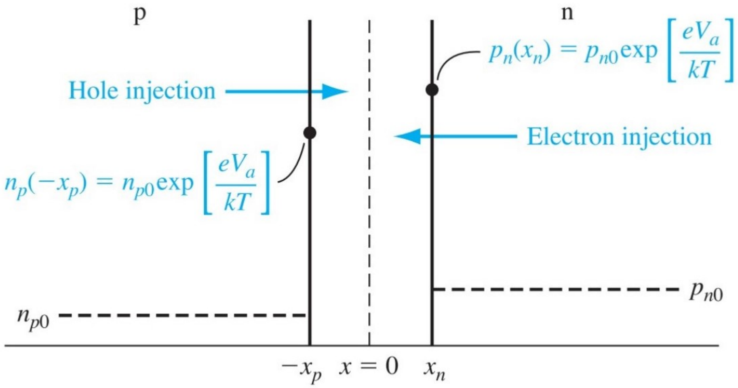
Relates the minority carrier electron concentration on the p side of the junction to the majority carrier electron concentration on the n side of the junction in thermal equilibrium.

# Forward Biased




$$n_p(-x_p) = n_{n0} \exp\left(-\frac{e\left(V_{bi} - V_a\right)}{kT}\right)$$
$$= n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

# Forward Biased



# Minority Carrier Distribution

$$D_p \frac{\mathrm{d}^2 (\delta p_n)}{\mathrm{d}x^2} - \mu_p \left( E \frac{\mathrm{d}(\delta p_n)}{\mathrm{d}x} + p \frac{\mathrm{d}E}{\mathrm{d}x} \right) + g' - \frac{\delta p_n}{\tau_{pt}} = 0$$

Assumption: the electric field is zero in both the neutral $p$ and $n$ regions. In $n$ region for $x > x_n$, we have $g' = 0$. The equation becomes

$$\frac{\mathrm{d}^2 (\delta p_n)}{\mathrm{d}x^2} - \frac{\delta p_n}{L_p^2} = 0, \quad (x > x_n), \quad L_n^2 = D_n \tau_{n0}$$

Solve it with boundary conditions

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

$$p_n(x \to +\infty) = p_{n0}$$

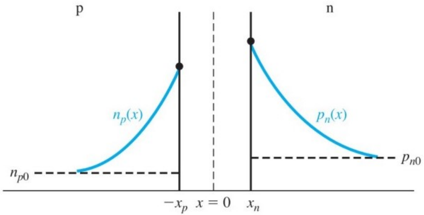
# Minority Carrier Distribution

The solution is

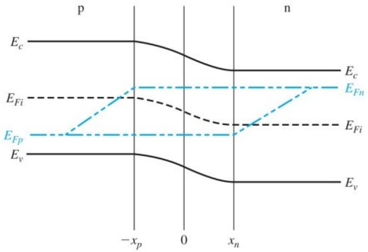
$$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right), \quad x \geq x_n$$

Similarly,

$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right), \quad x \leq -x_p$$



# Quasi-Fermi Level



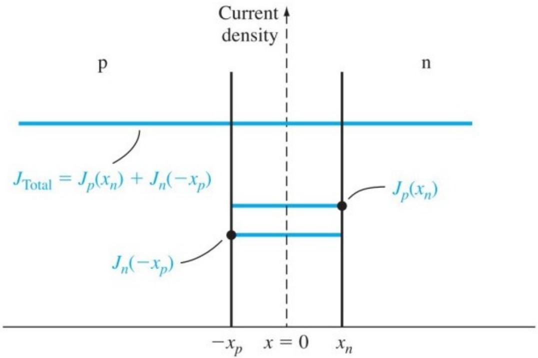

$$p = p_0 + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Therefore the quasi-Fermi levels are linear functions of distance in the neutral $p$ and $n$ regions. Also,

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

# Ideal pn Junction Current

Assumption 4(a): The total current is a constant throughout the entire pn structure.




$$J_p(x_n) = -eD_p \frac{\mathrm{d}(\delta p_n(x))}{\mathrm{d}x}\bigg|_{x=x_n}$$

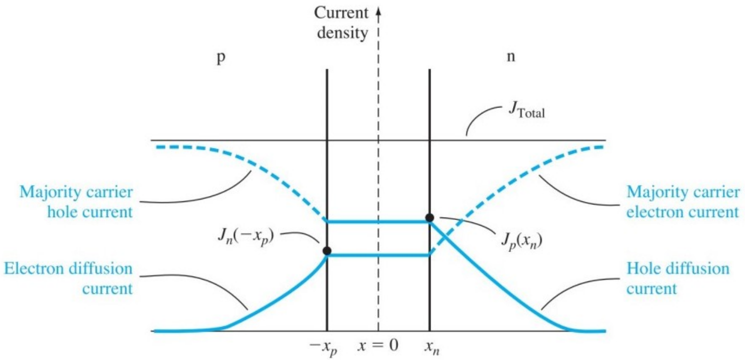
# Ideal pn Junction Current

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$
$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Then the total current density

$$J = J_s\left[\exp\left(\tfrac{eV_a}{kT}\right) - 1\right]$$
$$\text{where } J_s = \left[\tfrac{eD_p p_{n0}}{L_p} + \tfrac{eD_n n_{p0}}{L_n}\right]$$

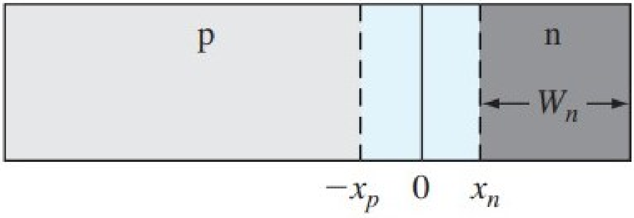
# Ideal pn Junction Current




The above figure shows idea electron and hole current components through a pn junction under forward bias.

# Example: Short Diode

We assumed in the previous analysis that both p and n regions were long compared with the minority carrier diffusion lengths. In many pn junction structures, one region may, in fact, be short compared with the minority carrier diffusion length. Assume the length $W_n$ is much smaller than the minority carrier hole diffusion length, $L_p$. Assume an infinite surface recombination velocity and therefore an excess minority carrier concentration of zero. Calculate the minority carrier hole diffusion current density $J_p$.



# Example: Short Diode

The steady-state excess minority carrier hole concentration in the $n$ region:

$$\frac{d^2\left(\delta p_n\right)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

Boundary conditions:

$$p_n\left(x_n\right) = p_{n0}\exp\left(\frac{eV_a}{kT}\right)$$

$$p_n\left(x = x_n + W_n\right) = p_{n0}$$

The general solution is then

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \quad (x \geq x_n)$$

# Example: Short Diode

We get the solution

$$\delta p_n(x) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\frac{\sinh\left[\left(x_n + W_n - x\right)/L_p\right]}{\sinh\left[W_n/L_p\right]}$$

If $W_n \ll L_p$, we can approximate the hyperbolic sine terms by

$$\sinh\left(\frac{x_n + W_n - x}{L_p}\right) \approx \left(\frac{x_n + W_n - x}{L_p}\right)$$

and

$$\sinh\left(\frac{W_n}{L_p}\right) \approx \left(\frac{W_n}{L_p}\right)$$

Then

$$\delta p_n(x) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\left(\frac{x_n + W_n - x}{W_n}\right)$$

# Example: Short Diode

The minority carrier hole diffusion current density is given by

$$J_p = -eD_p \frac{d\left[\delta p_n(x)\right]}{dx}$$

so that in the short $n$ region, we have

$$J_p(x) = \frac{eD_p p_{n0}}{W_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

# Questions?