
VE320 – Summer 2021

Introduction to Semiconductor Devices

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Chapter 11 Metal-Oxide-Semiconductor Field Effect
Transistors: More Concepts



Outline

Nonideal Effects:

11.1 Channel length modulation

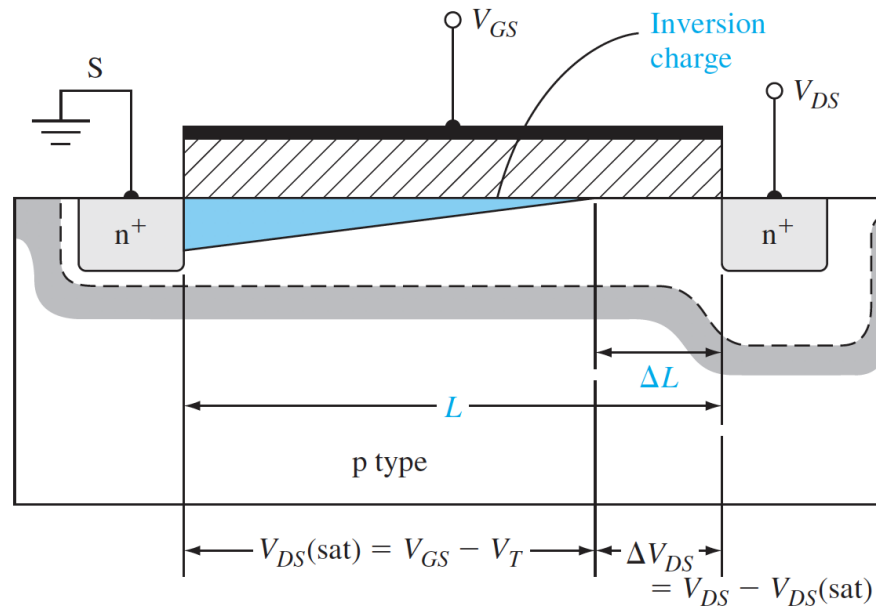
11.2 Conductance and transconductance

11.3 Subthreshold conduction

11.4 Velocity Saturation

11.5 Short Channel Effect

11.1 Channel length modulation



$$I_D' = \frac{k_n'}{2} \cdot \frac{W}{L} \cdot [(V_{GS} - V_T)^2 (1 + \lambda V_{DS})]$$

11.1 Channel length modulation

$$I_D = \begin{cases} 0 & V_{GS} - V_T < 0 \\ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 & 0 \leq V_{GS} - V_T < V_{DS} \\ \mu_n C_{ox} \frac{W}{L} [(V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2] & V_{GS} - V_T \geq V_{DS} \end{cases}$$

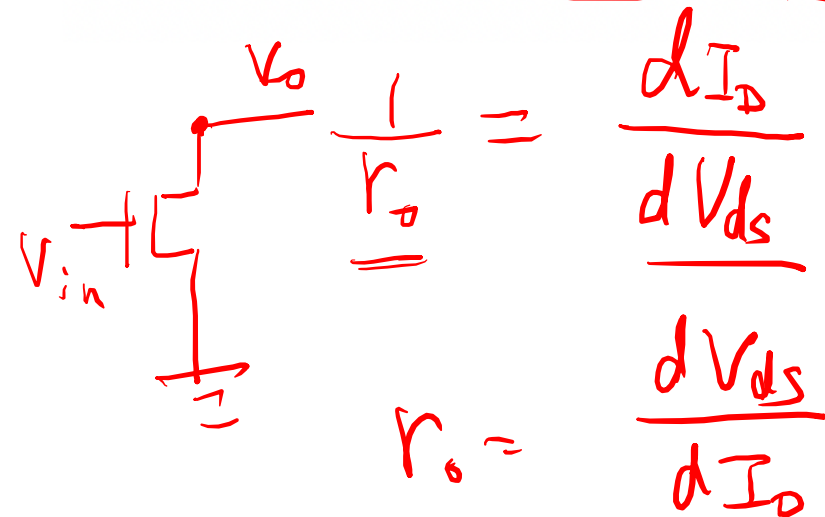


$$I_D = \begin{cases} 0 & V_{GS} - V_T < 0 \\ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) & 0 \leq V_{GS} - V_T < V_{DS} \\ \mu_n C_{ox} \frac{W}{L} [(V_{GS} - V_T) V_{DS} - \frac{1}{2} V_{DS}^2] & V_{GS} - V_T \geq V_{DS} \end{cases}$$

Check your understanding

Problem Example 1

Consider an n-channel silicon MOSFET. The parameters are $k'_n = 75 \mu\text{A}/\text{V}^2$, $W/L = 10$, and $V_T = 0.35 \text{ V}$. The applied drain-to-source voltage is $V_{DS} = 1.5 \text{ V}$. (a) For $V_{GS} = 0.8 \text{ V}$, find (i) the ideal drain current, (ii) the drain current if $\lambda = 0.02 \text{ V}^{-1}$, and (iii) the output resistance for $\lambda = 0.02 \text{ V}^{-1}$. (b) Repeat part (a) for $V_{GS} = 1.25 \text{ V}$.



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Nonideal Effects:

11.1 Channel length modulation

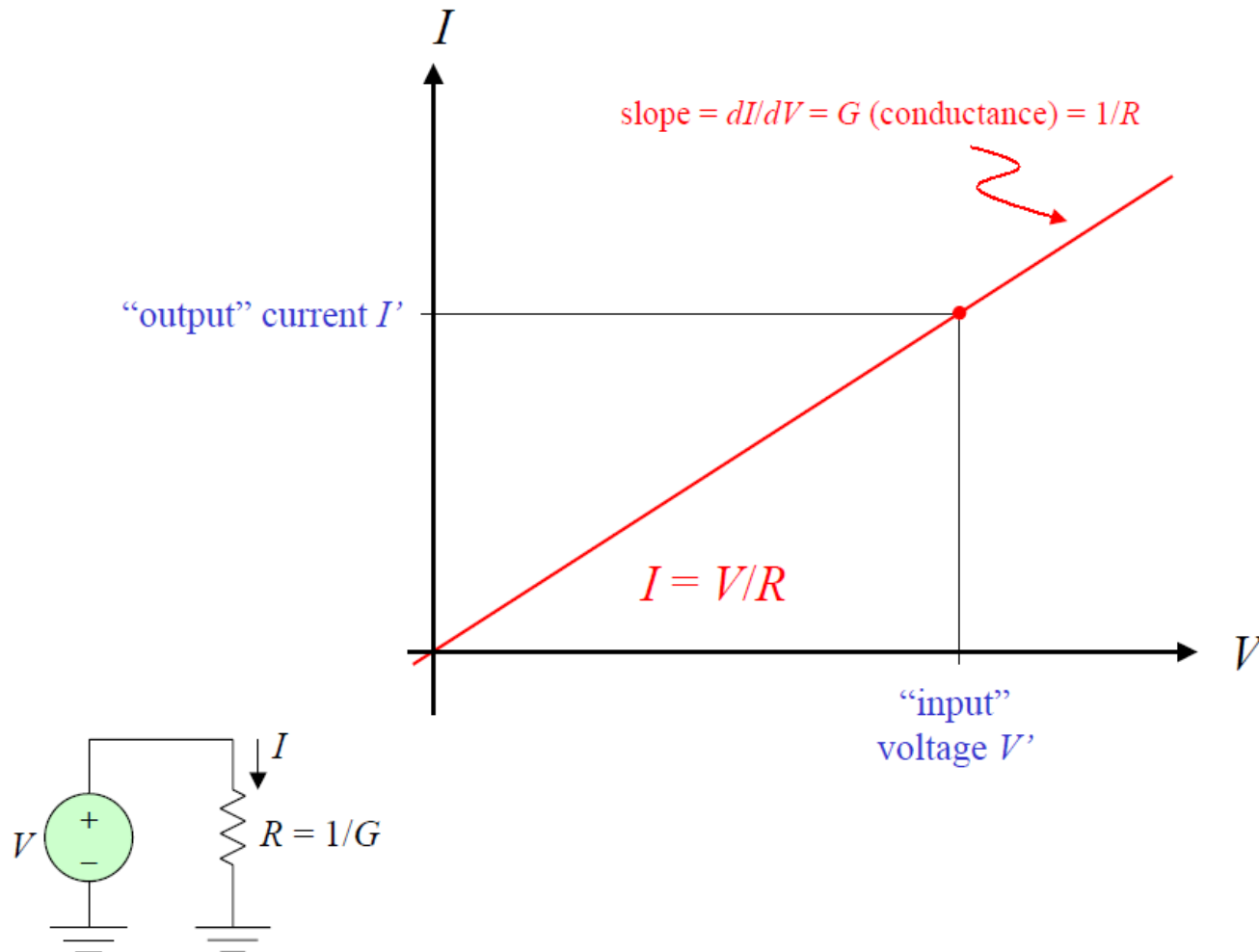
11.2 Conductance and transconductance

11.3 Subthreshold conduction

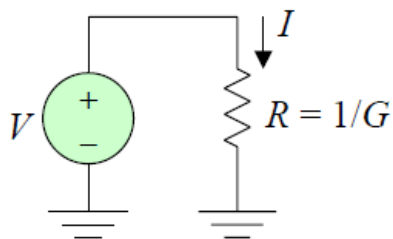
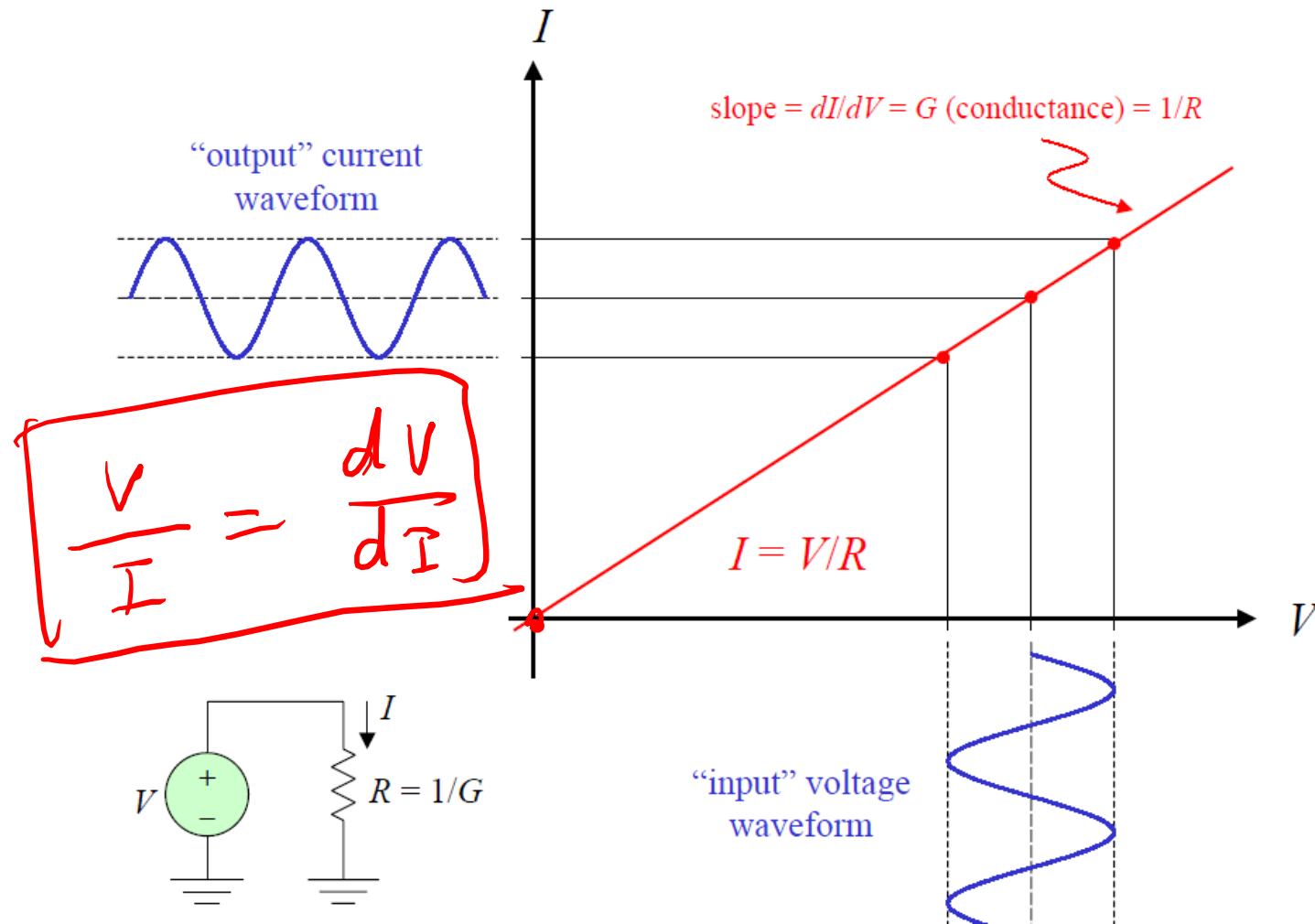
11.4 Velocity Saturation

11.5 Short Channel Effect

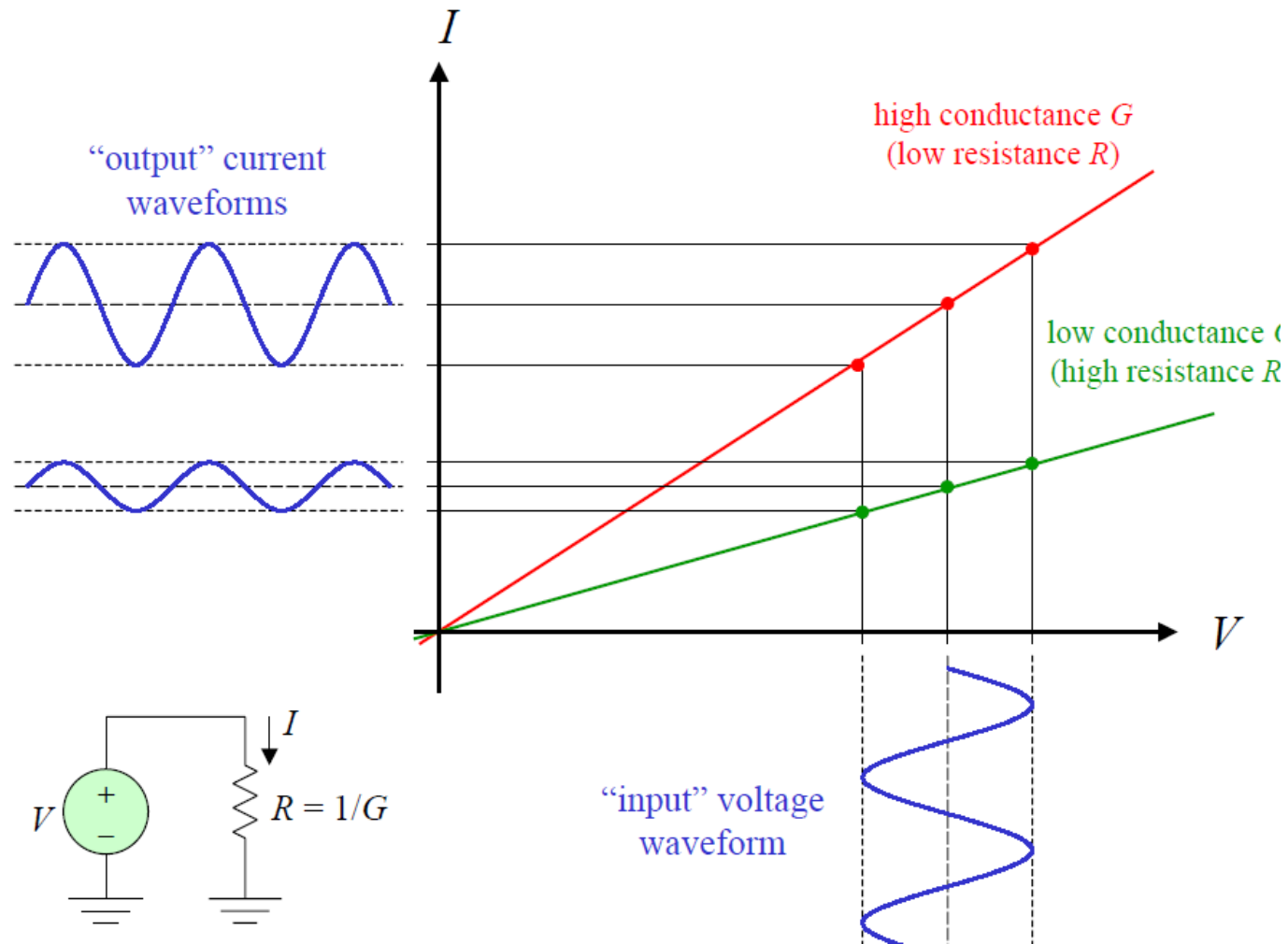
11.2 conductance and transconductance



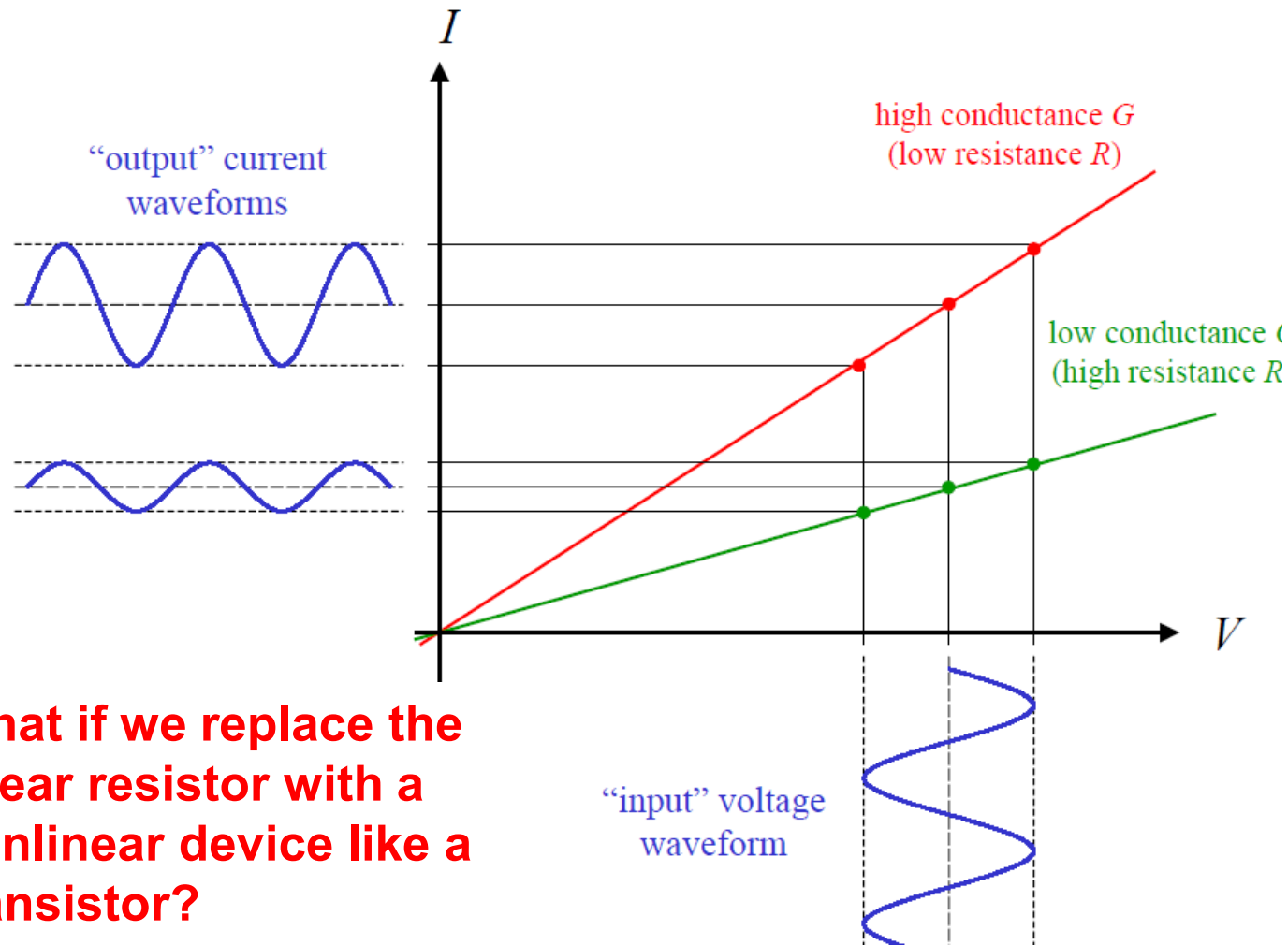
11.2 conductance and transconductance



11.2 conductance and transconductance

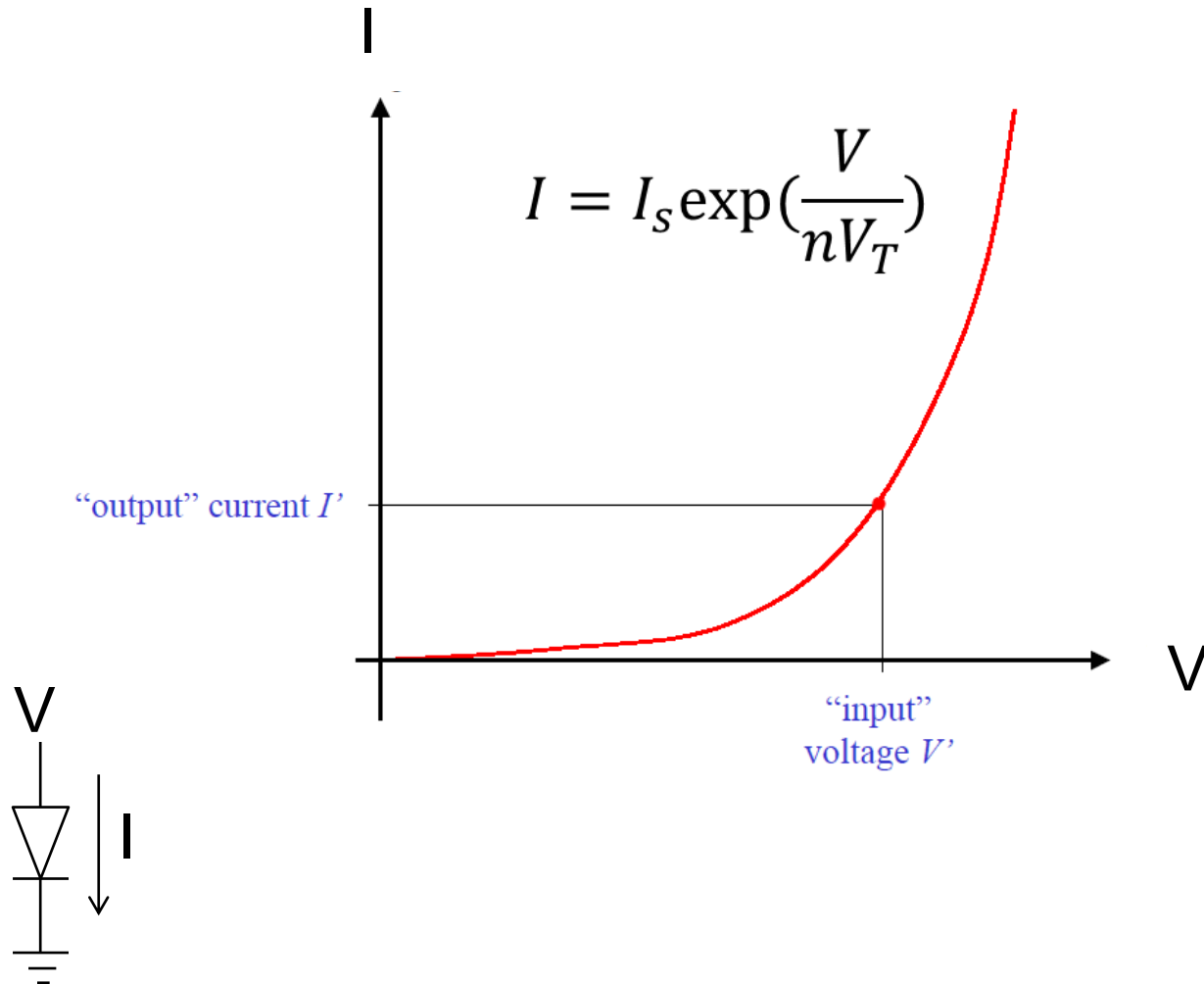


11.2 conductance and transconductance

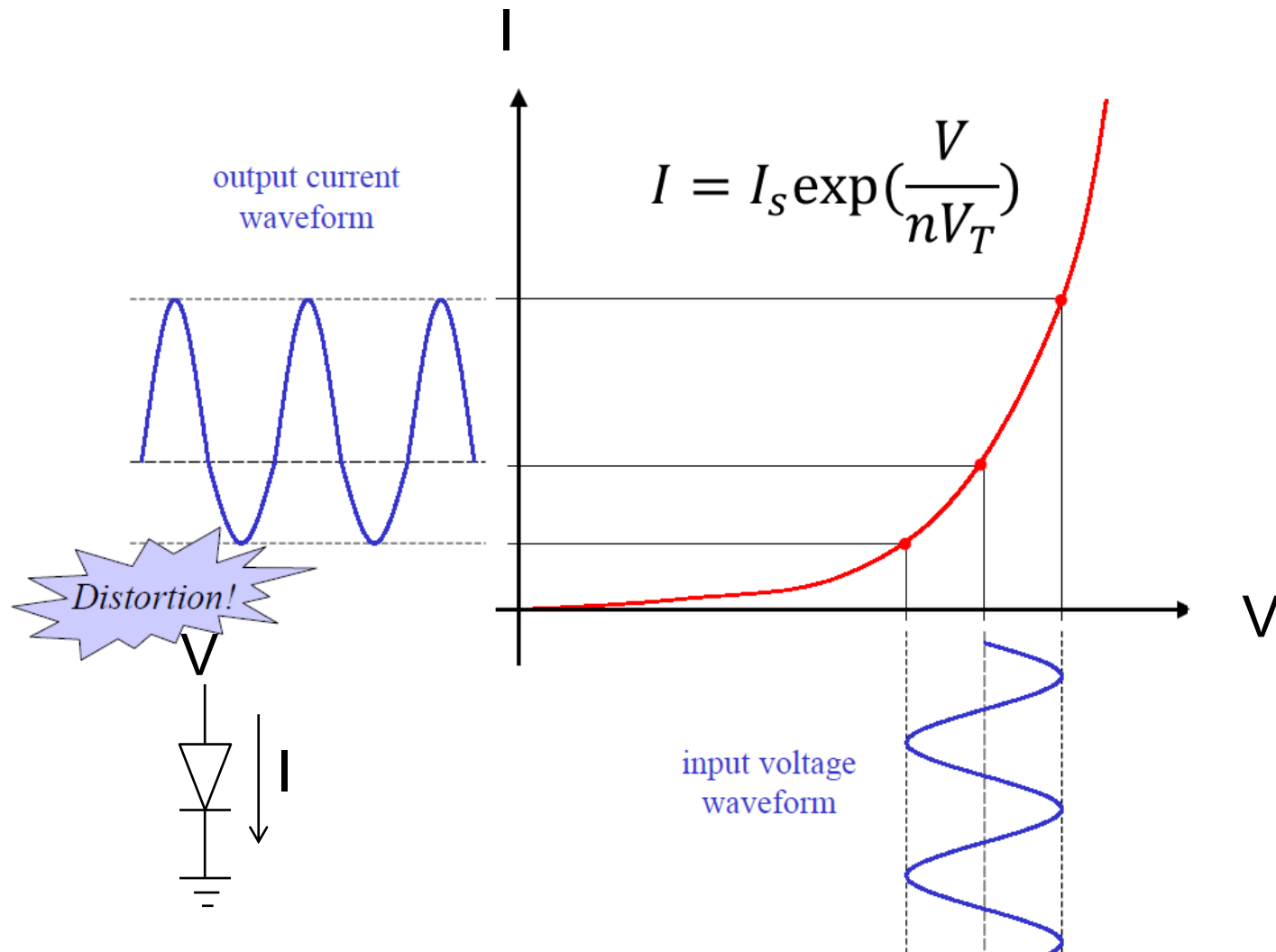


What if we replace the linear resistor with a nonlinear device like a transistor?

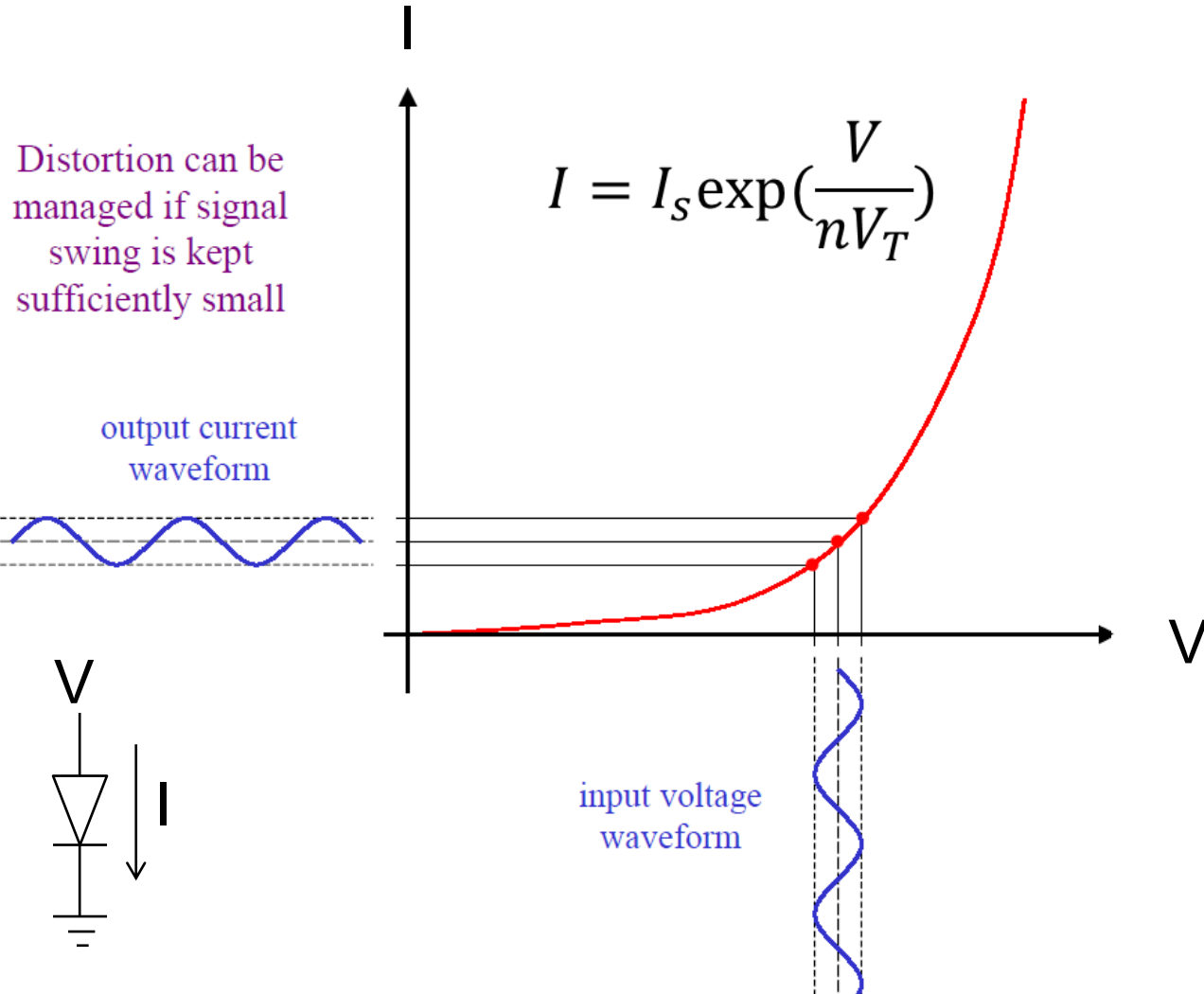
11.2 conductance and transconductance



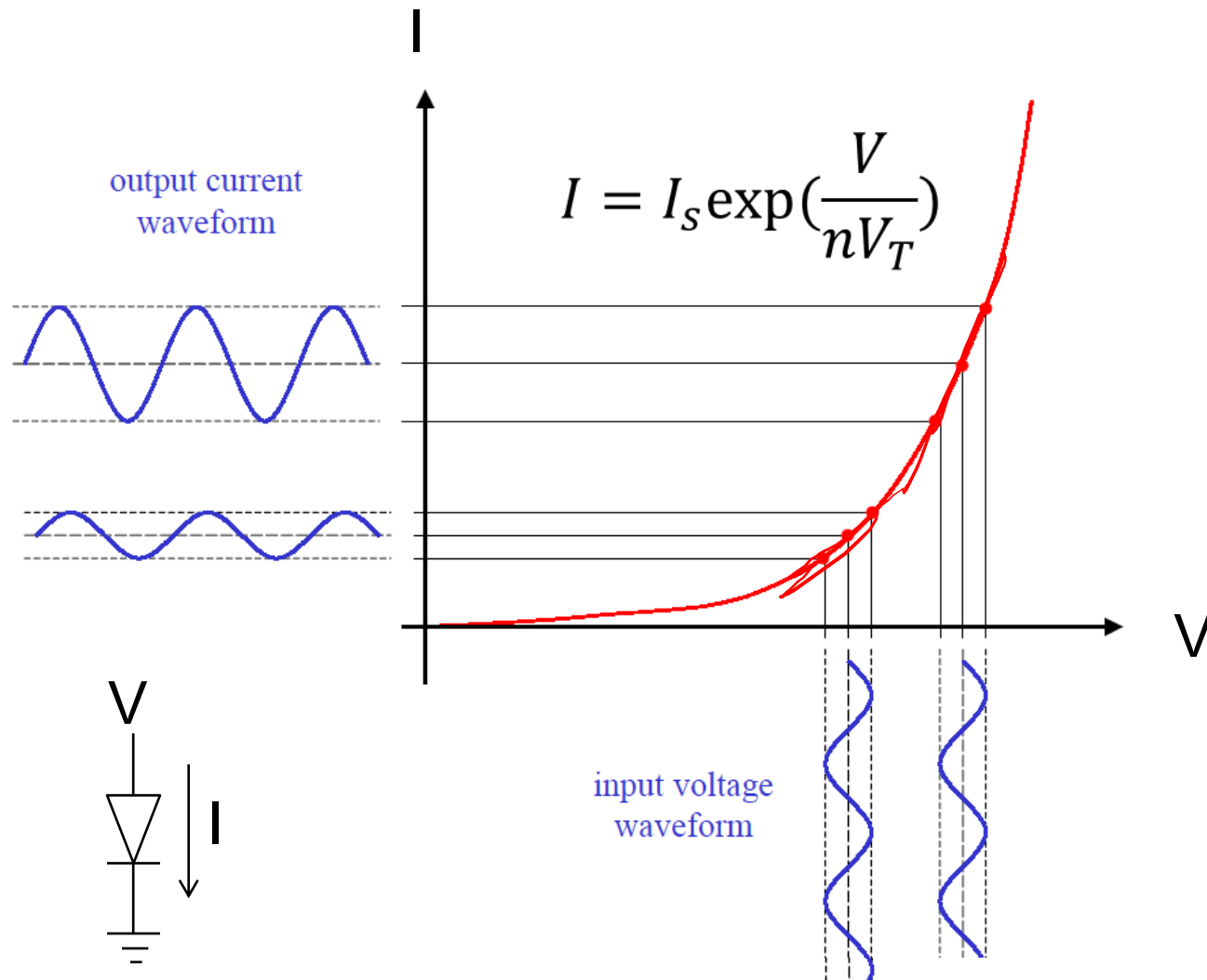
11.2 conductance and transconductance



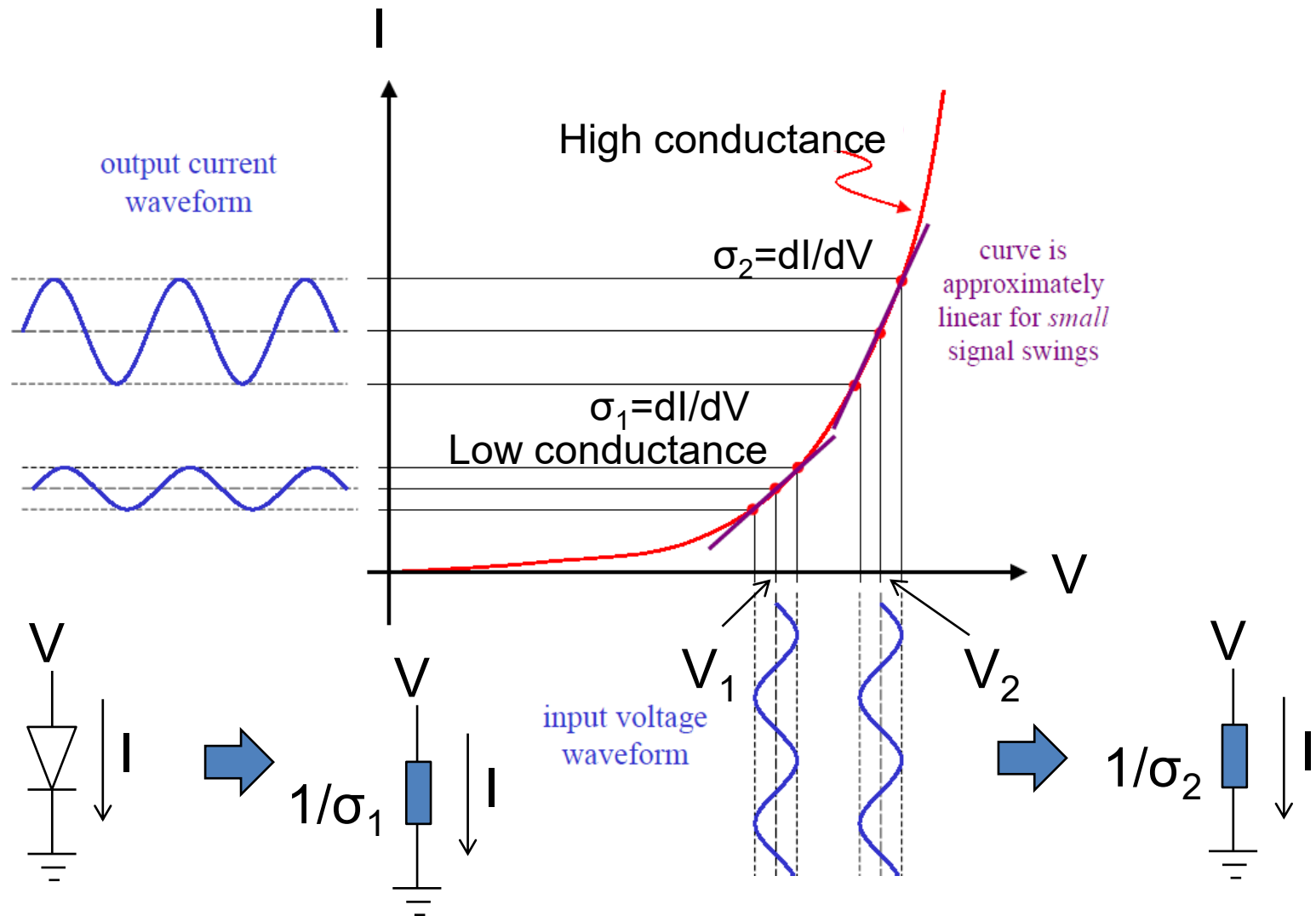
11.2 conductance and transconductance



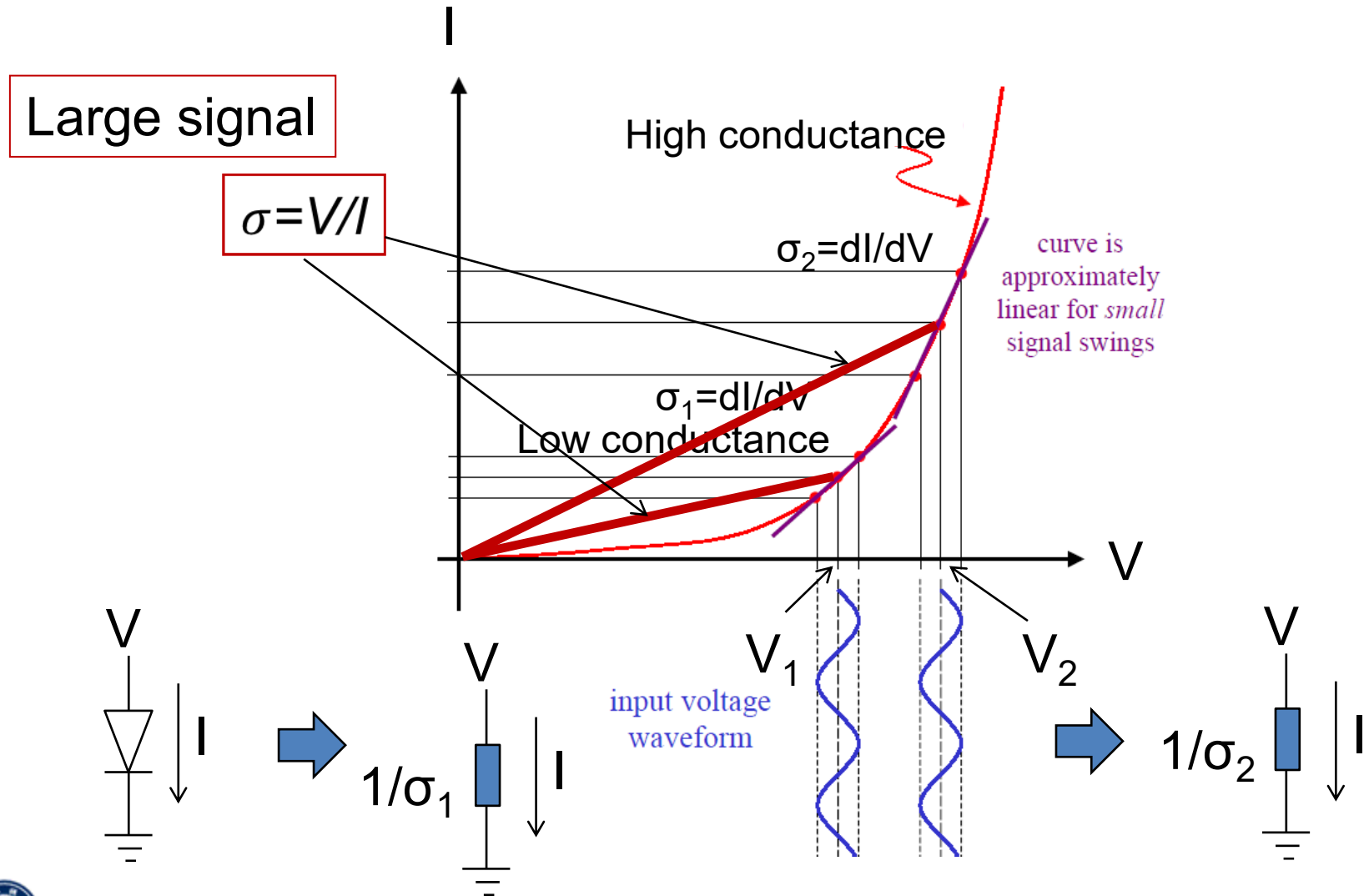
11.2 conductance and transconductance



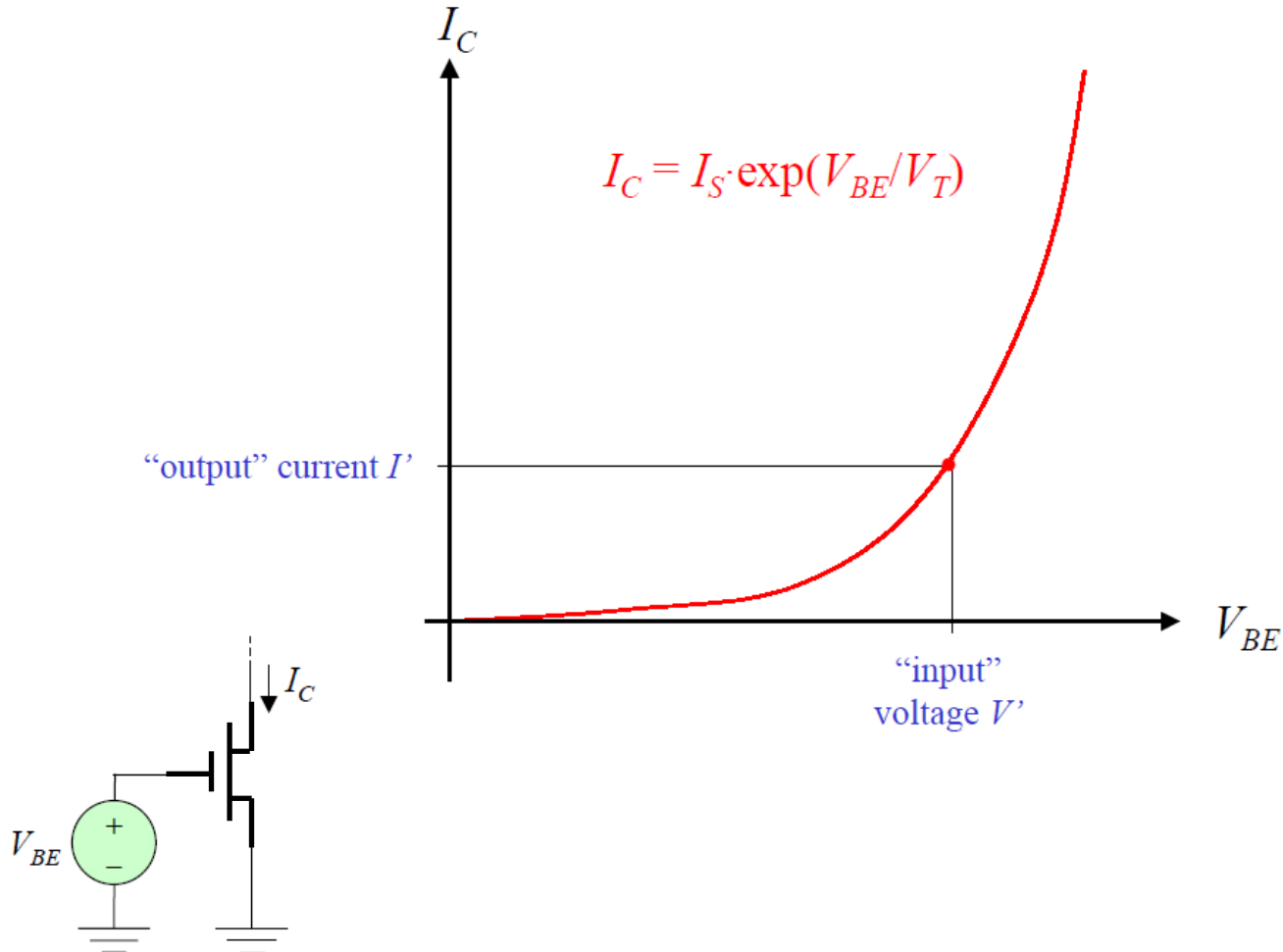
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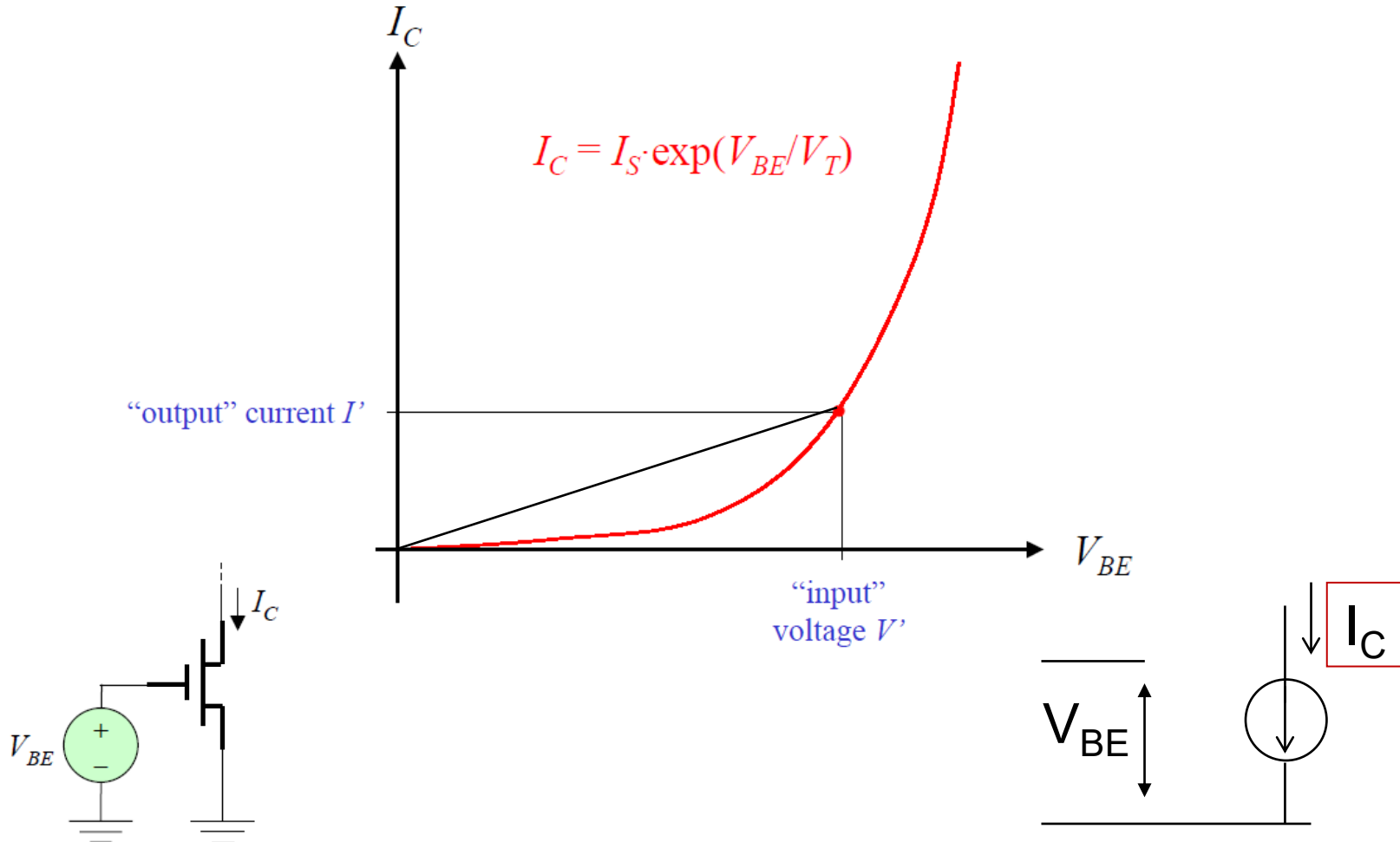
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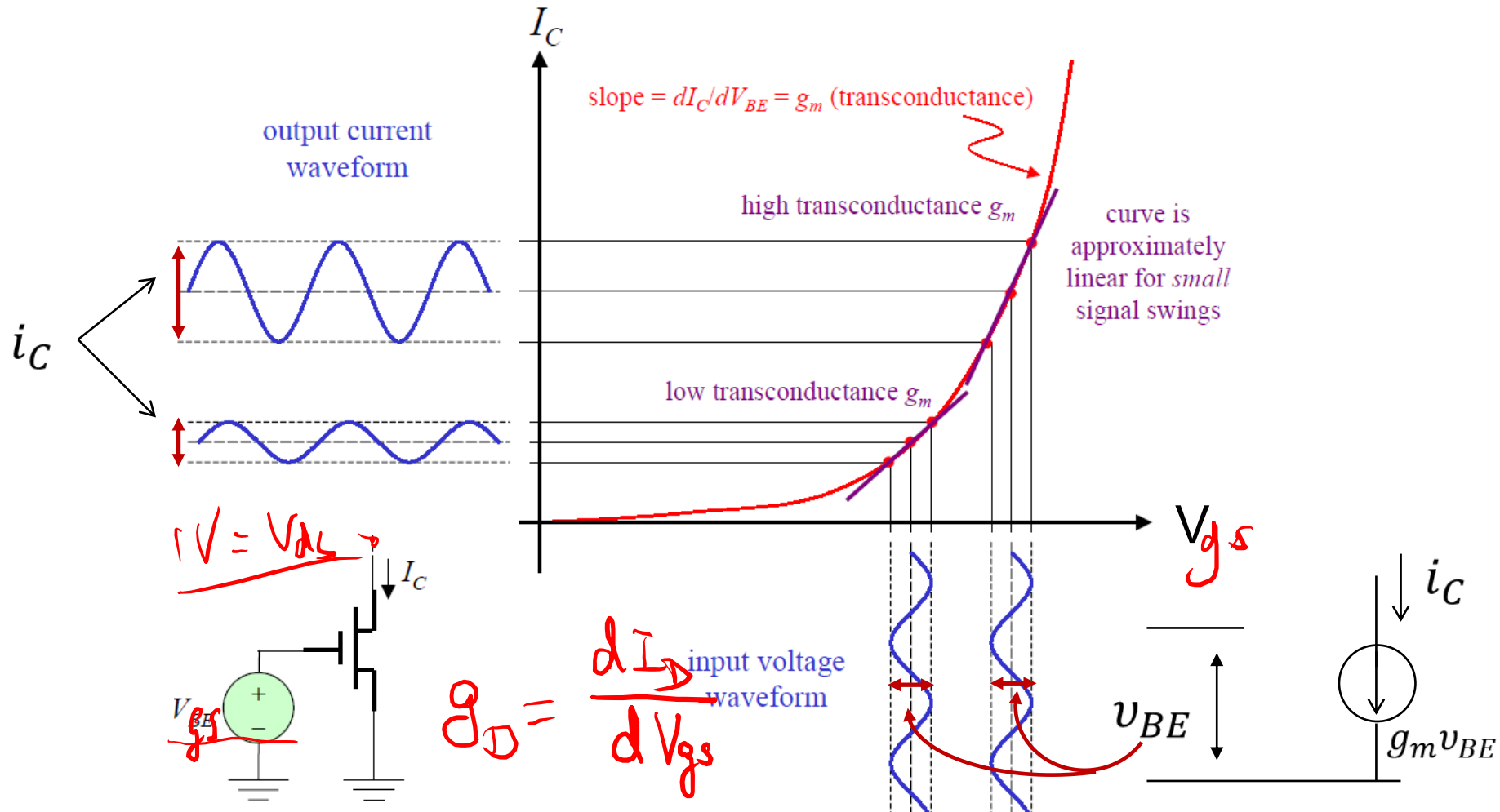
11.2 conductance and transconductance



11.2 conductance and transconductance



11.2 conductance and transconductance



11.2 conductance and transconductance

$$I_d = \begin{cases} 0 & V_{gs} - V_T < 0 \\ \frac{1}{2} k_n (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) & 0 \leq V_{gs} - V_T < V_{ds} \\ k_n [(V_{gs} - V_T) V_{ds} - \frac{1}{2} V_{ds}^2] & V_{gs} - V_T \geq V_{ds} \end{cases}$$

output conductance $g_D = \frac{dI_D}{dV_{ds}} = \begin{cases} \frac{1}{2} k_n (V_{gs} - V_T)^2 \cdot \lambda V_{ds} \\ k_n [V_{gs} - V_T - V_{ds}] \end{cases}$

transconductance $g_m = \frac{dI_D}{dV_{gs}} = \begin{cases} k_n (V_{gs} - V_T) (1 + \lambda V_{ds}) \\ k_n \cdot V_{ds} \end{cases} = \frac{2I_D}{1 + \lambda V_{ds}}$

11.2 conductance and transconductance

Transconductance

$$I_d = \begin{cases} 0 & V_{gs} - V_T < 0 \\ \frac{1}{2} k_n (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) & 0 \leq V_{gs} - V_T < V_{ds} \\ k_n [(V_{gs} - V_T) V_{ds} - \frac{1}{2} V_{ds}^2] & V_{gs} - V_T \geq V_{ds} \end{cases}$$

11.2 conductance and transconductance

Output Impedance

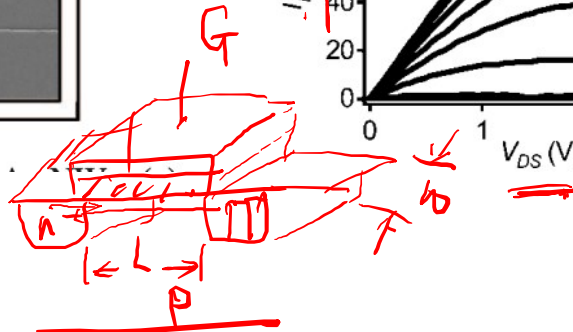
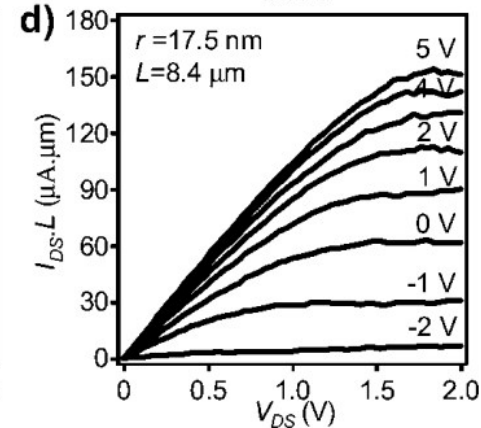
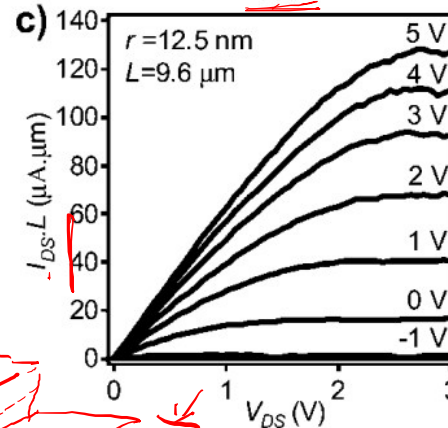
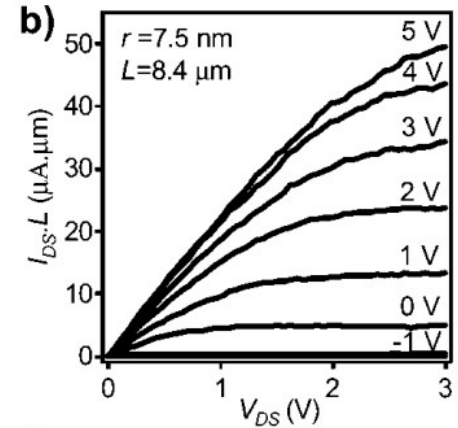
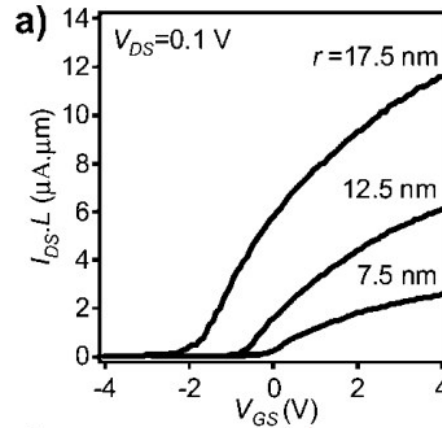
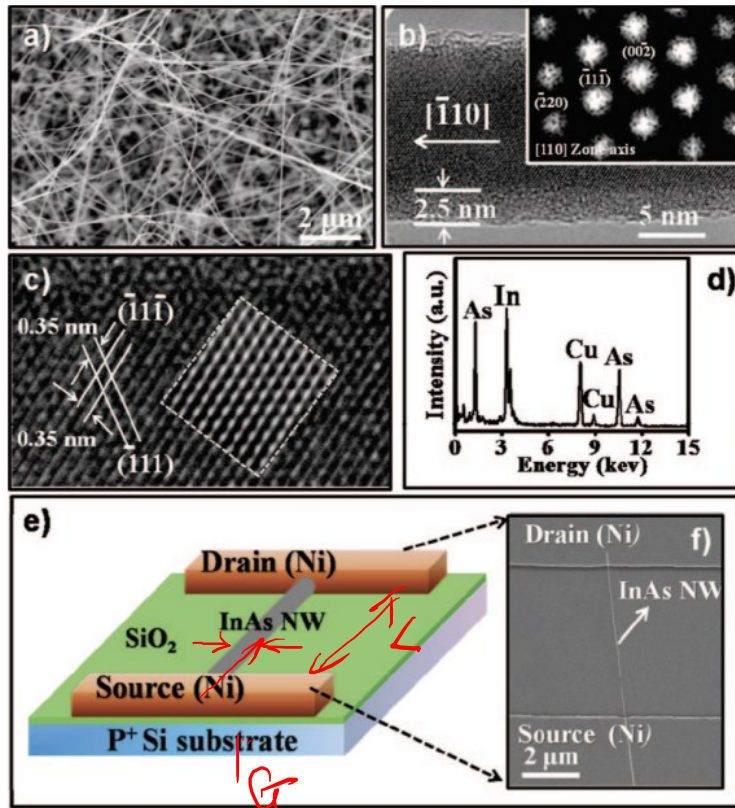
$$I_d = \begin{cases} 0 & V_{gs} - V_T < 0 \\ \frac{1}{2} k_n (V_{gs} - V_T)^2 (1 + \lambda V_{ds}) & 0 \leq V_{gs} - V_T < V_{ds} \\ k_n [(V_{gs} - V_T) V_{ds} - \frac{1}{2} V_{ds}^2] & V_{gs} - V_T \geq V_{ds} \end{cases}$$

Diameter-Dependent Electron Mobility of InAs Nanowires

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Zhiyong Fan,^{‡,\$,||} Jing Guo,[⊥] Jeffrey Bokor,^{‡,\$} and Ali Javey^{*,‡,\$,||}**

Department of Electrical Engineering and Computer Sciences, University of California at Berkeley, Berkeley, California 94720, Materials Sciences Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, Berkeley Sensor and Actuator Center, University of California at Berkeley, Berkeley, California 94720, and Department of Electrical and Computer Engineering, University of Florida, Gainesville, Florida 32611

How the knowledge used by engineers and scientists



How the knowledge used by engineers and scientists

We next assess the field-effect electron mobility of InAs NW FETs by using the low-bias ($V_{DS} = 0.1$ V) transconductance

$$g_m = \left. \frac{dI_{DS}}{dV_{GS}} \right|_{V_{DS}}$$

and the analytical expression

$$\mu_n = g_m \frac{L^2}{C_{ox}} \frac{1}{V_{DS}}$$

$$g_m = k_n V_{DS} = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

total

MOSFETs. The field-effect and effective mobilities are, however, deduced from the I - V characteristics by using different analytical models. Specifically, the effective mobility is deduced from the drain conductance.

$$g_D = \left. \frac{dI_{DS}}{dV_{DS}} \right|_{V_{GS}}$$

with

$$\mu_{n,eff} = g_D \frac{L^2}{C_{ox}} \frac{1}{(V_{GS} - V_t)}$$

$$\mu_n =$$

$$= \frac{g_m}{C_{ox} \cdot \frac{1}{L^2} \cdot V_{DS}} = \frac{g_m}{C_{ox}} \cdot \frac{L^2}{V_{DS}}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$$

$$C_{ox}^t = \frac{\epsilon_{ox} \cdot W \cdot L}{t_{ox}} = C_{ox} \cdot W \cdot L$$

$$C_{ox} = \frac{C_{ox}^t}{W \cdot L}$$

$$g_D = \mu_n \cdot C_{ox} \cdot \frac{W}{L} (V_{GS} - V_t)$$

Check your understanding

Problem example 2

The transconductance of an n-channel MOSFET is found to be $g_m = \partial I_D / \partial V_{GS} = 1.25 \text{ mA/V}$ when measured at $V_{DS} = 50 \text{ mV}$. The threshold voltage is $V_T = 0.3 \text{ V}$. (a) Determine the conductance parameter K_n . (b) What is the current at $V_{GS} = 0.8 \text{ V}$ and $V_{DS} = 50 \text{ mV}$? (c) Determine the current at $V_{GS} = 0.8 \text{ V}$ and $V_{DS} = 1.5 \text{ V}$.

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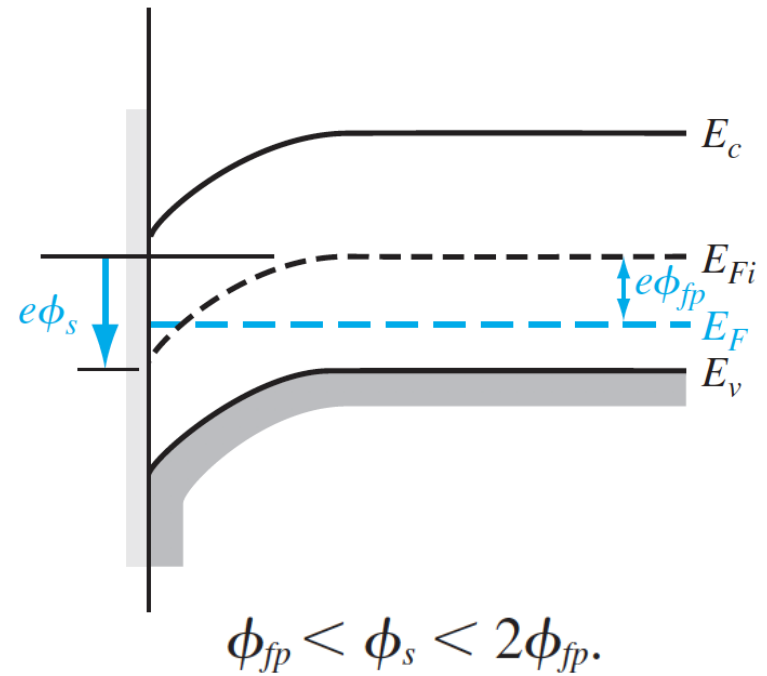
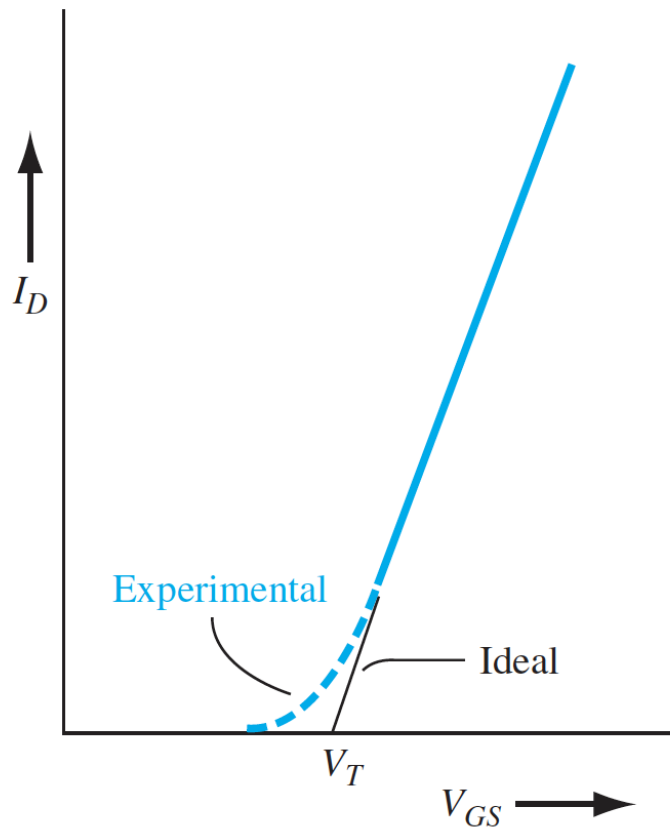
11.3 Subthreshold conduction

11.4 Velocity Saturation

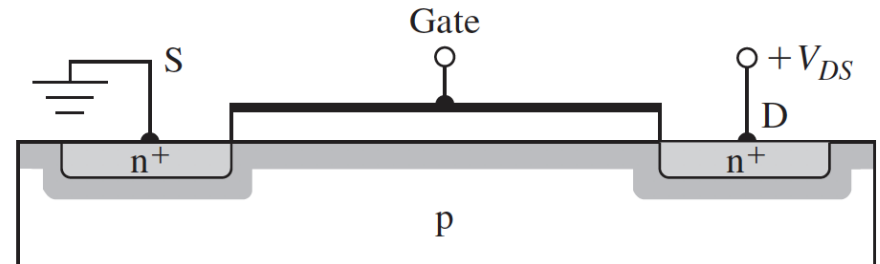
11.5 Short Channel Effect

11.3 Subthreshold conduction

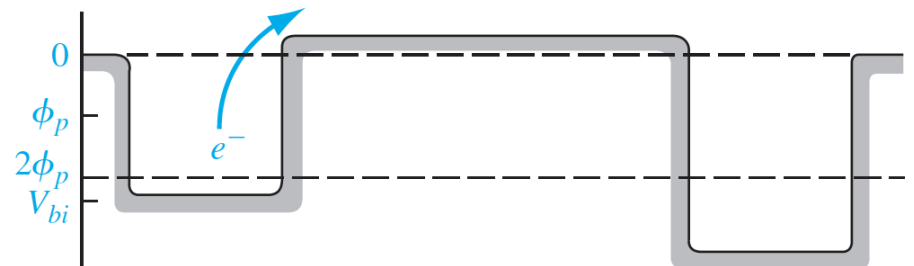
$$V_{GS} < V_T$$



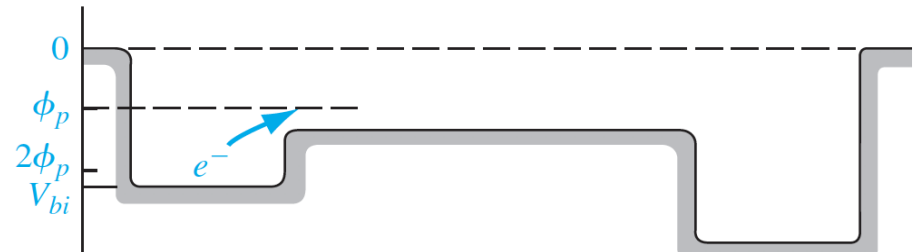
11.3 Subthreshold conduction



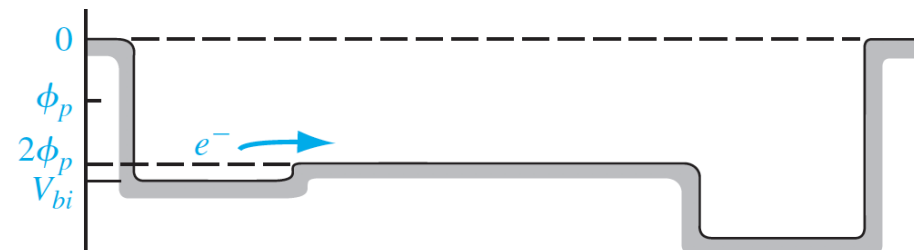
Accumulation



Weak inversion



Inversion



11.3 Subthreshold conduction

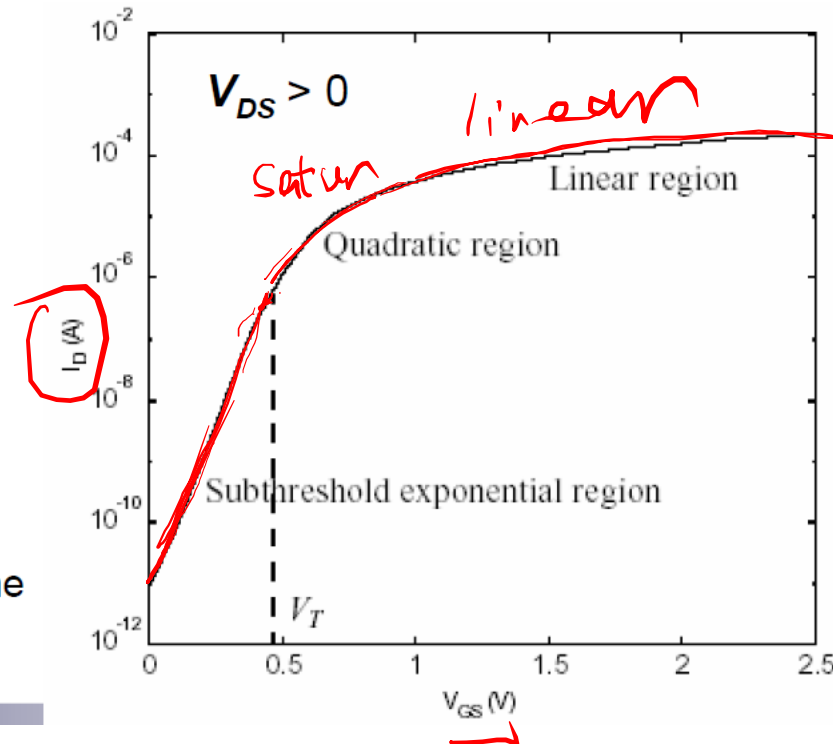
Subthreshold Conduction (Leakage Current)

- The transition from the ON state to the OFF state is gradual. This can be seen more clearly when I_D is plotted on a logarithmic scale:

- In the subthreshold ($V_{GS} < V_T$) region,

$$I_D \propto \exp\left(\frac{qV_{GS}}{nkT}\right)$$

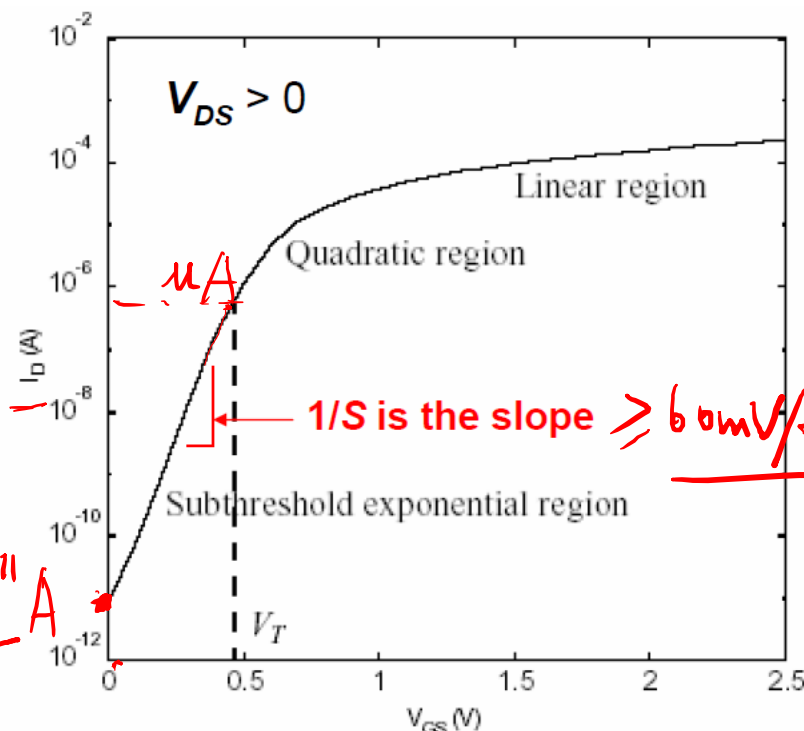
$C_{ox} C_s$ This is essentially the channel-source pn junction current. (Some electrons diffuse from the source into the channel, if this pn junction is forward biased.)



11.3 Subthreshold conduction

Slope Factor (or Subthreshold Swing) S

- S is defined to be the inverse slope of the $\log(I_D)$ vs. V_{GS} characteristic in the subthreshold region:



$$S \equiv n \left(\frac{kT}{q} \right) \ln(10)$$

Units: Volts per decade

Note that $S \geq 60 \text{ mV/dec}$ at room temperature:

$$\left(\frac{kT}{q} \right) \ln(10) = 60 \text{ mV}$$

$$10^{-11} \times 10^9 = 10^{-2} \text{ A} \times 2.5 \text{ V} = 25 \text{ mW}$$

11.3 Subthreshold conduction

V_T Design Trade-Off

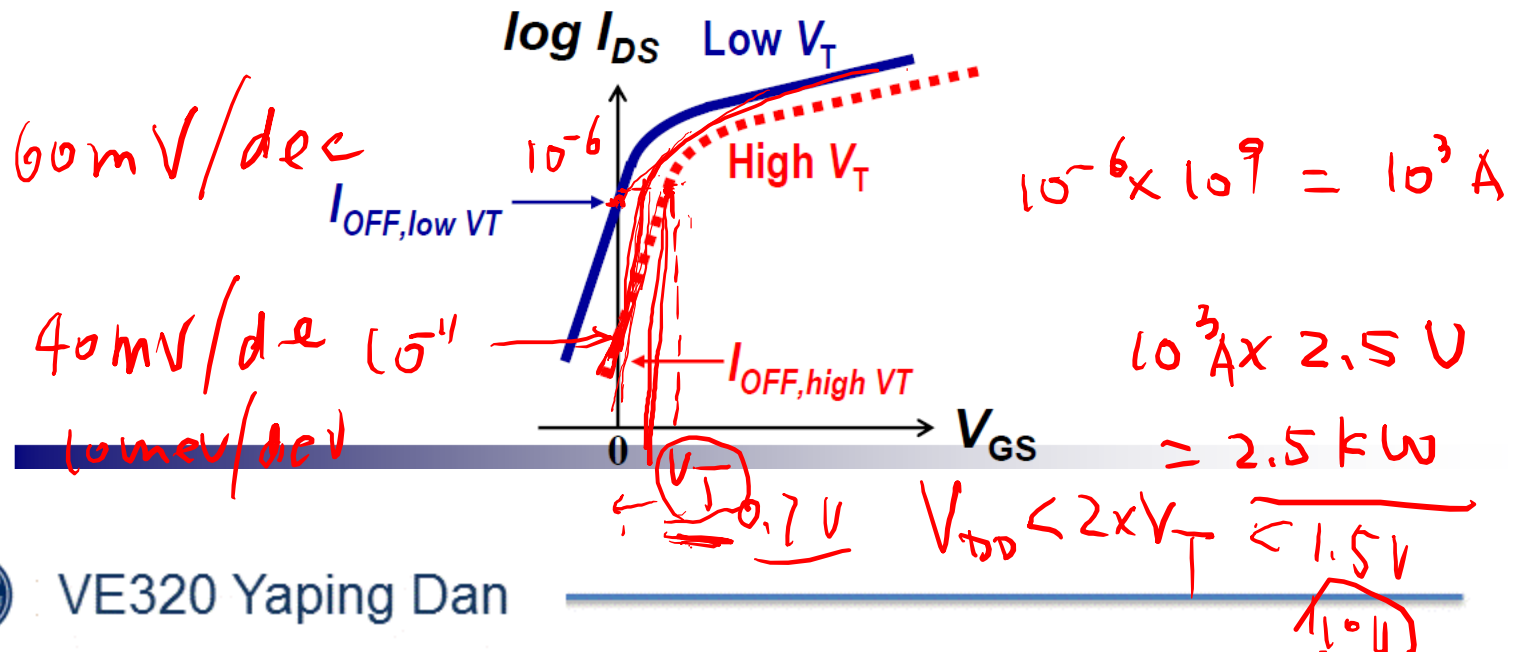
(Important consideration for digital-circuit applications)

- Low V_T is desirable for high ON current

$$I_{DSAT} \propto (V_{DD} - V_T)^\eta \quad 1 < \eta < 2$$

where V_{DD} is the power-supply voltage

...but high V_T is needed for low OFF current



Check your understanding

Problem example 3

Assume that the subthreshold current of a MOSFET is given by

$$I_D = 10^{-15} \exp\left(\frac{V_{GS}}{(2.1)V_t}\right)$$

over the range $0 \leq V_{GS} \leq 1$ volt and where the factor 2.1 takes into account the effect of interface states. Assume that 10^6 identical transistors on a chip are all biased at the same V_{GS} and at $V_{DD} = 5$ V. (a) Calculate the total current that must be supplied to the chip at $V_{GS} = 0.5$, 0.7, and 0.9 V. (b) Calculate the total power dissipated in the chip for the same V_{GS} values.

$V_{GS} = 0.5$ V

$$I_D^t = 10^6 I_D = 10^6 \times 10^{-15} \exp\left(\frac{0.5}{2.1 \times 0.0259}\right) = 9.8 \times 10^{-6} \text{ A}$$

$$P = 50 \mu\text{W}$$

$$V_{GS} = 0.7 \text{ V} \quad I_D^t = 3.88 \times 10^{-4} \text{ A} \quad P = 1.9 \text{ mW}$$

$$V_{GS} = 0.9 \text{ V} \quad I_D^t = 1.54 \times 10^{-2} \text{ A} \quad P = 75 \text{ mW}$$

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11.4 Velocity Saturation

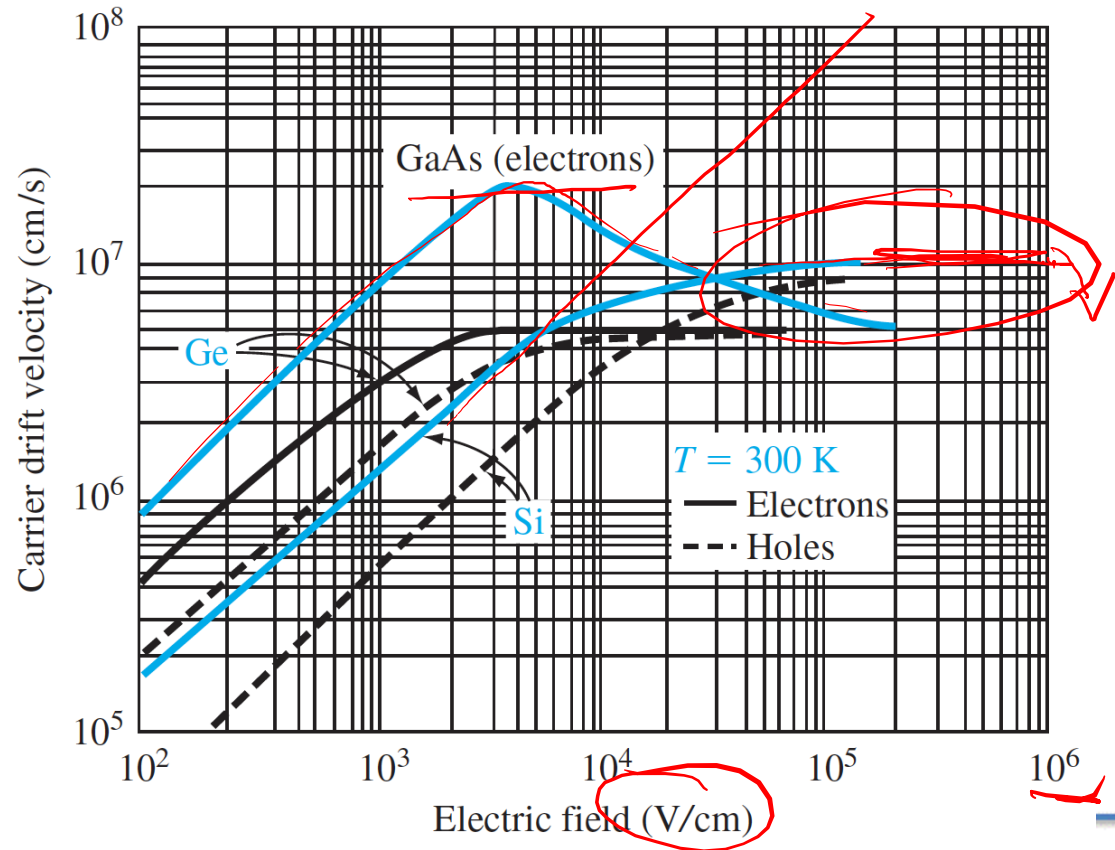
$$\frac{3}{2} kT = \frac{1}{2} m_e v_{th}^2$$

$$v \sim v_{th}$$



$$v_d \rightarrow v_{th}$$

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium



$$v_n = \frac{v_s}{\left[1 + \left(\frac{E_{on}}{E} \right)^2 \right]^{1/2}}$$

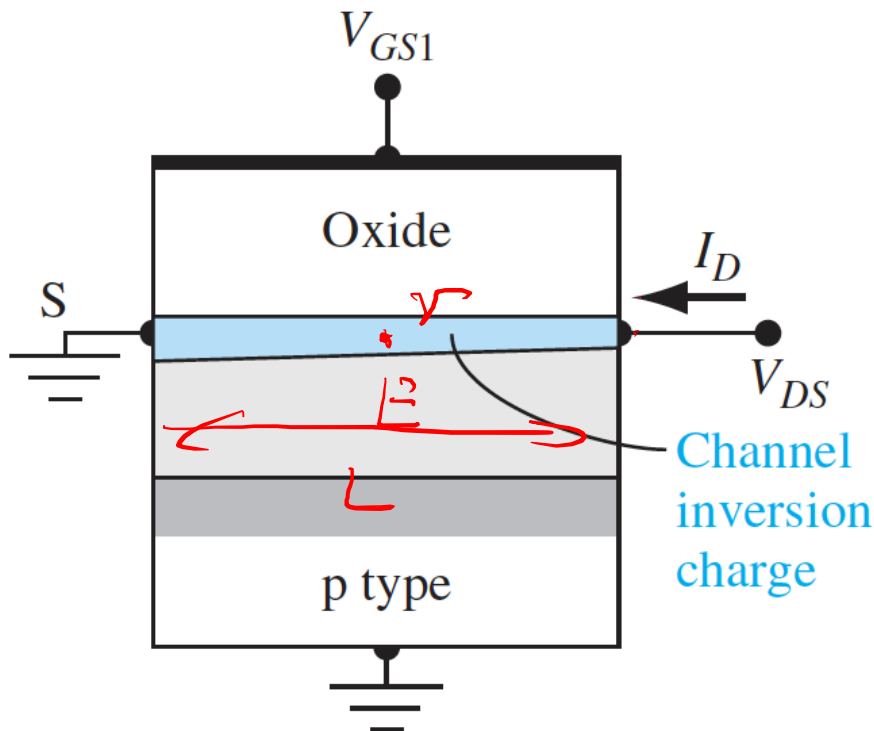
$$v_p = \frac{v_s}{\left[1 + \left(\frac{E_{op}}{E} \right)^2 \right]^{1/2}}$$

Probably a typo in textbook

11.4 Velocity Saturation

$$v_d \rightarrow v_{th}$$

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium



$$\underline{E_{DS}} = \frac{V_{DS}}{\underline{L}}$$

As the transistor size scales down,
the electric field intensity E increases.

$$E_{on} = \frac{V_{DSAT}}{L}$$

11.4 Velocity Saturation

$$v_d \rightarrow v_{th}$$

As the transistor size scales down, the electric field intensity E increases.

- Electric field is heating up electrons
- Electrons transfer energy to lattice to reach thermal equilibrium

$$I_{DSAT} = \mu_n C_{ox} \frac{W}{L} \left(V_{GS} - V_T - \frac{1}{2} V_{DS} \right) V_{DS}$$

linear

$$= C_{ox} W \left(V_{GS} - V_T - \frac{1}{2} V_{DSAT} \right) \frac{V_{DSAT}}{L} \mu_n = E_{on} v$$

$$I_{DSAT} = W C_{ox} \left[V_{GS} - V_T - \frac{V_{DSAT}}{2} \right] v_{sat}$$

where $V_{DSAT} = \frac{L}{\mu_n} v_{sat}$

$\mu E_{on} = v$

11.4 Velocity Saturation

Unified model

$$I_D = 0 \text{ for } V_{GT} \leq 0$$

$$I_D = \underline{k'} \frac{W}{L} \left(\underline{V_{GT}} \underline{V_{min}} - \frac{V_{min}^2}{2} \right)$$

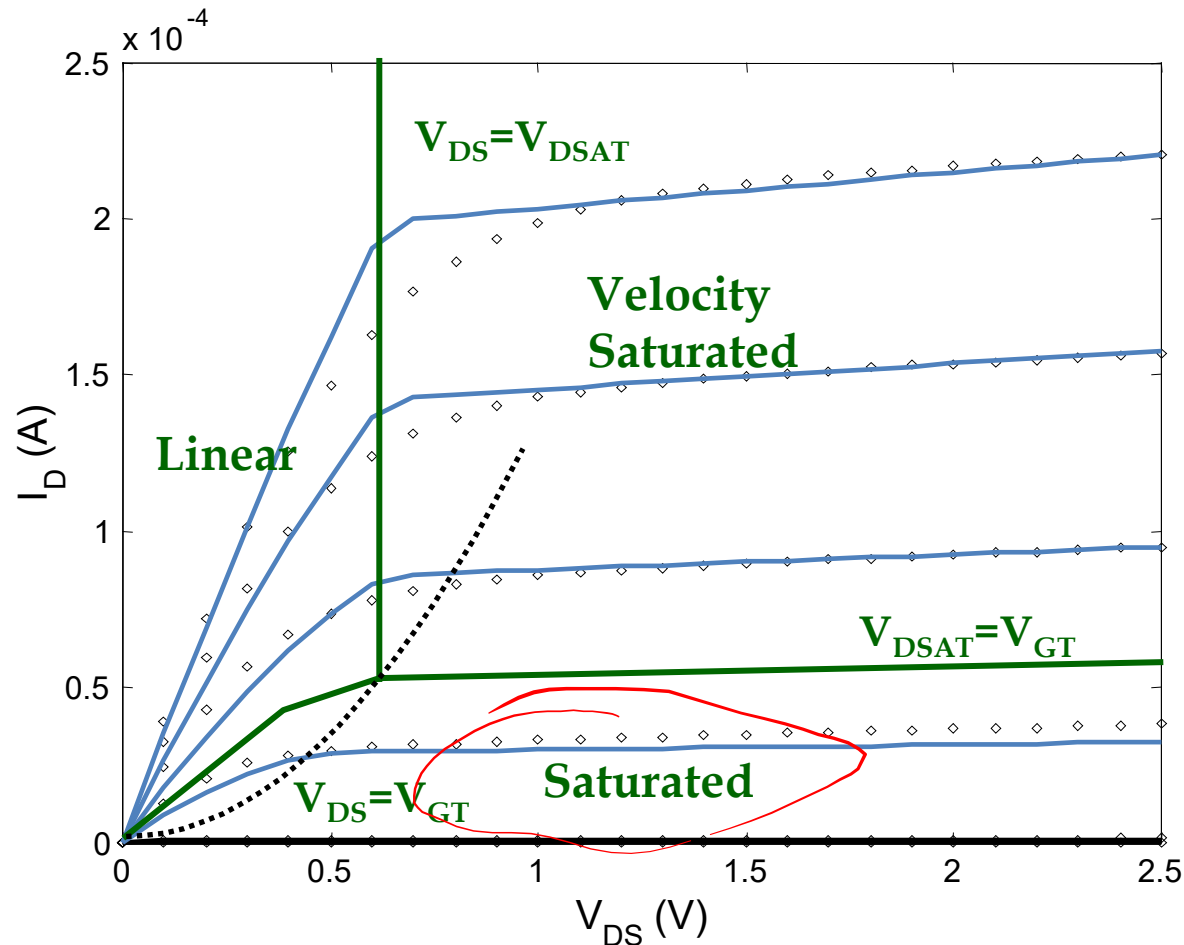
$$\begin{aligned} k'_n &= \mu_n C_{ox} \\ \underline{\mu C_{ox}} \quad k'_p &= \mu_p C_{ox} \\ &\text{for } V_{GT} \geq 0 \end{aligned}$$

$$\text{with } V_{min} = \underline{\min(V_{GT}, V_{DS}, V_{DSAT})},$$

$$V_{GT} = \underline{V_{GS}} - \underline{V_T},$$

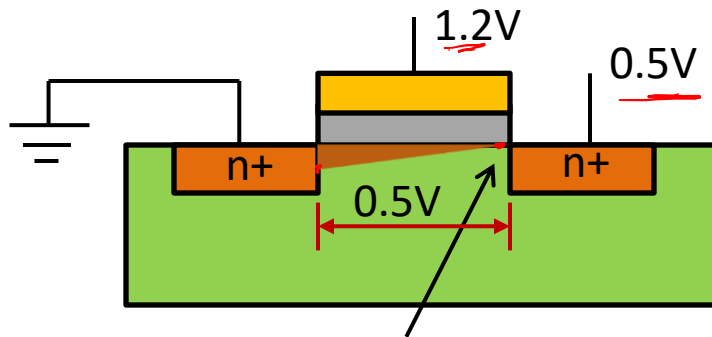
$$\text{and } \underline{V_T} = \underline{V_{T0}} + \underline{\gamma} (\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|-2\phi_F|})$$

11.4 Velocity Saturation



Example

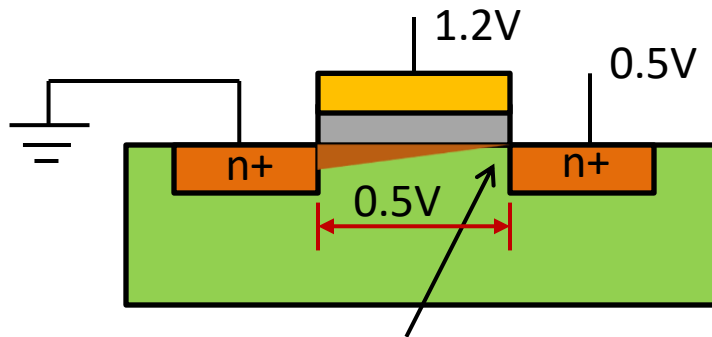
$$V_T = 0.7\text{V}, V_{gs} = 1.2\text{V}, V_{sat} = 1\text{V}$$



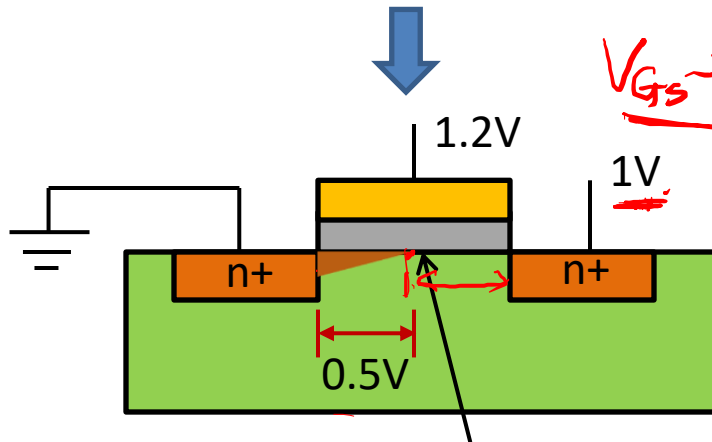
$$V_{gs} - V_T = V_{ds}$$

Example

$$V_T = 0.7, \quad V_{gs} = 1.2 \text{ V}, \quad V_{sat} = \underline{1 \text{ V}}$$



Start to pinch off

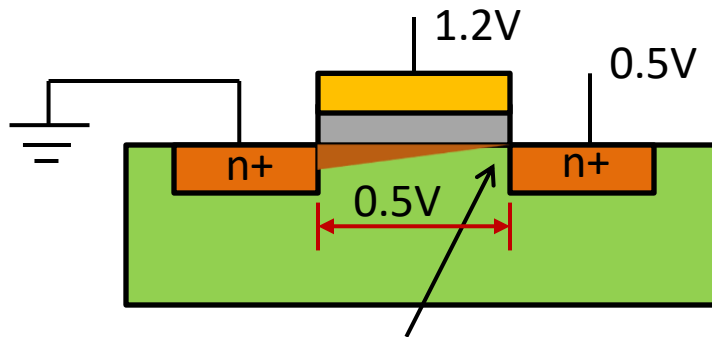


Start to pinch off

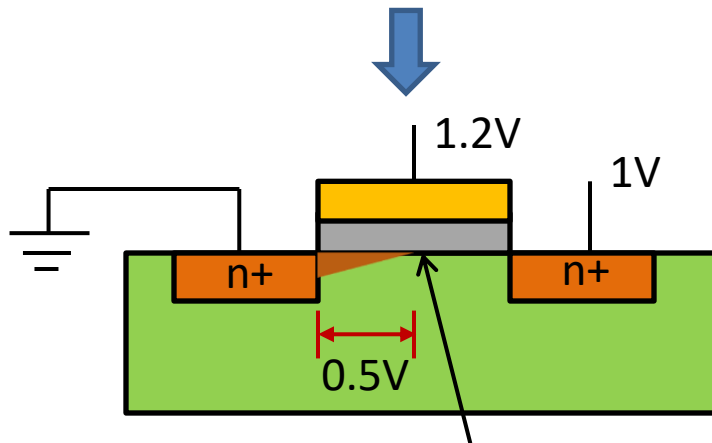
$$V_{gs} - V_T = V_{ds}$$

Example

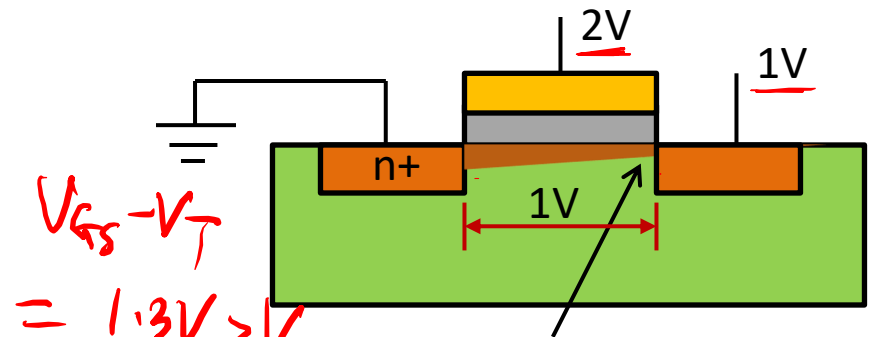
$$V_T = 0.7, \quad V_{gs} = 1.2 \text{ V}, \quad V_{sat} = 1 \text{ V}$$



Start to pinch off



Start to pinch off

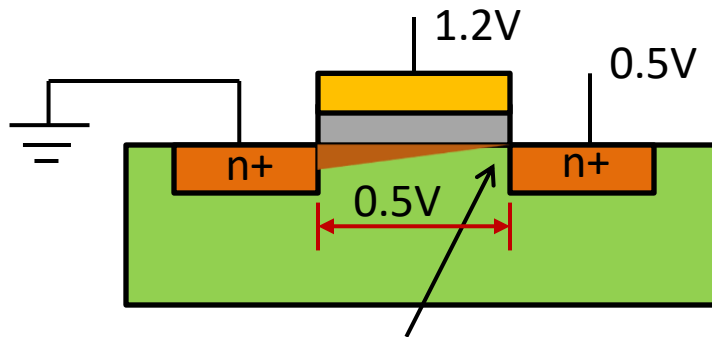


$$V_{gs} - V_T = 1.3 \text{ V} > V_{ds}$$

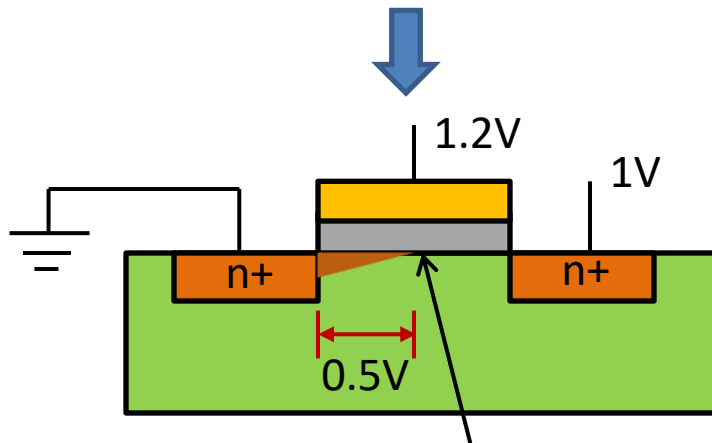
- NO pinch off
- Velocity saturation starts

Example

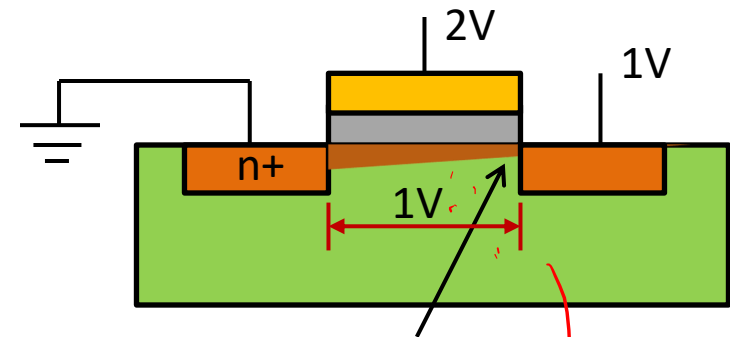
$$V_T = 0.7, \quad V_{gs} = 1.2 \text{ V}, \quad V_{sat} = 1 \text{ V}$$



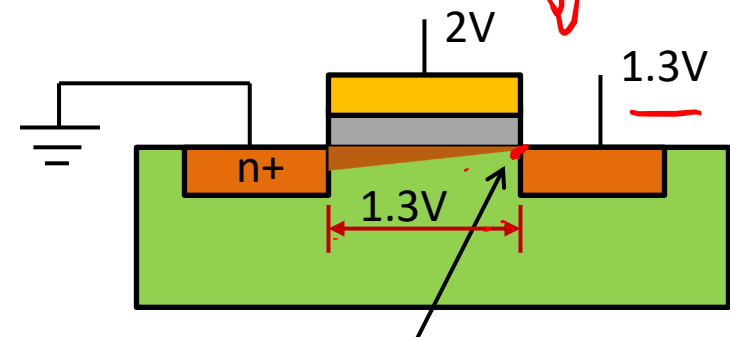
Start to pinch off



Start to pinch off



- NO pinch off
- Velocity saturation starts



- Starts to pinch off
- Velocity saturation

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11.2 Conductance and transconductance

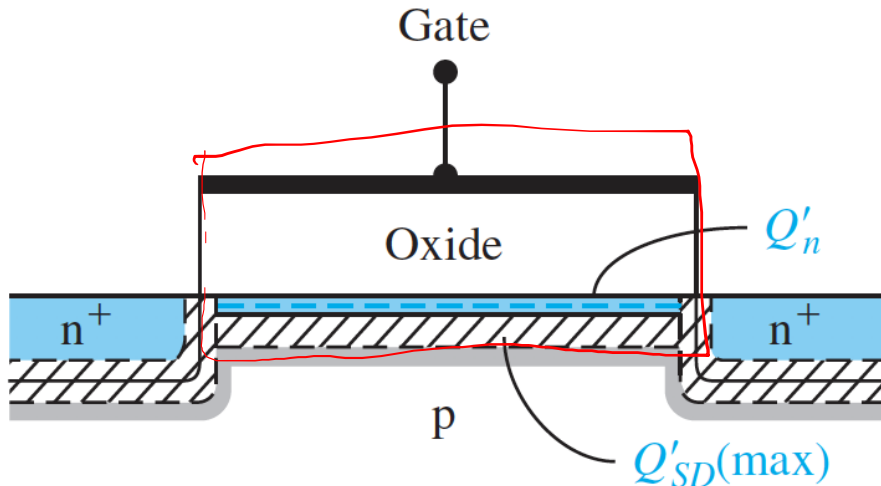
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11.5 Short Channel Effect

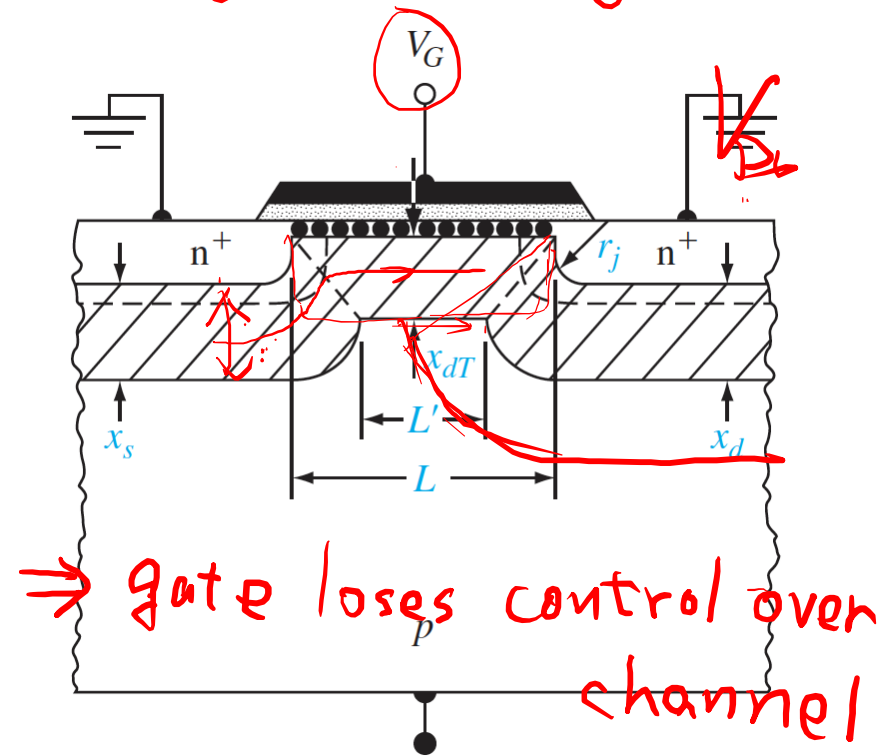
11.5 Short Channel Effect

MOSFET



A long channel device

charge sharing effect



A short channel device

11.5 Short Channel Effect

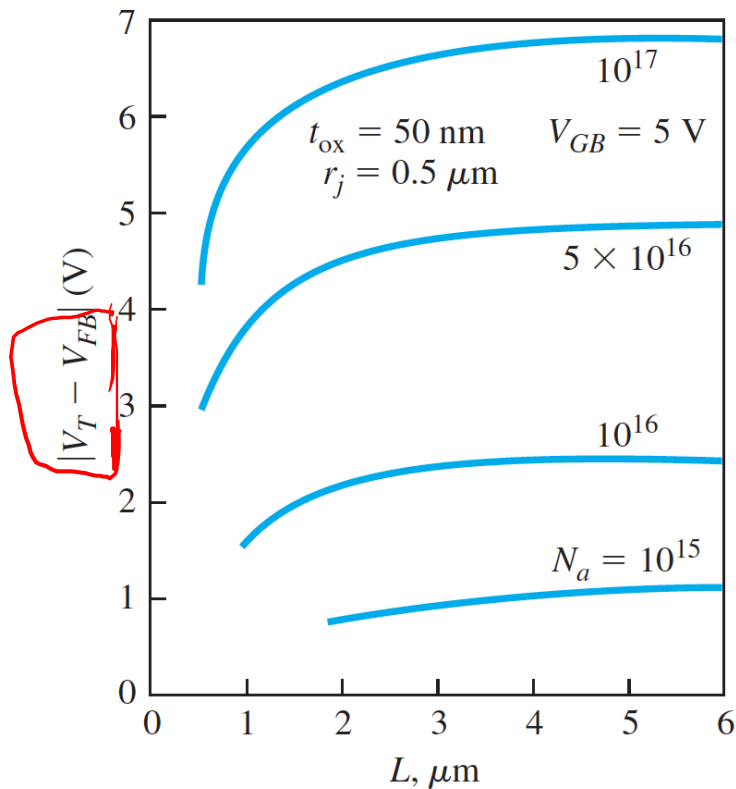


Figure 11.16 | Threshold voltage versus channel length for various substrate dopings. (From Yau [26].)

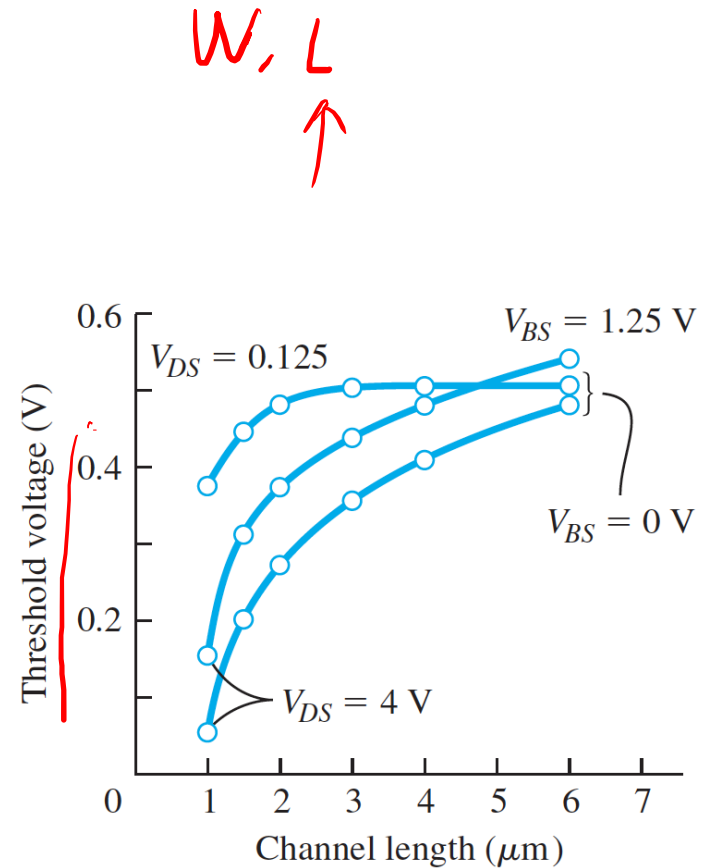
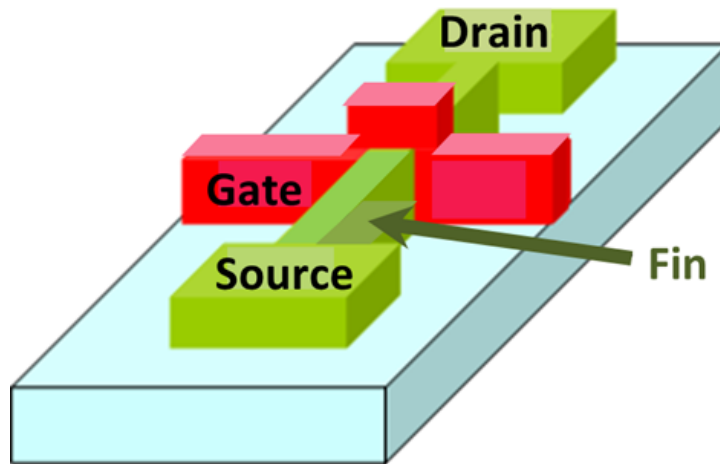


Figure 11.17 | Threshold voltage versus channel length for two values of drain-to-source and body-to-source voltage. (From Yang [25].)

11.5 Short Channel Effect



chengming H₄

Fin Gate to suppress short channel effect