# VE320 – Summer 2021

# Introduction to Semiconductor Devices

Instructor: Yaping Dan （但亚平）
yaping.dan@sjtu.edu.cn

## Chapter 7 The pn Junction

# Outline

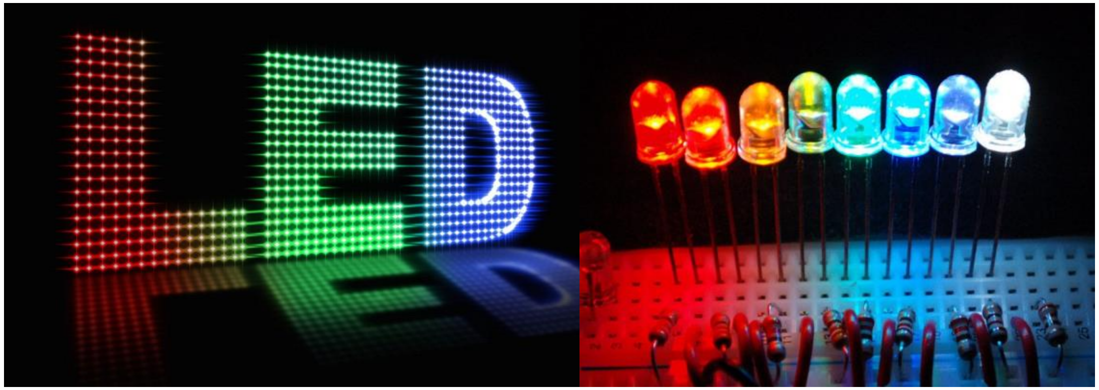
7.0 Introduction to semiconductor devices

7.1 Basic structure of the pn junction

7.2 Zero applied bias

7.3 Reverse applied bias

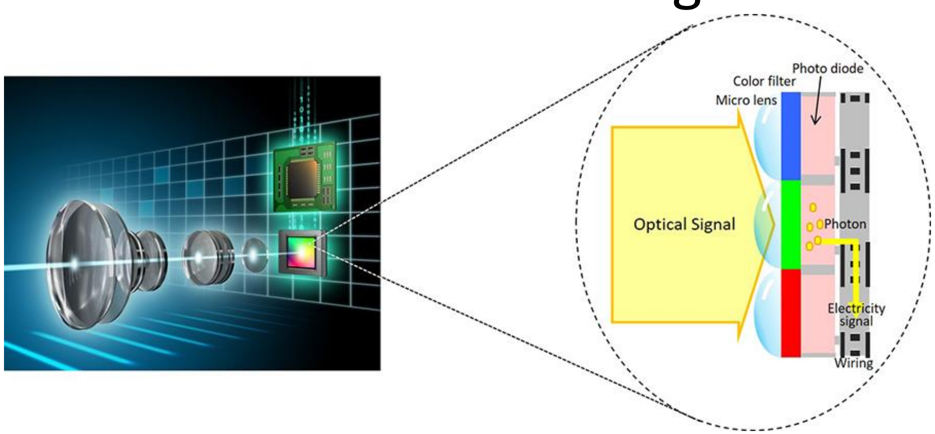
# 7.0 Introduction to semiconductor devices

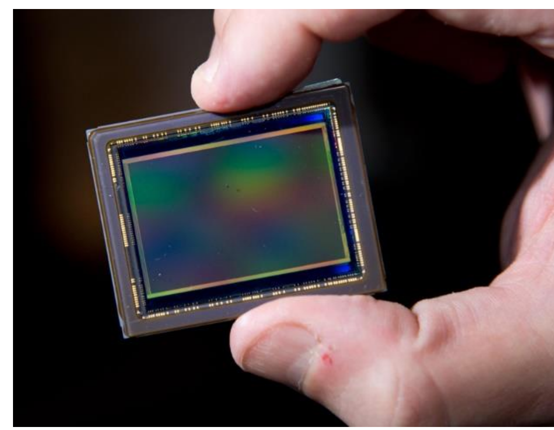



Light emitting diodes

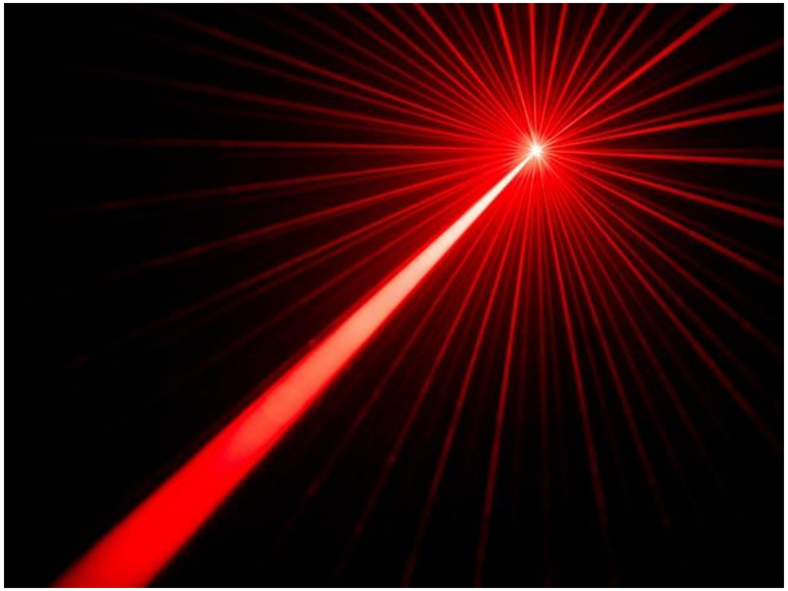
Cold light source

Photodetector: CMOS image sensor



# 7.0 Introduction to semiconductor devices



Semiconductor lasers

Solar cells

# Outline

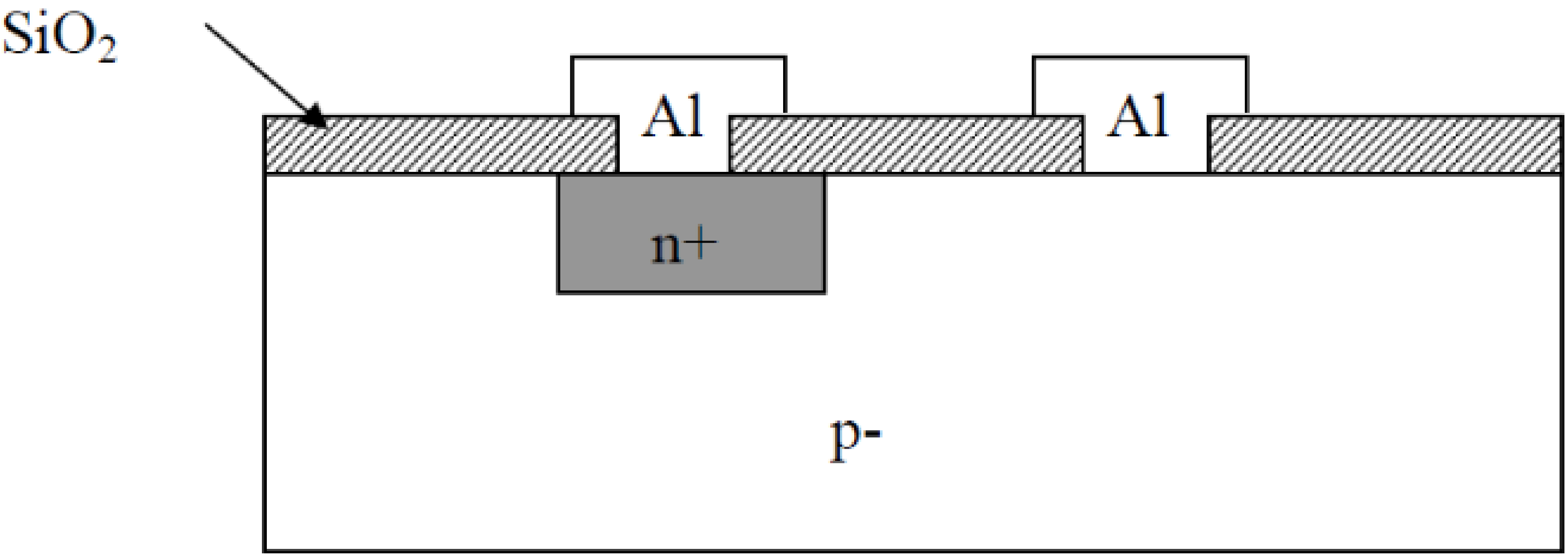
7.0 Introduction to semiconductor devices

**7.1 Basic structure of the pn junction**

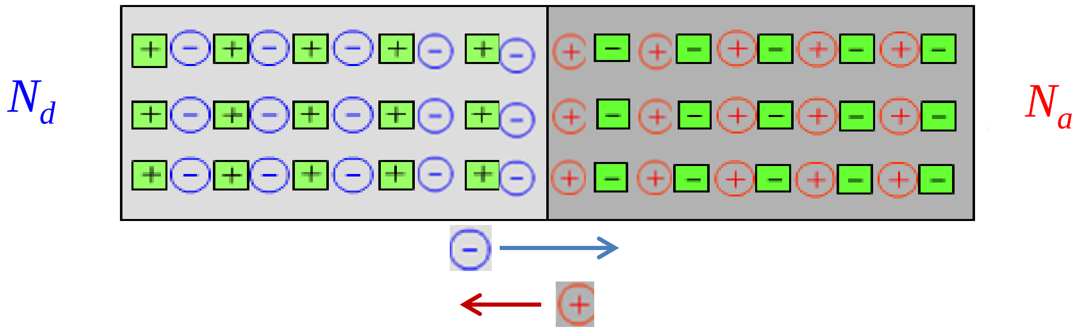
7.2 Zero applied bias

7.3 Reverse applied bias

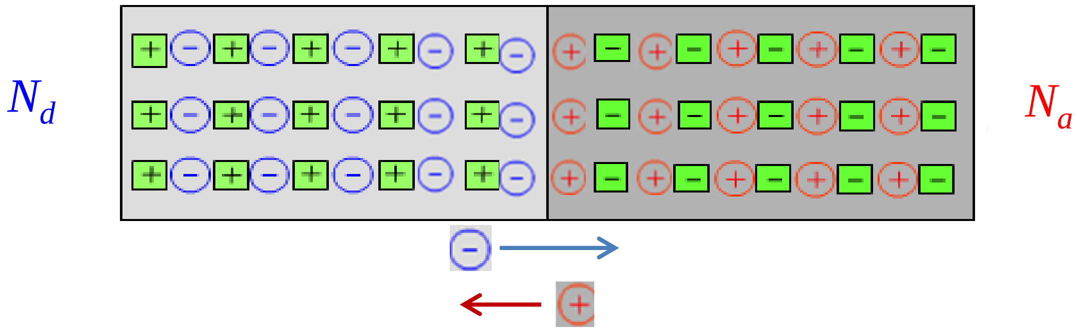
# 7.1 Basic structure of pn junction

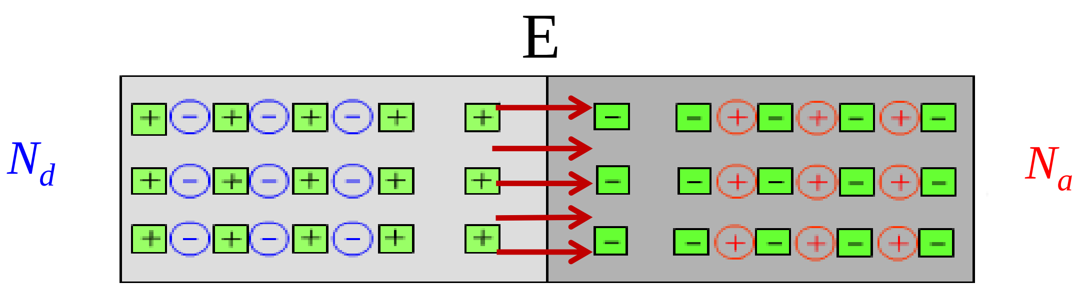


# 7.1 Basic structure of pn junction



# 7.1 Basic structure of pn junction



# Outline

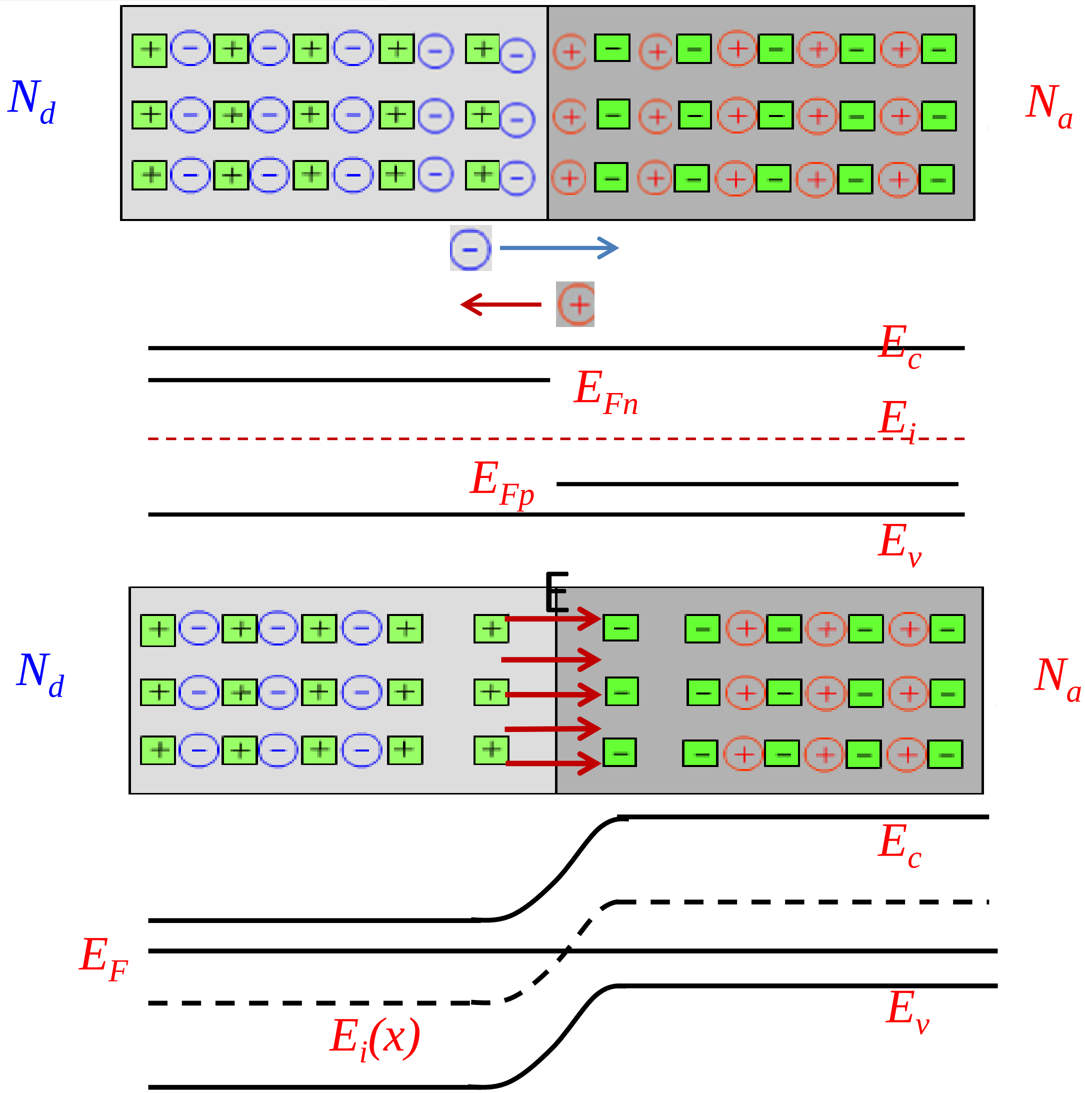
7.1 Basic structure of the pn junction

**7.2 Zero applied bias**

7.3 Reverse applied bias

# 7.2 Zero applied bias

## Built-in potential barrier



# 7.2 Zero applied bias

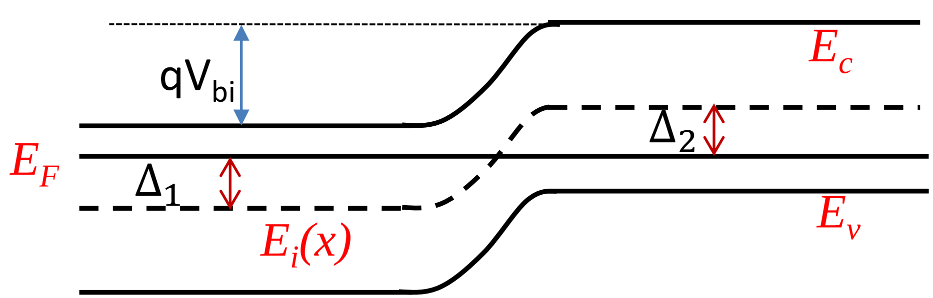
## Built-in potential barrier

$$n_{n0} = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = n_i \exp\left(\frac{q\Delta_1}{kT}\right)$$

$$p_{p0} = n_i \exp\left(\frac{E_i - E_F}{kT}\right) = n_i \exp\left(\frac{q\Delta_2}{kT}\right)$$

$$\Rightarrow V_{bi} = kTln\left(\frac{n_{n0}}{n_i}\right) + kTln\left(\frac{P_{p0}}{n_i}\right) = kTln\left(\frac{n_{n0}P_{p0}}{n_i^2}\right) = kTln\left(\frac{N_aN_d}{n_i^2}\right)$$

$$Example\colon\ N_a = 10^{17}cm^{-3}, N_d = 10^{17}cm^{-3}, \Rightarrow\ V_{bi} = 0.026/q * \ln\left(\frac{10^{17}10^{17}}{10^{20}}\right) = 0.84V$$



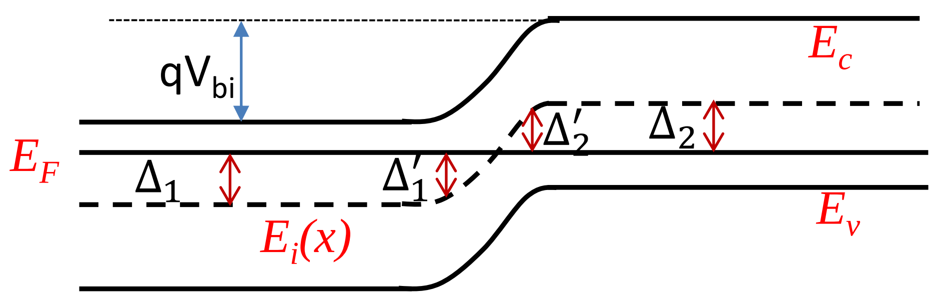
# 7.2 Zero applied bias

## Charge carrier distribution

$$n = n_i \exp\left(\frac{E_F - E_i(x)}{kT}\right) = N_d exp\left(-\frac{0.1eV}{0.026eV}\right) \approx \frac{N_d}{50}$$

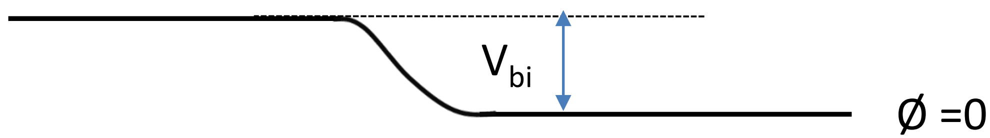
$$p = n_i \exp\left(\frac{E_i(x) - E_F}{kT}\right) = N_a exp\left(-\frac{0.1eV}{0.026eV}\right) \approx \frac{N_a}{50}$$

$$\Delta_1 - \Delta_1' = 0.1eV \qquad \Delta_2 - \Delta_2' = 0.1eV$$

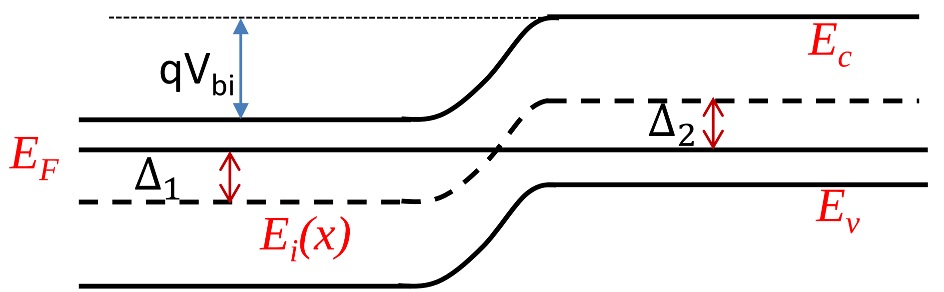


# 7.2 Zero applied bias

## Potential profile




Potential profile

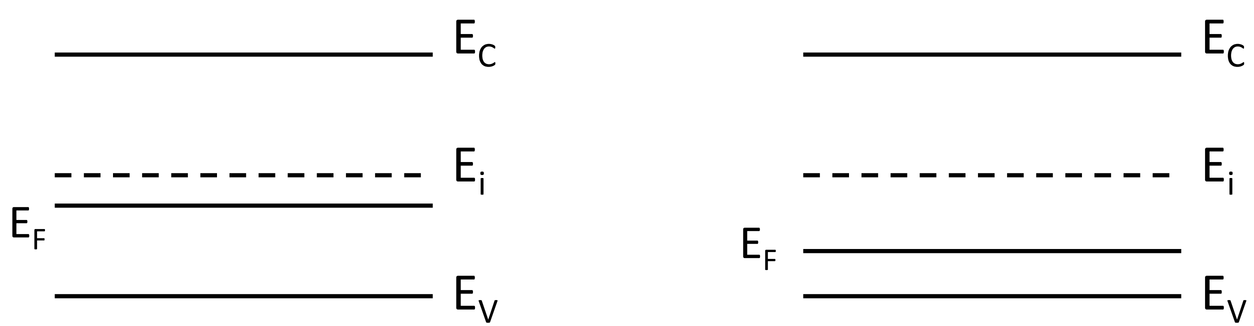



Energy band diagram

# Check your understanding

## Problem Example #1

Two pieces of p-type silicon are in contact. The doping concentrations are $10^{16}$ $cm^{-3}$ and $10^{18}$ $cm^{-3}$. Calculate the built-in potential between these two pieces of silicon and plot the energy band bending diagram.



# 7.2 Zero applied bias

Poisson's equation

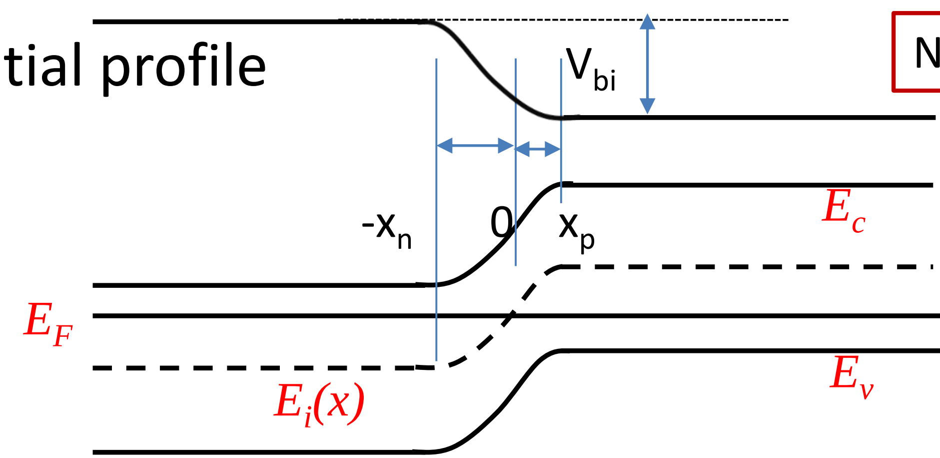
$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i\exp(\frac{E_i(x) - E_F}{kT})\ - n_i\exp(\frac{E_F - E_i(x)}{kT})]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i\exp(\frac{-qV(x) - E_F}{kT})\ - n_i\exp(\frac{E_F + qV(x)}{kT})]$$

Potential profile




NO analytical solution

$\emptyset = 0$

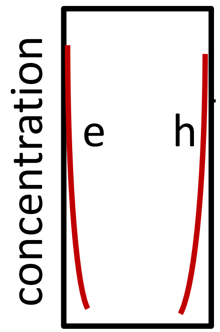
# 7.2 Zero applied bias

Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x)$$

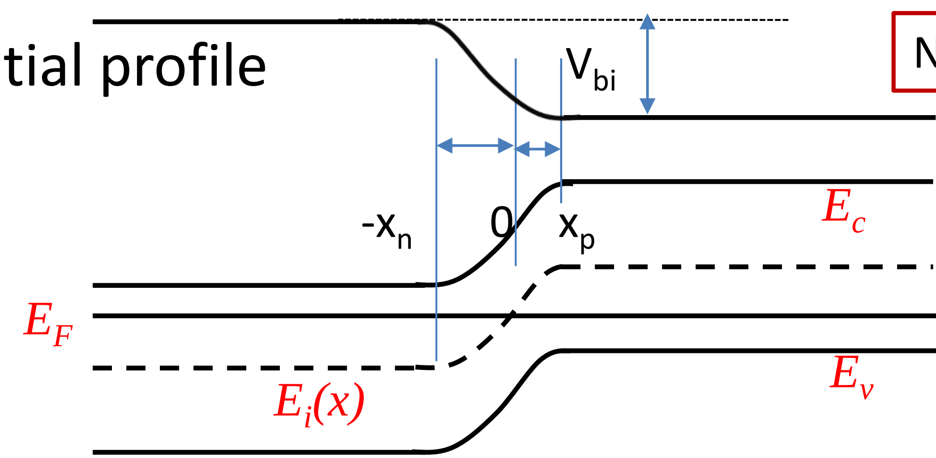



$$n_i\exp(\frac{E_i(x) - E_F}{kT})\ - n_i\exp(\frac{E_F - E_i(x)}{kT})]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x$$

$$+ n_i\exp(\frac{-qV(x) - E_F}{kT})\ - n_i\exp(\frac{E_F + qV(x)}{kT})]$$

Potential profile




NO analytical solution

Ø =0

# 7.2 Zero applied bias

Poisson's equation

Third time approximation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

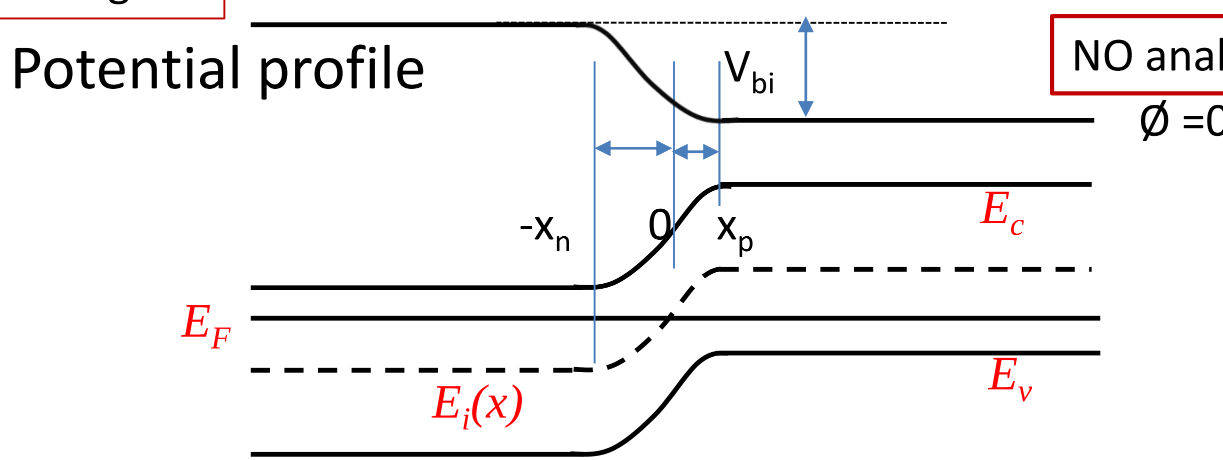
$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i\exp(\frac{E_i(x) - E_F}{kT}) - n_i\exp(\frac{E_F - E_i(x)}{kT})]$$

$\boxed{-x_n \le x \le x_p}$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + \cancelto{0}{n_i\exp(\frac{-qV(x) - E_F}{kT})} - \cancelto{0}{n_i\exp(\frac{E_F + qV(x)}{kT})}]$$

Depletion region

NO analytical solution



# 7.2 Zero applied bias

Poisson's equation

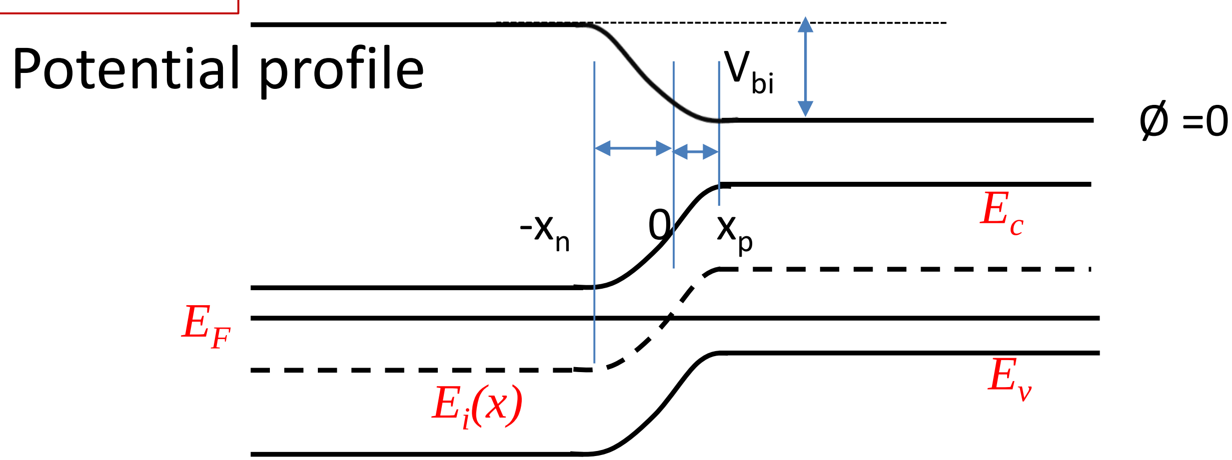
$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + p(x) - n(x)]$$

$$= -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x) + n_i \exp(\frac{E_i(x) - E_F}{kT}) - n_i \exp(\frac{E_F - E_i(x)}{kT})]$$

$$\boxed{-x_n \le x \le x_p} = -\frac{q}{\varepsilon}[N_d\ (x) - N_a\ (x)] = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

Depletion region



# 7.2 Zero applied bias

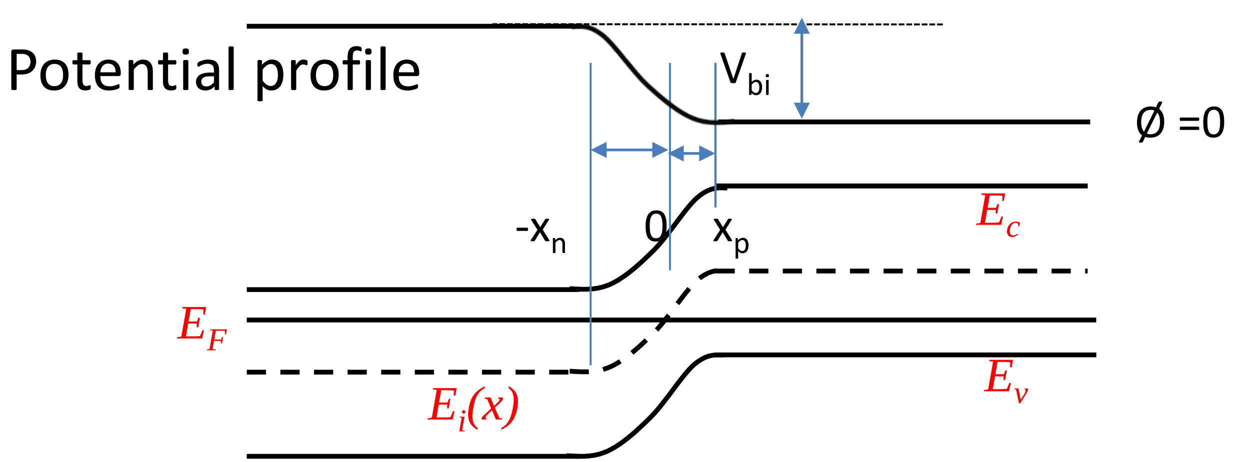
$$\frac{d^2V(x)}{dx^2} = \begin{cases} \frac{q}{\varepsilon}N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon}N_d & -x_p \le x < 0 \end{cases}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a\ x + A_1 & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d\ x + A_2 & -x_n \le x < 0 \end{cases}$$

Boundary condition:

$$\boxed{E(x = x_p) = 0}$$

$$\boxed{E(x = -x_n) = 0}$$



# 7.2 Zero applied bias

$$\frac{d^2V(x)}{dx^2} = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

Boundary condition:

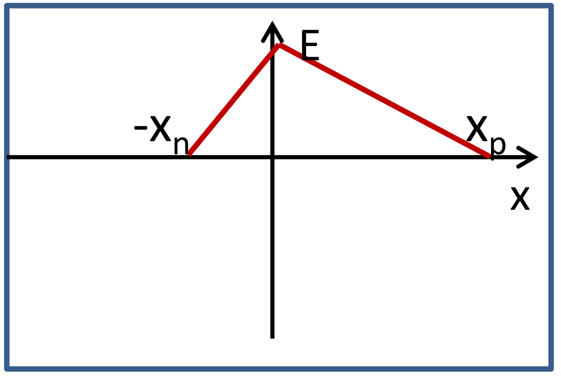
$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a\ x + A_1 & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d\ x + A_2 & -x_n \le x < 0 \end{cases}$$

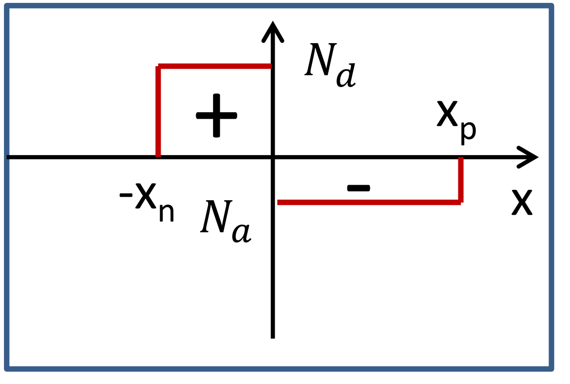
$$\boxed{E(x = x_p) = 0}$$

$$\boxed{E(x = -x_n) = 0}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a\ x + \frac{q}{\varepsilon} N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d\ x + \frac{q}{\varepsilon} N_d\ x_n & -x_n \le x < 0 \end{cases}$$

$$\boxed{x = 0 \Rightarrow\ N_a\ x_p = N_d\ x_n}$$



# 7.2 Zero applied bias

## Space charge width

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a \ x + \frac{q}{\varepsilon} N_a \ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d \ x + \frac{q}{\varepsilon} N_d \ x_n & -\mathrm{x}_n \le x < 0 \end{cases}$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon} N_a \ (\frac{1}{2} x^2 - x_p x + C_1) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d \ (\frac{1}{2} x^2 + x_n x + C_2) & -x_n \le x < 0 \end{cases}$$

Boundary condition:

$$V(x = x_p) = 0 \ \Rightarrow \ C_1 = \frac{{x_p}^2}{2}$$

$$V(x = 0) \ is \ continuous$$

$$\Rightarrow \ C_2 = -\frac{N_a}{2N_d} {x_p}^2$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon} N_a \ (\frac{1}{2} x^2 - x_p x + \frac{{x_p}^2}{2}) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d \ (\frac{1}{2} x^2 + x_n x - \frac{N_a}{N_d} \frac{{x_p}^2}{2}) & -x_n \le x < 0 \end{cases}$$

# 7.2 Zero applied bias

## Space charge width

$$\boxed{x = 0 \Rightarrow N_d \ x_n = N_a \ x_p}$$

$$\boxed{V(x = x_n) = V_{bi} = \frac{kT}{q} ln(\frac{N_d \ N_a}{n_i^2})}$$

$$V(x) = \begin{cases} \frac{q}{\varepsilon} N_a \ (\frac{1}{2}x^2 - x_p x + \frac{{x_p}^2}{2}) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d \ (\frac{1}{2}x^2 + x_n x - \frac{N_a}{N_d}\frac{{x_p}^2}{2}) & -x_n \le x < 0 \end{cases}$$

$$\frac{q}{\varepsilon} N_d \left(\frac{1}{2}{x_n}^2 + \frac{N_a}{2N_d}{x_p}^2\right) = V_{bi}$$

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}}$$

$$\boxed{N_d \ x_n = N_a \ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}}}$$

# 7.2 Zero applied bias

## Space charge width

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_d}{N_a} \frac{1}{N_a + N_d}}$$

$$N_d\ x_n = N_a\ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_a}{N_d} \frac{1}{N_a + N_d}}$$

$$W = x_p + x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_d + N_a}{N_a N_d}}$$

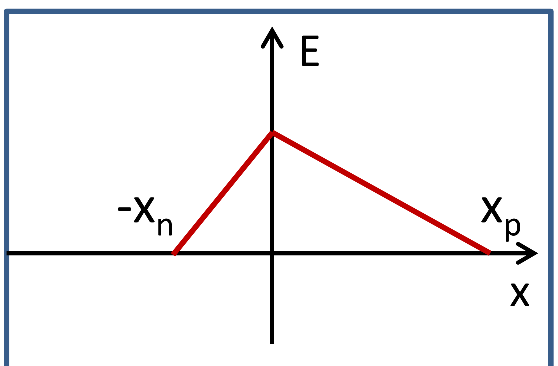
# 7.2 Zero applied bias

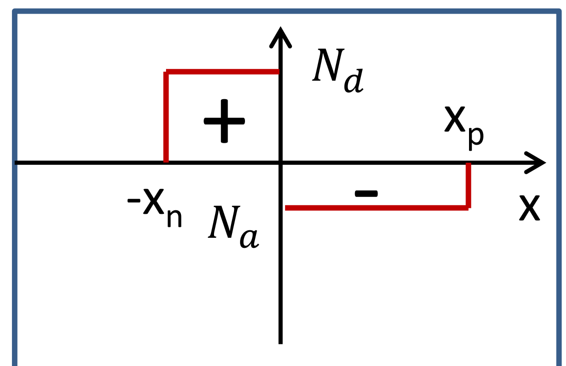
Space charge width

$$\boxed{x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}}$$

$$\boxed{N_d\ x_n = N_a\ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a\ x + \frac{q}{\varepsilon}N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d\ x + \frac{q}{\varepsilon}N_d\ x_n & -x_n \le x < 0 \end{cases}$$



# Check your understanding

## Problem Example #2

A silicon pn junction at T=300K with zero applied bias has doping concentration of $N_d = 5 \times 10^{16}$ cm$^{-3}$ and $N_a = 5\times10^{15}$ cm$^{-3}$. Determine $x_n$, $x_p$, W and $|E_{max}|$.

# Outline

7.1 Basic structure of the pn junction

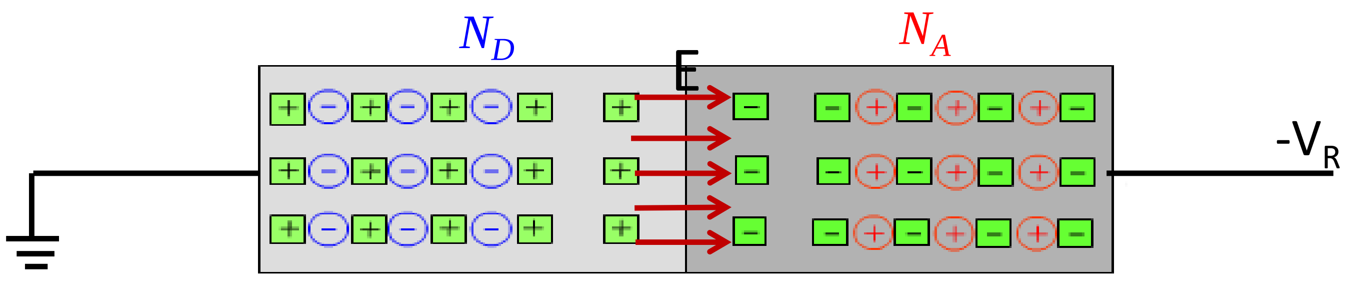
7.2 Zero applied bias

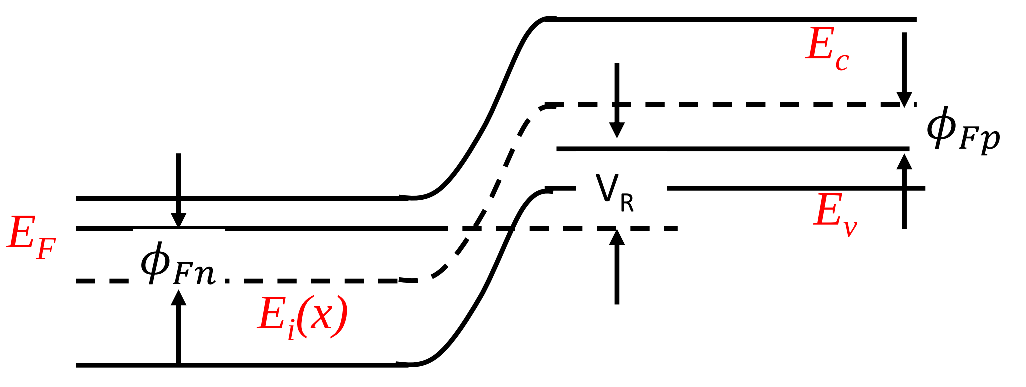
**7.3 Reverse applied bias**

# 7.3 Reverse applied bias

Space charge width and electric field

$$V_{\text{total}} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$



# 7.3 Reverse applied bias

Space charge width and electric field

$$x_p = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}$$

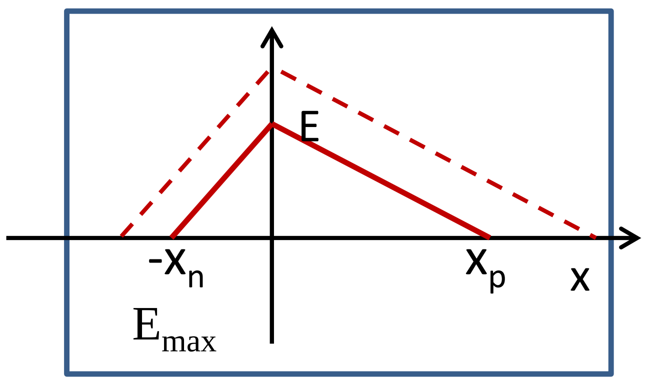
$$N_a^- x_n = N_d^+ x_p \Rightarrow x_n = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

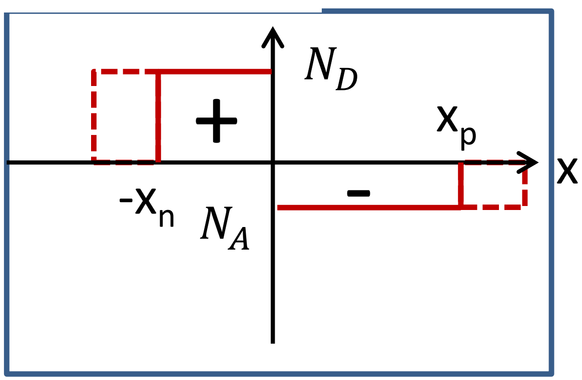
$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_d+N_a}{N_aN_d}}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a\ x + \frac{q}{\varepsilon}N_a\ x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_d\ x + \frac{q}{\varepsilon}N_d\ x_n & -x_n \le x < 0 \end{cases}$$

$$E_{max} = \left\{\frac{2e(V_{bi}+V_R)}{\epsilon_s}\left(\frac{N_aN_d}{N_a+N_d}\right)\right\}^{1/2}$$

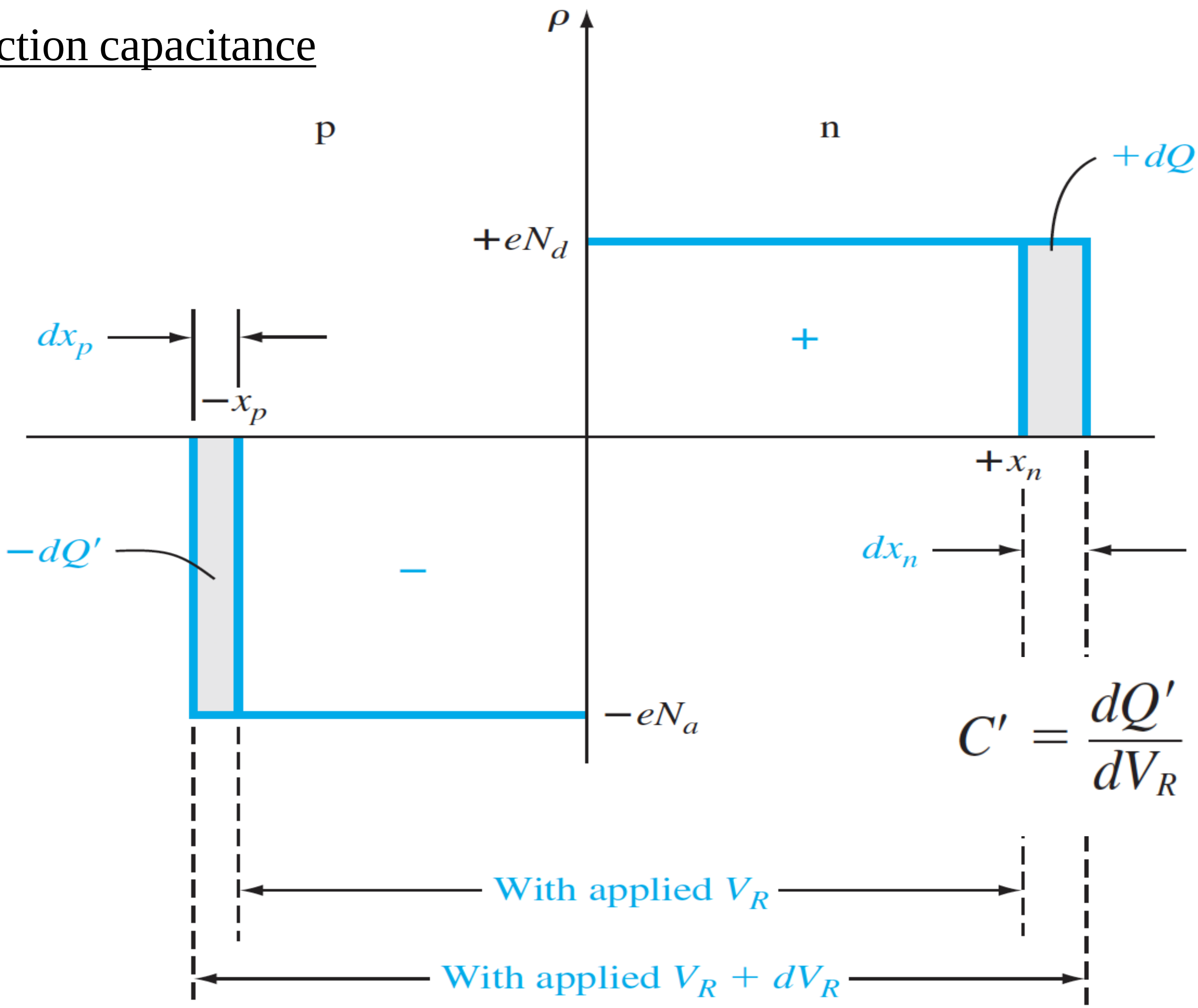
$$E_{max} = \frac{2(V_{bi}+V_R)}{W}$$



# 7.3 Reverse applied bias

Junction capacitance




$$C' = \frac{dQ'}{dV_R}$$

# 7.3 Reverse applied bias

Junction capacitance

$$C' = \frac{dQ'}{dV_R}$$

$$dQ' = eN_d\, dx_n = eN_a\, dx_p$$

$$x_n = \left\{\frac{2\epsilon_s(V_{bi} + V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2}$$

$$\boxed{C' = \frac{dQ}{dV_\mathrm{R}}|_{V_R=V_{R_0}} = qN_d\frac{db}{dV_R}|_{V_R=V_{R_0}} = \sqrt{\frac{q\varepsilon}{2(V_{bi}+V_{R_0})}\frac{N_dN_a}{N_a+N_d}} = \frac{\varepsilon}{W}}$$

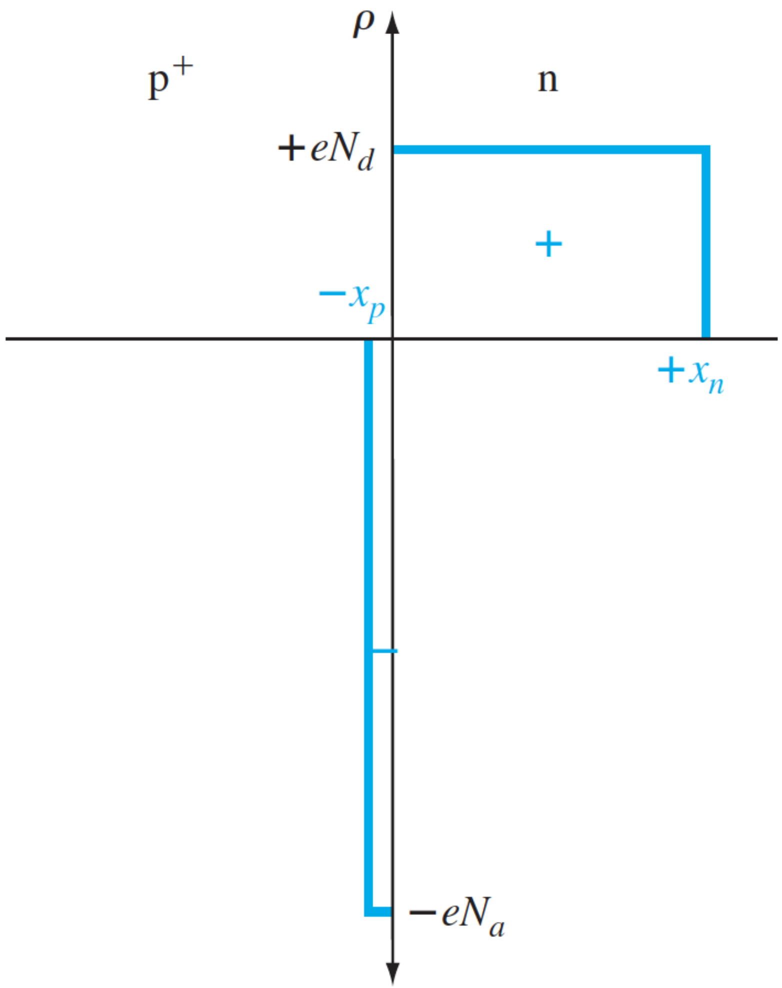
# Check your understanding

<u>Problem Example #3</u>

Consider a GaAs pn junction at T = 300K doped to $N_a = 5 \times 10^{15}$ cm$^{-3}$ and $N_d = 2 \times 10^{16}$ cm$^{-3}$. (a) Calculate $V_{bi}$. (b) Determine the junction capacitance C' for $V_R$ =4V.

# 7.3 Reverse applied bias

One-sided junction




$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{N_d+N_a}{N_aN_d}}$$

$$\Big\downarrow\ N_a \to \infty$$

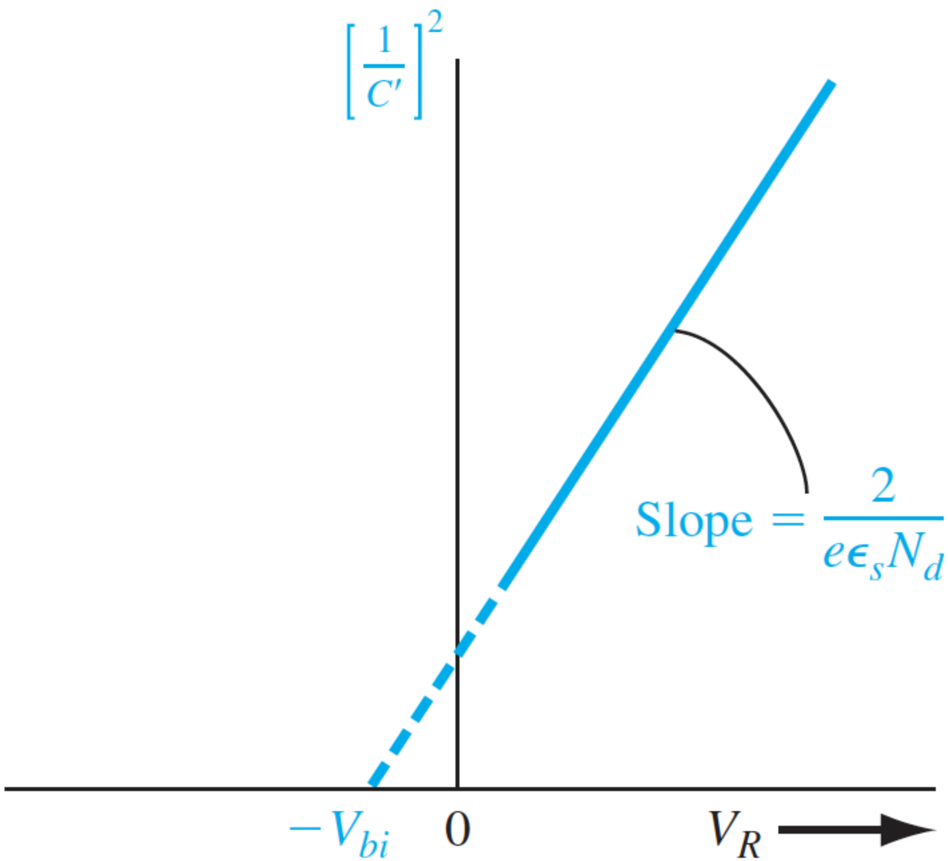
$$W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{q}\frac{1}{N_d}} \approx x_n$$

$$\Big\downarrow$$

$$C' = \frac{\varepsilon}{W} = \sqrt{\frac{q\varepsilon N_d}{2\,(V_{bi}+V_R)}}$$

# 7.3 Reverse applied bias

<u>One-sided junction</u>




$$C' = \frac{\varepsilon}{W} = \sqrt{\frac{q\varepsilon N_d}{2\,(V_{bi} + V_R)}}$$

$$\frac{1}{C'^2} = \frac{2\,(V_{bi} + V_R)}{q\varepsilon N_d}$$