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**VE320 – Summer 2021**

**Semiconductor Physics**

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yaping.dan@sjtu.edu.cn

**Chapter 10 Fundamentals of Metal-Oxide-Semiconductor  
Field Effect Transistors**

# Outline

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10.1 The two-terminal MOS structure

10.2 Capacitance-voltage characteristics

10.3 Non-ideal effects

10.4 The basic MOSFET operation

# Outline

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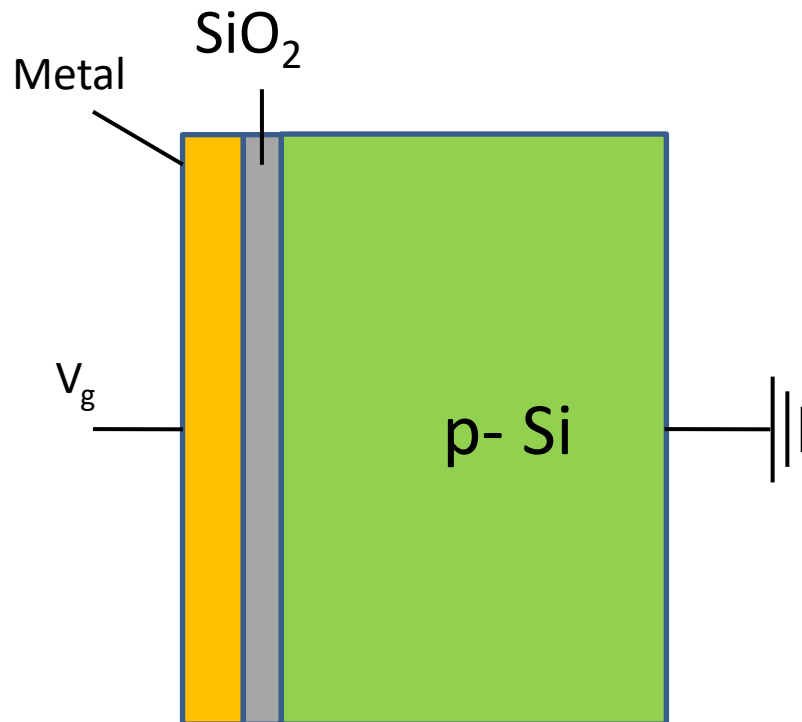
**10.1 The two-terminal MOS structure**

10.2 Capacitance-voltage characteristics

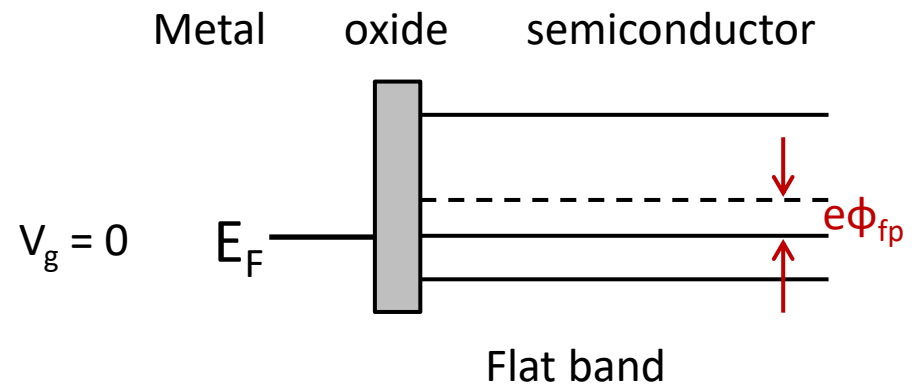
10.3 Non-ideal effects

10.4 The basic MOSFET operation

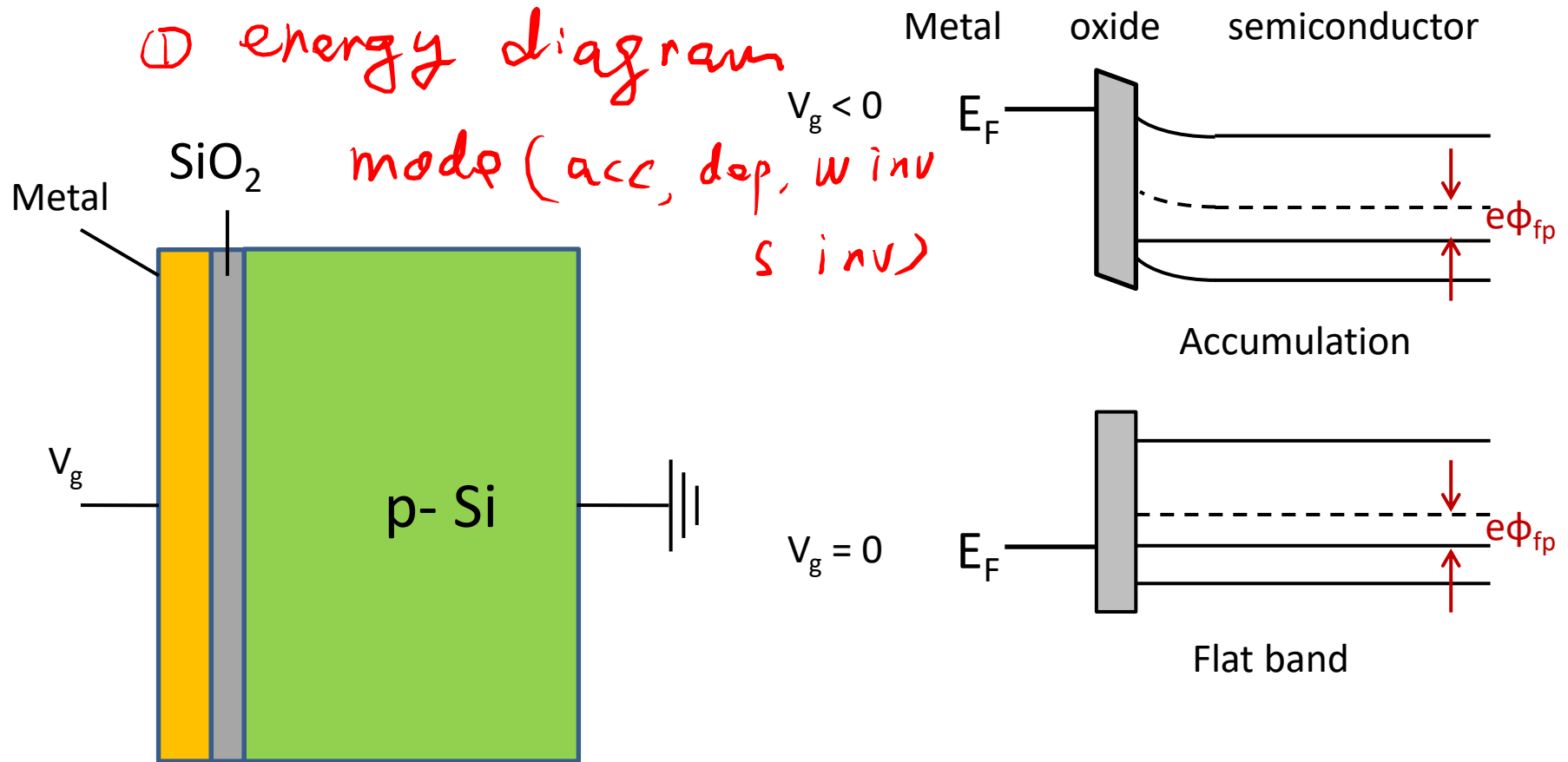
# 10.1 The two-terminal MOS structure



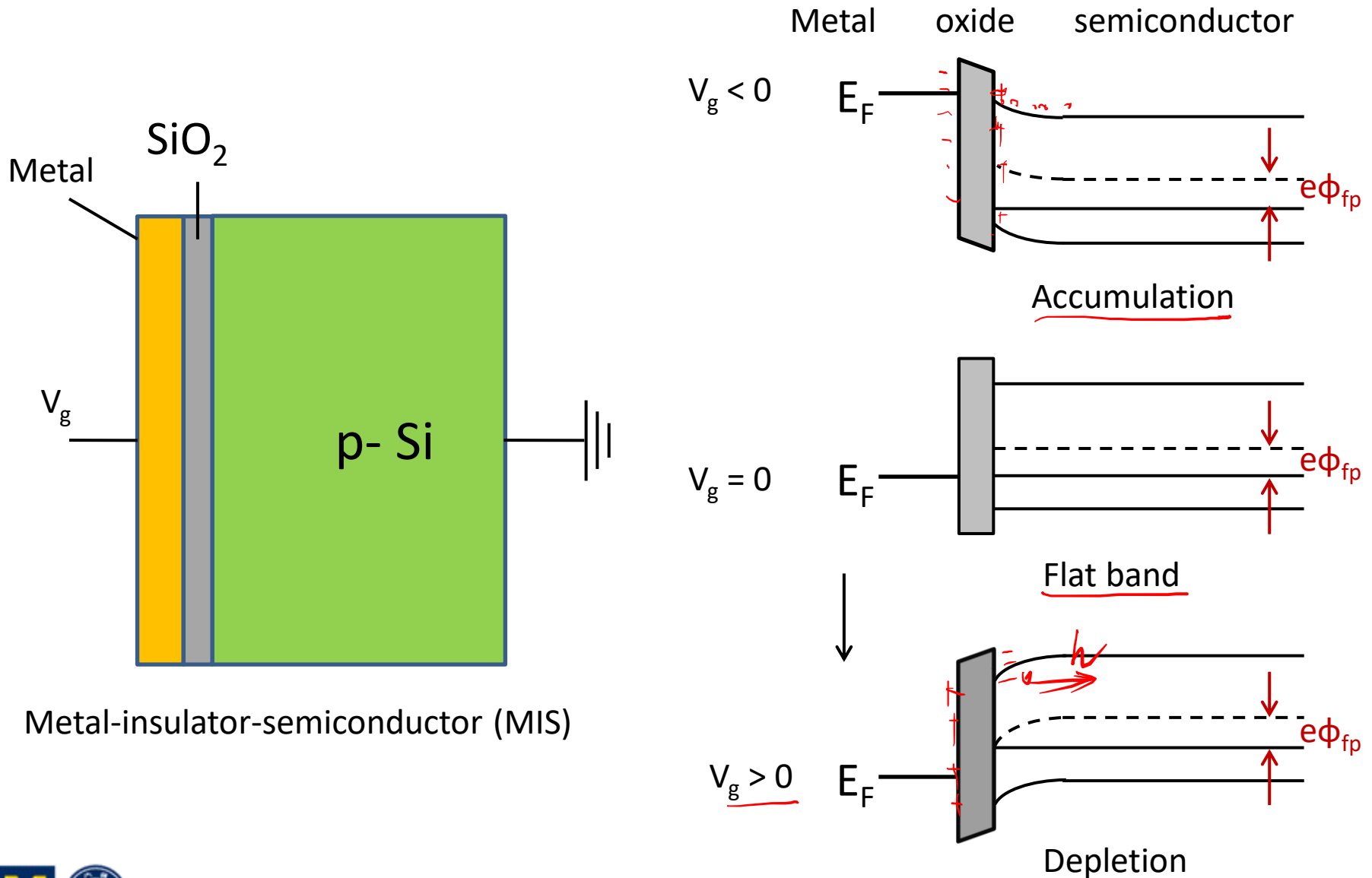
Metal-insulator-semiconductor (MIS)



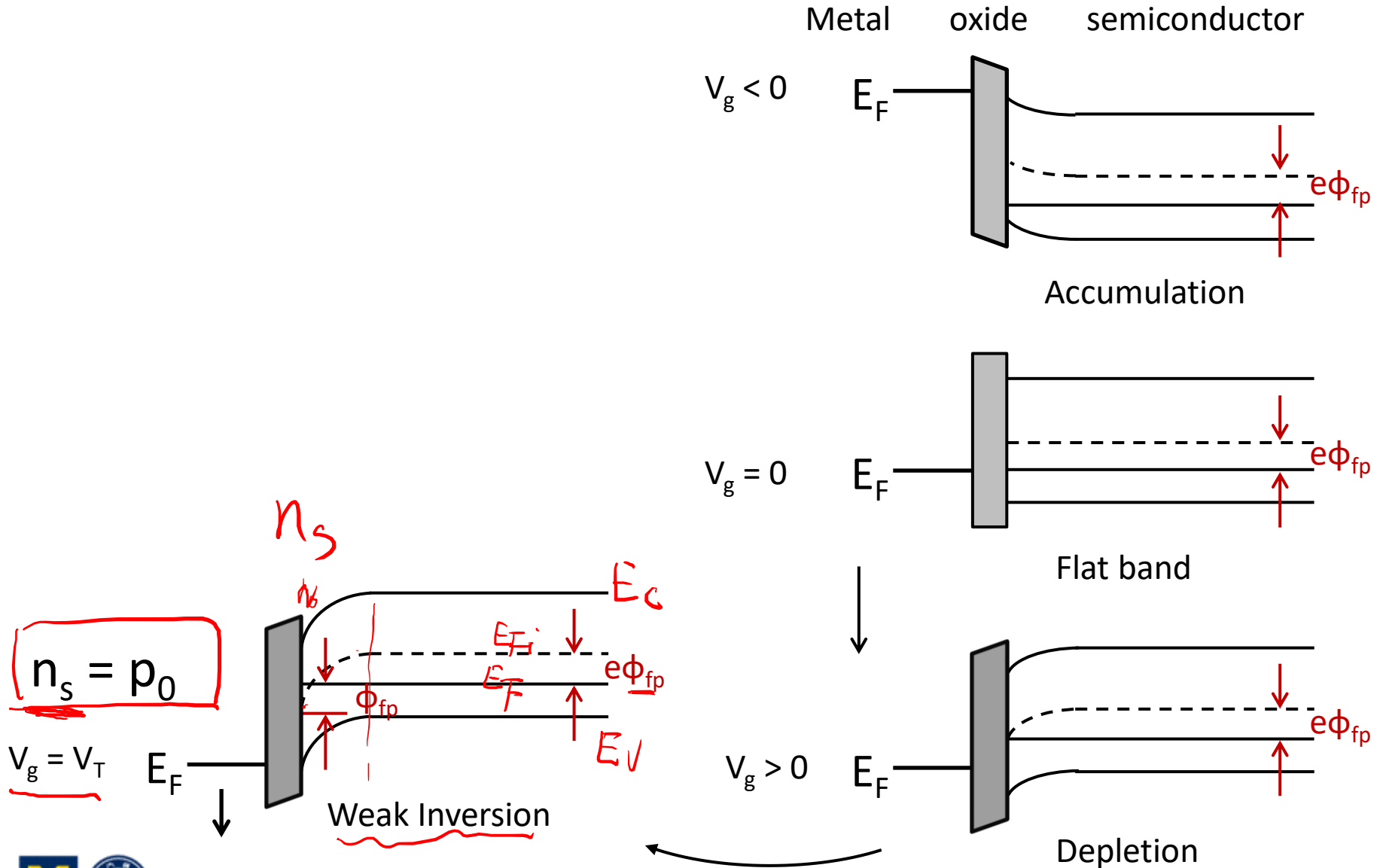
# 10.1 The two-terminal MOS structure



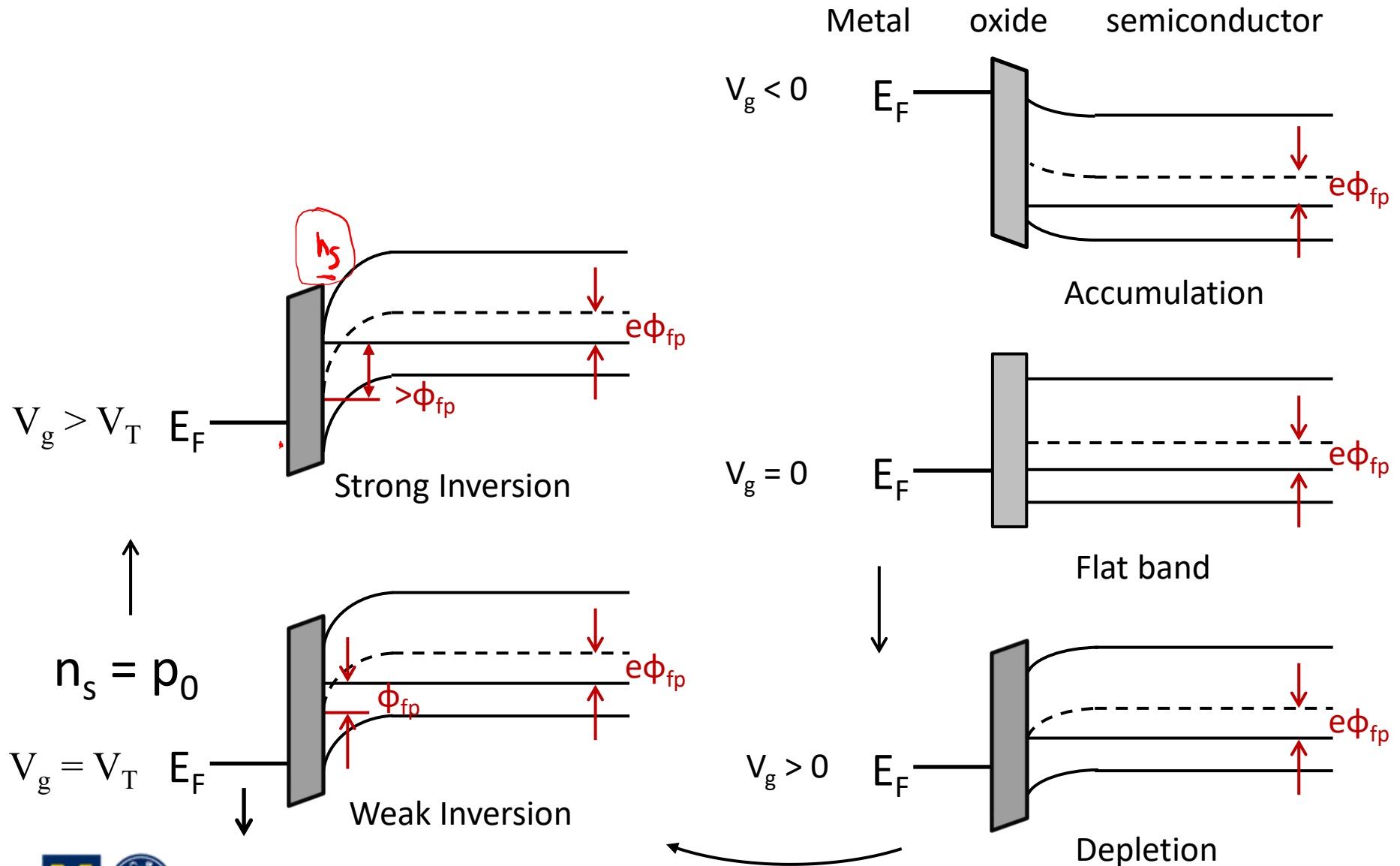
# 10.1 The two-terminal MOS structure



## 10.1 The two-terminal MOS structure



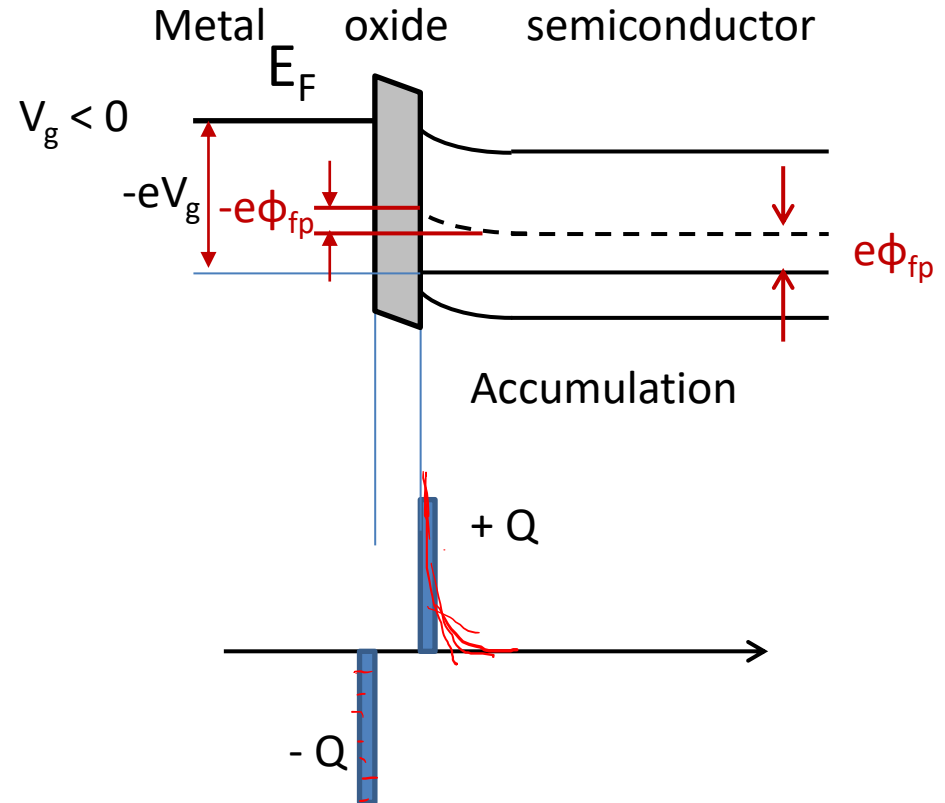
# 10.1 The two-terminal MOS structure





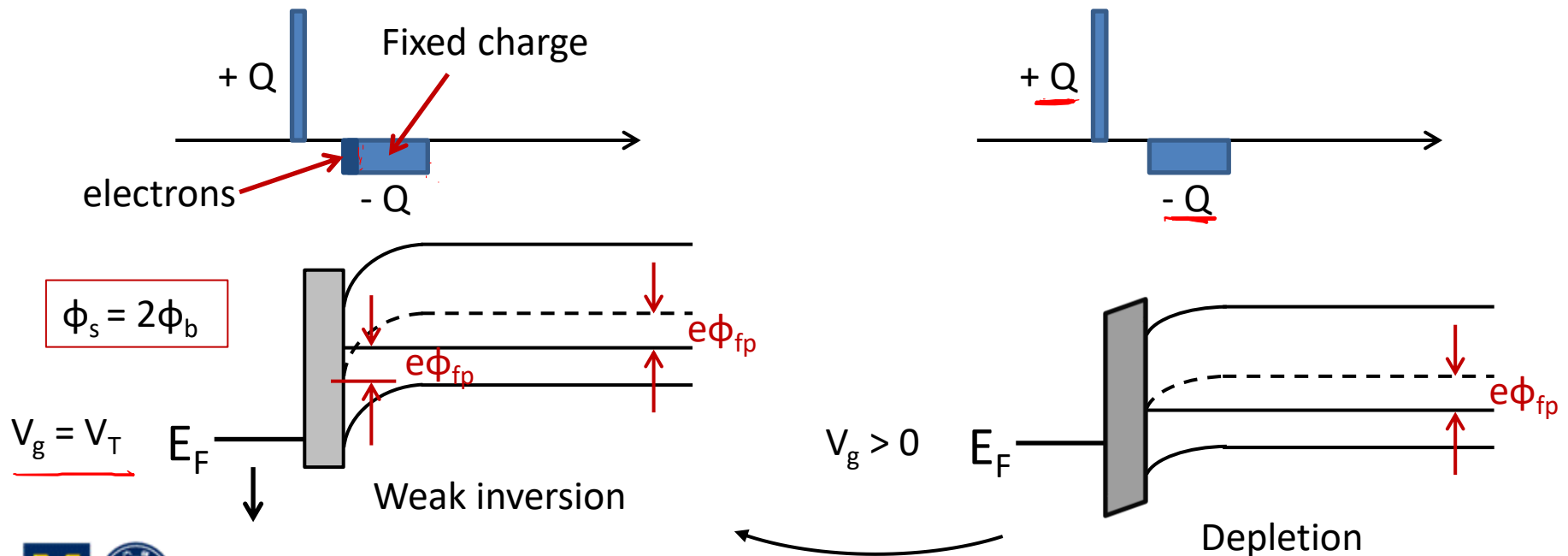
# 10.1 The two-terminal MOS structure

## ② Charge distribution



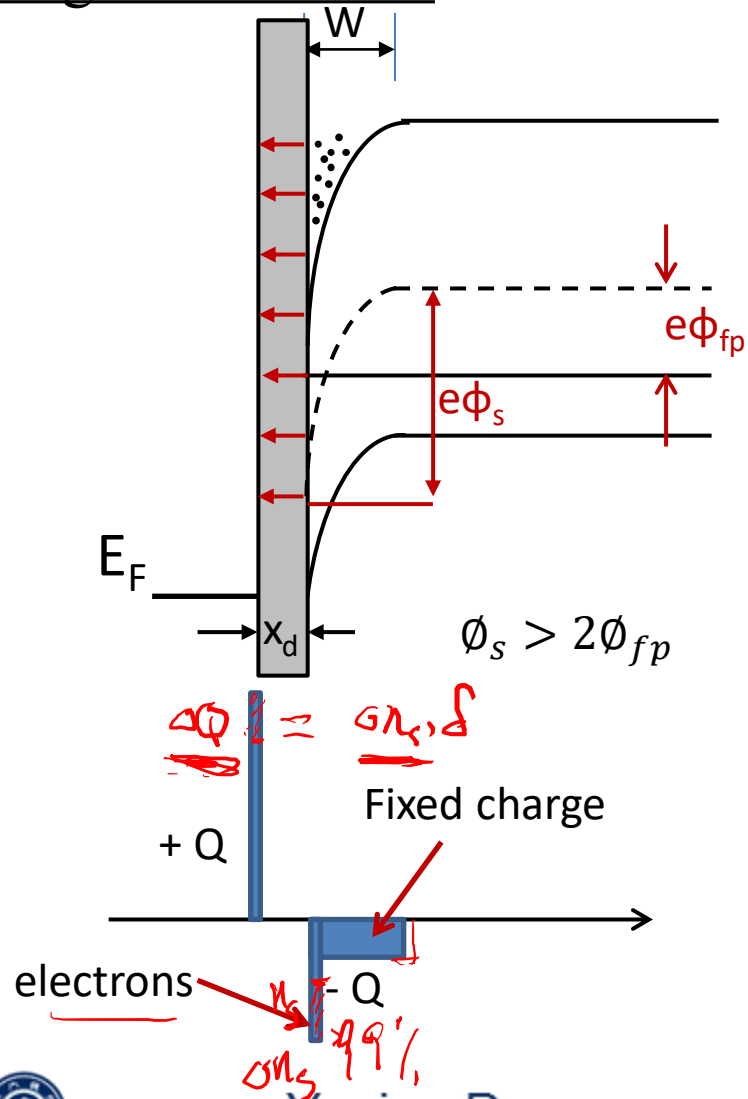
# 10.1 The two-terminal MOS structure

## Charge distribution



# 10.1 The two-terminal MOS structure

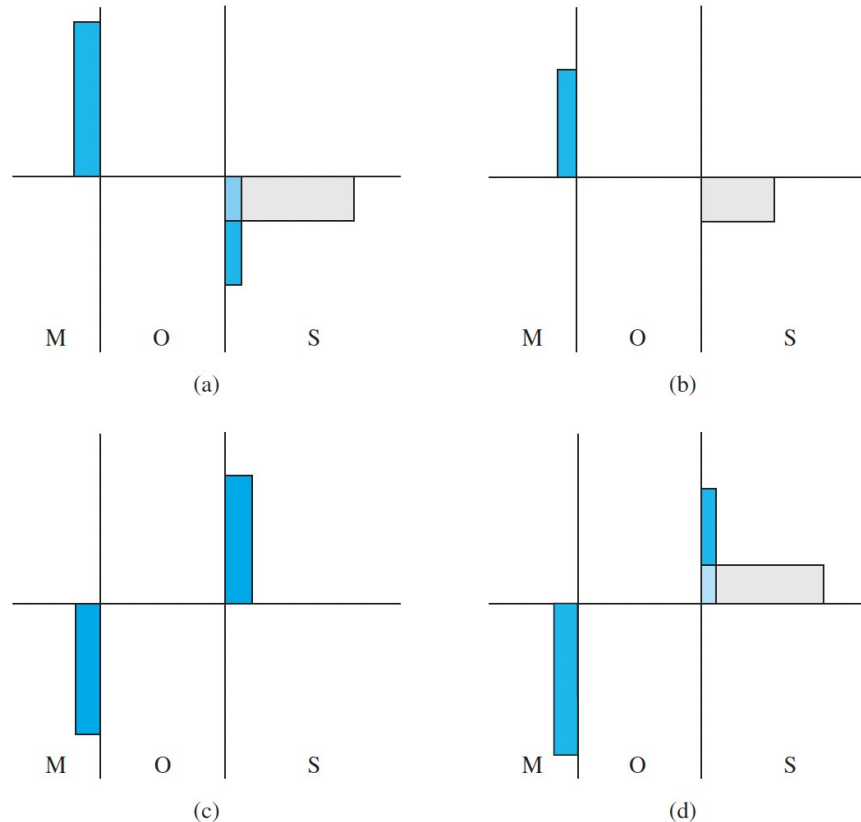
## Charge distribution



# Check your understanding

## Example Problem #1

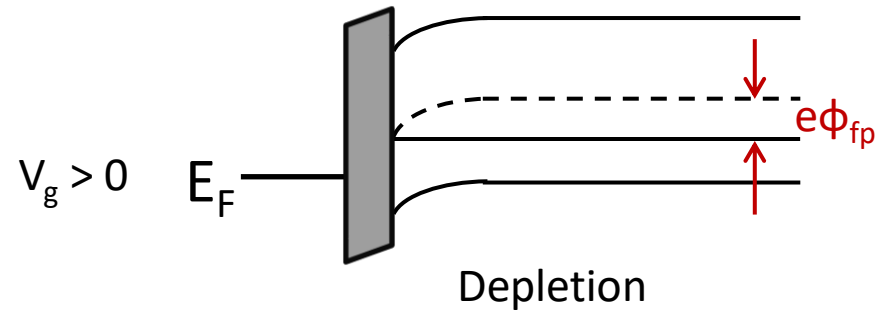
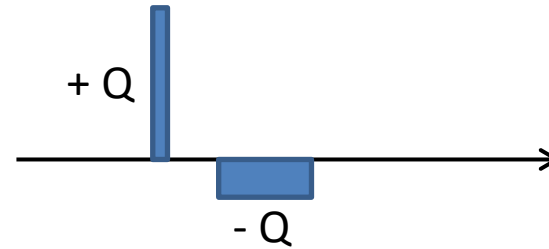
The dc charge distributions of four ideal MOS capacitors are shown in Figure P10.1. For each case: (a) Is the semiconductor n or p type? (b) Is the device biased in the accumulation, depletion, or inversion mode? (c) Draw the energy-band diagram in the semiconductor region.



# 10.1 The two-terminal MOS structure

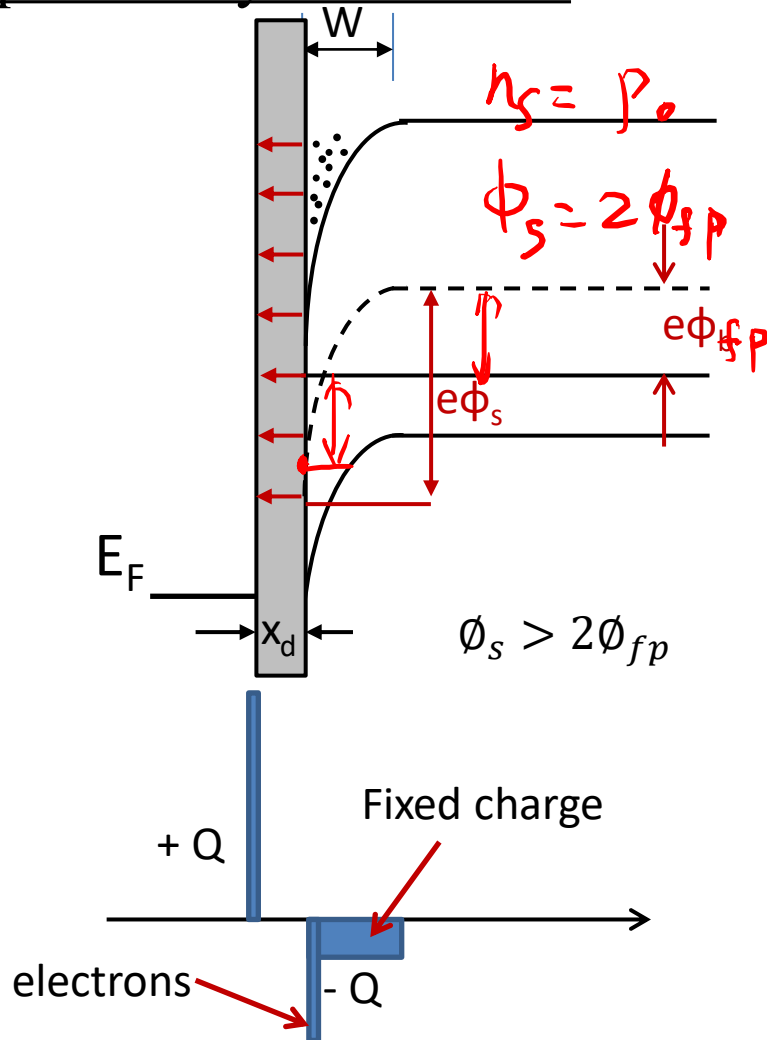
## ③ Depletion layer thickness

$$x_d = \left( \frac{2\epsilon_s \phi_s}{eN_a} \right)^{1/2}$$



# 10.1 The two-terminal MOS structure

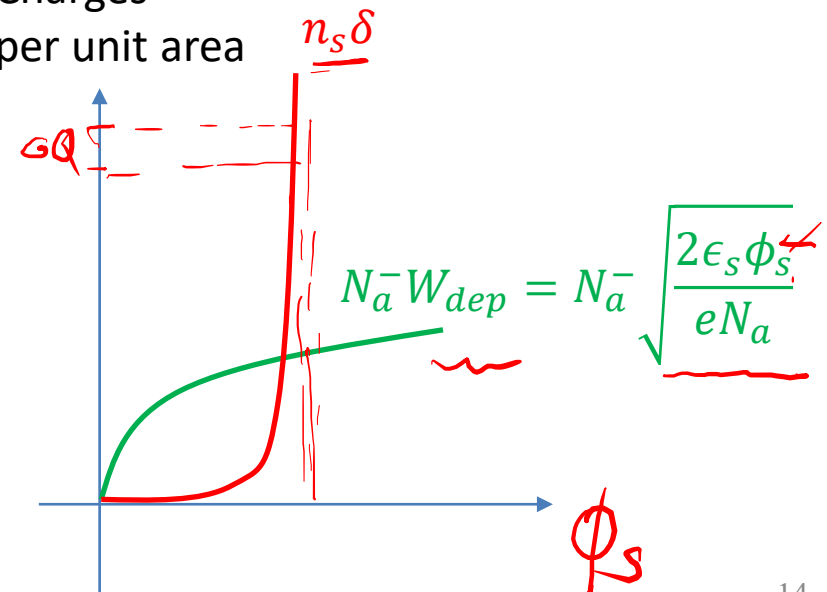
## Depletion layer thickness



Maximum depletion layer

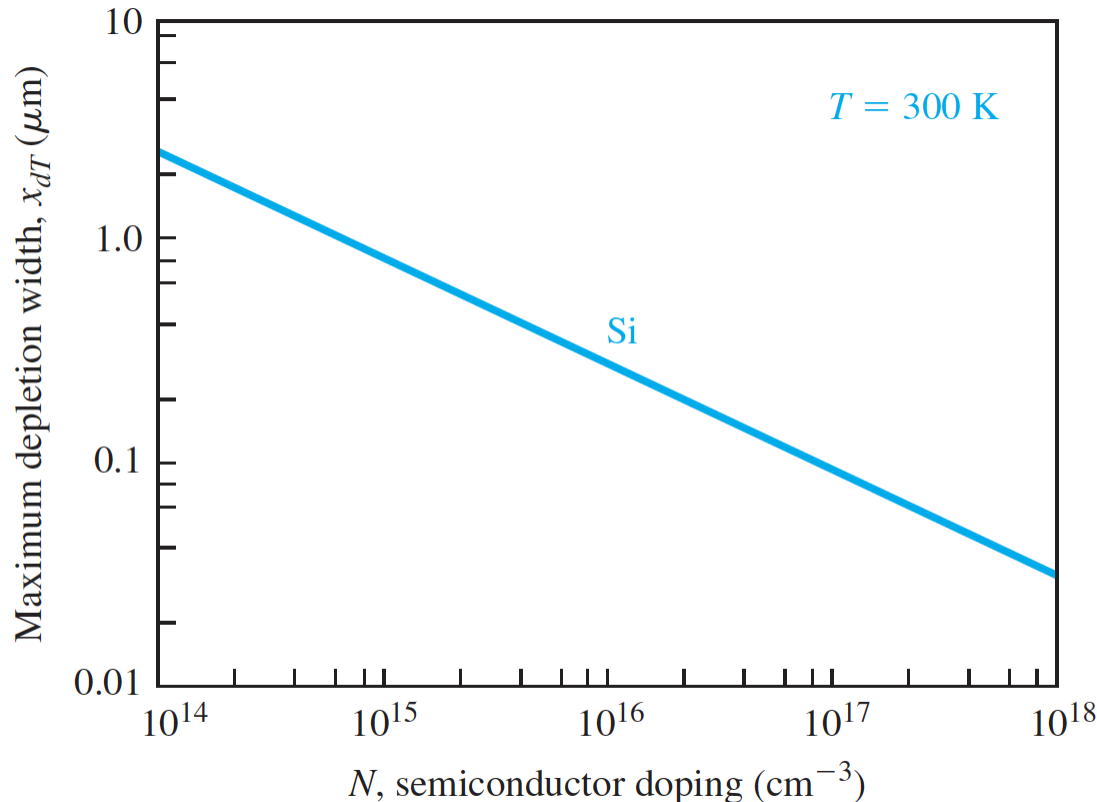
$$x_{dT} = \left( \frac{4\epsilon_s \phi_{fp}}{eN_a} \right)^{1/2}$$

Charges per unit area



# 10.1 The two-terminal MOS structure

## Depletion layer thickness



Maximum depletion layer

$$\underline{x_{dT}} = \left( \frac{4\epsilon_s \phi_{fp}}{e \underline{N_a}} \right)^{1/2}$$

# Outline

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10.1 The two-terminal MOS structure

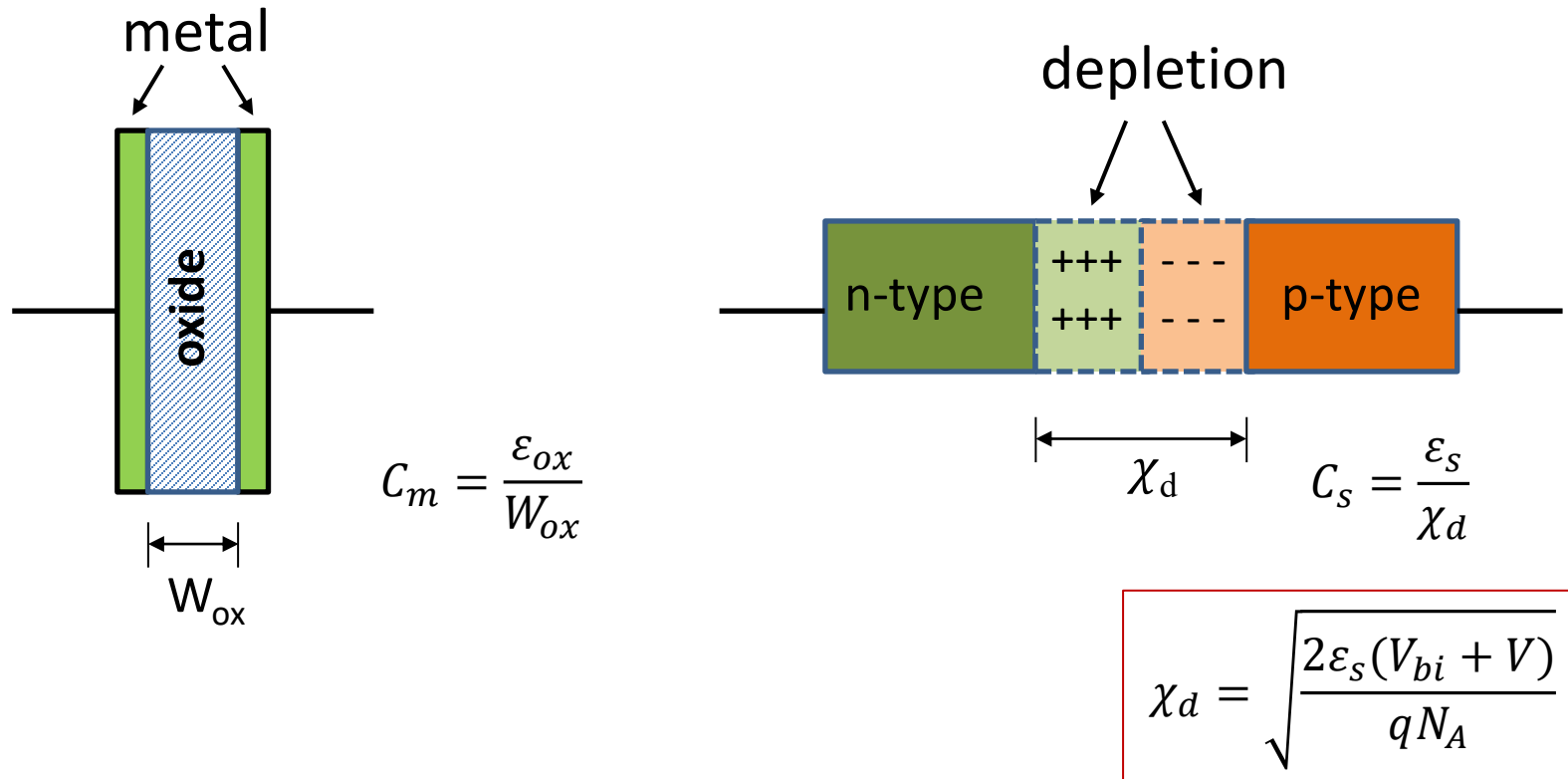
**10.2 Capacitance-voltage characteristics**

10.3 Non-ideal effects

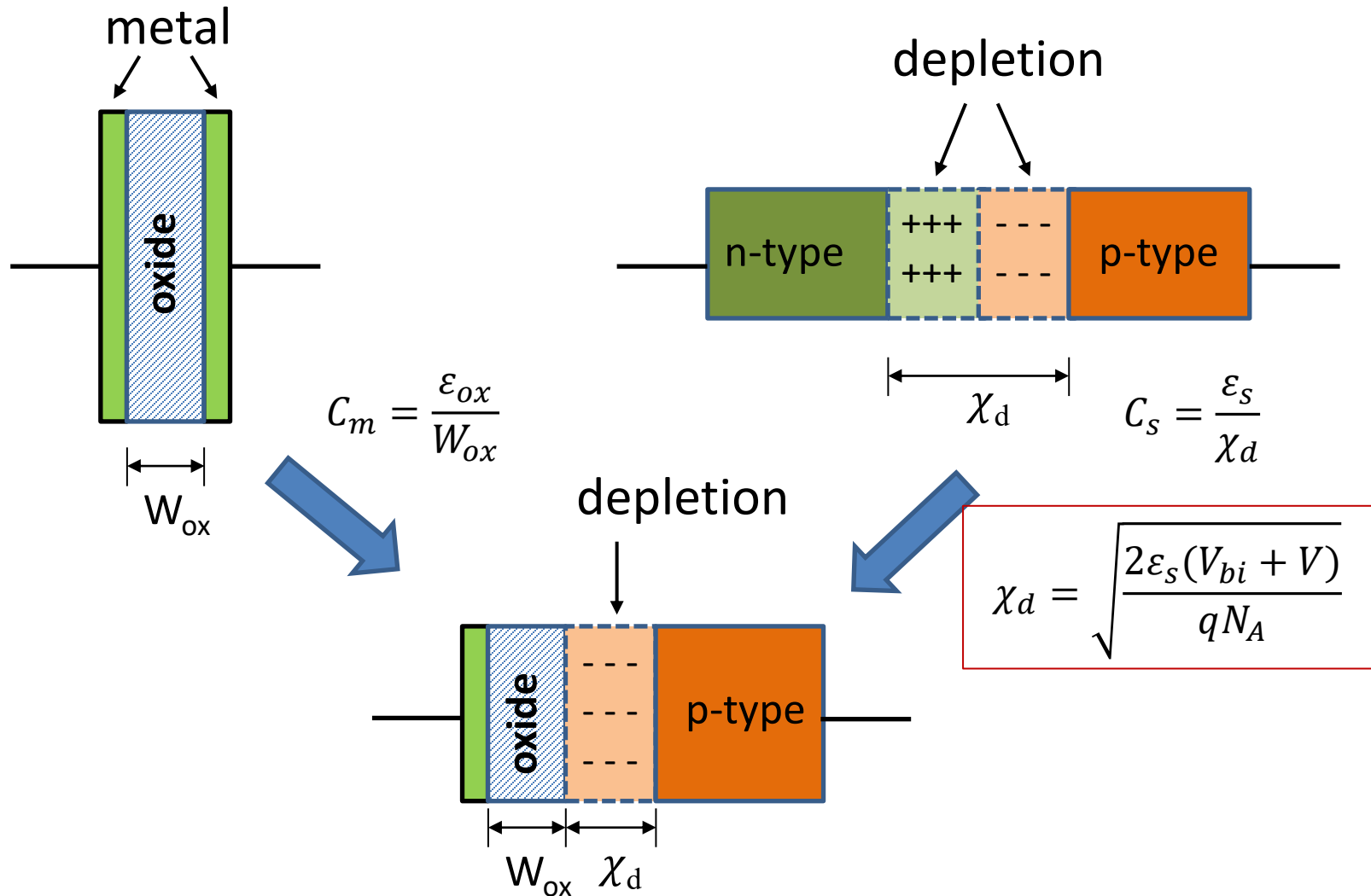
10.4 The basic MOSFET operation



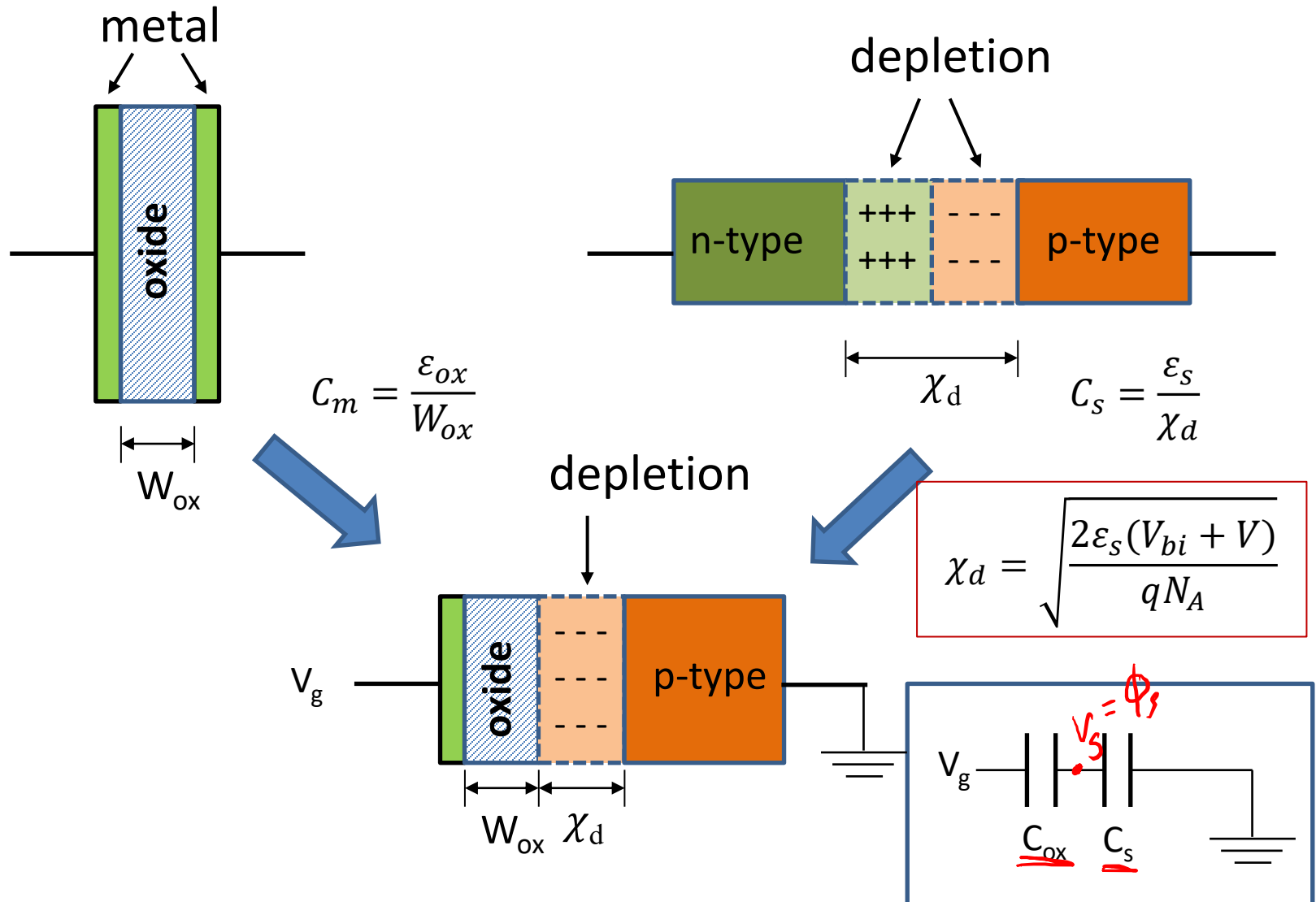
## 10.2 The capacitance-voltage characteristics



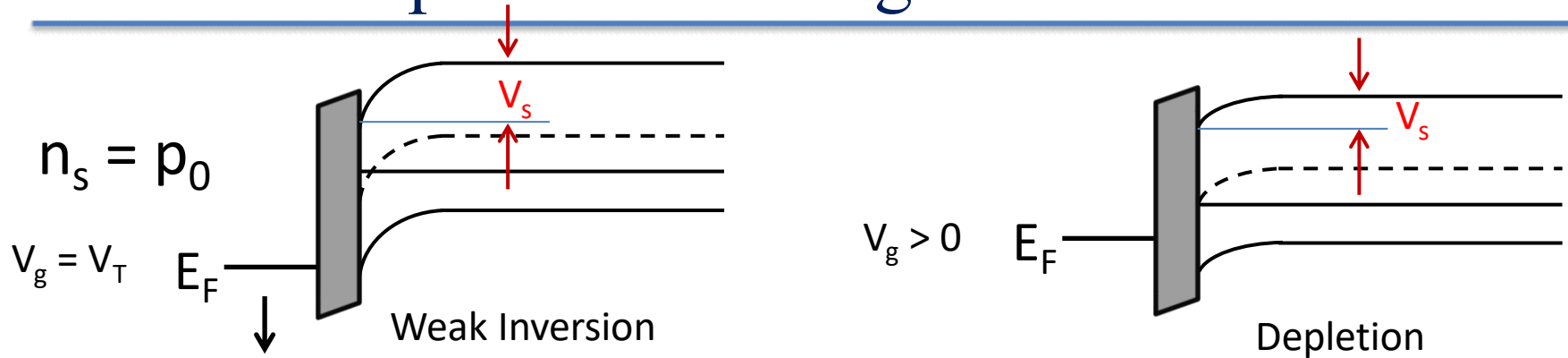
## 10.2 The capacitance-voltage characteristics



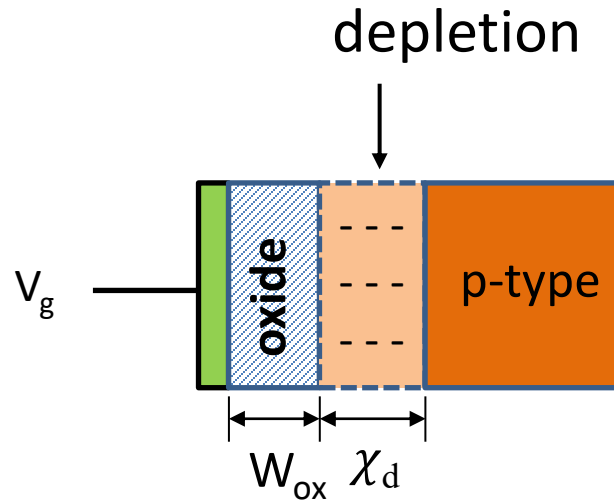
# 10.2 The capacitance-voltage characteristics



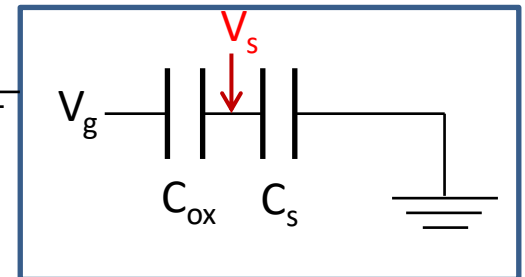
# 10.2 The capacitance-voltage characteristics



$$C_s = \frac{\epsilon_s}{\chi_d}$$



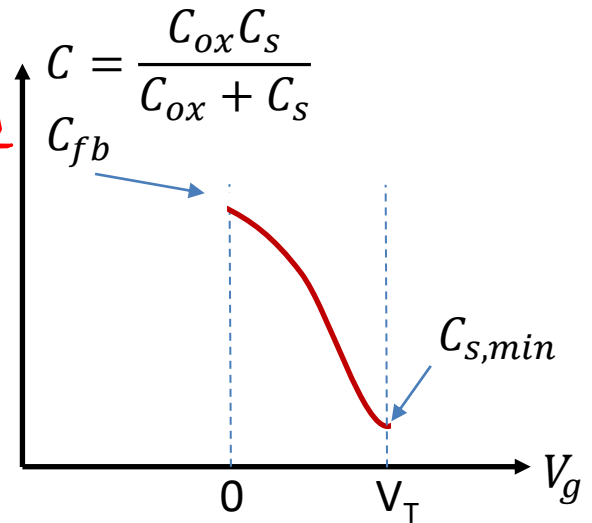
$$\chi_d = \sqrt{\frac{2\epsilon_s(V_{bi} + V)}{qN_A}}$$



## 10.2 The capacitance-voltage characteristics

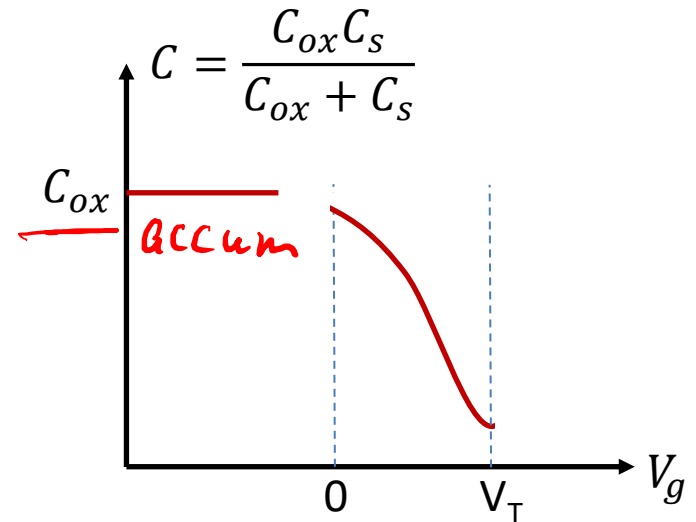
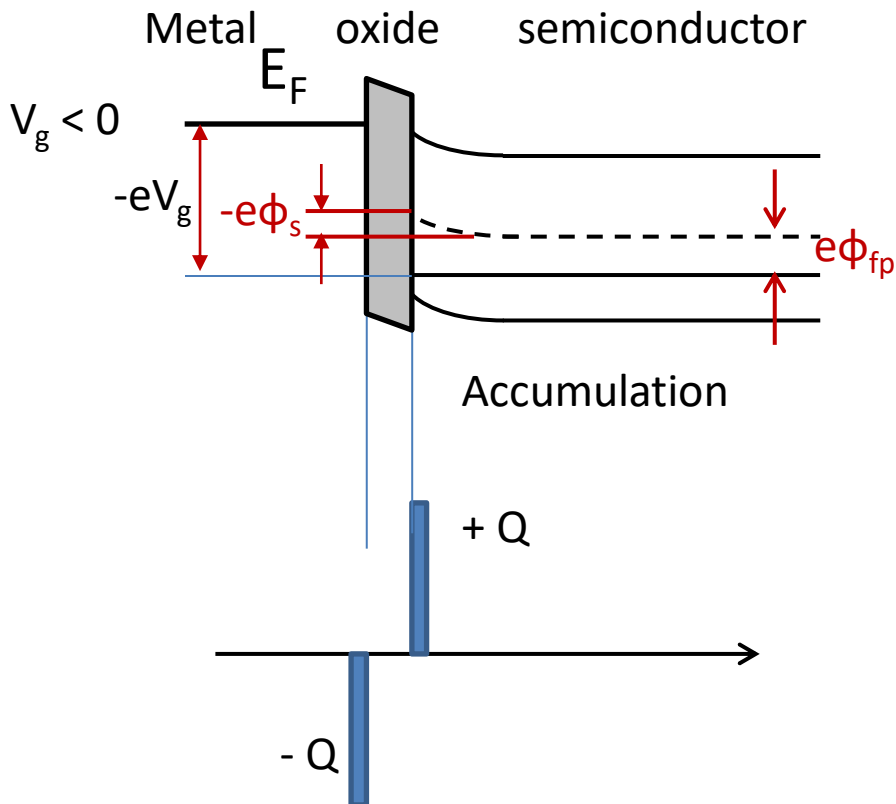
$$A C_s^3 + B C_s^2 + C C_s + D = 0$$

$$\left\{ \begin{array}{l} V_s = \frac{C_g \cdot C_{ox}}{C_{ox} + C_s} \\ C_s = \frac{\epsilon_s}{\sqrt{\frac{2 \epsilon_s V_s}{q N_A}}} \end{array} \right.$$



# 10.2 The capacitance-voltage characteristics

## Accumulation



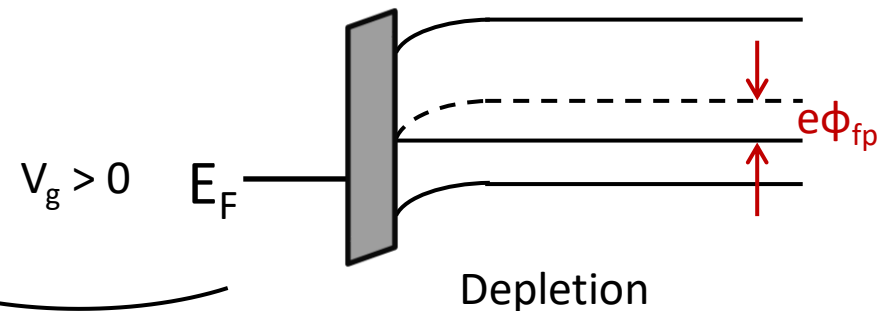
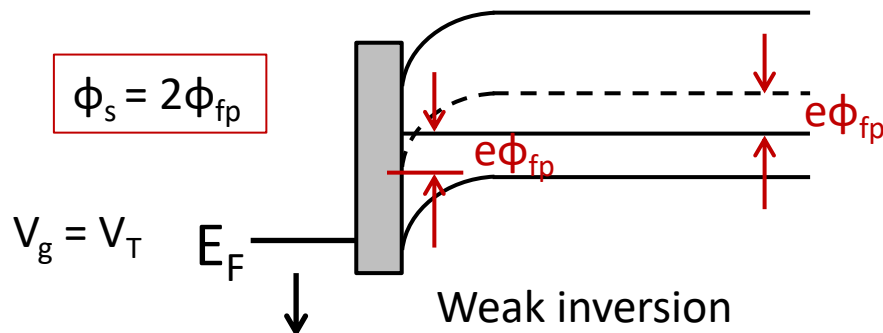
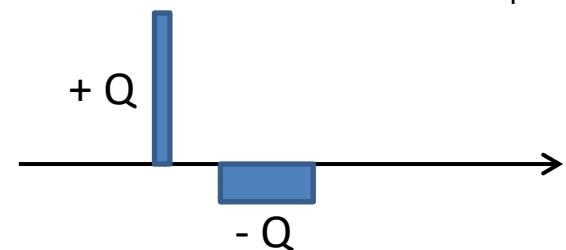
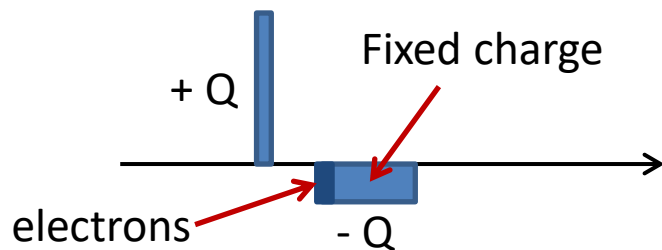
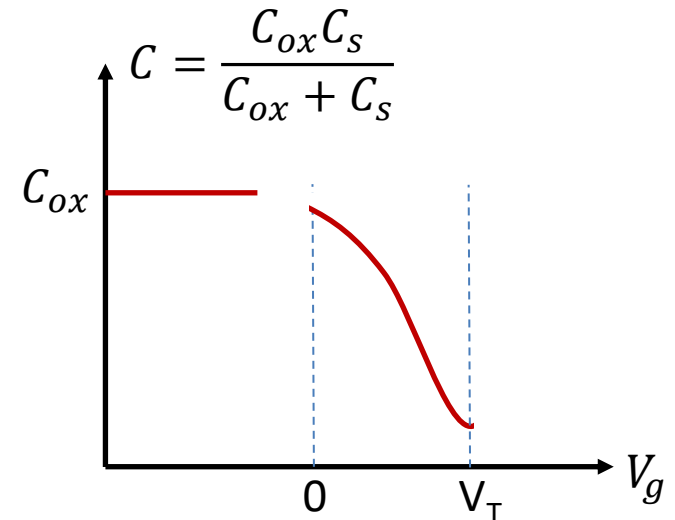
$$C_s \rightarrow \infty$$

$$C = \frac{C_s C_{ox}}{C_s + C_{ox}} \approx C_{ox}$$

# 10.2 The capacitance-voltage characteristics

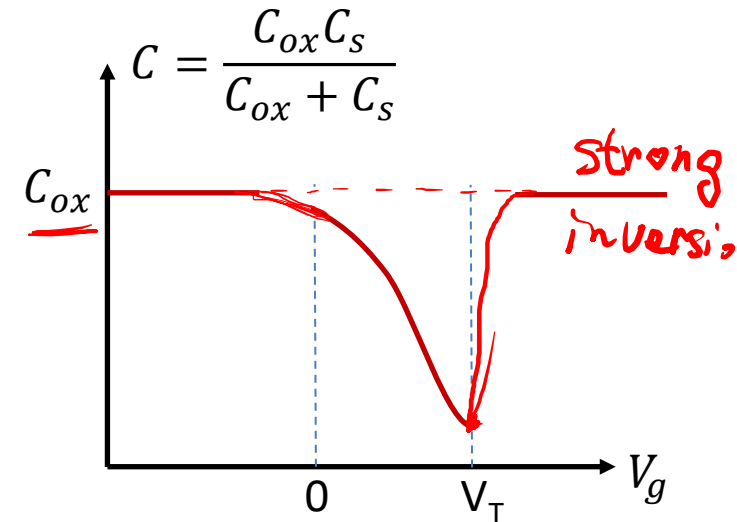
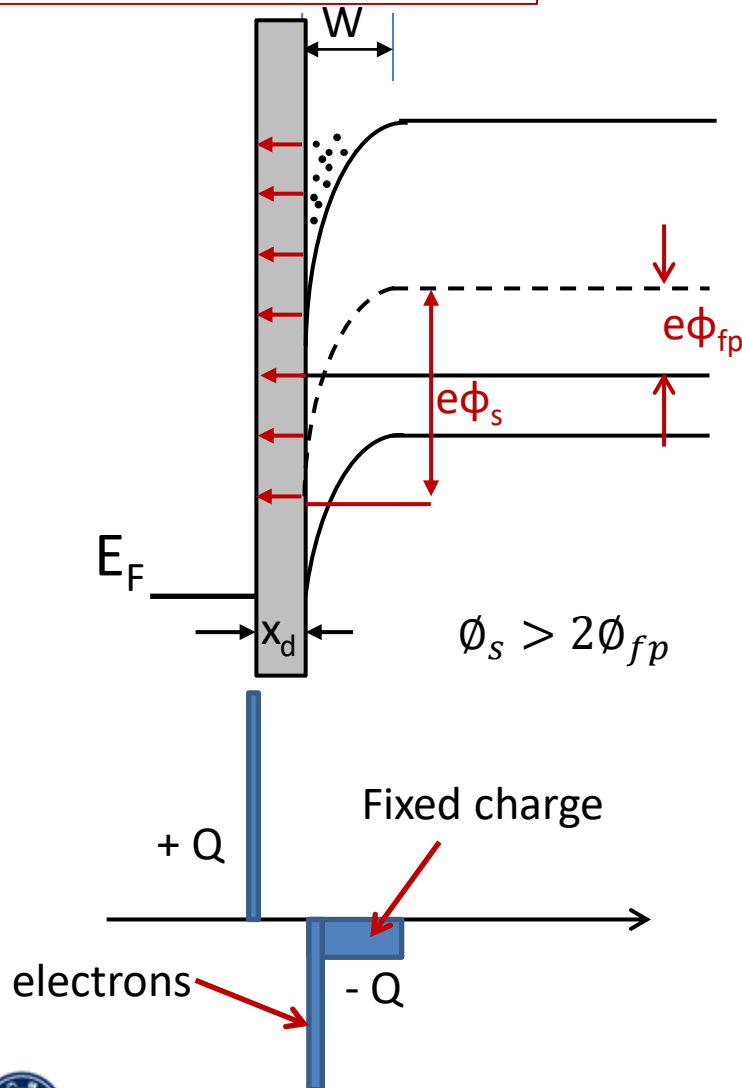
Depletion and weak inversion

$$C_s = \frac{\epsilon_s}{\sqrt{\frac{2\epsilon_s(V_s)}{qN_A}}}$$



# 10.2 The capacitance-voltage characteristics

Strong inversion



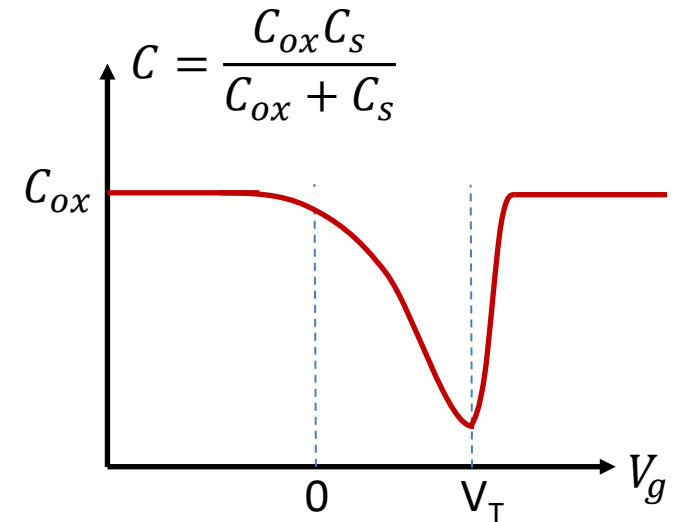
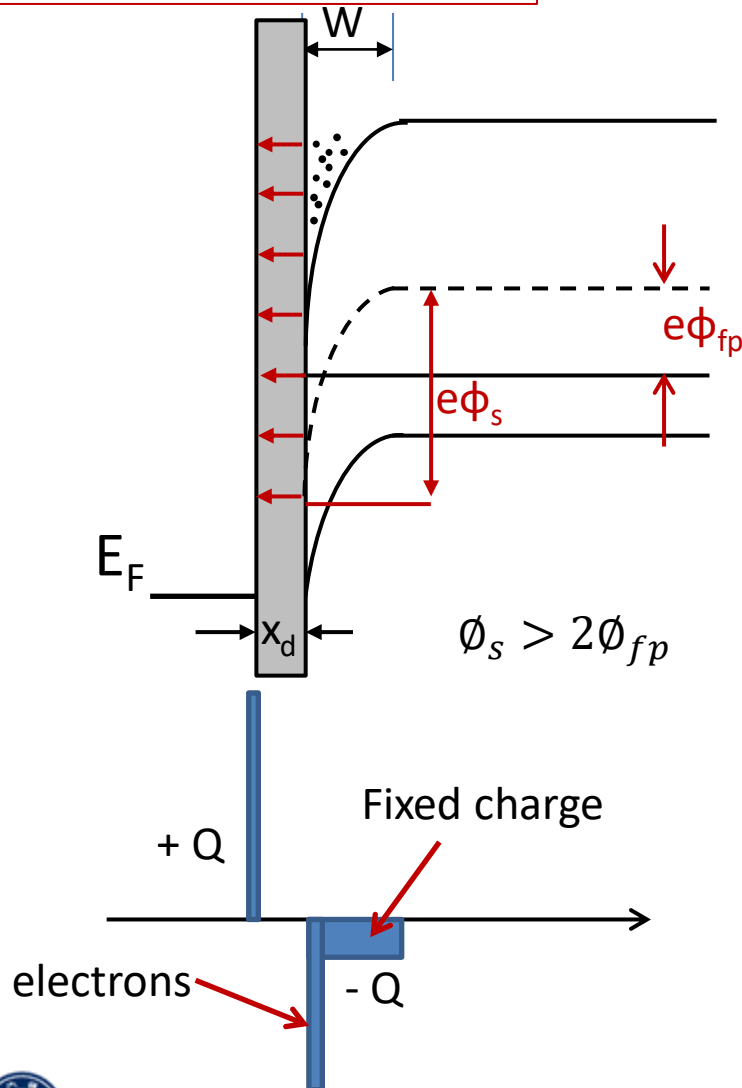
$$C_s \rightarrow \infty$$

$$C = \frac{C_s C_{ox}}{C_s + C_{ox}} \approx C_{ox}$$



# 10.2 The capacitance-voltage characteristics

Strong inversion



$$C_s \rightarrow \infty$$

$$C = \frac{C_s C_{ox}}{C_s + C_{ox}} \approx C_{ox}$$

# Check your understanding

## Problem Example #2

Consider a p-type silicon substrate at  $T = 300$  K doped to  $N_a = 10^{16} \text{ cm}^{-3}$ .

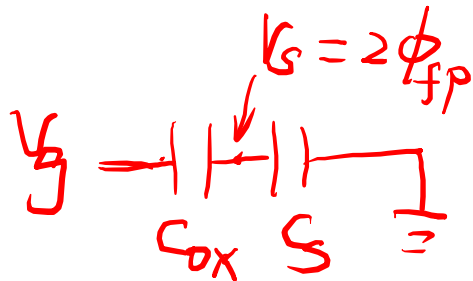
The oxide is silicon dioxide with a thickness of  $t_{ox} = 18 \text{ nm} = 180 \text{ \AA}$ , and the gate is aluminum.

Calculate  $C_{ox}$ ,  $C'_{min}$ , and  $C'_{FB}$  for a MOS capacitor.

$$C_{min} = \frac{\epsilon_{ox} t_{ox} \chi_{dT}}{\epsilon_{ox} \chi_{dT} + \epsilon_s t_{ox}}$$

$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} = \frac{3.9 \times 8.85 \times 10^{-14}}{18 \times 10^{-7}} = 1.92 \times 10^{-7} \text{ F/cm}^2$$

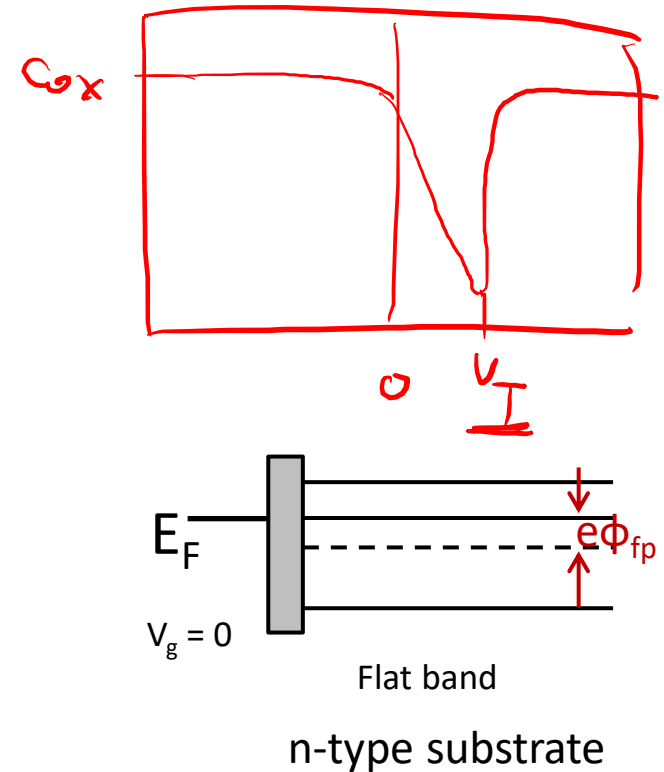
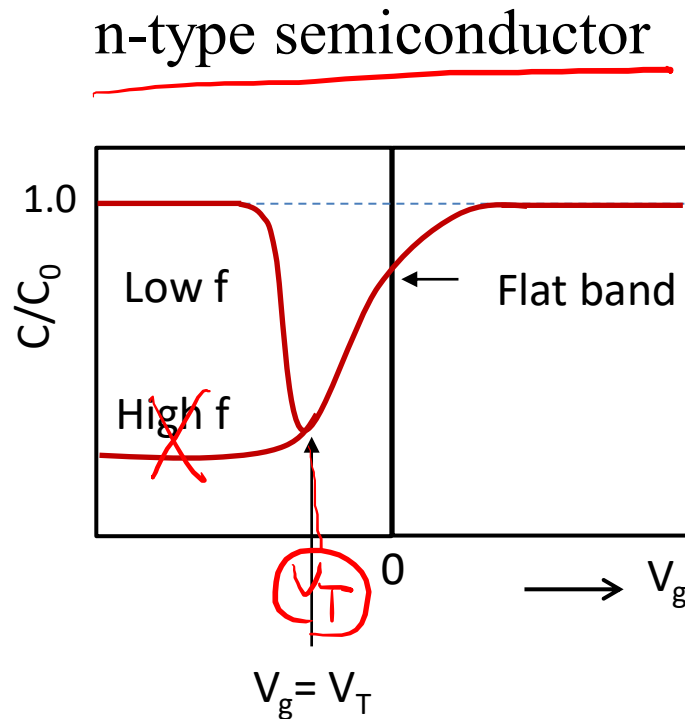
$$\frac{1}{C_t} = \frac{1}{C_{ox}} + \frac{1}{C_{smin}} = \frac{\epsilon_{ox}}{t_{ox}} + \frac{\epsilon_s}{\chi_{dT}}$$



$$\chi_{dT} = \left( \frac{4\epsilon_s \phi_{fp}}{eN_a} \right)^{1/2}$$

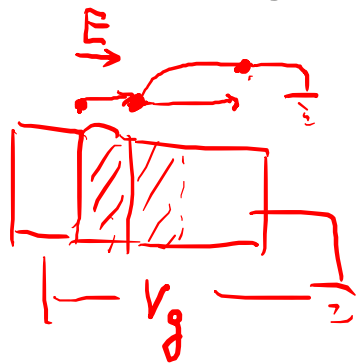
$$C_{smin} = \frac{\epsilon_s}{\chi_{dT}}$$

## 10.2 The capacitance-voltage characteristics



# 10.2 The capacitance-voltage characteristics

## Threshold voltage

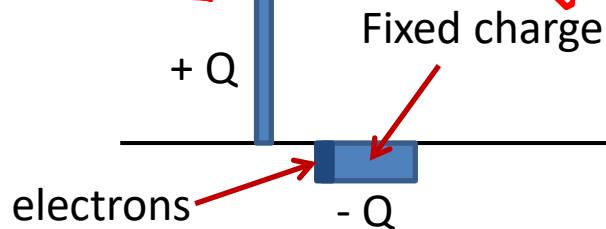


$$V_T = V_g = \underbrace{E \cdot t_{ox}}_{V_{ox} = N_a \cdot \chi_{dT} / C_{ox}} + \phi_s$$

$$C_s = \frac{\epsilon_s}{\sqrt{\frac{2\epsilon_s(V_s)}{qN_A}}} = \frac{\epsilon_s}{\sqrt{\frac{2\epsilon_s(2\phi_{fp})}{qN_A}}}$$

$$C \cdot V = Q \quad V = \frac{Q}{C}$$

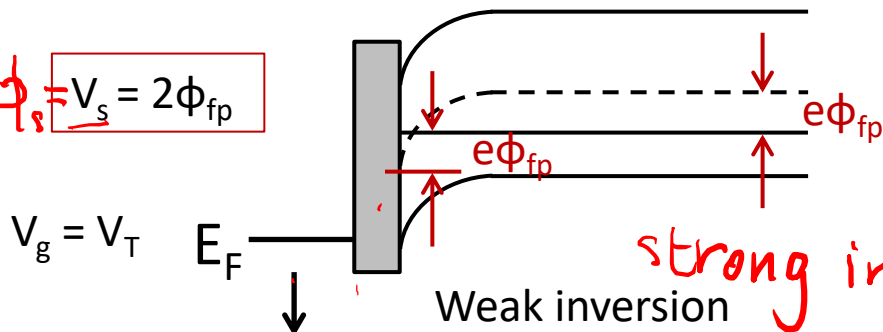
$$2\phi_{fp} = \frac{V_T C_{ox}}{C_{ox} + C_s} \Rightarrow V_T = 2\phi_{fp} + 2\phi_{fp} \frac{C_s}{C_{ox}}$$



$$Q = \chi_{dT} \cdot N_a$$

$$V_T = 2\phi_{fp} + \frac{2\sqrt{e\epsilon_s N_A \phi_{fp}}}{C_{ox}}$$

$$\phi_s = V_s = 2\phi_{fp}$$

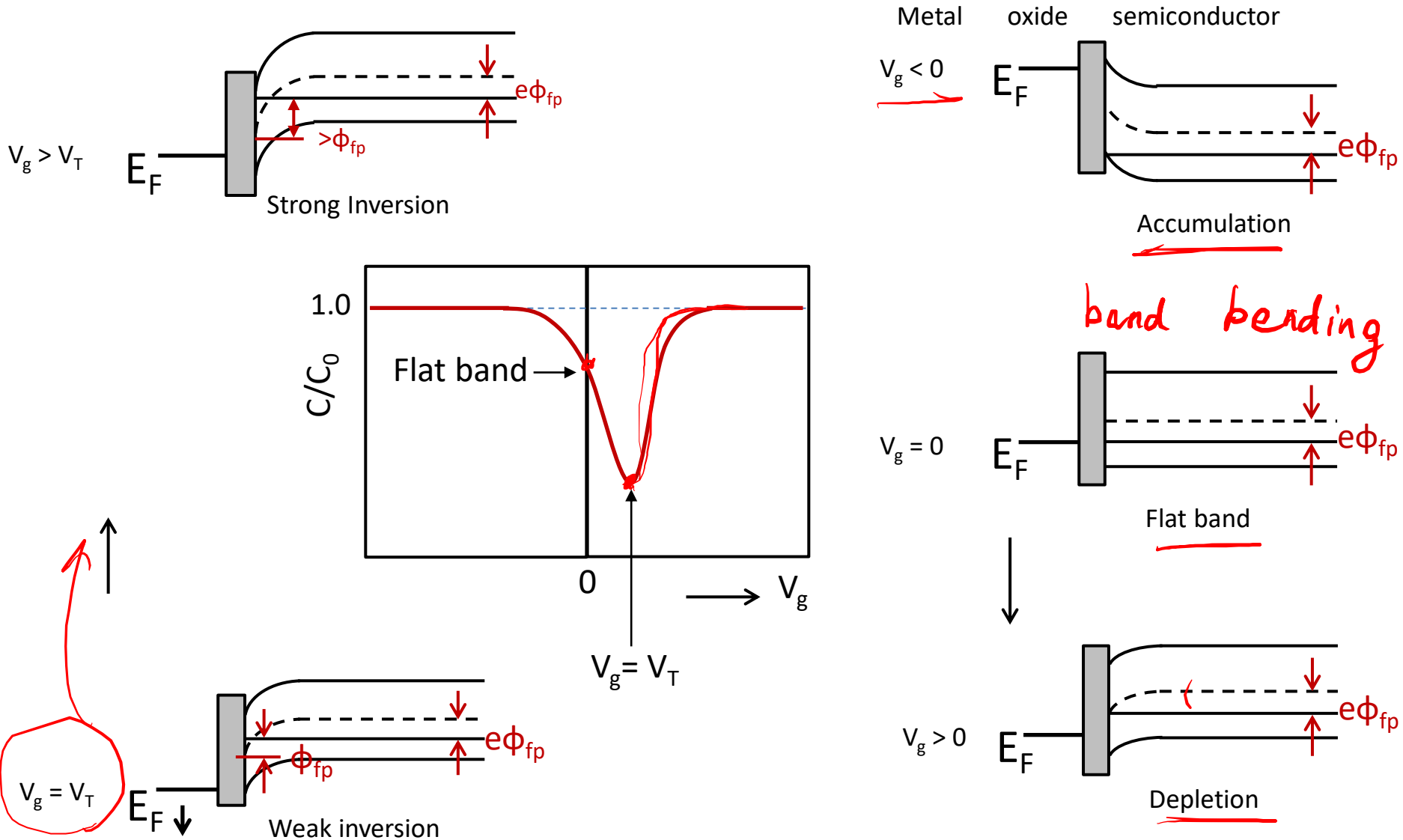


strong inversion

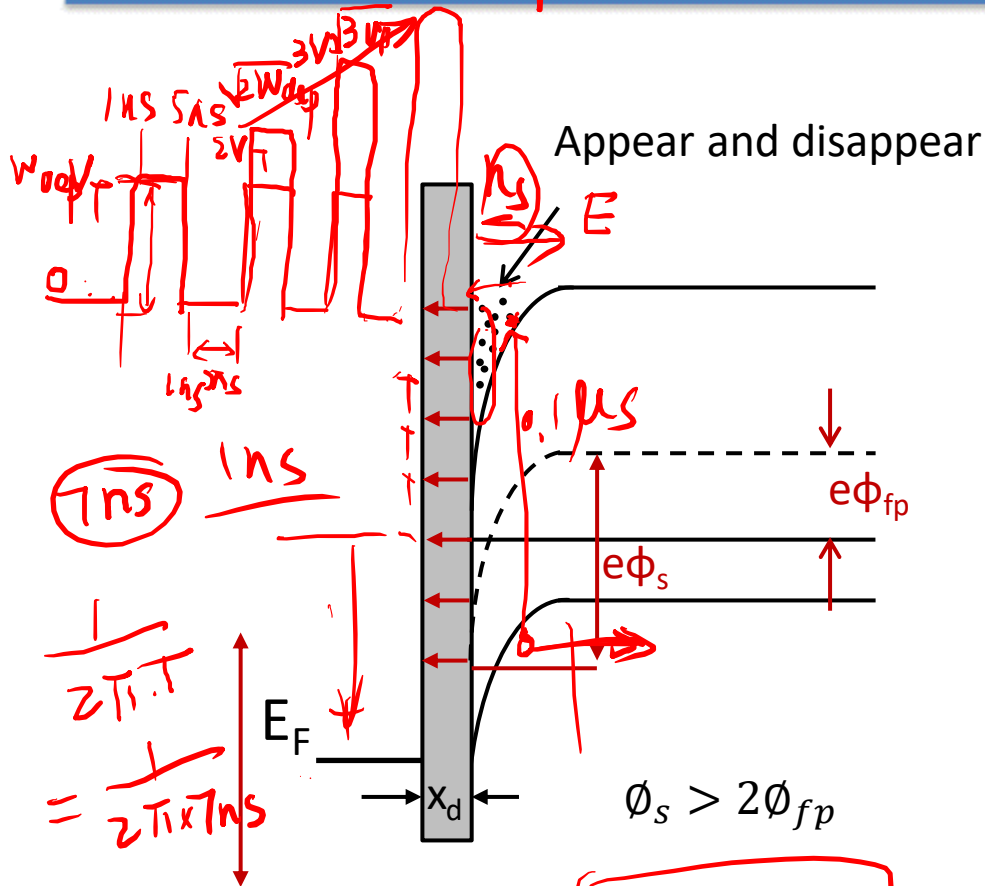
$$V_T = 2\phi_{fp} + \frac{N_a \cdot \chi_{dT}}{C_{ox}}$$

$$\chi_{dT} = \sqrt{\frac{A \epsilon_s \cdot \phi_{fp}}{e N_a}} \cdot C_{ox}$$

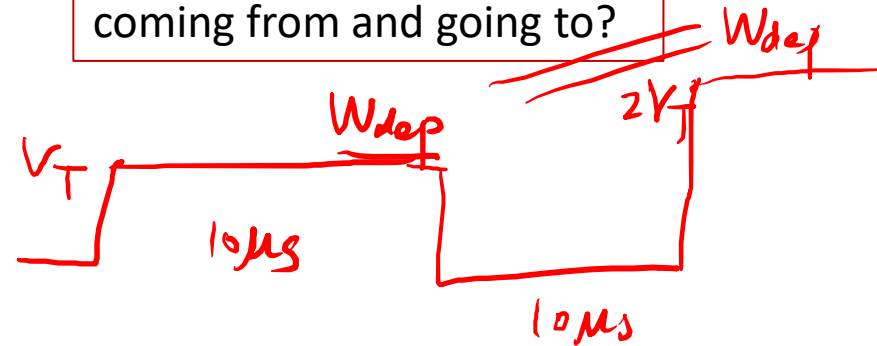
# 10.2 The capacitance-voltage characteristics: summary



# Question: $W_{dep}$



Where are the electrons coming from and going to?

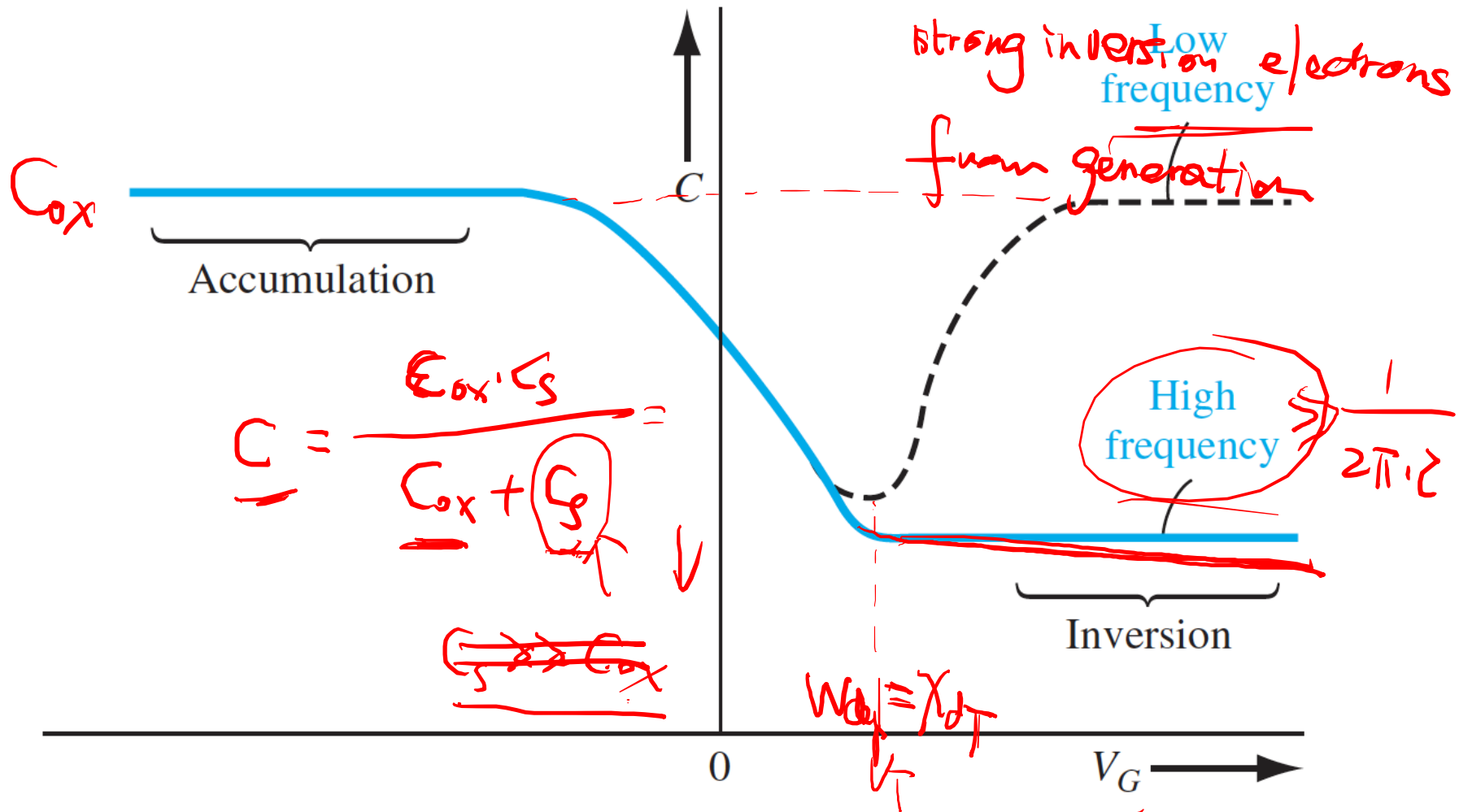


$$= 2 \times 3.14 \times \frac{1}{2\pi \times 7 ns} = 2.4 \times 10^9 Hz = 2.4 GHz$$

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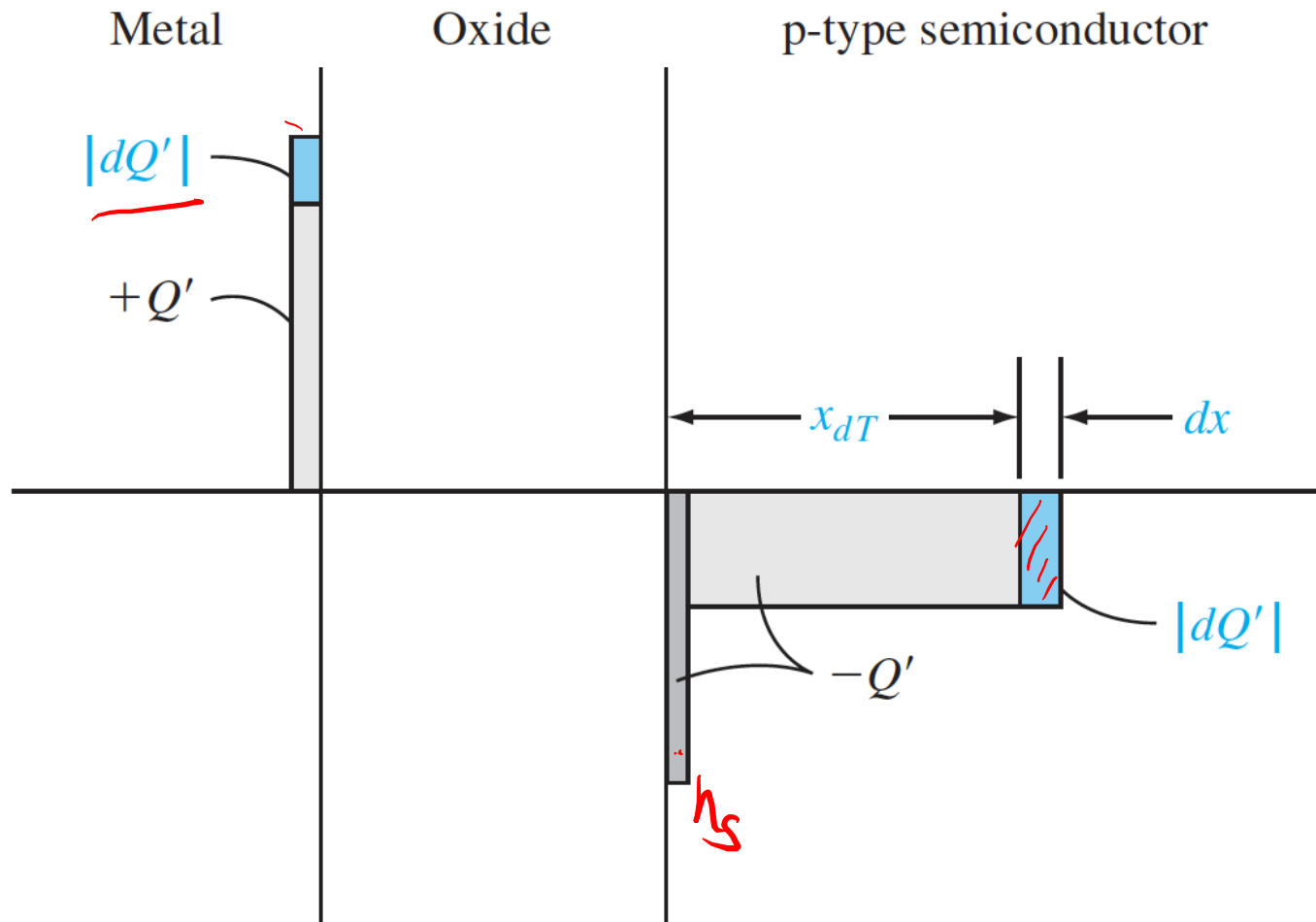
# 10.2 The capacitance-voltage characteristics

## Frequency dependence



# 10.2 The capacitance-voltage characteristics

## Frequency dependence





# Outline

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10.1 The two-terminal MOS structure

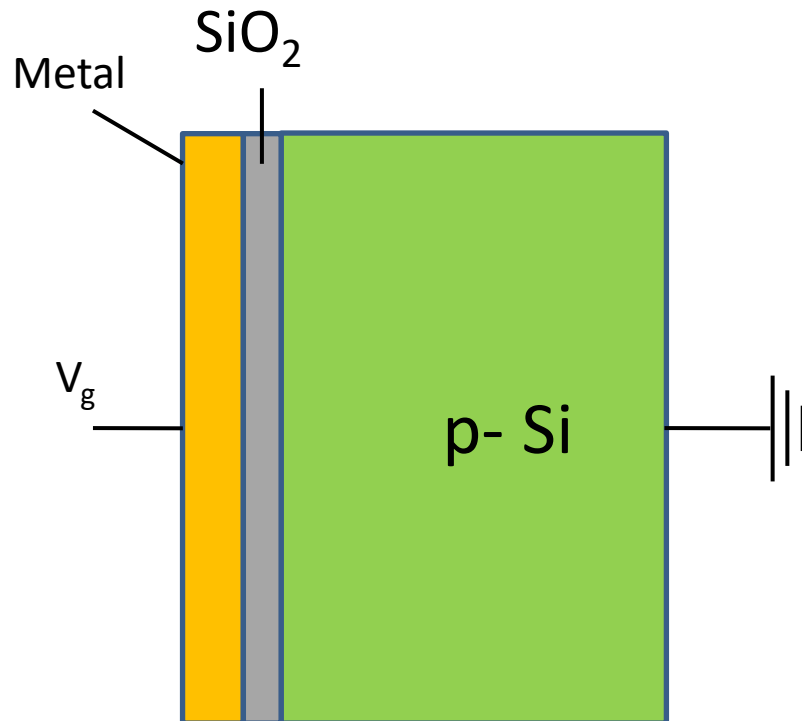
10.2 Capacitance-voltage characteristics

**10.3 Non-ideal effects**

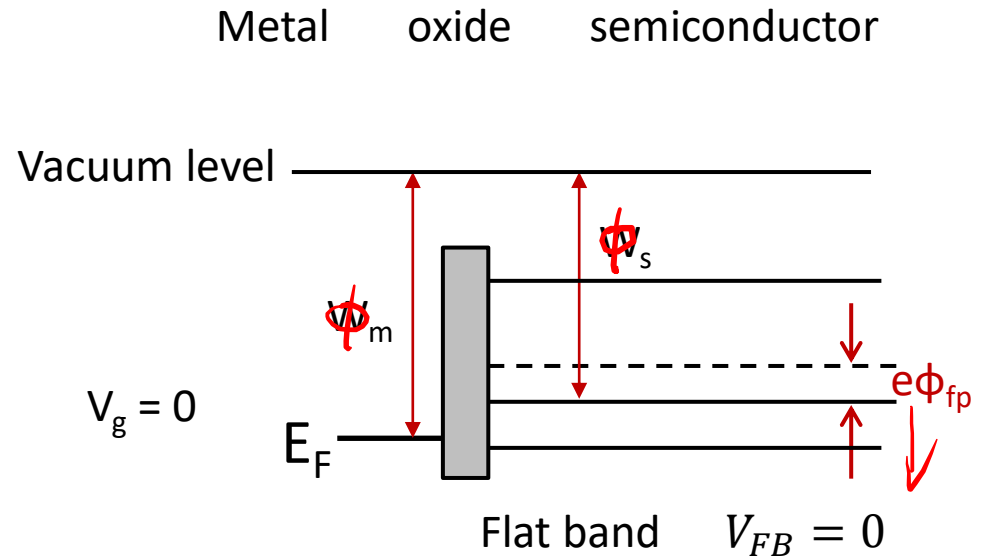
10.4 The basic MOSFET operation

# 10.3 Non-ideal effects

## Work function difference

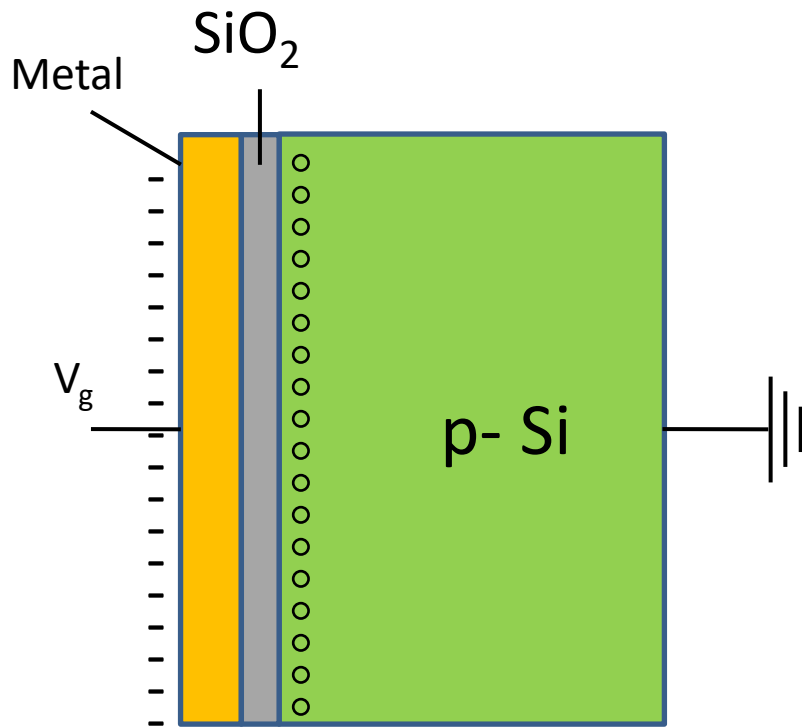


Metal-insulator-semiconductor (MIS)

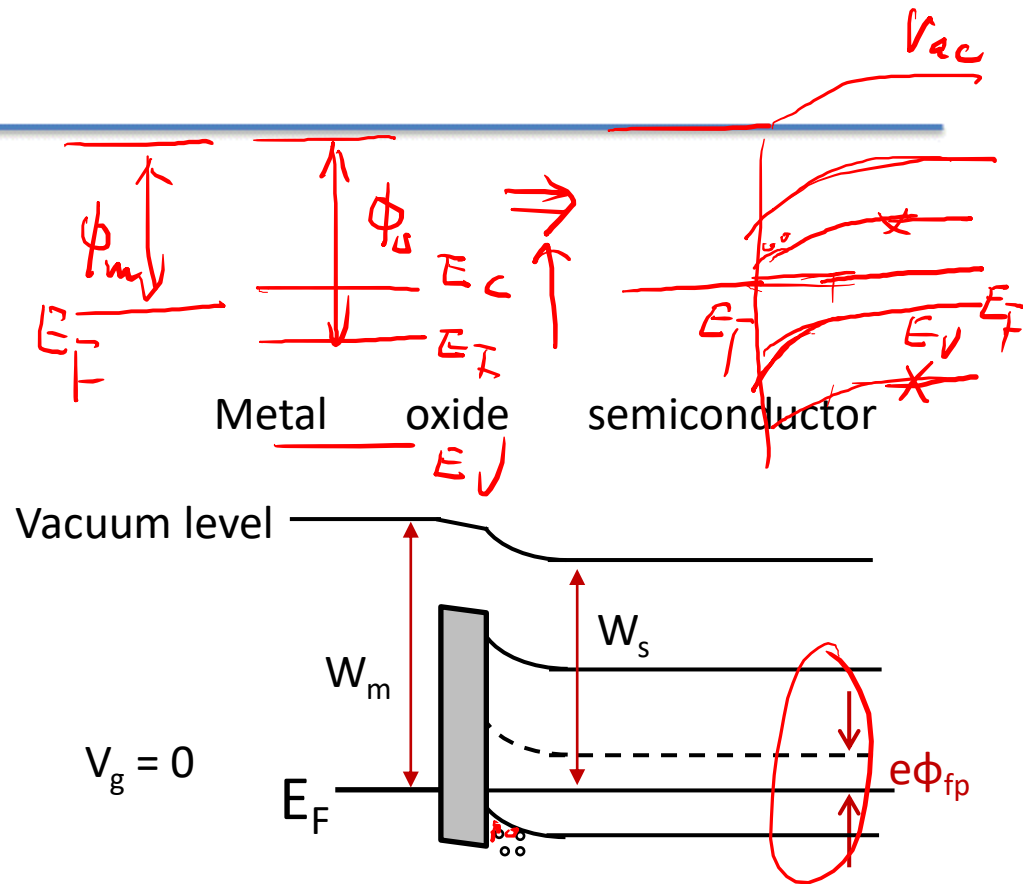


# 10.3 Non-ideal effects

## Work function difference

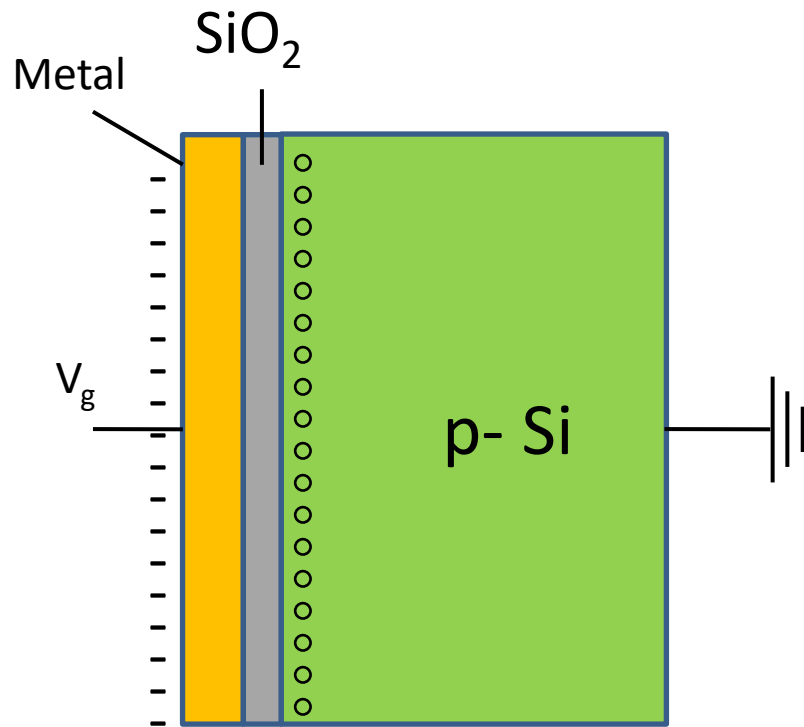


Metal-insulator-semiconductor (MIS)

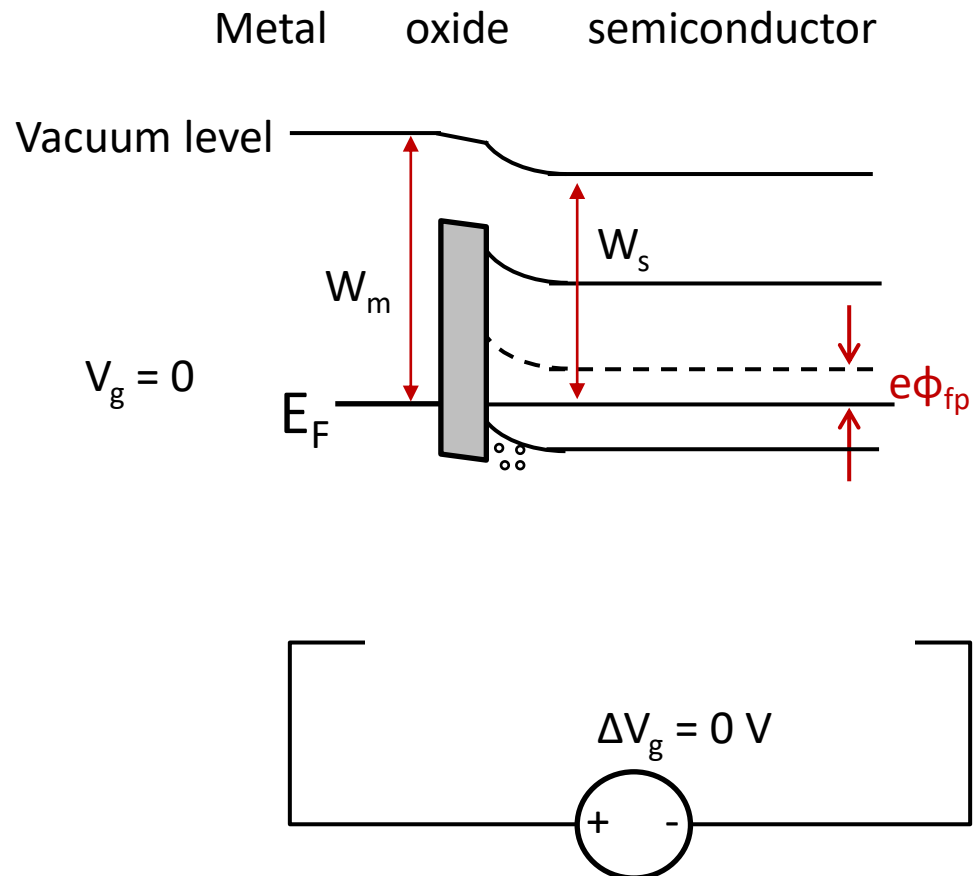


## 10.3 Non-ideal effects

### Work function difference

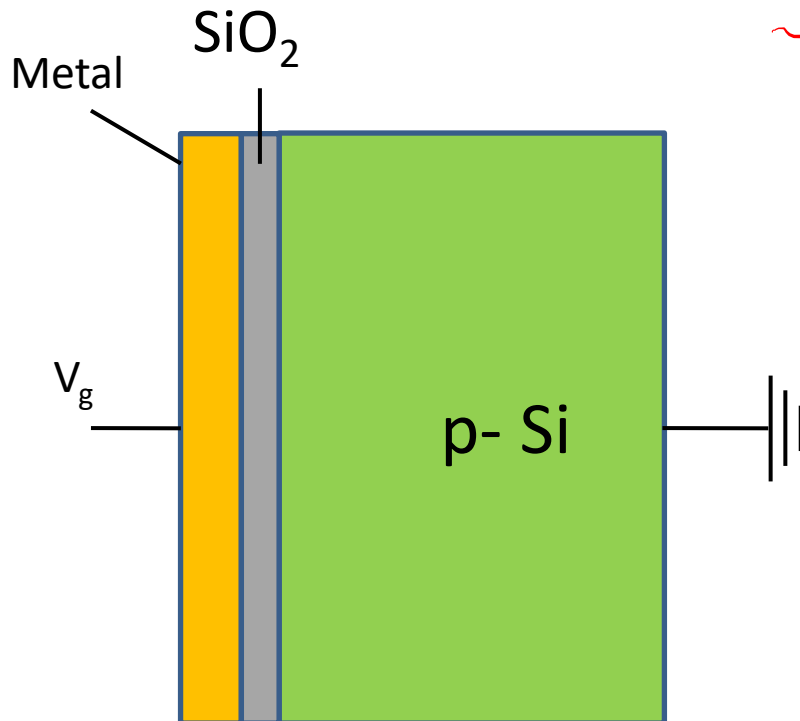


Metal-insulator-semiconductor (MIS)



# 10.3 Non-ideal effects

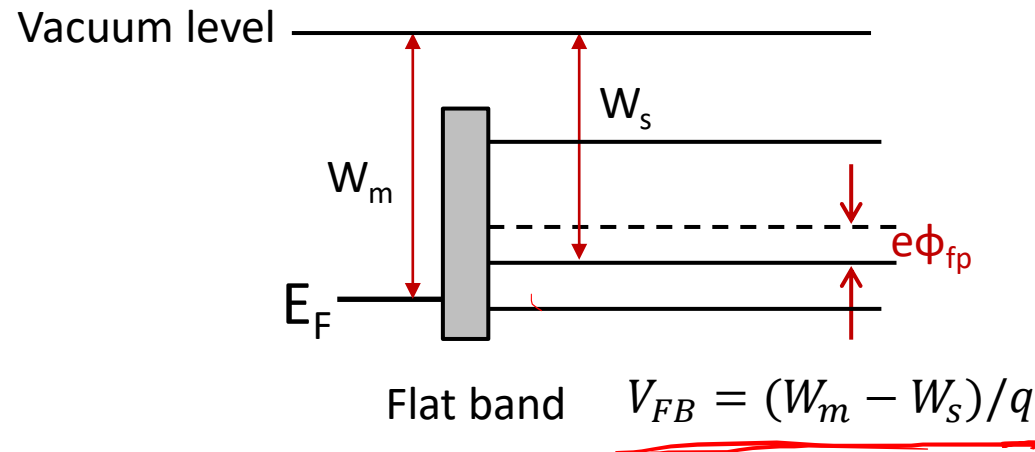
## Work function difference



Metal-insulator-semiconductor (MIS)

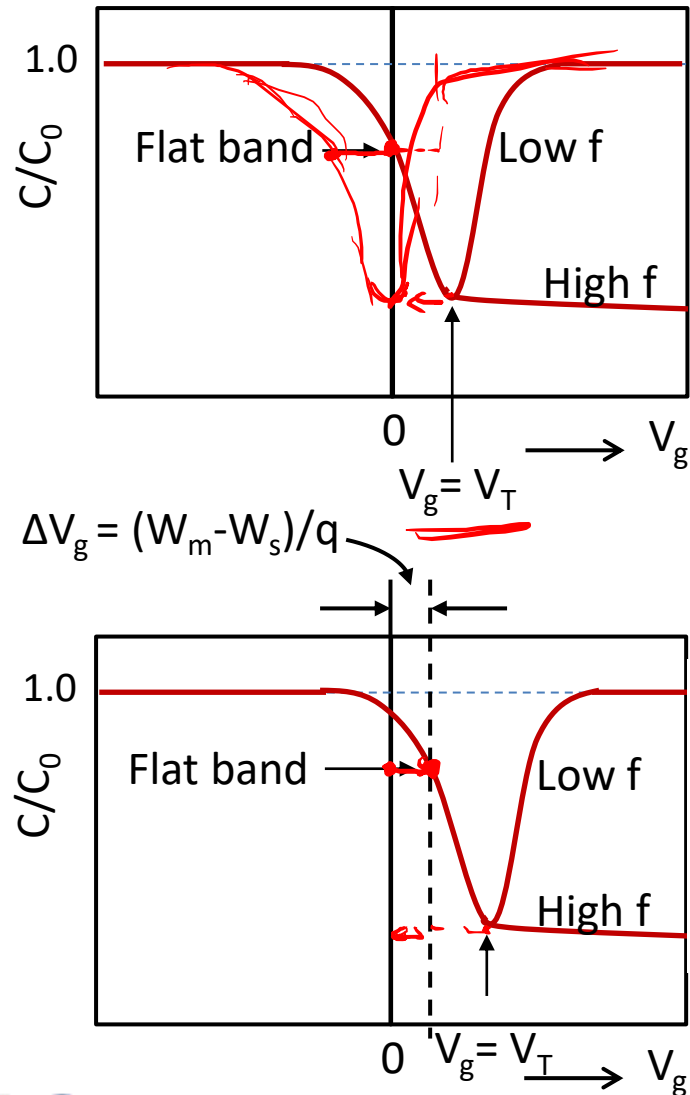
$$V_T = 2\phi_{fp} + \frac{N_a \cdot \chi_{IT}}{C_{ox}} + \frac{\phi_m - \phi_s}{q}$$

Metal      oxide      semiconductor

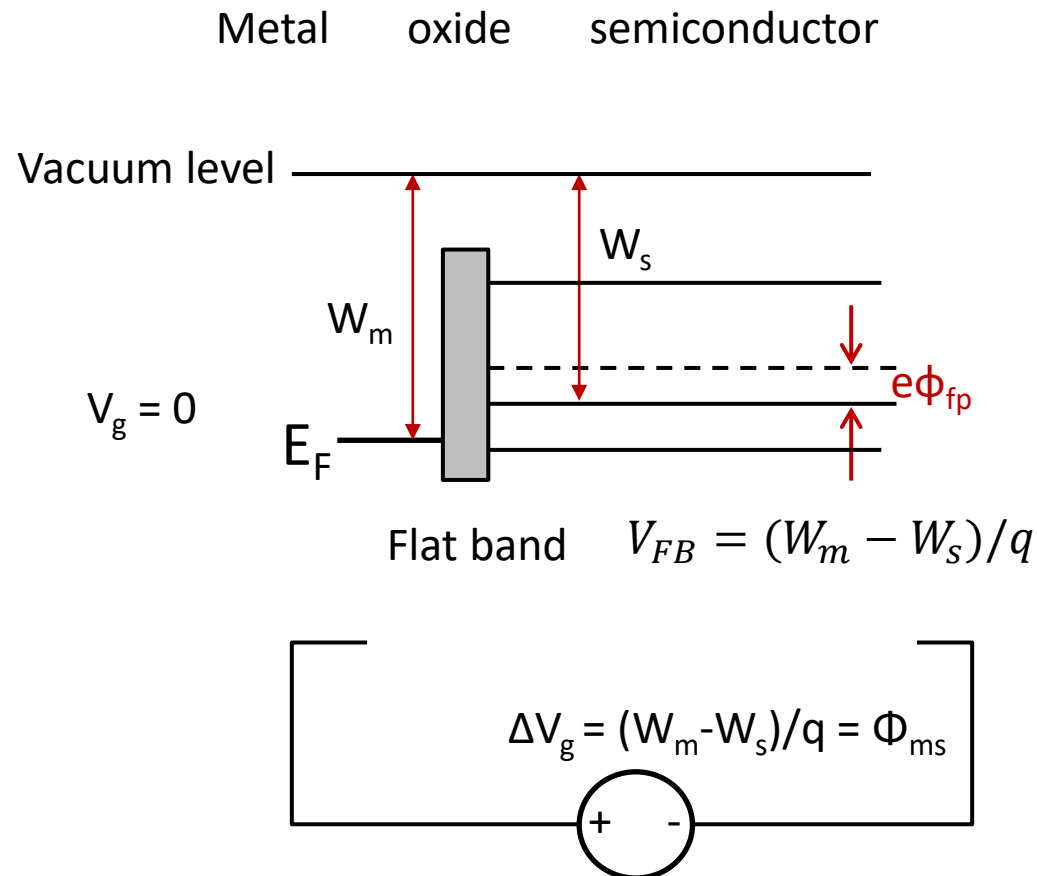


$$\Delta V_g = (W_m - W_s)/q = \Phi_{ms}$$

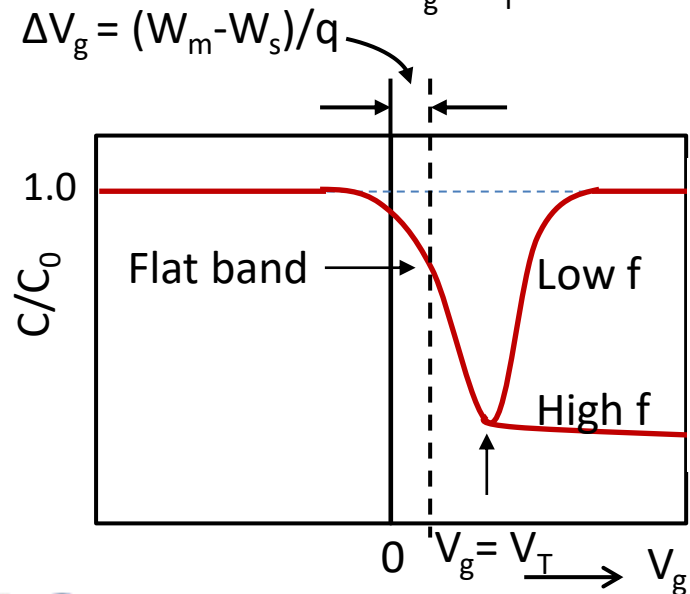
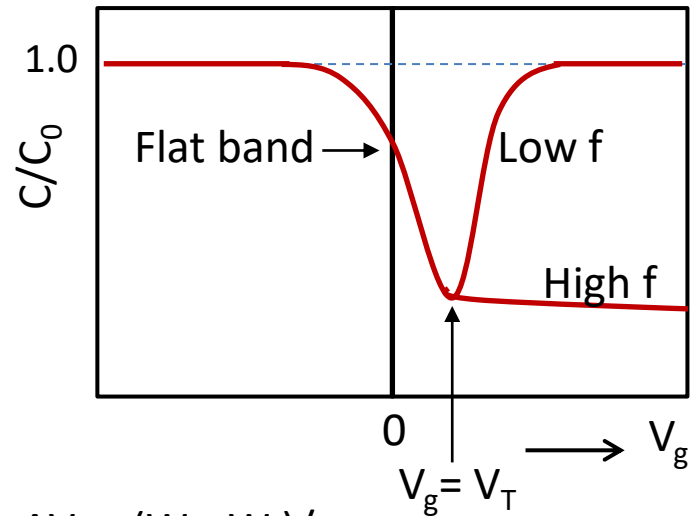
# 10.3 Non-ideal effects



## Work function difference



# 10.3 Non-ideal effects



## Work function difference

$$V_T = 2\phi_{fp} + t_{ox} \sqrt{\frac{4eN_a\epsilon_{Si}\phi_{fp}}{\epsilon_{ox}^2}} = 2\phi_b + \frac{|Q_{SD}|}{C_{ox}}$$

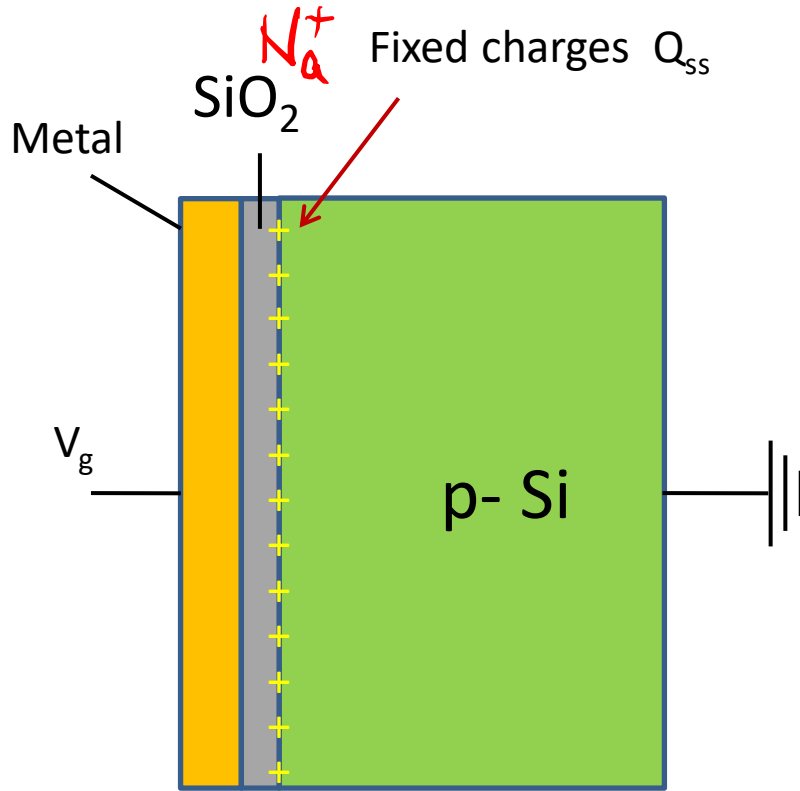
$$V_T = 2\phi_{fp} + t_{ox} \sqrt{\frac{4eN_a\epsilon_{Si}\phi_{fp}}{\epsilon_{ox}^2}} + V_{FB}$$

$$= 2\phi_{fp} + \frac{|Q_{SD}|}{C_{ox}} + \phi_{ms}$$

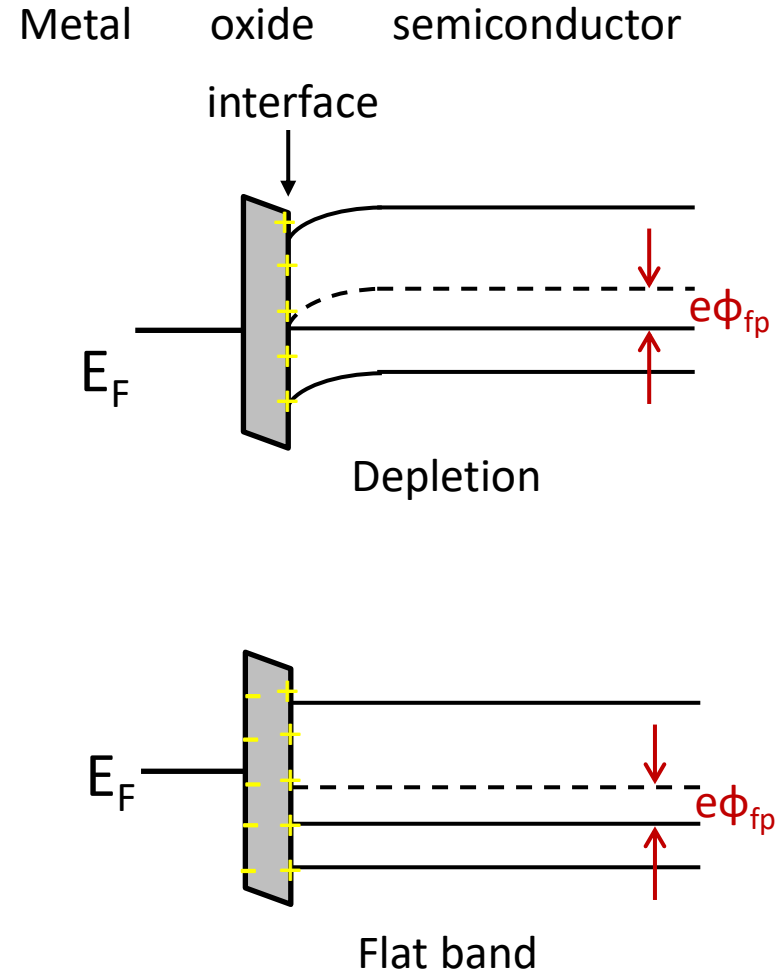
$\frac{\phi_m - \phi_s}{\epsilon}$

# 10.3 Non-ideal effects

## Fixed charges



Metal-insulator-semiconductor (MIS)

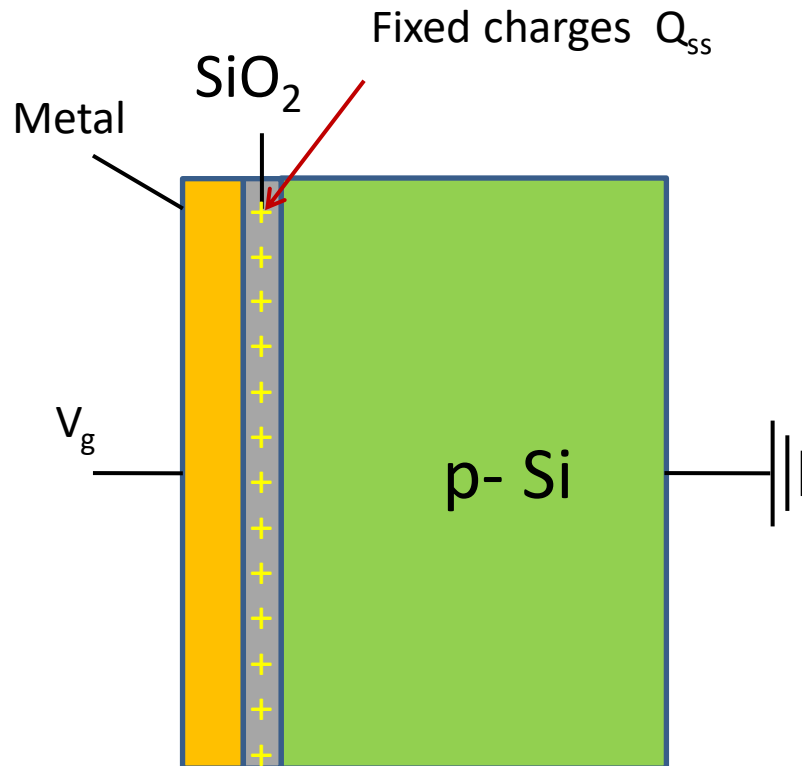


$$V_g = V_{FB} = -Q_{ss}/C$$

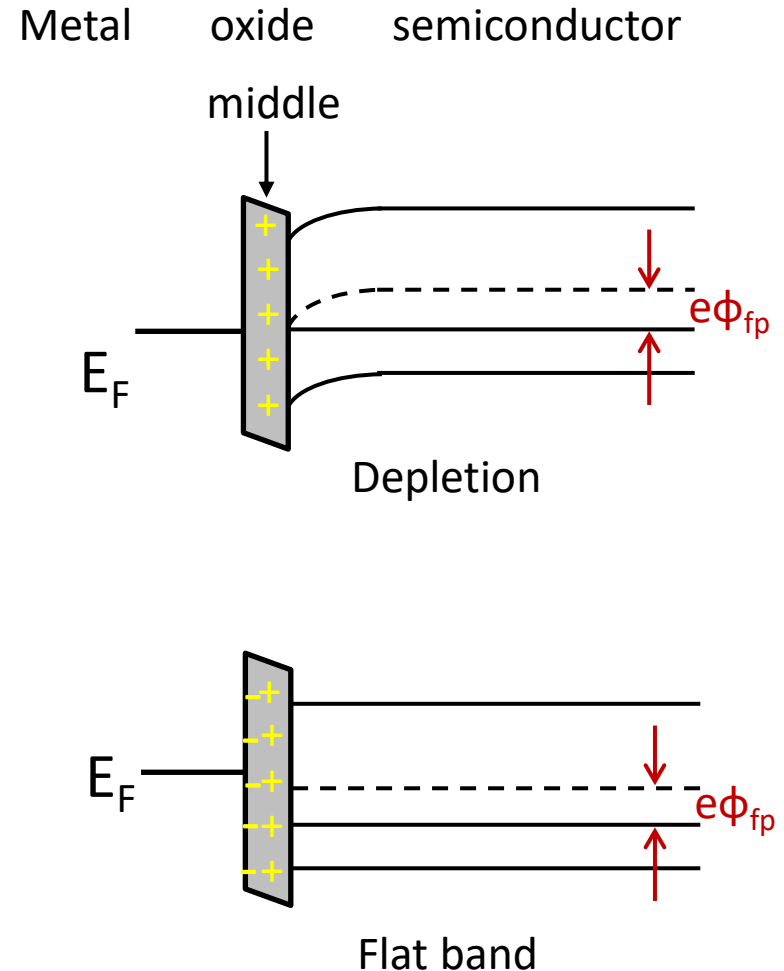


# 10.3 Non-ideal effects

## Fixed charges



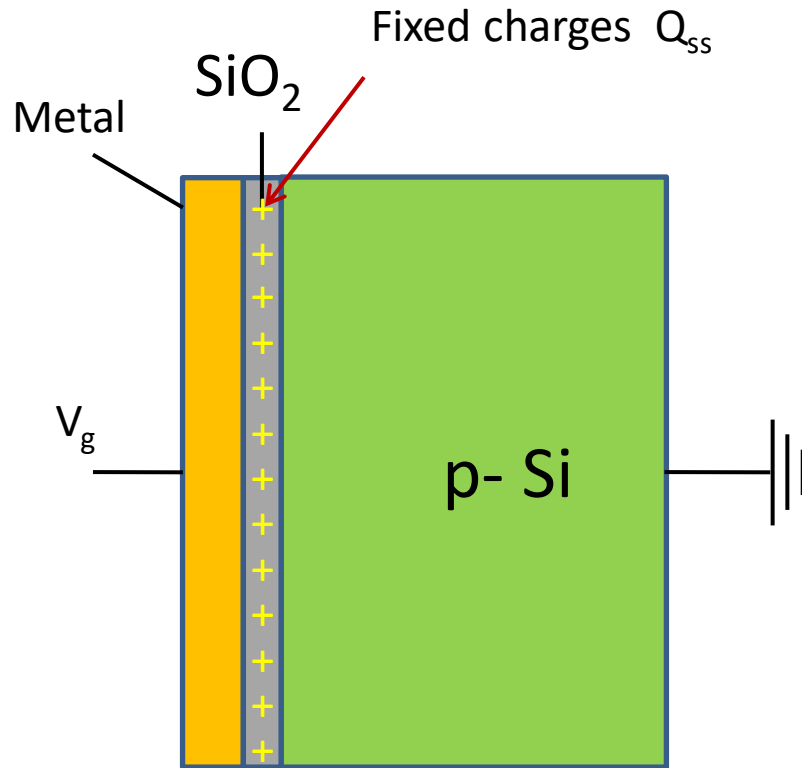
Metal-insulator-semiconductor (MIS)



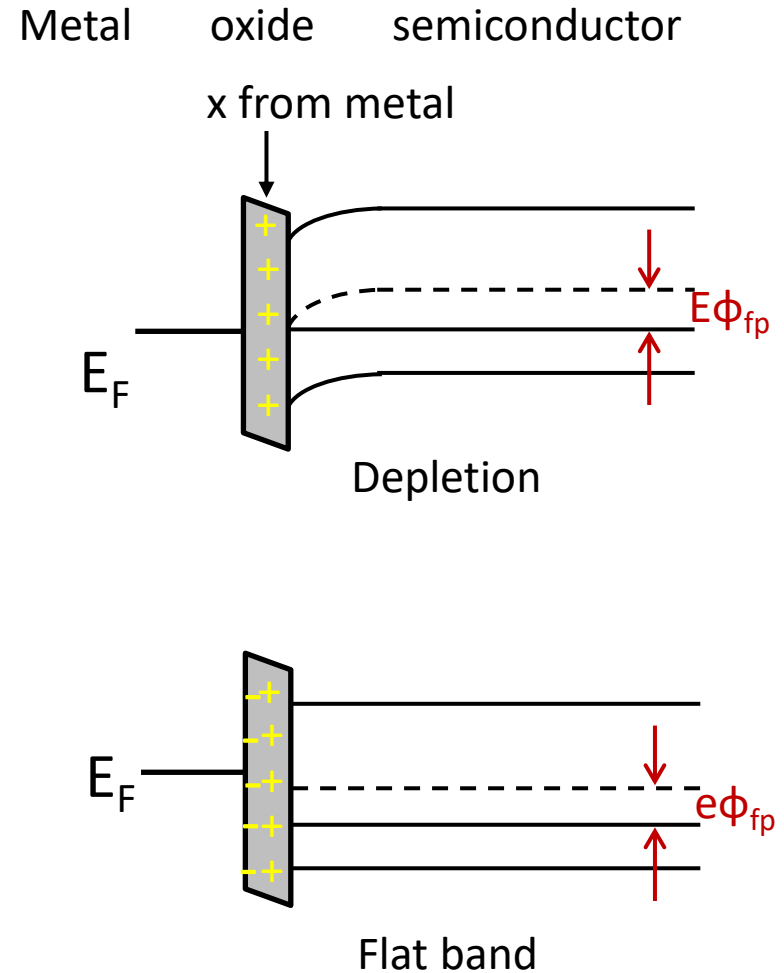
$$V_g = V_{FB} = -Q_{ss}/2C$$

# 10.3 Non-ideal effects

## Fixed charges



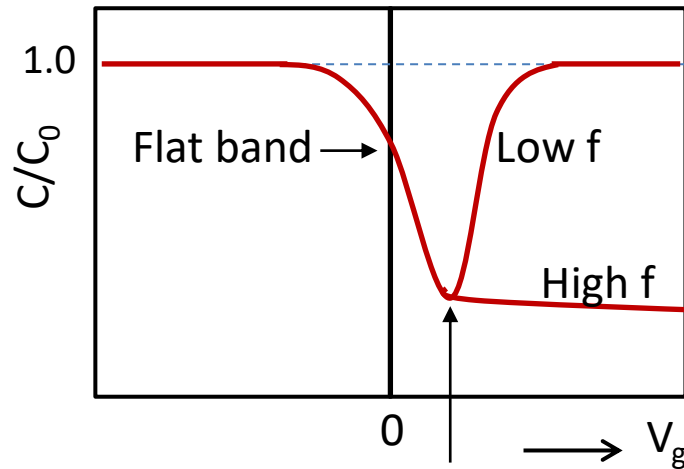
Metal-insulator-semiconductor (MIS)



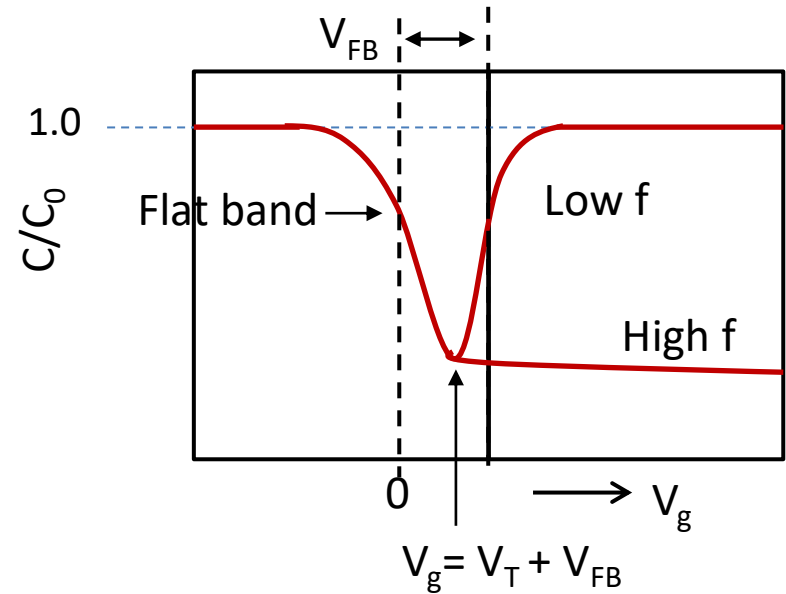
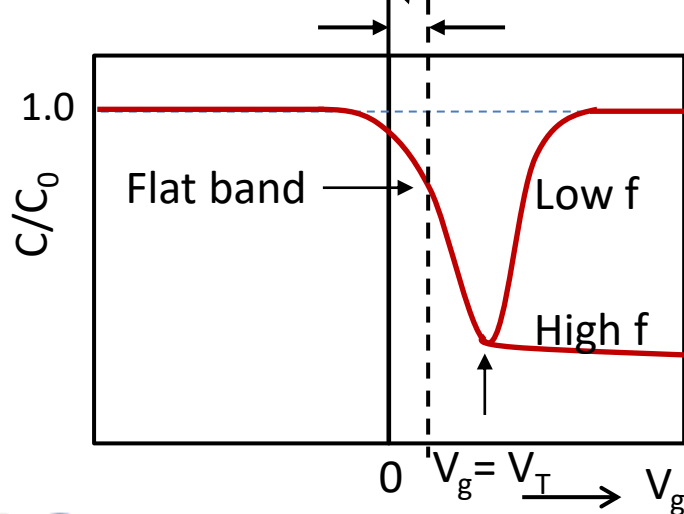
$$V_g = V_{FB} = -\frac{Q_{ss}}{C} \cdot \frac{x}{d}$$

# 10.3 Non-ideal effects

## Fixed charges



$$\Delta V_g = (W_m - W_s)/q$$



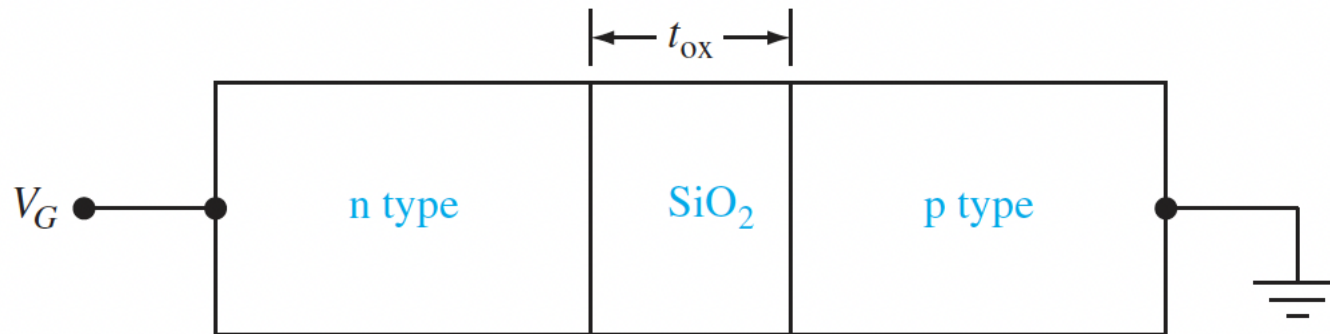
$$V_T = 2\phi_{fp} + t_{ox} \sqrt{\frac{4eN_a\epsilon_{Si}\phi_{fp}}{\epsilon_{ox}^2}} + V_{FB}$$

$$= 2\phi_{fp} + \frac{|Q_{SD}|}{C_{ox}} + \phi_{ms} - \frac{Q_{ss}}{C_{ox}}$$

# Check your understanding

## Problem Example #3

Consider an SOS capacitor as shown in Figure P10.29. Assume the  $\text{SiO}_2$  is ideal (no trapped charge) and has a thickness of  $t_{\text{ox}} = \underline{500 \text{ \AA}}$ . The doping concentrations are  $N_d = 10^{16} \text{ cm}^{-3}$  and  $N_a = 10^{16} \text{ cm}^{-3}$ . (a) Sketch the energy-band diagram through the device for (i) flat band, (ii)  $V_G = +3 \text{ V}$ , and (iii)  $V_G = -3 \text{ V}$ . (b) Calculate the flat-band voltage. (c) Estimate the voltage across the oxide for (i)  $V_G = +3 \text{ V}$  and (ii)  $V_G = -3 \text{ V}$ . (d) Sketch the high-frequency  $C$ - $V$  characteristic curve.



# Check your understanding

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## Problem Example #3

# Check your understanding

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## Problem Example #3

# Check your understanding

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## Problem Example #3

# Check your understanding

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## Problem Example #4

**Objective:** Calculate the threshold voltage of a MOS system using an aluminum gate.

Consider a p-type silicon substrate at  $T = 300$  K doped to  $N_a = 10^{15} \text{ cm}^{-3}$ . Let  $Q'_{ss} = 10^{10} \text{ cm}^{-2}$ ,  $t_{ox} = 12 \text{ nm} = 120 \text{ \AA}$ , and assume the oxide is silicon dioxide.



# Check your understanding

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## Problem Example #4

# Outline

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10.1 The two-terminal MOS structure

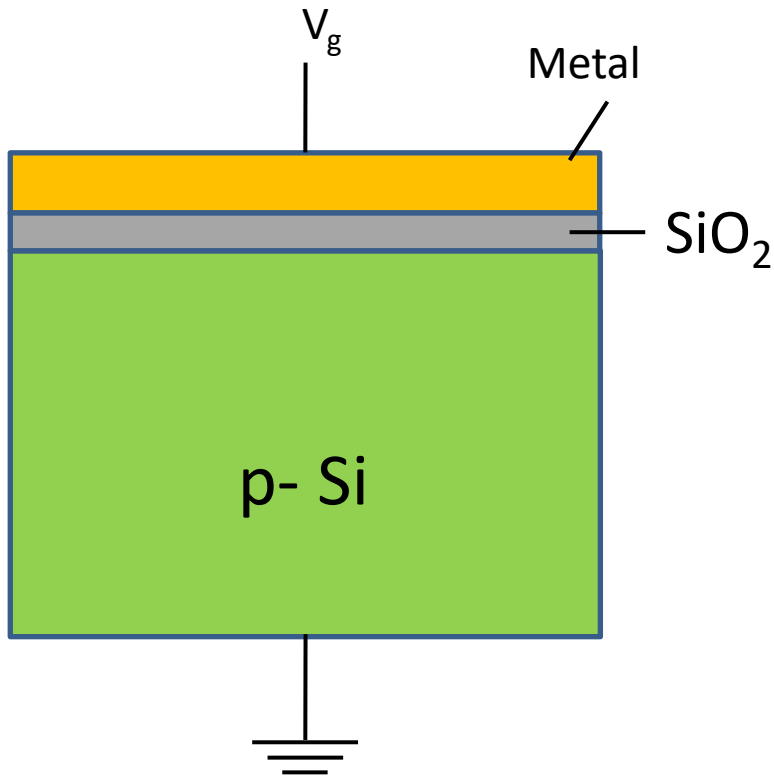
10.2 Capacitance-voltage characteristics

10.3 Non-ideal effects

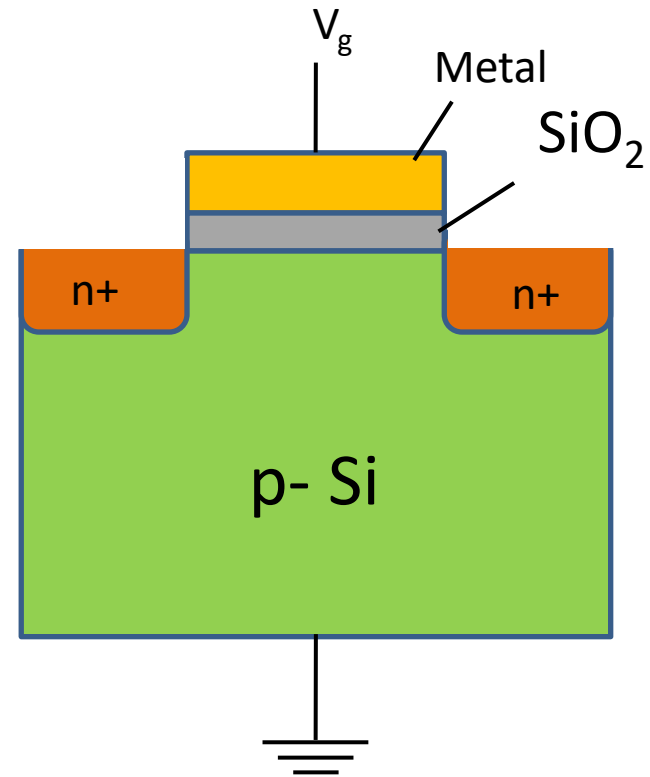
**10.4 The basic MOSFET operation**

# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: MOSFET



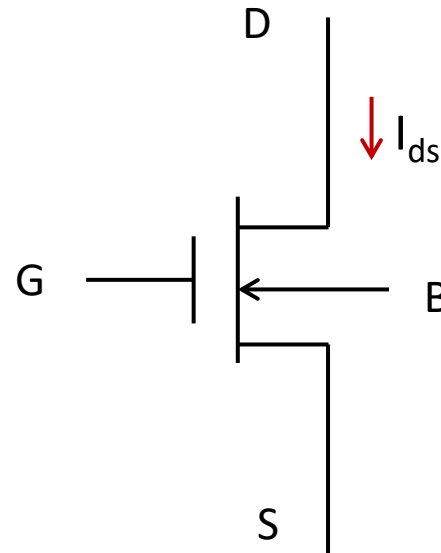
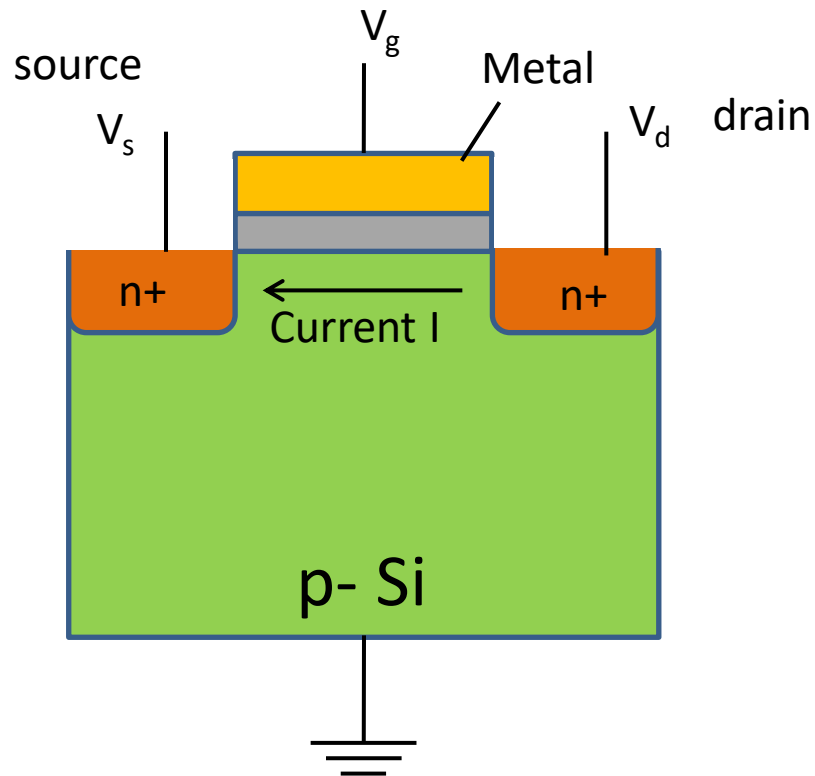
MOS capacitor



MOSFET

# 10.4 The basic MOSFET operation

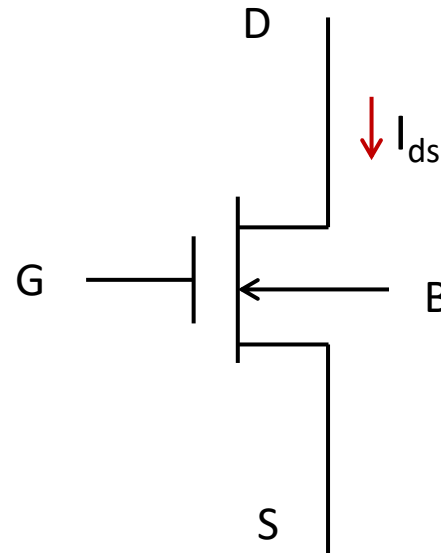
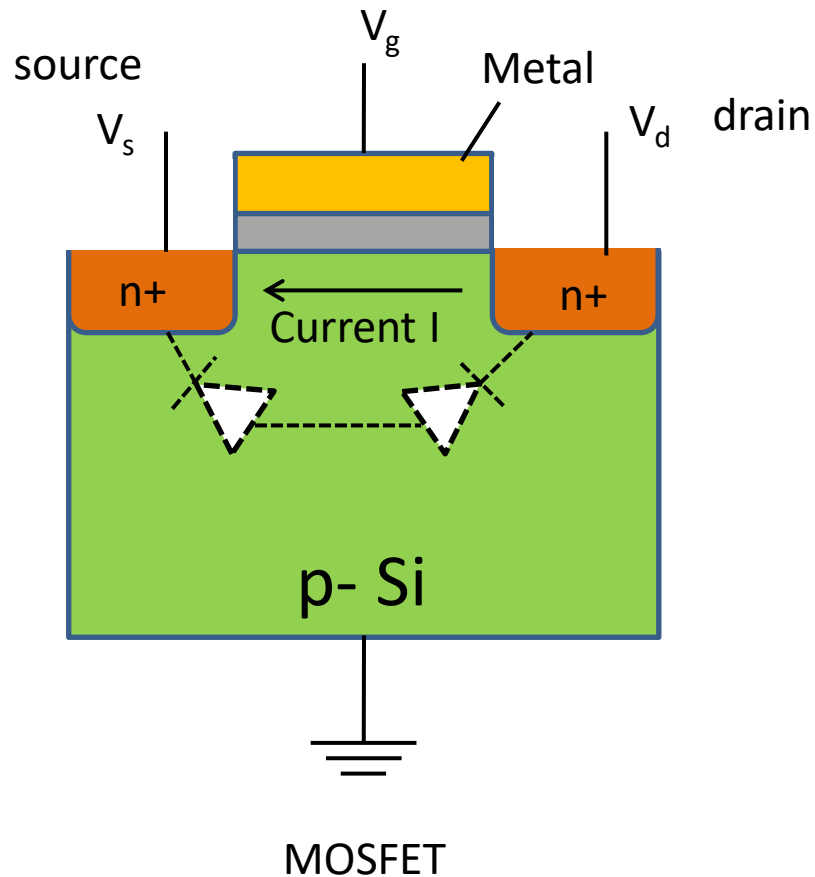
Metal-Oxide-Semiconductor field effect transistor: MOSFET



Metal-oxide-semiconductor (MOS)

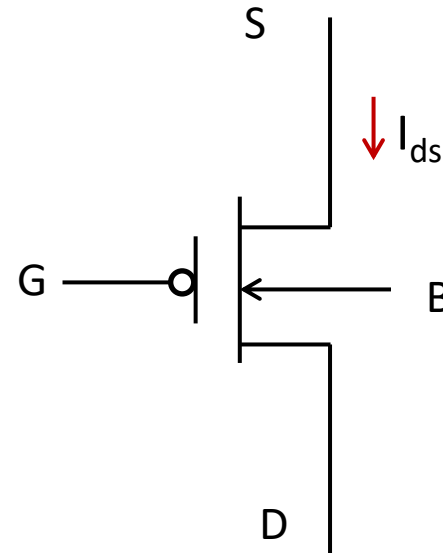
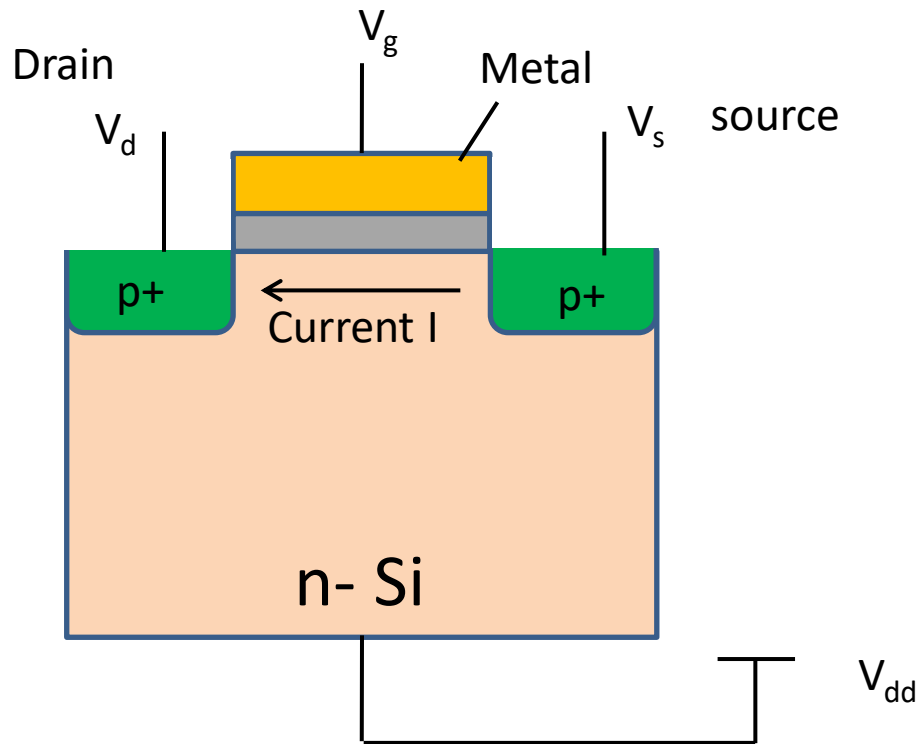
# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: MOSFET



# 10.4 The basic MOSFET operation

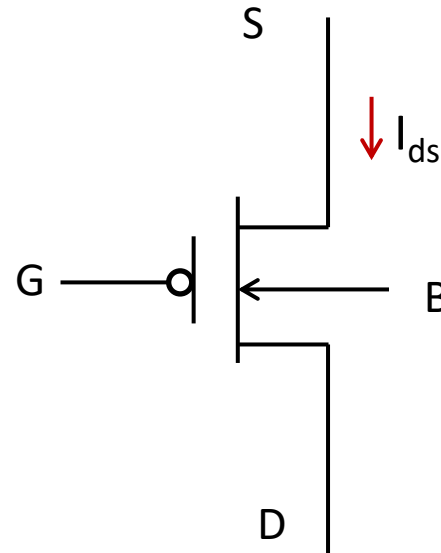
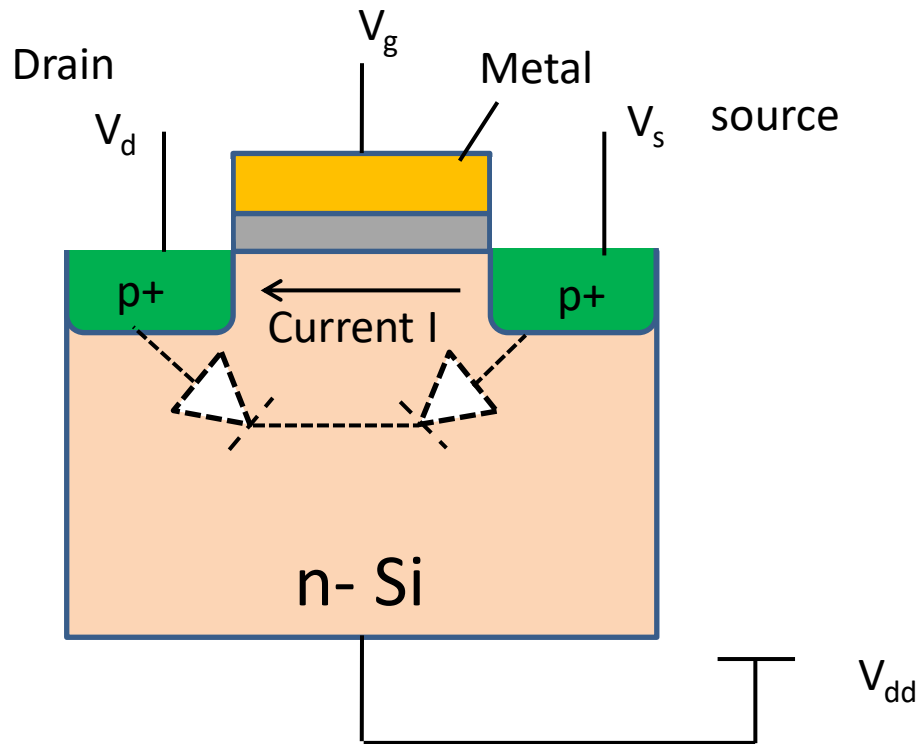
Metal-Oxide-Semiconductor field effect transistor: p-type MOSFET



P-type MOSFET

# 10.4 The basic MOSFET operation

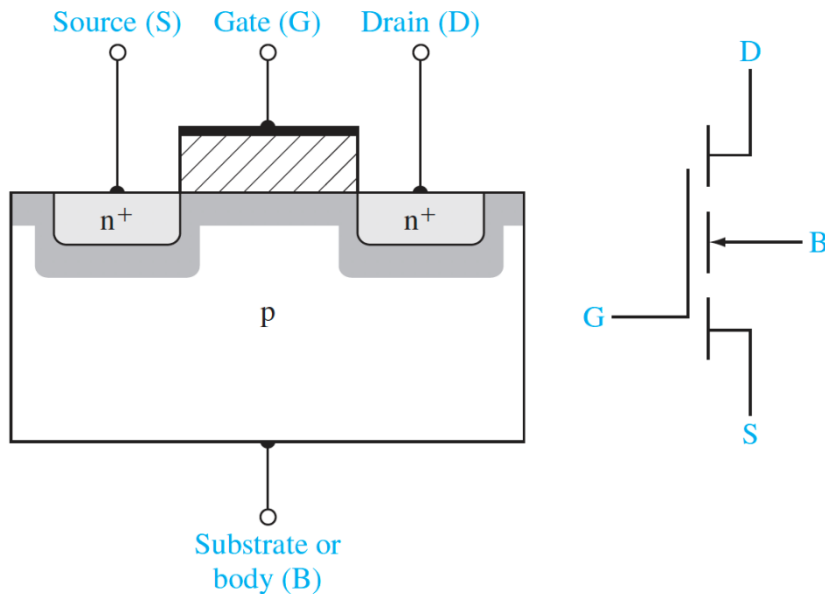
Metal-Oxide-Semiconductor field effect transistor: P MOSFET



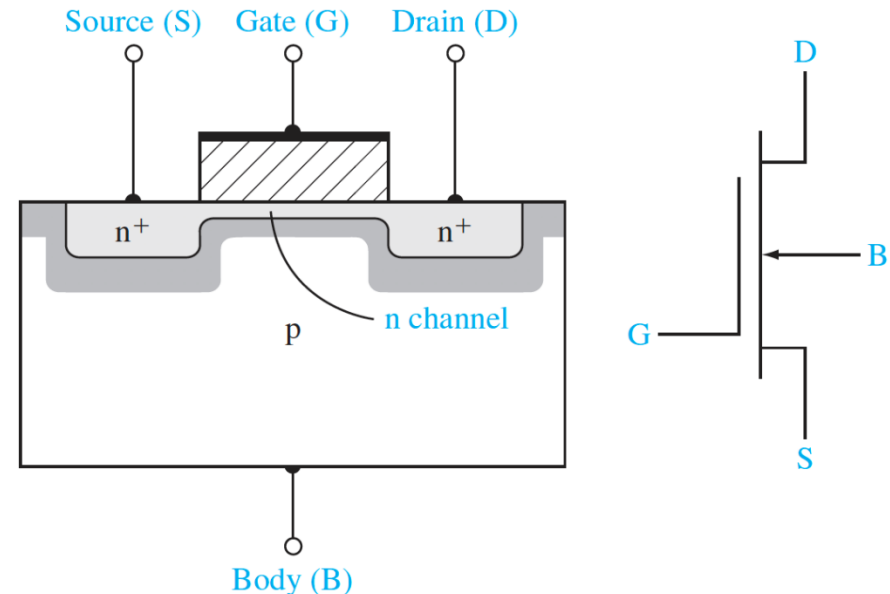
P-type MOSFET

# 10.4 The basic MOSFET operation

## MOSFET structures



NMOS Enhancement mode

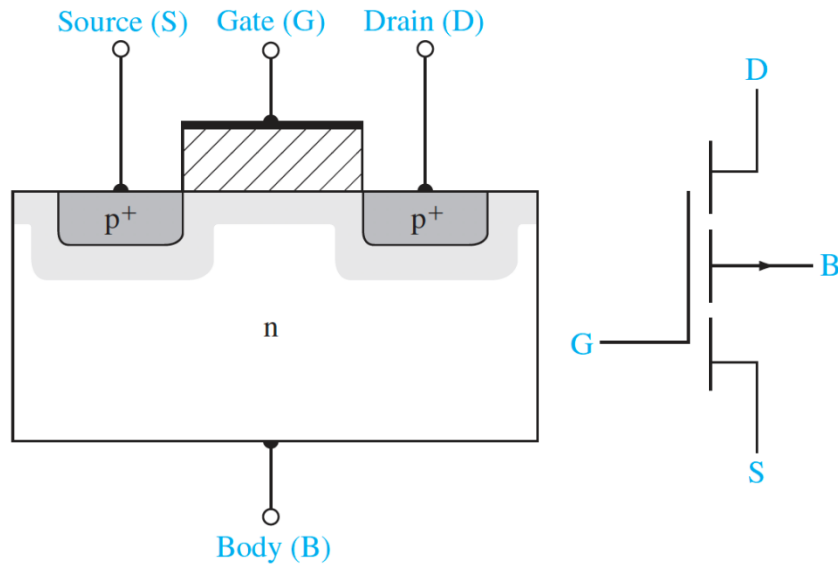


NMOS Depletion mode

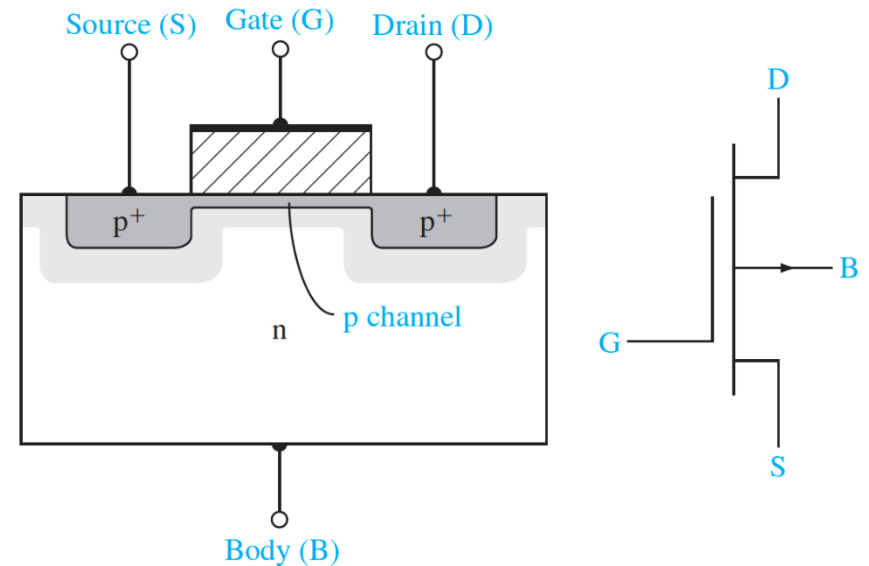


# 10.4 The basic MOSFET operation

## MOSFET structures



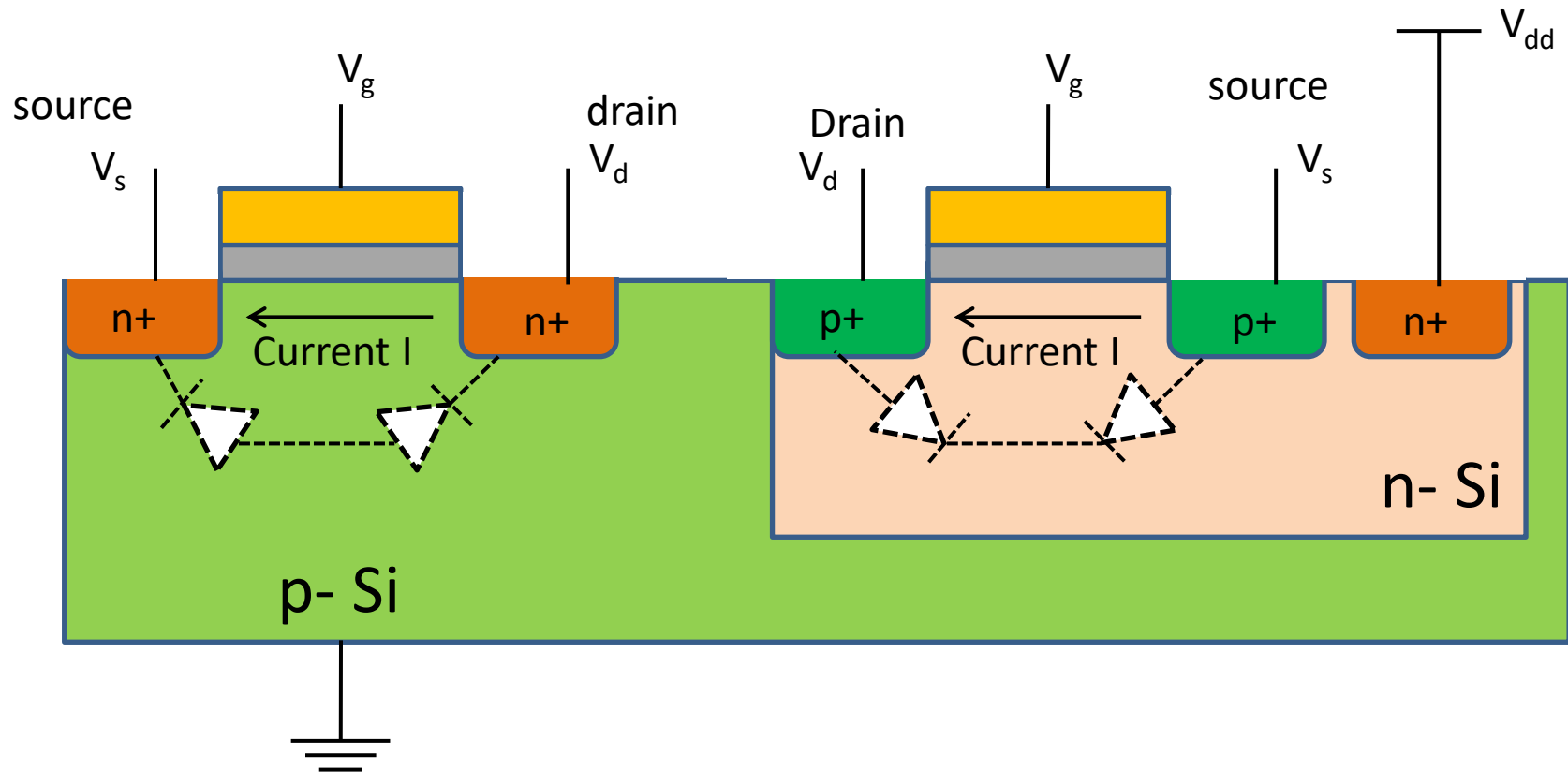
PMOS Enhancement mode



PMOS Depletion mode

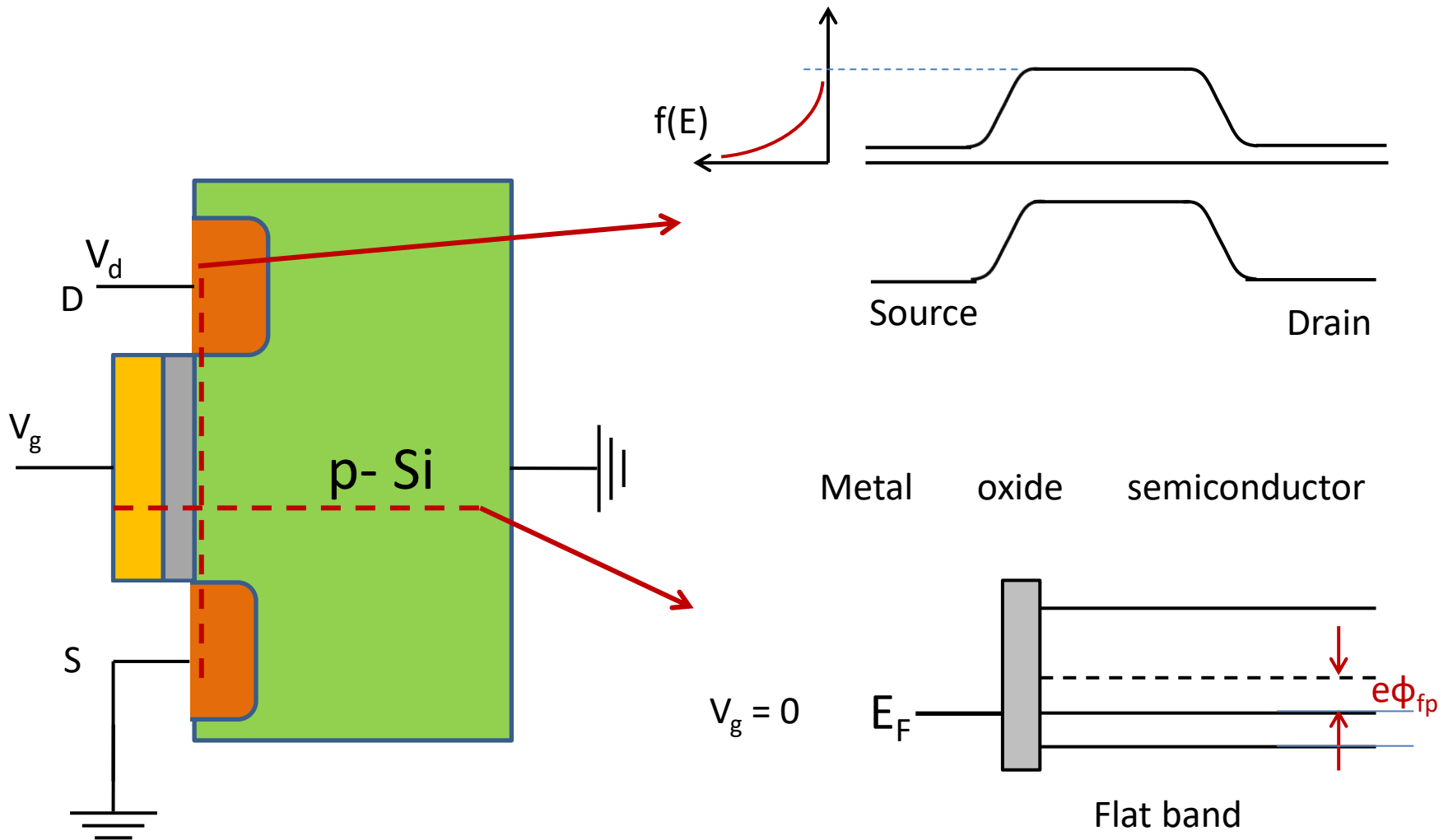
# 10.4 The basic MOSFET operation

Metal-Oxide-Semiconductor field effect transistor: CMOS

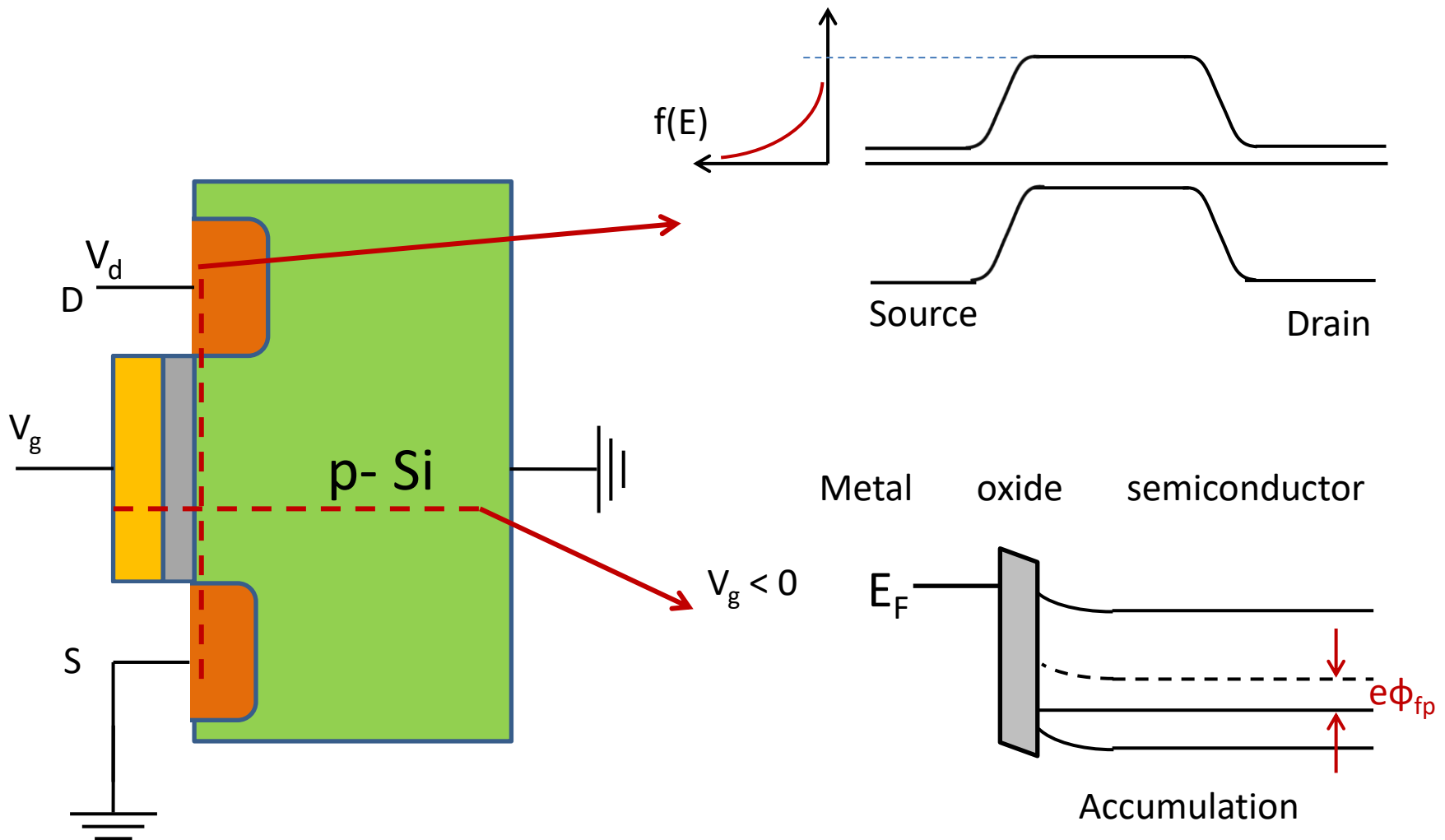


Complementary Metal-oxide-semiconductor (CMOS) field effect transistors

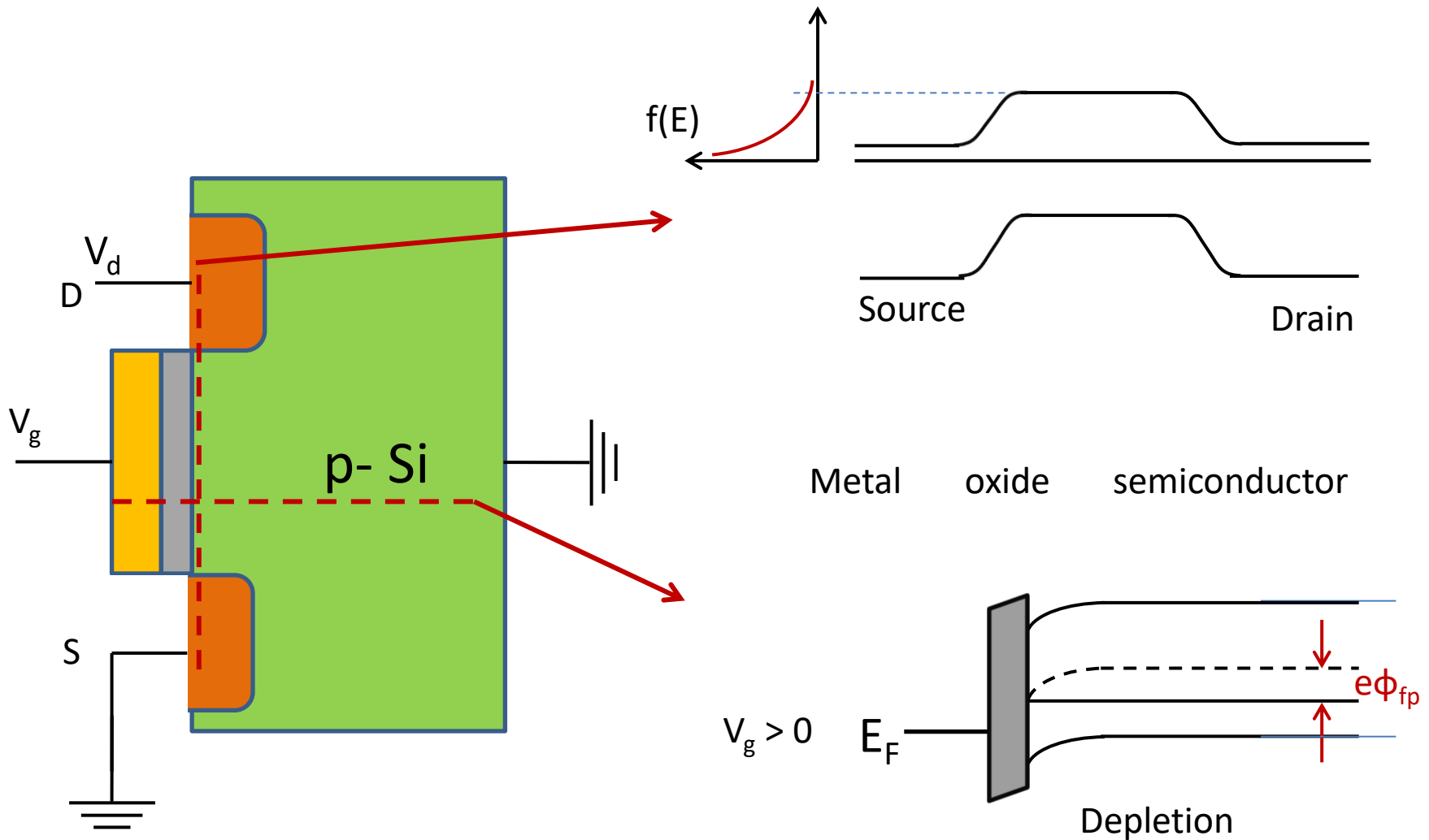
# 10.4 The basic MOSFET operation



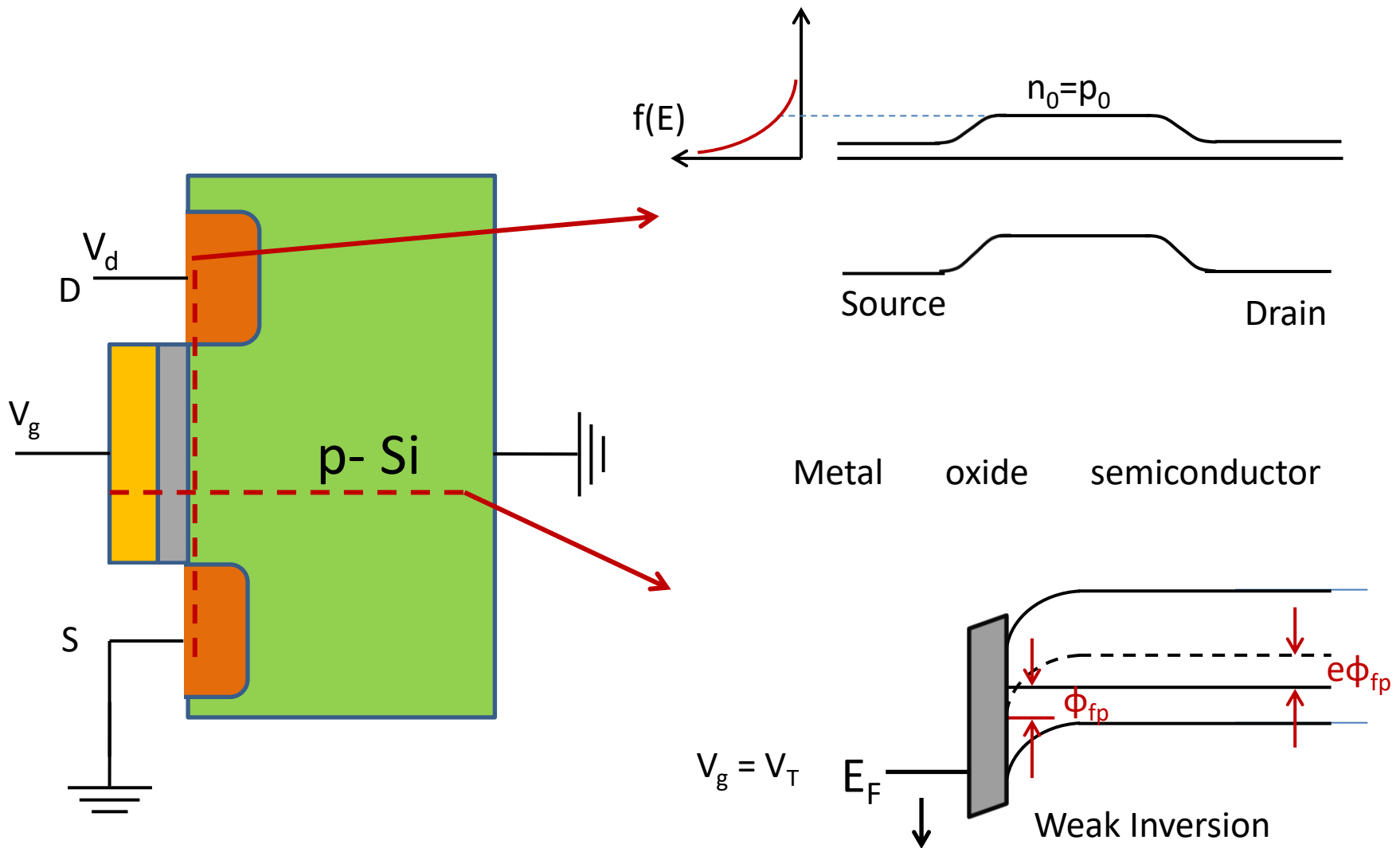
# 10.4 The basic MOSFET operation



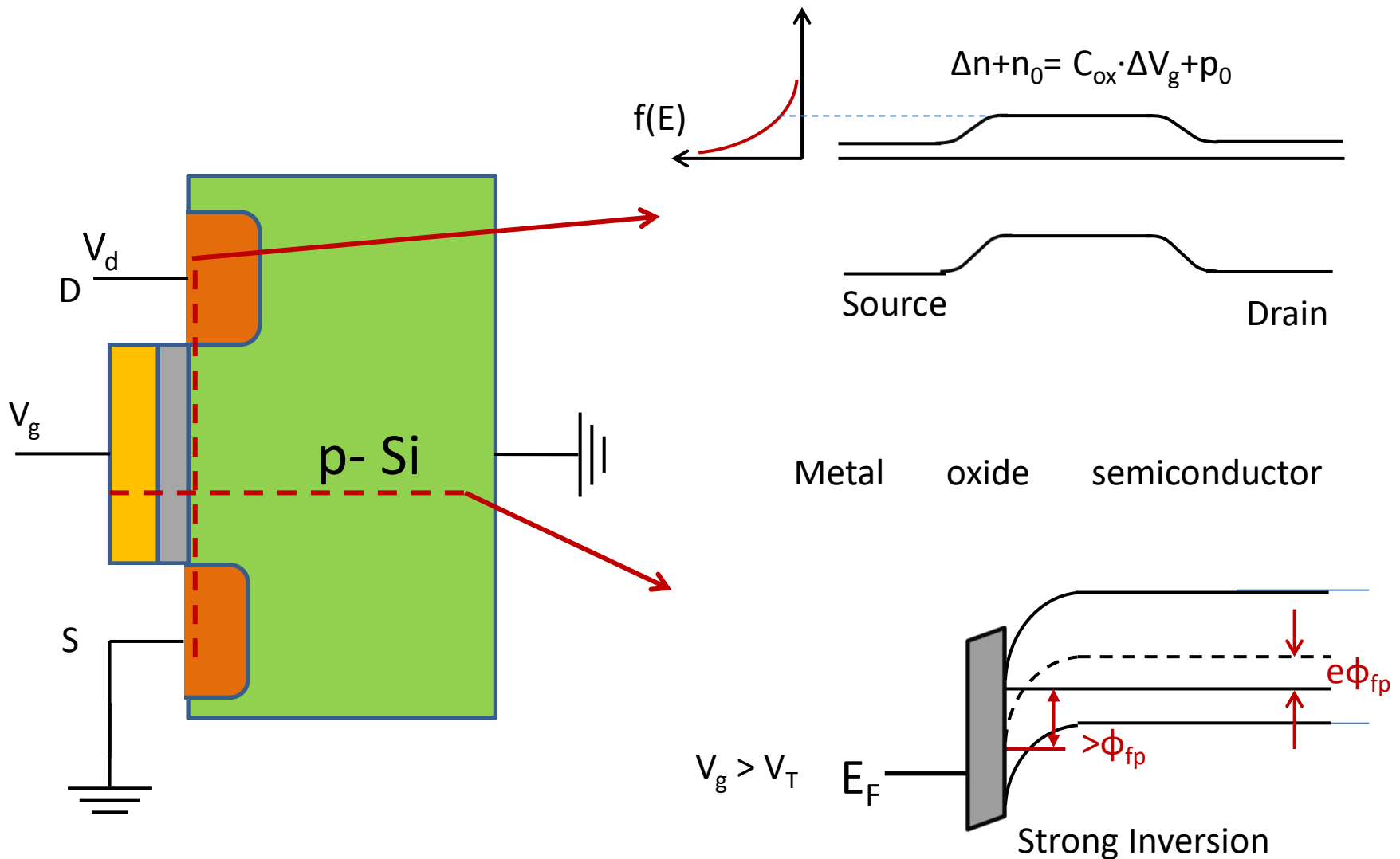
# 10.4 The basic MOSFET operation



# 10.4 The basic MOSFET operation



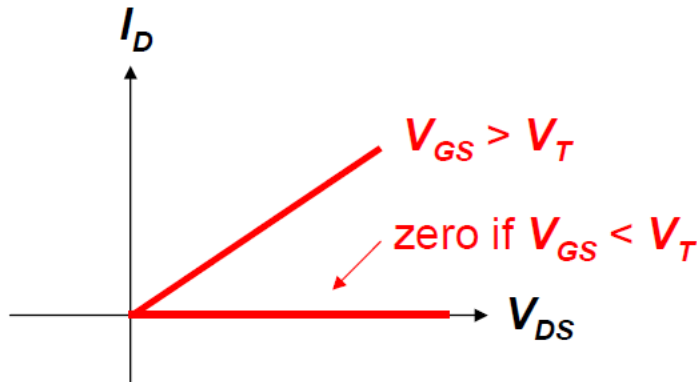
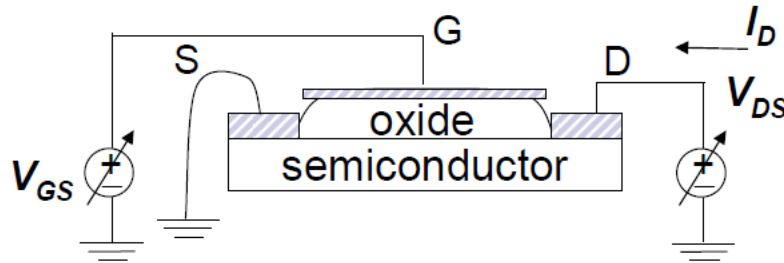
# 10.4 The basic MOSFET operation



## 10.4 The basic MOSFET operation

### NMOSFET $I_D$ vs. $V_{DS}$ Characteristics

Next consider  $I_D$  (flowing into **D**) versus  $V_{DS}$ , as  $V_{GS}$  is varied:



**Above threshold ( $V_{GS} > V_T$ ):**  
“inversion layer” of electrons  
appears, so conduction  
between **S** and **D** is possible

**Below “threshold” ( $V_{GS} < V_T$ ):**  
no charge  $\rightarrow$  no conduction



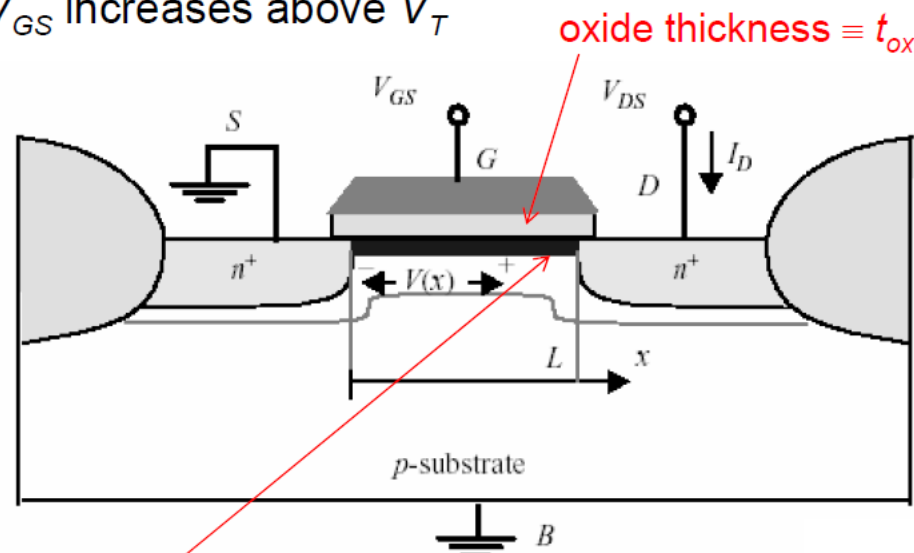
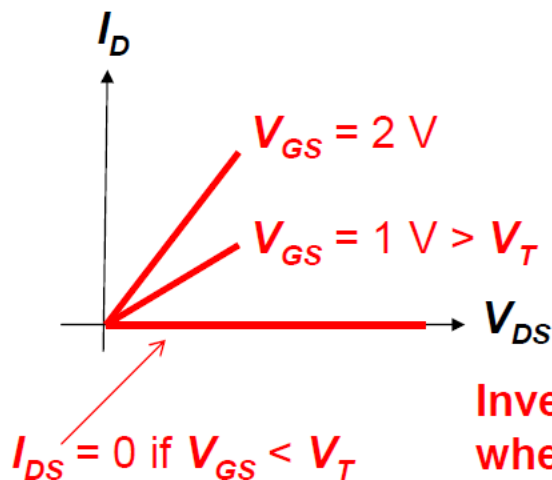
# 10.4 The basic MOSFET operation

## Current-voltage characteristics

### The MOSFET as a Controlled Resistor

- The MOSFET behaves as a resistor when  $V_{DS}$  is low:
  - Drain current  $I_D$  increases linearly with  $V_{DS}$
  - Resistance  $R_{DS}$  between SOURCE & DRAIN depends on  $V_{GS}$ 
    - $R_{DS}$  is lowered as  $V_{GS}$  increases above  $V_T$

#### NMOSFET Example:



Inversion charge density  $Q_i(x) = -C_{ox}[V_{GS} - V_T - V(x)]$   
where  $C_{ox} \equiv \epsilon_{ox} / t_{ox}$

# 10.4 The basic MOSFET operation

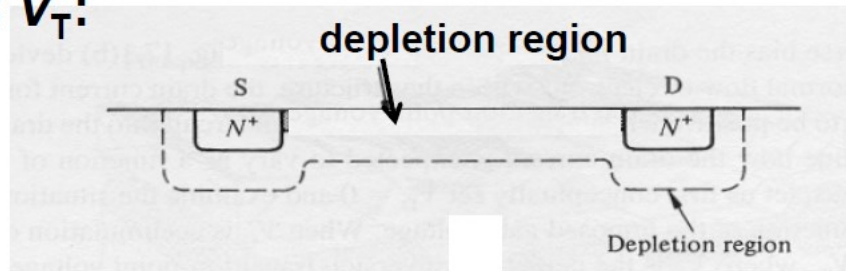
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Derivation of  $I_{ds}$  vs  $V_{ds}$  and  $V_{gs}$

## 10.4 The basic MOSFET operation

### Charge in an N-Channel MOSFET

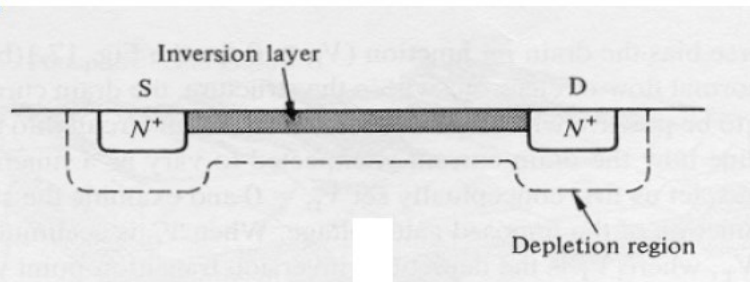
$$V_{GS} < V_T:$$



(no inversion layer at surface)

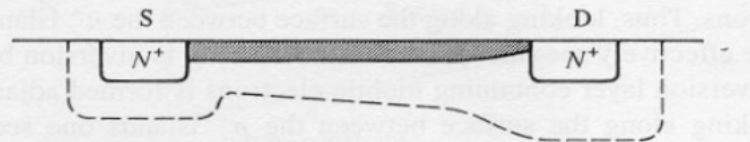
$$V_{GS} > V_T:$$

$$V_{DS} \approx 0$$



$$V_{DS} > 0$$

(small)



$$\begin{aligned} I_D &= WQ_{inv}v \\ &= WQ_{inv}\mu_n E \\ &= WQ_{inv}\mu_n \left( \frac{V_{DS}}{L} \right) \end{aligned}$$

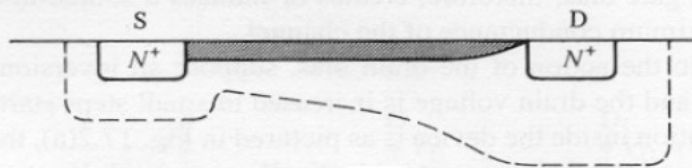
Average electron velocity  $v$  is proportional to lateral electric field  $E$

## 10.4 The basic MOSFET operation

### What Happens at Larger $V_{DS}$ ?

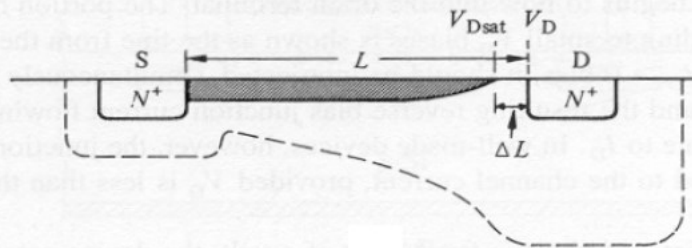
$$V_{GS} > V_T:$$

$$V_{DS} = V_{GS} - V_T$$



Inversion-layer is “pinched-off” at the drain end

$$V_{DS} > V_{GS} - V_T$$



As  $V_{DS}$  increases above  $V_{GS} - V_T \equiv V_{DSAT}$ ,

the length of the “pinch-off” region  $\Delta L$  increases:

- “extra” voltage ( $V_{DS} - V_{DSAT}$ ) is dropped across the distance  $\Delta L$
- the voltage dropped across the inversion-layer “resistor” remains  $V_{DSAT}$

$\Rightarrow$  the drain current  $I_D$  saturates

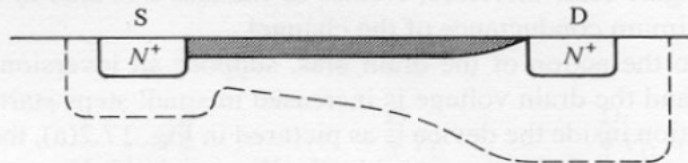
Note: Electrons are swept into the drain by the  $E$ -field when they enter the pinch-off region.

# 10.4 The basic MOSFET operation

## What Happens at Larger $V_{DS}$ ?

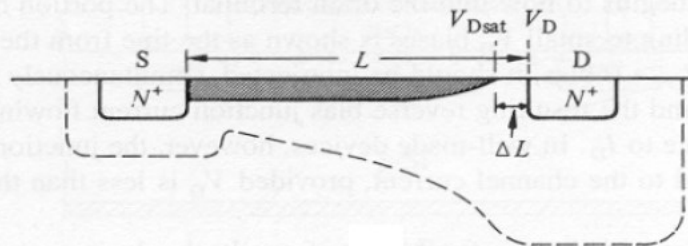
$$V_{GS} > V_T:$$

$$V_{DS} = V_{GS} - V_T$$



Inversion-layer  
is “pinched-off”  
at the drain end

$$V_{DS} > V_{GS} - V_T$$

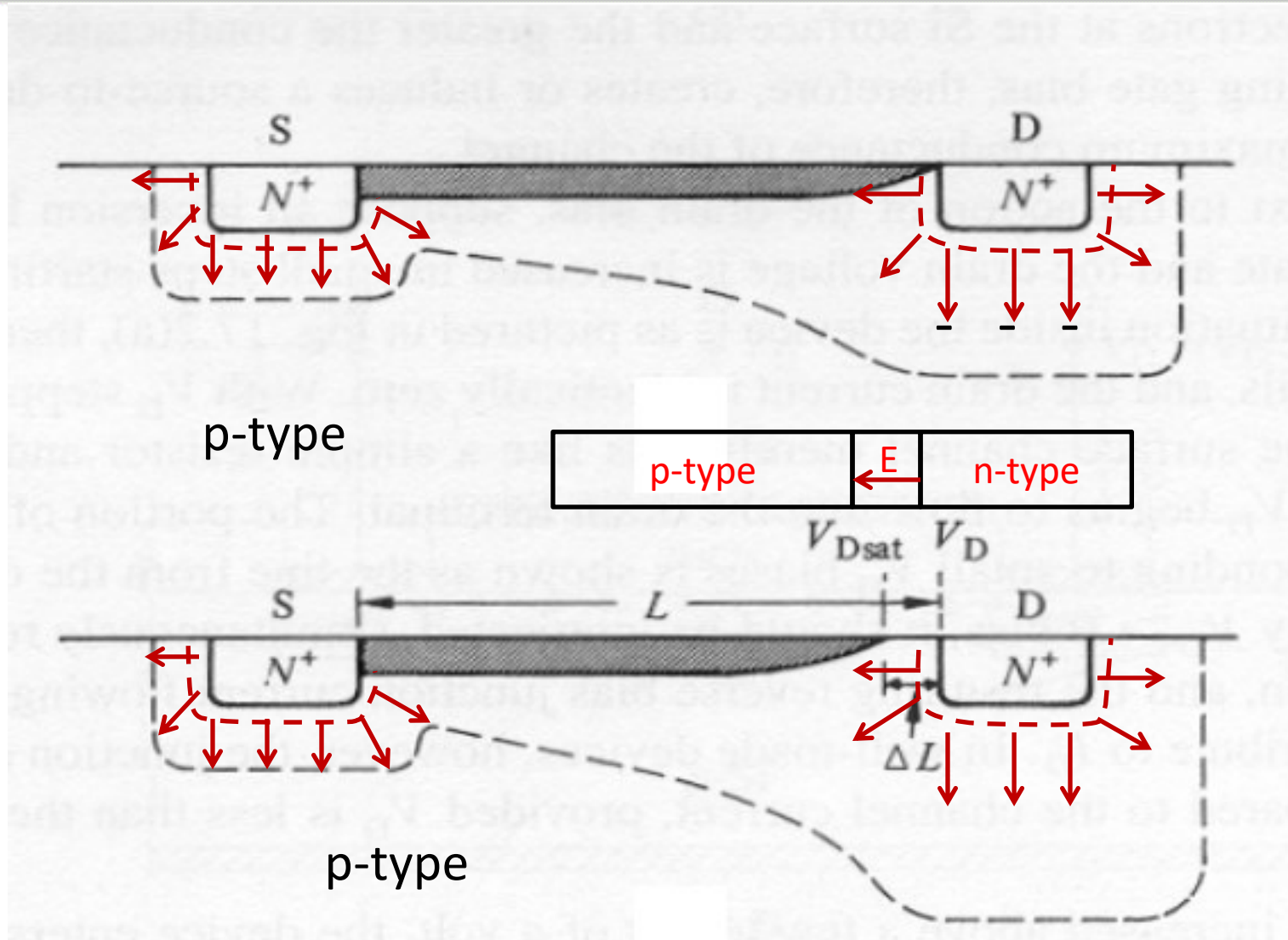


As  $V_{DS}$  increases above  $V_{GS} - V_T \equiv V_{DSAT}$ ,

$$I_D = \mu_n C_{ox} \frac{W}{L} \left( V_{GS} - V_T - \frac{1}{2} V_{DS} \right) V_{DS}$$

$I_D$  will not increase after  
 $V_{DS} \geq V_{GS} - V_T$

## 10.4 The basic MOSFET operation

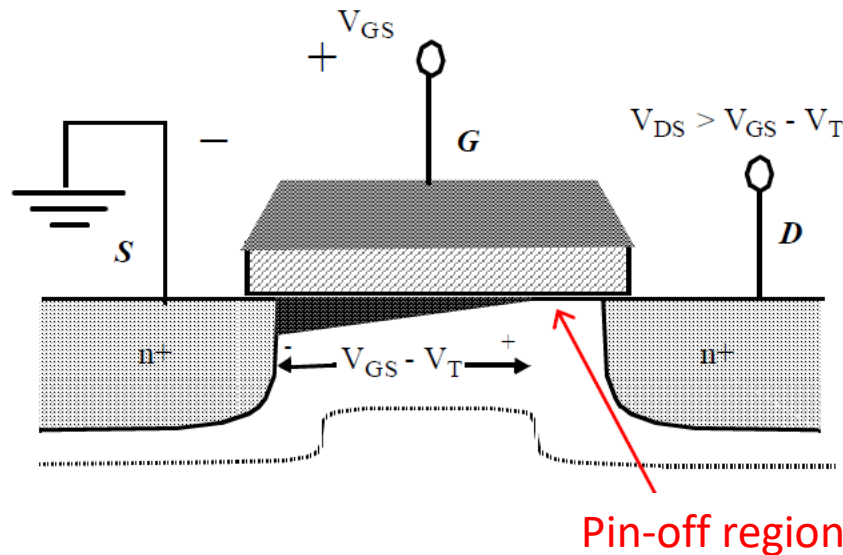




# 10.4 The basic MOSFET operation

## Summary of $I_D$ vs. $V_{DS}$

- As  $V_{DS}$  increases, the inversion-layer charge density at the drain end of the channel is reduced; therefore,  $I_D$  does not increase linearly with  $V_{DS}$ .
- When  $V_{DS}$  reaches  $V_{GS} - V_T$ , the channel is “pinched off” at the drain end, and  $I_D$  saturates (*i.e.* it does not increase with further increases in  $V_{DS}$ ).



$$I_{DSAT} = \mu_n C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2$$

# 10.4 The basic MOSFET operation

## $I_D$ vs. $V_{DS}$ Characteristics

The MOSFET  $I_D$ - $V_{DS}$  curve consists of two regions:

1) Resistive or “Triode” Region:  $0 < V_{DS} < V_{GS} - V_T$

$$I_D = k'_n \frac{W}{L} \left[ V_{GS} - V_T - \frac{V_{DS}}{2} \right] V_{DS}$$

where  $k'_n = \mu_n C_{ox}$

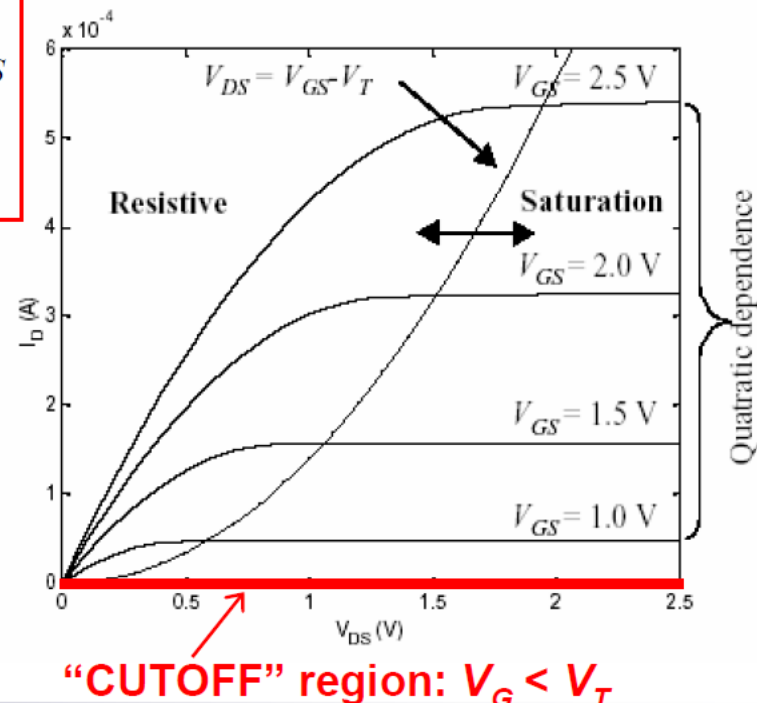
process transconductance parameter

2) Saturation Region:

$$V_{DS} > V_{GS} - V_T$$

$$I_{DSAT} = \frac{k'_n}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

where  $k'_n = \mu_n C_{ox}$

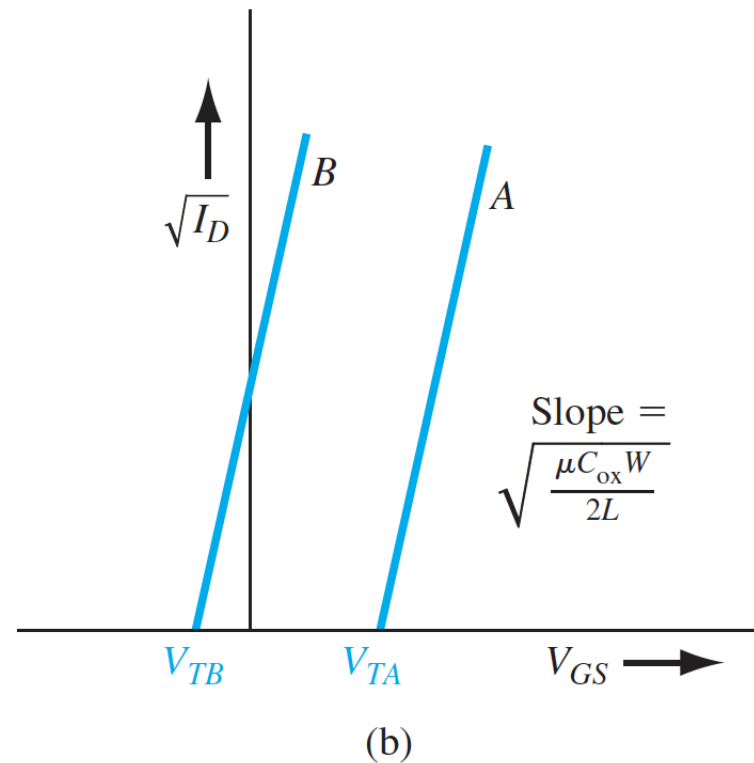
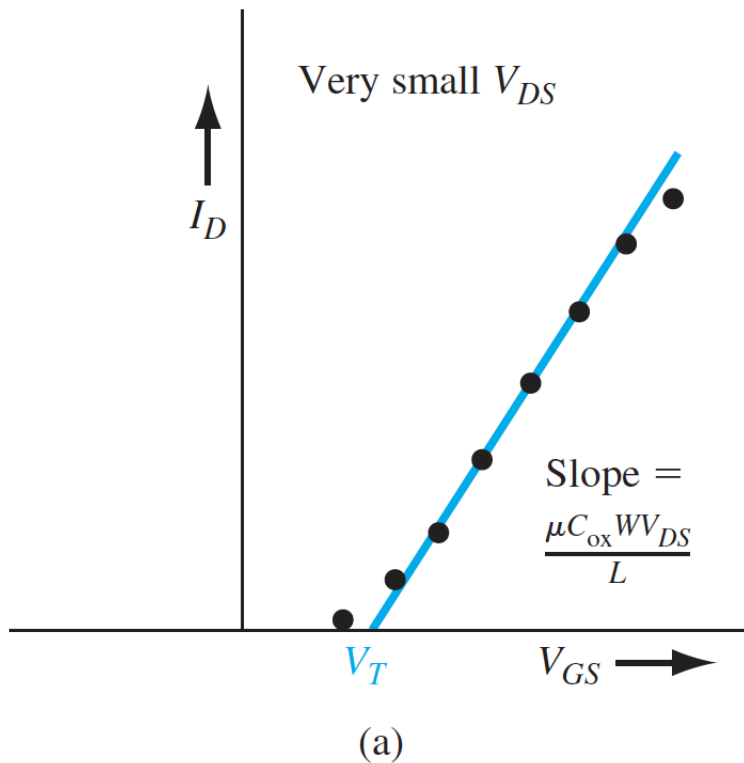




## 10.4 The basic MOSFET operation

$$I_D = \frac{W\mu_n C_{ox}}{L} (V_{GS} - V_T) V_{DS}$$

$$\sqrt{I_{D(sat)}} = \sqrt{\frac{W\mu_n C_{ox}}{2L}} (V_{GS} - V_T)$$



# Check your understanding

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## Problem example 5

**Objective:** Design the width of a MOSFET such that a specified current is induced for a given applied bias.

Consider an ideal n-channel MOSFET with parameters  $L = 1.25 \mu\text{m}$ ,  $\mu_n = 650 \text{ cm}^2/\text{V}\cdot\text{s}$ ,  $C_{\text{ox}} = 6.9 \times 10^{-8} \text{ F/cm}^2$ , and  $V_T = 0.65 \text{ V}$ . Design the channel width  $W$  such that  $I_D(\text{sat}) = 4 \text{ mA}$  for  $V_{GS} = 5 \text{ V}$ .

# Check your understanding

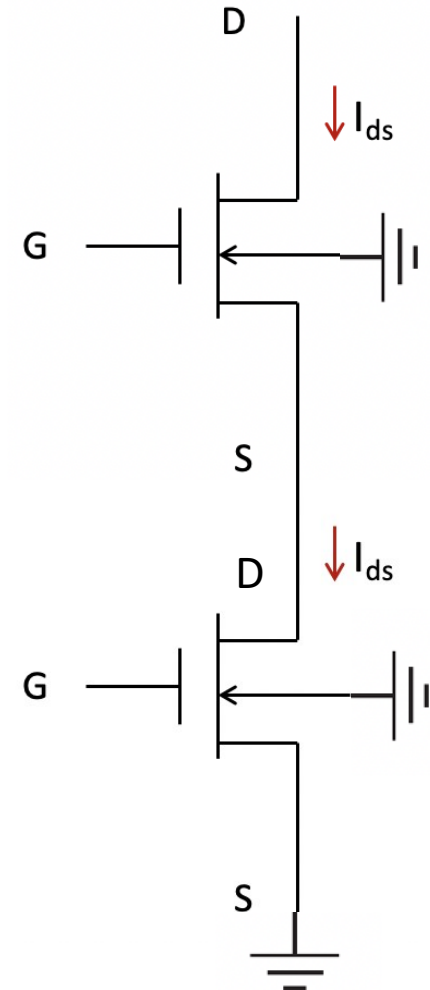
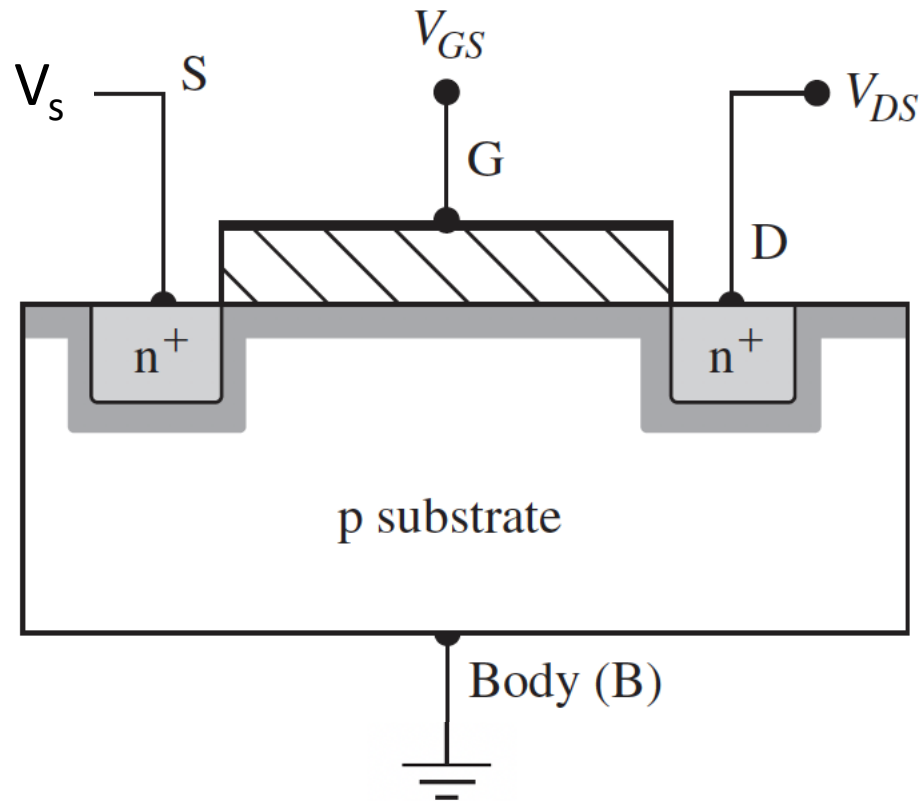
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Problem example 5



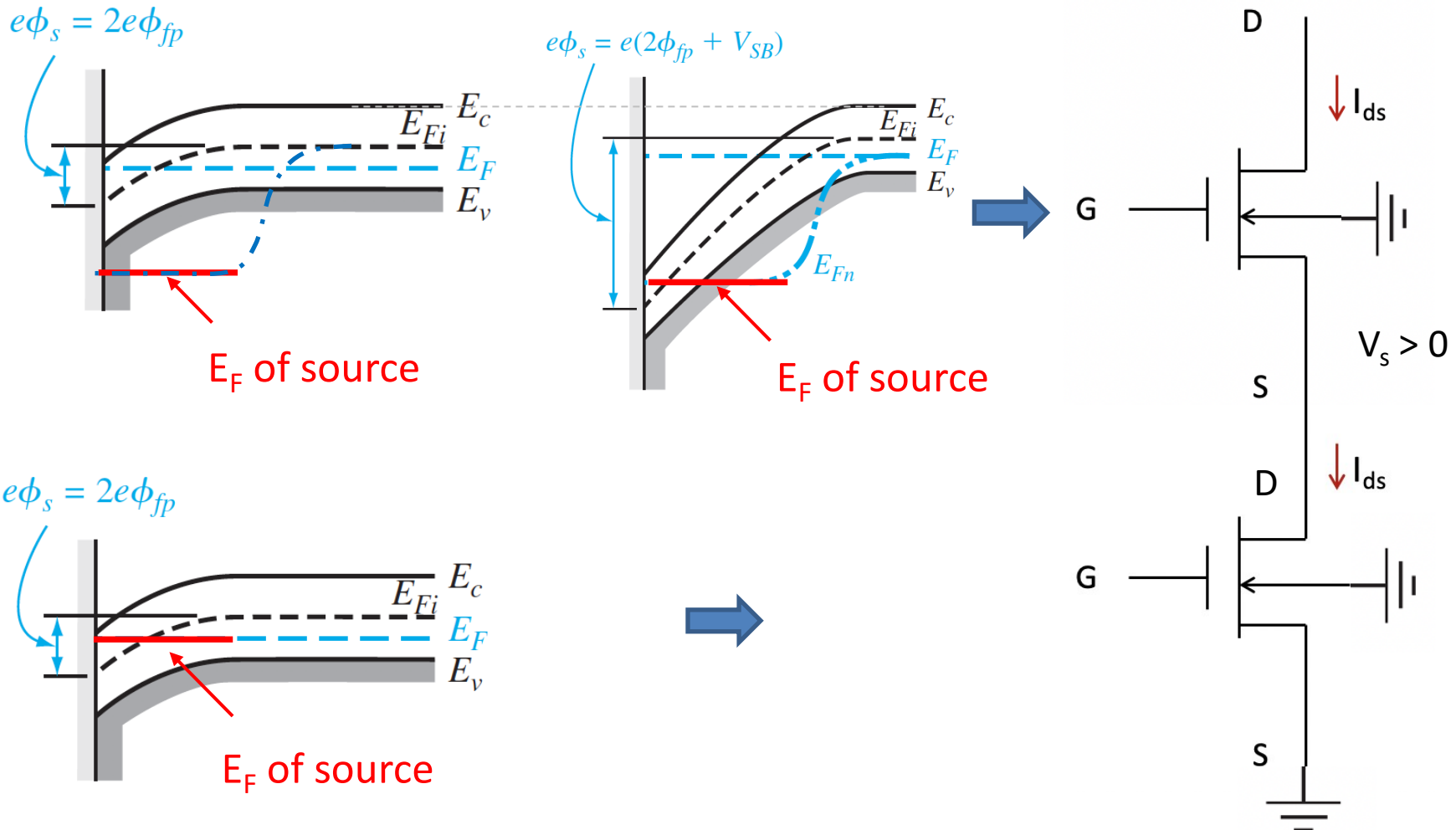
# 10.4 The basic MOSFET operation

## Substrate bias effect



# 10.4 The basic MOSFET operation

## Substrate bias effect



## 10.4 The basic MOSFET operation

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### Substrate bias effect

When  $V_{SB} = 0$ , we had

When  $V_{SB} > 0$ , the space charge width increases and we now have

The change in the space charge density is then

## 10.4 The basic MOSFET operation

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### Substrate bias effect

$$\Delta V_T = -\frac{\Delta Q'_{SD}}{C_{ox}} = \frac{\sqrt{2e\epsilon_s N_a}}{C_{ox}} \left[ \sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$$

$$\gamma = \frac{\sqrt{2e\epsilon_s N_a}}{C_{ox}} \quad \Delta V_T = \gamma \left[ \sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}} \right]$$

# Check your understanding

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## Problem example 6

**Objective:** Calculate the body-effect coefficient and the change in the threshold voltage due to an applied source-to-body voltage.

Consider an n-channel silicon MOSFET at  $T = 300$  K. Assume the substrate is doped to  $N_a = 3 \times 10^{16} \text{ cm}^{-3}$  and assume the oxide is silicon dioxide with a thickness of  $t_{ox} = 20 \text{ nm} = 200 \text{ \AA}$ . Let  $V_{SB} = 1 \text{ V}$ .



# Check your understanding

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Problem example 7

