VE320 – Summer 2021

Introduction to Semiconductor Devices

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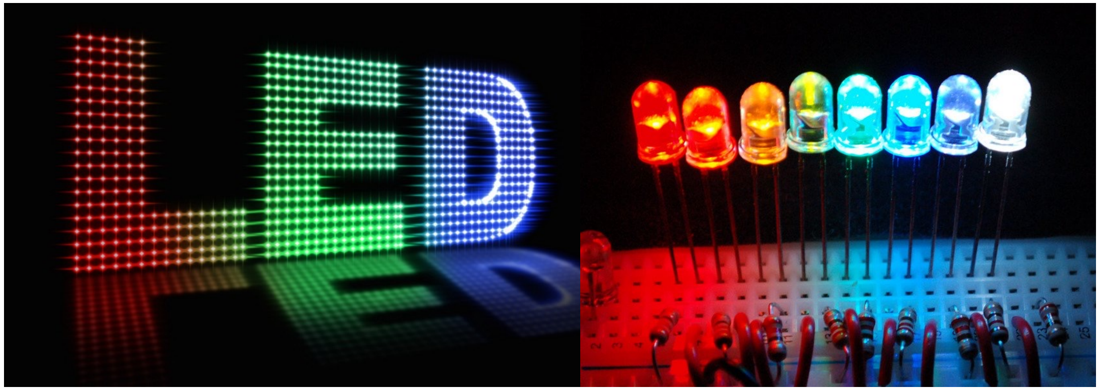
Chapter 7 The pn Junction

Outline

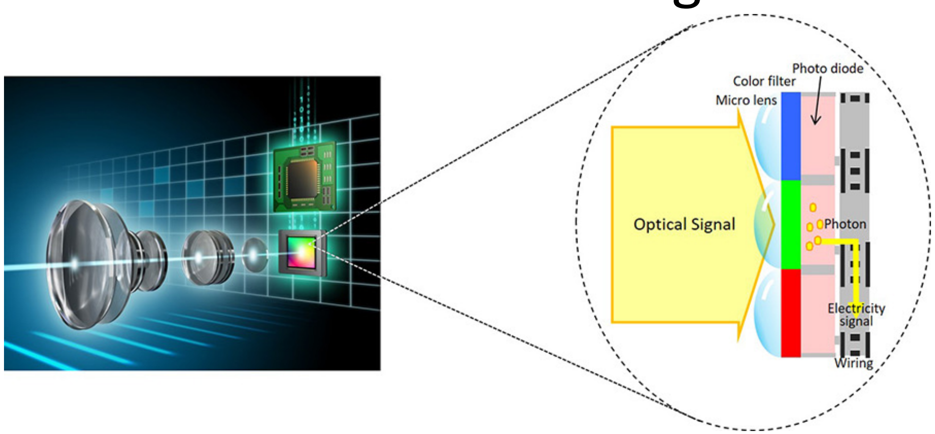
7.0 Introduction to semiconductor devices

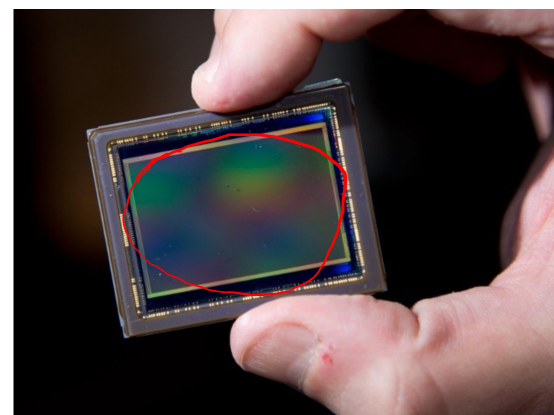


Light emitting diodes

Cold light source

Photodetector: CMOS image sensor





7.0 Introduction to semiconductor devices

Semiconductor lasers

Solar cells

PN

Outline

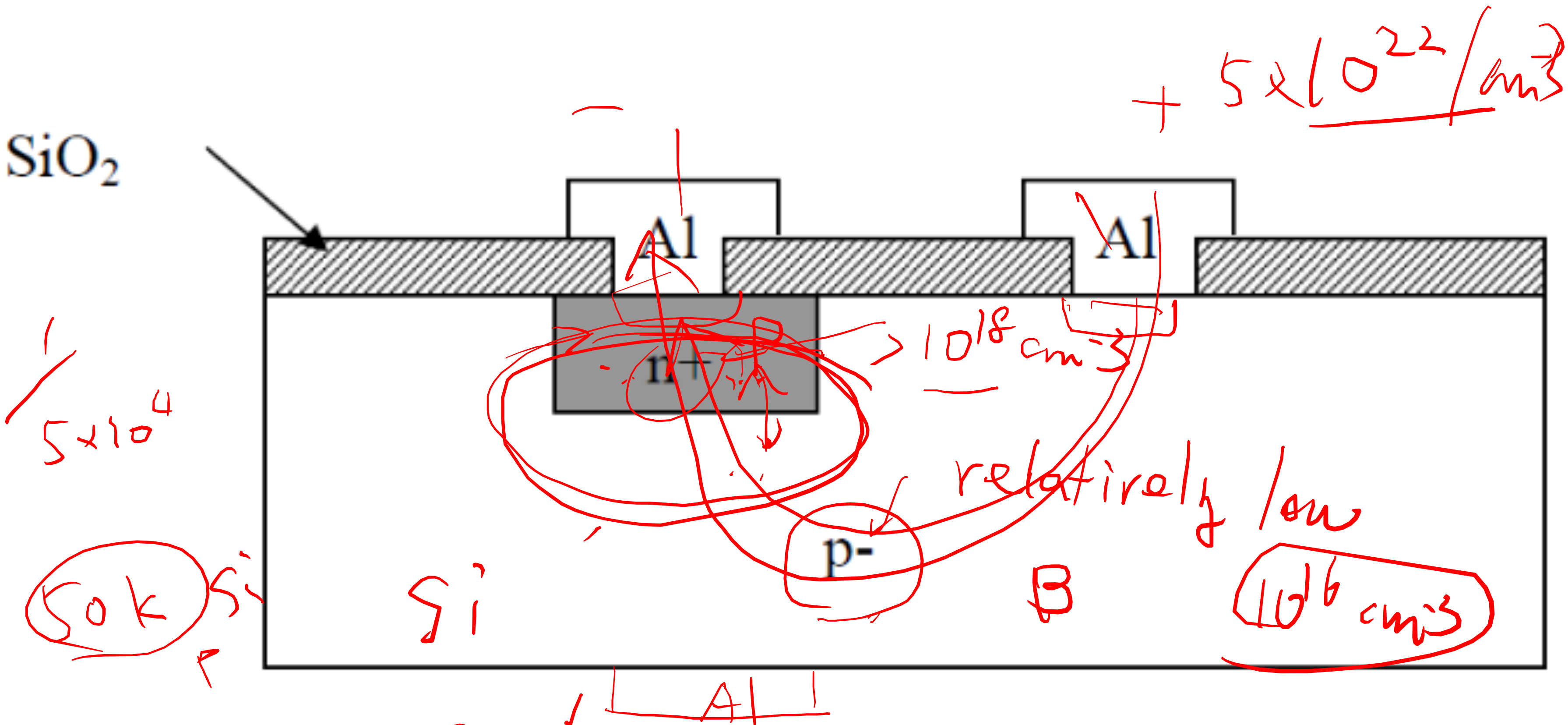
7.0 Introduction to semiconductor devices

7.1 Basic structure of the pn junction

7.2 Zero applied bias

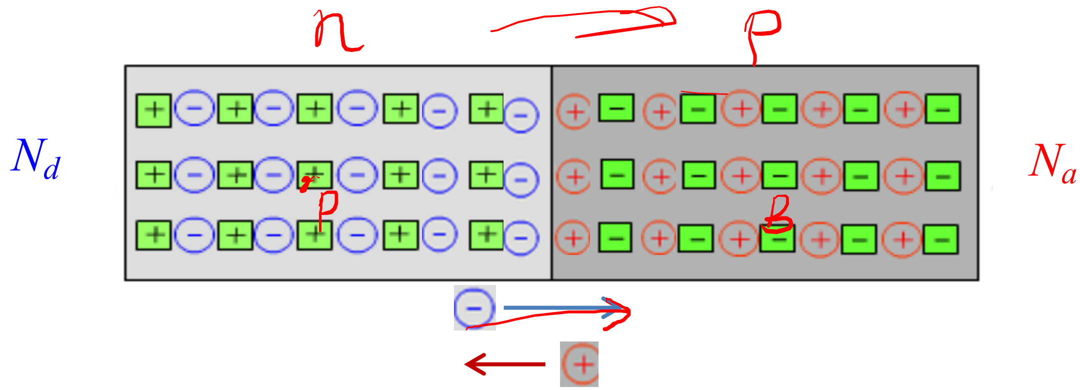
7.3 Reverse applied bias

7.1 Basic structure of pn junction

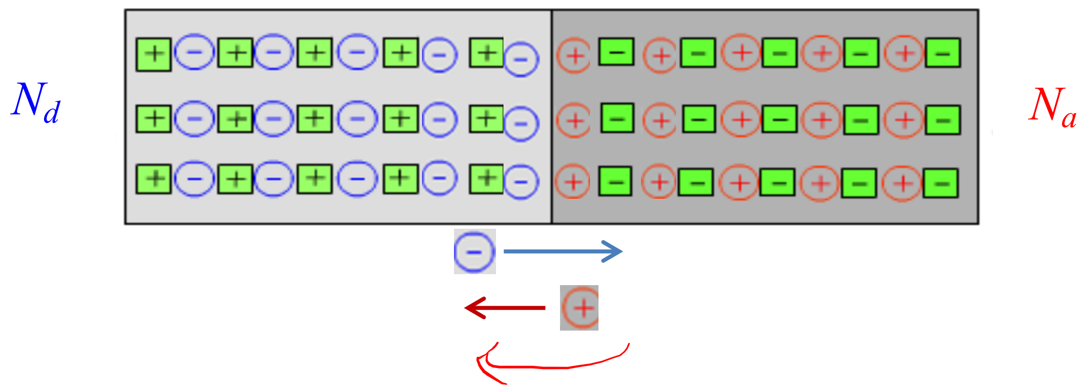


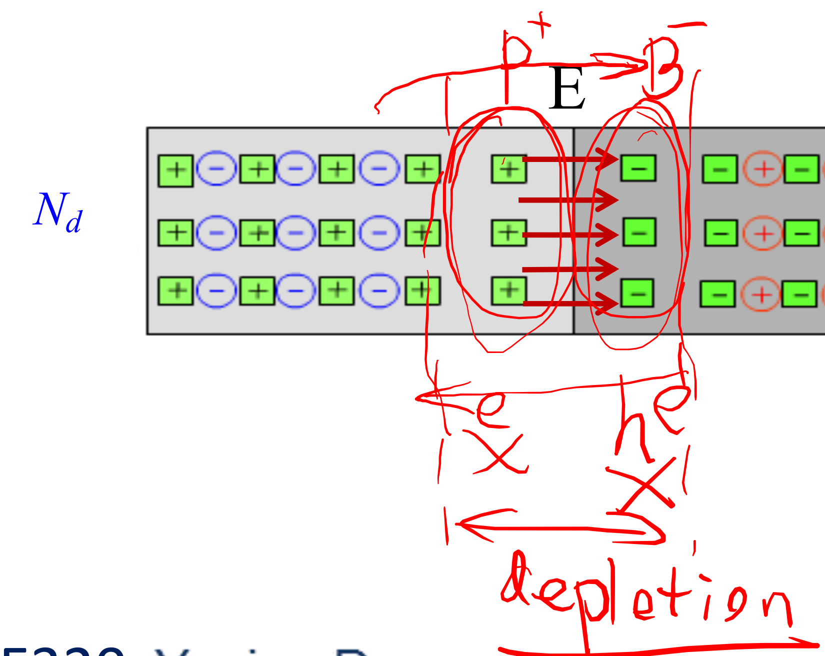
GaAs

7.1 Basic structure of pn junction



7.1 Basic structure of pn junction





$$D = \mu \frac{kT}{q}$$

Outline

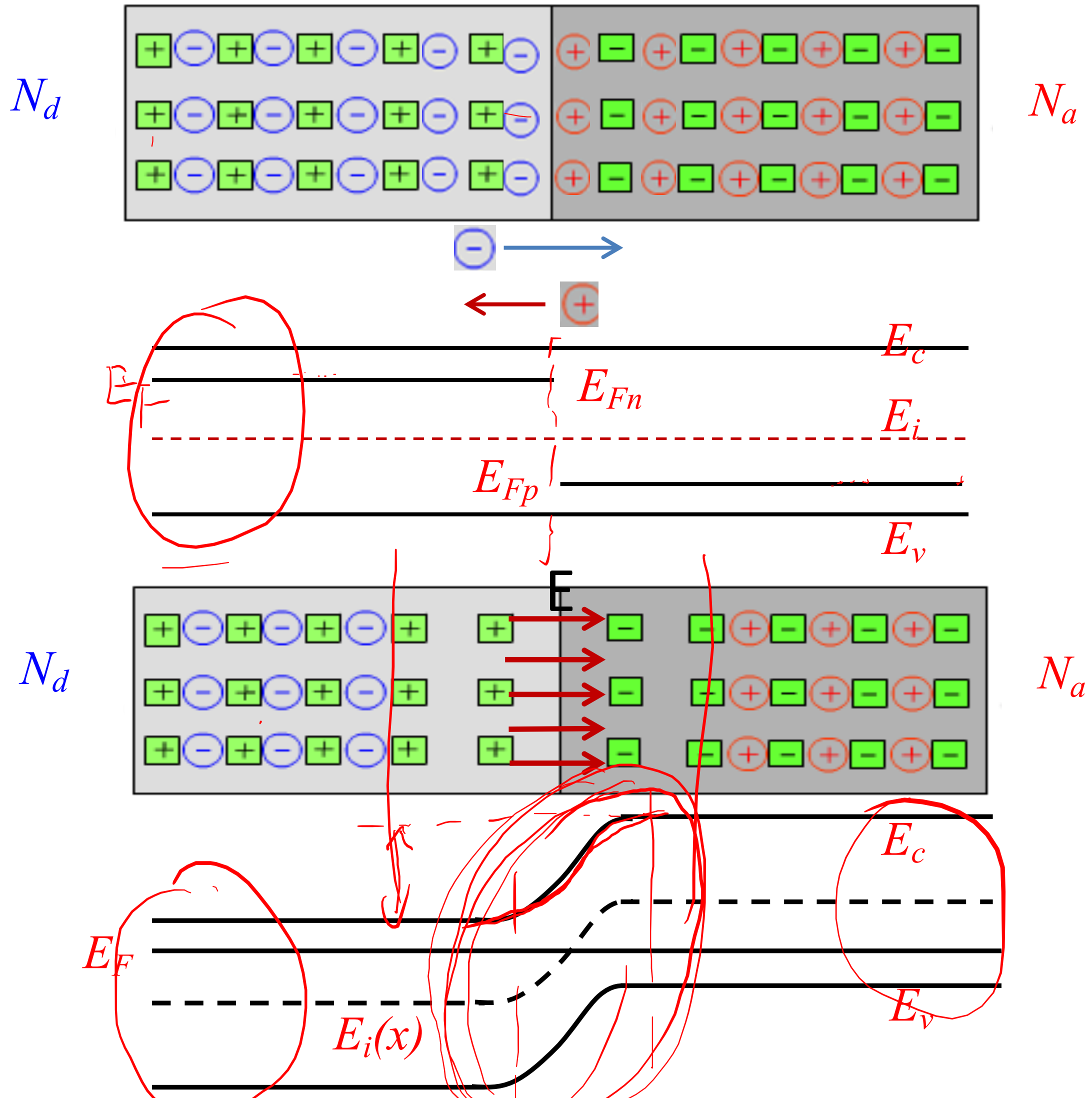
7.1 Basic structure of the pn junction

7.2 Zero applied bias

7.3 Reverse applied bias

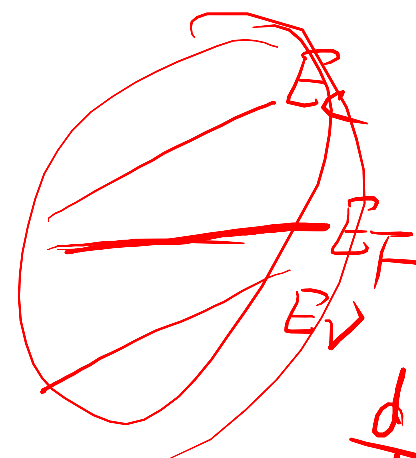
7.2 Zero applied bias

Built-in potential barrier



$\phi(x)$

$E =$



$\frac{dE_F}{dx} = 0$

7.2 Zero applied bias

$\ln x + \ln y = \ln(xy)$

Built-in potential barrier

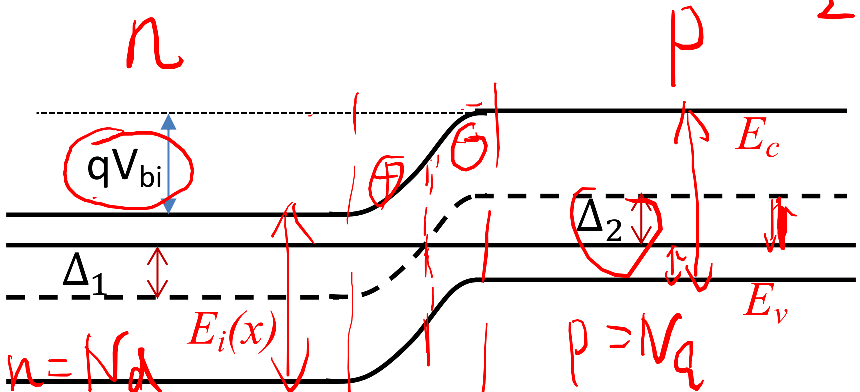
$$n_{n0} = N_d = n_i \exp\left(\frac{E_F - E_i}{kT}\right) = n_i \exp\left(\frac{q\Delta_1}{kT}\right) \qquad \Delta_1 = \frac{kT}{q}\ln\frac{n_{n0}}{n_i}$$

$$p_{p0} = N_a = n_i \exp\left(\frac{E_i - E_F}{kT}\right) = n_i \exp\left(\frac{q \cdot \Delta_2}{kT}\right) \qquad \Delta_2 = \frac{kT}{q}\ln\frac{p_{p0}}{n_i}$$

$$V_{bi} = \Delta_1 + \Delta_2 = \frac{kT}{q}\ln\frac{n_{n0}}{n_i} + \frac{kT}{q}\ln\frac{p_{p0}}{n_i} = \frac{kT}{q}\ln\frac{n_{n0} \cdot p_{p0}}{n_i^2} = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$$

$\Delta_2 = E_i - E_F$

$p = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$



$E_F - E_v = kT\ln\frac{N_v}{N_a}$

7.2 Zero applied bias

Charge carrier distribution

0.1eV

$$n_{n0} = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \Rightarrow \boxed{n(x) = n_i \exp\left(\frac{E_F - E_i(x)}{kT}\right)}$$

0.0259eV

$$n_{p0} = n_i \exp\left(\frac{-\Delta_2}{kT}\right)$$

$$p_{p0} = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$$n_{p0} \cdot p_{p0} = n_i^2$$

$$n_{n0} = N_d = n_i \exp\left(\frac{\Delta_1 - \Delta E_i(x)}{kT}\right)$$

$$n_{n0} \cdot p_{n0} = n_i^2$$

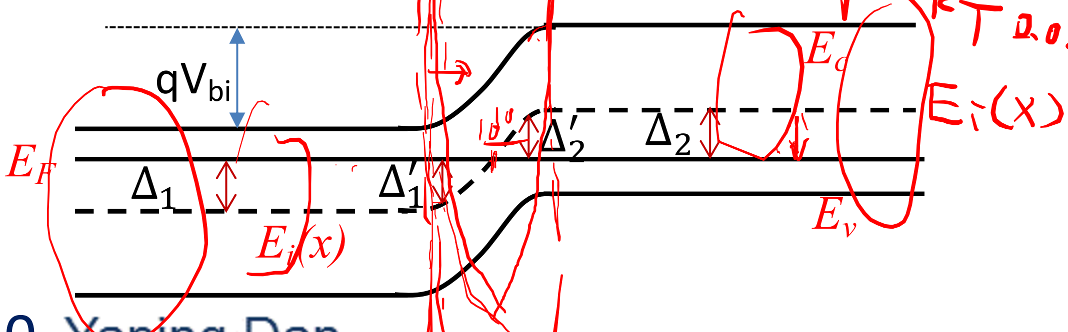
$$n(x) = n_i \exp\left(\frac{\Delta_1}{kT}\right) \exp\left(\frac{-\Delta E_i(x)}{kT}\right) \qquad \boxed{n(x) \cdot p(x) = n_i^2}$$

$$= n_{n0} \exp\left(\frac{-\Delta E_i(x)}{kT}\right)$$

$N_d = n_{n0}$ 10^{17} $n(x)$ 10^{16} p_{p0}

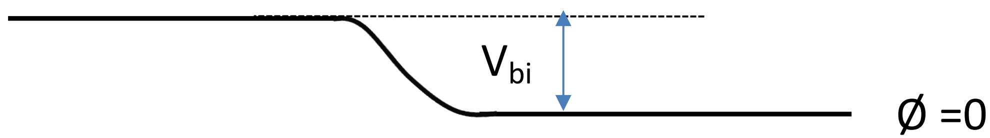
$$= n_{n0} \exp\left(\frac{-0.1}{kT}\right) \approx \frac{n_{n0}}{50} \quad 2\%$$

kT 0.0259

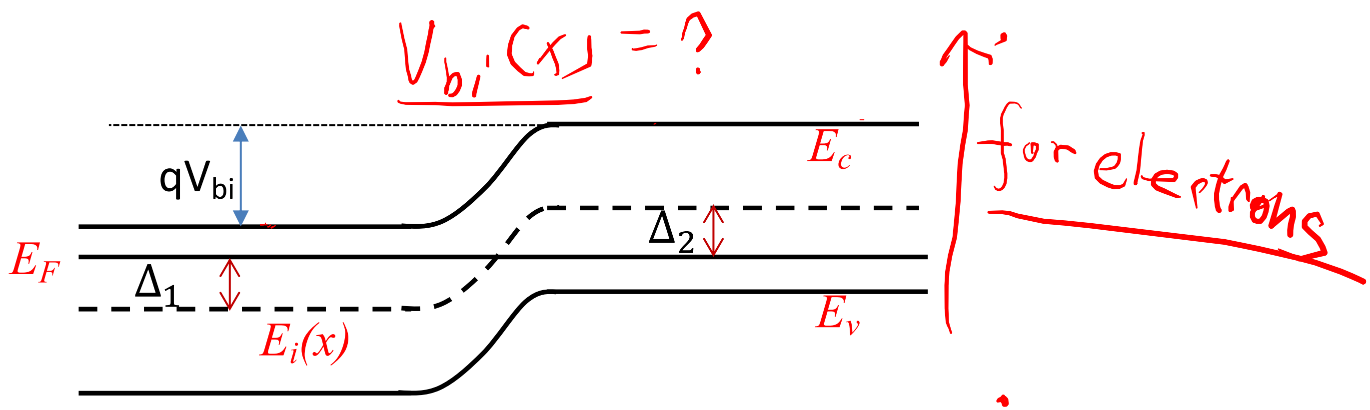


7.2 Zero applied bias

Potential profile



Potential profile



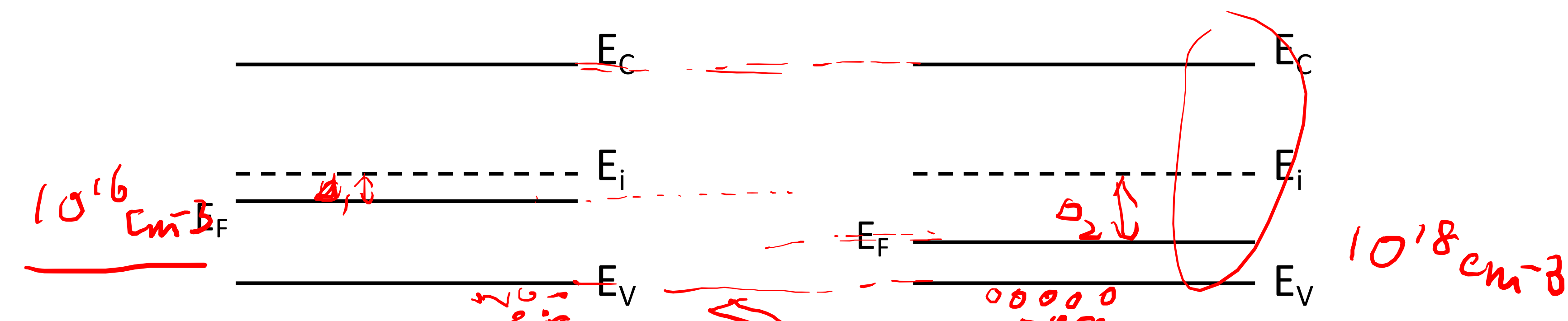
Energy band diagram

Check your understanding

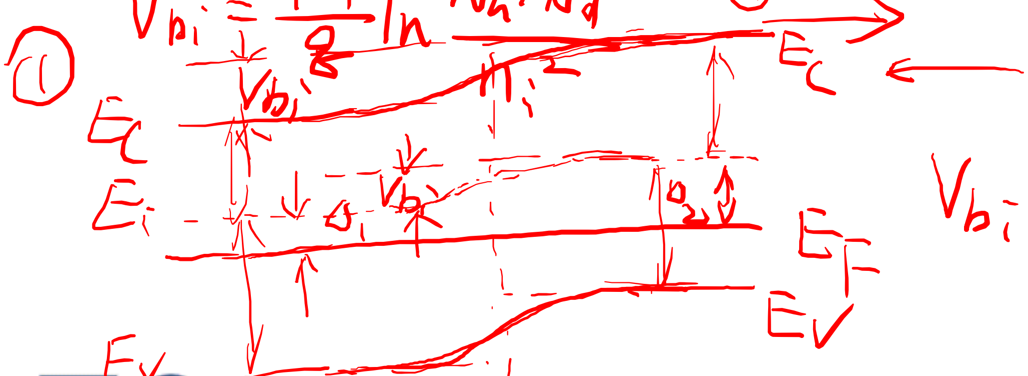
Problem Example #1

Two pieces of p-type silicon are in contact. The doping concentrations are 10^{16} cm^{-3} and 10^{18} cm^{-3}. Calculate the built-in potential between these two pieces of silicon and plot the energy band bending diagram.

N_{a2} N_{a1}



$$V_{bi} = \frac{kT}{q} \ln \frac{N_a \cdot N_d}{n_i^2}$$



$$\Delta_2 = \frac{kT}{q} \ln \frac{N_{a1}}{n_i}$$

$$V_{bi} = \Delta_2 - \Delta_1$$

$$\Delta_1 = \frac{kT}{q} \ln \frac{N_{a2}}{n_i}$$

$$= \frac{kT}{q} \ln \frac{N_{d1}}{N_{a2}} = \frac{kT}{q} \ln 100 = 0.12V$$

7.2 Zero applied bias

Poisson's equation

$$\boxed{\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}}$$

$n(x) \quad N_a^-(x) \qquad N_d^+(x) \quad p(x)$

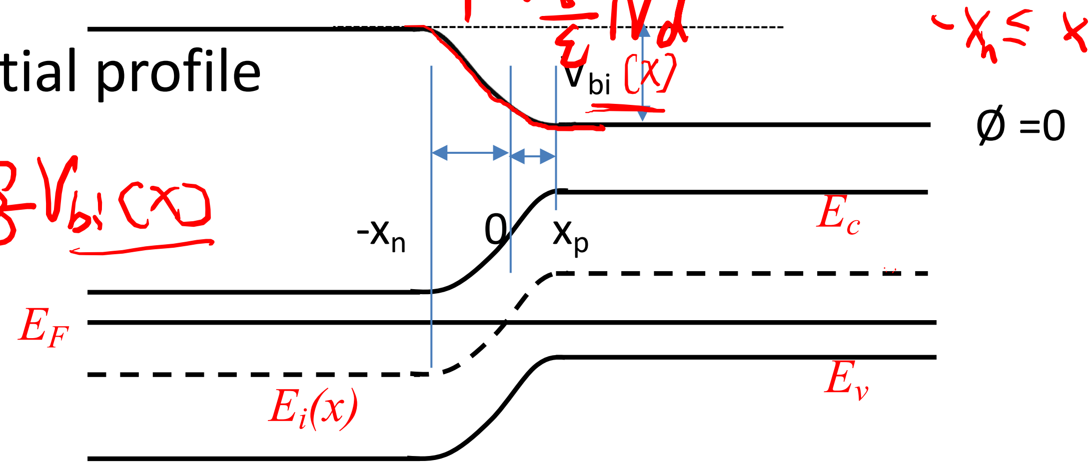
$$= -\frac{q\left[-n(x) - N_a^-(x) + p(x) + N_d^+(x)\right]}{\varepsilon}$$

$$= -\frac{q}{\varepsilon}\left[N_d(x) - N_a(x) + n_i \exp\left(\frac{E_i(x)-E_F}{kT}\right) - n_i \exp\left(\frac{E_F - E_i(x)}{kT}\right)\right]$$

$$= -\frac{q}{\varepsilon}\left[N_d(x) - N_a(x) + n_i \exp\left(\frac{-qV(x) - E_F}{kT}\right) - n_i \exp\left(\frac{E_F + qV(x)}{kT}\right)\right]$$

$$= \begin{cases} \dfrac{q}{\varepsilon} N_a & 0 \le x < x_p \\ -\dfrac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

Potential profile



$E_i(x) = -qV_{bi}(x)$

7.2 Zero applied bias

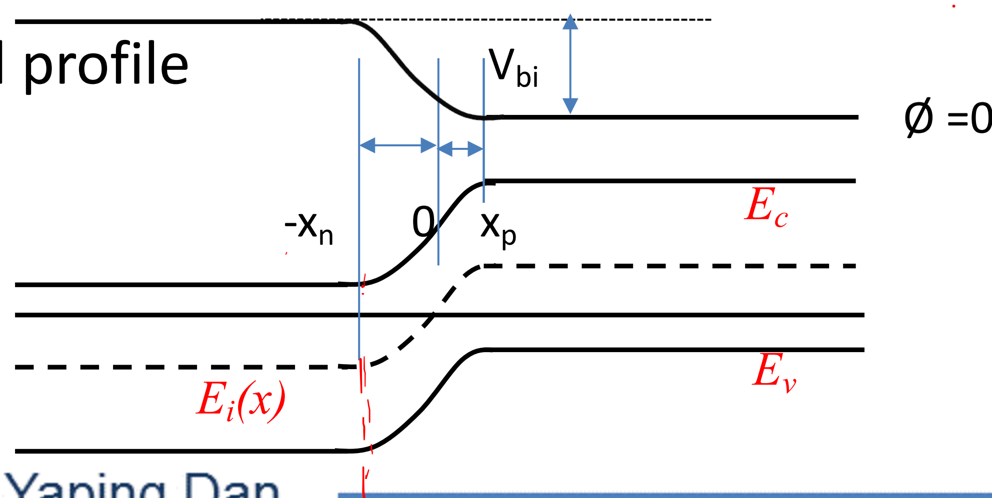
Poisson's equation

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon} = \begin{cases} \frac{q}{\varepsilon} N_a & 0 \le x \le x_p \\ -\frac{q}{\varepsilon} N_d & -x_n \le x < 0 \end{cases}$$

$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon} N_a x + \frac{q}{\varepsilon} N_a x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon} N_d x + \frac{q}{\varepsilon} N_a x_n & -x_n \le x < 0 \end{cases}$$

$E(x_p) = 0 \quad E(-x_n) = 0$

Potential profile



7.2 Zero applied bias

Third time approximation

Poisson's equation

$V(x)$

$V(x_p)=0$

$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

E_c

E_v

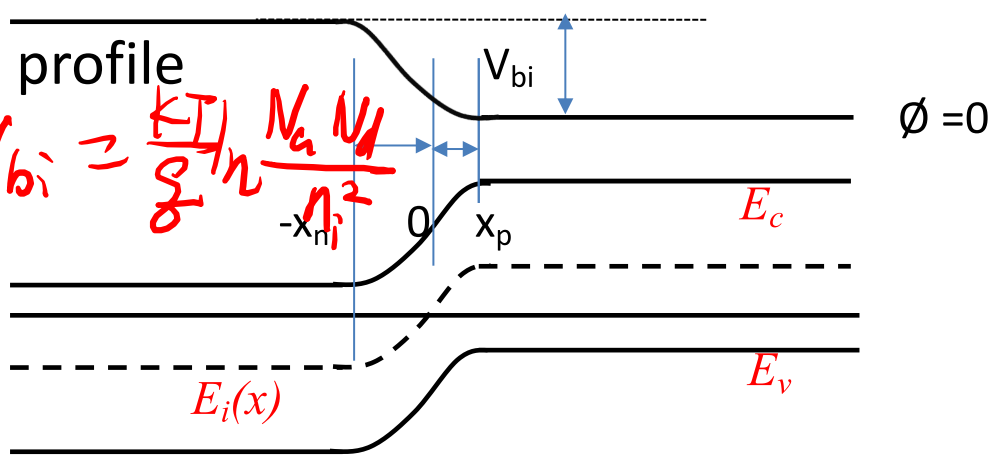
$$E = -\frac{dV(x)}{dx} = \begin{cases} -\frac{q}{\varepsilon}N_a x + \frac{q}{\varepsilon}N_a x_p & 0 \le x \le x_p \\ \frac{q}{\varepsilon}N_a x + \frac{q}{\varepsilon}N_d x_n & -x_n \le x < 0 \end{cases}$$

$$V(x) = \int_{-x_n}^{x_p} \frac{dV(x)}{dx} = \begin{cases} \frac{q}{\varepsilon}N_a\left(\frac{1}{2}x^2 - x_p \cdot x + C_1\right) & 0 \le x \le x_p \\ -\frac{q}{\varepsilon}N_a\left(\frac{1}{2}x^2 + x_n \cdot x + C_2\right) & -x_n \le x < 0 \end{cases}$$

$V(x_p) = 0$

Potential profile

$$V(-x_n) = V_{bi} = \frac{kT}{q}\ln\frac{N_a N_d}{n_i^2}$$



7.2 Zero applied bias

Poisson's equation

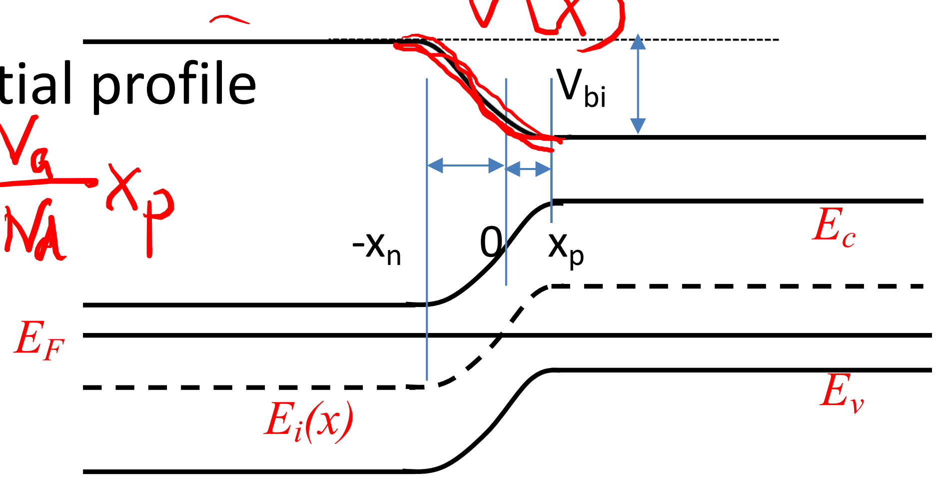
$$\frac{d^2V(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon}$$

$$V(x) = \begin{cases} \dfrac{q}{\varepsilon} N_a \left(\dfrac{1}{2}x^2 - x_p \cdot x + \dfrac{x_p^2}{2}\right) & 0 \le x \le x_p \\ -\dfrac{q}{\varepsilon} N_d \left(\dfrac{1}{2}x^2 + \dfrac{N_a}{N_d} x_p \cdot x \, \cancel{x_n \cdot x} - \dfrac{N_a}{N_d}\dfrac{x_p^2}{2}\right) & -x_n \le x < 0 \end{cases}$$

$$N_a \cdot x_p = N_d \cdot x_n$$

$$V(x=-x_n) = V_{bi} = \frac{kT}{q} \ln \frac{N_a N_d}{n_i^2}$$

Potential profile



$$\Rightarrow x_n = \frac{N_a}{N_d} x_p$$

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q} \frac{N_d}{N_a} \frac{1}{N_a + N_d}}$$

$$x_n = \frac{N_a}{N_d} x_p$$

7.2 Zero applied bias

$$0 \le x \le x_p$$

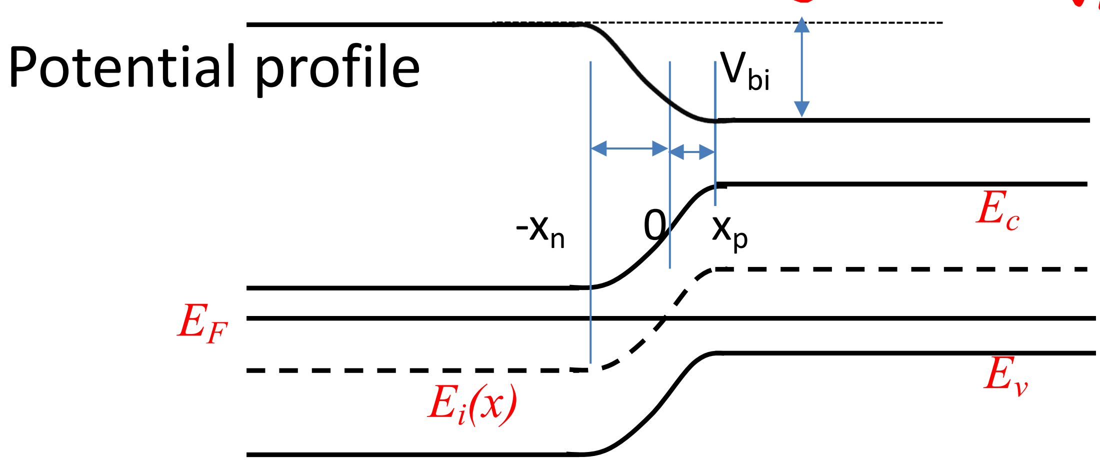
$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_d}{N_a}\frac{1}{N_d+N_a}}$$

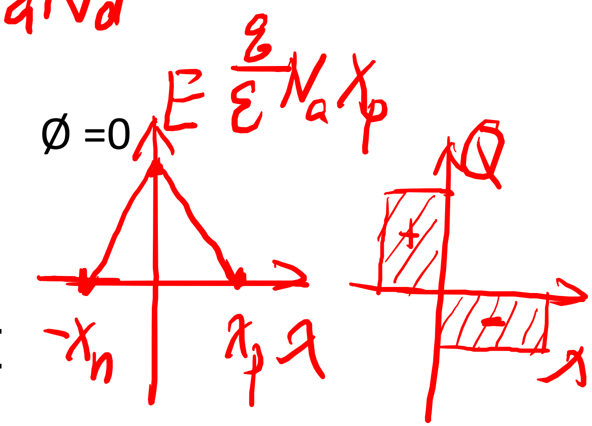
$$x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_d+N_a}}$$

$$E = \begin{cases} -\frac{q}{\varepsilon}N_a x + \frac{q}{\varepsilon}N_a x_p \\ \frac{q}{\varepsilon}N_d x + \frac{q}{\varepsilon}N_d x_n \end{cases}$$

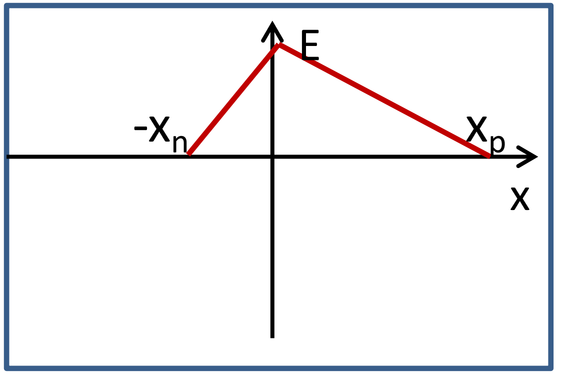
$$-x_n \le x < 0$$

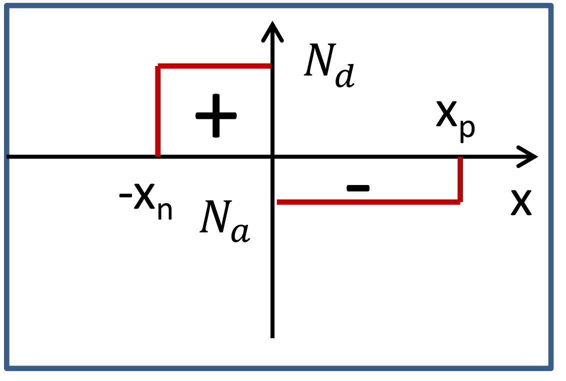
$$\underline{W_{dep}} = x_n + x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a+N_d}{N_a N_d}}$$





7.2 Zero applied bias





7.2 Zero applied bias

Space charge width

$$W_{dep} = x_n + x_p = \sqrt{\frac{2\varepsilon V_{bi}}{q}\,\frac{N_d + N_a}{N_d \cdot N_a}}$$

7.2 Zero applied bias

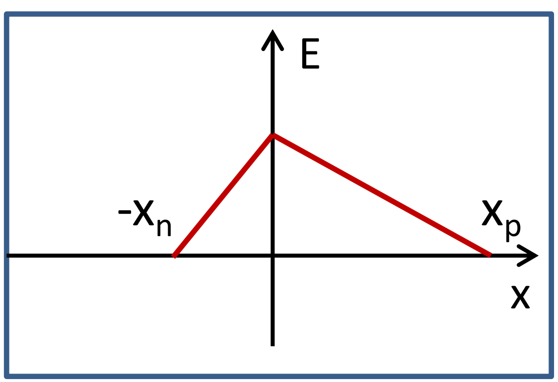
Space charge width

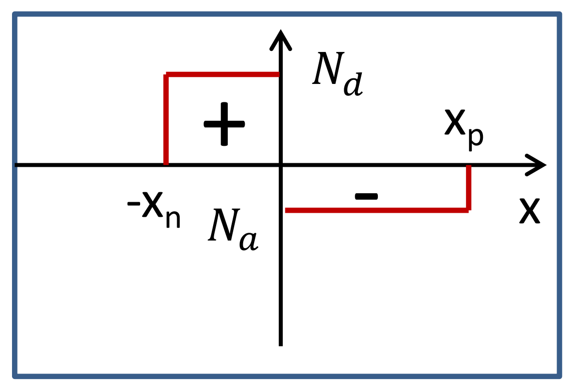
7.2 Zero applied bias

<u>Space charge width</u>

7.2 Zero applied bias

Space charge width





Check your understanding

Problem Example #2

A silicon pn junction at T=300K with zero applied bias has doping concentration of $N_d = 5 \times 10^{16}$ cm^{-3} and $N_a = 5\times10^{15}$ cm^{-3}. Determine x_n, x_p, W and $|E_{max}|$.

$$x_n = \sqrt{\frac{2\varepsilon V_{bi}}{q}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

$$V_{bi} = \frac{kT}{q}\ln\frac{N_a \cdot N_d}{n_i^2}$$

$$= \sqrt{\frac{2\times11.7\times8.85\times10^{-14}\times0.7183}{1.6\times10^{-19}}\times\frac{5\times10^{15}}{5\times10^{16}}\times\frac{1}{5\times10^{15}+5\times10^{16}}}$$

$$= 4.11\times10^{-6}\text{ cm} = 41.1\text{ nm}$$

$$= 0.0259\times\ln\frac{5\times10^{16}\times5\times10^{15}}{(1.5\times10^{10})^2}$$

$$= 0.7183\text{ V}$$

$$|E| = \frac{q}{\varepsilon}N_a \cdot x_p$$

$$= 3.18\times10^4\text{ V/cm}$$

$$x_p = 4.11\times10^{-5}\text{ cm}$$

$$= 411\text{ nm}$$

$$W = x_n + x_p = 452\text{ nm}$$

Outline

7.1 Basic structure of the pn junction

7.2 Zero applied bias

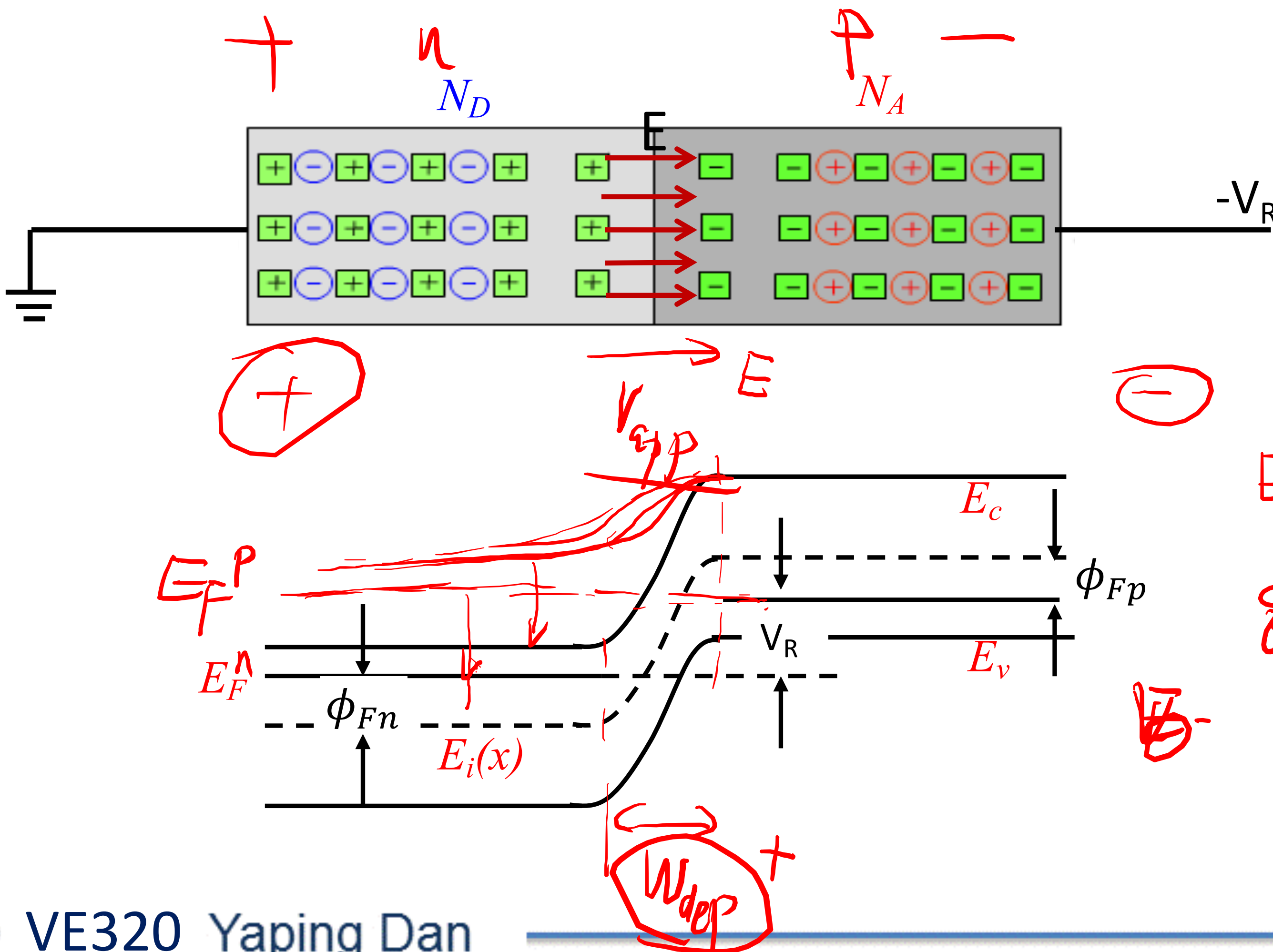
7.3 Reverse applied bias

7.3 Reverse applied bias

<u>Space charge width and electric field</u>

$$V_{\text{total}} = |\phi_{Fn}| + |\phi_{Fp}| + V_R$$

$V_{bi}' = V_{bi} + V_R$



$E_F(p) - E_F(n)$

$q \quad = q \cdot V_{app}$

7.3 Reverse applied bias

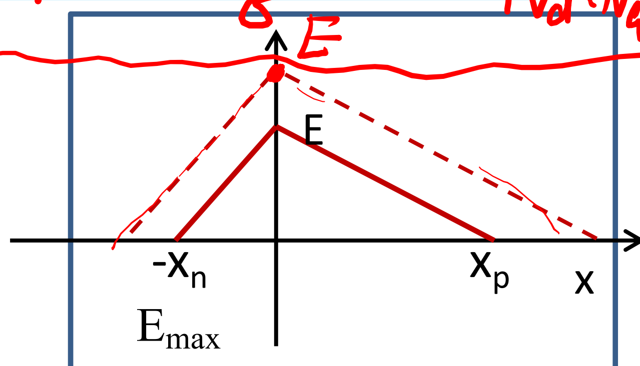
<u>Space charge width and electric field</u>

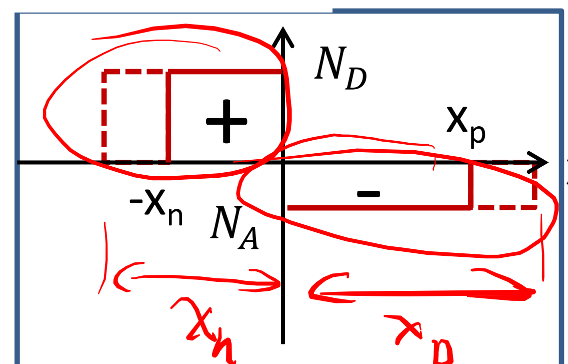
$$V_{bi}' = \underbrace{|\phi_{Fn}| + |\phi_{Fp}|}_{V_{bi}} + V_R$$

$$x_p = \sqrt{\frac{2\varepsilon V_{bi}}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}}$$

$$x_p' = \sqrt{\frac{2\varepsilon (V_{bi}+V_R)}{e}\frac{N_d}{N_a}\frac{1}{N_a+N_d}} \sim \sqrt{V_R}$$

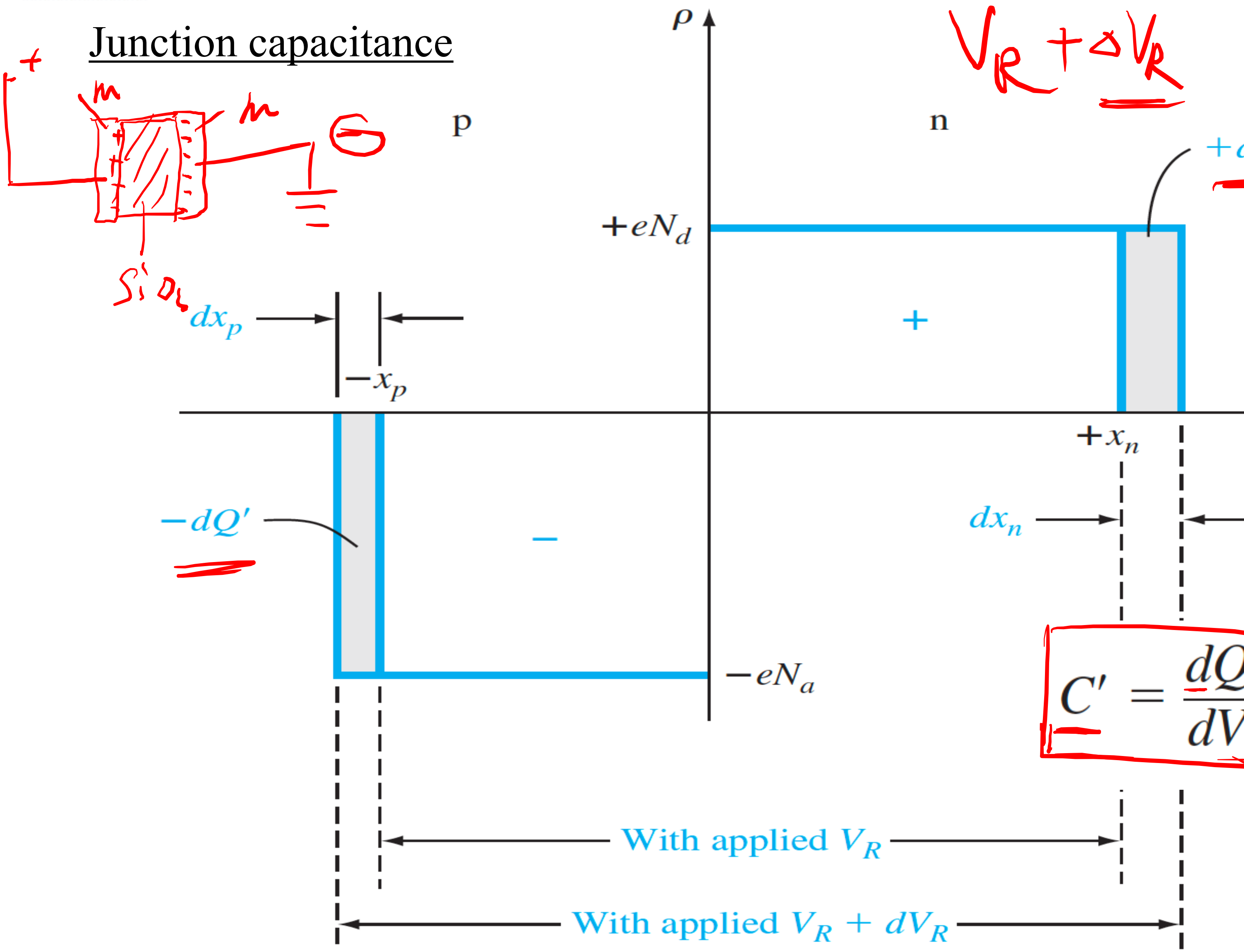
$$W_{dep} = \sqrt{\frac{2\varepsilon (V_{bi}+V_R)}{e}\frac{N_d+N_a}{N_d \cdot N_a}}$$





7.3 Reverse applied bias

Junction capacitance



$$\boxed{C' = \frac{dQ'}{dV_R}}$$

$V_R + \Delta V_R$

$C' = \dfrac{\varepsilon}{W(V_R)}$

ΔV_{app}

ΔV_b

$+V_R + \Delta V_R$

n p

$W(V_R)$

C'

7.3 Reverse applied bias

Junction capacitance

$$\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$$

$$x_n = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{e}\frac{N_a}{N_d}\frac{1}{N_a+N_d}}$$

$$C' = \frac{dQ'}{dV_R}$$

$$Q' = e \cdot N_d \cdot x_n = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)e^2 \cdot N_d^2}{e}\frac{N_a}{N_d}\frac{1}{N_a+N_d}} = \sqrt{\frac{2\varepsilon e \cdot N_d N_a (V_{bi}+V_R)}{N_a+N_d}}$$

$$= \sqrt{\frac{2\varepsilon e N_a N_d}{N_a+N_d}}\frac{d\sqrt{V_{bi}+V_R}}{dV_R}$$

$$= \sqrt{\frac{2\varepsilon e N_a N_d}{N_a+N_d}}\frac{1}{2\sqrt{V_{bi}+V_R}}$$

$$= \sqrt{\frac{e\varepsilon}{2(V_{bi}+V_R)}\frac{N_a \cdot N_d}{N_a+N_d}}$$

$$C' = \varepsilon / W \qquad W = \sqrt{\frac{2\varepsilon(V_{bi}+V_R)}{e}\frac{N_a+N_d}{N_a N_d}}$$

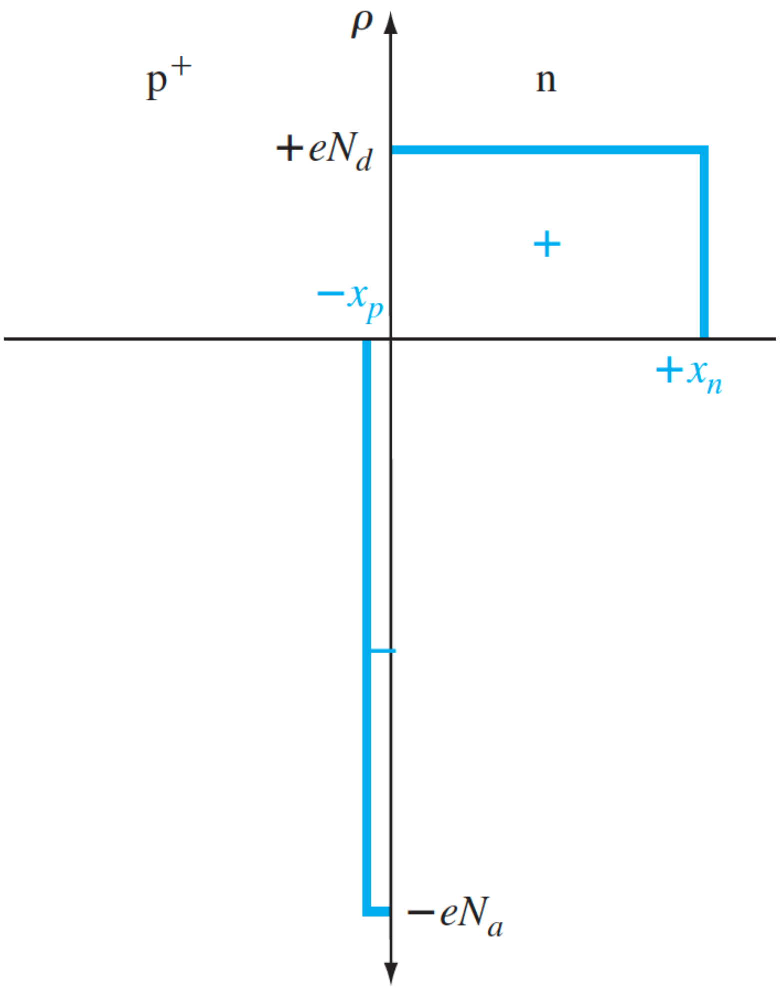
Check your understanding

Problem Example #3

Consider a GaAs pn junction at $T = 300K$ doped to $N_a = 5 \times 10^{15}\ cm^{-3}$ and $N_d = 2 \times 10^{16}\ cm^{-3}$. (a) Calculate V_{bi}. (b) Determine the junction capacitance C' for V_R =4V.

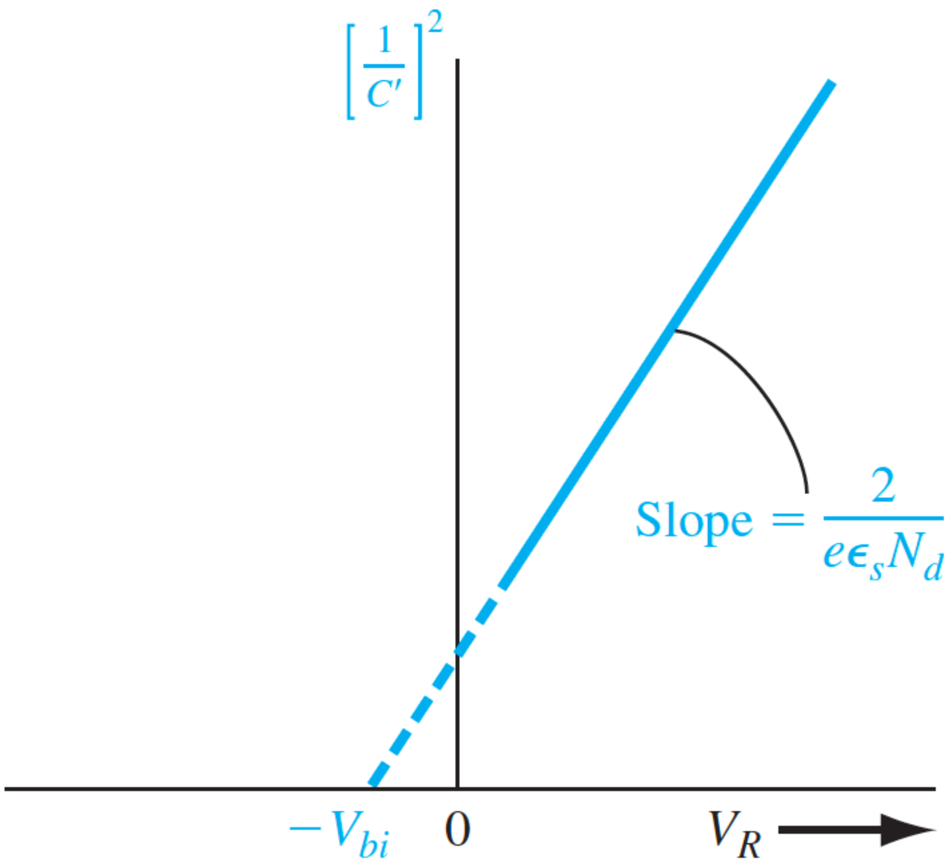
7.3 Reverse applied bias

One-sided junction



7.3 Reverse applied bias

<u>One-sided junction</u>



Check your understanding

Problem Example #4

nature COMMUNICATIONS

ARTICLE

DOI: 10.1038/s41467-017-02564-3 OPEN

Control sample： Au is in contact with a uniform doped n-type Si substrate forming a device similar to pn junction. Find the substrate dopnig concentration and built-in potential.

SAMM-doped sample: Au is in contact with Si that is doped with SAMM (n-type doping)

Take the Au as p++ doping

Deep level transient spectroscopic investigation of phosphorus-doped silicon by self-assembled molecular monolayers

Xuejiao Gao[1], Bin Guan[1], Abdelmadjid Mesli[2], Kaixiang Chen[1] & Yaping Dan[1]