VE320 Intro to Semiconductor Devices
Summer 2022 — Problem Set 4

June 28, 2022

Exercise 4.1

GaAs, at $T = 300$ K, is uniformly doped with acceptor impurity atoms to a concentration of $N_a = 2 \times 10^{16}$ cm^{-3}. Assume an excess carrier lifetime of 5×10^{-7} s.

Determine the electron-hole recombination rate if the excess electron concentration is $\delta n = 5 \times 10^{14}$ cm^{-3}.

Answer:

$$
\begin{aligned}
p_o &= N_a = 2 \times 10^{16} \text{ cm}^{-3} \\
n_o &= \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{2 \times 10^{16}} = 1.62 \times 10^{-4} \text{ cm}^{-3} \\
R' &= \frac{\delta n}{\tau_{n0}} = \frac{5 \times 10^{14}}{5 \times 10^{-7}} = 10^{21} \text{ cm}^{-3} \text{ s}^{-1}
\end{aligned}
$$

Exercise 4.2

Consider an infinitely large, homogeneous n-type semiconductor with a zero applied electric field. Assume that, for $t < 0$, the semiconductor is in thermal equilibrium and that, for $t \geq 0$, a uniform generation rate exists in the crystal.

(a) Calculate the excess carrier concentration as a function of time assuming the condition of low injection.

(b) Consider n-type silicon at $T = 300$ K doped to $N_d = 5 \times 10^{16}$ cm^{-3}. Assume that $g' = 5 \times 10^{21}$ cm^{-3} s^{-1} and let $\tau_{p0} = 10^{-7}$ s.

Determine $\delta p(t)$ at (i) $t = 0$, (ii) $t = 10^{-7}$ s, (iii) $t = 5 \times 10^{-7}$ s, and (iv) $t \to \infty$.

Answer:

(a) The condition of a uniform generation rate and a homogeneous semiconductor again implies that $\partial^2(\delta p)/\partial x^2 = \partial(\delta p)/\partial x = 0$. The equation, for this case, reduces to

$$
g' - \frac{\delta p}{\tau_{p0}} = \frac{d(\delta p)}{dt}
$$

The solution to this differential equation is

$$
\delta p(t) = g' \tau_{p0} \left(1 - e^{-t/\tau_{p0}}\right)
$$

(b) $\delta p(t)=g'\tau_{po}\left(1-e^{-t/\tau_{po}}\right)=\left(5\times 10^{21}\right)\left(10^{-7}\right)\left(1-e^{-t/\tau_{p0}}\right)=5\times 10^{14}\left(1-e^{-t/\tau_{p}}\right)$

(i) $\delta p(0)=5\times 10^{14}\left(1-e^{-0}\right)=0$

(ii) $\delta p\left(10^{-7}\right)=5\times 10^{14}\left(1-e^{-1/1}\right)=3.16\times 10^{14}\ \mathrm{cm}^{-3}$

(iii) $\delta p\left(5\times 10^{-7}\right)=5\times 10^{14}\left(1-e^{-5/1}\right)=4.966\times 10^{14}\ \mathrm{cm}^{-3}$

(iv) $\delta p(\infty)=5\times 10^{14}\left(1-e^{-\infty}\right)=5\times 10^{14}\ \mathrm{cm}^{-3}$

Exercise 4.3

Consider a silicon sample at $T=300$ K that is uniformly doped with acceptor impurity atoms at a concentration of $N_a=10^{16}\ \mathrm{cm}^{-3}$. At $t=0$, a light source is turned on generating excess carriers uniformly throughout the sample at a rate of $g'=8\times\ 10^{20}\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1}$. Assume the minority carrier lifetime is $\tau_{n0}=5\times 10^{-7}$ s, and assume mobility values of $\mu_n=900\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$ and $\mu_p=380\ \mathrm{cm}^2/\mathrm{V}\cdot\mathrm{s}$.

(a) Determine the conductivity of the silicon as a function of time for $t\geq 0$.

(b) What is the value of conductivity at $(i) t=0$ and (ii) $t=\infty$?

Answer:

(a)

$$
\begin{aligned}
p_o &= N_a=10^{16}\ \mathrm{cm}^{-3}\\
n_o =& \frac{n_i^2}{p_o}=\frac{\left(1.5\times 10^{10}\right)^2}{10^{16}}=2.25\times 10^{4}\ \mathrm{cm}^{-3}\\
&\sigma=e\mu_n\left(n_o+\delta n\right)+e\mu_p\left(p_o+\delta p\right)\\
&\cong e\mu_p p_o+e\left(\mu_n+\mu_p\right)\delta n\\
&\text{Now } \delta n=\delta p=g'\tau_{n0}\left(1-e^{-t/\tau_{n0}}\right)\\
&=\left(8\times 10^{20}\right)\left(5\times 10^{-7}\right)\left(1-e^{-t/\tau_{n0}}\right)\\
&=4\times 10^{14}\left(1-e^{-t/\tau_{n0}}\right)\mathrm{cm}^{-3}\\
&\sigma=\left(1.6\times 10^{-19}\right)(380)\left(10^{16}\right)\\
&\qquad+\left(1.6\times 10^{-19}\right)(900+380)\\
&\qquad\times\left(4\times 10^{14}\right)\left(1-e^{-t/\tau_{n0}}\right)\\
\sigma &= 0.608+0.0819\left(1-e^{-t/\tau_{n0}}\right)(\Omega-\mathrm{cm})^{-1}
\end{aligned}
$$

(b)

(i) $\sigma(0)=0.608(\Omega-\mathrm{cm})^{-1}$

(ii) $\sigma(\infty)=0.690(\Omega-\mathrm{cm})^{-1}$

Exercise 4.4

A p-type gallium arsenide semiconductor at $T=300$ K is doped at $N_a=10^{16}\ \mathrm{cm}^{-3}$. The excess carrier concentration varies linearly from $10^{14}\ \mathrm{cm}^{-3}$ to zero over a distance of 50μm. Plot the position of the quasi-Fermi levels with respect to the intrinsic Fermi level versus distance.

Answer:
Quasi-Fermi level for minority carrier electrons:

$$E_{Fn} - E_{Fi} = kT \ln \left(\frac{n_o + \delta n}{n_i} \right)$$
$$n_o = \frac{n_i^2}{p_o} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4} \text{ cm}^{-3}$$

We have

$$\delta n = \left(10^{14}\right) \left(\frac{x}{50}\right)$$

Then

$$E_{Fn} - E_{Fi} = kT \ln \left[\frac{3.24 \times 10^{-4} + (10^{14} x/50)}{1.8 \times 10^6} \right]$$

We find

$x(\mu\text{m})$	$(E_{Fn} - E_{Fi})$ (eV)
0	-0.581
1	$+0.361$
2	$+0.379$
10	$+0.420$
20	$+0.438$
50	$+0.462$

Quasi-Fermi level for holes: we have

$$E_{Fi} - E_{Fp} = kT \ln \left(\frac{p_o + \delta p}{n_i} \right)$$

We have $p_o = 10^{16}$ cm^{-3} and $\delta n = \delta p$. We find

$x(\mu\text{m})$	$(E_{Fi} - E_{Fp})$ (eV)
0	$+0.58115$
50	$+0.58140$

Exercise 4.5

In a GaAs material at $T = 300$ K, the doping concentrations are $N_d = 8 \times 10^{15}$ cm^{-3} and $N_a = 2 \times 10^{15}$ cm^{-3}. The thermal equilibrium recombination rate is $R_o = 4 \times 10^4$ cm^{-3} s^{-1}.

1. A uniform generation rate for excess carriers results in an excess carrier recombination rate of $R' = 2 \times 10^{21}$ cm^{-3} s^{-1}. What is the steady-state excess carrier concentration?

2. What in the excess carrier lifetime?

Answer:

$$\begin{aligned} n_o &= N_d - N_a = 8 \times 10^{15} - 2 \times 10^{15} \\ &= 6 \times 10^{15}\ \text{cm}^{-3} \end{aligned}$$

$$p_o = \frac{n_i^2}{n_o} = \frac{(1.8 \times 10^6)^2}{6 \times 10^{15}} = 5.4 \times 10^{-4}\ \text{cm}^{-3}$$

$$R_o = \frac{p_o}{\tau_{p0}} \Rightarrow 4 \times 10^4 = \frac{5.4 \times 10^{-4}}{\tau_{p0}}$$

so

$$\tau_{p0} = 1.35 \times 10^{-8}\ \text{s}$$

(1) $\delta p = g' \tau_{p0} = (2 \times 10^{21})\,(1.35 \times 10^{-8})$

$$= 2.7 \times 10^{13}\ \text{cm}^{-3}$$

(2) $\tau = \tau_{p0} = 1.35 \times 10^{-8}$ s

Exercise 4.6

Consider a bar of n-type silicon that is uniformly doped to a value of $N_\text{d} = 2 \times 10^{16}$ cm^{-3} at $T = 300$ K.

The applied electric field is zero.

A light source is incident on the end of the semiconductor ($x = 0$).

The steady-state concentration of excess carriers generated at $x = 0$ is $\Delta n(0) = \Delta p(0) = 3 \times 10^{14}$ cm^{-3}.

Assume the following parameters: $\mu_\text{n} = 1100\ \text{cm}^2/\text{Vs}, \mu_\text{p} = 500\ \text{cm}^2/\text{Vs}, \tau_\text{n0} = 2 \times 10^{-6}$ s, and $\tau_\text{p0} = 8 \times 10^{-7}$ s.

Neglecting surface effects

(a) Determine the steady-state excess electron and hole concentrations as a function of distance into the semiconductor from the surface ($x = 0$).

(b) Calculate the steady-state hole diffusion current density as a function of distance into the surface from the surface ($x = 0$).

Answer:

(a) It's n-type semiconductor, so we choose τ_{p0} and D_P in (a).

$$\delta n(x) = \delta p(x) = \delta n(0) e^{-\frac{x}{L_p}} = \delta n(0) e^{-\frac{x}{\sqrt{D_p \tau_p}}} = 3 \times 10^{14} e^{\frac{-x}{3.22 \times 10^{-3}}}\ \text{cm}^{-3}$$

(b) $J_p = -eD_p \frac{dp}{dx} = -eD_p \frac{d(\delta p)}{dx} = 0.193 \times e^{-\frac{x}{3.22 \times 10^{-3}}} \text{A/cm}^2$

Reference

1. Neamen, Donald A. Semiconductor physics and devices: basic principles. McGraw-hill, 2003.