# VE320 Intro to Semiconductor Devices
# Summer 2022 — Problem Set 2

June 10, 2022

## Exercise 2.1

Two possible valence bands are shown in the $E$ versus $k$ diagram given in Figure 1. State which band will result in the heavier hole effective mass; state why.

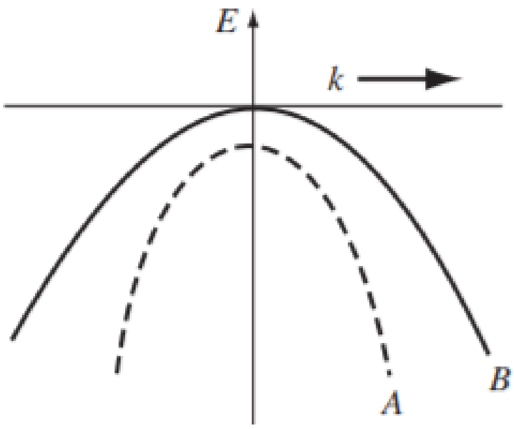


Figure 1: Valence bands for Problem 2.1.

**Answer:**

The effective mass for a hole is given by

$$m_p^* = \left(\frac{1}{\hbar^2} \cdot \left|\frac{d^2 E}{dk^2}\right|\right)^{-1}$$

We have that

$$\left|\frac{d^2 E}{dk^2}\right| (\text{ curve } A) > \left|\frac{d^2 E}{dk^2}\right| (\text{ curve } B)$$

so that $m_p^*(\text{ curve } A) < m_p^*(\text{ curve } B)$

# Exercise 2.2

(a) The forbidden bandgap energy in GaAs is 1.42eV. (i) Determine the minimum frequency of an incident photon that can interact with a valence electron and elevate the electron to the conduction band. (ii) What is the corresponding wavelength?

(b) Repeat part ( $a$ ) for silicon with a bandgap energy of 1.12eV.

**Answer:**

(a) (i)

$$E = hv$$

or

$$\begin{aligned} v = \frac{E}{h} &= \frac{(1.42)\left(1.6 \times 10^{-19}\right)}{6.625 \times 10^{-34}} \\ &= 3.429 \times 10^{14} \text{ Hz} \end{aligned}$$

(ii)

$$\begin{aligned} \lambda = \frac{hc}{E} = \frac{c}{v} &= \frac{3 \times 10^{10}}{3.429 \times 10^{14}} \\ &= 8.75 \times 10^{-5} \text{ cm} = 875 \text{ nm} \end{aligned}$$

(b) (i)

$$v = \frac{E}{h} = \frac{(1.12)\left(1.6 \times 10^{-19}\right)}{6.625 \times 10^{-34}}$$

$$= 2.705 \times 10^{14} \text{ Hz}$$

(ii)

$$\begin{aligned} \lambda = \frac{c}{v} &= \frac{3 \times 10^{10}}{2.705 \times 10^{14}} \\ &= 1.109 \times 10^{-4} \text{ cm} = 1109 \text{ nm} \end{aligned}$$

# Exercise 2.3

The energy-band diagram for silicon is shown in Figure 2. The minimum energy in the conduction band is in the [100] direction. The energy in this one-dimensional direction near the minimum value can be approximated by

$$E = E_0 - E_1 \cos \alpha \left(k - k_0\right)$$

where $k_0$ is the value of $k$ at the minimum energy. Determine the effective mass of the particle at $k = k_0$ in terms of the equation parameters.

**Answer:**

$$E = E_O - E_1 \cos\left[\alpha\left(k - k_O\right)\right]$$

Then

$$\begin{aligned} \frac{dE}{dk} &= \left(-E_1\right)(-\alpha) \sin\left[\alpha\left(k - k_O\right)\right] \\ &= +E_1 \alpha \sin\left[\alpha\left(k - k_O\right)\right] \end{aligned}$$

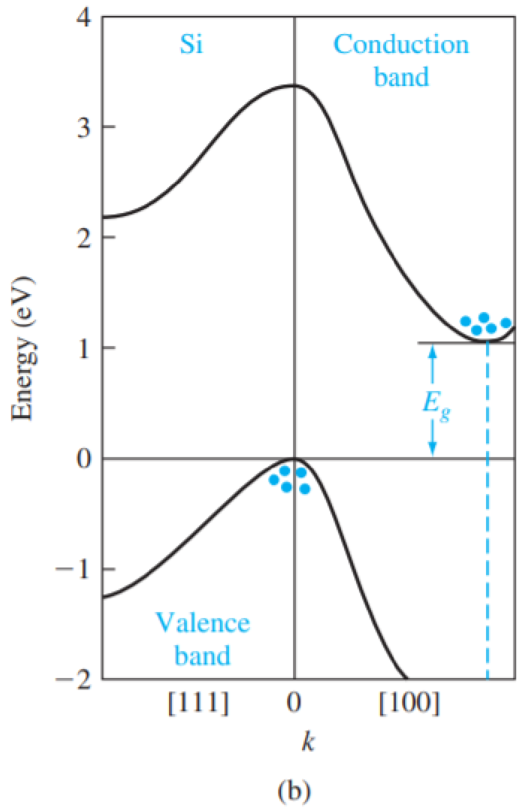


Figure 2: Energy-band structures of Si

and

$$\frac{d^2E}{dk^2} = E_1\alpha^2\cos\left[\alpha\left(k - k_O\right)\right]$$

Then

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\cdot\left.\frac{d^2E}{dk^2}\right|_{k=k_o} = \frac{E_1\alpha^2}{\hbar^2}$$

or

$$m^* = \frac{\hbar^2}{E_1\alpha^2}$$

# Exercise 2.4

(a) Determine the total number ($\#/\mathrm{cm}^3$) of energy states in silicon between $E_v$ and $E_v - 3kT$ at (i) $T = 300$ K and (ii) $T = 400$ K.

(b) Repeat part (a) for GaAs.

**Answer:**

(a) Silicon, $m_p^* = 0.56 m_o$

$$\begin{aligned}
g_v(E) &= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_v - E} \\
g_v &= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\int_{E_v-3kT}^{E_v}\sqrt{E_v - E}\cdot dE \\
&= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left(E_v - E\right)^{3/2}\Big|_{E_v-3kT}^{E_v} \\
&= \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\left(\frac{-2}{3}\right)\left[-(3kT)^{3/2}\right] \\
&= \frac{4\pi\left[2(0.56)\left(9.11\times10^{-31}\right)\right]^{3/2}\left(\frac{2}{3}\right)(3kT)^{3/2}}{\left(6.625\times10^{-34}\right)^3} \\
&= \left(2.969\times10^{55}\right)(3kT)^{3/2}
\end{aligned}$$

(i)At $T = 300\ \mathrm{K}, kT = 4.144\times10^{-21}\ \mathrm{J}$

$$\begin{aligned}
g_v &= \left(2.969\times10^{55}\right)\left[3\left(4.144\times10^{-21}\right)\right]^{3/2} \\
&= 4.116\times10^{25}\ \mathrm{m}^{-3} \\
\text{or } g_v &= 4.12\times10^{19}\ \mathrm{cm}^{-3}
\end{aligned}$$

(ii)

$$\begin{aligned}
&\text{At } T = 400\ \mathrm{K}, kT = 5.5253\times10^{-21}\ \mathrm{J} \\
g_v &= \left(2.969\times10^{55}\right)\left[3\left(5.5253\times10^{-21}\right)\right]^{3/2} \\
&= 6.337\times10^{25}\ \mathrm{m}^{-3} \\
\text{or } g_v &= 6.34\times10^{19}\ \mathrm{cm}^{-3}
\end{aligned}$$

(b) GaAs, $m_p^* = 0.48 m_o$

$$\begin{aligned}
g_v &= \frac{4\pi\left[2(0.48)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\left(\frac{2}{3}\right)(3kT)^{3/2} \\
&= \left(2.3564\times10^{55}\right)(3kT)^{3/2}
\end{aligned}$$

(i) At $T = 300\ \mathrm{K}, kT = 4.144\times10^{-21}\ \mathrm{J}$

$$\begin{aligned}
g_v &= \left(2.3564\times10^{55}\right)\left[3\left(4.144\times10^{-21}\right)\right]^{3/2} \\
&= 3.266\times10^{25}\ \mathrm{m}^{-3} \\
\text{or } g_v &= 3.27\times10^{19}\ \mathrm{cm}^{-3}
\end{aligned}$$

(ii)

$$\begin{aligned}
&\text{At } T = 400\ \mathrm{K}, kT = 5.5253\times10^{-21}\ \mathrm{J} \\
g_v &= \left(2.3564\times10^{55}\right)\left[3\left(5.5253\times10^{-21}\right)\right]^{3/2} \\
&= 5.029\times10^{25}\ \mathrm{m}^{-3} \\
\text{or } g_v &= 5.03\times10^{19}\ \mathrm{cm}^{-3}
\end{aligned}$$

# Exercise 2.5

(a) For silicon, find the ratio of the density of states in the conduction band at $E = E_c + kT$ to the density of states in the valence band at $E = E_v - kT$.

(b) Repeat part (a) for GaAs.

**Answer:**

(a) $\frac{g_c}{g_v} = \frac{(m_n^*)^{3/2}}{(m_p^*)^{3/2}} = \left(\frac{1.08}{0.56}\right)^{3/2} = 2.68$

(b) $\frac{g_c}{g_v} = \frac{(m_n^*)^{3/2}}{(m_p^*)^{3/2}} = \left(\frac{0.067}{0.48}\right)^{3/2} = 0.0521$

# Exercise 2.6

Consider the energy levels shown in Figure 3. Let $T = 300$ K.

(a) If $E_1 - E_F = 0.30$eV, determine the probability that an energy state at $E = E_1$ is occupied by an electron and the probability that an energy state at $E = E_2$ is empty.

(b) Repeat part (a) if $E_F - E_2 = 0.40$eV.

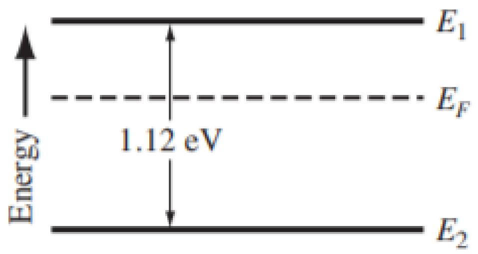


Figure 3: Energy levels for Problem 2.6

**Answer:**

(a) For $E = E_1$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E_1 - E_F}{kT}\right)} \cong \exp\left[\frac{-(E_1 - E_F)}{kT}\right]$$

Then

$$f(E_1) = \exp\left(\frac{-0.30}{0.0259}\right) = 9.32 \times 10^{-6}$$

For $E = E_2, E_F - E_2 = 1.12 - 0.30 = 0.82$eV Then

$$1 - f(E) = 1 - \frac{1}{1 + \exp\left(\frac{-0.82}{0.0259}\right)}$$

or

$$1 - f(E) \cong 1 - \left[1 - \exp\left(\frac{-0.82}{0.0259}\right)\right]$$
$$= \exp\left(\frac{-0.82}{0.0259}\right) = 1.78 \times 10^{-14}$$

(b) For $E_F - E_2 = 0.4\text{eV}$,

$$E_1 - E_F = 0.72\text{eV}$$

At $E = E_1$,

$$f(E) = \exp\left[\frac{-(E_1 - E_F)}{kT}\right] = \exp\left(\frac{-0.72}{0.0259}\right)$$

or

$$f(E) = 8.45 \times 10^{-13}$$

At $E = E_2$,

$$\begin{aligned} 1 - f(E) &= \exp\left[\frac{-(E_F - E_2)}{kT}\right] \\ &= \exp\left(\frac{-0.4}{0.0259}\right) \end{aligned}$$

or

$$1 - f(E) = 1.96 \times 10^{-7}$$

# Exercise 2.7

(a) The carrier effective masses in a semiconductor are $m_n^* = 1.21m_0$ and $m_p^* = 0.70m_0$. Determine the position of the intrinsic Fermi level with respect to the center of the bandgap at $T = 300$ K.

(b) Repeat part (a) if $m_n^* = 0.080m_0$ and $m_p^* = 0.75m_0$.

**Answer:**

(a)

$$\begin{aligned} E_{Fi} - E_{\text{midgap}} &= \frac{3}{4}kT \ln\left(\frac{m_p^*}{m_n^*}\right) \\ &= \frac{3}{4}(0.0259) \ln\left(\frac{0.70}{1.21}\right) \\ &\Rightarrow -10.63\text{meV} \end{aligned}$$

(b)

$$\begin{aligned} E_{Fi} - E_{\text{midgap}} &= \frac{3}{4}(0.0259) \ln\left(\frac{0.75}{0.080}\right) \\ &\Rightarrow +43.47\text{meV} \end{aligned}$$

# Exercise 2.8

Silicon at $T = 300$ K is doped with boron atoms such that the concentration of holes is $p_0 = 5 \times 10^{15}\ \text{cm}^{-3}$.

(a) Find $E_F - E_v$.

(b) Determine $E_c - E_F$.

(c) Determine $n_0$.

(d) Which carrier is the majority carrier?

(e) Determine $E_{Fi} - E_F$.

**Answer:**

(a)

$$\begin{aligned}
E_F - E_v &= kT \ln\left(\frac{N_v}{p_o}\right) \\
&= (0.0259)\ln\left(\frac{1.04\times 10^{19}}{5\times 10^{15}}\right) \\
&= 0.1979\text{eV}
\end{aligned}$$

(b)

$$\begin{aligned}
E_c - E_F &= E_g - (E_F - E_v) \\
&= 1.12 - 0.19788 = 0.92212\text{eV}
\end{aligned}$$

(c)

$$\begin{aligned}
n_o &= \left(2.8\times 10^{19}\right)\exp\left[\frac{-0.92212}{0.0259}\right] \\
&= 9.66\times 10^{3}\ \text{cm}^{-3}
\end{aligned}$$

(d) Holes

(e)

$$\begin{aligned}
E_{Fi} - E_F &= kT \ln\left(\frac{p_o}{n_i}\right) \\
&= (0.0259)\ln\left(\frac{5\times 10^{15}}{1.5\times 10^{10}}\right) \\
&= 0.3294\text{eV}
\end{aligned}$$

# Reference


1. Neamen, Donald A. Semiconductor physics and devices: basic principles. McGraw-hill, 2003.