VE320 Intro to Semiconductor Devices Summer 2022 — Problem Set for Chapter 7

June 28, 2022

Homework description

This is an extra exercise for Chapter 7, and you don't need to hand in.

Exercise 5.1

(a) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine $(i) N_a$, $(ii) N_d$, (iii) x_n, (iv) x_p, and $(v)\ |\mathrm{E}_{\max}|$.

(b) Repeat part (a) for a GaAs pn junction with $V_{bi} = 1.180$ V.

Answer:

$$x_n = 0.25W = 0.25\left(x_n + x_p\right)$$
$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$
$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3$$

So $N_d = 3N_a$

(a) $V_{bi} = (0.0259) \ln\left[\frac{N_a N_d}{(1.5\times10^{10})^2}\right]$

$$0.710 = (0.0259) \ln\left[\frac{3N_a^2}{(1.5\times10^{10})^2}\right]$$

or $3N_a^2 = (1.5\times10^{10})^2 \exp\left(\frac{0.710}{0.0259}\right)$ which yields

$$N_a = 7.766\times10^{15}\ \mathrm{cm}^{-3}$$
$$N_d = 2.33\times10^{16}\ \mathrm{cm}^{-3}$$

$$\begin{aligned} x_n &= \left\{\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2} \\ &= \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.710)}{1.6\times10^{-19}}\right. \\ &\quad \left.\times\left(\frac{1}{3}\right)\left[\frac{1}{4\left(7.766\times10^{15}\right)}\right]\right\}^{1/2} \\ &\Rightarrow x_n = 9.93\times10^{-6}\ \mathrm{cm} \end{aligned}$$

or $x_n = 0.0993\mu\mathrm{m}$

$$
\begin{aligned}
x_p &= \left\{ \frac{2(11.7)\,(8.85\times 10^{-14})\,(0.710)}{1.6\times 10^{-19}} \right. \\
&\quad \left. \times \left(\frac{3}{1}\right)\left[\frac{1}{4\,(7.766\times 10^{15})}\right] \right\}^{1/2} \\
&= 2.979\times 10^{-5}\ \text{cm} \\
&\text{or } x_p = 0.2979\mu\text{m}
\end{aligned}
$$

Now

$$
\begin{aligned}
|\mathrm{E}_{\max}| &= \frac{eN_d x_n}{\epsilon_s} \\
&= \frac{(1.6\times 10^{-19})\,(2.33\times 10^{16})\,(0.0993\times 10^{-4})}{(11.7)\,(8.85\times 10^{-14})} \\
&= 3.58\times 10^{4}\ \text{V/cm}
\end{aligned}
$$

(b)

From part (a), we can write

$$3N_a^2 = \left(1.8\times 10^{6}\right)^2 \exp\left(\frac{1.180}{0.0259}\right)$$

which yields $N_a = 8.127\times 10^{15}\ \text{cm}^{-3}$

$$N_d = 2.438\times 10^{16}\ \text{cm}^{-3}$$

$$
\begin{aligned}
x_n &= \left\{ \frac{2(13.1)\,(8.85\times 10^{-14})\,(1.180)}{1.6\times 10^{-19}} \right. \\
&\quad \left. \times \left(\frac{1}{3}\right)\left[\frac{1}{4\,(8.127\times 10^{15})}\right] \right\}^{1/2} \\
&= 1.324\times 10^{-5}\ \text{cm}
\end{aligned}
$$

or $x_n = 0.1324\mu\text{m}$

$$
\begin{aligned}
x_p &= \left\{ \frac{2(13.1)\,(8.85\times 10^{-14})\,(1.180)}{1.6\times 10^{-19}} \right. \\
&\quad \left. \times \left(\frac{3}{1}\right)\left[\frac{1}{4\,(8.127\times 10^{15})}\right] \right\}^{1/2} \\
&= 3.973\times 10^{-5}\ \text{cm}
\end{aligned}
$$

or $x_p = 0.3973\mu\text{m}$

$$
\begin{aligned}
|\mathrm{E}_{\max}| &= \frac{eN_d x_n}{\epsilon_s} \\
&= \frac{(1.6\times 10^{-19})\,(2.438\times 10^{16})\,(0.1324\times 10^{-4})}{(13.1)\,(8.85\times 10^{-14})} \\
&= 4.45\times 10^{4}\ \text{V/cm}
\end{aligned}
$$

Exercise 5.2

An "isotype" step junction is one in which the same impurity type doping changes from one concentration value to another value. An n-n isotype doping profile is shown in Figure 1.

(a) Sketch the thermal equilibrium energy-band diagram of the isotype junction.
(b) Using the energy-band diagram, determine the built-in potential barrier.
(c) Discuss the charge distribution through the junction.

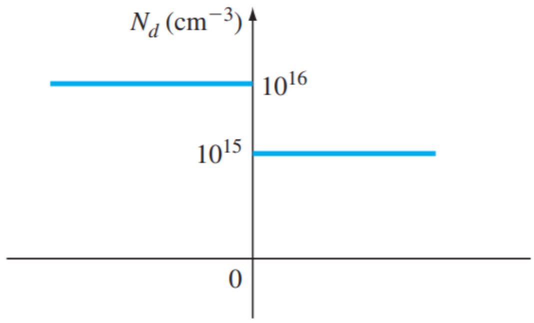

Figure 1: Figure for Problem 5.2

Answer:
(a)

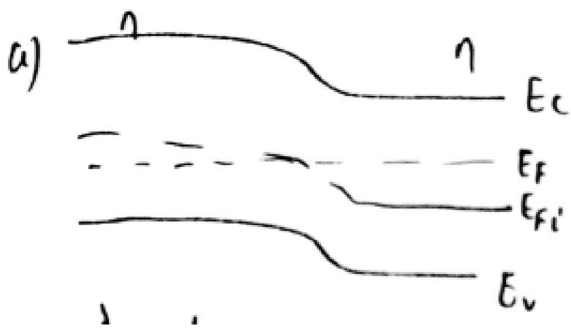

Figure 2: Figure for Problem 5.2

(b) For $N_d = 10^{16}$ cm^{-3}, $E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$

$$= (0.0259) \ln\left(\frac{10^{16}}{1.5 \times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.3473\text{eV}$$

For $N_d = 10^{15}\ \text{cm}^{-3}$

$$E_F - E_{Fi} = (0.0259)\ln\left(\frac{10^{15}}{1.5\times 10^{10}}\right)$$

or

$$E_F - E_{Fi} = 0.2877\text{eV}$$

Then

$$V_{bi} = 0.34732 - 0.28768$$

or

$$V_{bi} = 0.0596\ \text{V}$$

(c) Since $\phi(x) = \frac{eN_d}{\epsilon_x}\left(x_n \cdot x - \frac{x^2}{2}\right) + \frac{e_a}{2\epsilon_s}x_p^2(0 \sim x_n\right)$ The electric field is consecutive - $n - n$ negative charge and positive charge is the same

Exercise 5.3

An ideal one-sided silicon p^+n junction at $T = 300$ K is uniformly doped on both sides of the metallurgical junction. It is found that the doping relation is $N_a = 80N_d$ and the built-in potential barrier is $V_{bi} = 0.740$ V. A reverse-biased voltage of $V_R = 10$ V is applied. Determine

(a) N_a, N_d ;

(b) x_p, x_n ;

(c)$|\mathrm{E}_{\max}|$;

(d) C_j'.

Answer:

(a) $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= V_t \ln\left(\frac{80N_d^2}{n_i^2}\right)$$

We find

$$80N_d^2 = n_i^2 \exp\left(\frac{V_{bi}}{V_t}\right)$$

$$= \left(1.5\times 10^{10}\right)^2 \exp\left(\frac{0.740}{0.0259}\right)$$

$$= 5.762\times 10^{32}$$

$$\Rightarrow N_d = 2.684\times 10^{15}\ \text{cm}^{-3}$$

$$N_a = 2.147\times 10^{17}\ \text{cm}^{-3}$$

(b)

$$x_n = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_a}{N_d}\right) \left(\frac{1}{N_a + N_d}\right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.740 + 10)}{1.6 \times 10^{-19}} \right) \Bigg\}^{1/2}$$

$$\times \left(\frac{80}{1}\right) \left(\frac{1}{2.147 \times 10^{17} + 2.684 \times 10^{15}}\right) \Bigg\}^{1/2}$$

$$= 2.262 \times 10^{-4} \text{ cm}$$

or $x_n = 2.262 \mu$m

$$x_p = \left\{ \frac{2 \in_s (V_{bi} + V_R)}{e} \left(\frac{N_d}{N_a}\right) \left(\frac{1}{N_a + N_d}\right) \right\}^{1/2}$$

$$= \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.740 + 10)}{1.6 \times 10^{-19}} \right) \Bigg\}^{1/2}$$

$$\times \left(\frac{1}{80}\right) \left(\frac{1}{2.147 \times 10^{17} + 2.684 \times 10^{15}}\right)$$

$$= 2.83 \times 10^{-6} \text{ cm}$$

or $x_p = 0.0283 \mu$m

(c) $$|\mathrm{E}_{\max}| = \frac{2(V_{bi} + V_R)}{W}$$

$$= \frac{2(0.740 + 10)}{(2.262 + 0.0283) \times 10^{-4}}$$

$$= 9.38 \times 10^4 \text{ V/cm}$$

(d) $$C' = \left\{ \frac{e \in_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}$$

$$= \left\{ \frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})}{2(0.740 + 10)} \right.$$

$$\left. \times \left[\frac{(2.147 \times 10^{17})(2.684 \times 10^{15})}{2.147 \times 10^{17} + 2.684 \times 10^{15}} \right] \right\}^{1/2}$$

$$C' = 4.52 \times 10^{-9} \text{ F/cm}^2$$

Exercise 5.4

A silicon p$^+$n junction has doping concentrations of $N_a = 2 \times 10^{17}$ cm^{-3} and $N_d = 2 \times 10^{15}$ cm^{-3}. The cross-sectional area is 10^{-5} cm^2. Calculate

(a) V_{bi}

(b) the junction capacitance at (i) $V_R = 1$ V, (ii) $V_R = 3$ V, and (iii) $V_R = 5$ V.

(c) Plot $1/C^2$ versus V_R and show that the slope can be used to find N_d and the intercept at the voltage axis yields V_{bi}.

Answer:

(a) $V_{bi} = (0.0259) \ln \left[\frac{(2 \times 10^{17})(2 \times 10^{15})}{(1.5 \times 10^{10})^2} \right] = 0.7305$ V

(b)

$$C = AC' \cong A \cdot \left[\frac{e \in_s N_d}{2\left(V_{bi}+V_R\right)}\right]^{1/2}$$

$$= \left(10^{-5}\right)) \left\{\frac{\left(1.6 \times 10^{-19}\right)}{2\left(V_{bi}+V_R\right)}\right.$$

$$\left. \times (11.7)\left(8.85 \times 10^{-14}\right)\left(2 \times 10^{15}\right)\right\}^{1/2}$$

$$C = \frac{1.287 \times 10^{-13}}{\sqrt{V_{bi}+V_R}}$$

(i) For $V_R = 1$ V, $C = 9.783 \times 10^{-14}$ F
(ii) For $V_R = 3$ V, $C = 6.663 \times 10^{-14}$ F
(iii) For $V_R = 5$ V, $C = 5.376 \times 10^{-14}$ F

Exercise 5.5

A silicon pn junction at $T = 300$ K has the doping profile shown in Figure 2. Calculate
(a) V_{bi},
(b) x_n and x_p at zero bias, and
(c) the applied bias required so that $x_n = 30\mu$m

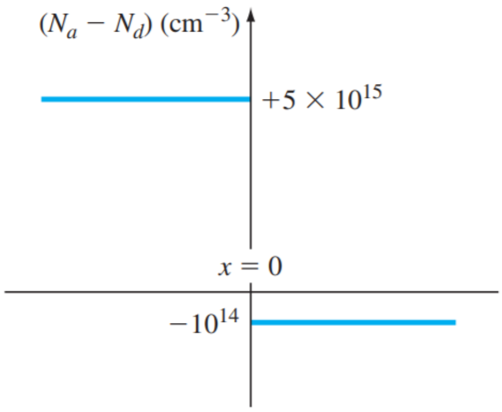

Figure 3: Figure for Problem 5.5

Answer:
(a) $V_{bi} = (0.0259) \ln \left[\frac{\left(5 \times 10^{15}\right)\left(10^{14}\right)}{\left(1.5 \times 10^{10}\right)^2}\right]$ or

$$V_{bi} = 0.5574 \text{ V}$$

(b)

$$x_p = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \left[\frac{2(11.7)\left(8.85 \times 10^{-14}\right)(0.5574)}{1.6 \times 10^{-19}}\right.$$

$$\times \left(\frac{10^{14}}{5 \times 10^{15}}\right)\left(\frac{1}{10^{14}+5 \times 10^{15}}\right)\Bigg]^{1/2}$$

or

$$x_p = 5.32 \times 10^{-6} \text{ cm}$$

Also

$$x_n = \left[\frac{2 \in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}$$

$$= \Bigg[\frac{2(11.7)\,(8.85 \times 10^{-14})\,(0.5574)}{1.6 \times 10^{-19}}$$

$$\times \left(\frac{5 \times 10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5 \times 10^{15}}\right)\Bigg]^{1/2}$$

or

$$x_n = 2.66 \times 10^{-4} \text{ cm}$$

(c) For $x_n = 30\mu$m, we have

$$30 \times 10^{-4} = \Bigg[\frac{2(11.7)\,(8.85 \times 10^{-14})\,(V_{bi}+V_R)}{1.6 \times 10^{-19}}$$

$$\times \left(\frac{5 \times 10^{15}}{10^{14}}\right)\left(\frac{1}{10^{14}+5 \times 10^{15}}\right)\Bigg]^{1/2}$$

which becomes

$$9 \times 10^{-6} = 1.269 \times 10^{-7}\,(V_{bi}+V_R)$$

We find

$$V_R = 70.4 \text{ V}$$

Exercise 5.6

Consider a silicon pn junction with the doping profile shown in Figure 3. $T = 300$ K.

(a) Calculate the applied reverse-biased voltage required so that the space charge region extends entirely through the p region.

(b) Determine the space charge width into the n$^+$region with the reverse-biased voltage calculated in part (a).

(c) Calculate the peak electric field for this applied voltage.

Answer:

An n^+p junction with $N_a = 10^{14}$ cm^{-3}, (a) A one-sided junction and assume $V_R \gg> V_{bi}$. Then

$$x_p \cong \left[\frac{2\epsilon_s V_R}{eN_a}\right]^{1/2}$$

or

$$\left(50 \times 10^{-4}\right)^2 = \frac{2(11.7)\,(8.85 \times 10^{-14})\,V_R}{(1.6 \times 10^{-19})\,(10^{14})}$$

which yields

$$V_R = 193 \text{ V}$$

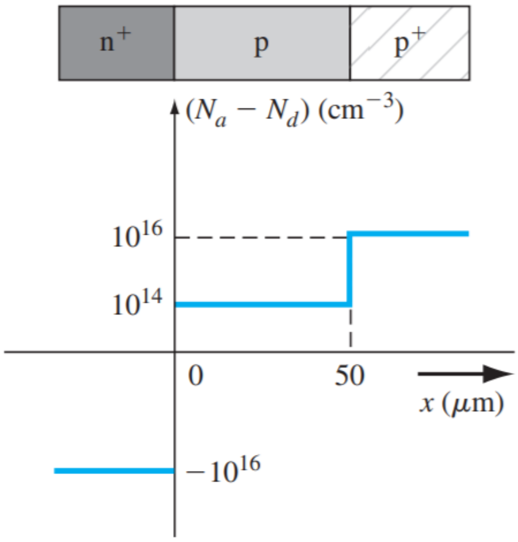

Figure 4: Figure for Problem 5.6

(b)

$$\frac{x_p}{x_n} = \frac{N_d}{N_a} \Rightarrow x_n = x_p \left(\frac{N_a}{N_d}\right)$$

So

$$\begin{aligned} x_n &= \left(50 \times 10^{-4}\right)\left(\frac{10^{14}}{10^{16}}\right) \\ &= 0.50 \times 10^{-4} \text{ cm} = 0.50 \mu\text{m} \end{aligned}$$

(c)

$$|\text{E}_{\text{max}}| \cong \frac{2V_R}{W} = \frac{2(193.15)}{50.5 \times 10^{-4}}$$

or

$$|E_{\text{max}}| = 7.65 \times 10^4 \text{ V/cm}$$

Exercise 5.7

Consider a silicon n$^+$p junction diode. The critical electric field for breakdown in silicon is approximately $\text{E}_{\text{crit}} = 4 \times 10^5$ V/cm. Determine the maximum p-type doping concentration such that the breakdown voltage is

(a) 40 V and

(b) 20 V.

Answer:

(a) $V_B = \frac{\epsilon_s \text{E}_{\text{crit}}^2}{2eN_B}$ or

$$N_B = \frac{\epsilon_s \text{ E}_{\text{crit}}^2}{2eV_B} = \frac{(11.7)\left(8.85 \times 10^{-14}\right)\left(4 \times 10^5\right)^2}{2\left(1.6 \times 10^{-19}\right)(40)}$$

Then $N_B = N_a = 1.294 \times 10^{16}$ cm^{-3}

(b) $N_B = \frac{(11.7)\left(8.85\times10^{-14}\right)\left(4\times10^{5}\right)^{2}}{2(1.6\times10^{-19})(20)}$

Or $N_B = N_a = 2.59 \times 10^{16}\ \mathrm{cm}^{-3}$

Reference

1. Neamen, Donald A. Semiconductor physics and devices: basic principles. McGraw-hill, 2003.