VE320
Intro to Semiconductor Devices
MID2 RC

Yucheng Huang

University of Michigan
Shanghai Jiao Tong University
Joint Institute

July 8, 2022

1. The pn Junction
   - Reverse Applied Bias

2. The pn Junction Diode
   - pn Junction Current
   - Generation–Recombination Currents
   - High-Level Injection

Reverse Applied Bias

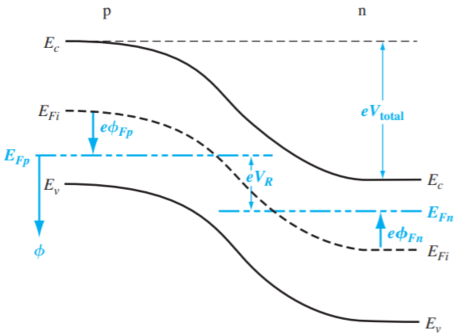

Figure: Energy-band diagram of a pn junction under reverse bias

Reverse Applied Bias

If we apply a potential between the p and n regions, we will no longer be in an equilibrium condition—the Fermi energy level will no longer be constant through the system. Figure shows the energy-band diagram of the pn junction for the case when a positive voltage is applied to the n region with respect to the p region. As the positive potential is downward, the Fermi level on the n side is below the Fermi level on the p side. The difference between the two is equal to the applied voltage in units of energy.

Reverse Applied Bias

The total potential barrier, indicated by V_{total} , has increased. The applied potential is the reverse-biased condition. The total potential barrier is now given by

$$V_{\text{total}} = |\phi_{rn}| + |\phi_{rp}| + V_R$$

where V_R is the magnitude of the applied reverse-biased voltage. Equation can be rewritten as

$$V_{\text{total}} = V_{bi} + V_R$$

where V_{bi} is the same built-in potential barrier we had defined in thermal equilibrium.

Space Charge Width and Electric Field

In all of the previous equations, the built-in potential barrier can be replaced by the total potential barrier. The total space charge width can be written from Equation as

$$W = \left\{ \frac{2\epsilon_s \left(V_{bi} + V_R\right)}{e} \left[\frac{N_a + N_d}{N_a N_d}\right] \right\}^{1/2}$$

$$\mathrm{E}_{\mathsf{max}} = -\left\{ \frac{2e \left(V_{bi} + V_R\right)}{\epsilon_s} \left(\frac{N_a N_d}{N_a + N_d}\right) \right\}^{1/2}$$

We can show that the maximum electric field in the pn junction can also be written as

$$\mathrm{E}_{\mathsf{max}} = \frac{-2\left(V_{bi} + V_R\right)}{W}$$

where W is the total space charge width.

Example 1

The maximum electric field in a reverse-biased GaAs pn junction at $T = 300$ K is to be limited to $|\mathrm{E}_{\mathsf{max}}| = 7.2 \times 10^4$ V/cm. The doping concentrations are $N_d = 5\times\ 10^{15}\ \mathrm{cm}^{-3}$ and $N_a = 3 \times 10^{16}\ \mathrm{cm}^{-3}$. Determine the maximum reverse-biased voltage that can be applied.

Example 1 Solution

$$
\begin{aligned}
V_{bi} &= V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) \\
&= (0.0259) \ln\left[\frac{\left(5\times 10^{15}\right)\left(3\times 10^{16}\right)}{\left(1.8\times 10^{6}\right)^2}\right] \\
&= 1.173 \text{ V} \\
|\mathrm{E}_{\max}| &= \left\{\frac{2e\left(V_{bi}+V_R\right)}{\epsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right\}^{1/2}
\end{aligned}
$$

Example 1 Solution

$$\text{Now}\ \left(7.2\times 10^{4}\right)^{2} = \left\{ \frac{2\left(1.6\times 10^{-19}\right)\left(V_{bi}+V_{R}\right)}{(13.1)\left(8.85\times 10^{-14}\right)} \right.$$

$$\left. \times \left[\frac{\left(5\times 10^{15}\right)\left(3\times 10^{16}\right)}{5\times 10^{15}+3\times 10^{16}} \right] \right\}$$

$$5.184\times 10^{9} = 1.1829\times 10^{9}\left(V_{bi}+V_{R}\right)$$

$$V_{bi}+V_{R} = 1.173 + V_{R} = 4.382$$

Junction Capacitance

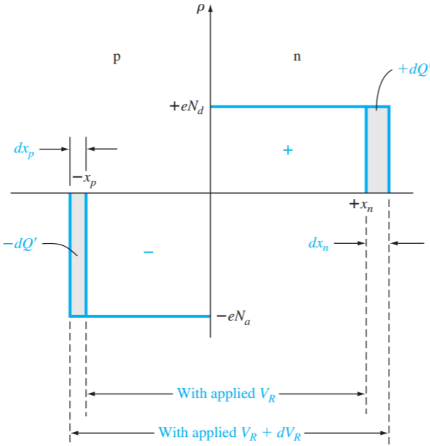

Figure: Differential change in the space charge width with a differential change in reverse-biased voltage for a uniformly doped pn junction

Junction Capacitance

The junction capacitance is defined as

$$C' = \frac{dQ'}{dV_R}$$

where

$$dQ' = eN_d dx_n = eN_a dx_p$$

The differential charge dQ' is in units of $\mathrm{C/cm^2}$ so that the capacitance C' is in units of farads per square centimeter $\mathrm{F/cm^2}$), or capacitance per unit area. For the total potential barrier, Equation may be written as

$$x_n = \left\{ \frac{2\epsilon_s \left(V_{bi} + V_R\right)}{e} \left[\frac{N_a}{N_d}\right] \left[\frac{1}{N_a + N_d}\right] \right\}^{1/2}$$

The junction capacitance can be written as

$$C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}$$

Junction Capacitance

so that

$$C' = \left\{ \frac{e\epsilon_s N_a N_d}{2\left(V_{bi} + V_R\right)\left(N_a + N_d\right)} \right\}^{1/2}$$

Exactly the same capacitance expression is obtained by considering the space charge region extending into the p region x_p. The junction capacitance is also referred to as the depletion layer capacitance.

One-Sided Junctions

Consider a special pn junction called the one-sided junction. If, for example, $N_a \gg N_d$, this junction is referred to as a $\mathrm{p^+n}$ junction. The total space charge width, from Equation, reduces to

$$W \approx \left\{ \frac{2\epsilon_s \left(V_{bi} + V_R\right)}{eN_d} \right\}^{1/2}$$

Considering the expressions for x_n and x_p, we have for the $\mathrm{p^+n}$ junction

$$x_p \ll x_n$$

$$W \approx x_n$$

Almost the entire space charge layer extends into the low-doped region of the junction. This effect can be seen in Figure. The junction capacitance of the $\mathrm{p^+n}$ junction reduces to

$$C' \approx \left\{ \frac{e\epsilon_s N_d}{2\left(V_{bi} + V_R\right)} \right\}^{1/2}$$

One-Sided Junctions

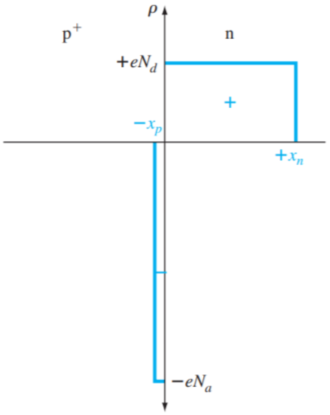

Figure: Space charge density of a one-sided pn junction

One-Sided Junctions

The depletion layer capacitance of a one-sided junction is a function of the doping concentration in the low-doped region. Equation may be manipulated to give

$$\left(\frac{1}{C'}\right)^2 = \frac{2\left(V_{bi} + V_R\right)}{e\epsilon_s N_d}$$

which shows that the inverse capacitance squared is a linear function of applied reverse-biased voltage.

One-Sided Junctions

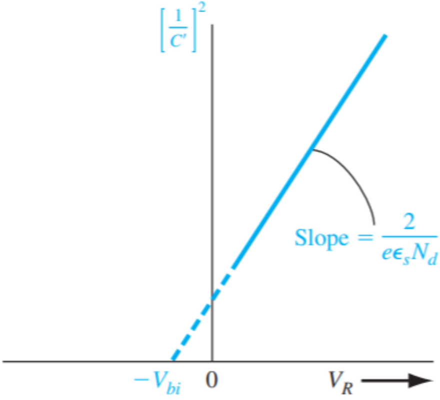

Figure: $(1/C')^2$ versus V_R of a uniformly doped pn junction.

Example 2

The experimentally measured junction capacitance of a one-sided silicon $\mathrm{n^+p}$ junction biased at $V_R = 3\ \mathrm{V}$ and at $T = 300\ \mathrm{K}$ is $C = 0.105\mathrm{pF}$. The built-in potential barrier is found to be $V_{bi} = 0.765\ \mathrm{V}$. The cross-sectional area is $A = 10^{-5}\ \mathrm{cm}^2$. Find the doping concentrations.

Example 2 Solution

For a one-sided junction

$$C' = \left\{ \frac{e \in_s N_a}{2\left(V_{bi} + V_R\right)} \right\}^{1/2}$$

$$C = A \cdot C' = \left(10^{-5}\right) C'$$

$$0.105 \times 10^{-12}$$

$$= \left(10^{-5}\right) \left\{ \frac{\left(1.6 \times 10^{-19}\right)(11.7)\left(8.85 \times 10^{-14}\right) N_a}{2(3 + 0.765)} \right\}^{1/2}$$

$$\left(0.105 \times 10^{-12}\right)^2 = \left(10^{-5}\right)^2 \left(2.20 \times 10^{-32}\right) N_a$$

So $N_a = 5.01 \times 10^{15}\ \mathrm{cm}^{-3}$

Example 2 Solution

We have $V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$ Then

$$N_d = \frac{n_i^2}{N_a}\exp\left(\frac{V_{bi}}{V_t}\right)$$

$$= \frac{\left(1.5\times 10^{10}\right)^2}{5.01\times 10^{15}}\exp\left(\frac{0.765}{0.0259}\right)$$

$$N_d = 3.02\times 10^{17}\ \mathrm{cm}^{-3}$$

pn Junction Current

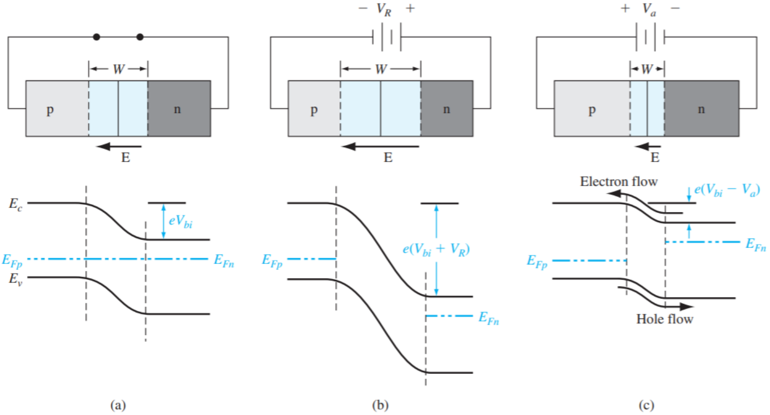

Figure: A pn junction and its associated energy-band diagram for (a) zero bias, (b) reverse bias, and (c) forward bias

pn Junction Current

The ideal current-voltage relationship of a pn junction is derived on the basis of four assumptions. (The last assumption has three parts, but each part deals with current.) They are:

1. The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.

2. The Maxwell-Boltzmann approximation applies to carrier statistics.

3. The concepts of low injection and complete ionization apply.

4a. The total current is a constant throughout the entire pn structure.

4b. The individual electron and hole currents are continuous functions through the pn structure.

4c. The individual electron and hole currents are constant throughout the depletion region.

Boundary Conditions

An expression for the built-in potential barrier was derived in the last chapter and was given by Equation (7.10) as

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

If we divide the equation by $V_t = kT/e$, take the exponential of both sides, and then take the reciprocal, we obtain

$$\frac{n_i^2}{N_a N_d} = \exp\left(\frac{-eV_{bi}}{kT}\right)$$

Boundary Conditions

If we assume complete ionization, we can write

$$n_{n0} \approx N_d$$

where n_{n0} is the thermal-equilibrium concentration of majority carrier electrons in the n region. In the p region, we can write

$$n_{p0} \approx \frac{n_i^2}{N_a}$$

where n_{p0} is the thermal-equilibrium concentration of minority carrier electrons. Substituting Equations into Equation 1, we obtain

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right)$$

Boundary Conditions

The potential barrier V_{bi} in Equation can be replaced by $(V_{bi} - V_a)$ when the junction is forward biased. Equation becomes

$$n_p = n_{n0} \exp\left(\frac{-e\left(V_{bi} - V_a\right)}{kT}\right) = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \exp\left(\frac{+eV_a}{kT}\right)$$

If we assume low injection, the majority carrier electron concentration n_{n0}, for example, does not change significantly. However, the minority carrier concentration, n_p, can deviate from its thermal-equilibrium value n_{p0} by orders of magnitude. Using Equation, we can write Equation as

$$n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right)$$

Boundary Conditions

Exactly the same process occurs for majority carrier holes in the p region, which are injected across the space charge region into the *n* region under a forward-bias voltage. We can write that

$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

Example 1

Consider a silicon pn junction at $T = 300\ \mathrm{K}$. Assume the doping concentration in the n region is $N_d = 10^{16}\ \mathrm{cm}^{-3}$ and the doping concentration in the p region is $\mathrm{N}_a = 6 \times 10^{15}\ \mathrm{cm}^{-3}$, and assume that a forward bias of $0.60\ \mathrm{V}$ is applied to the pn junction. Calculate the minority carrier concentrations at the edge of the space charge regions in a forward-biased pn junction.

Example 1 Solution

$$n_p(-x_p) = n_{po}\exp\left(\frac{eV_a}{kT}\right) \quad \text{and} \quad p_n(x_n) = p_{no}\exp\left(\frac{eV_a}{kT}\right)$$

The thermal-equilibrium minority carrier concentrations are

$$n_{po} = \frac{n_i^2}{N_a} = \frac{\left(1.5\times 10^{10}\right)^2}{6\times 10^{15}} = 3.75\times 10^4\ \mathrm{cm}^{-3}$$

$$p_{no} = \frac{n_i^2}{N_d} = \frac{\left(1.5\times 10^{10}\right)^2}{10^{16}} = 2.25\times 10^4\ \mathrm{cm}^{-3}$$

We then have

$$n_p(-x_p) = 3.75\times 10^4\exp\left(\frac{0.60}{0.0259}\right) = 4.31\times 10^{14}\ \mathrm{cm}^{-3}$$

$$p_n(x_n) = 2.25\times 10^4\exp\left(\frac{0.60}{0.0259}\right) = 2.59\times 10^{14}\ \mathrm{cm}^{-3}$$

Minority Carrier Distribution

We developed, in Chapter 6, the ambipolar transport equation for excess minority carrier holes in an n region. This equation, in one dimension, is

$$D_p \frac{\partial^2 (\delta p_n)}{\partial x^2} - \mu_p \mathrm{E} \frac{\partial (\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial (\delta p_n)}{\partial t}$$

In the n region for $x > x_n$, we have that $\mathrm{E} = 0$ and $g' = 0$. If we also assume steady state so $\partial (\delta p_n) / \partial t = 0$, then Equation reduces to

$$\frac{d^2 (\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \quad (x > x_n)$$

where $L_p^2 = D_p \tau_{p0}$.

Minority Carrier Distribution

For the same set of conditions, the excess minority carrier electron concentration in the p region is determined from

$$\frac{d^2\left(\delta n_p\right)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \quad (x < x_p)$$

where $L_n^2 = D_n \tau_{n0}$. The general solution to Equation 1 is

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \quad (x \geq x_n)$$

and the general solution to Equation 2 is

$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \quad (x \leq -x_p)$$

Minority Carrier Distribution

The excess carrier concentrations are then found to be, for $(x \geq x_n)$,

$$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right)$$

and, for $(x \leq -x_p)$,

$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right)$$

Minority Carrier Distribution

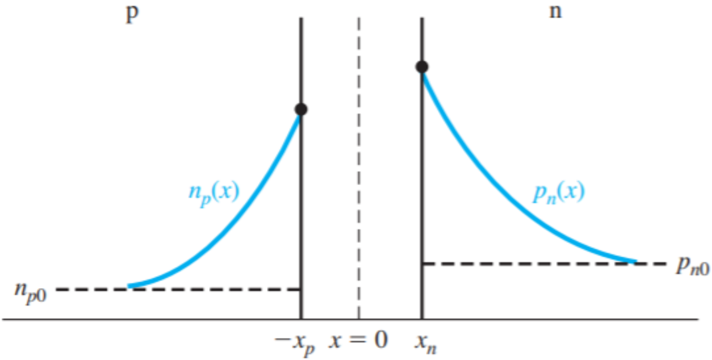

Figure: Steady-state minority carrier concentrations in a pn junction under forward bias.

Minority Carrier Distribution

In Chapter 6, we discussed the concept of quasi-Fermi levels, which apply to excess carriers in a nonequilibium condition. Since excess electrons exist in the neutral p region and excess holes exist in the neutral n region, we can apply quasi-Fermi levels to these regions. We had defined quasi-Fermi levels in terms of carrier concentrations as

$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

and

$$n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

Combining them, we can write

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right)$$

Minority Carrier Distribution

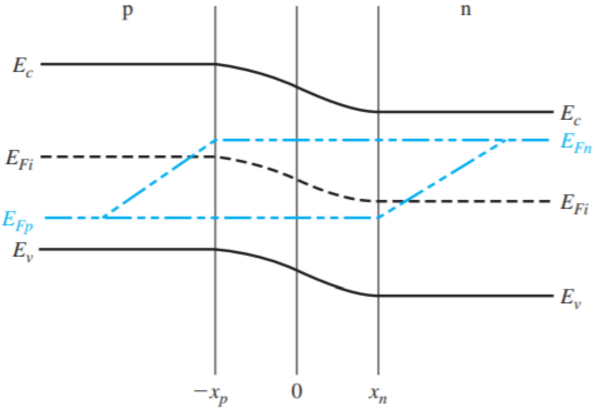

Figure: Quasi-Fermi levels through a forward-biased pn junction.

Ideal pn Junction Current

$$J_p\left(x_n\right) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

$$J_n\left(-x_p\right) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

The total current density in the pn junction is then

$$J = J_p\left(x_n\right) + J_n\left(-x_p\right) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Ideal pn Junction Current

We may define a parameter J_s as

$$J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]$$

so that Equation may be written as

$$J = J_s \left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Example 2

Consider a GaAs pn junction diode at $T = 300\ \mathrm{K}$. The parameters of the device are $N_d = 2 \times 10^{16}\ \mathrm{cm}^{-3}, N_a = 8 \times 10^{15}\ \mathrm{cm}^{-3}, D_n = 210\ \mathrm{cm}^2/\mathrm{s}, D_p = 8\ \mathrm{cm}^2/\mathrm{s}, \tau_{no} = 10^{-7}\ \mathrm{s}$, and $\tau_{po} = 5 \times 10^{-8}\ \mathrm{s}$. Determine the ideal reverse-saturation current density.

Example 2 Solution

The ideal reverse-saturation current density is given by

$$J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$$

which may be rewritten as

$$\begin{aligned} J_s &= e n_i^2 \left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}} \right] \\ &= \left(1.6 \times 10^{-19}\right)\left(1.8 \times 10^{6}\right)^2 \\ &\times \left[\frac{1}{8 \times 10^{15}}\sqrt{\frac{210}{10^{-7}}} + \frac{1}{2 \times 10^{16}}\sqrt{\frac{8}{5 \times 10^{-8}}} \right] \\ J_s &= 3.30 \times 10^{-18}\ \mathrm{A/cm^2} \end{aligned}$$

Ideal I–V characteristic

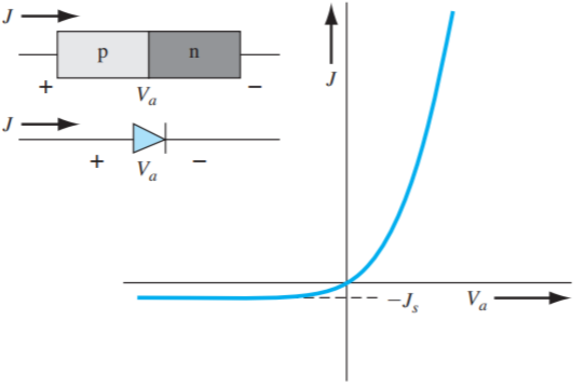

Figure: Ideal I–V characteristic of a pn junction diode

Reverse-Biased Generation Current

$$R = \frac{-n_i}{\tau_{p0} + \tau_{n0}}$$

If we define a new lifetime as the average of τ_{p0} and τ_{n0}, or

$$\tau_0 = \frac{\tau_{p0} + \tau_{n0}}{2}$$

then the recombination rate can be written as

$$R = \frac{-n_i}{2\tau_0} \equiv -G$$

Reverse-Biased Generation Current

The negative recombination rate implies a generation rate, so G is the generation rate of electrons and holes in the space charge region. The generation current density may be determined from

$$J_{\text{gen}} = \int_0^W eG dx$$

where the integral is over the space charge region. If we assume that the generation rate is constant throughout the space charge region, then we obtain

$$J_{\text{gen}} = \frac{en_i W}{2\tau_0}$$

Forward-Bias Recombination Current

The recombination current density may be calculated from

$$J_{\mathrm{rec}} = \int_0^W eRdx$$

where again the integral is over the entire space charge region. In this case, however, the recombination rate is not a constant through the space charge region. We have calculated the maximum recombination rate at the center of the space charge region, so we may write

$$J_{\mathrm{rec}} = ex' \frac{n_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right)$$

where x' is a length over which the maximum recombination rate is effective. However, since τ_0 may not be a well-defined or known parameter, it is customary to write

$$J_{\mathrm{rec}} = \frac{eWn_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right) = J_{r0} \exp\left(\frac{eV_a}{2kT}\right)$$

Total Forward-Bias Current

In general, the diode current-voltage relationship may be written as

$$I = I_s \left[\exp\left(\frac{eV_a}{nkT} \right) - 1 \right]$$

where the parameter n is called the ideality factor. For a large forward-bias voltage, $n \approx 1$ when diffusion dominates, and for low forward-bias voltage, $n \approx 2$ when recombination dominates. There is a transition region where $1 < n < 2$.

Example 3

Consider a silicon pn junction diode at $T = 300\ \mathrm{K}$ with parameters $N_a = 2\times 10^{15}\ \mathrm{cm}^{-3}$, $N_d = 8\times 10^{16}\ \mathrm{cm}^{-3}$, $D_p = 10\ \mathrm{cm}^2/\mathrm{s}$, $D_n = 25\ \mathrm{cm}^2/\mathrm{s}$, and $\tau_0 = \tau_{p0} = \tau_{n0} = 10^{-7}\ \mathrm{s}$. The diode is forward biased at $V_a = 0.35\ \mathrm{V}$.

(a) Calculate the ideal diode current density.

(b) Find the forward-biased recombination current density.

(c) Determine the ratio of recombination current to the ideal diffusion current.

Example 3 Solution

(a)

$$J \cong J_s \exp\left(\frac{V_a}{V_t}\right)$$

$$J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{po}}}\right]$$

$$= \left(1.6 \times 10^{-19}\right)\left(1.5 \times 10^{10}\right)^2$$

$$\times \left[\frac{1}{2 \times 10^{15}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{8 \times 10^{16}}\sqrt{\frac{10}{10^{-7}}}\right]$$

$$J_s = 2.891 \times 10^{-10}\ \mathrm{A/cm^2}$$

$$\text{Then } J \cong \left(2.891 \times 10^{-10}\right)\exp\left(\frac{0.35}{0.0259}\right)$$

$$= 2.137 \times 10^{-4}\ \mathrm{A/cm^2}$$

Example 3 Solution

(b) $V_{bi} = (0.0259)\ln\left[\frac{(2\times10^{15})(8\times10^{16})}{(1.5\times10^{10})^2}\right]$

$$= 0.7068 \text{ V}$$

We find

$$W = \left\{\frac{2(11.7)\left(8.85\times10^{-14}\right)(0.7068-0.35)}{1.6\times10^{-19}} \times \left[\frac{2\times10^{15}+8\times10^{16}}{(2\times10^{15})(8\times10^{16})}\right]\right\}^{1/2}$$

$$= 4.865\times10^{-5} \text{ cm}$$

Example 3 Solution

Then

$$J_{rec} = \frac{en_i W}{2\tau_o} \exp\left(\frac{V_a}{2V_t}\right)$$

$$= \frac{\left(1.6 \times 10^{-19}\right)\left(1.5 \times 10^{10}\right)\left(4.865 \times 10^{-5}\right)}{2\left(10^{-7}\right)} \times \exp\left[\frac{0.35}{2(0.0259)}\right]$$

$$J_{rec} = 5.020 \times 10^{-4}\ \mathrm{A/cm^2}$$

(c)

$$\frac{J_{\text{rec}}}{J} = \frac{5.020 \times 10^{-4}}{2.137 \times 10^{-4}} = 2.35$$

High-Level Injection

As the forward-bias voltage increases, the excess carrier concentrations increase and may become comparable or even greater than the majority carrier concentration.

$$np = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

Under high-level injection, we may have $\delta n > n_o$ and $\delta p > p_o$ so that Equation becomes approximately

$$(\delta n)(\delta p) \cong n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

$$\delta n = \delta p \cong n_i \exp\left(\frac{V_a}{2V_t}\right)$$

The diode current is proportional to the excess carrier concentration so that, under high-level injection, we have

$$I \propto \exp\left(\frac{V_a}{2V_t}\right)$$

High-Level Injection

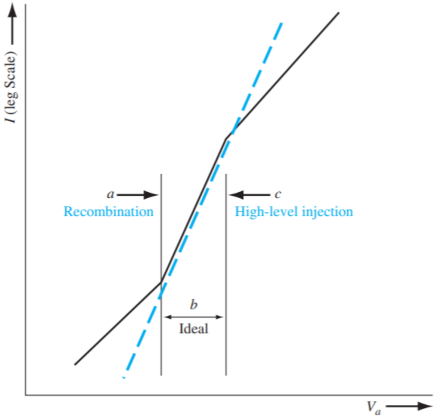

Figure: Forward-bias current versus voltage from low forward bias to high forward bias

END

Thanks