VE320 RC3

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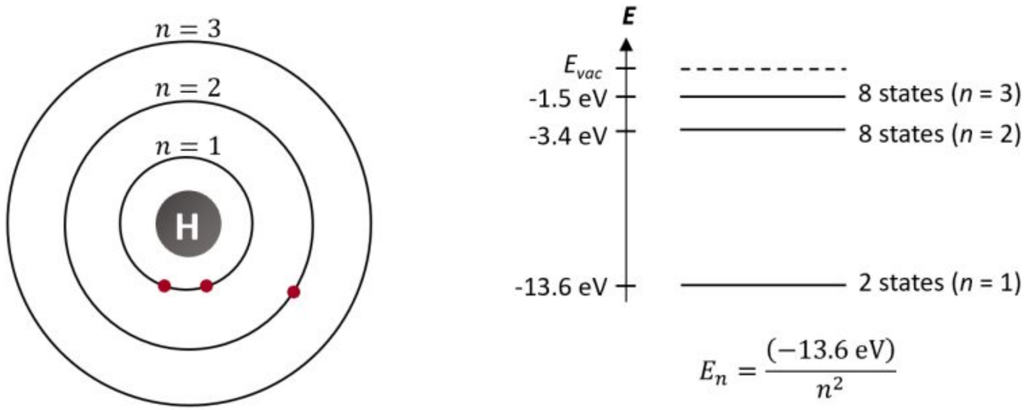
Outline

- Fermi-Dirac Distribution
- The semiconductor in equilibrium
- Carrier Transport Phenomena

Fermi-Dirac Distribution



Hydrogen Atom

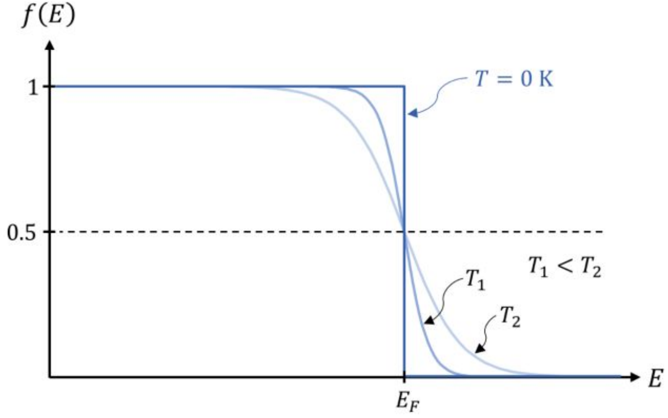
- For Hydrogen at $T = 0K$, electrons occupy lowest possible states
- At higher temperatures, there is some probability that e– are excited to other states.
- At a given temperature, given the total number of e– in the atom, one can calculate the average (i.e., expected) number of e– in each state using Fermi-Dirac statistics.

Fermi-Dirac Distribution

Fermi-Dirac Distribution Function $f_E(E)$:

$\approx e^{-(E-E_F)/kT}$

$$f_E(E) = \frac{1}{1 + e^{(E-E_F)/kT}} =$$ Probability of occupancy of a discrete energy state E



$E_F \equiv$ Fermi Energy (or Fermi Level), represents the "filling level"

$k(k_B) \equiv$ Boltzmann constant

$kT = 0.0259$ eV at $T = 300$K

Temperature T determines shape of $f_E(E)$

The semiconductor in equilibrium

- $n_0 \equiv$ concentration of ***occupied*** states in conduction band

- $p_0 \equiv$ concentration of ***empty*** states in valence band

- Density of states $N(E)$ in conduction and valence bands, respectively, are ***continuous functions***:

  - Conduction band: $N(E) = \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{3/2} \cdot (E - E_C)^{1/2}$

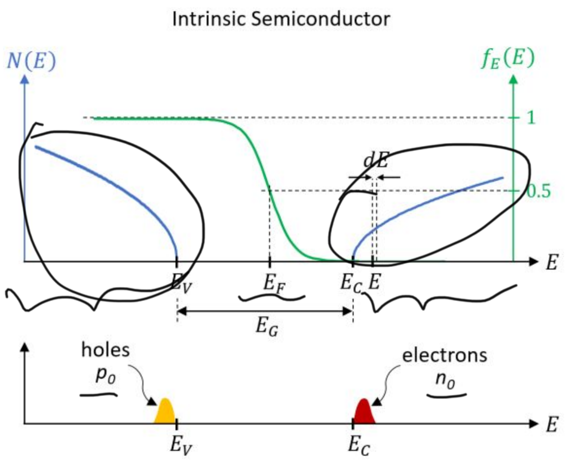
  - Valence band: $N(E) = \frac{1}{2\pi^2}\left(\frac{2m_h^*}{\hbar^2}\right)^{3/2} \cdot (E_V - E)^{1/2}$

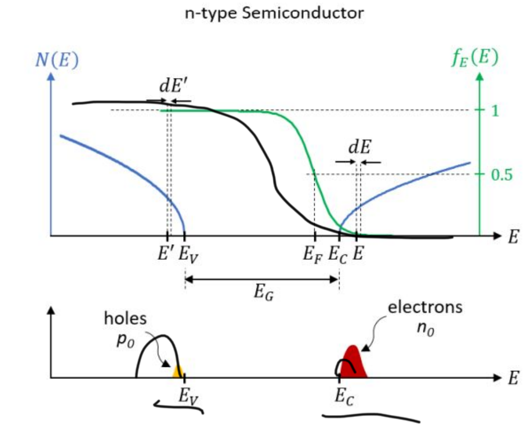
The semiconductor in equilibrium

- $N(E)dE \;\equiv$ number of states per unit volume between E and $E + dE$
- $f_E(E)N(E)dE \;\equiv$ number of occupied states per unit volume between E and $E + dE$

$$n_0 = \int_{E_C}^{\infty} N(E) \cdot f_E(E) \cdot dE$$

$$p_0 = \int_{-\infty}^{E_V} N(E) \cdot \underbrace{[1 - f_E(E)]}_{\text{Probability that state is } empty} \cdot dE$$





The semiconductor in equilibrium

Define $N_C \equiv 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} =$ Effective density of states in the conduction band at $E = E_C$

$$\boxed{n_0 = N_C \cdot f_E(E_C) = N_C \cdot e^{-(E_C - E_F)/kT}} = f_E(\text{근}) \qquad [6]$$

$$\boxed{p_0 = N_V \cdot \underbrace{[1 - f_E(E_V)]} = N_V \cdot e^{-(E_F - E_V)/kT}} \qquad [7]$$

where

$N_V \equiv 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} =$ Effective density of states in the valence band at $E = E_V$

$$1 - f_E(E_V) = 1 - \frac{1}{1 + e^{-(E_F - E_V)/kT}} \approx 1 - \left(1 - e^{-(E_F - E_V)/kT}\right)$$

$$= e^{-(E_F - E_V)/kT}$$

The semiconductor in equilibrium

$$n_0 \cdot p_0 = \left(N_C \cdot e^{-(E_C - E_F)/kT}\right)\left(N_V \cdot e^{-(E_F - E_V)/kT}\right)$$

Bandgap

$$= N_C N_V e^{-(E_C - E_V)/kT} = \underline{N_C N_V e^{-E_G/kT}} \quad \text{(independent of doping)}$$

$$n_0 \cdot p_0 = n_i^2 \quad \rightarrow \quad \boxed{n_i = \sqrt{N_C N_V}\, e^{-E_G/2kT}} \qquad [8]$$

The semiconductor in equilibrium - Intrinsic

$$n_i = \underbrace{N_C e^{-(E_C-E_i)/kT}}_{=n_0} = \underbrace{N_V e^{-(E_i-E_V)/kT}}_{=p_0}$$

$$e^{-(E_C+E_V-2E_i)/kT} = \left(\frac{N_V}{N_C}\right) \rightarrow \text{Take ln of both sides}$$

$$2E_i - E_C - E_V = kT \cdot \ln\left(\frac{N_V}{N_C}\right)$$

$$\boxed{E_i = \frac{E_C + E_V}{2} + \frac{1}{2}kT \cdot \ln\left(\frac{N_V}{N_C}\right)} \quad [10]$$

For intrinsic semiconductor material:

- $n_0 = p_0 = n_i$
- $n_0 = N_C e^{-(E_C-E_i)/kT} = n_i$ [9a]
- $p_0 = N_V e^{-(E_i-E_V)/kT} = n_i$ [9b]

The semiconductor in equilibrium - Extrinsic

e^- h^+

$$n + N_A^- = p + N_D^+$$

$$n.p = n_i^2$$

$$n = n_i \exp\left(\frac{E_f - E_i}{kT}\right)$$

$$n = N_C \exp\left(\frac{E_f - E_C}{kT}\right)$$

$$p = n_i \exp\left(-\frac{E_f - E_i}{kT}\right)$$

$$p = N_V \exp\left(\frac{E_V - E_f}{kT}\right)$$

$$E_f = E_C + kT \ln\left(\frac{n}{N_C}\right)$$

$$E_f = E_V - kT \ln\left(\frac{p}{N_V}\right)$$

$$E_f - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

$$E_i - E_f = kT \ln\left(\frac{p}{n_i}\right)$$

The semiconductor in equilibrium - Example 1

- n-type Si ($E_G = 1.1$ eV, $n_i = 1.5 \times 10^{10}$ cm^{-3}, $N_C = 2.8 \times 10^{19}$ cm^{-3}, $N_V = 1.8 \times 10^{19}$ cm^{-3})
- $N_D = 10^{17}$ cm^{-3}
- $E_C - E_D = 10$ meV
- T = 300 K (kT = 25.9 meV)

$n_0 p_0 = n_i^2$

$p_0 \ll n_0$

Shallow Donor

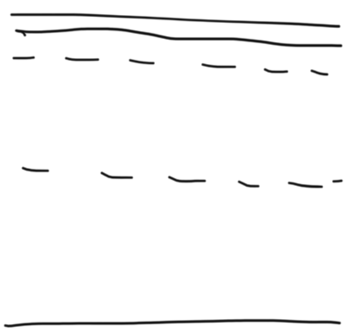
$E_C - E_D \ll kT$

$$n_0 + \cancel{N_A} = \cancel{p_0} + N_D^+$$

$$n_0 \approx N_D^+ \approx N_D$$

$E_D - E_F \gg kT$

3kT



$$N_D^+ = N_D[1 - f_F(E_D)] = N_D\left[\frac{e^{(E_D-E_F)/kT}}{1+e^{(E_D-E_F)/kT}}\right] \approx N_D$$

full ionization

$$n_0 = N_D = N_C e^{-(E_C-E_F)/kT}$$

$$n_0 = N_D = 10^{17} \text{ cm}^{-3}$$

$$p_0 = \frac{n_i^2}{N_D} = 2.25 \times 10^{3} \text{ cm}^{-3}$$

$$E_C - E_F = kT \cdot \ln\left(\frac{N_C}{N_D}\right)$$

The semiconductor in equilibrium - Example 2

- *n*-type Si (properties given on last page)
- $N_D = 10^{17}\ \text{cm}^{-3}$
- $E_C - E_D = 155\ \text{meV}$
- $T = 300$ K ($kT = 25.9$ meV)

$$N_D^+ = N_D[1 - f_F(E_D)] = N_D\left[\frac{e^{(E_D-E_F)/kT}}{1+e^{(E_D-E_F)/kT}}\right]$$

$$n_0 = p_0 + N_D^+ \qquad n_0 \approx N_D^+$$

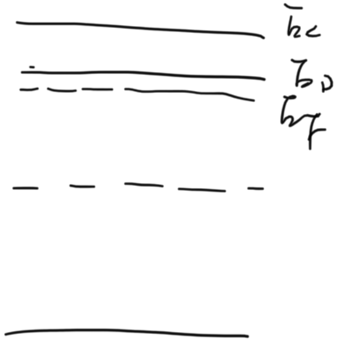
$$n_0 = N_c \cdot e^{-(E_c-E_F)/kT} = N_D\left[\frac{e^{(E_D-E_F)/kT}}{1+e^{(E_D-E_F)/kT}}\right]$$

$$e^{-(E_c-E_F)/kT} + e^{-(E_c-E_D)/kT} = \frac{N_D}{N_c}\left[e^{(E_D-E_F)/kT}\right]$$

$$E_c - E_F =$$

$$n_0 =$$

$$p_0 = \frac{n_i^2}{n_0}$$

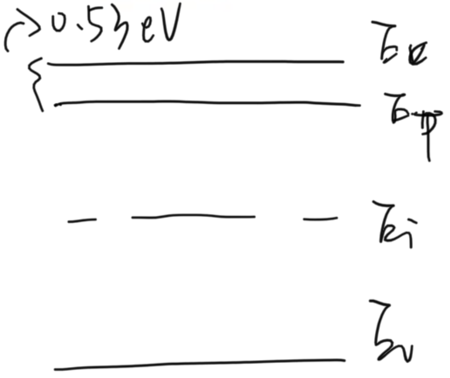


Deep donor

$E_c - E_D > kT$

The semiconductor in equilibrium - Example 3

Prob 1) Manganese makes a donor trap state 0.53 eV below the conduction band edge in silicon. If the silicon is doped with $N_D = 10^{16}$ cm^{-3}, what is the probability of occupancy of the trap state? Assume the concentration of Mn is small enough not to affect the overall doping and that the trap state is single-valued (**40 points**).



$$p_0 = \frac{n_i^2}{n_0}$$

$$N_D = 10^{16}\ cm^{-3} \qquad n_0 \approx N_D$$

$$n_0 = N_c \cdot e^{-(E_c - E_F)/kT} \Rightarrow E_c - E_F = kT \ln\left(\frac{N_c}{n_0}\right) = 0.205\ eV$$

$$f(E_T) = \frac{1}{1 + e^{(E_T - E_F)/kT}} = 0.999 \approx 1$$

$$E_T - E_F = (E_T - E_c) + (E_c - E_F) = -0.53 + 0.205$$

Carrier Transport Phenomena

Drift: Carriers in an E-field move as a result of the force from the E-field

Diffusion: Carriers move from region of high concentration to region of low concentration (statistical process)

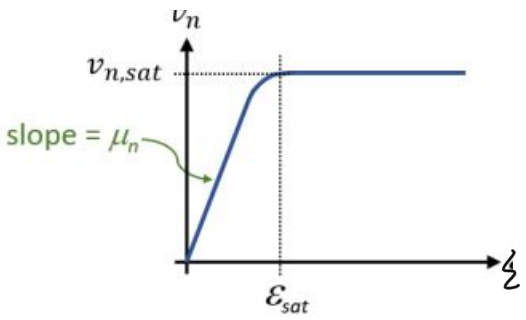
Carrier Transport Phenomena - Drift

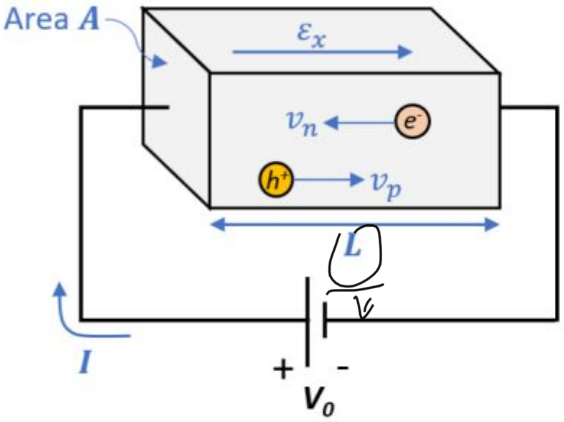
- $\varepsilon_x = \dfrac{V_0}{L}$
- At lower voltages V_0 (small ε_x), v_n and v_p increase linearly with V_0
  - $v_n = \mu_n \varepsilon_x$
  - $v_p = \mu_p \varepsilon_x$

$\left.\begin{array}{l} v_n = \mu_n \varepsilon_x \\ v_p = \mu_p \varepsilon_x \end{array}\right\} \varepsilon_x < \varepsilon_{sat} \quad \text{(Ohm's Law)}$

$\mu_n \equiv$ electron mobility

$\mu_p \equiv$ hole mobility





Carrier Transport Phenomena - Drift

$$I_n = \frac{(\text{charge } q) \cdot (\text{\# of electrons in block})}{(\text{time } t_0 \text{ for electron to transit block})}$$

$$\text{\# of electrons in block} = n_0 \cdot L \cdot A$$

$$t_0 = \frac{L}{v_n} = \frac{L}{\mu_n \varepsilon_x}$$

$$I_n = q \cdot n_0 \cdot L \cdot A \cdot \left(\frac{\mu_n \varepsilon_x}{L}\right) = \boxed{A(q\mu_n n_0)\varepsilon_x}$$

$$J_n = (q\mu_n n_0)\varepsilon_x = \boxed{\sigma_n \varepsilon_x} \rightarrow \sigma_n \equiv \text{electron conductivity} = \boxed{q\mu_n n_0}$$

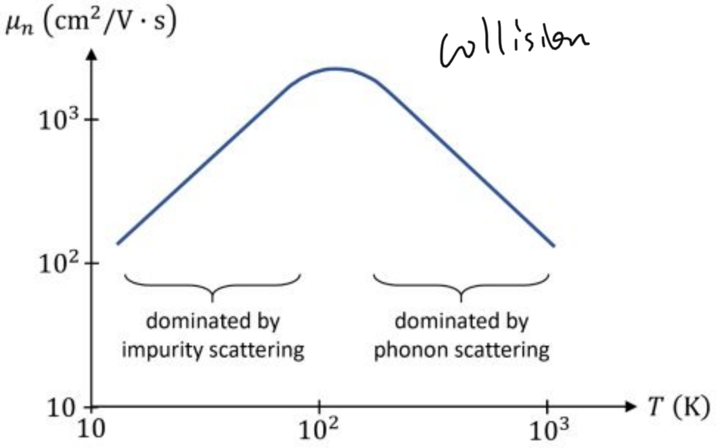
$$I_p = \boxed{A(q\mu_p p_0)\varepsilon_x} \text{ and } J_p = (q\mu_p p_0)\varepsilon_x = \boxed{\sigma_p \varepsilon_x}$$

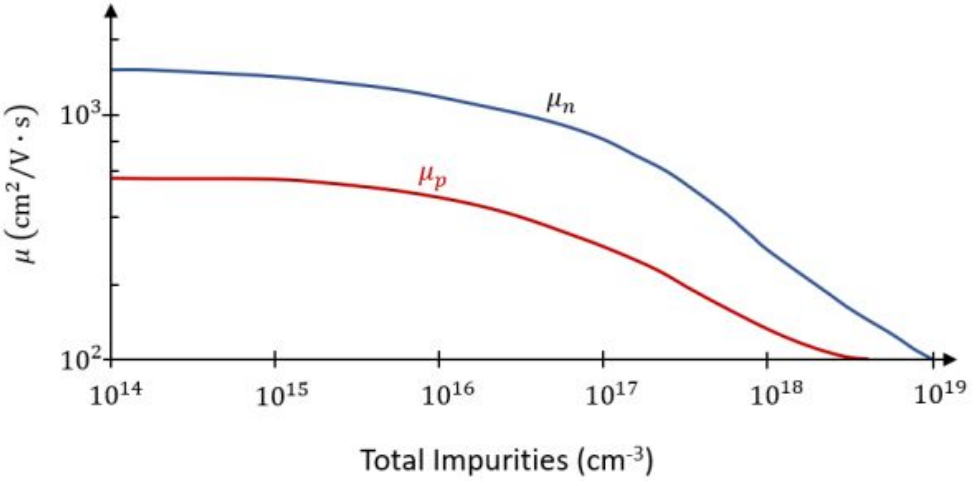
$$J_{tot} = J_n + J_p = \boxed{\sigma \varepsilon_x} \rightarrow \sigma = \sigma_n + \sigma_p = \boxed{q(n_0\mu_n + p_0\mu_p)}$$

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)}$$

ρ : resistivity

Carrier Transport Phenomena - Drift





Thanks