VE320
Intro to Semiconductor Devices
RC Week3

Yucheng Huang

University of Michigan
Shanghai Jiao Tong University
Joint Institute

May 27, 2022

Electrical Conduction In Solids

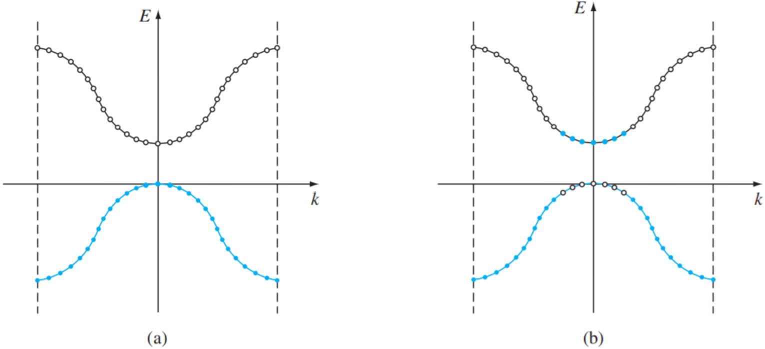

Figure: The E versus k diagram of the conduction and valence bands of a semiconductor

Electrical Conduction In Solids

$$F_{\text{total}} = F_{\mathrm{ext}} + F_{\mathrm{int}} = ma$$

Since it is difficult to take into account all of the internal forces, we will write the equation

$$F_{\mathrm{ext}} = m^* a$$

Relating momentum to velocity, Equation can be written as

$$\frac{1}{\hbar}\frac{dE}{dk} = \frac{p}{m} = v$$

If we now take the second derivative of E with respect to k, we have

$$\frac{d^2E}{dk^2} = \frac{\hbar^2}{m}$$

We may rewrite Equation as

$$\frac{1}{\hbar^2}\frac{d^2E}{dk^2} = \frac{1}{m}$$

Exercise 1

The E versus k diagram for a particular allowed energy band is shown in Figure 1. Determine (a) the sign of the effective mass and (b) the direction of velocity for a particle at each of the four positions shown.

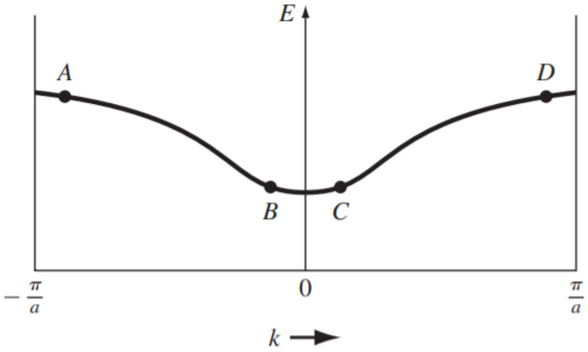

Figure: Figure for Problem 1

Exercise 1 Solution

Points A, B: $\frac{dE}{dk} < 0 \Rightarrow$ velocity in $-\mathrm{x}$ direction
Points C, D: $\frac{dE}{dk} > 0 \Rightarrow$ velocity in $+\mathrm{x}$ direction
Points A, D: $\frac{d^2E}{dk^2} < 0 \Rightarrow$ negative effective mass
Points B, C: $\frac{d^2E}{dk^2} > 0 \Rightarrow$ positive effective mass

Exercise 2

A simplified E versus k curve for an electron in the conduction band is given. The value of a is $10\mathring{A}$. Determine the relative effective mass m^*/m_0.

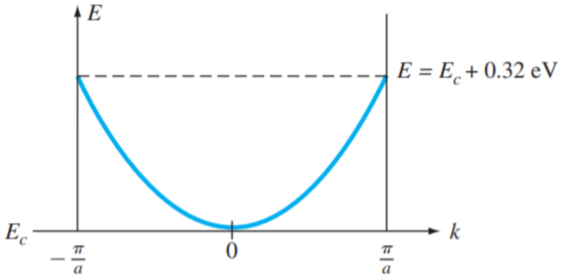

Figure: Figure for Problem 2

Exercise 2 Solution

We have $E - E_c = C_1 k^2$

$$\left(E_c + 0.32 - E_c\right)\left(1.6 \times 10^{-19}\right)$$
$$= C_1 \left(\frac{\pi}{10 \times 10^{-10}}\right)^2$$

so that $C_1 = 5.1876 \times 10^{-39}$ We have

$$m^* = \frac{\hbar^2}{2C_1} \Rightarrow \frac{m^*}{m_o} = \frac{\hbar^2}{2m_o C_1}$$
$$= \frac{\left(1.054 \times 10^{-34}\right)^2}{2\left(9.11 \times 10^{-31}\right)\left(5.1876 \times 10^{-39}\right)}$$

Or

$$\frac{m^*}{m_o} = 1.175$$

1. Quantum Theory of Solids
   - Electrical Conduction In Solids
   - Density of States Function
2. The Semiconductor in Equilibrium
   - The Semiconductor in Equilibrium
   - The Extrinsic Semiconductor

Density of States Function

A general expression for the density of allowed electron quantum states using the model of a free electron with mass m bounded in a three-dimensional infi nite potential well:

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}$$

The density of quantum states is a function of energy E

Density of States Function

Consider the density of states for a free electron given by Equation. Calculate the density of states per unit volume with energies between 0 and 1 eV.

The volume density of quantum states is

$$\mathrm{N}=\int_0^{1\mathrm{eV}} g(E)dE=\frac{4\pi(2m)^{3/2}}{h^3}\cdot\int_0^{1\mathrm{eV}}\sqrt{E}dE$$

or

$$N=\frac{4\pi(2m)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot E^{3/2}$$

The density of states is now

$$N=\frac{4\pi\left[2\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\cdot\frac{2}{3}\cdot\left(1.6\times10^{-19}\right)^{3/2}=4.5\times10^{27}\ \mathrm{m}^{-3}$$

or

$$N=4.5\times10^{21}\text{ states }/\mathrm{cm}^3$$

Extension to Semiconductors

The density of allowed electronic energy states in the conduction band:

$$g_c(E) = \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E - E_c}$$

Similarly,

$$g_v(E) = \frac{4\pi\left(2m_p^*\right)^{3/2}}{h^3}\sqrt{E_v - E}$$

Exercise 3

Determine the number $(\#/\mathrm{cm}^3)$ of quantum states in silicon between E_c and $E_c + kT$ at $T = 300\ \mathrm{K}$.

Exercise 3 Solution

$$
\begin{aligned}
N &= \int_{E_c}^{E_c+kT} \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\sqrt{E-E_c}\cdot dE \\
&= \frac{4\pi\left(2m_n^*\right)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot\left(E-E_c\right)^{3/2}\Big|_{E_c}^{E_c+kT} \\
&= \frac{4\pi\left[2(1.08)\left(9.11\times10^{-31}\right)\right]^{3/2}}{\left(6.625\times10^{-34}\right)^3}\cdot\frac{2}{3}\cdot\left[(0.0259)\left(1.6\times10^{-19}\right)\right]^{3/2} \\
&= 2.12\times10^{25}\ \mathrm{m}^{-3}
\end{aligned}
$$

Or

$$N = 2.12\times10^{19}\ \mathrm{cm}^{-3}$$

The Fermi–Dirac Probability Function

The actual number of independent ways of realizing a distribution of N_i particles in the i th level is

$$W_i = \frac{g_i!}{N_i!\,(g_i - N_i)!}$$

The Fermi–Dirac Probability Function

We may write the most probable distribution function as

$$\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

The number density $N(E)$ is the number of particles per unit volume per unit energy and the function $g(E)$ is the number of quantum states per unit volume per unit energy. The function $f_F(E)$ is called the Fermi-Dirac distribution or probability function and gives the probability that a quantum state at the energy E will be occupied by an electron. The energy E_F is called the Fermi energy.

Example 1

Let $T = 300\ \mathrm{K}$. Determine the probability that an energy level $3kT$ above the Fermi energy is occupied by an electron.

From Equation above, we can write

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{3kT}{kT}\right)}$$

which becomes

$$f_F(E) = \frac{1}{1 + 20.09} = 0.0474 = 4.74\%$$

Exercise 4

Assume that the Fermi energy level for a particular material is $6.25\mathrm{eV}$ and that the electrons in this material follow the Fermi-Dirac distribution function. Calculate the temperature at which there is a 1 percent probability that a state $0.30\mathrm{eV}$ below the Fermi energy level will not contain an electron.

Exercise 4 Solution

The probability that a state is empty is

$$1 - f_F(E) = 1 - \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Then

$$0.01 = 1 - \frac{1}{1 + \exp\left(\frac{5.95 - 6.25}{kT}\right)}$$

Solving for kT, we find $kT = 0.06529\mathrm{eV}$, so that the temperature is $T = 756\ \mathrm{K}$.

Maxwell–Boltzmann approximation

Consider the case when $E - E_F \gg kT$, where the exponential term in the denominator of Equation is much greater than unity. We may neglect the 1 in the denominator, so the Fermi-Dirac distribution function becomes

$$f_F(E) \approx \exp\left[\frac{-(E - E_F)}{kT}\right]$$

is known as the Maxwell-Boltzmann approximation, or simply the Boltzmann approximation, to the Fermi-Dirac distribution function. Figure 3.35 shows the Fermi-Dirac probability function and the Boltzmann approximation. This figure gives an indication of the range of energies over which the approximation is valid.

1. Quantum Theory of Solids
   - Electrical Conduction In Solids
   - Density of States Function

2. The Semiconductor in Equilibrium
   - The Semiconductor in Equilibrium
   - The Extrinsic Semiconductor

The Semiconductor in Equilibrium

The distribution (with respect to energy) of electrons in the conduction band is given by the density of allowed quantum states times the probability that a state is occupied by an electron. This statement is written in equation form as

$$n(E) = g_c(E) f_F(E)$$

Similarly,

$$p(E) = g_v(E) \left[1 - f_F(E)\right]$$

The Semiconductor in Equilibrium

The equation for the thermalequilibrium concentration of electrons may be found by integrating Equation above over the conduction band energy, or

$$n_0 = \int g_c(E) f_F(E) dE$$

$$n_0 = N_c \exp\left[\frac{-\left(E_c - E_F\right)}{kT}\right]$$

Exercise 5

Calculate the probability that a quantum state in the conduction band at $E = E_c + kT/2$ is occupied by an electron, and calculate the thermal-equilibrium electron concentration in silicon at $T = 300\ \mathrm{K}$.

Assume the Fermi energy is $0.25\mathrm{eV}$ below the conduction band. The value of N_c for silicon at $T = 300\ \mathrm{K}$ is $N_c = 2.8 \times 10^{19}\ \mathrm{cm}^{-3}$ (see Appendix B).

Exercise 5 Solution

The probability that a quantum state at $E = E_c + kT/2$ is occupied by an electron is given by

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \exp\left[\frac{-\left(E_c + (kT/2) - E_F\right)}{kT}\right]$$

or

$$f_F(E) = \exp\left[\frac{-(0.25 + (0.0259/2))}{0.0259}\right] = 3.90 \times 10^{-5}$$

The electron concentration is given by

$$n_0 = N_c \exp\left[\frac{-\left(E_c - E_F\right)}{kT}\right] = \left(2.8 \times 10^{19}\right) \exp\left[\frac{-0.25}{0.0259}\right]$$

or

$$n_0 = 1.80 \times 10^{15}\ \mathrm{cm}^{-3}$$

The Semiconductor in Equilibrium

The thermalequilibrium concentration of holes in the valence band may now be written as

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

The Intrinsic Carrier Concentration

For an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valence band. We may denote n_i and p_i as the electron and hole concentrations, respectively, in the intrinsic semiconductor.
The Fermi energy level for the intrinsic semiconductor is called the intrinsic Fermi energy, or $E_F = E_{Fi}$. We can write

$$n_0 = n_i = N_c \exp\left[\frac{-\left(E_c - E_{Fi}\right)}{kT}\right]$$

and

$$p_0 = p_i = n_i = N_v \exp\left[\frac{-\left(E_{Fi} - E_v\right)}{kT}\right]$$

The Intrinsic Carrier Concentration

If we take the product of above, we obtain

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \cdot \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

or

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right]$$

Table: Commonly accepted values of n_i at $T = 300\ \mathrm{K}$

Silicon	$n_i = 1.5 \times 10^{10}\ \mathrm{cm}^{-3}$
Gallium arsenide	$n_i = 1.8 \times 10^{6}\ \mathrm{cm}^{-3}$
Germanium	$n_i = 2.4 \times 10^{13}\ \mathrm{cm}^{-3}$

Example 3

Calculate the intrinsic carrier concentration in silicon at $T = 250\ \mathrm{K}$.

The values of N_c and N_v for silicon at $T = 300\ \mathrm{K}$ are $2.8 \times 10^{19}\ \mathrm{cm}^{-3}$ and $1.04 \times 10^{19}\ \mathrm{cm}^{-3}$, respectively. Both N_c and N_v vary as $T^{3/2}$. Assume the bandgap energy of silicon is $1.12\mathrm{eV}$ and does not vary over this temperature range.

We find, at $T = 250\ \mathrm{K}$

$$
\begin{aligned}
n_i^2 &= \left(2.8 \times 10^{19}\right)\left(1.04 \times 10^{19}\right)\left(\frac{250}{300}\right)^3 \exp\left[\frac{-1.12}{(0.0259)(250/300)}\right] \\
&= 4.90 \times 10^{15}
\end{aligned}
$$

or

$$
n_i = 7.0 \times 10^7\ \mathrm{cm}^{-3}
$$

The Intrinsic Fermi-Level Position

Since the electron and hole concentrations are equal

$$N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right]$$

After a series of operation

$$E_{Fi} - E_{\text{midgap}} = \frac{3}{4}kT \ln\left(\frac{m_p^*}{m_n^*}\right)$$

The Extrinsic Semiconductor

The thermal-equilibrium electron concentration can be written as

$$n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$$

Similarly, if we add and subtract an intrinsic Fermi energy in the exponent of Equation, we will obtain

$$p_0 = n_i \exp\left[\frac{-\left(E_F - E_{Fi}\right)}{kT}\right]$$

The Extrinsic Semiconductor

If $n_0 > p_0$, the semiconductor is n type. In an n-type semiconductor, electrons are referred to as the majority carrier and holes as the minority carrier. By comparing the relative values of n_0 and p_0 in the example, it is easy to see how this designation came about. Similarly, in a p-type semiconductor where $p_0 > n_0$, holes are the majority carrier and electrons are the minority carrier.

Exercise 6

The electron concentration in silicon at $T = 300\ \mathrm{K}$ is $n_0 = 2 \times 10^5\ \mathrm{cm}^{-3}$.

(a) Determine the position of the Fermi level with respect to the valence band energy level.

(b) Determine p_0.

(c) Is this n- or p-type material?

Exercise 6 Solution

(a)

$$\begin{aligned} E_c - E_F &= kT \ln\left(\frac{N_c}{n_o}\right) \\ &= (0.0259)\ln\left(\frac{2.8\times 10^{19}}{2\times 10^{5}}\right) \\ &= 0.8436\mathrm{eV} \\ E_F - E_v &= E_g - (E_c - E_F) \\ &= 1.12 - 0.8436 \\ E_F - E_v &= 0.2764\mathrm{eV} \end{aligned}$$

(b)

$$\begin{aligned} p_o &= \left(1.04\times 10^{19}\right)\exp\left(\frac{-0.27637}{0.0259}\right) \\ &= 2.414\times 10^{14}\ \mathrm{cm}^{-3} \end{aligned}$$

(c) p-type

END

Thanks