
VE320 – Summer 2022

Introduction to Semiconductor Devices

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**Chapter 6 Non-Equilibrium Excess Carriers in
Semiconductors**

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

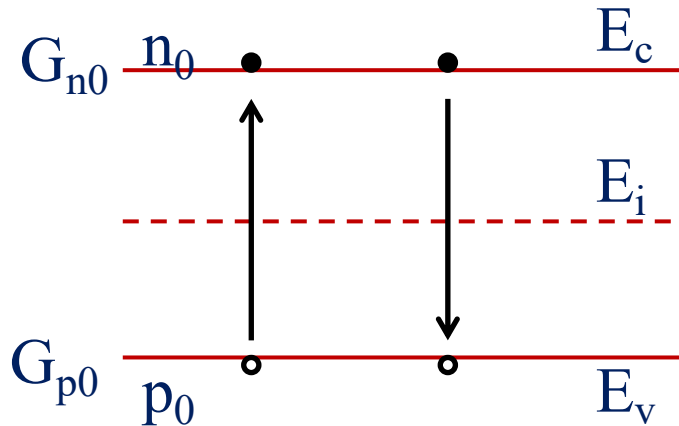
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

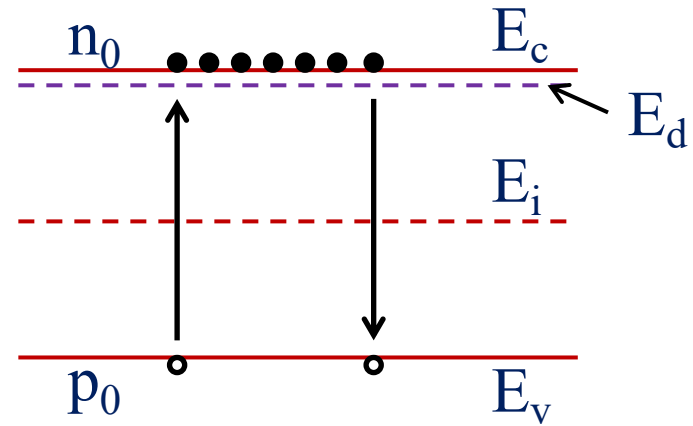
6.5 Surface effects

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0 = p_0 = n_i$



n type : $n_0 \gg n_i \gg p_0$

G_{n0} : the thermal generation rate of electrons

G_{p0} : the thermal generation rate of holes

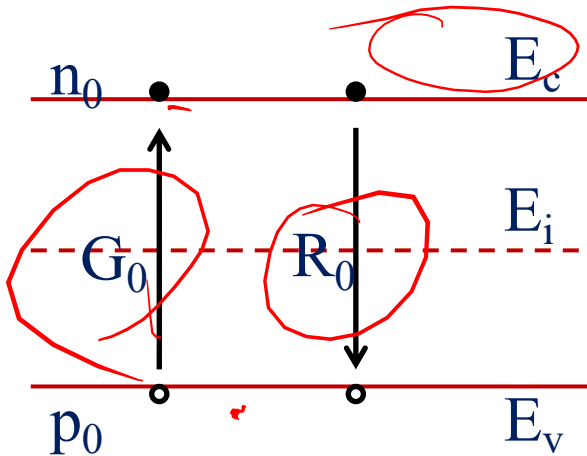
R_{n0} : the recombination rate of electrons

R_{p0} : the recombination rate of holes

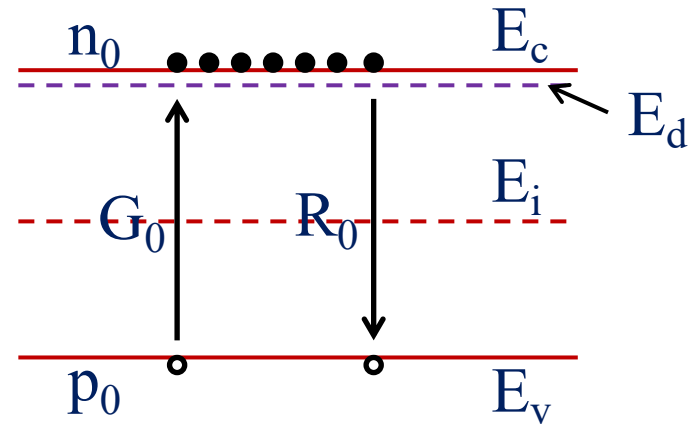
$$G_{n0} = G_{p0} = R_{n0} = R_{p0} \quad (\text{direct G and R from band to band})$$

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0 = p_0 = n_i$



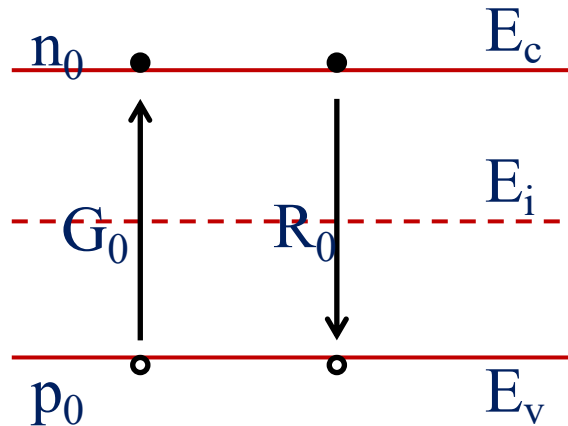
n type : $n_0 \gg n_i \gg p_0$

$$R_0 \sim \underbrace{n \cdot p}_{n_i + \Delta n} = \underline{n_0 \cdot p_0}$$

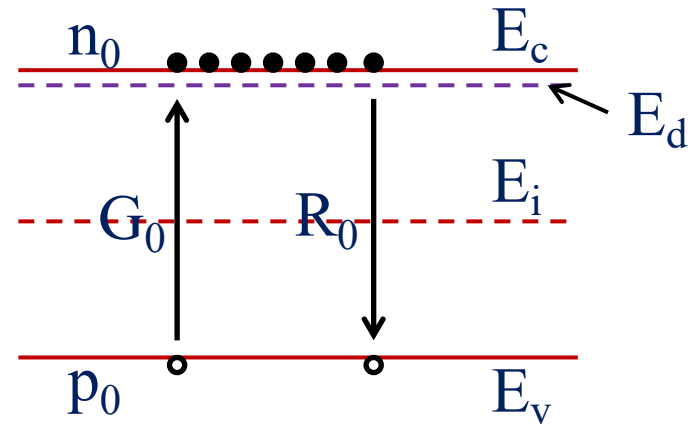
$$\cancel{G_0 \sim n_0 \cdot p_0} \quad G_0 = R_0 = n_0 \cdot p_0$$

6.1 Carrier generation and recombination

The semiconductor in equilibrium



Intrinsic: $n_0 = p_0 = n_i$

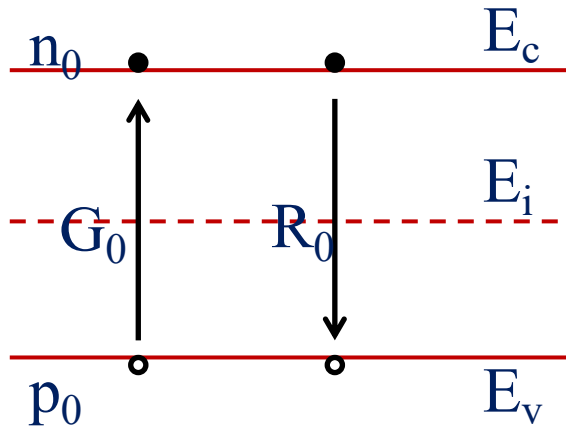


n type : $n_0 \gg n_i \gg p_0$

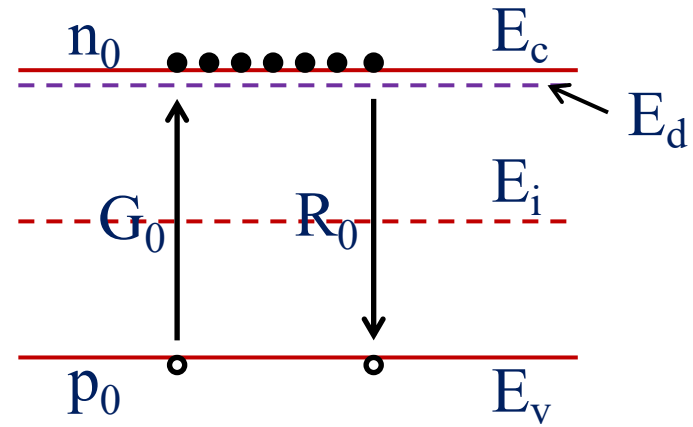
$$G_0 = R_0 = \alpha_r \cdot n_0 \cdot p_0$$

6.1 Carrier generation and recombination

The semiconductor in equilibrium



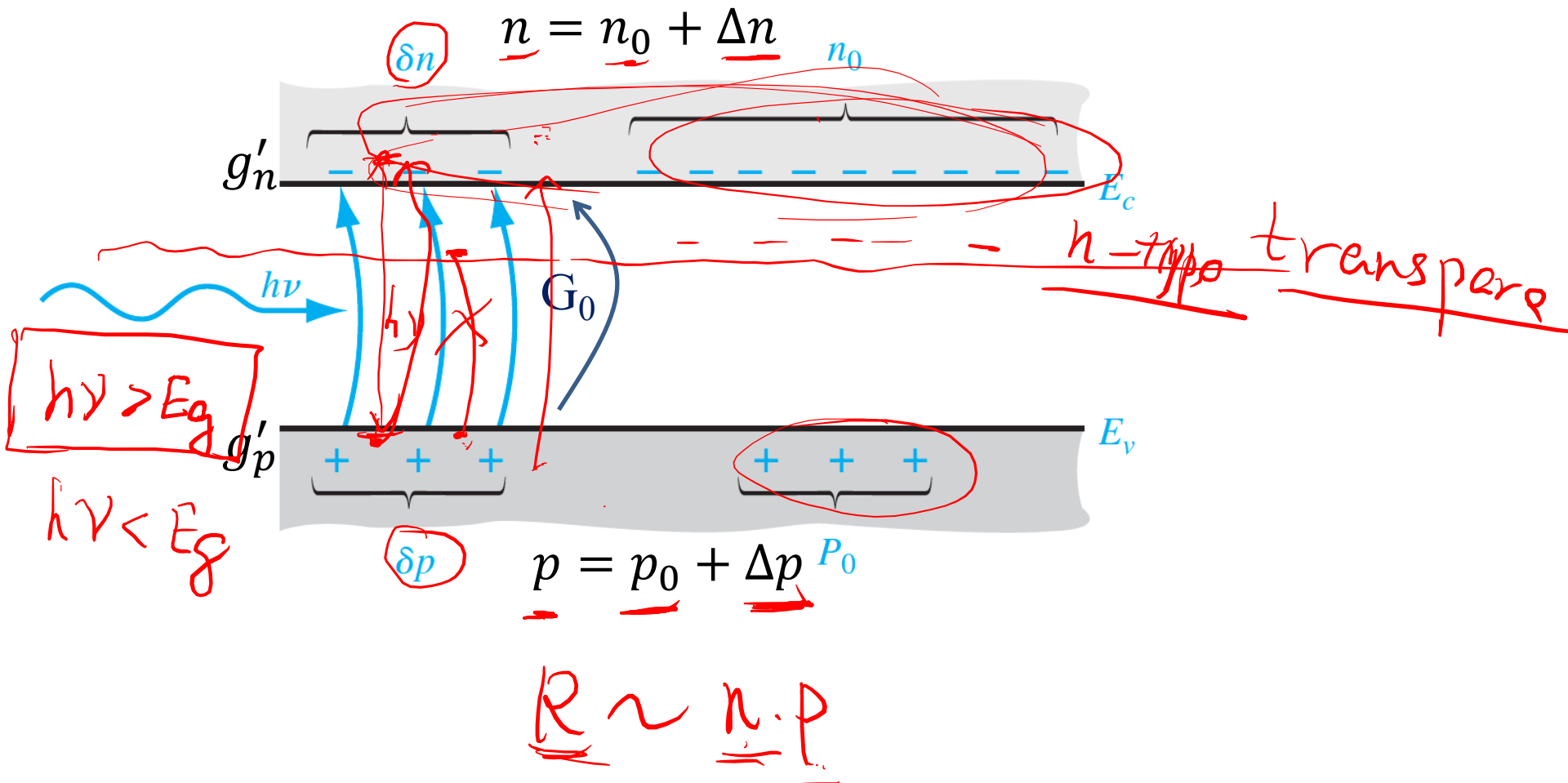
Intrinsic: $n_0 = p_0 = n_i$



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6.1 Carrier generation and recombination

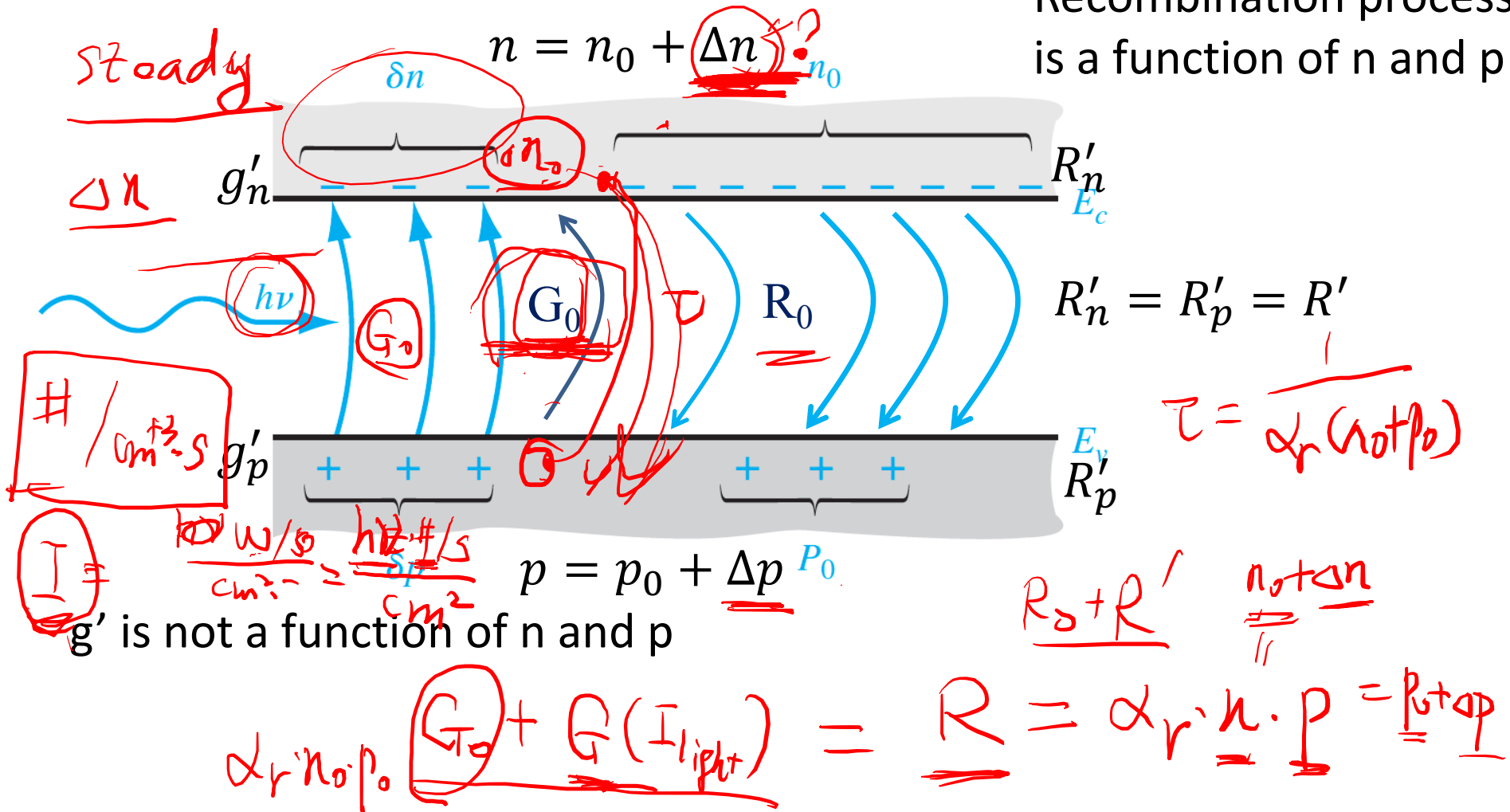
Excess carrier generation and recombination



6.1 Carrier generation and recombination

Excess carrier generation and recombination

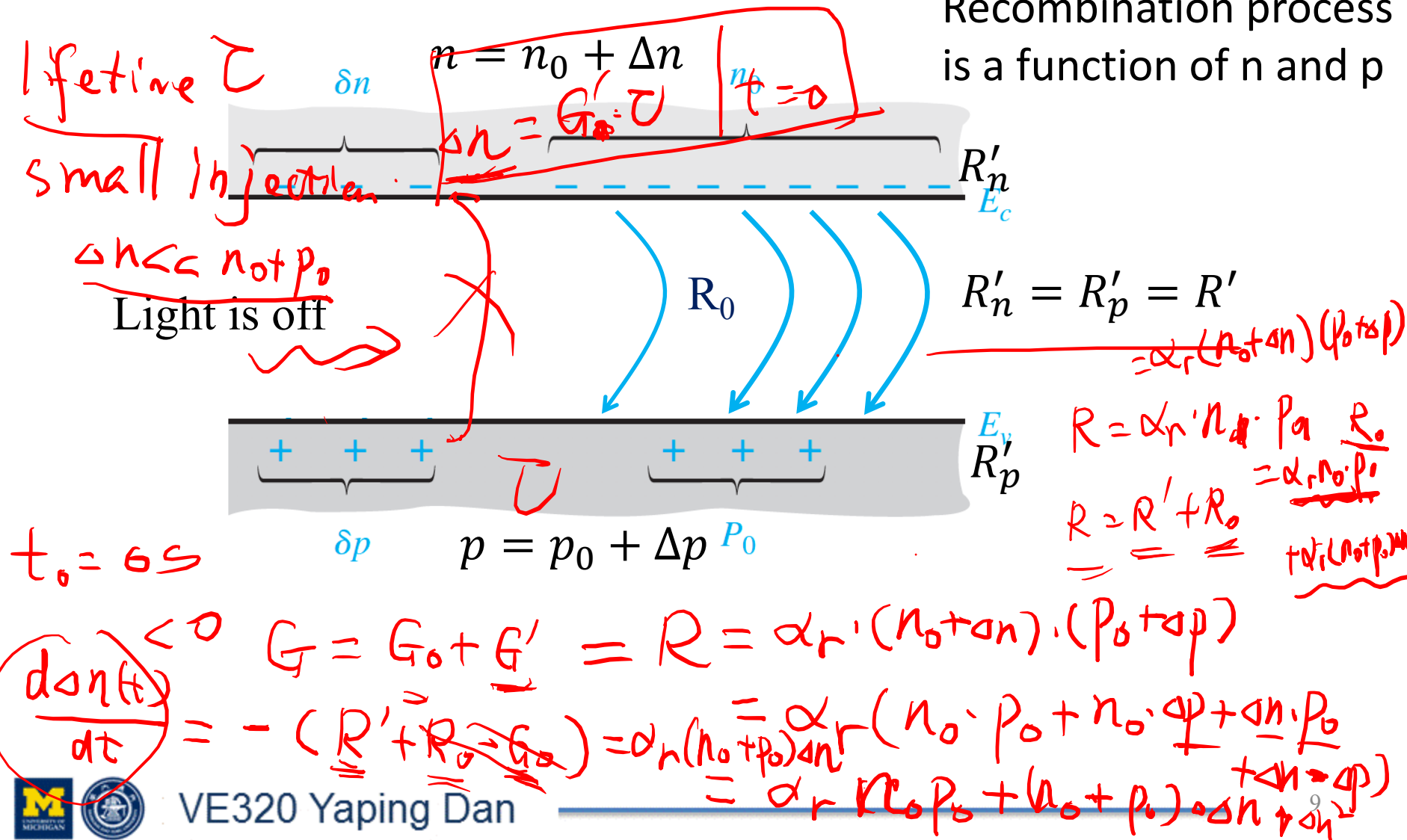
Recombination process is a function of n and p



6.1 Carrier generation and recombination

Excess carrier generation and recombination

Recombination process is a function of n and p



6.1 Carrier generation and recombination

Excess carrier generation and recombination

Net recombination rate

$$\cancel{G_0} + \underline{G'} = R = \cancel{\alpha_r n_0 p_0} + \alpha_r \cdot (n_0 + p_0) \Delta n + \alpha_r (\Delta n)^2$$

$$G' = \alpha_r (n_0 + p_0) \cdot \Delta n + \alpha_r \cdot (\Delta n)^2$$

$$\underline{\Delta n} = \frac{G'}{\alpha_r (n_0 + p_0 + \underline{\Delta n})}$$

$$= G' \cdot \frac{1}{\alpha_r (n_0 + p_0 + \underline{\Delta n})}$$

Small injection
condition
if $\Delta n \ll n_0 + p_0$

$$\boxed{\alpha_r (n_0 + p_0 + \underline{\Delta n})}$$

τ lifetime

6.1 Carrier generation and recombination

Excess carrier generation and recombination

$$\frac{d\Delta p(t)}{dt} = -\alpha_r (n_0 + p_0) \Delta n(t) = \frac{\Delta n}{\tau_n}$$

$n_i = 10^{10} \text{ cm}^{-3}$
 $p_0 = 10^{15} \text{ cm}^{-3}$
 $R' = R - R_0$
 $\Delta n = 10^{12} \text{ cm}^{-3}$
 $\alpha_r = \frac{1}{\tau_n}$

For n-type semiconductor, net recombination rate

$$R'_n = R'_p = \frac{\Delta p(t)}{\tau_{p0}}$$

$$\frac{d\Delta n(t)}{dt} = -\alpha_r (n_0 + p_0) \Delta n(t)$$

$$d(\ln \Delta n(t)) = -\alpha_r (n_0 + p_0) dt$$

For p-type semiconductor, net recombination rate

$$R'_n = R'_p = \frac{\Delta n(t)}{\tau_{n0}}$$

$$\ln \Delta n(t) = -\alpha_r (n_0 + p_0) t + C$$

$$\Delta n(t) = C' e^{-\alpha_r (n_0 + p_0) t}$$

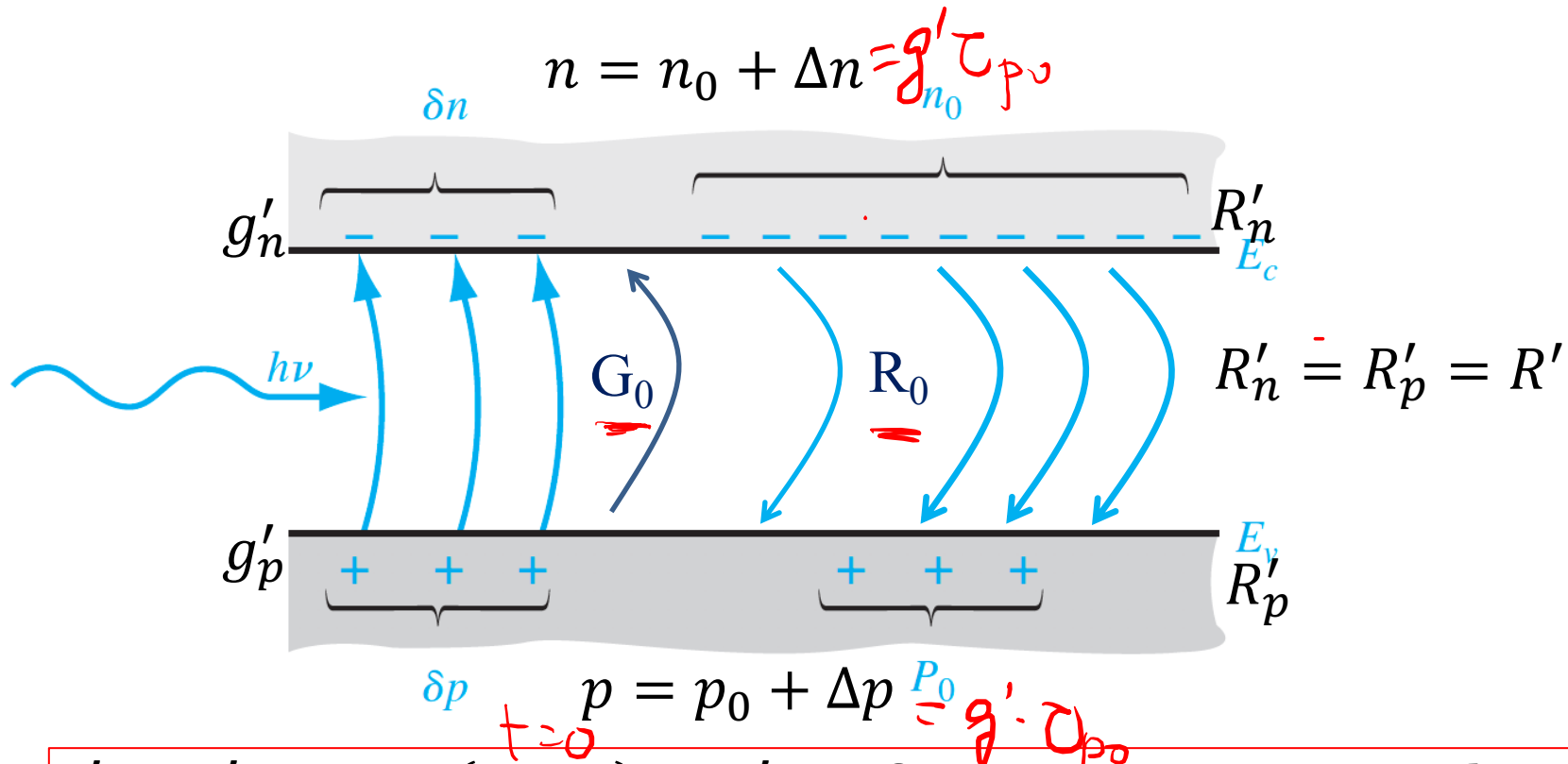
$$= C' e^{-t/\tau} = G' \tau$$

$$\Delta n(t) = G' \tau e^{-t/\tau}$$

$$\tau = \frac{1}{\alpha_r (n_0 + p_0)} \text{ lifetime}$$

6.1 Carrier generation and recombination

Excess carrier generation and recombination



$$g' = R' \Rightarrow \Delta p(t \leq 0) = g' \tau_{p0} \text{ for } n\text{-type semiconductors}$$

$$g' = R' \Rightarrow \Delta n(t \leq 0) = g' \tau_{n0} \text{ for } p\text{-type semiconductors}$$

Check your understanding

Problem Example #1

Assume that excess carriers have been generated uniformly in a semiconductor to a concentration of $\Delta n(0) = 10^{15} \text{ cm}^{-3}$. The generation of the excess carriers turns off at time $t=0$. Assuming the excess carrier lifetime is $\tau_{n0} = 10^{-6} \text{ s}$, $\approx 1 \mu\text{s}$, calculate the recombination rate of excess carriers for $t = 4 \mu\text{s}$.

$$R' = \alpha_r (n_0 + p_0) \Delta n = \frac{\Delta n}{\tau_{n0}}$$

$$R = \alpha_r \cdot n \cdot p$$

$$\frac{d\Delta n(t)}{dt} = -R' = -\frac{\Delta n}{\tau_{n0}}$$

$$R_0 = \alpha_r \cdot n_0 \cdot p_0 = G_0$$

$$\Delta n(t) = \Delta n(t=0) e^{-t/\tau_{n0}}$$

$$g' \rightarrow \Delta n = g' / \alpha_r (n_0 + p_0)$$

$$\begin{aligned} &= 10^{15} e^{-4/1} \quad \left(\Delta n = g' e^{-\alpha_r (n_0 + p_0) t} \right) \quad \frac{1}{\alpha_r (n_0 + p_0)} = \tau \\ &= 10^{15} e^{-4} \\ &= \boxed{1.83 \times 10^3 \text{ cm}^{-3}} e^{-4} e^{-t/\tau} \end{aligned}$$

$$\frac{d\Delta n}{dt} = -(R' + R_0 - G_0)$$

$$\tau = \frac{1}{\alpha_r (n_0 + p_0)} = -\alpha_r (n_0 + p_0 + \Delta n)$$

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

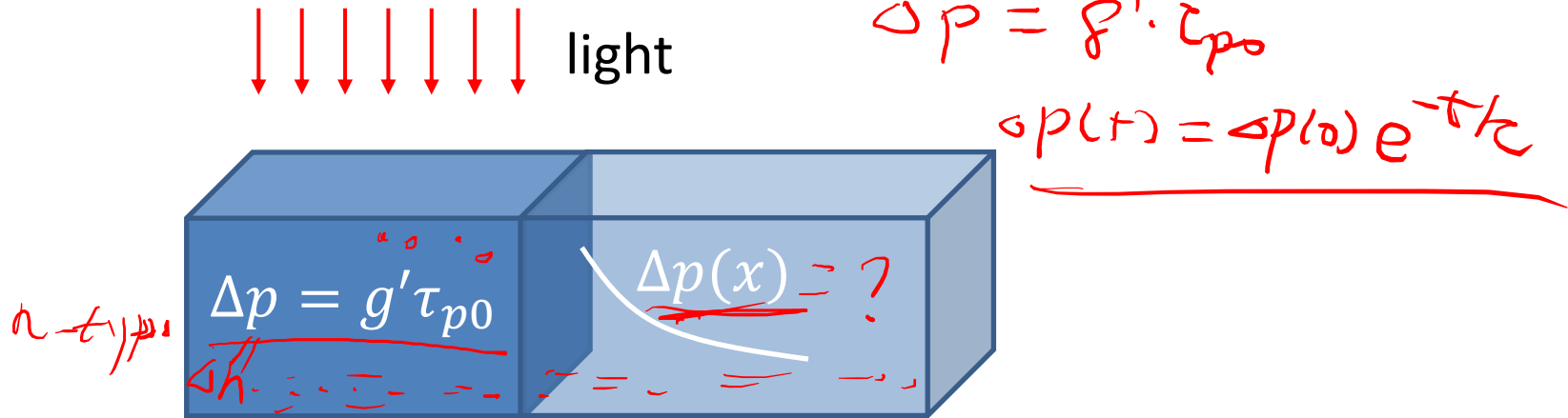
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

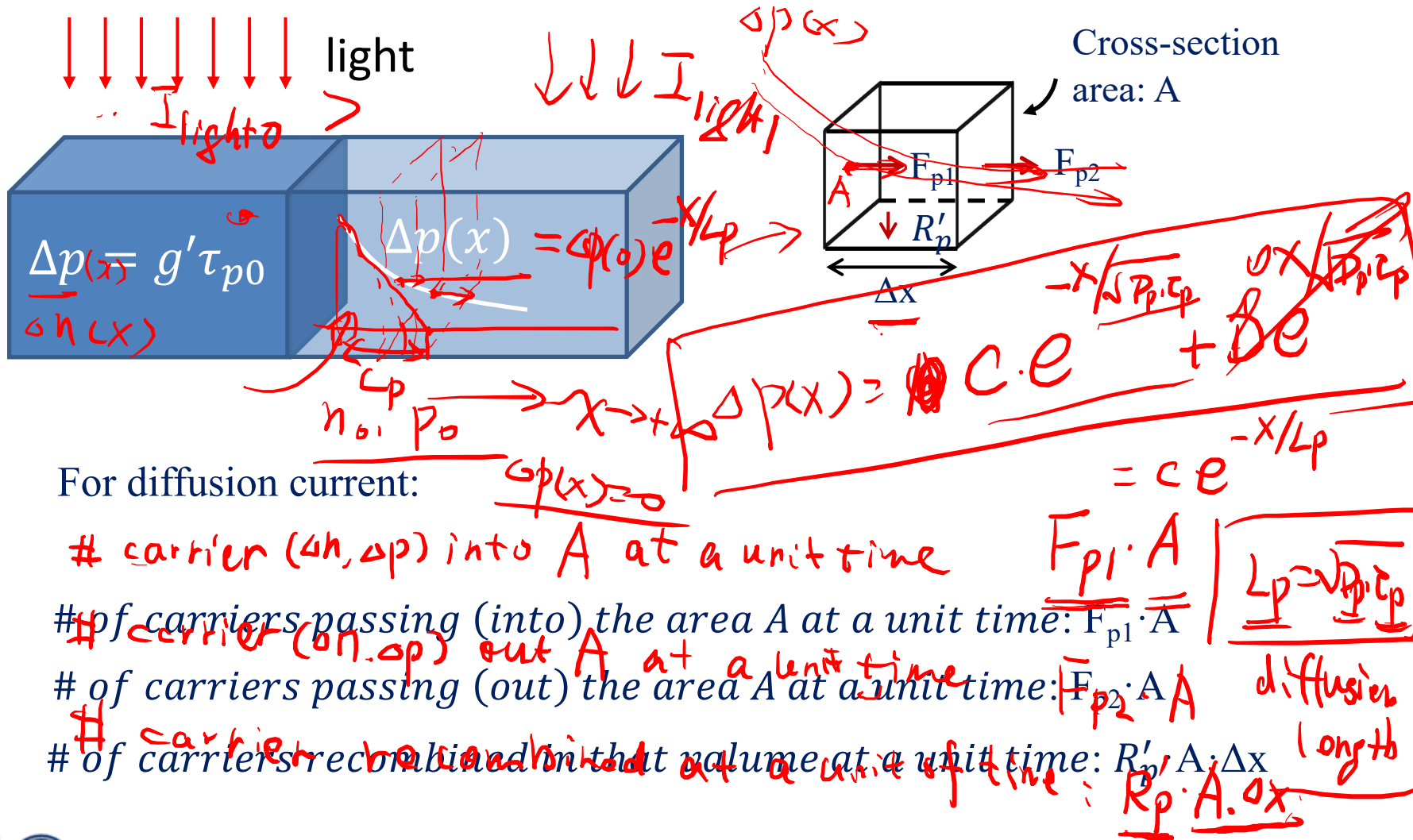
6.2 Characteristics of excess carriers

Continuity equation at steady state



6.2 Characteristics of excess carriers

Continuity equation at steady state



For diffusion current:

carrier ($\delta n, \delta p$) into A at a unit time

of carriers passing (into) the area A at a unit time: $F_{p1} \cdot A$

of carriers passing (out) the area A at a unit time: $F_{p2} \cdot A$

of carriers recombined in that volume at a unit of time: $R_p' \cdot A \cdot \Delta x$

6.2 Characteristics of excess carriers

Continuity equation at steady state

diffusion

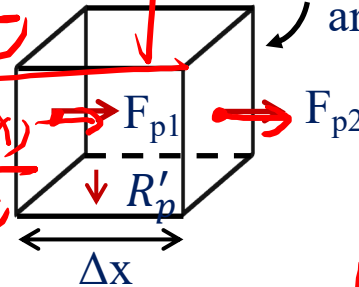
Cross-section
area: A

$$\Delta p(x) = A e^{-x/\sqrt{D_p \tau}} + B e^{x/\sqrt{D_p \tau}}$$

For diffusion current:

$$\frac{d^2 \Delta p(x)}{dx^2} = \frac{\Delta p(x)}{L_p^2}$$

$$\frac{dJ_p(x)}{dx} = \frac{\Delta p(x)}{D_p \tau}$$



$$F_{p2} \cdot A - F_{p1} \cdot A = -R'_p \cdot A \Delta x$$

$$(F_{p2} - F_{p1}) A = -\frac{\Delta n}{\tau} \cdot A \cdot \Delta x$$

$$R'_p = \frac{\Delta n}{\tau} = \alpha_r (n_0 + p_0) \Delta n$$

$$F_p = -D_p \frac{d\Delta p}{dx}$$

$$p = p_0 + \Delta p$$

$$= -D_p \frac{d\Delta p}{dx}$$

$$\lim_{\Delta x \rightarrow 0} \frac{F_{p2} - F_{p1}}{\Delta x}$$

$$\frac{dF_p}{dx} = -\left(\frac{\Delta n}{\tau}\right) + g'_p$$

6.2 Characteristics of excess carriers

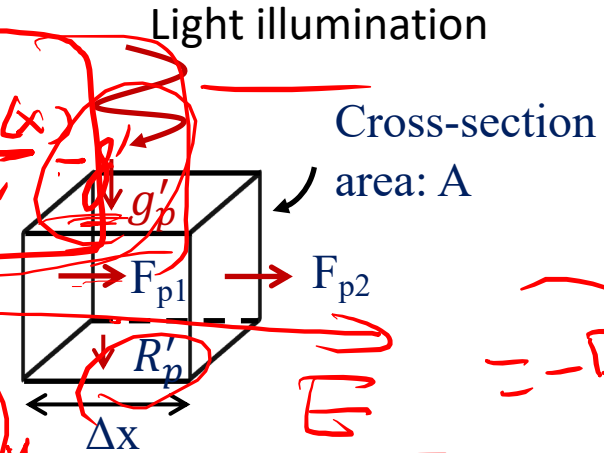
Steady-state continuity equation

diffusion equation

$$\frac{d^2 \delta p(x)}{dx^2} = \frac{\delta p(x)}{\tau}$$

$$F = \frac{J}{q}$$

$$F_{p2}A - F_{p1}A = -R_p' \cdot A \cdot \Delta x + G_p' \cdot A \cdot \Delta x$$



$$F = -D_p \frac{d\delta p}{dx}$$

$$F = -D_p \frac{d\delta p}{dx}$$

$$\lim_{\Delta x \rightarrow 0} \frac{F_{p2} - F_{p1}}{\Delta x} = -R_p' + G_p' = -\frac{\delta p}{\tau} + g'$$

$$-D_p \frac{d^2 \delta p}{dx^2} + \frac{d(p \mu_p E)}{dx} = -\frac{\delta p}{\tau} + g'$$

$$-D_p \frac{d^2 \delta p}{dx^2} = -\frac{\delta p}{\tau} + g'$$

6.2 Characteristics of excess carriers

Steady-state continuity equation

Steady state:

$$D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - p \mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g'_p = 0$$

Handwritten notes: Above the equation, $\frac{dE}{dx}$ is written in red. The term $\mu_p E \frac{dp}{dx}$ is circled in red. The term $p \mu_p \frac{dE}{dx}$ is circled in red. The term $\frac{\Delta p}{\tau_{p0}}$ is circled in red. The term g'_p has a red arrow pointing to it.

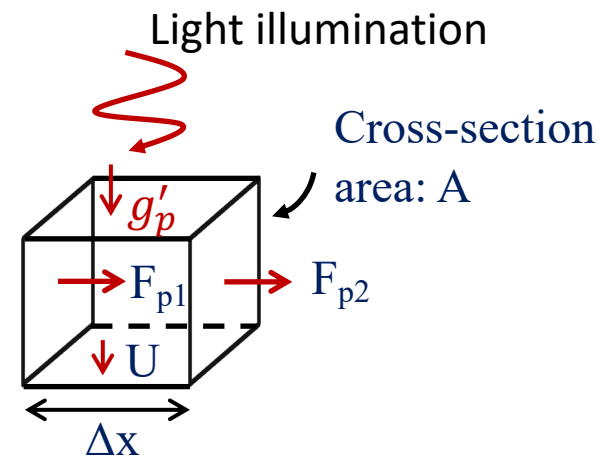
When the n-type semiconductor is uniformly doped,

$$p(x) = p_0 + \Delta p(x)$$

$$D_p \frac{d^2 \Delta p}{dx^2} - \mu_p E \frac{d\Delta p}{dx} - \Delta p \mu_p \frac{dE}{dx} - \frac{\Delta p}{\tau_{p0}} + g'_p = 0$$

6.2 Characteristics of excess carriers

Time-dependent continuity equation



6.2 Characteristics of excess carriers

Time-dependent continuity equation

For an n-type semiconductor,

$$\frac{d\Delta p}{dt} = D_p \frac{d^2 p}{dx^2} - \mu_p E \frac{dp}{dx} - \bar{p} \mu_p \frac{dE}{dx} - R'_p + g'_p$$

(minority carriers)

$$\frac{\Delta p}{\tau_{p0}}$$

holes as minority

$$R'_p = \frac{\Delta p}{\tau_{p0}}$$

$$\frac{d\Delta n}{dt} = D_n \frac{d^2 n}{dx^2} + \mu_n E \frac{dn}{dx} + n \mu_n \frac{dE}{dx} - R'_n + g'_n$$

(majority carriers)

$$\frac{\Delta p}{\tau_{p0}}$$

$$R'_n = R'_p = \frac{\Delta p}{\tau_{p0}}$$

$$g'_n = g'_p$$

nonlinear

Summary

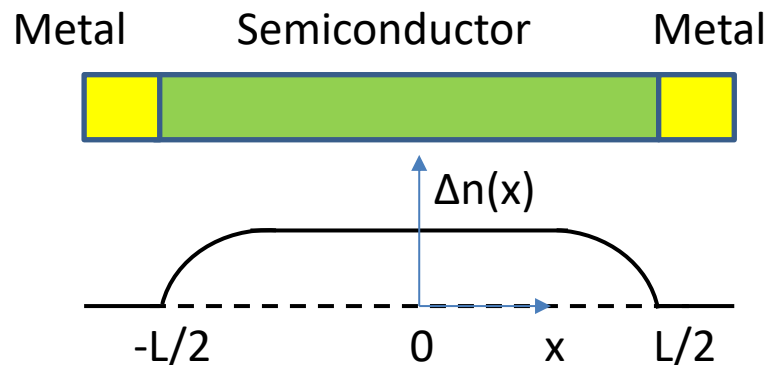
Table 6.2 |

Specification	Effect
Steady state	$\frac{\partial(\delta n)}{\partial t} = 0, \quad \frac{\partial(\delta p)}{\partial t} = 0$
Uniform distribution of excess carriers (uniform generation rate) + no boundary confinement	$D_n \frac{\partial^2(\delta n)}{\partial x^2} = 0, \quad D_p \frac{\partial^2(\delta n)}{\partial x^2} = 0$
Zero electric field	$E \frac{\partial(\delta n)}{\partial x} = 0, \quad E \frac{\partial(\delta p)}{\partial x} = 0$
No excess carrier generation	$g' = 0$
No excess carrier recombination (infinite lifetime)	$\frac{\delta n}{\tau_{n0}} = 0, \quad \frac{\delta p}{\tau_{p0}} = 0$

Check your understanding

Problem Exmple #2

Given a piece of p-type uniformly doped semiconductor in contact with two metal electrodes separated by a length of L , forming a photoconductor device. The light illumination will create electron-hole pairs at a generation rate of g . The minority carrier recombination lifetime is τ_0 . Find the analytical distribution of the excess minority electrons at zero external bias. Note that light illumination will not create excess carriers in metals.



Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

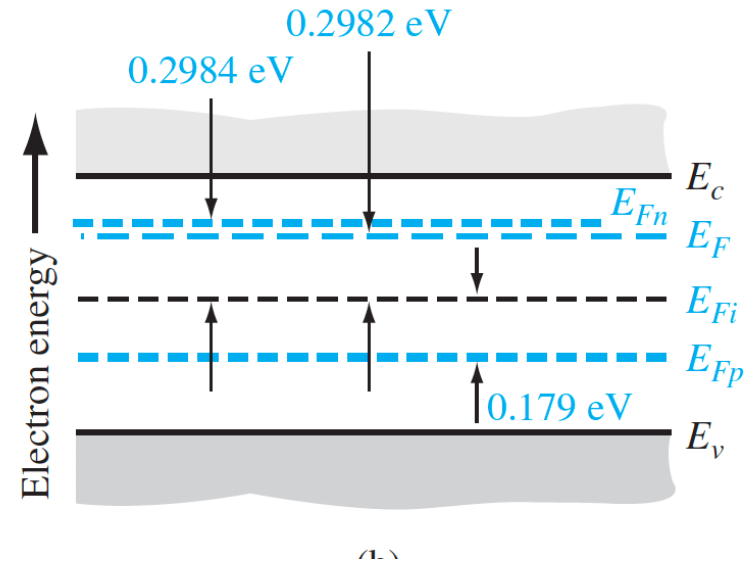
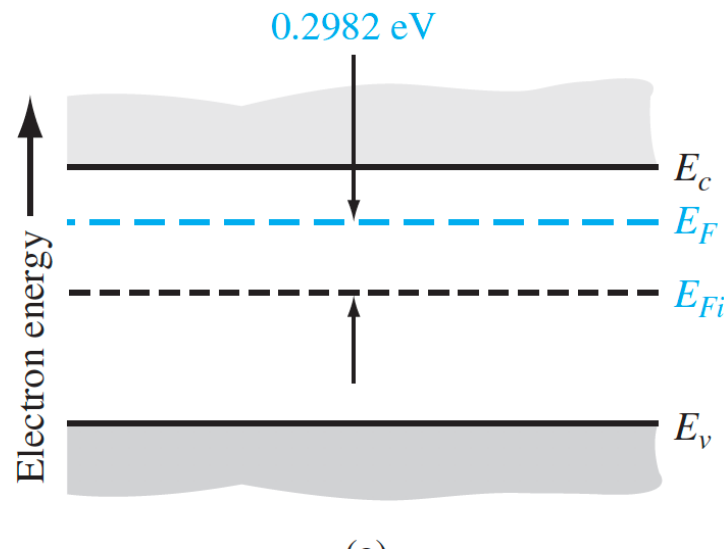
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

6.3 Quasi-Fermi energy level

$$n_0 = n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) \longrightarrow n_0 + \Delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right)$$

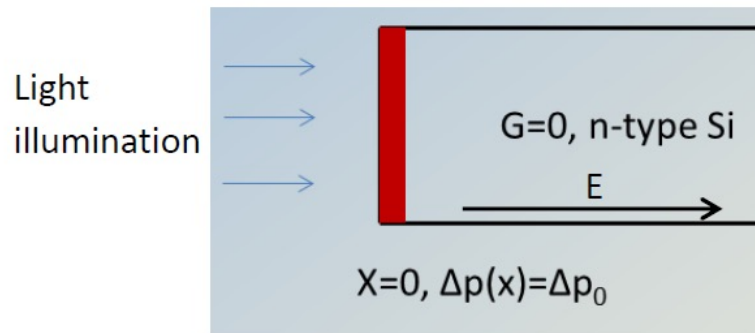


$$p_0 = n_i \exp\left(\frac{E_{Fi} - E_F}{kT}\right) \longrightarrow p_0 + \Delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right)$$

Check your understanding

Problem example #3

A light beam is illuminated on the surface of a silicon wafer, generating excess carriers Δp_0 at the surface ($x=0$). The wafer is placed in a constant electric field with a known intensity E . We assume there is no external generation inside the wafer. The thickness of the wafer is infinite. Find the excess minority carriers at equilibrium as a function of the distance away from the surface ($x=0$). Small injection condition is always maintained and the wafer is uniformly doped as N_d .



* Find the quasi Fermi level of holes.

$$\frac{\partial p}{\partial t} = D_p \frac{\partial^2 p}{\partial x^2} - \mu_p E \frac{\partial p}{\partial x} - p \mu_p \frac{\partial E}{\partial x} + -\frac{\Delta p}{\tau} + G_{ex}$$

Check your understanding

Check your understanding

Check your understanding

Outline

6.1 Carrier generation and recombination

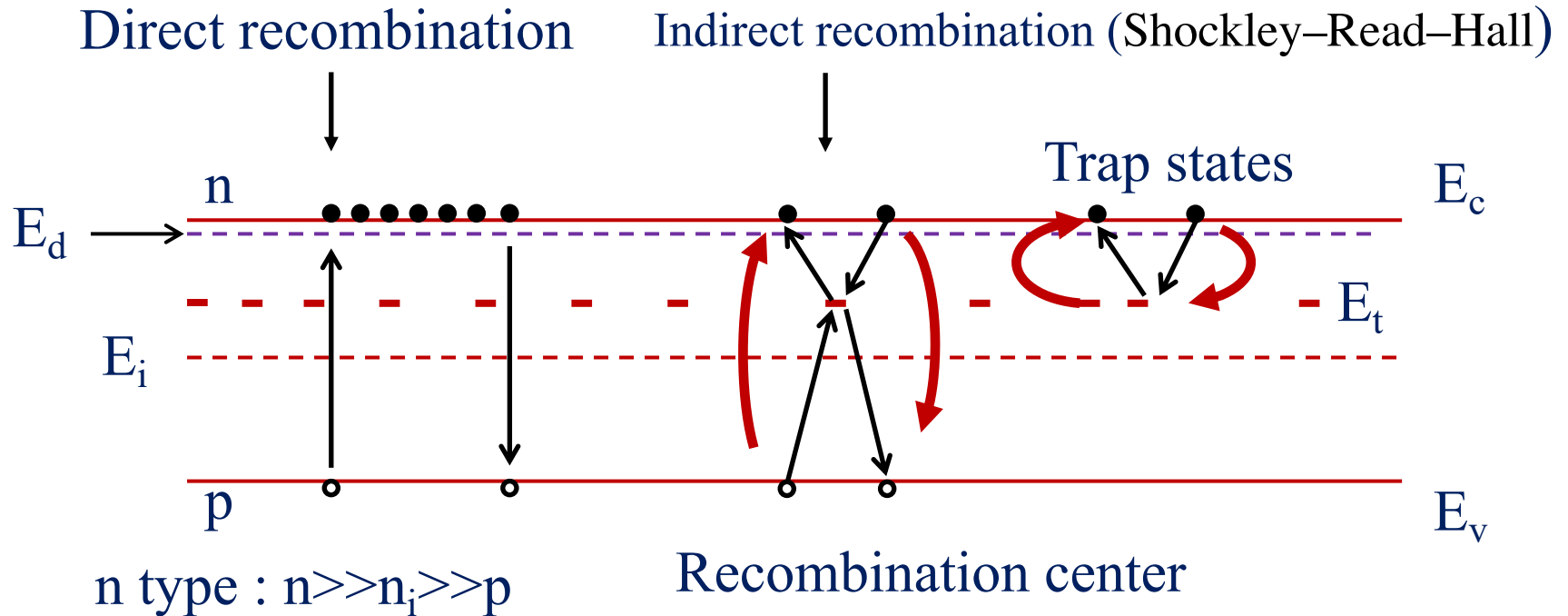
6.2 Characteristics of excess carriers

6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

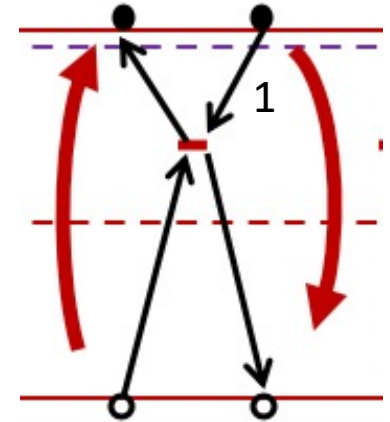
6.5 Surface effects

6.4 Excess carrier lifetime



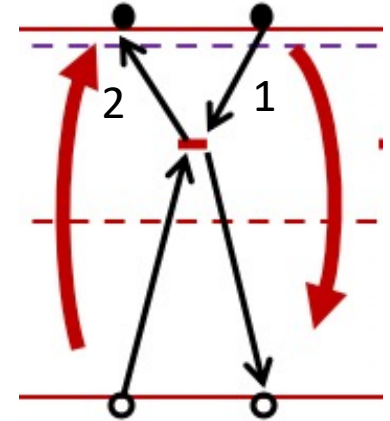
6.4 Excess carrier lifetime

1. Capture of an electron from conductance band by an initially neutral empty trap



6.4 Excess carrier lifetime

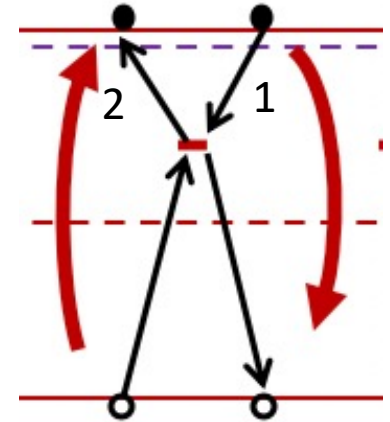
2. Inverse of process 1—the emission of an electron that is initially occupying a trap level back into the conduction band



6.4 Excess carrier lifetime

3. Capture of an hole from valence band by a trap containing an electron (Or we may consider the process to be the emission of an electron from the trap into the valence band.)

4. Inverse of process 3—the emission of a hole from a neutral trap into the valence band. (Or we may consider this process to be the capture of an electron from the valence band.)



6.4 Excess carrier lifetime

Check your understanding

Problem Example #4

A PN junction consisting an n-type semiconductor in contact with another p-type semiconductor (to be covered later) has a depletion region in which n_0 and p_0 are nearly zero. Suppose a silicon PN junction has defects located at the middle of the semiconductor. The defect concentration is 10^{16} cm^{-3} and the capture rate C_n and C_p for electrons and holes are $10^{-10} \text{ cm}^3/\text{s}$. Find the recombination rate of charge carriers in the depletion region of the Si PN junction.



Depletion region

$$N_t = 10^{16} \text{ cm}^{-3}$$
$$C_n = C_p = 10^{-10} \text{ cm}^3/\text{s}$$

Outline

6.1 Carrier generation and recombination

6.2 Characteristics of excess carriers

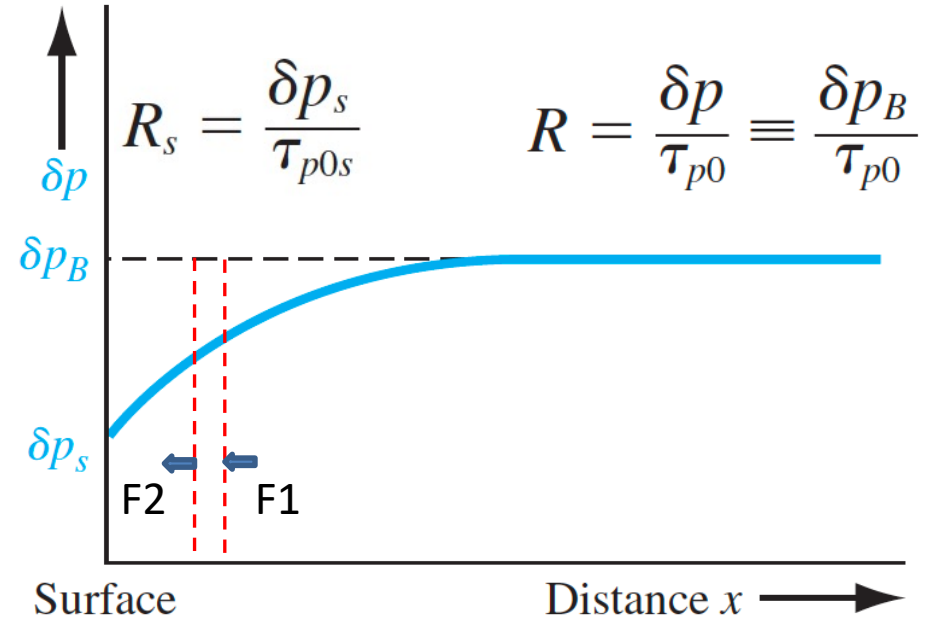
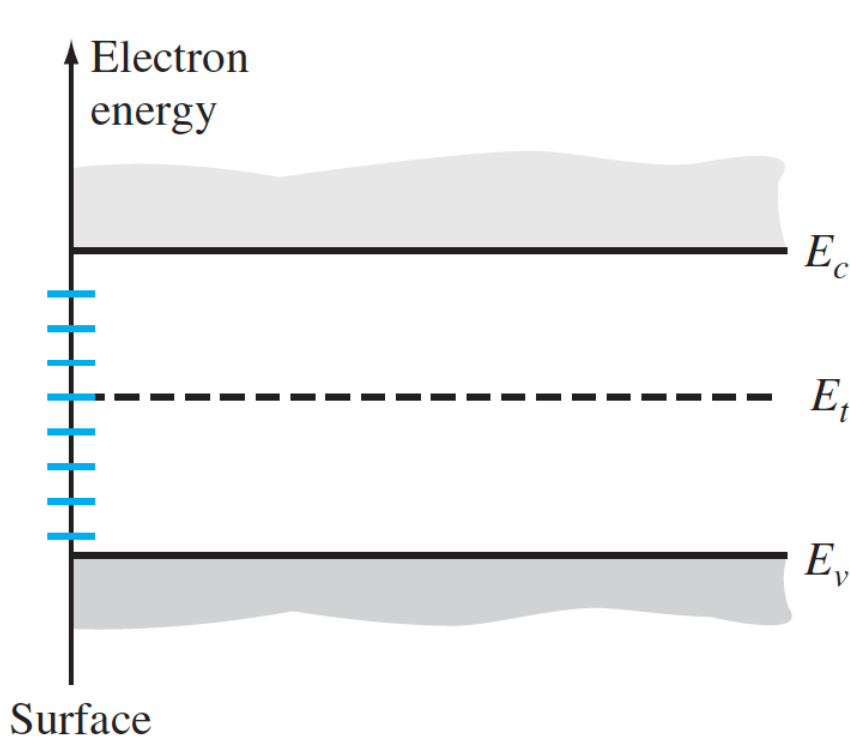
6.3 Quasi-Fermi levels

6.4 Excess carrier lifetime

6.5 Surface effects

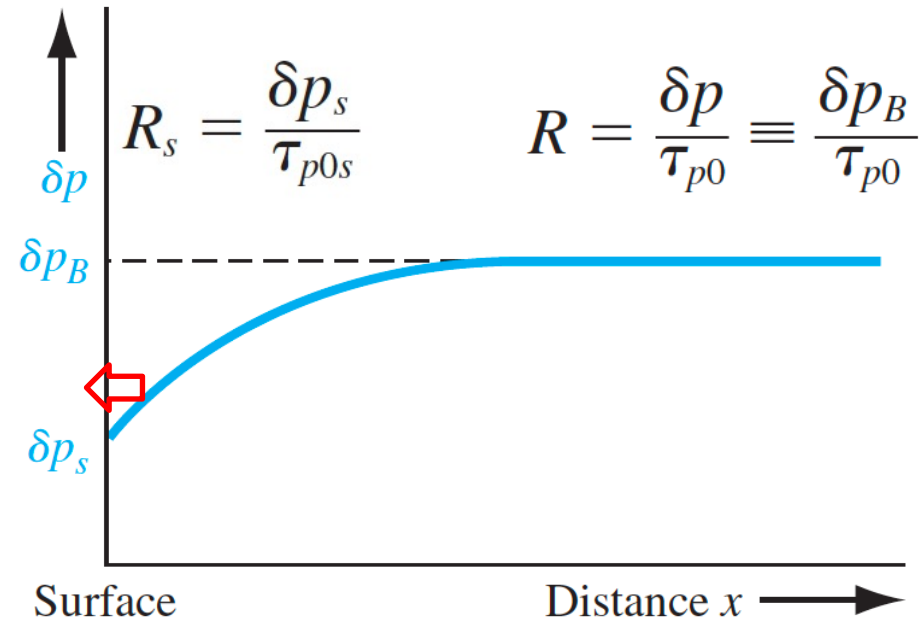
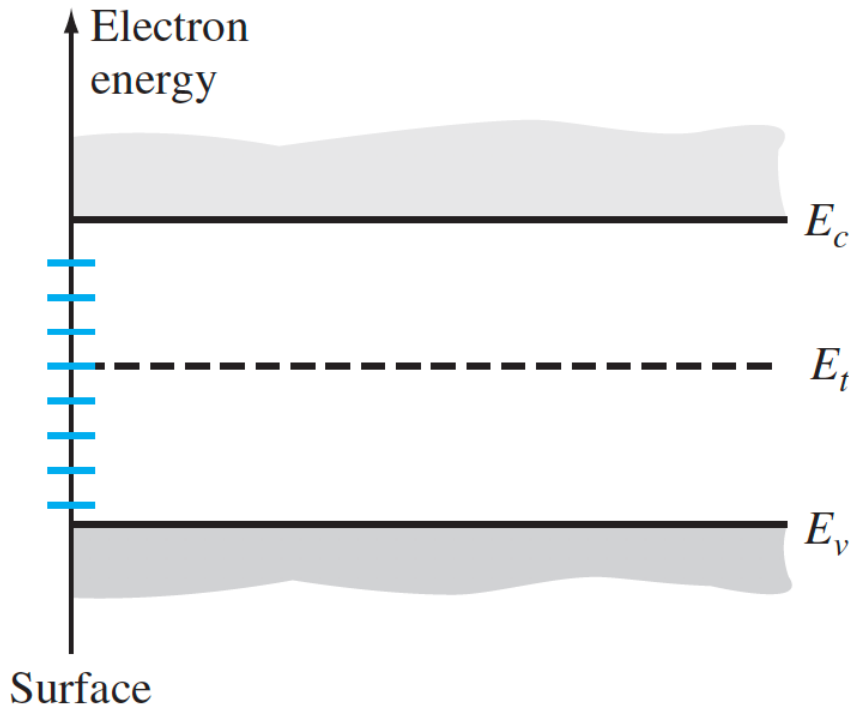
6.5 Surface effects

Surface States



6.5 Surface effects

Surface recombination velocity



Surface recombination rate:

number of recombined carriers in a unit surface area at a give unit time

Check your understanding

Problem example #5

A n-type semiconductor wafer is uniformly doped and uniformly illuminated by light. There is no electric field. The illumination generation rate is g and the minority carrier lifetime is τ . The semiconductor is long enough on the right side. On the left side edge, the surface recombination velocity is s . Find how does the concentration of the excess minority carriers change along x coordinate at equilibrium. Small injection condition is always maintained.

