Vv156 Honors Calculus II (Fall 2021)

Assignment 1

Date Due: 8:00 AM, Tuesday, Sep. 28, 2021
This assignment has a total of **(31 points)**.

Exercise 1.1 [Ste10, p. 23] Given $f, g : \mathbb{R} \to \mathbb{R}$, determine the parity of $f + g$ and $f \cdot g$ based on the parities of f and g. Fill in the following table.

f	g	$f+g$	$f \cdot g$
even	even		
even	odd		
odd	even		
odd	odd		

(4 points)

Exercise 1.2 [Ste10, p. 44] Given linear functions $f, g : \mathbb{R} \to \mathbb{R}$ with $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$. Is $f \circ g$ also a linear function? If so, what is the slope of its graph?
(2 points)

Exercise 1.3 [Ste10, p. 57] Given $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 5^x$, show that for $h \neq 0$,

$$\frac{f(x+h) - f(x)}{h} = 5^x \left(\frac{5^h - 1}{h} \right)$$

(2 points)

Exercise 1.4 Given functions $e, \tau, \tau', \tau'', \sigma, \sigma' : \{1, 2, 3\} \to \{1, 2, 3\}$ as follows,

x	$e(x)$	$\tau(x)$	$\tau'(x)$	$\tau''(x)$	$\sigma(x)$	$\sigma'(x)$
1	1	2	1	3	2	3
2	2	1	3	2	3	1
3	3	3	2	1	1	2

(i) `(3 points)` Complete the following composition table of functions using elements from the set $\{e, \tau, \tau', \tau'', \sigma, \sigma'\}$.

$\circ$	e	τ	τ'	τ''	σ	σ'
e						
τ						
τ'						
τ''						
σ'				$\sigma' \circ \tau''$		
σ						

(For example, $\sigma' \circ \tau''$ should be replaced with τ.)

(ii) `(6 points)` Let $f^{\circ n} := \underbrace{f \circ f \circ \cdots \circ f}_{n \text{ times}}$, $n \in \mathbb{N}$. For each $f \in \{e, \tau, \tau', \tau'', \sigma, \sigma'\}$, find the smallest number $n \in \mathbb{N}$ such that $f^{\circ n} = e$.

(9 points)

Exercise 1.5 Given $f : \mathbb{R} \to \mathbb{R}$, $f_0(x) = 3 + x/2$, and f_i, $i = 1, \ldots, 4$ as follows,

x	$f_1(x)$	x	$f_2(x)$	x	$f_3(x)$	x	$f_4(x)$
10.0	8.04	10.0	9.14	10.0	7.46	8.0	6.58
8.0	6.95	8.0	8.14	8.0	6.77	8.0	5.76
13.0	7.58	13.0	8.74	13.0	12.74	8.0	7.71
9.0	8.81	9.0	8.77	9.0	7.11	8.0	8.84
11.0	8.33	11.0	9.26	11.0	7.81	8.0	8.47
14.0	9.96	14.0	8.10	14.0	8.84	8.0	7.04
6.0	7.24	6.0	6.13	6.0	6.08	8.0	5.25
4.0	4.26	4.0	3.10	4.0	5.39	19.0	12.50
12.0	10.84	12.0	9.13	12.0	8.15	8.0	5.56
7.0	4.82	7.0	7.26	7.0	6.42	8.0	7.91
5.0	5.68	5.0	4.74	5.0	5.73	8.0	6.89

(i) `(5 points)` Sketch the graph of f_i, $i = 0, \ldots, 4$ (by hand or software).

(ii) `(4 points)` Calculate $\sum_{k=1}^{11} |f_0(x_k) - f_i(x_k)|^2$ for each $i = 1, \ldots, 4$, where $x_1, \ldots, x_{11}$ are taken from the x-column of the above table for different f_i's respectively.

(9 points)

Exercise 1.6 [Ste10, p. 44] The **Heaviside function** H is defined by

$$H(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

It is used in the study of electric circuits to represent the sudden surge of electric current, or voltage, when a switch is instantaneously turned on.

(a) `(1 point)` Sketch the graph of the Heaviside function.

(b) `(1 point)` Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and 120 volts are applied instantaneously to the circuit. Write a formula for $V(t)$ in terms of $H(t)$.

(2 points)

Exercise 1.7 [Ste10, p. 44] The Heaviside function defined in Exercise 1.6 can also be used to define the **ramp function** $y = ctH(t)$, which represents a gradual increase in voltage or current in a circuit.

(a) `(1 point)` Sketch the graph of the ramp function $y = tH(t)$.

(b) `(1 point)` Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 0$ and the voltage is gradually increased to 120 volts over a 60-second time interval. Write a formula for $V(t)$ in terms of $H(t)$ for $t \leq 60$.

(c) `(1 point)` Sketch the graph of the voltage $V(t)$ in a circuit if the switch is turned on at time $t = 7$ seconds and the voltage is gradually increased to 100 volts over a period of 25 seconds. Write a formula for $V(t)$ in terms of $H(t)$ for $t \leq 32$.

(3 points)

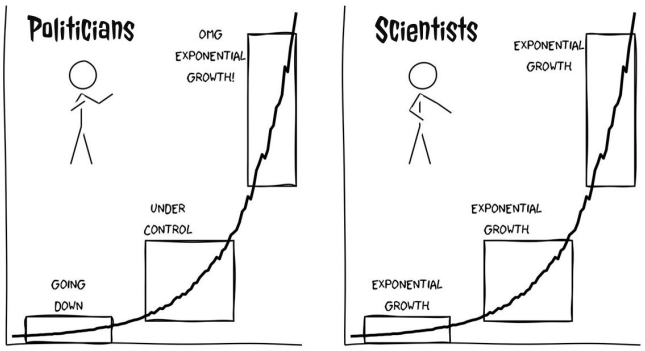

image from internet.

References

[Ste10] J. Stewart. *Calculus: Early Transcendentals*. 7th ed. Cengage Learning, 2010 (Cited on pages 1, 2).