Vv156 Honors Calculus II (Fall 2021)

Assignment 2

Date Due: 22:00 PM, Monday, Oct. 11, 2021
This assignment has a total of **(40 points)**.
Note: Unless specified otherwise, you must show the details of your work via logical reasoning for each exercise. Simply writing a final result (whether correct or not) will receive **0 point**.

Exercise 2.1 [Ste10, p. 189]

(i) `(2 points)` The curve $y = 1/(1+x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$.

(ii) `(2 points)` The curve $y = x/(1+x^2)$ is called a **serpentine**. Find an equation of the tangent line to this curve at the point $(3, 0.3)$.

(4 points)

Exercise 2.2 [Ste10, p. 190] If f is a differentiable function, find an expression for the derivative of each of the following functions.

(i) $y = x^2 f(x)$ (ii) $y = \dfrac{f(x)}{x^2}$ (iii) $y = \dfrac{x^2}{f(x)}$ (iv) $y = \dfrac{1 + xf(x)}{\sqrt{x}}$

(4 points)

Exercise 2.3 [Ste10, p. 191]

(a) If g is differentiable, the **Reciprocal Rule** says that

$$\frac{d}{dx}\left[\frac{1}{g(x)}\right] = -\frac{g'(x)}{[g(x)]^2}$$

Use the Quotient Rule to prove the Reciprocal Rule.

(b) Use the Reciprocal Rule to verify that the Power Rule is valid for negative integers, that is,

$$\frac{d}{dx}(x^{-n}) = -nx^{-n-1}$$

(4 points)

Exercise 2.4 [Ste10, p. 197] Calculate the **first** and **second** derivatives of the following functions.

(i) $f(x) = \sqrt{x}\sin x$ (ii) $f(x) = \sin x + \dfrac{1}{2}\cot x$ (iii) $f(x) = 2\sec x - \csc x$ (iv) $f(x) = \dfrac{x}{2 - \tan x}$

(v) $f(x) = \dfrac{\sec x}{1 + \sec x}$ (vi) $f(x) = \dfrac{x \sin x}{1 + x}$ (vii) $f(x) = \dfrac{1 - \sec x}{\tan x}$ (viii) $f(x) = x^2 \sin x \tan x$

(8 points)

Exercise 2.5 [Ste10, p. 197]

(a) Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan x - 1}{\sec x}$$

(b) Simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, and then find $f'(x)$.

(c) Show that your answers to parts (a) and (b) are equivalent.

(3 points)

Exercise 2.6 [Ste10, p. 198] Find the limit (use whatever method you like, but show the details of your work)

(i) $\lim_{x \to 0} \frac{\sin 3x}{x}$ (ii) $\lim_{x \to 0} \frac{\sin 4x}{\sin 6x}$ (iii) $\lim_{t \to 0} \frac{\tan 6t}{\sin 2t}$ (iv) $\lim_{\theta \to 0} \frac{\cos \theta}{\sin \theta}$

(v) $\lim_{x \to 0} \frac{\sin 3x}{5x^3 - 4x}$ (vi) $\lim_{x \to 0} \frac{\sin 3x \sin 5x}{x^2}$ (vii) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta + \tan \theta}$ (viii) $\lim_{x \to 0} \frac{2x}{x + \sin x}$

(ix) $\lim_{x \to \pi/4} \frac{1 - \tan x}{\sin x - \cos x}$ (x) $\lim_{x \to 1} \frac{\sin(x-1)}{x^2 + x - 2}$

(10 points)

Exercise 2.7 [Ste10, p. 198]

(a) Evaluate $\lim_{x \to \infty} x \sin \frac{1}{x}$.

(b) Evaluate $\lim_{x \to 0} x \sin \frac{1}{x}$.

(c) Illustrate parts (a) and (b) by graphing $y = \sin(1/x)$.

(3 points)

Exercise 2.8 [Ste10, p. 198] Find constants A and B such that the function $y = A \sin x + B \cos x$ satisfies the differential equation $y'' + y' - 2y = \sin x$.

(2 points)

Exercise 2.9 Given function f satisfying $|f(x)| \leq x^2$, calculate $f'(0)$.

(2 points)

References

[Ste10] J. Stewart. *Calculus: Early Transcendentals.* 7th ed. Cengage Learning, 2010 (Cited on pages 1, 2).