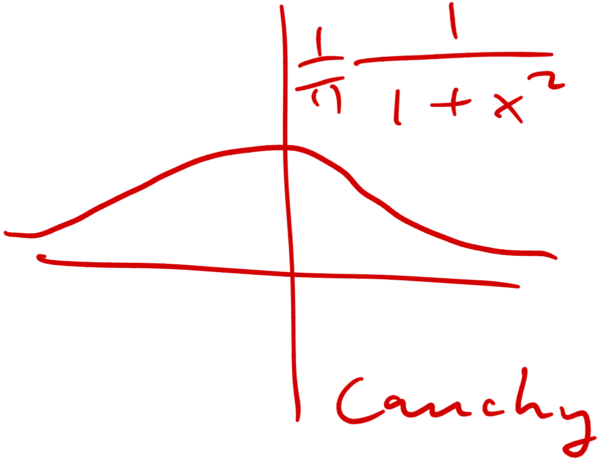
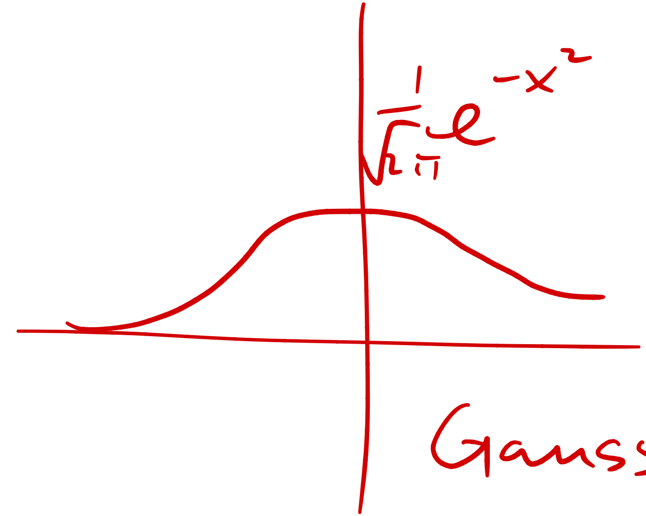
$$\int_{-\infty}^{\infty} x\,dx := \underbrace{\int_{-\infty}^{0} x\,dx}_{\to\infty} + \underbrace{\int_{0}^{\infty} x\,dx}_{\to\infty}$$

divergent

$$\underline{PV}\int_{-\infty}^{\infty} x\,dx := \lim_{c\to\infty}\int_{-c}^{c} x\,dx = 0$$





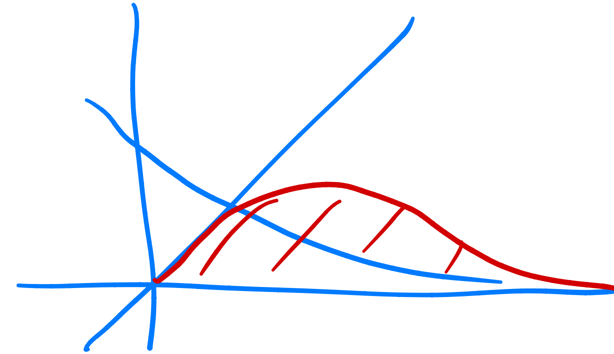
$$\text{mean} = \int_{-\infty}^{\infty} \frac{1}{\pi}\,\frac{x}{1+x^2}\,dx$$

does not exist!

mean = 0

variance = 1

Ex 2 Evaluate $\int_{-\infty}^{0} x e^{x}\, dx$



Let $y = -x$

$$= \int_{+\infty}^{0} (-y) e^{-y}\, d(-y)$$

$$= + \int_{0}^{\infty} (-y) e^{-y}\, dy = \int_{0}^{\infty} y e^{-y}\, dy$$

$$= -\int_{0}^{\infty} x e^{-x}\, dx$$

$$-\int_0^\infty x e^{-x}\,dx = \int_0^\infty x\,d(e^{-x})$$

$$= x e^{-x}\Big|_0^\infty - \int_0^\infty e^{-x}\,dx$$

$$= \lim_{t\to\infty}\left(\underline{t e^{-t}} - 0\right) - \lim_{t\to\infty}\int_0^t e^{-x}\,dx$$

$$= 0 - \lim_{t\to\infty}\left[-e^{-x}\right]_0^t$$

$$= +\left[\lim_{t\to\infty} e^{-t} - e^{-0}\right] = -1$$

$$e^x := 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$$

$$\lim_{x \to \infty} \frac{x^4}{e^x} = 0$$

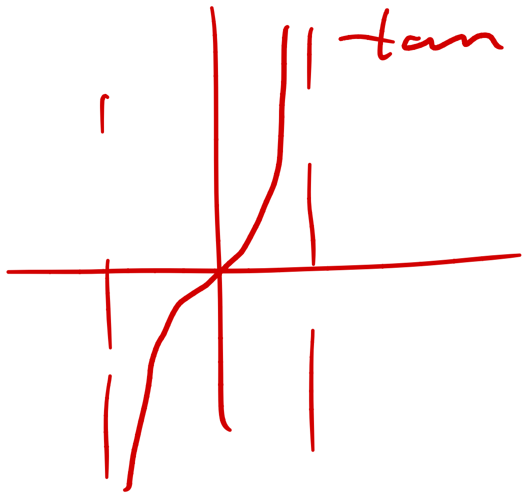
Ex 3 Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

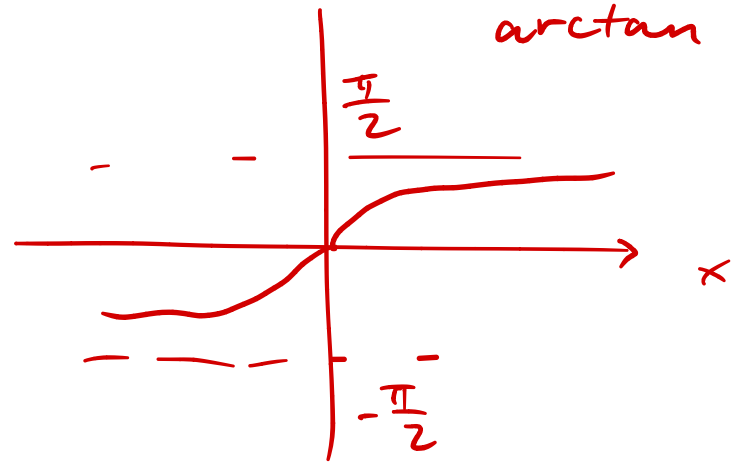
$$= \int_0^{\infty} \frac{dx}{1+x^2} + \int_{-\infty}^{0} \frac{dx}{1+x^2}$$

$$= \lim_{t\to\infty} \arctan x \Big|_0^t$$

$$+ \lim_{s\to-\infty} \arctan x \Big|_{-\infty}^0$$

$$= \cdots = \pi$$

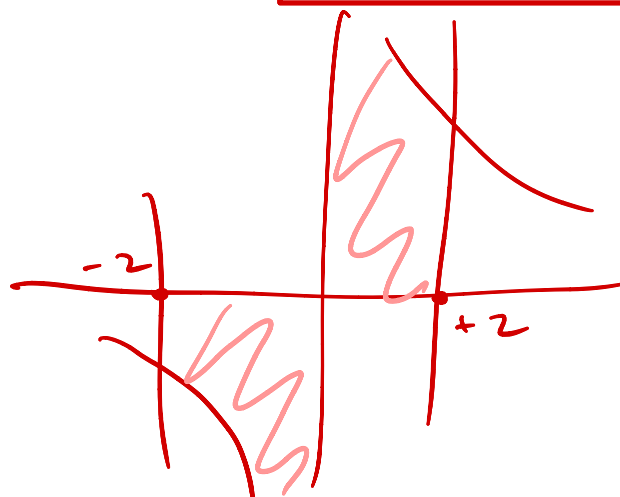




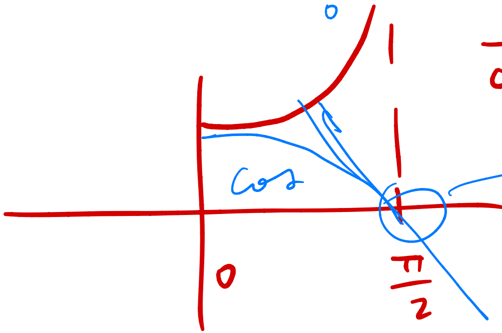
$$\lim_{c \to \infty} \int_{-c}^{2c} f(x)\,dx \neq \lim_{c \to \infty} \int_{-c}^{c} f(x)\,dx$$

$\frac{1}{x^2}$

$$\boxed{\int_{-2}^{2} \frac{1}{x}\,dx = ?}$$



Ex6 $\displaystyle\int_0^{\pi/2} \sec(x)\,dx$ **diverges**



$\dfrac{1}{\cos(x)}$

$$\int \frac{1}{-(x-\frac{\pi}{2})} = \ln|\cdot|$$

$k = -1$

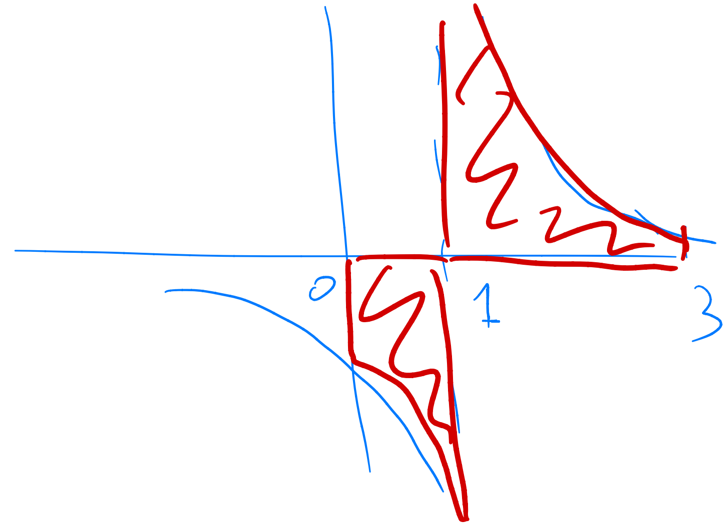
$y = -(x - \frac{\pi}{2})$

Ex 7 $\int_0^3 \frac{dx}{x-1}$

$$:= \int_0^1 \frac{dx}{x-1} + \int_1^3 \frac{dx}{x-1}$$

$$= \lim_{t \to 1^-} \int_0^t \frac{dx}{x-1}$$

$$+ \lim_{t \to 1^+} \int_t^3 \frac{dx}{x-1}$$



Ex 8

$$\int_0^1 \ln(x)\,dx = -\int_{-\infty}^0 e^x\,dx = -1$$