Vv156 Honors Calculus II (Fall 2021)

Assignment 7

Date Due: 22:00 PM, Friday, Dec. 10, 2021
This assignment has a total of **(54 points)**.
Note: Unless specified otherwise, you must show the details of your work via logical reasoning for each exercise. Simply writing a final result (whether correct or not) will receive **0 point**.

Exercise 7.1 (8 pts) [Ste10, p. 720] Use integral test to determine whether the series is convergent or divergent

(i) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ (ii) $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$ (iii) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ (iv) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

Exercise 7.2 (2 pts) [Ste10, p. 727] For what values of $p \in \mathbb{R}$ does the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$ converge?

Exercise 7.3 (2 pts) [Ste10, p. 727] Show that if $a \geq 0$ and $\sum a_n < \infty$, then $\sum a_n^2 < \infty$.

Exercise 7.4 Work out the details of using Shanks transformation to calculate $\mathscr{S}^{\circ 3}(S_3)$ of the series

$$\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

Exercise 7.5 (8 pts) [Ste10, p. 737] Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(i) $\sum_{n=1}^{\infty} \frac{n}{5^n}$ (ii) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$ (iii) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ (iv) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

Exercise 7.6 (8 pts) [Ste10, p. 745]

(i) $\sum_{n=1}^{\infty} (-1)^n n x^n$ (ii) $\sum_{n=1}^{\infty} \frac{(-x)^n}{n^2}$ (iii) $\sum_{n=2}^{\infty} \frac{(-x)^n}{4^n \ln n}$ (iv) $\sum_{n=2}^{\infty} \frac{x^{2n}}{n(\ln n)^2}$

Exercise 7.7 (4 pts) [Ste10, p. 751] Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

(i) $f(x) = \frac{3}{x^2 - x - 2}$ (ii) $f(x) = \frac{x+2}{2x^2 - x - 1}$

Exercise 7.8 (8 pts) [Ste10, p. 752] Find a power series representation for the function and determine the radius of convergence.

(i) $f(x) = \ln(5-x)$ (ii) $f(x) = x^2 \arctan(x^3)$ (iii) $f(x) = \frac{x}{(1+4x)^2}$ (iv) $f(x) = \frac{x^2 - x}{(1-x)^3}$

Exercise 7.9 (8 pts) [Ste10, p. 765] Find the Taylor series for $f(x)$ centered at the given value of a. [Assume that f has a power series expansion.] Also find the associated radius of convergence.

(i) $f(x) = x - x^3$, $a = -2$. (ii) $f(x) = 1/x$, $a = -3$. (iii) $f(x) = \sin x$, $a = \pi/2$. (iv) $f(x) = \sqrt{x}$, $a = 16$.

Exercise 7.10 (4 pts) Find general solution $x(t)$ to the following ODE's

(i) $\ddot{x} + 4\dot{x} + 5x = e^{5t} + te^{-2t} \cos t$ (ii) $\ddot{x} + 4\dot{x} + 4x = t^2 e^{-2t}$

References

[Ste10] J. Stewart. *Calculus: Early Transcendentals*. 7th ed. Cengage Learning, 2010 (Cited on page 1).