Vv156 Honors Calculus II (Fall 2021)

Assignment 4

Date Due: 22:00 PM, Monday, Nov. 8, 2021
This assignment has a total of **(34 points)**.
Note: Unless specified otherwise, you must show the details of your work via logical reasoning for each exercise. Simply writing a final result (whether correct or not) will receive **0 point**.

Exercise 4.1 [Ste10, p. 385]

(a) `(1 pt)` If f is continuous on $[a, b]$, use $-|f(x)| \leq f(x) \leq |f(x)|$ to show that

$$\left| \int_a^b f(x)\,\mathrm{d}x \right| \leq \int_a^b |f(x)|\,\mathrm{d}x$$

(b) `(1 pt)` Use the result of prevous part to show that

$$\left| \int_0^{2\pi} f(x)\sin(2x)\,\mathrm{d}x \right| \leq \int_0^{2\pi} |f(x)|\,\mathrm{d}x$$

(2 pts)

Exercise 4.2 [Ste10, p. 395] The **error function**

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,\mathrm{d}t$$

is used in probability, statistics, and engineering.

(a) `(1 pt)` Show that

$$\int_a^b e^{-t^2}\,\mathrm{d}t = \frac{1}{2}\sqrt{\pi}[\mathrm{erf}(b) - \mathrm{erf}(a)]$$

(b) `(1 pt)` Show that the function $y = e^{x^2}\mathrm{erf}(x)$ satisfies the differential equation $y' = 2xy + 2/\sqrt{\pi}$.

(2 pts)

Exercise 4.3 [Ste10, p. 396] The **sine integral function**

$$\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,\mathrm{d}t$$

is important in electrical engineering. [The integrand $f(t) = (\sin t)/t$ is not defined when $t = 0$, but we know that its limit is 1 when $t \to 0$. So we define $f(0) = 1$ and this makes f a continuous function everywhere.]

(a) `(1 pt)` Sketch the graph of Si.

(b) `(1 pt)` At what values of x does this function have local maximum values?

(c) `(1 pt)` Find the coordinates of the first inflection point to the right of the origin.

(d) `(1 pt)` Does this function have horizontal asymptotes?

(e) `(1 pt)` Solve the following equation (for x) correct to one decimal place:

$$\int_0^x \frac{\sin t}{t}\,\mathrm{d}t = 1$$

(5 pts)

Exercise 4.4 [Ste10, p. 396] Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on $[0, 1]$.

(a) `(1 pt)` $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$.

(b) `(1 pt)` $\displaystyle\lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \sqrt{\frac{i}{n}}$

(2 pts)

Exercise 4.5 [Ste10, p. 396] If f is continuous and g and h are differentiable functions, find a formula for

$$\frac{\mathrm{d}}{\mathrm{d}x}\int_{g(x)}^{h(x)} f(t)\,\mathrm{d}t$$

(2 pts)

Exercise 4.6 [Ste10, p. 396] Find a function f and a number a such that

$$6+\int_a^x \frac{f(t)}{t^2}\,\mathrm{d}t = 2\sqrt{x} \qquad \text{for all } x>0.$$

(2 pts)

Exercise 4.7 [Ste10, p. 414] Evaluate the indefinite and definite integral.

(i) $\displaystyle\int e^{\tan x}\sec^2 x\,\mathrm{d}x$ (ii) $\displaystyle\int \frac{\sin(\ln x)}{x}\,\mathrm{d}x$ (iii) $\displaystyle\int \sqrt{\cot x}\csc^2 x\,\mathrm{d}x$ (iv) $\displaystyle\int \frac{\mathrm{d}x}{\sqrt{1-x^2}\arcsin x}$

(v) $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,\mathrm{d}x$ (vi) $\displaystyle\int_{-\pi/3}^{\pi/3} x^4\sin x\,\mathrm{d}x$ (vii) $\displaystyle\int_e^{e^4}\frac{\mathrm{d}x}{x\sqrt{\ln x}}$ (viii) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{(1+\sqrt{x})^4}$

(8 pts)

Exercise 4.8 [Ste10, p. 412] If $f\in C^0(\mathbb{R})$, show that

(a) `(2 pts)` $\displaystyle\int_a^b f(-x)\,\mathrm{d}x = \int_{-b}^{-a} f(x)\,\mathrm{d}x.$

(b) `(2 pts)` $\displaystyle\int_a^b f(x+c)\,\mathrm{d}x = \int_{a+c}^{b+c} f(x)\,\mathrm{d}x.$

(c) `(2 pts)` $\displaystyle\int_0^\pi x f(\sin x)\,\mathrm{d}x = \frac{\pi}{2}\int_0^\pi f(\sin x)\,\mathrm{d}x.$

(d) `(2 pts)` $\displaystyle\int_0^{\pi/2} f(\cos x)\,\mathrm{d}x = \int_0^{\pi/2} f(\sin x)\,\mathrm{d}x.$

(8 pts)

Exercise 4.9 [Ste10, p. 412] Evaluate the definite integral.

(a) `(1 pt)` $\displaystyle\int_0^\pi \frac{x\sin x}{1+\cos^2 x}\,\mathrm{d}x.$

(b) `(1 pt)` $\displaystyle\int_0^{\pi/2}\cos^2 x\,\mathrm{d}x.$

(c) `(1 pt)` $\displaystyle\int_0^{\pi/2}\sin^2 x\,\mathrm{d}x.$

(3 pts)

References

[Ste10] J. Stewart. *Calculus: Early Transcendentals.* 7th ed. Cengage Learning, 2010 (Cited on pages 1, 2).