Vv156 Honors Calculus II (Fall 2021)

Assignment 5

Date Due: 22:00 PM, Monday, Nov. 17, 2021
This assignment has a total of **(40 points)**.
Note: Unless specified otherwise, you must show the details of your work via logical reasoning for each exercise. Simply writing a final result (whether correct or not) will receive **0 point**.

Exercise 5.1 [Ste10, p. 427] Sketch the region enclosed by the given curves and find its (unsigned) area.

(i) $y = \cos x$, $y = 2 - \cos x$, $0 \le x \le 2\pi$.

(ii) $x = 2y^2$, $x = 4 + y^2$.

(2 pts)

Exercise 5.2 [Ste10, p. 427] Evaluate the integral and interpret it as the area of a region. Sketch the region.

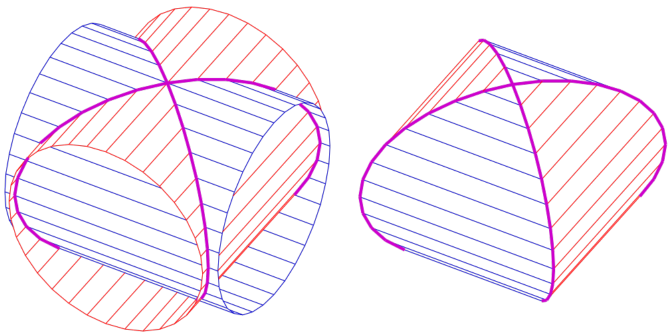
(i) $\displaystyle\int_0^{\pi/2} |\sin x - \cos 2x| \,\mathrm{d}x$

(ii) $\displaystyle\int_{-1}^{1} |3^x - 2^x| \,\mathrm{d}x$

(2 pts)

Exercise 5.3 [Ste10, p. 440] Find the volume common to two circular cylinders, each with radius r, if the axes of the cylinders intersect at right angles.[1]

(2 pts)



Exercise 5.4 [Ste10, p. 445] Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

(a) `(1 pt)` $y = x^3$, $y = 8$, $x = 0$; about $y = 0$.

(b) `(1 pt)` $x = 4y^2 - y^3$, $x = 0$; about $y = 0$.

(c) `(1 pt)` $y = x^4$, $y = 0$, $x = 1$; about $x = 2$.

(3 pts)

Exercise 5.5 [Ste10, p. 453]

(a) `(2 pts)` If f is continuous and $\displaystyle\int_1^3 f(x)\,\mathrm{d}x = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.

(b) `(2 pts)` Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

(4 pts)

Exercise 5.6 [Ste10, p. 470]

(a) `(2 pts)` Use integration by parts to show that

$$\int f(x)\,\mathrm{d}x = xf(x) - \int xf'(x)\,\mathrm{d}x$$

[1] `https://en.wikipedia.org/wiki/Steinmetz_solid`

(b) (2 pts) If f and g are inverse functions and f' is continuous, show that

$$\int_a^b f(x)\,\mathrm{d}x = bf(b) - af(a) - \int_{f(a)}^{f(b)} g(y)\,\mathrm{d}y$$

(c) (2 pts) In the case where f and g are positive functions and $b > a > 0$, draw a diagram to give a geometric interpretation of part (b).

(d) (2 pts) Use part (b) to evaluate $\displaystyle\int_1^e \ln x\,\mathrm{d}x$.

(8 pts)

Exercise 5.7 [Ste10, p. 478] Prove the formula, where m and n are positive integers.

(i) $\displaystyle\int_{-\pi}^{\pi} \sin mx \cos nx\,\mathrm{d}x = 0.$

(ii) $\displaystyle\int_{-\pi}^{\pi} \sin mx \sin nx\,\mathrm{d}x = \begin{cases} 0, & \text{if } m \neq n \\ \pi, & \text{if } m = n \end{cases}$

(iii) $\displaystyle\int_{-\pi}^{\pi} \cos mx \cos nx\,\mathrm{d}x = \begin{cases} 0, & \text{if } m \neq n \\ \pi, & \text{if } m = n \end{cases}$

(3 pts)

Exercise 5.8 [Ste10, p. 478] A finite fourier sine series is given by the sum $f(x) = \displaystyle\sum_{n=1}^{N} a_n \sin nx$. Show that the mth coefficient a_m is given by

$$a_m = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x) \sin mx\,\mathrm{d}x$$

(2 pts)

Exercise 5.9 [Ste10, p. 528] Evaluate the integral

(i) $\displaystyle\int_0^{\infty} \frac{\mathrm{d}x}{\sqrt{x}(1+x)}$

(ii) $\displaystyle\int_0^{\infty} \frac{\ln x}{1+x^2}\,\mathrm{d}x$

(4 pts)

Exercise 5.10 [Ste10, p. 543] Find the exact length of the curve.

(i) $y = \ln(\sec x)$, $1 \leq x \leq 2$.

(ii) $y = 3 + \frac{1}{2}\cosh 2x$, $0 \leq x \leq 1$.

(2 pts)

Exercise 5.11 [Ste10, p. 544] Find the arc length function for the curve $y = \arcsin x + \sqrt{1-x^2}$ with starting point $(0, 1)$.

(2 pts)

Exercise 5.12 [Ste10, p. 550] Find the exact area of the surface obtained by rotating the curve about the x-axis.

(i) $y = x^3$, $0 \leq x \leq 2$.

(ii) $9x = y^2 + 18$, $2 \leq x \leq 6$.

(2 pts)

Exercise 5.13 [Ste10, p. 573] Let $f(x) = 30x^2(1-x)^2$ for $0 \leq x \leq 1$ and $f(x) = 0$ otherwise.

(a) (1 pt) Verify that f is a probability density function.

(b) (1 pt) Find $P(X \leq \frac{1}{3})$.

(2 pts)

Exercise 5.14 [Ste10, p. 573] Let $f(x) = c/(1+x^2)$.

(a) (1 pt) For what value of c is f a probability density function?

(b) (1 pt) For that value of c, find $P(-1 < X < 1)$.

(2 pts)

References

[Ste10] J. Stewart. *Calculus: Early Transcendentals*. 7th ed. Cengage Learning, 2010 (Cited on pages 1, 2).