Vv156 Honors Calculus II (Fall 2021)

Assignment 6

Date Due: 22:00 PM, Thursday, Dec. 2, 2021
This assignment has a total of **(30 points)**.
Note: Unless specified otherwise, you must show the details of your work via logical reasoning for each exercise. Simply writing a final result (whether correct or not) will receive **0 point**.

Exercise 6.1 (8 pts) [Ste10, p. 641] Eliminate the parameter to find a Cartesian equation of the curve. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

(i) $x = 1 - t^2$, $y = t - 2$, $-2 \le t \le 2$.

(ii) $x = t - 1$, $y = t^3 + 1$, $-2 \le t \le 2$.

(iii) $x = \sin t$, $y = \csc t$, $0 < t < \pi/2$.

(iv) $x = \tan^2\theta$, $y = \sec\theta$, $-\pi/2 < \theta < \pi/2$.

Exercise 6.2 (4 pts) [Ste10, p. 651] Find $\mathrm{d}y/\mathrm{d}x$ and $\mathrm{d}^2y/\mathrm{d}x^2$. For which values of t is the curve convex?

(i) $x = 2\sin t$, $y = 3\cos t$, $0 < t < 2\pi$.

(ii) $x = \cos 2t$, $y = \cos t$, $0 < t < \pi$.

Exercise 6.3 (4 pts) [Ste10, p. 651] Given the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$, $a > 0$, $0 \le \theta < 2\pi$.

(a) (2 pts) Find the area of the region enclosed by the astroid.

(b) (2 pts) Find the total length of the astroid.

Exercise 6.4 (4 pts) [Ste10, p. 651] The **curvature** at a point P of a curve is defined as

$$\kappa = \left|\frac{\mathrm{d}\phi}{\mathrm{d}s}\right|$$

where ϕ is the angle of inclination of the tangent line at P. Thus the curvature is the absolute value of the rate of change of ϕ with respect to arc length.

(a) (2 pts) For a parametric curve $x = x(t)$, $y = y(t)$, show that

$$\kappa = \frac{|\dot{x}\ddot{y} - \ddot{x}\dot{y}|}{[\dot{x}^2 + \dot{y}^2]^{3/2}}$$

where the dots indicate derivatives with respect to t, i.e., $\dot{x} = \mathrm{d}x/\mathrm{d}t$.

(b) (2 pts) By regarding a curve $y = f(x)$ as the parametric curve $x = x$, $y = f(x)$, with parameter x, show that

$$\kappa = \frac{|\mathrm{d}^2y/\mathrm{d}x^2|}{[1 + (\mathrm{d}y/\mathrm{d}x)^2]^{3/2}}$$

Exercise 6.5 (2 pts) [Ste10, p. 664] Find the points on the given polar curve where the tangent line is horizontal or vertical.

(i) $r = 1 + \cos\theta$.

(ii) $r = e^\theta$.

Exercise 6.6 (2 pts) [Ste10, p. 669] Find the area enclosed by the loop of the **strophoid** $r = 2\cos\theta - \sec\theta$.

Exercise 6.7 (2 pts) [Ste10, p. 669] Find the area of the region that lies inside the first (polar) curve and outside the second (polar) curve.

(i) $r = 2\cos\theta$, $r = 1$.

(ii) $r = 1 - \sin\theta$, $r = 1$.

Exercise 6.8 (4 pts) [Ste10, p. 669] Find the exact length of the polar curve.

(i) $r = 2\cos\theta$, $0 \le \theta \le \pi$.

(ii) $r = 5^\theta$, $0 \le \theta \le 2\pi$.

(iii) $r = \theta^2$, $0 \le \theta \le 2\pi$.

(iv) $r = 2(1 + \cos\theta)$.

References

[Ste10] J. Stewart. *Calculus: Early Transcendentals*. 7th ed. Cengage Learning, 2010 (Cited on page 1).