Computational methods and applications (AMS 147)

Homework 4 - Due Sunday February 25

Please submit to CANVAS a .zip file that includes the following Matlab functions:

`poly_least_squares.m`
`test_least_squares.m`

Exercise 1 Write a Matlab function `poly_least_squares.m` that implements the least squares method we discussed in class to approximate a data set in terms of a polynomial model of degree M. The function should be of the form

```
function [a,err] = poly_least_squares(x,y,M)
```

Input:
`x`: vector of nodes `x=[x(1) ... x(N)]`
`y`: vector of data points `y=[y(1) ... y(N)]` corresponding to `[x(1) ... x(N)]`
`M`: degree of the polynomial model

$$\psi(x) = \texttt{a(1)} + \texttt{a(2)}x + \texttt{a(3)}x^2 + \cdots + \texttt{a(M+1)}x^M \tag{1}$$

Output:
`a`: Vector of coefficients representing the polynomial (1)
`err`: Error between the model and the data in the 2-norm

$$\texttt{err} = \sum_{i=1}^{N} \left[y_i - \psi(x_i)\right]^2 . \tag{2}$$

Exercise 2 Use the function you coded in Exercise 1 to determine the least squares polynomial approximants of the attached data set `AMD_data_2013_2018.dat` (daily closing prices of the Advanced Micro Devices Inc. stock from from 2/19/13 to 2/19/18). To this end, write a Matlab/Octave function `test_least_squares.m` of the form

```
function [x,p1,p2,p4,p8] = test_least_squares()
```

Output:
`x`: Vector of 1000 evenly-spaced evaluation nodes in $[0, 1]$ (including endpoints), i.e., `x(j) = (j-1)/999` with `j=1,...,1000`.

`p1, p2, p4, p8`: Least squares polynomial models (1) of the AMD stock price with degrees $M = 1, 2, 4, 8$, respectively, evaluated at `x`.

The function should also return one figure with the plots of the data points $\{x_i, y_i\}_{i=1,2,\ldots}$ in the file (in blue) and the least-squares polynomial models `p1, p2, p4` and `p8` you computed above (in red).

Hint: To load the AMD data in Matlab/Octave use the command `load()` (see the Matlab/Octave documentation for further details).