# CS264A: Automated Reasoning

Fall 2020

Homework 3

Due Date: Sunday, Nov 29

1. [12 pts] Consider a structured space of selecting at most $k-1$ items over $n$ items. There are in total $\sum_{0 \leq j \leq k} \binom{n}{j}$ possible selections. We want to construct a propositional sentence $\Delta$ such that there is a one-to-one correspondence between the selections of at most $k-1$ items and the satisfying assignments of $\Delta$. Suppose we use the propositional variable $A_i$ to indicate whether the item $i$ is selected. Please write a CNF which represents $\Delta$.

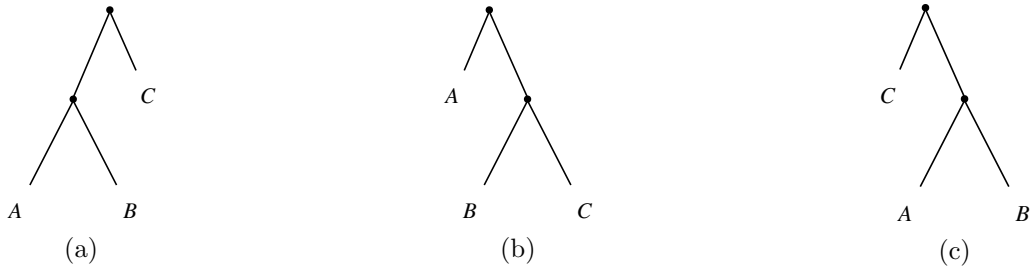


Figure 1: Vtree candidates

2. [10 pts] Consider the following knowledge base $\Delta$, represented as a CNF.

$$A \vee B \vee C$$
$$\neg B \vee \neg C$$

   (a) What is the **X**-**Y** partition of $\Delta$, where $\mathbf{X} = \{A, B\}$ and $\mathbf{Y} = \{C\}$?

   (b) To construct an SDD following the **X**-**Y** partition that you derived in the previous question, which of the vtrees in Figure 1 should be used?

3. [18 pts] Consider the following function $f$.
$f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$

   (a) What is the **X**-**Y** partition of $f$, where $\mathbf{X} = \{A, C\}$ and $\mathbf{Y} = \{B, D\}$?

   (b) What is the **X**-**Y** partition of $\neg f$, where $\mathbf{X} = \{A, C\}$ and $\mathbf{Y} = \{B, D\}$?

   (c) From part (a) and (b), can you come up with the general rule for finding a partition for $\neg f$ given a partition for $f$?

4. [10 pts] Consider the function in previous question, construct an SDD based on the vtree in Figure 2.

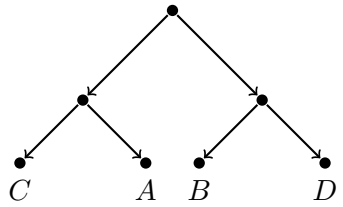


Figure 2: A vtree

5. [14 pts] Consider the following functions:
   $f = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D)$
   $g = (\neg A \wedge B) \vee (C \wedge \neg D)$
   $h = B \vee C$
   Let $\mathbf{X} = \{A, B\}$ and $\mathbf{Y} = \{C, D\}$

   (a) What is the **X**-**Y** partition of $f$?

   (b) What is the **X**-**Y** partition of $g$?

   (c) Compute the **X**-**Y** partition of $f \vee g$? Compare the result with the **X**-**Y** partition of $h$.

6. [10 pts] Consider the following CNF:
   $(\neg A \vee B \vee \neg C) \wedge (\neg A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee \neg B \vee C) \wedge (A \vee B \vee C)$
   What is the Prime Implicates of the CNF?

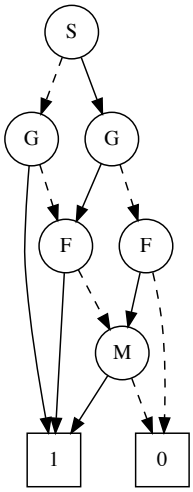


Figure 3: An OBDD

7. [14 pts] Consider a model that predicts a movie's box success based on four binary features, $S$ (Original Screenplay), $G$ (Great Cinematography), $F$ (Famous Cast), and $M$ (Marketing). The OBDD in Figure 3 describes the classification function, i.e. a feature configuration is evaluated to 1 if the corresponding movie is predicted to be a success. Please answer the following explanation queries on the OBDD.

   **Minimum Cardinality Explanation:** Consider a movie that is an original screenplay and has great cinematography, a famous cast, and poor marketing $\{S = 1, G = 1, F =$

$1, M = 0\}$. Identify a largest set of features that can be turned off (0) without changing the decision on this instance.

**Prime Implicant Explanation:** Consider a movie that is an original screenplay and has poor cinematography, a famous cast, and good marketing $\{S = 1, G = 0, F = 1, M = 1\}$. Identify a smallest set of features $\alpha$ that renders the remaining features $\beta$ irrelevant to the decision on this instance. That is, if we fix features $\alpha$ to their current values, we can change the values of features $\beta$ arbitrarily without changing the current decision.

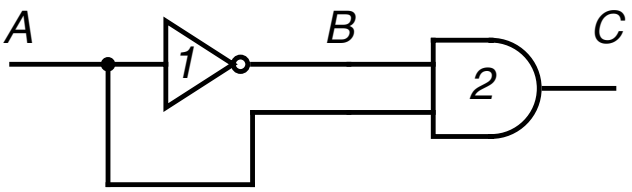


Figure 4: A Circuit

8. [12 pts] Please write the system description for the circuit in Figure 4. What is the kernel diagnosis and the minimal-cardinality diagnosis of the system when the output is observed to be *true*? What are the two diagnoses when the output is observed to be *false*?