CS264A: Homework #1

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Problem 1

1. **Solution:**
   w: A = true B = false
   In that case,
   $(\neg A \implies B)$ is True
   $(A \implies \neg B)$ is True
   so $(\neg A \implies B) \wedge (A \implies \neg B)$ is True
   Similarly $(A \wedge B) \implies (\neg A \vee \neg B)$ is also True

Problem 2

1. **Solution:**

A	B	$(A \implies B)$	$(\neg B \implies \neg A)$	$(A \implies B) \implies (\neg B \implies \neg A)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

2. **Solution:**

A	B	C	$(A \vee B) \wedge (A \implies C)$	$(B \vee C)$	$(A \vee B) \wedge (A \implies C) \implies (B \vee C)$
T	T	T	T	T	T
T	T	F	F	T	T
T	F	T	T	T	T
F	T	T	F	T	T
T	F	F	F	F	T
F	T	F	T	T	T
F	F	T	F	T	T
F	F	F	F	F	T

Problem 3

1. **Solution:**
   According to the definition of $\exists$
   $\exists P(\Delta \vee \Gamma)$ means There exists a P in knowledge base Δ or Γ
   Which is equivalent to There exists a P in knowledge base Δ or There exists a P in knowledge base Γ
   In that case, it is equivalent to $(\exists P \Delta) \vee (\exists P \Gamma)$

2. **Solution:**
   According to the definition of $\forall$
   $\forall P(\Delta \wedge \Gamma)$ means all P are in knowledge base Δ and Γ
   Which is equivalent to all P are in knowledge base Δ and all P are in knowledge base Γ
   In that case, it is equivalent to $(\forall P \Delta) \wedge (\forall P \Gamma)$

Problem 4

1. **Solution:** $(A \implies B) = \neg A \vee B = \{\neg A, B\}$
$\neg A \implies (\neg B \wedge C) = A \vee \neg(\neg B \wedge C) = A \vee (B \vee \neg C) = A \vee B \vee \neg C = \{A, B, \neg C\}$
$(B \vee C) \implies D = \neg(B \vee C) \vee D = (\neg B \wedge \neg C) \vee D = (\neg B \vee D) \wedge (\neg C \vee D) = \{\{\neg C, D\}, \{\neg B, D\}\}$

Problem 5

1. **Solution:** Since we have poly time procedure for clausal entailment, then we are able to show Gamma and Delta entail each other using the procedure. Finally we are able to show their equivalence in polynomial time.

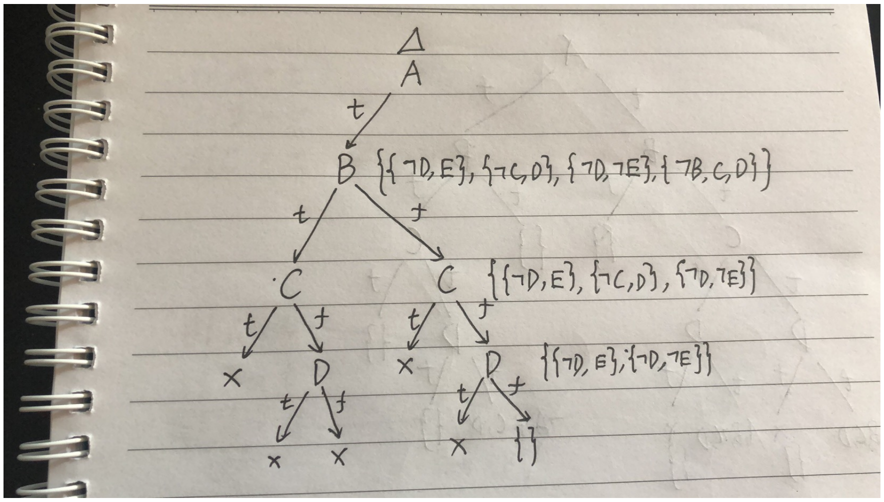
Problem 6

1. **Solution:** Convert the knowledge base to clausal form:
$\neg A \implies B = A \vee B = \{A, B\}$
$A \implies \neg C = \neg A \vee \neg C = \{\neg A, \neg C\}$
$\neg D \implies \neg B \wedge \neg C = D \vee (\neg B \wedge \neg C) = (D \vee \neg B) \wedge (D \vee \neg C) = \{\{\neg B, D\}, \{\neg C, D\}\}$ 3
$A \implies E = \{\neg A, E\}$
The clausal form of $\neg(D \vee E) = \neg D \wedge \neg E = \{\neg D\}, \{\neg E\}$
now we want to prove that $\Delta \wedge \{\neg D\}, \{\neg E\}$result in empty clause to prove $\Delta entails D \vee E$
1.$\{\neg D\}$ $\neg(D \vee E)$
2.$\{\neg B, D\}$ Δ
3.$\{\neg E\}$ $\neg(D \vee E)$
4.$\{\neg A, E\}$ Δ
5.$\{A, B\}$ Δ
6.$\{\neg B\}$ 1,2
7.$\{\neg A\}$ 3,4
8.$\{B\}$ 7,5
9,$\{\}$ 6,8
The above resolution trace proves that $\Delta| = (D \vee E)$, as it shows that the empty clause is derived from the clausal form of $\Delta \wedge \neg(D \vee E)$.

Problem 7

Solution: Convert to clausal form of Δ $\{\{\neg A, \neg D, E\}, \{\neg C, D\}.\{\neg D, \neg E\}, \{\neg B, C, D\}\}$

Termination Tree:

1. 

Problem 8

1. Convert to clausal form of Δ

   1.$\{\{\neg A, \neg D, E\}$,

   2.$\{\neg C, D\}$.

   3.$\{\neg D, \neg E\}$,

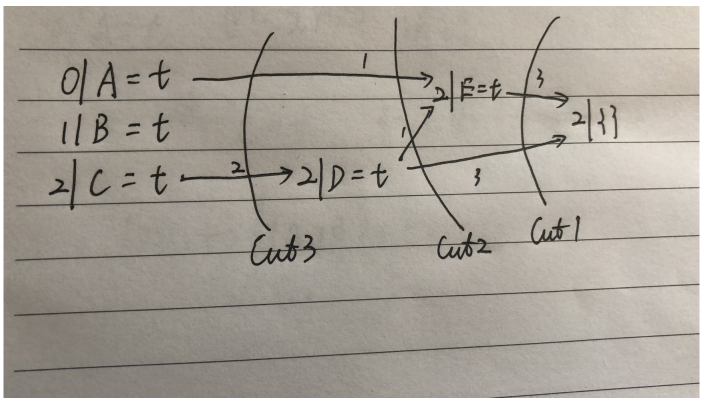
   4.$\{\neg B, C, D\}\}$

   $D = ()$ $\Gamma = \{\}$

   1).Using DPLL + Method in 3.5.2

   First Iteration: $D = (A = t, B = t, C = t)$

   According to the Implication graph:

   We have Conflict driven clause: $\{\neg A, \neg C\}$ al = 0, $\{\neg A, \neg D\}$ al = 0, $\{\neg E, \neg D\}$ not asserting

   then we pick $\alpha = \{\neg A, \neg C\}$

   m = 0

   D $\Leftarrow$ A =t

   $\Gamma \Leftarrow \{\{\neg A, \neg C\}\}$

   Second Iteration:

   $\Delta, \Gamma = \{\{\neg A, \neg C\}\}, D = (A = t)$

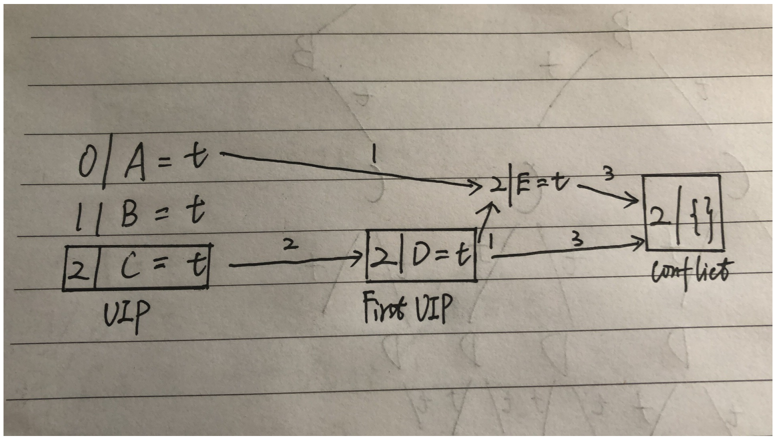
   No conflict sets detected,

   Finally $D \Leftarrow \{A = t, C = f, D = f, B = f, E = t\}$

2). Using DPLL + first UIP

First Iteration: $D = (A = t, B = t, C = t)$

According to the Implication graph:



Conflict driven clause generated via first UIP Method: From the above implication graph, $2|D = t$ would be a UIP as it lies on every path from decison $2|C = t$ to conflict.

Given the definitions, modern satisfiability solvers would generate an asserting conflict set which includes the first UIP at the current level (instead of the decision at the current level)

In that case, the conflict set $\{0|A = t, 2|D = t\}$ fits criteria. leading to the asserting conflict-driven clause $\neg A \vee \neg D$

We have the conflict driven clause: $\alpha = \{\neg A, \neg D\}$ at assertion level 0

$D \Leftarrow \{A = t\}$ $\Gamma = \{\{\neg A, \neg D\}\}$

Second Iteration: $D = (A = t)$

According to the implication graph, no conflict driven clause is detected. $D \Leftarrow \{A = t, C = f, D = f, B = f, E = t\}$

Problem 9

Solution: Clausal form CDPLL:

$Count(\Delta, 4) = Count(\Delta|A, 3) + Count(\Delta|\neg A, 3)$

$Count(\Delta|A, 3) = Count(\Delta|\{\{A\}, \{B\}\}, 2) + Count(\Delta|\{\{A\}, \{\neg B\}\}, 2)$

$Count(\Delta|\neg A, 3) = Count(\Delta|\{\{\neg A\}, \{B\}\}, 2) + Count(\Delta|\{\{\neg A\}, \{\neg B\}\}, 2)$

take $Count(\Delta|A, 3)$ as one example since iteration is similar:

$Count(\Delta|\{\{\neg A\}, \{B\}\}, 2) = Count(\Delta|\{\{\neg A\}, \{B\}, \{C\}\}, 1) + Count(\Delta|\{\{\neg A\}, \{B\}, \{\neg C\}\}, 1)$

$Count(\Delta|\{\{\neg A\}, \{\neg B\}\}, 2) = Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{C\}\}, 1) + Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{\neg C\}\}, 1)$

$Count(\Delta|\{\{\neg A\}, \{B\}, \{C\}\}, 1)$

$= Count(\Delta|\{\{\neg A\}, \{B\}, \{C\}, \{D\}\}, 0) + Count(\Delta|\{\{\neg A\}, \{B\}, \{C\}, \{\neg D\}\}, 0)$

$Count(\Delta|\{\{\neg A\}, \{B\}, \{\neg C\}\}, 1)$

$= Count(\Delta|\{\{\neg A\}, \{B\}, \{\neg C\}, \{D\}\}, 0) + Count(\Delta|\{\{\neg A\}, \{B\}, \{\neg C\}, \{\neg D\}\}, 0)$

$Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{C\}\}, 1)$

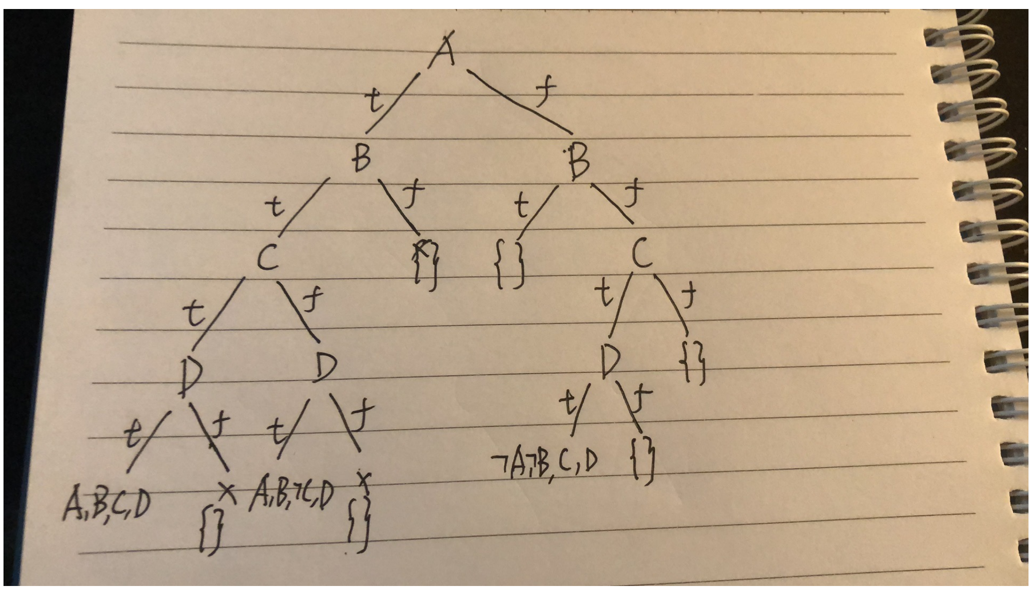
$= Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{C\}, \{D\}\}, 0) + Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{C\}, \{\neg D\}\}, 0)$

$Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{\neg C\}\}, 1)$

$= Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{\neg C\}, \{D\}\}, 0) + Count(\Delta|\{\{\neg A\}, \{\neg B\}, \{\neg C\}, \{\neg D\}\}, 0)$

Termination tree:

3 models in total

1. 

Problem 10

1. **Solution:** Convert to clausal form of Δ
$\{\{P_1, P_2, P_3\}, \{\neg P_1, Q\}.\{\neg P_2, Q\}, \{\neg P_3, Q\}\}$

2. **Solution:** Directed resolution:
$P_1 : \{\neg P, Q\}, \{\{P_1, P_2, P_3\}$
$P_2 : \{\neg P_2, Q\}, \{Q, P_2, P_3\}$
$P_3 : \{\neg P_3, Q\}\}, \{Q, P_3\}$
$Q : Q$
Done, So the knowledge base is satisfiable.

3. **Solution:** 7 models
The decision tree: