CS264A: Automated Reasoning

Fall 2020

Homework 2

Due Date: November 4th, 2020

1. [8 pts] Identify a minimal unsatisfiable core of the following CNF:

$$\begin{aligned}\Delta = &(X \vee Y),\\ &(X \vee Z),\\ &(\neg Y \vee W),\\ &(\neg Z \vee \neg W),\\ &(X \vee \neg Y),\\ &(\neg X \vee W),\\ &(\neg X \vee \neg W \vee V),\\ &(\neg X \vee \neg W \vee \neg V),\\ &(Z \vee \neg X),\\ &(\neg Z \vee W).\end{aligned}$$

2. [8 pts] Consider the CNF:

$$\Delta = \ (\neg X), (X \vee Y), (X \vee Z), (\neg Y \vee \neg Z).$$

   a. Assuming that every clause has weight 1, construct a table that shows the costs of all worlds. What is the optimal solution for this Max-Sat problem?
   b. Use Max-Sat resolution to derive an empty clause (show trace). What compensation clauses are generated?

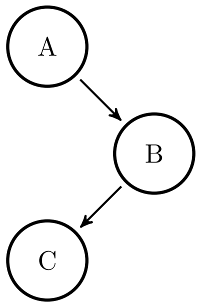
3. [8 pts] Consider the CNF: $\Delta = \ (A \vee B \vee C),\ (\neg A \vee D \vee E)$ and assume that each clause has weight 1. What is the CNF which results from applying Max-Sat resolution to Δ?

4. [16 pts] Consider the CNF:

$$\Delta = (\neg D \vee \neg E \vee B), (\neg B \vee E \vee \neg A), (\neg D \vee C \vee \neg B), (\neg B \vee C \vee E)$$

   (a) What is the solution of the Maj-Sat problem on this CNF? Justify your answer.
   (b) Using the split $\boldsymbol{X} = \{A, B, C\}, \boldsymbol{Y} = \{D, E\}$, what is the solution of the E-Maj-Sat problem on this CNF? Justify your answer.
   (c) Using the same split as in (b), what is the solution of the Maj-Maj-Sat problem on this CNF? Justify your answer.

5. [12 pts] Figure 1 shows a Bayesian network. Encode this Bayesian network as a Boolean formula Δ with weights on literals so that weighted model counting on Δ can be used to compute marginal



A	B: T	B: F
F	$\theta_{B\mid\neg A}$	$\theta_{\neg B\mid\neg A}$
T	$\theta_{B\mid A}$	$\theta_{\neg B\mid A}$

B	C: T	C: F
F	$\theta_{C\mid\neg B}$	$\theta_{\neg C\mid\neg B}$
T	$\theta_{C\mid B}$	$\theta_{\neg C\mid B}$

A: T	A: F
θ_A	$\theta_{\neg A}$

Figure 1: A Bayesian network with CPTs

probabilities on the Bayesian network. Compute the weighted model count of $A = T, C = F$ using Δ in terms of given parameters. Substitute the following values and report the final answer.

$$\theta_{B|\neg A} = 0.1, \theta_{\neg B|\neg A} = 0.9, \theta_{B|A} = 0.8, \theta_{\neg B|A} = 0.2$$

$$\theta_{C|\neg B} = 0.3, \theta_{\neg C|\neg B} = 0.7, \theta_{C|B} = 0.25, \theta_{\neg C|B} = 0.75$$

$$\theta_A = 0.6, \theta_{\neg A} = 0.4$$

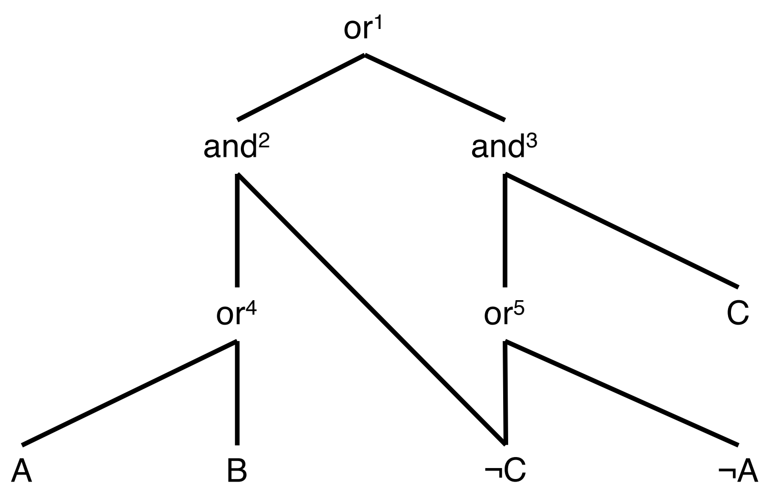


Figure 2: An NNF

6. [4 pts] Please identify and explain whether the NNF circuit shown in Figure 2 is decomposable and/or deterministic.

7. [12 pts] Consider a DNNF that is smooth in Figure 3.

   (a) Run the minimum cardinality query on this circuit.

   (b) Explain how to check whether the clause $(A \vee B)$ is entailed.

   (c) Explain how to existential quantify variable C, and show the resulting circuit.

8. [8 pts] Figure 4 shows an OBDD representing a sentence Δ with three propositions A, B, C.

   (a) Label the model count of every node in the circuit.

   (b) Does the OBDD entail the clause $(A \vee \neg B)$?

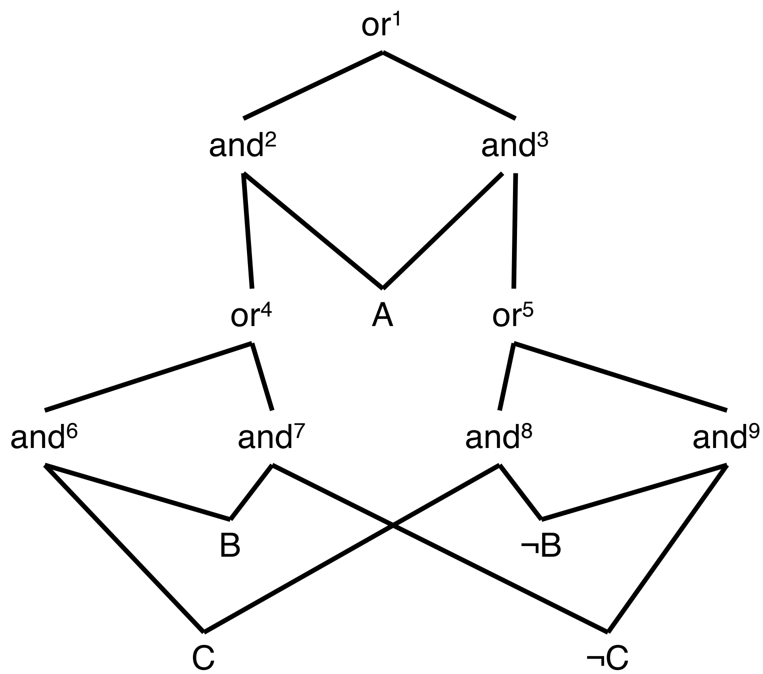


Figure 3: A DNNF

9. [16 pts] The cardinality of a world equals the number of positive assignments in it. For example, the world $A = T, B = T, C = F$ has the cardinality 2. Consider a knowledge base Δ whose models consist of all worlds, over variables $X_1, \cdots, X_N$, with odd cardinality.

   (a) Given a variable order $X_1, \cdots, X_N$, what are the sub-functions after setting variables $X_1, \cdots, X_{\lfloor N/2 \rfloor}$?

   (b) How many nodes in the reduced OBDD representing Δ with the variable order $X_1, \cdots X_N$?

   (c) Draw the OBDD over 4 variables with the order X_1, X_2, X_3, X_4?

10. [8 pts] Describe a property on NNF circuits which allows one to universally quantify variables in time linear in the circuit size. Justify your answer.

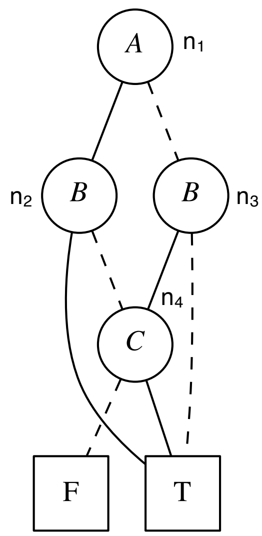


Figure 4: An OBDD