CS264A: Automated Reasoning

Fall 2020

Homework1

Due Date: Wednesday, Oct 21

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1. [8 pts] Show that the following sentences are consistent by identifying a world which satisfies each sentence:

   - $(\neg A \Rightarrow B) \wedge (A \Rightarrow \neg B)$.
   - $(A \wedge B) \Rightarrow (\neg A \vee \neg B)$.

2. [8 pts] Show that the following sentences are valid by showing that each is true at every world:

   - $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$.
   - $((A \vee B) \wedge (A \Rightarrow C)) \Rightarrow (B \vee C)$.

3. [8 pts] Prove from the definitions of Boolean quantifiers $\exists$ and $\forall$ that (a) $\exists P(\Delta \vee \Gamma)$ is equivalent to $(\exists P \Delta) \vee (\exists P \Gamma)$, and (b) $\forall P(\Delta \wedge \Gamma)$ is equivalent to $(\forall P \Delta) \wedge (\forall P \Gamma)$.

4. [8 pts] Convert the following knowledge base to clausal form:

$$\Delta = \ A \Rightarrow B, \ \neg A \Rightarrow (\neg B \wedge C), \ (B \vee C) \Rightarrow D.$$

5. [8 pts] Show that if we have a polynomial procedure for model counting, and another for clausal entailment on a knowledge base Γ, then we have a polynomial procedure for testing the equivalence between Γ and any CNF Δ.

6. [10 pts] Show using resolution that $D \vee E$ is entailed by the knowledge base:

$$\Delta = \ \neg A \Rightarrow B, \ A \Rightarrow \neg C, \ \neg D \Rightarrow \neg B \wedge \neg C, \ A \Rightarrow E.$$

7. [12 pts] Show the termination tree for DPLL when run on the following KB, assuming that variables are tested according to the order A, B, C, D, E and *true* expanded before *false*:

$$\Delta = \begin{array}{l} 1.\ A \wedge D \Rightarrow E \\ 2.\ C \Rightarrow D \\ 3.\ D \Rightarrow \neg E \\ 4.\ B \wedge \neg C \Rightarrow D \end{array}$$

   Note that DPLL does not use conflict–directed backtracking.

8. [12 pts] Show a trace of DPLL+ on the above KB, assuming that decisions are made according to the constraints given above. At each conflict, show the decision sequence, implication graph, conflict–drive clause, and its assertion level. Perform one trace of DPLL+which assuming that conflict–driven clauses are generated using the first UIP method of Section 3.6.2.

9. [12 pts] Consider the following knowledge base.

$$\Delta = \ A \Rightarrow B, \ \neg A \Rightarrow (\neg B \wedge C), \ (B \vee C) \Rightarrow D.$$

Show how you can count the number of models of Δ using CDPLL and draw the termination tree. Assume that you are expanding variables according to the order A, B, C, D and always expand *true* before *false*.

10. [14 pts] Consider the following knowledge base:

$$\Delta = P_1 \vee P_2 \vee P_3, \ P_1 \Rightarrow Q, \ P_2 \Rightarrow Q, \ P_3 \Rightarrow Q.$$

a. Convert Δ into clausal form.

b. Apply directed resolution to the clausal form using the order P_1, P_2, P_3, Q.

c. Construct a decision tree for Δ and use it to count the number of models of Δ.