CS264A: Homework #3

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Nov 27, 2020

Problem 1

Solution:

1. When we select $k-1$ items and there is a one to one correspondence and Δ is true, we need to specify that ~~it is~~ $k-1$ item makes it true but k items will make it false. For example, if we pick A_1 to A_{k-1}, the Δ is true but if we pick A_1 to A_k then Δ is false.
To satisfy this, we need to have

$$\Delta = (-A_1 \lor -A_2 \lor \cdots -A_{k-1} \lor -A_k)$$

So when A_k is false the Δ is true, but if all items in $-A_1$ to $-A_k$ are true, the Δ is false.
Now it is just one scenario of $k-1$ selection in n items. We need to find all possible combination of k items in n items space. For example if we select first $k-1$ items but ~~to~~ randomly select last item, we will have

$$\Delta = \underbrace{(-A_1 \lor -A_2 \lor \cdots -A_{k-1} \lor -A_k) \lor (-A_1 \cdots \lor -A_{k-1} \lor -A_{k+1}) \cdots \lor (-A_1 \lor \cdots \lor -A_{k-1} \lor -A_n)}_{n-k+1 \text{ clauses in total}}$$

Similarly, we can have first $k-2$ items selected and so on. The component in clauses will be all possible combination of k items.

$$\Delta = \underbrace{(-A_1 \lor -A_2 \lor \cdots \lor -A_{k-1} \lor -A_k) \cdots \lor (-A_1 \lor \cdots \lor -A_{k-1} \lor -A_n) \cdots (-A_{n-k+1} \lor -A_{n-k+2} \cdots -A_{n-1} \lor -A_n)}_{\binom{n}{k} \text{ clauses in total}}$$

Problem 2

1. **Solution:**

(2)

a.

Prime	Sub
$A \wedge B$	$\neg C$
$\bar{A} \wedge B$	$\neg C$
$A \wedge \bar{B}$	True
$\bar{A} \wedge \bar{B}$	C

$$f = (A \wedge \bar{B})(\text{true}) + (\bar{A} \wedge \bar{B})(C) + (B)(\neg C)$$

b.

Vtree has to have X on left and Y on right which is (a).

Problem 3

1. **Solution:**

(3).

(a).

Prime	Sub
$A \wedge C$	$B \vee D$
$\bar{A} \wedge C$	$B \vee D$
$A \wedge \bar{C}$	B
$\bar{A} \wedge \bar{C}$	False

$f = (C)(B \vee D) + (A \wedge \bar{C})(B) + (\bar{A} \wedge \bar{C})(\text{False})$

(b)

Prime	Sub
$A \wedge C$	$\neg B \wedge \neg D$
$\bar{A} \wedge C$	$\neg B \wedge \neg D$
$A \wedge \bar{C}$	$\neg B$
$\bar{A} \wedge \bar{C}$	True

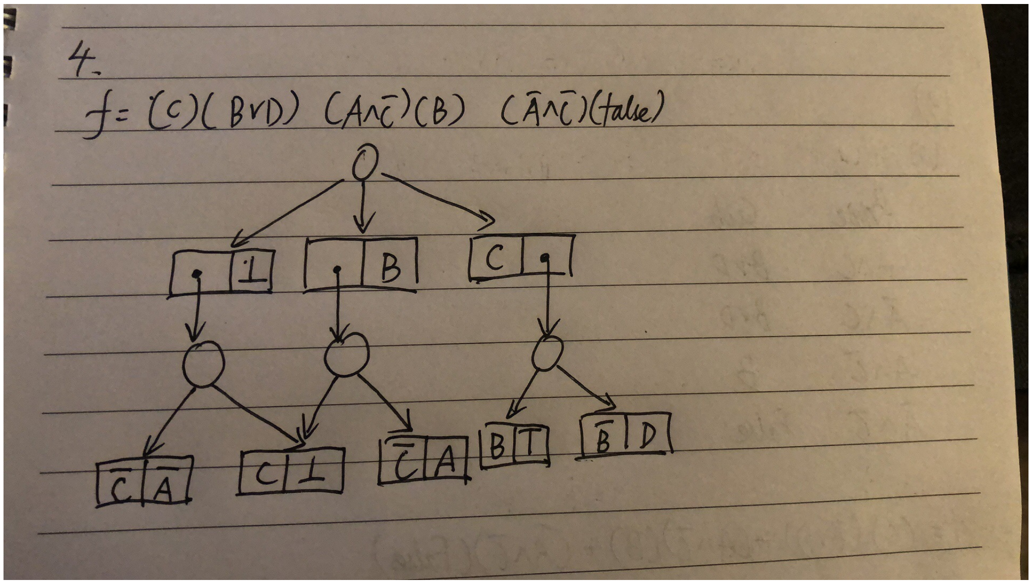
X-Y partition

$\neg f = (C)(\bar{B} \wedge \bar{D}) + (A \wedge \bar{C})(\bar{B}) + (\bar{A} \wedge \bar{C})(\text{True})$

(c). To get the partition for $\neg f$ we can just negate every sub in the X-Y partition of f.

Problem 4

Solution:

1. 

Problem 5

Solution:

1.

Memo No. ______
Date / /

Mo Tu We Th Fr Sa Su

5.

(a). X-Y partition

$f = (A \wedge B)(\text{true}) + (\bar{A} \wedge B)(C) + (\bar{B})(C \wedge D)$

(b). X-Y partition

Prime.	Sub
$A \wedge B$	$C \wedge \neg D$
$\bar{A} \wedge B$	true
$A \wedge \bar{B}$	$C \wedge \neg D$
$\bar{A} \wedge \bar{B}$	$C \wedge \neg D$

$f = (\bar{A} \wedge B)(\text{true}) + (A \vee \bar{B})(C \wedge \neg D)$

(c).

Prime.	Sub
$A \wedge B$	true
$\bar{A} \wedge B$	true
$A \wedge \bar{B}$	C
$\bar{A} \wedge \bar{B}$	C

$f \vee g = (A \wedge B) \vee (B \wedge C) \vee (C \wedge D) \vee (\neg A \wedge B) \vee (C \wedge \neg D)$

$f \vee g = (B)(\text{true}) + (\bar{B})(C)$

X-Y partition of h

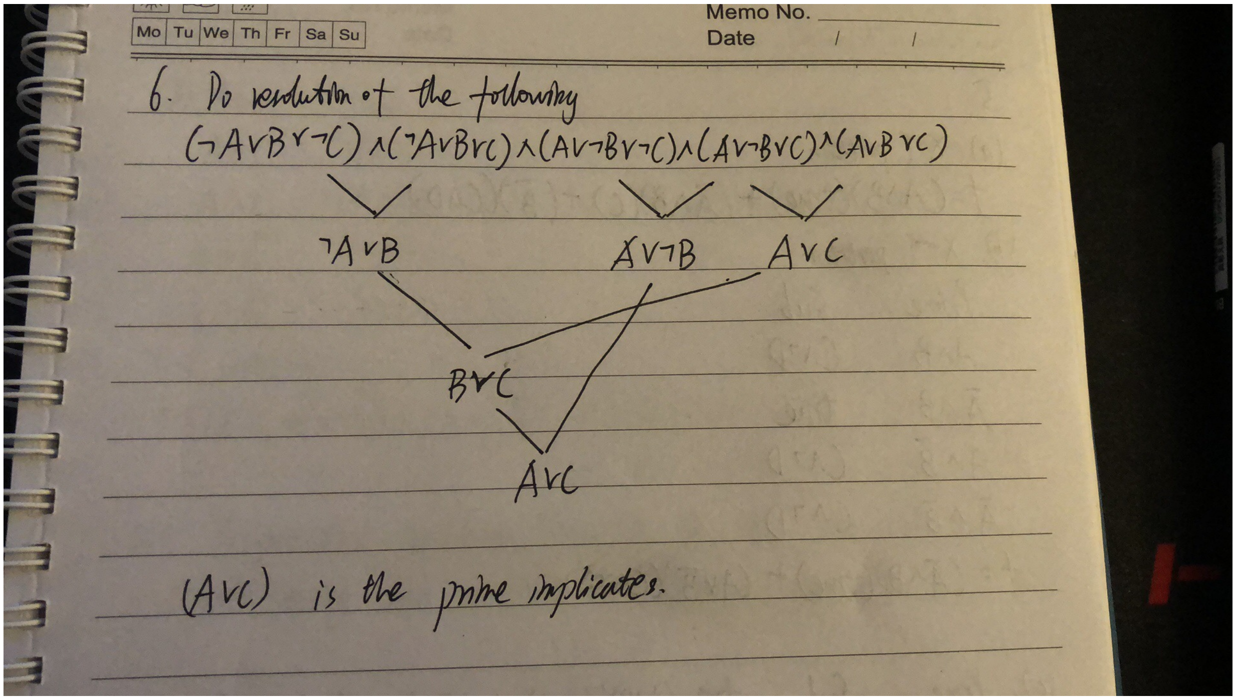
Prime	Sub
$A \wedge B$	true
$\bar{A} \wedge B$	true
$A \wedge \bar{B}$	C
$\bar{A} \wedge \bar{B}$	C

$f = (B)(\text{true}) + (\bar{B})(C)$

They are the same.

Problem 6

Solution:

1. 

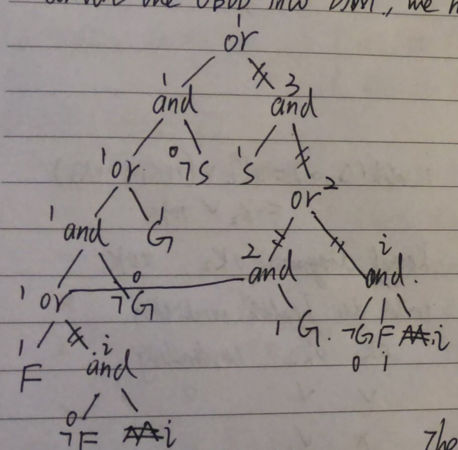
Problem 7

Solution:

success.

7. ①

Convert the OBDD into DNNF, we have



S, G, F, ¬M.

Since the root or is 1, which means there is no instance that all features are 0, but if we remove all edges ~~to~~ with higher value. There are two scenarios:

¬S, G, ¬F, ¬M.

¬S, ¬G, F, ¬M.

The largest set is 3.

with two possibilities:

{S=0, F=0, M=0}

{S=0, G=0, M=0}

7②.

S	G	F	M		Explanation
1	0	1	1	Yes	(GFM)(FM)(SFM)(GM)(GF)

The smallest set of features α is {F=1, M=1} {G=1, M=1} {G=1, F=1}

1.

Problem 8

Solution:

8. system description: $\Delta \begin{cases} ok_1 \Rightarrow (A \Leftrightarrow \neg B) \\ ok_2 \Rightarrow (A \wedge B) \Leftrightarrow C \end{cases}$

① α: $\cancel{A \wedge C}$ C

ok_1	ok_2	diagnosis
✓	✓	No
✓	×	Yes.
×	✓	Yes
×	×	No

$Health(\Delta, \alpha) = (ok_1 \wedge \neg ok_2) \vee (\neg ok_1 \wedge ok_2)$

Kernel diagnosis: $(ok_1 \wedge \neg ok_2)$ $(\neg ok_1 \wedge ok_2)$

Under this health condition:

ok_1	ok_2	cardinality
✓	×	1
×	✓	1

They are both 1-cardinality so
min-cardinality diagnosis is $(ok_1 \wedge \neg ok_2)$ $(\neg ok_1 \wedge ok_2)$

② when C is false

α: $\cancel{A}$ $\neg C$

ok_1	ok_2	diagnosis.
✓	✓	Yes.
✓	×	No
×	✓	~~No~~ Yes.
×	×	Yes.

$Health(\Delta, \alpha) = ok_2 \vee (\neg ok_1 \wedge \neg ok_2)$
$= ok_2 \vee \neg ok_1$

Kernel diagnosis: ok_2, $\neg ok_1$

under this health condition

ok_1	ok_2	cardinality
✓	✓	0
×	✓	1
×	×	2

min-cardinality diagnosis : $ok_1 \wedge ok_2$

1.