CS267A: Probabilistic Programming and Relational Learning **Spring 2020**

Homework 4: Due Friday May 8, 11:59PM

Instructions: Upload one file to CCLE: a PDF typeset using LaTeX containing your solutions (**including code**). Consider using package *listings* to embed the code. No late submissions will be accepted. See the syllabus for policies about collaboration and academic honesty.

A few things to consider when using ProbLog:

- One can install ProbLog by following the instructions at `https://github.com/ML-KULeuven/problog` or one can use its online editor through `https://dtai.cs.kuleuven.be/problog/editor.html#`.
- Logical variables in ProbLog need to be capitalized.

Problem 1

Topics: Logic

Let α, β, and γ be Boolean formulae, and let $\texttt{MC}$ denote the model count of a Boolean formula. Select whether the following is a true or false statement about model counts and provide a brief justification for your choice.

1. $\texttt{MC}(\alpha) \geq \texttt{MC}(\alpha \wedge \beta)$.
2. Suppose $\alpha \Rightarrow \beta$ and $\beta \Rightarrow \gamma$. Then, $\texttt{MC}(\alpha \vee \beta \vee \gamma) = \texttt{MC}(\gamma)$.
3. Suppose $\alpha \Rightarrow \beta$ and $\alpha \Rightarrow \gamma$. Then, $\texttt{MC}(\alpha) \leq \texttt{MC}(\gamma \wedge \beta)$.

Problem 2

Topics: First-order logic and grounding

Consider the following first-order vocabulary:

- $\mathsf{Friends}(x, y)$ is a predicate which says x is friends with y (one-directional)
- $\mathsf{Smokes}(x)$ is a predicate which says x is a smoker.

Then, from this vocabulary we can build the following first-order sentence:

$$\forall x.\forall y.(\mathsf{Smokes}(x) \wedge \mathsf{Friends}(x, y)) \implies \mathsf{Smokes}(y). \tag{1}$$

Answer the following for the above sentence:

1. Assume that there is a finite domain of people $\{\mathsf{Alice}, \mathsf{Bob}\}$ that each x and y variable is drawn from. Compute the propositional grounding Δ for the first-order sentence with this finite domain.
2. How many models of Δ are there, still assuming there is a finite domain of people $\{\mathsf{Alice}, \mathsf{Bob}\}$?

Problem 3

Topics: Modeling with first-order logic

Consider a vocabulary with the following symbols:

- $\mathsf{Occupation}(p, o)$: A predicate which states person p has occupation o.
- $\mathsf{Customer}(p_1, p_2)$: A predicate which states p_1 is a customer of p_2.
- $\mathsf{Boss}(p_1, p_2)$: A predicate which states p_1 the boss of p_2.
- $\mathsf{Doctor}, \mathsf{Person}, \mathsf{Lawyer}, \mathsf{Actor}, \mathsf{Surgeon}$: constants which denote occupations.
- $\mathsf{Emily}, \mathsf{Joe}$: constants denoting people.

Using the above symbols, translate the following sentences into first-order logic:

1. Emily is either a surgeon or an actor.
2. Joe is an actor, but he also has at least one other job.
3. All surgeons are doctors.
4. Emily has a boss who is a lawyer.
5. There exists a lawyer whose customers are all doctors.
6. Every surgeon has a lawyer.

Translate the following first-order sentences into English:

i. $\forall x.\mathsf{Occupation}(x, \mathsf{Doctor}) \implies \exists y.\mathsf{Customer}(x, y)$

ii. $\exists x.\mathsf{Occupation}(x, \mathsf{Doctor}) \implies \forall y.\mathsf{Customer}(x, y)$

iii. $\exists x.\forall y.\mathsf{Occupation}(x, \mathsf{Lawyer}) \wedge \mathsf{Customer}(x, y) \wedge \mathsf{Occupation}(y, \mathsf{Doctor})$

Problem 4

Flipping coins in ProbLog

Suppose you have an unbiased coin (50% of the time it will show heads) which you flip M times in sequence. What is the probability that N consecutive heads appear at some in this sequence? Implement this as a function in ProbLog. Submit your code, as well as answers to the following queries:

1. 2 consecutive heads in 5 flips
2. 5 consecutive heads in 6 flips
3. 7 consecutive heads in 10 flips

Problem 5

Topics: Modeling with ProbLog

Suppose you sample N classes from the following list of classes:

- CS267A: 0, 1, and 2 midterms with probability 0.25, 0.7, and 0.05, respectively.
- CS31: 0, 1, and 2 midterms with probability 0.1, 0.1, and 0.8, respectively.
- Math61: 0, 1, and 2 midterms with probability 0.01, 0.1, and 0.89, respectively.
- History101: 0, 1, and 2 midterms with probability 0.49, 0.5, and 0.01, respectively.
- English101: 0, 1, and 2 midterms with probability 0.7, 0.3, and 0, respectively.

What is the probability that there are at least M midterms in a sampled set of size N? Implement this as a function in ProbLog. Submit your code as well as answers to the following queries:

1. at least 3 midterms in 2 classes
2. at least 7 midterms in 4 classes