CS267A: Homework #4

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May 6, 2020

Acknowledgements: If you have discussed with other students in the class regarding this homework, please acknowledge their names. See the syllabus for detailed policies about collaboration and academic honesty.

Problem 1

1. **Solution:**
   What is the probability that there is a burglary or earthquake given both John and Mary call : 0.092088197
   What is the probability that there is an earthquake given no burglary and Mary calls: 0.14564687

2. Please find the following my code that generates the solution.

```
Myprogram.pl:

t(0.001)::burglary.
t(0.002)::earthquake.
t(0.95)::p_alarm1.
t(0.94)::p_alarm2.
t(0.29)::p_alarm3.
t(0.001)::p_alarm4.

alarm :- burglary, earthquake, p_alarm1.
alarm :- burglary, \+earthquake, p_alarm2.
alarm :- \+burglary, earthquake, p_alarm3.
alarm :- \+burglary, \+earthquake, p_alarm4.

t(0.9)::call_j1.
t(0.05)::call_j2.
call_john :- alarm,call_j1.
call_john :- \+alarm,call_j2.

t(0.7)::call_m1.
t(0.01)::call_m2.
call_mary :- alarm,call_m1.
call_mary :- \+alarm,call_m2.

Myevidence.pl:


%%% The data:
```

```
evidence(burglary,false).
evidence(earthquake,false).
evidence(call_john,false).
evidence(call_mary,false).
-----
evidence(burglary,true).
evidence(earthquake,false).
evidence(call_john,true).
evidence(call_mary,false).
-----
evidence(burglary,false).
evidence(earthquake,false).
evidence(call_john,false).
evidence(call_mary,false).
-----
evidence(burglary,false).
evidence(earthquake,false).
evidence(call_john,false).
evidence(call_mary,true).
-----
evidence(burglary,true).
evidence(earthquake,false).
evidence(call_john,true).
evidence(call_mary,true).
-----
evidence(burglary,true).
evidence(earthquake,true).
evidence(call_john,true).
evidence(call_mary,true).
-----
evidence(burglary,false).
evidence(earthquake,false).
evidence(call_john,false).
evidence(call_mary,false).
-----
evidence(burglary,false).
evidence(earthquake,false).
evidence(call_john,false).
evidence(call_mary,true).

question1.pl:

% Your model here

0.375::burglary.
0.125::earthquake.
1::p_alarm1.
1::p_alarm2.
0.29::p_alarm3.
0::p_alarm4.

alarm :- burglary, earthquake, p_alarm1.
alarm :- burglary, \+earthquake, p_alarm2.
alarm :- \+burglary, earthquake, p_alarm3.
```

```
alarm :- \+burglary, \+earthquake, p_alarm4.

1::call_j1.
0::call_j2.
call_john :- alarm,call_j1.
call_john :- \+alarm,call_j2.

0.66666667::call_m1.
0.4::call_m2.
call_mary :- alarm,call_m1.
call_mary :- \+alarm,call_m2.

%run one at a time
evidence(burglary,false).
evidence(call_mary,true).
query(earthquake).

%run one at a time
out1 :- burglary.
out1 :- earthquake.
evidence(call_john,false).
evidence(call_mary,true).
query(out1).
```

Problem 2

1. **Solution:** There is one in T means, 1 minus the possibility that no one is in T:
$\Pr(\exists x.T(x)) = 1-$ Pr $(\omega_8) = 1 - 0.06 = 0.94$.

2. **Solution:** Add up all the scenarios of A,B,C appearing in the case:
Pr(A) = Pr(ω_1) + Pr(ω_2) + Pr(ω_4) + Pr(ω_5) = 0.16 + 0.16 + 0.04 + 0.04 = 0.40.
Pr(B) = Pr(ω_1) + Pr(ω_2) + Pr(ω_3) + Pr(ω_6) = 0.16 + 0.16 + 0.24 + 0.24 = 0.80.
Pr(C) = Pr(ω_1) + Pr(ω_3) + Pr(ω_4) + Pr(ω_7) = 0.16 + 0.24 + 0.04 + 0.06 = 0.50.

Problem 3

1. **Solution:**
(a) Pr(T(Alice,Pixar)) = Pr(ω_1) + Pr(ω_2) = 3/8.
(b) Pr($\exists x$. T(Alice,x)) = Pr(ω_1) + Pr(ω_2) + Pr(ω_3) = 3/4.
(c) Pr($\exists x, y$. T(x, y)) = Pr(ω_1) + Pr(ω_2) + Pr(ω_3) = 3/4.
(d) Pr(x. T(x,Brown)) = Pr(T(Alice,Brown) $\wedge$ T(Carol,Brown)) = 0

Problem 3 continued on next page. . .

2.
Solution: this probabilistic database is not tuple-independent.

For tuple-independence database:
Pr(T(Alice,Pixar) $\wedge$ T(Carol,UPenn)) = Pr(T(Alice,Pixar)) X Pr(T(Carol,UPenn)).
Pr(T(Alice,Pixar) $\wedge$ T(Carol,INRIA)) = Pr(T(Alice,Pixar)) X Pr(T(Carol,INRIA)).

However, Pr(T(Alice,Pixar)) = Pr(ω_1) + Pr(ω_2) = 1/8 + 1/4 = 3/8,
Pr(T(Carol,UPenn)) = Pr(ω_1) = 1/8,
Pr(T(Alice,Pixar)) X Pr(T(Carol,UPenn)) = (3/8)(1/8) = 3/64
Pr(T(Alice,Pixar)) $\wedge$ Pr(T(Carol,UPenn)) = 1/8
Pr(T(Alice,Pixar)) X Pr(T(Carol,INRIA)) = (3/8)(1/4) = 3/32
Pr(T(Alice,Pixar)$\wedge$ T(Carol,INRIA)) = 1/4
They are not equal. So it is not tuple-independent.

Problem 4

1.
Solution: Let A = Alice, B = Bob, C = Charlie.
The set of all possible world databases of T_H is {{A,B,C},{A,B},{A,C},{B,C},{A},{B},{C},{}}.
The probabilities of each world are:
Pr($\{A, B, C\}$) = Pr(A, B, C) = 0.7 X 0.4 X 0.9 = 0.252.
Pr($\{A, B\}$) = Pr($A, B, \neg C$) = 0.7 X 0.4 X 0.1 = 0.028.
Pr($\{A, C\}$) = Pr($A, \neg B, C$) = 0.7 X 0.6 X 0.9 = 0.378.
Pr($\{B, C\}$) = Pr($\neg A, B, C$) = 0.3 X 0.4 X 0.9 = 0.108.
Pr($\{A\}$) = Pr($A, \neg B, \neg C$) = 0.7 X 0.6 X 0.1 = 0.042.
Pr($\{B\}$) = Pr($\neg A, B, \neg C$) = 0.3 X 0.4 X 0.1 = 0.012.
Pr($\{C\}$) = Pr($\neg A, \neg B, C$) = 0.3 X 0.6 x 0.9 = 0.162.
Pr($\{\}$) = Pr($\neg A, \neg B, \neg C$) = 0.3 X 0.6 X 0.1 = 0.018.

Problem 4 continued on next page...

Solution: 1. Pr$(\exists x.H(x))$
= 1$-$ Pr$(\forall x.\neg H(x))$
= 1 - (1 - H(A))(1 - H(B))(1 - H(C))
= 1 - (1 - 0.7)(1 - 0.4)(1 - 0.9)
= 1 - 0.3 X 0.6 X 0.1
= 1 - 0.018 = 0.982

2. Pr$(\exists x.H(x) \wedge E(x,$ AI))
= 1 - Pr$(\forall x.\neg H(x) \vee \neg E(x, AI))$
= 1 - $\prod_i Pr(\neg H(i) \vee \neg E(i, AI))$
= 1 - $\prod_i$ (1 - Pr (H(i) $\wedge$ E(i, AI)))
= 1 - $\prod_i$ (1 - Pr (H(i)) X Pr (E(i, AI)))
= 1 - (1 - Pr (H(A)) Pr (E(A,AI))) (1 - Pr(H(B)) Pr (E(B, AI))) (1 - Pr (H(C)) Pr (E(C, AI)))
= 1 - (1 - 0.7 X 0.7)(1 - 0.4 X 0.3)(1 - 0.9 X 0)
= 1 - (1 - 0.49)(1 - 0.12) = 1 - 0.51 X 0.88 = 1 - 0.45
= 0.55.

3. Pr$(\exists x \exists y.H(x) \wedge E(x, y))$
= 1 - $Pr(\forall x \forall y.\neg H(x) \vee \neg E(x, y))$
= 1 - $\prod_i$(Pr$(\forall y.\neg H(i) \vee \neg E(i, y)))$
= 1 - $\prod_i$(Pr$(\neg H(i) \vee \forall y.\neg E(i, y)))$
= 1 - $\prod_i$(1$-$ Pr$(H(i) \wedge \exists y.E(i, y)))$
= 1 - $\prod_i$(1$-$ Pr$(H(i))\times$Pr$(\exists y.E(i, y)))$
= 1 - $\prod_i$(1$-$ Pr$(H(i))X(1-$Pr$(\forall y.\neg E(i, y))))$
= 1 - $\prod_i$ (1 - Pr(H(i)) X (1 - $\prod_j$ (1- Pr(E(i,j)))))
= 1 - (1 - Pr(H(A)) X (1 - $\prod_j$ (1- Pr(E(A,j)))))(1 - Pr(H(B)) X (1 - $\prod_j$ (1- Pr(E(B,j)))))(1 - Pr(H(C)) X (1 - $\prod_j$ (1- Pr(E(C,j)))))
= 1 - (1 - Pr(H(A)) X (1 - (1- Pr(E(A,AI)))(1- Pr(E(A,PL)))))(1 - Pr(H(B))
X (1 - (1- Pr(E(B,AI)))(1- Pr(E(B,PL)))))(1 - Pr(H(C))
X (1 - (1- Pr(E(C,AI)))(1- Pr(E(C,PL)))))
= 1 - (1 - 0.7 X (1 - (1- 0.7)(1- 0.4)))(1 - 0.4 X (1 - (1- 0.3)(1- 0)))
= 1 - (1 - 0.7 X (1 - 0.3 X 0.6))(1 - 0.4 X (1 - 0.7))
= 1 - 0.426 X 0.88

2. = 0.625

Solution:

3. 0.982

```
0.70::alice.
0.40::bob.
0.90::charlie.

0.70::alice_AI.
0.40::alice_PL.
0.30::bob_AI.

out1 :- \+(\+alice,\+bob,\+charlie).
```

```
query(out1).
```

4. **Solution:**
0.5512

```
0.70::alice.
0.40::bob.
0.90::charlie.

0.70::alice_AI.
0.40::alice_PL.
0.30::bob_AI.

p1 :- bob, bob_AI.
p2 :- alice, alice_AI.
out2 :- \+(\+p1,\+p2).

query(out2).
```

5. **Solution:**
(3.) 0.62512

```
0.70::alice.
0.40::bob.
0.90::charlie.

0.70::alice_AI.
0.40::alice_PL.
0.30::bob_AI.

p1 :- alice, alice_AI.
p2 :- alice, alice_PL.
p3 :- bob, bob_AI.

out3 :- \+(\+p1,\+p2,\+p3).

query(out3).
```