CS267A: Probabilistic Programming and Relational Learning **Spring 2020**

Homework 7: Solution for Question 3

Instructions: This is the solution for Question 3 in Homework 3. It is intended as a reference to clarify what we intended for this question. Furthermore, we also provide the right number if this question is perceived under a different interpretation. For more about this, please see our announcement.

Problem 1

Consider the following Markov logic network:

$$\begin{aligned} &1.5 \quad \forall x. S(x) \Rightarrow C(x). \\ &2.0 \quad \forall x, y. S(x) \wedge F(x, y) \Rightarrow S(y). \end{aligned}$$

Suppose there are two constants, labeled a and b.

1. What is the *un-normalized* weight of $\omega_1 = S(a) \wedge C(a) \wedge S(b) \wedge C(b) \wedge F(a,b) \wedge F(b,a) \wedge F(a,a) \wedge F(b,b)$?
   Solution Every constraint in the MLN is satisfied, so the weight is:
   $$\exp(1.5 + 1.5 + 2 + 2 + 2 + 2) \approx 59874.14$$

2. What is the *un-normalized* weight of $\omega_2 = S(a) \wedge \neg C(a) \wedge S(b) \wedge C(b) \wedge F(a,b) \wedge F(b,a) \wedge F(a,a) \wedge F(b,b)$?
   Solution The constraint $S(a) \Rightarrow C(a)$ is not satisfied, so we do not include this weight in the sum:
   $$\exp(1.5 + 0 + 2 + 2 + 2 + 2) \approx 13359.72$$

3. What is $\Pr(C(a) \mid S(a))$?
   Solution There are a two ways of computing this. They both rely on the fact that $C(a)$ is independent of all other factors given $S(a)$. Let $\omega = S(a) \wedge S(b) \wedge F(a,b) \wedge F(b,a) \wedge F(a,a) \wedge F(b,b) \wedge C(b)$. Then we can compute:

$$\begin{aligned}
\Pr(C(a) \mid S(a)) =& \frac{\Pr(C(a) \wedge S(a))}{\Pr(S(a))} \\
=& \frac{\Pr(C(a) \wedge S(a))}{\Pr(C(a) \wedge S(a)) + \Pr(\neg C(a) \wedge S(a))} && \text{By mutual exclusivity} \\
=& \frac{\Pr(C(a) \mid S(a)) \cdot \cancel{\Pr(S(a))}}{\Pr(C(a) \mid S(a)) \cdot \cancel{\Pr(S(a))} + \Pr(\neg C(a) \mid S(a)) \cdot \cancel{\Pr(S(a))}} \\
=& \frac{\Pr(C(a) \mid \omega)}{\Pr(C(a) \mid \omega) + \Pr(\neg C(a) \mid \omega)} && \text{By indep.} \\
=& \frac{\frac{\Pr(C(a) \wedge \omega)}{\Pr(\omega)}}{\frac{\Pr(C(a) \wedge \omega)}{\Pr(\omega)} + \frac{\Pr(\neg C(a) \wedge \omega)}{\Pr(\omega)}} && \text{Bayes rule} \\
=& \frac{\frac{1}{Z} 59874.14}{\frac{1}{Z}(59874.14 + 13359.72)} = 0.8176
\end{aligned}$$

The other way is to realize that, by conditioning on $S(a)$, we only need to consider the following MLN (again using conditional independence):

$$1.5 \quad S(a) \Rightarrow C(a).$$

It is easy to compute $\Pr(C(a) \mid S(a))$ for this MLN:

$$\Pr(C(a) \mid S(a)) = \frac{\frac{1}{Z}\exp(1.5)}{\frac{1}{Z}(\exp(1.5) + \exp(0))} = 0.8176.$$

4. If under the interpretation that there are no free variables, then the answer to 3.1 would be $e^{3.5}$; the answer to 3.2 would be e^2; and the answer to the 3.3 would be 0.77.