**Abstract:**

To-be-filled.

**Introduction:**

Market making at market-close is a popular trading strategy. A market maker is someone who quotes both a buy (bid) and a sell (ask) price in a financial instrument, such as stocks, hoping to make a profit on the bid-ask spread. To quote a price is to post a price, and the bid-ask spread is the difference between the prices quoted for an immediate sale and an immediate purchase of a financial instrument. A market maker tends to be most active during market-close because there is a spike in liquidity during market-close. Market liquidity of a financial instrument refers to how rapidly the instrument can be bought or sold, which is usually measured by its trading volume. A liquid market would actively move back-and-forth and thus provide more opportunities for a market maker to capitalize.

A key decision for a market maker to decide is how far they would like to quote above and below the market. Suppose that the current price of a financial instrument is $x$. To quote $a$ above the market is to quote a sell (ask) price of $x+a$. Similarly, to quote $b$ below the market is to quote a buy (bid) price of $x-b$. In other words, by quoting $a$ above the market and $b$ below the market, we hope to buy low at $x-b$ and sell high at $x+a$. For each successful trade, we would make a profit of $(x+a)-(x-b)=a+b$. The further we quote above and below the market (the larger the values of $a$ and $b$), the more profit we would capitalize on each successful trade. However, the likelihood of success decreases as we quote further away from the market. For example, Google currently trades at \$1,750 per share. If we quote \$500 both above and below the market, surely we would make \$1,000 profit on each successful trade, but in reality, we would not make any successful trades at all because no one is willing to buy Google at \$1,250 per share and sell Google at \$2,250 per share. In contrast, if we quote \$1 both above and below the market, then our trades are likely to be successful, but we would only make a small profit of \$2 from each successful trade. In other words, we need to keep a balance between profit per trade and likely hood of success when we make markets.

When we make markets, it would be ideal if we could set our quoting range according to the then-current market condition. If we believe that prices are going to move rapidly, then we could quote far away from the market to maximize our gains per trade and still achieve a decent success rate. On the contrary, if we believe that prices are going to move slowly, then we could quote near the market to make sure that we could still achieve a decent success rate. Therefore, it is very helpful for a market maker to be able to forecast the market condition, at least to a certain degree.

**Problem Statement and Data Source**

For our final project, we would like to predict the volatility of the S\&P 500 Value Index during market-close. The ticker for the S\&P 500 Value Index is IVE, so we shall refer to it as ES from this point onward. A ticker is an abbreviation of the name of a financial instrument. We chose to study IVE because it is the only free second-by-second data that we could found online, and it has a decent amount of trading volume. We focus market-close because in real life many market makers participate during market-close.

We obtained IVE second-by-second trading data from 09/28/2009 9:30:00 to 11/20/2020 20:00:00. We found the data from <http://www.kibot.com/free_historical_data.aspx>. The data is stored in a .txt file, which includes the last traded price, bid price, ask price, and trading volume for each second. The dataset is comprehensive because it covers volatile periods such as 2009, 2015, 2020 as well as quiet periods such as 2012, 2013. All kinds of market conditions could be found in our dataset, and this makes sure that our models would not be biased towards any particular kind of market conditions, such as bullish market or a bearish market.

The closing price for IVE is released at 15:00:00 CST every trading day. The closing price is used as the baseline for market making. For example, to quote \$5 above the market is to quote \$5 above the closing price. The closing price is calculated based on the volume-weighted average price of all transactions during the interval 14:59:30 CST and 15:00:00 CST. Consequently, the market price for ES at 15:00:00 CST is almost always different from the closing price. For our final project, the volatility of IVE during market-close shall be defined as the normalized absolute difference between the closing price and the market price at 15:00:00 CST. For example, if the closing price is \$100 but IVE trades at \$105 at market close, then the volatility shall be $|100 – 105| / 100 $. It is essential to normalize the volatility because our data ranges across more than ten years. During this time, the IVE has grown from the initial \$57 per share to \$120 per share. We must normalize the volatility term in order to sensibly compare them across different time periods.

To measure the performance of our models, we decided to use the symmetric mean absolute percentage error (SMAPE) metric. The formula for calculating SMAPE is [**To-be-filled**], where [**To-be-filled, explain each parameter in SMAPE, see** [**https://towardsdatascience.com/choosing-the-correct-error-metric-mape-vs-smape-5328dec53fac**](https://towardsdatascience.com/choosing-the-correct-error-metric-mape-vs-smape-5328dec53fac)]. We chose the SMAPE method because it is one of the most popular metrics for evaluating the forecasting performance.

**Prior Work**

Predicting the stock market is a popular topic that many researchers have investigated. However, our problem is different from most of the prior works for two reasons. Firstly, researchers in this area tend to study classification problems, whereas we are looking for an accurate forecast. For example, there are many prior works that used neural networks to predict whether the stock market is going to go up or down in some time interval, which is a classification problem with two outcomes (up and down). In our final project, we would like to accurately forecast the volatility, which is a number and not a classification problem. Secondly, our problem statement is focused on the real-life implications of market-making only, which is not commonly studied by prior works. Though there are prior works that try to predict the volatility of the stock market, our definition of volatility is unique to us and is only useful for market-making during market-close.

**Models**

For our final project, we developed three neural network models using Pytorch: (1) Baseline model (2) CNN model (3) Recurrent Network model. Before we delve into more details. The following features are common across all three models:

1. All three models use the mean squared error loss function. We chose the MSE loss function because it is popular, easy to implement, interpret, and serves our purpose.
2. All three models use the ReLU activation function at each hidden layer. We chose the ReLU activation function because it is popular, easy to implement, and serves our purpose.
3. All three models used all data. The dataset is randomly split into a training set (30%) and a testing set (70%) using Pytorch data loader. As mentioned above, this hinders the problem of biased models.
4. For all three models, the inputs are volatilities calculated at the end of each minute for the 30 minutes before the market close. In other words, the inputs are volatility at 14:30:00 CST, 14:31:00, …, 14:59:00 CST. The method for calculating the volatility at the end of each minute is the same as calculating the volatility for the end of day. Consequently, the input has a size of 29. For each trading day we have in the dataset, the model shall predict the volatility at market close. Then we, will compare the predicted volatility to the actual observed volatility. As mentioned above, the performance of all three models is measured using the SMAPE metric.

Now, we shall present each model in mode details and show the corresponding results.

Baseline Model

Our baseline model is a fully connected neural network with 1 hidden layer that contains 10 nodes. Thus, the structure of the baseline model is 29-10-1. The baseline model is simple and shall be used to compare with the other two more sophisticated model.

CNN Model

The CNN model is a convolution neural network with 1 one-dimensional-convolution layer, 1 max-pool layer, and 1 fully connected hidden layer. The convolution layer has 1 input channel, and 2 output channels, with a kernel size of 3 and stride size of 1. The max-pool layer has a kernel size of 3 and stride size of 1. After the convolutional layer and the max-pool layer, we flatten data, and it now has a size of 18. Then, we feed it into the fully connected hidden layer, which has 10 nodes. Finally, we produce a single prediction from the fully connected layer.

We chose a single convolution layer and a single pooling layer with very small kernel and stride sizes because unlike an image data with 3 channels of colors each with thousands of pixels, our input only has a single channel of size 29. We tried to add more convolution layers and increase the kernel and stride size, but that would drastically shrink the size of our data to something around 3 to 5 by the time they reach the fully connected layer.

We believed that a CNN model would be suitable for our problem because CNN works well with data that has spatial relationship. We believe that the trading activities before the market-close is correlated with the trading activities at the market-close, and each minute shortly before the market close is also correlated with one another.

Recurrent Network Model

Future Work

For our final project, we initially wanted to choose the SNP 500 index (SPY) as our instrument of study. SPY is the most popular stock index in the world. However, because SPY is a very popular product, data on SPY is costly, especially second-by-second data, so we chose to study IVE instead, which has free data. If we could obtain data on SPY, we would love to see how our models perform on SPY. Since SPY is more popular, its price fluctuations are also more volatile, which makes it a more difficult instrument to predict than IVE. Our models were able to accurately predict the volatility at market close for IVE, but the same result might not be true for SPY. Testing our models on SPY is something that we could explore in the future.

Summary