

The psycho-logic of universal quantifiers

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LiMe Lab meeting @ Cornell

10.28.21

Big picture: Linguistic meaning in the mind



What sorts of instructions do meanings provide to cognition?

- ➔ To what extent do they constrain the thought that gets built?
- ➔ At what grain-size are they shared by speakers?

Why *each* and *every*?

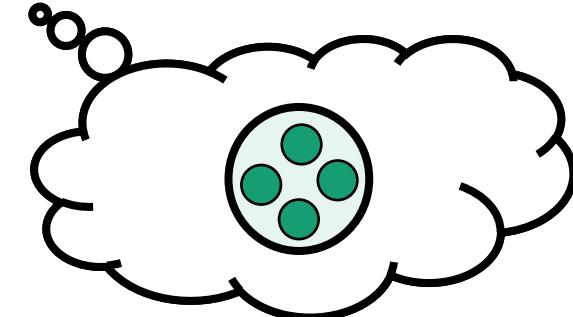
→ Can state precise hypotheses about their meaning representations

→ Can leverage an understanding of supporting cognitive systems

e.g., those for representing number, groups, individuals

“Every circle is green”

TheX:Circle(X)
[Green(X)]



(Other case studies I'm working on:
superlatives vs. comparatives; English & Cantonese *most*)

Roadmap: How are *each* & *every* mentally represented?

Three hypotheses

- Two (psycho)logical distinctions

Relational
Second-order

Restricted
Second-order

Restricted
First-order

Relational vs. Restricted

- Number cognition as a probe into which arguments are represented
- The “conservativity” universal

First-order vs. Second-order (individual- vs. group- implicating)

- Object-files vs. Ensembles as a probe into how arguments are represented
- Consequences for language acquisition

Each/Every circle is green – possible representations

The circles_x are among the green-things_y

$\text{TheX:Circle}(X) \subseteq \text{TheY:Green}(Y)$

The circles_x are such that they_x are green

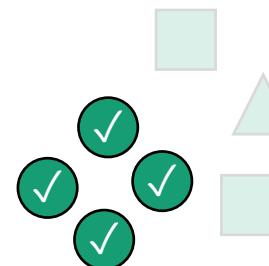
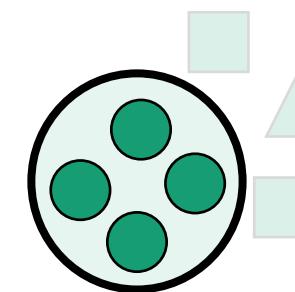
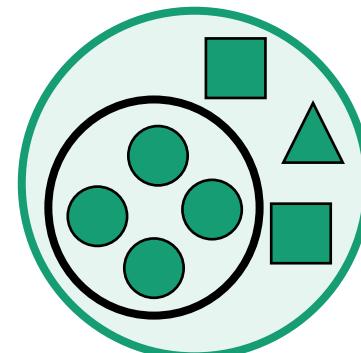
$\text{TheX:Circle}(X)[\text{Green}(X)]$

Any individual circle_x is such that it_x is green

$\forall x:\text{Circle}(x)[\text{Green}(x)]$

every

each



**Relational
Second-order**

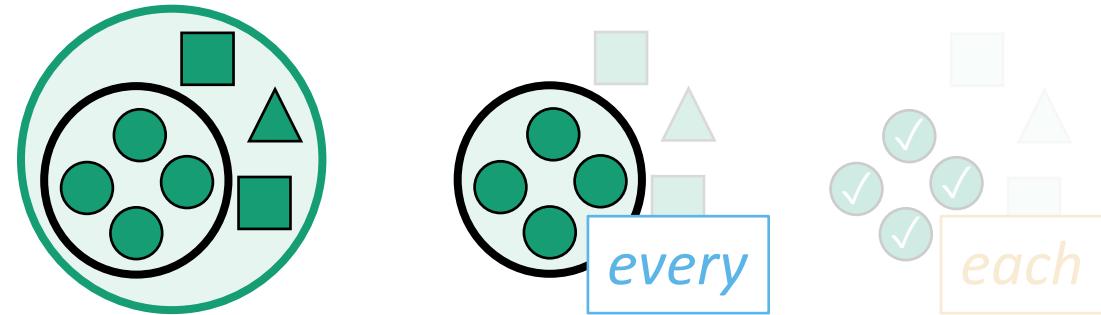
**Restricted
Second-order**

**Restricted
First-order**

Roadmap: How are *each* & *every* mentally represented?

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Different representations & behavioral predictions

Linking hypothesis (Interface Transparency): In evaluating a sentence, people are biased toward strategies that **directly compute the relations & operations expressed** by the semantic representation under evaluation

Relational

$\text{TheX:Circle}(X) \subseteq \text{TheY:Green}(Y)$

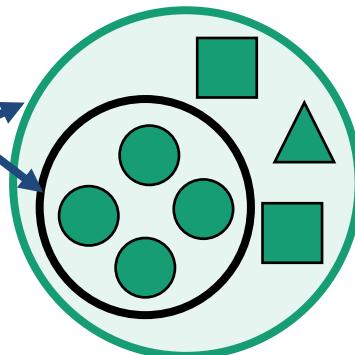
\approx The circles_X are among
the green-things_Y

Restricted

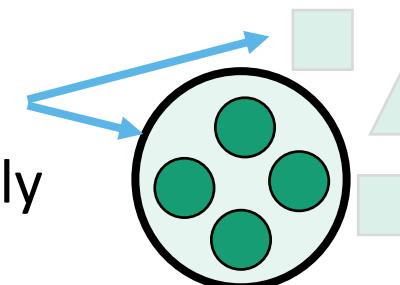
$\text{TheX:Circle}(X)[\text{Green}(X)]$

\approx The circles_X are such that
they_X are green

Represent &
compare both
arguments



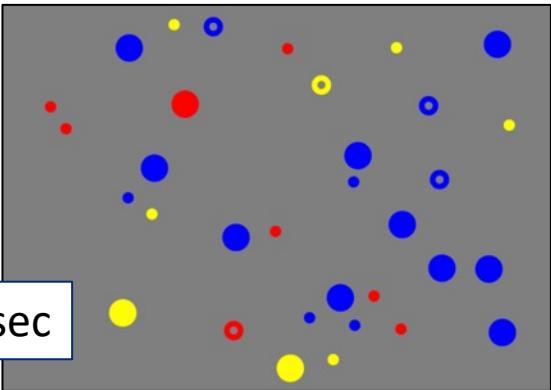
Treat
arguments
asymmetrically



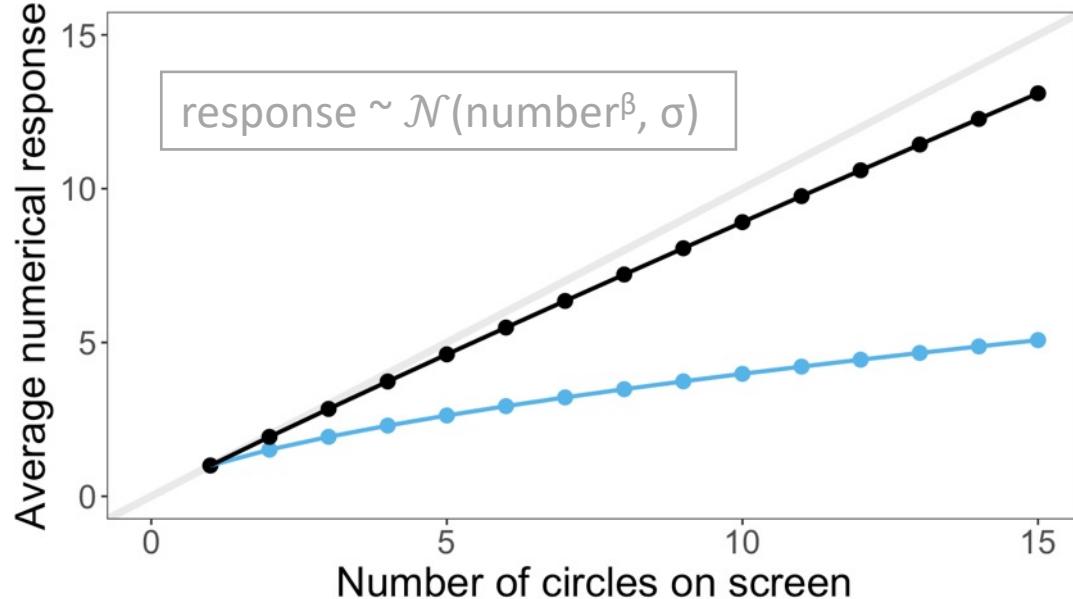
Every big circle is blue

TRUE

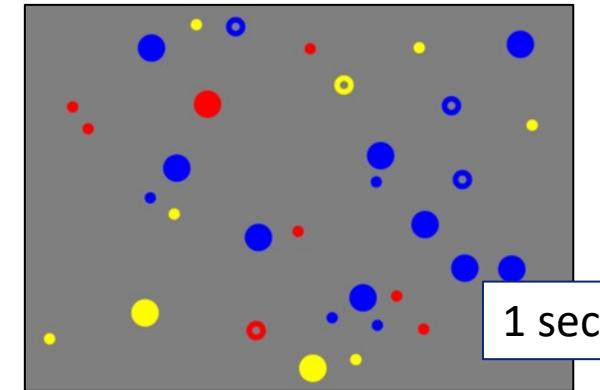
FALSE



Accuracy on “how many” question



How many big circles
are there?



How many big circles
were there?

#-knowledge following *every*
vs.
#-knowledge **baseline**

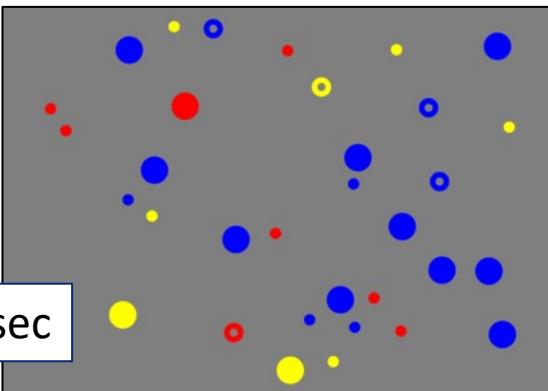
How many big circles
were there?

Every big circle is blue

TRUE

FALSE

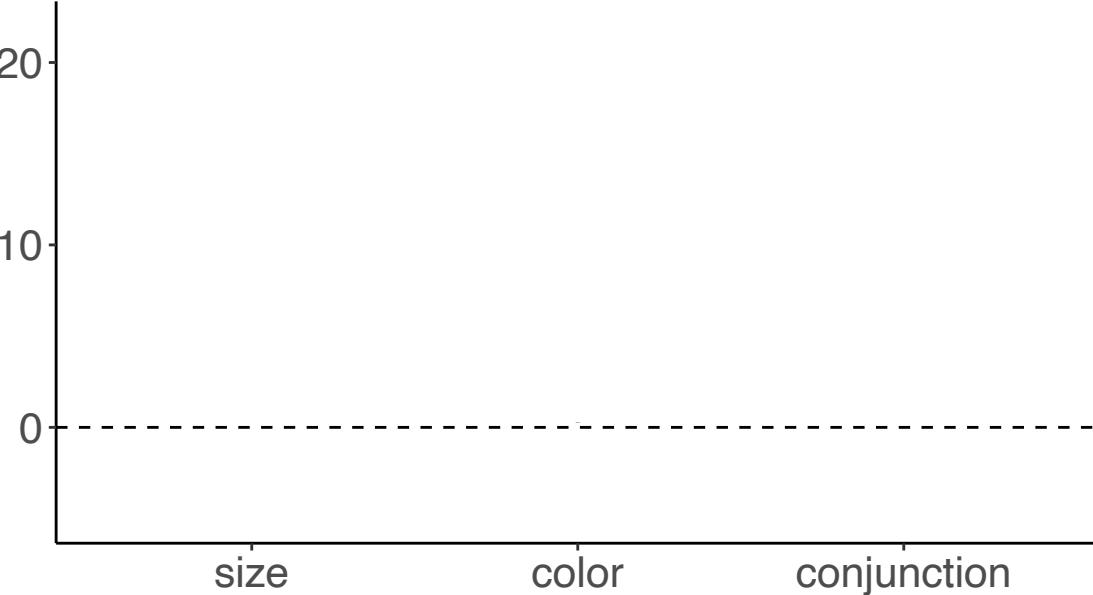
1 sec



How many
{big/blue/big blue}
circles were there?

Cardinality estimation error
"Every [size] circle is [color]"

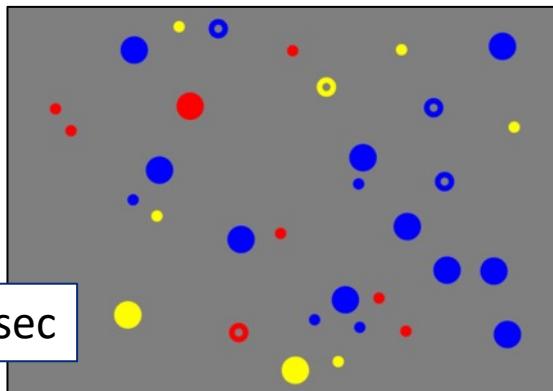
Difference in % error
(post-verification – baseline)



Every big circle is blue

TRUE

FALSE



Relational

The big-circles_x are among the blue-circles_y

Represent both arguments

How many
{big/blue/big blue}
circles were there?

Cardinality estimation error
"Every [size] circle is [color]"

Difference in % error
(post-verification – baseline)

size

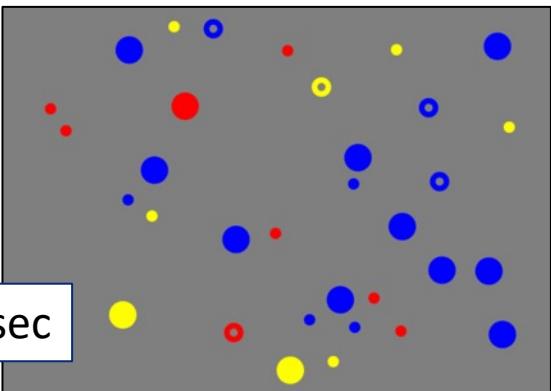
color

conjunction

Every big circle is blue

TRUE

FALSE



How many
{big/blue/big blue}
circles were there?

Relational

*The big-circles_x are
among the blue-circles_y*

Represent both
arguments

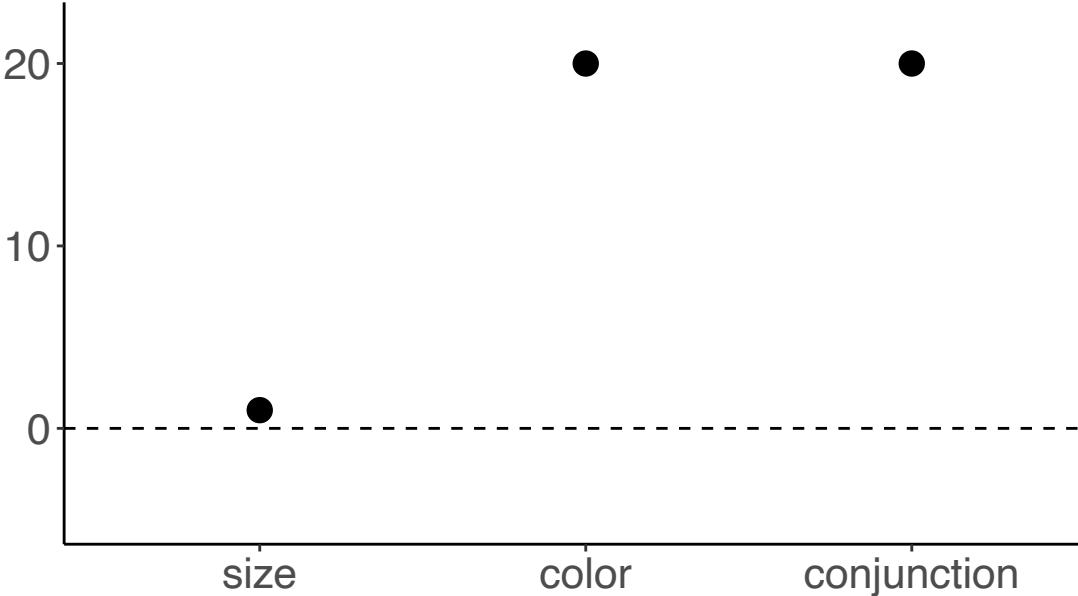
Restricted

*The big-circles_x are such
that they_x are blue*

Treat arguments
asymmetrically

Cardinality estimation error
"Every [size] circle is [color]"

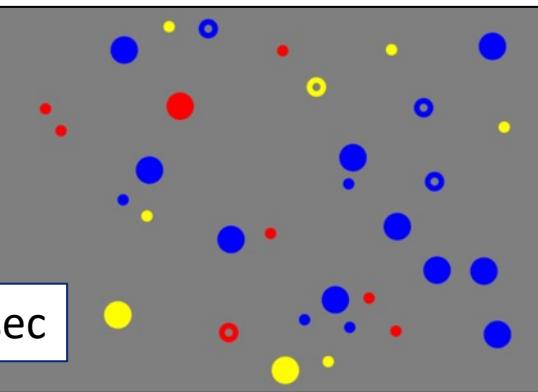
Difference in % error
(post-verification – baseline)



Every big circle is blue

TRUE

FALSE



How many
{big/blue/big blue}
circles were there?

Relational

The big-circles_x are among the blue-circles_y

Represent both arguments

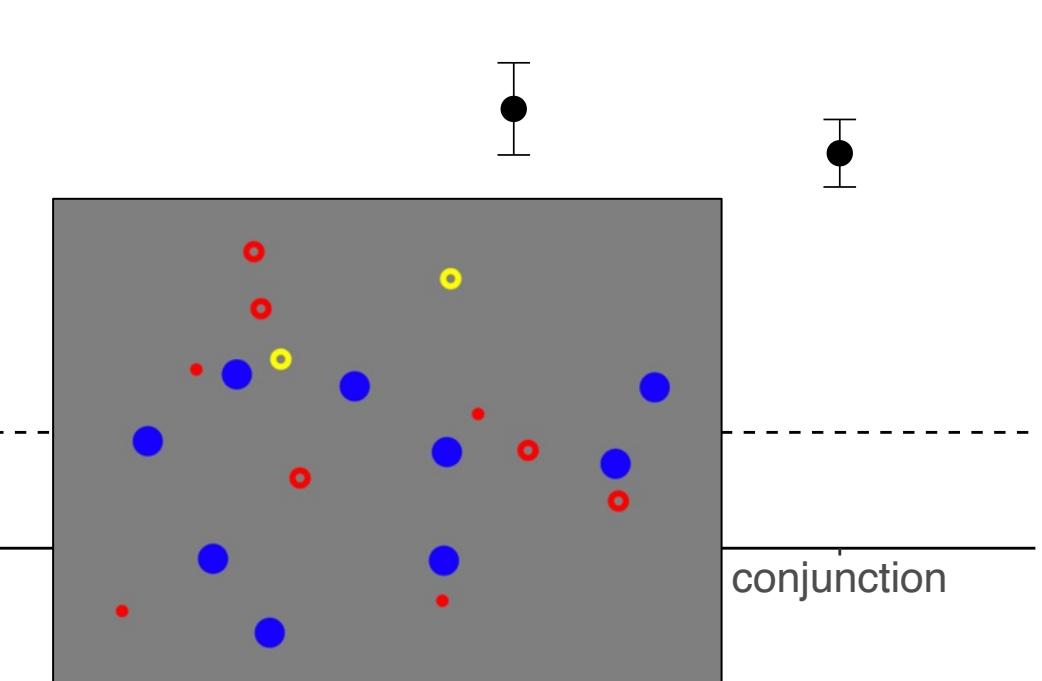
Restricted

The big-circles_x are such that they_x are blue

Treat arguments asymmetrically

Cardinality estimation error
"Every [size] circle is [color]"

Difference in % error
(post-verification – baseline)



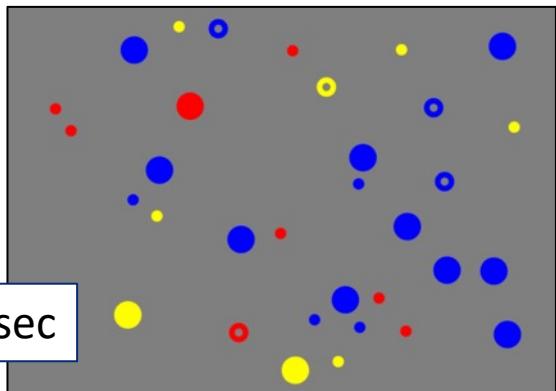
$|\text{BIG}| = |\text{BIG} \& \text{ BLUE}|$

n = 48

Every big one
is a blue one

TRUE

FALSE



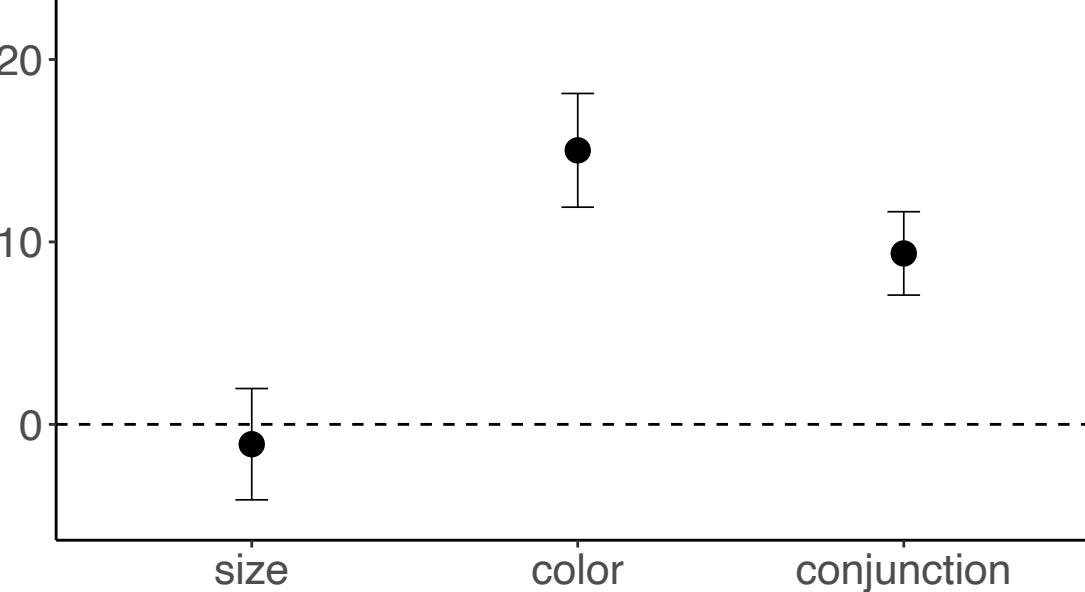
How many
{big/blue/big blue}
circles were there?

Does this reflect the
two arguments being
introduced in
different ways?

No!

Cardinality estimation error
"Every [size] one is a [color] one"

Difference in % error
(post-verification – baseline)

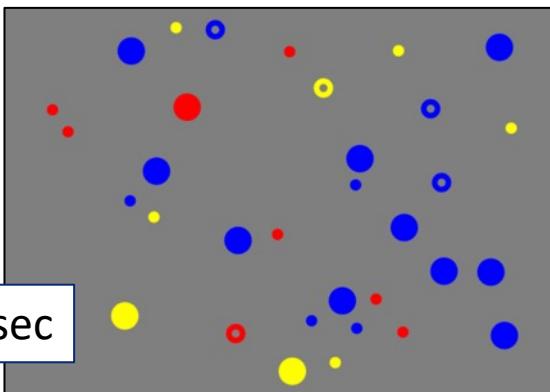


n = 48

Every circle that is big
is blue

TRUE

FALSE



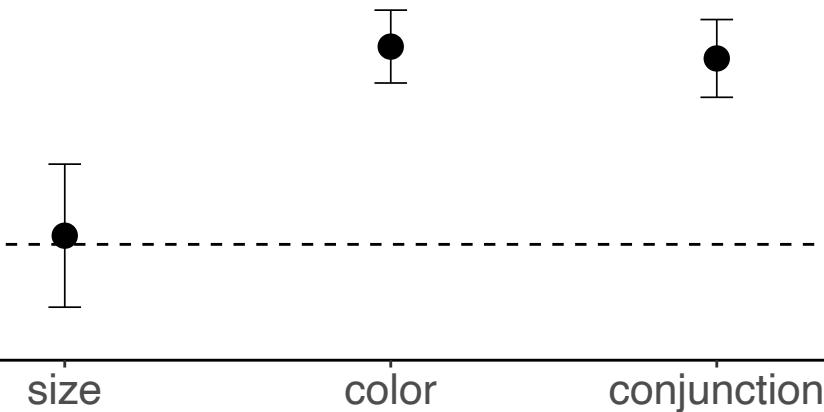
How many
{big/blue/big blue}
circles were there?

Does this reflect the
two arguments being
introduced in
different ways?

No!

Cardinality estimation error
"Every circle that is [size] is [color]"

Difference in % error
(post-verification – baseline)

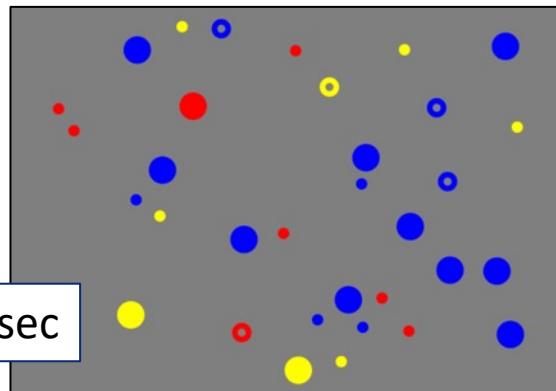


n = 48

Every big circle is blue

TRUE

FALSE



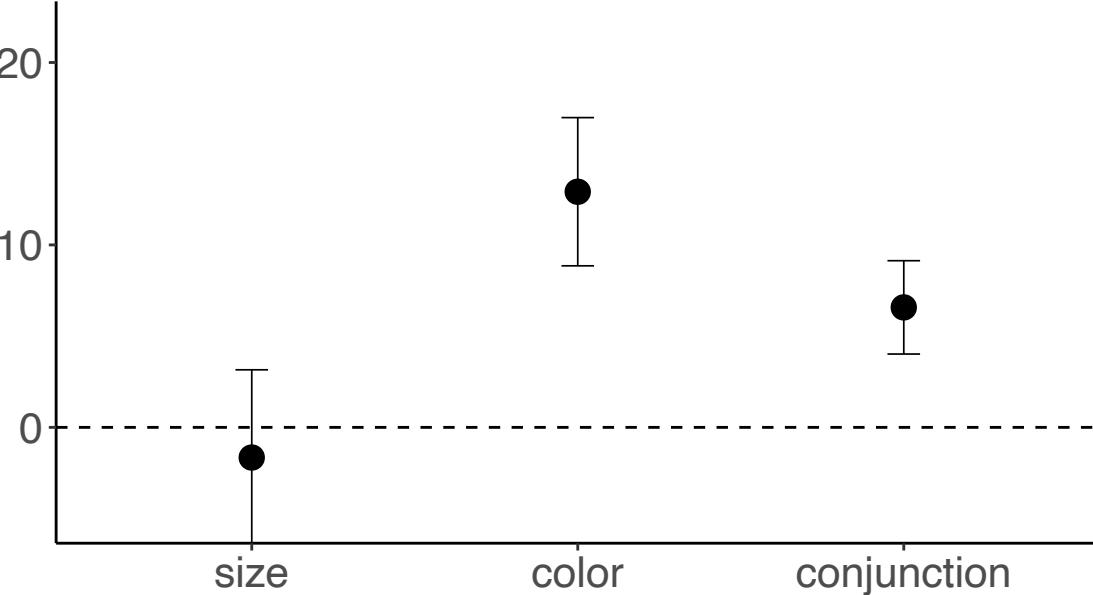
How many
{big/blue/big blue}
circles were there?

I don't know!

Another signature of
the asymmetry:
opting not to answer

Rate of pressing IDK button
"Every [size] circle is [color]"

Difference in % Opting out
(post-verification – baseline)

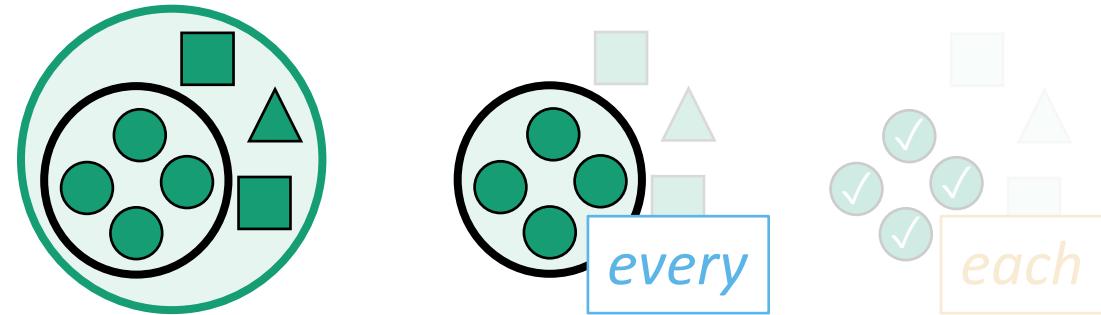


n = 48

Roadmap: How are *each* & *every* mentally represented?

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Relational vs. Restricted

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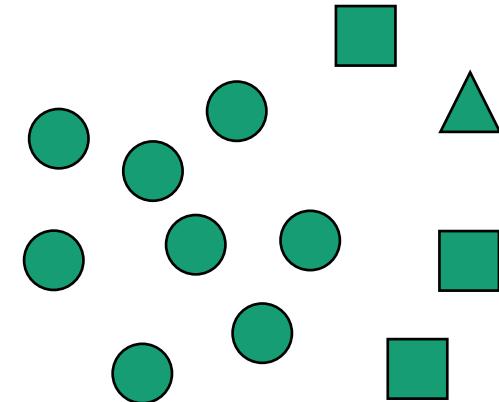
Natural language determiners are “conservative”

A determiner **DET** is conservative iff

- (1) $[[\text{DET } \text{NP}] \text{ PRED}] =$
- (2) $[[\text{DET } \text{NP}] [\text{be NP that PRED}]]$

every circle is green (TRUE) =

every circle is a circle that is green (TRUE)



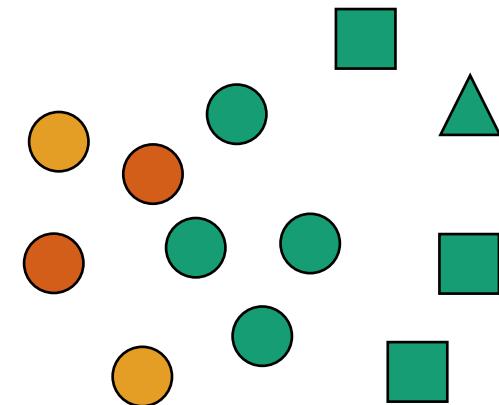
Natural language determiners are “conservative”

A determiner **DET** is conservative iff

- (1) $[[\text{DET } \text{NP}] \text{ PRED}] =$
- (2) $[[\text{DET } \text{NP}] [\text{be NP that PRED}]]$

every circle is green (FALSE) =

every circle is a circle that is green (FALSE)



Natural language determiners are “conservative”

A determiner **DET** is conservative iff

- (1) $[[\text{DET } \text{NP}] \text{ PRED}] =$
- (2) $[[\text{DET } \text{NP}] [\text{be } \text{NP} \text{ that PRED}]]$

- Cross-linguistically, all determiners are conservative
- 5year-olds can learn novel conservative determiners
but not novel non-conservative ones!

“Conservativity” is puzzling on the relational view

What rules out all the non-conservative relations?

$$|\text{CIRCLES} \cap \text{GREEN}| > |\text{CIRCLES} - \text{GREEN}|$$

≈ most circles are green

$$\text{CIRCLES} \subseteq \text{GREEN}$$

≈ every circle is green

$$|\text{CIRCLES}| = |\text{GREEN}|$$

$$\text{CIRCLES} \supseteq \text{GREEN}$$

“Conservativity” is entailed on the restricted view

Relative to the circles, *is green* applies to

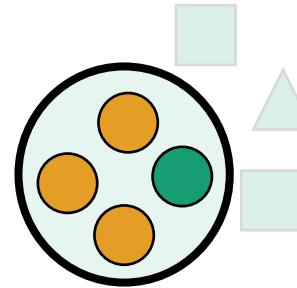
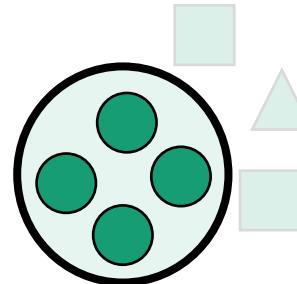
...**all** things

...**most** things

...**at least 2 & at most 4** things

...**???** things

(intended: $|\text{CIRCLES}| = |\text{GREEN}|$)

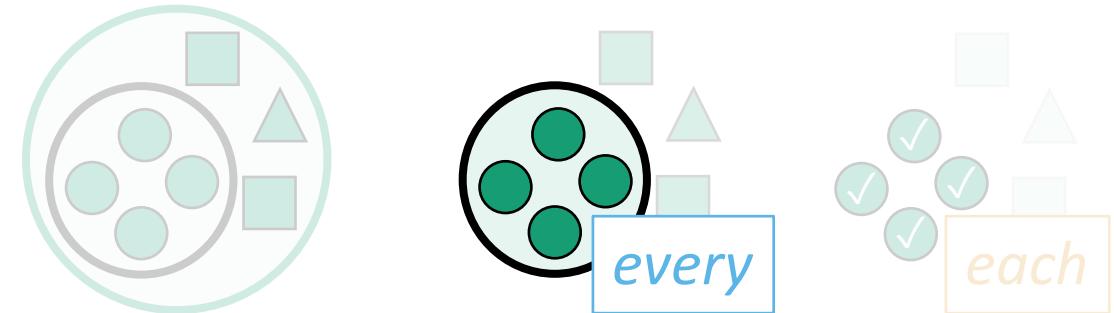


- Non-conservative meanings are not stateable if the first argument restricts the domain of quantification

Roadmap: How are *each* & *every* mentally represented?

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Each highlights individuals more than *every*

(1) Which book did you loan to **each** student?

a. *Frankenstein* to Frank, *Persuasion* to Paula, and *Moby Dick* to Mary

(2) Which book did you loan to **every** student?

a. #*Frankenstein* to Frank, *Persuasion* to Paula, and *Moby Dick* to Mary

b. There's no one book that I loaned to every student

Each highlights individuals more than *every*

(3) **Each** old fashioned needs an orange peel

a. *some particular cocktails are in need of garnishes*

(4) **Every** old fashioned needs an orange peel

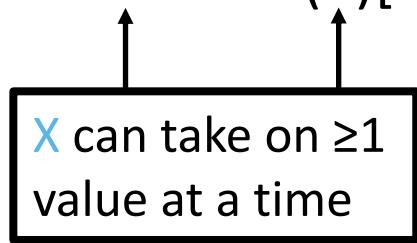
a. *some particular cocktails are in need of garnishes*

b. *in general, the recipe calls for an orange peel*

Different representations

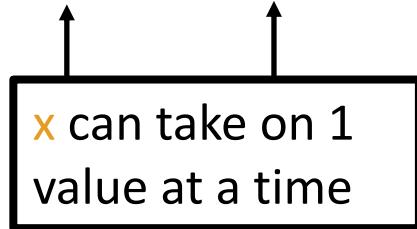
Second-order representation (*every circle is green*)

The $\exists x: \text{Circle}(x) [\text{Green}(x)]$ \approx The circles are such that they are green



First-order representation (*each circle is green*)

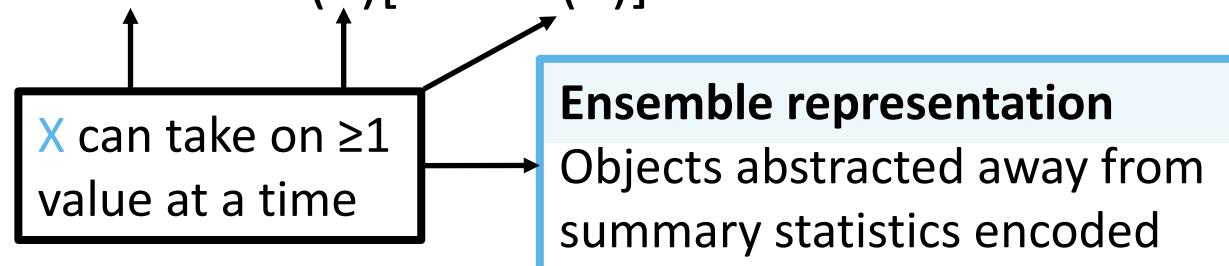
$\forall x: \text{Circle}(x) [\text{Green}(x)]$ \approx Any individual circle is such that it is green



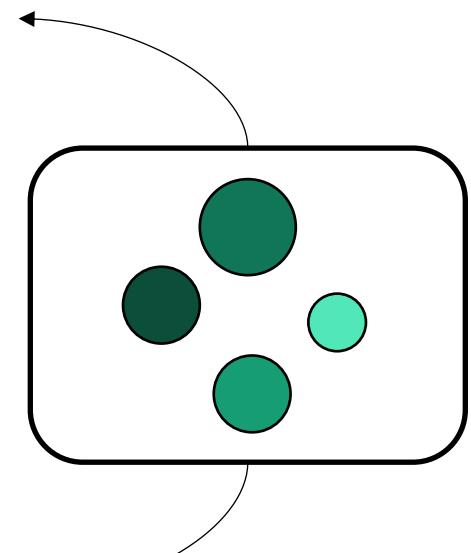
Different representations & cognitive systems

Second-order representation (*every circle is green*)

The $\forall X: \text{Circle}(X)[\text{Green}(X)]$ \approx The circles are such that they are green



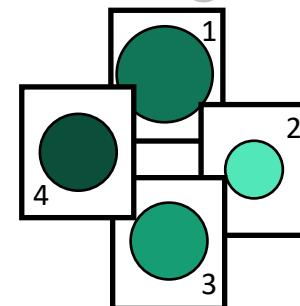
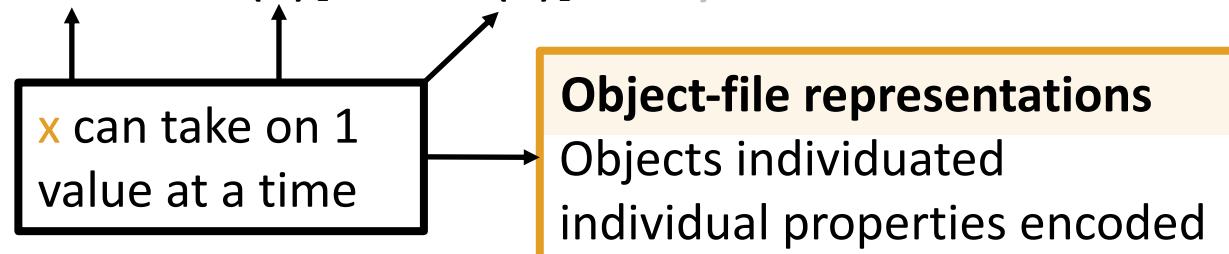
Center: (0,0)
Avg. Hue:
Avg. Size:



e.g., Ariely 2001; Feigenson, Dehaene & Spelke 2004; Alvarez 2011;
Haberman, Brady & Alvarez 2015; Ward, Bear & Scholl 2016; Whitney & Leib 2018

First-order representation (*each circle is green*)

$\forall x: \text{Circle}(x)[\text{Green}(x)]$ \approx Any individual circle is such that it is green



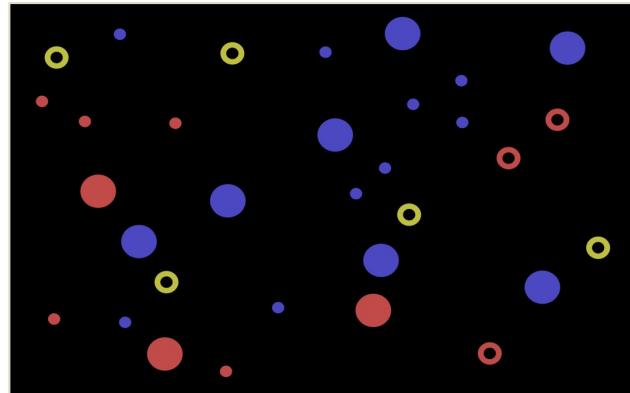
e.g., Kahnemann et al. 1992; Pylyshyn & Storm 1998; Scholl, Pylyshyn & Feldman 2001; Scholl 2002; Feigenson, Dehaene & Spelke 2004; Carey 2009

Cardinality (group property)

{Each/Every} big circle is blue

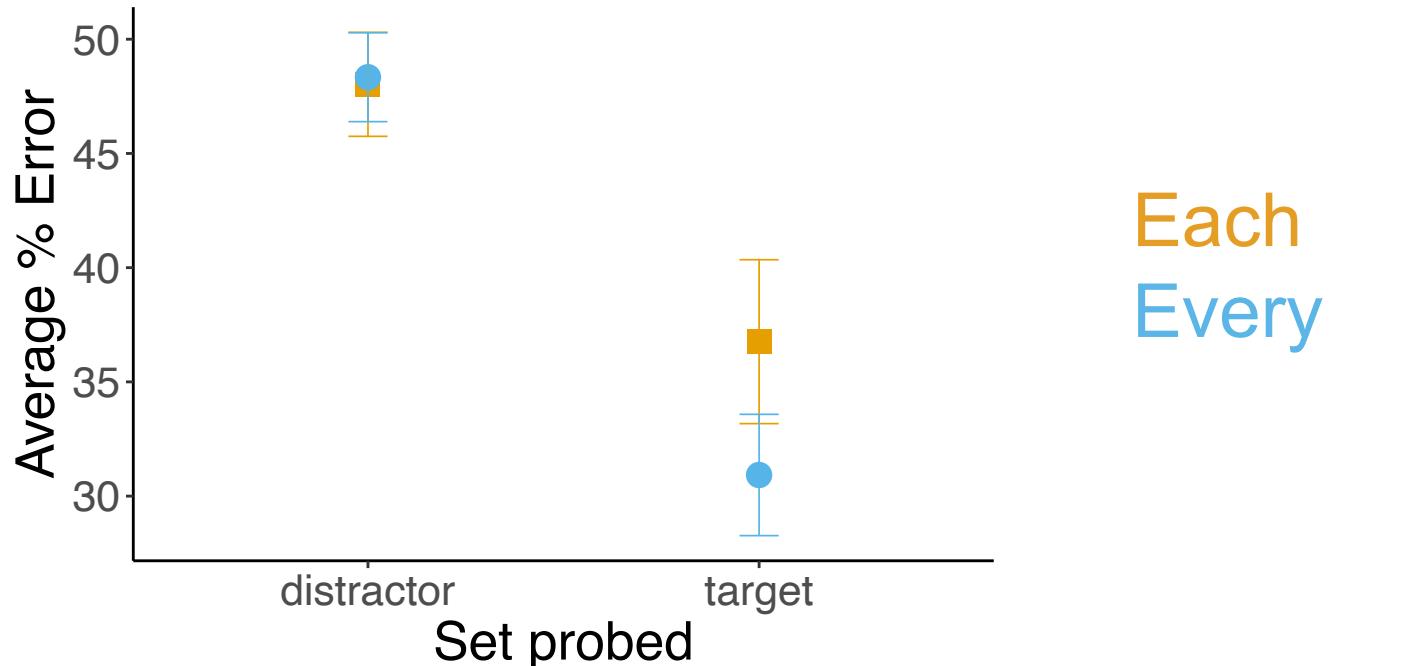
TRUE

FALSE



How many
{big/medium/small}
circles were there?

Percent error – "each" vs. "every"



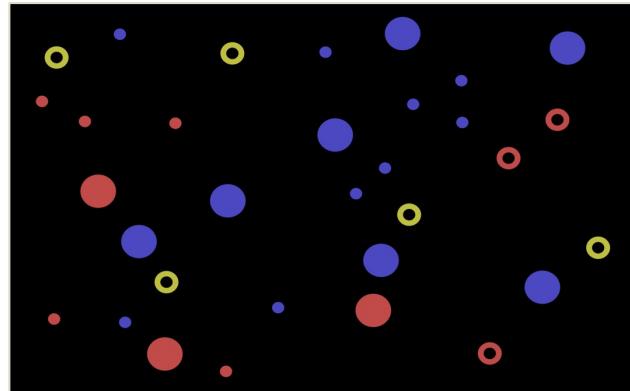
n = 24

Cardinality (group property)

{Each/Every} big circle is blue

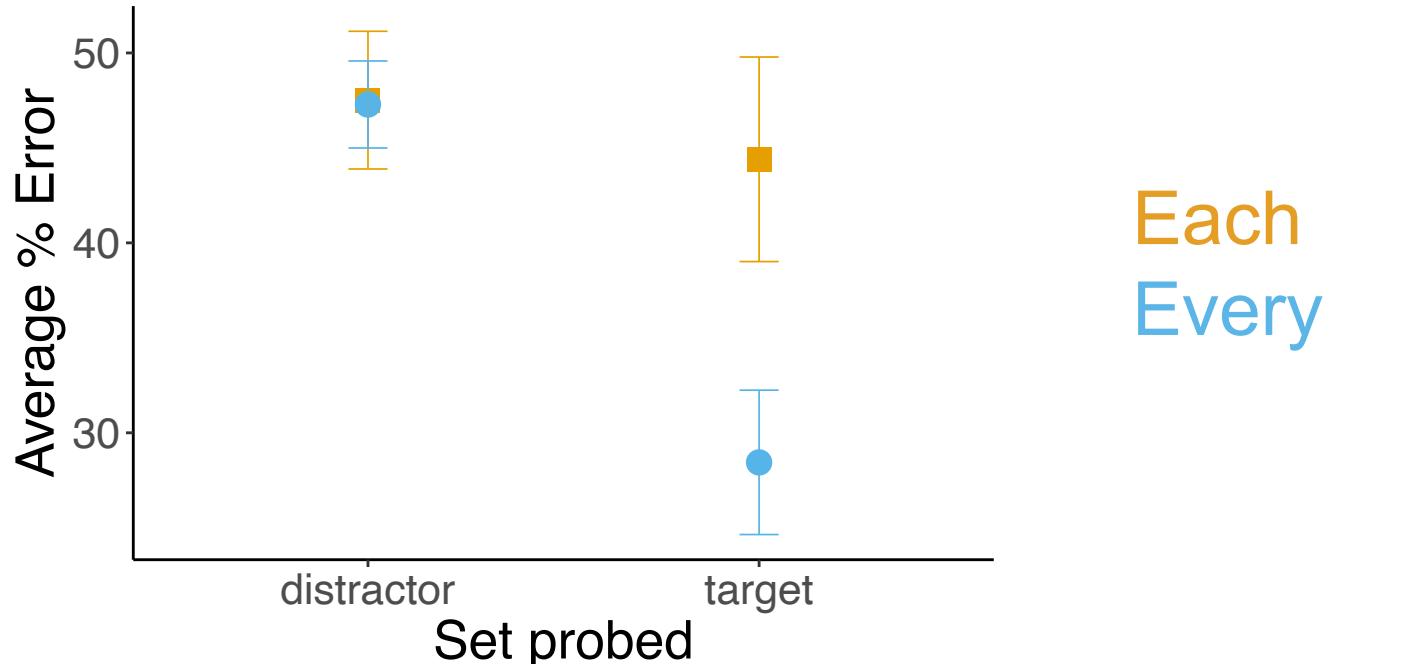
TRUE

FALSE



How many
{big/medium/small}
circles were there?

Percent error (initial condition "each")



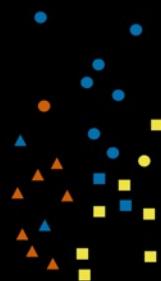
n = 12

Center of Mass (group property)

Is {each/every} circle blue?

“Yes”

“No”



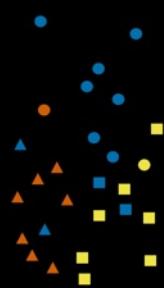
(with 3- to 8-year-olds)

Where was the middle
of the circles?

Is {each/every} circle blue?

“Yes”

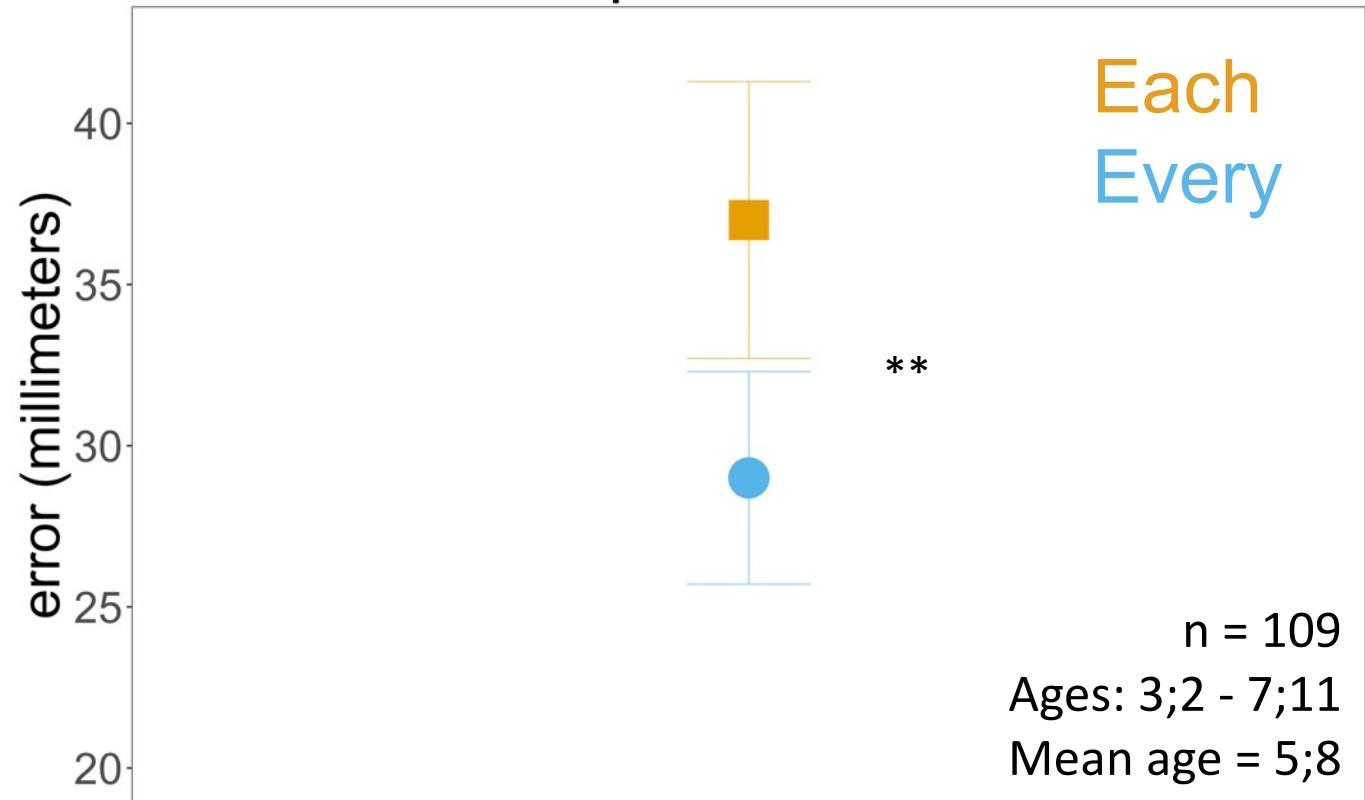
“No”



Where was the middle
of the circles?

Center of Mass (group property)

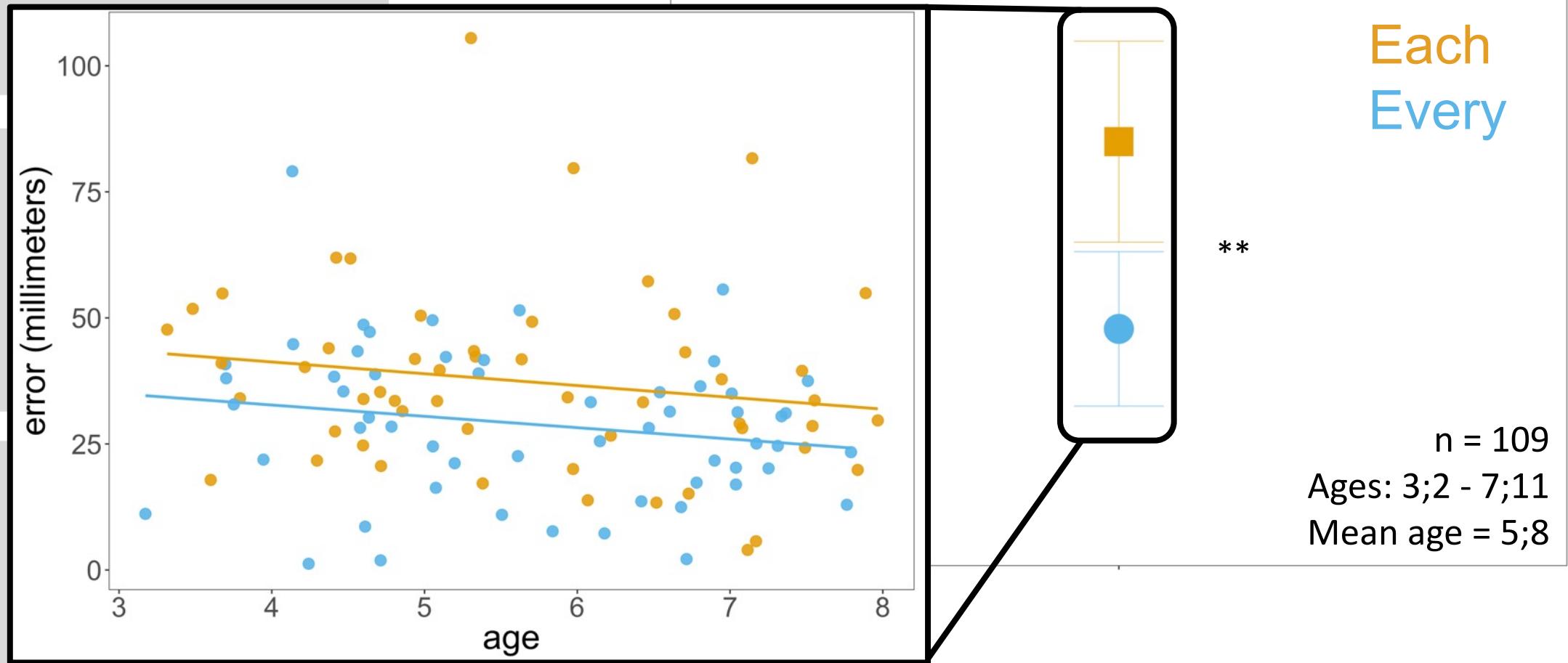
Distance from tap to actual set center



Center of Mass (group property)

Is {each/every} circle blue?

Distance from tap to actual set center



{Each/Every}
circle is green



TRUE

FALSE



One circle
changed its color

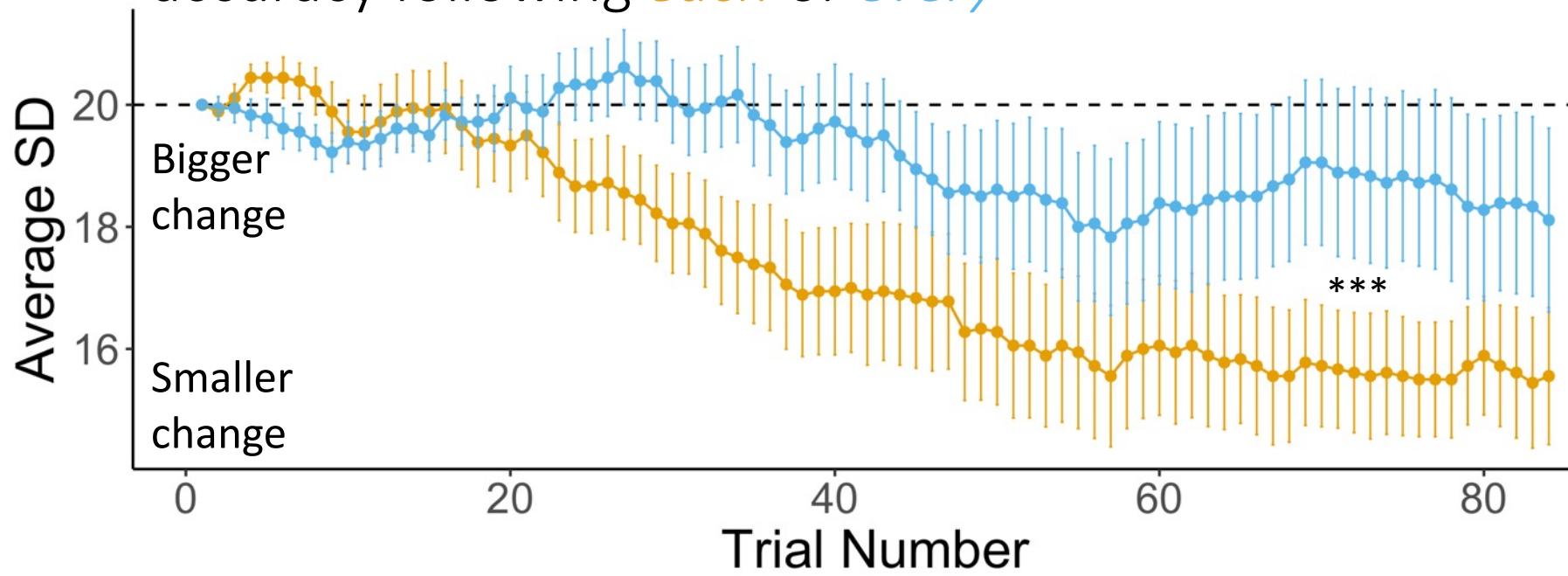


TRUE

FALSE

Color (individual property)

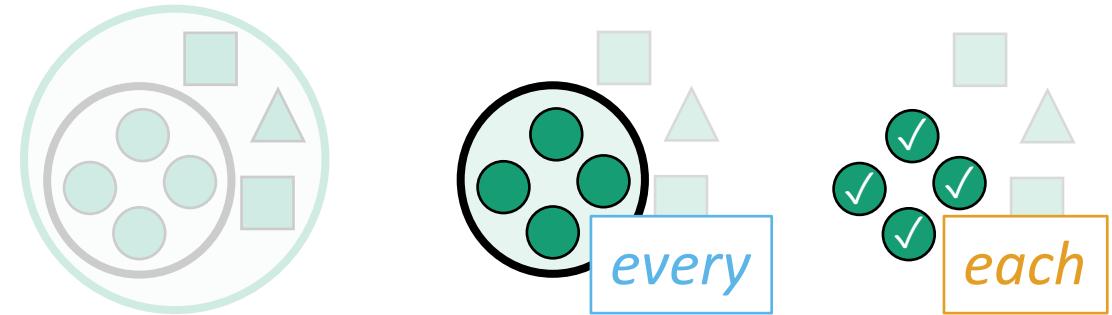
Color change detection: difficulty required for 70% accuracy following *each* or *every*



Roadmap: How are *each* & *every* mentally represented?

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First-order vs. Second-order (individual- vs. group- implicating)

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- ➔ Consequences for language acquisition

How are *each* & *every* acquired?

e.g., to acquire *every*, learners need to figure out its:

Syntactic category:

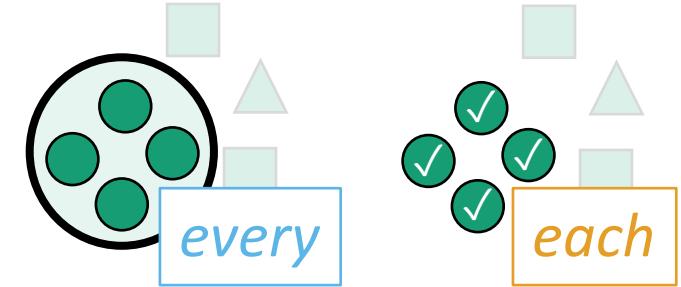
DET (not ADJ)

Quantificational content:

Universal (not proportional, existential, etc.)

Representational format:

Second-order restricted (not first-order restricted)



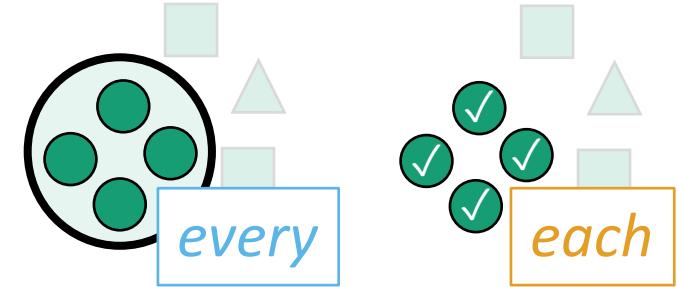
What leads learners to pair
“each” and “every” with
the right representations?



What information is in learners' input?

- "You have to ring up **each** thing"
- "Could you put a flower on **each** plate?"
- "Put sugar in **each** coffee"
- "We'll put one finger on **each** thing we count"
- "We **each** have three"

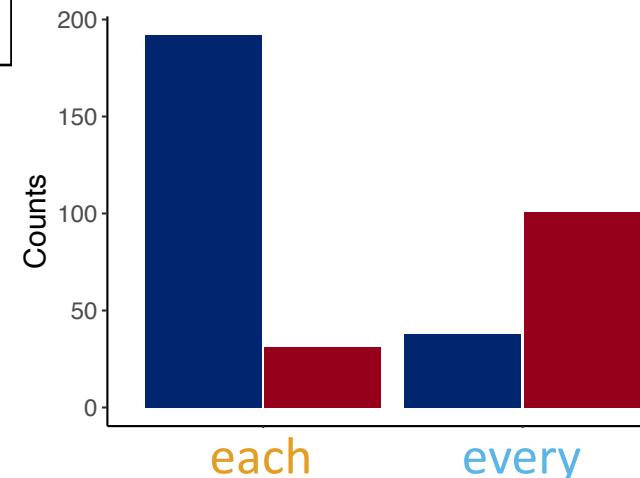
} Child ambient speech



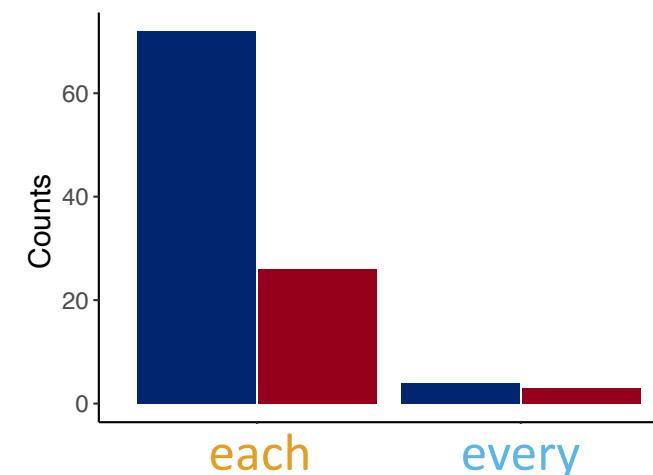
Generalize over local domain



Domain present vs. **not**



Within working memory limit (≤ 3) vs. **not** (≥ 4)



How are *each* & *every* acquired?



"You want one bite
of **each** piece, huh?"

Domain present in small #s

Triggers

Object-files



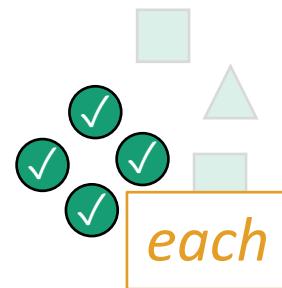
Every time you color,
you get better."

Project beyond local domain

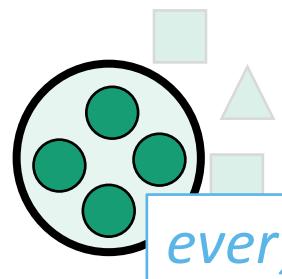
Triggers

Ensembles

Diversity: low
Avg. Hue: Avg. Size:



$\forall x: \text{Circle}(x)$
[Green(x)]

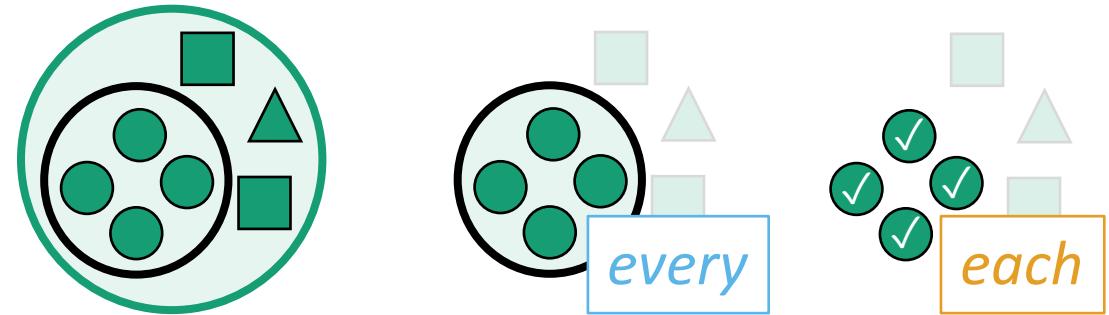


The $x: \text{Circle}(x)$
[Green(x)]

How are *each* & *every* mentally represented?

Three hypotheses

- ✓ Two (psycho)logical distinctions



Relational vs. Restricted

- ✓ Number cognition as a probe
- ✓ The “conservativity” universal

First-order vs. Second-order (individual- vs. group- implicating)

- ✓ Object-files vs. Ensembles as a probe
- ✓ Consequences for language acquisition

Thanks!

Special thanks to:

Jeffrey Lidz

Alexander Williams

Anna Papafragou

Zoe Ovans

Ellen Lau

Audiences at: UMD; Penn; USC; ELM1; SALT30; CUNY/HSP; BUCLD

NSF #NRT-1449815 & NSF #BCS-2017525

Paul Pietroski

Victor Gomes

John Trueswell

Alexis Wellwood

Darko Odic

Justin Halberda

Nicolò Cesana-Arlotti

Florian Schwarz

Valentine Hacquard

Laurel Perkins

And to each and every one of you!