

The mental representation of universal quantifiers: evidence from verification

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²Rutgers University, Philosophy

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Are meanings invariant across people?

- *Cat*

Are meanings invariant across people?

- *Cat, car, carburetor, ...*: no good hypotheses about what they mean
- Quantifiers: too many candidate hypotheses!

[[Every circle is blue]] = TRUE iff

$$\forall x : \text{circle}(x)[\text{blue}(x)]$$

$$\neg \exists x : \text{circle}(x)[\neg \text{blue}(x)]$$

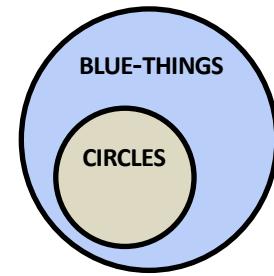
$$\text{CIRCLES} \subseteq \text{BLUE-THINGS}$$

$$\text{CIRCLES} = \text{CIRCLES} \cap \text{BLUE-THINGS}$$

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3

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[[Every circle is blue]] = TRUE iff

First-order

$$\left[\begin{array}{l} \forall x : \text{circle}(x)[\text{blue}(x)] \\ \neg \exists x : \text{circle}(x)[\neg \text{blue}(x)] \end{array} \right]$$

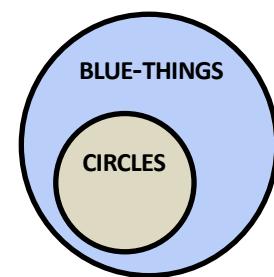
Second-order

$$\left[\begin{array}{l} \text{CIRCLES} \subseteq \text{BLUE-THINGS} \\ \text{CIRCLES} = \text{CIRCLES} \cap \text{BLUE-THINGS} \end{array} \right]$$

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4

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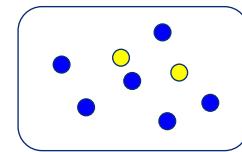
First-order $\boxed{\begin{array}{l} \text{Every circle is blue}] = \text{TRUE iff} \\ \forall x : \text{circle}(x)[\text{blue}(x)] \\ \neg \exists x : \text{circle}(x)[\neg \text{blue}(x)] \end{array}}$ } *each*

Second-order $\boxed{\begin{array}{l} \text{CIRCLES} \subseteq \text{BLUE-THINGS} \\ \text{CIRCLES} = \text{CIRCLES} \cap \text{BLUE-THINGS} \\ \cdot \\ \cdot \\ \cdot \end{array}}$ } *every/all*

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Linking hypothesis

Pietroski et al. 2009; Lidz et al. 2011; Pietroski et al. 2011

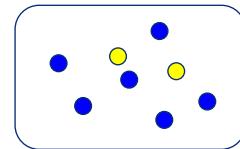


Each/*Every* circle is blue

6

Linking hypothesis

Pietroski et al. 2009; Lidz et al. 2011; Pietroski et al. 2011



Each/Every circle is blue

$\forall x: Circle(x)[Blue(x)]$

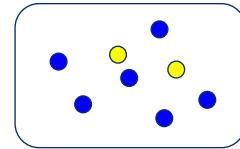
CIRCLE \subseteq BLUE

7

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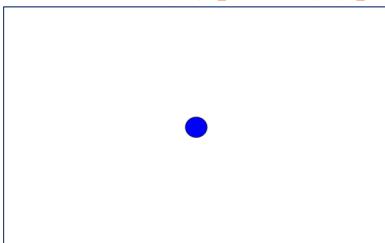
A **first-order** format will invite representation of **individuals**



Each/Every circle is blue

$\forall x: Circle(x)[Blue(x)]$

CIRCLE \subseteq BLUE



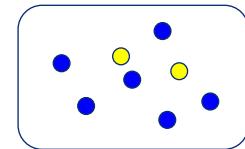
8

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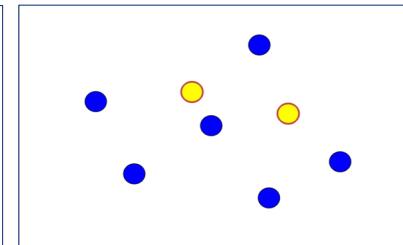
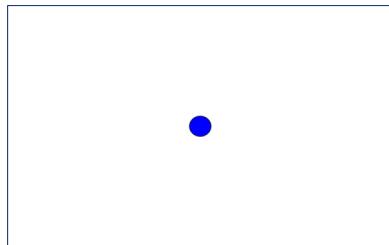
A **second-order** format will invite representation of **groups**



Each/Every circle is blue

$\forall x: Circle(x)[Blue(x)]$

CIRCLE \subseteq BLUE



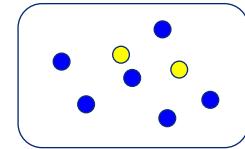
9

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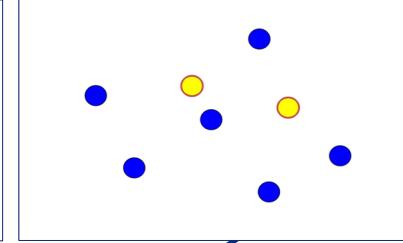
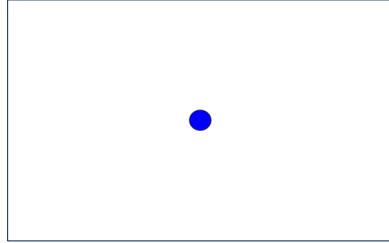
A **second-order** format will invite representation of **groups** → leading to knowledge of group properties



Each/Every circle is blue

$\forall x: Circle(x)[Blue(x)]$

CIRCLE \subseteq BLUE



Good estimate of summary statistics
(number, avg. size, center of mass, ...)
Ariely 2001; Cohen & Treisman 2003; Feigenson et al.
2004; Burr & Ross 2008; Alvarez 2011; ao

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Roadmap

Diagnosing set representation

- Accuracy and precision of number thoughts

Manipulating set representation with language

- *Most-* vs. existential-statements

Deploying the method with universals

- *Each* vs. *every* vs. *all*

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Roadmap

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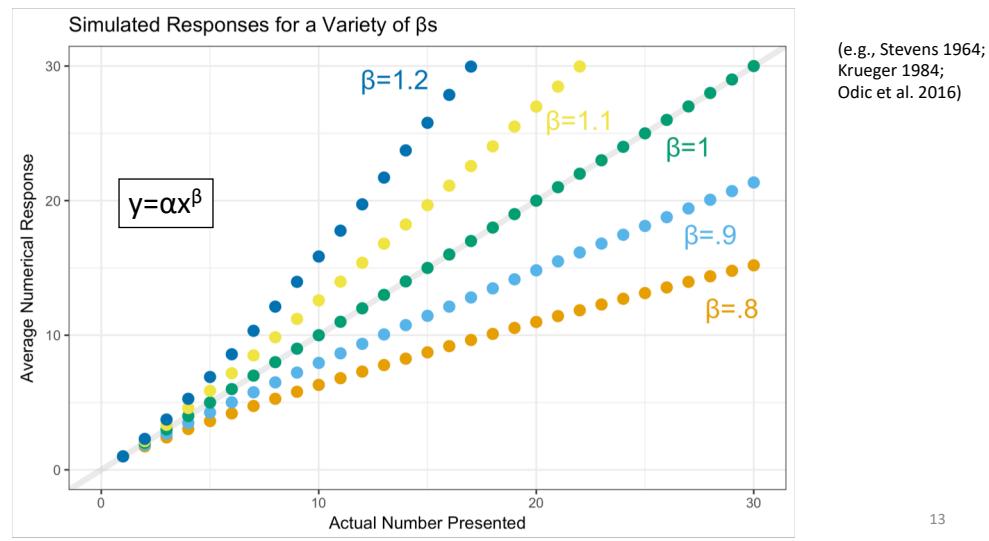
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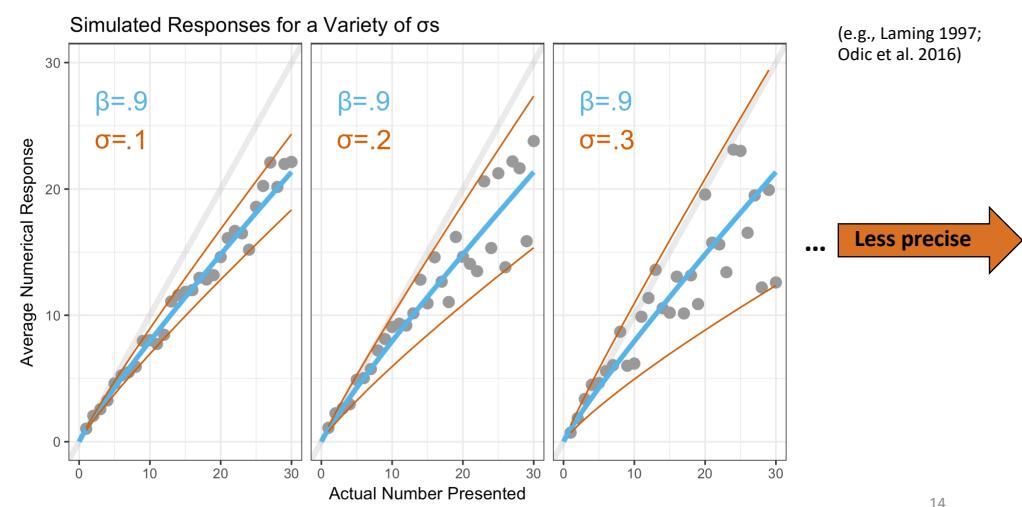
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12

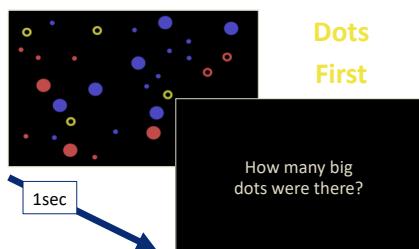
Measuring accuracy of number thoughts (β)



Measuring precision of number thoughts (σ)

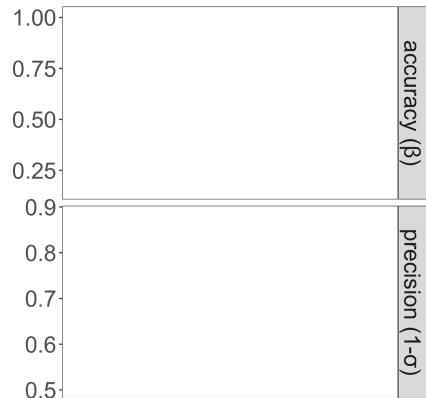


Baseline task



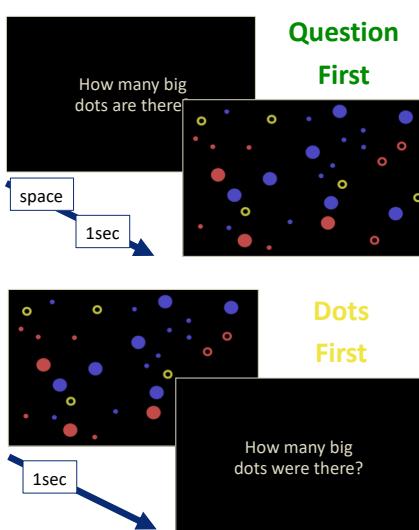
n=12

Baseline task model fits



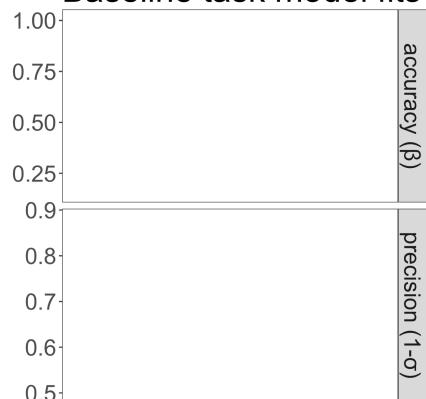
15

Baseline task



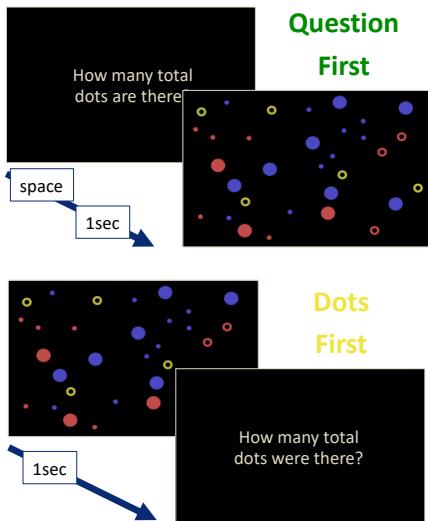
n=12

Baseline task model fits

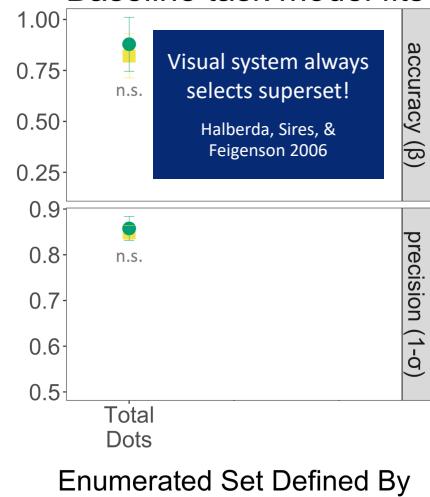


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Baseline task



Baseline task model fits

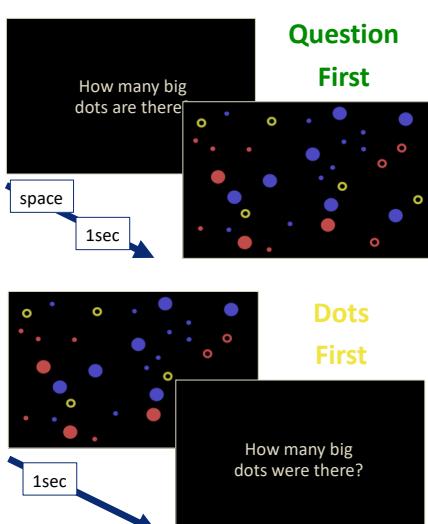


n=12

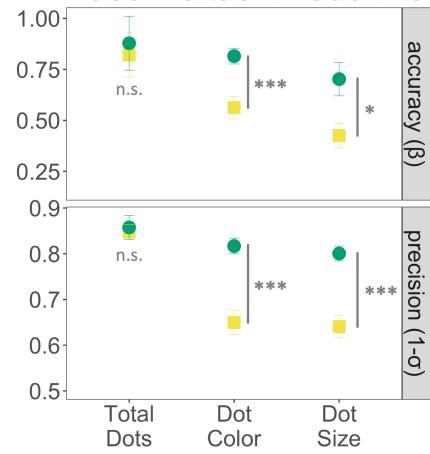
Enumerated Set Defined By

17

Baseline task



Baseline task model fits



n=12

Enumerated Set Defined By

18

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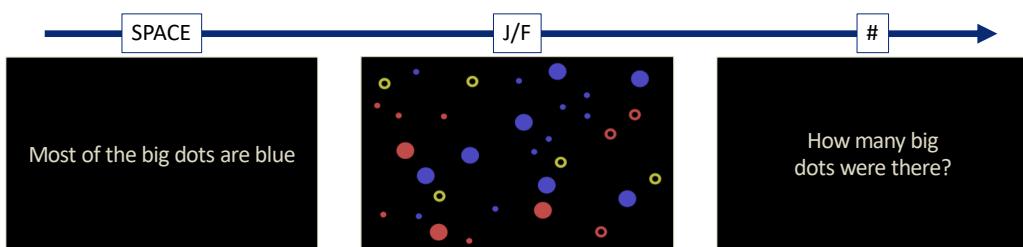
Deploying the method with universals

- *Each* vs. *every* vs. *all*

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Adding language

- Same pictures, but now evaluate some quantificational statement first

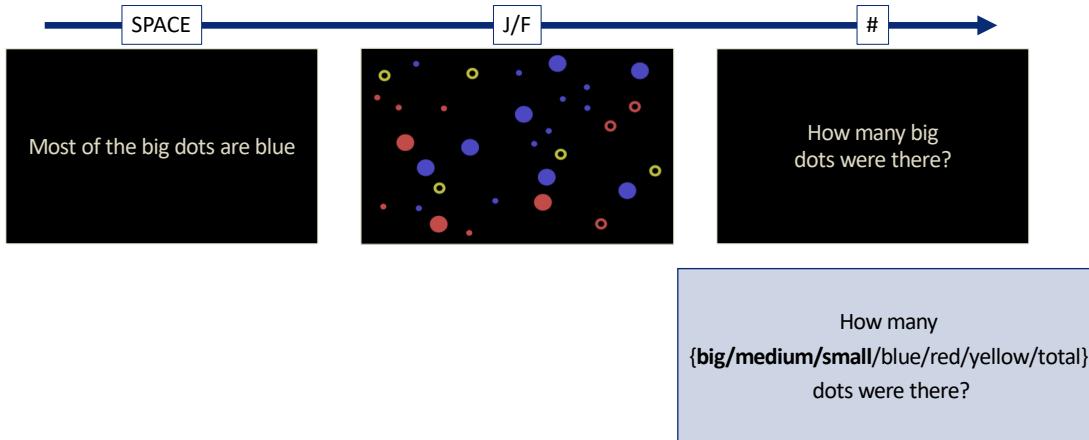


20

10

Adding language

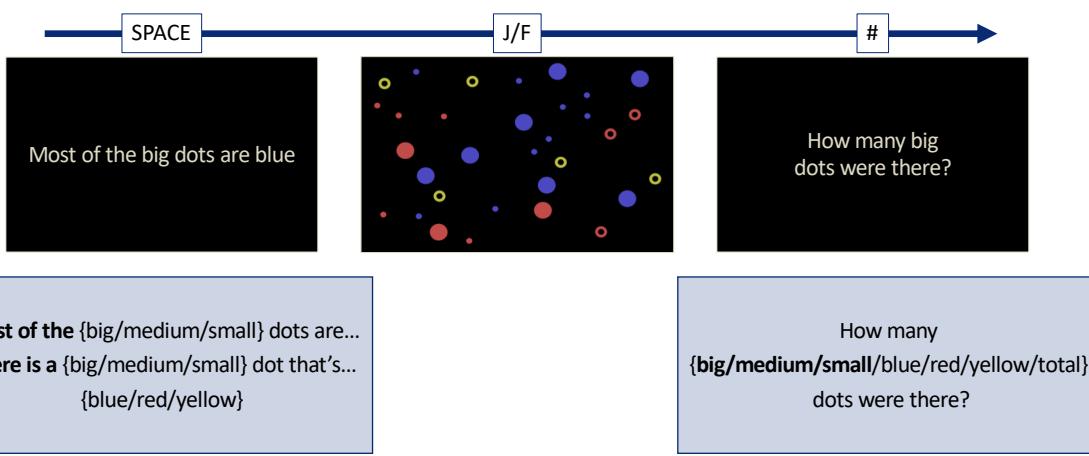
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21

Adding language

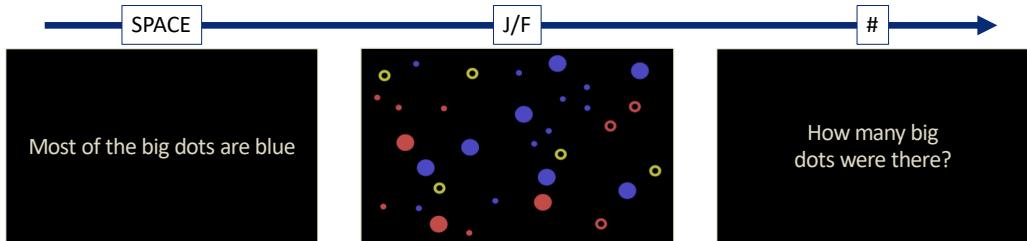
- Same pictures, but now evaluate some quantificational statement first



22

Adding language

- Same pictures, but now evaluate some quantificational statement first



Most of the {big/medium/small} dots are...
There is a {big/medium/small} dot that's...
{blue/red/yellow}

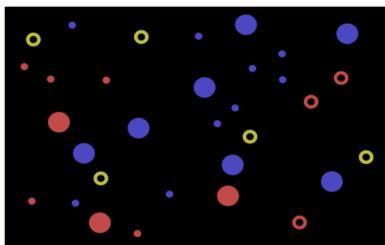
Most:
-Provably second-order
(Rescher 1962; Wiggins 1980; Barwise & Cooper 1981)
-Empirically shown to invite # thoughts
(Pietroski et al. 2009; Lidz et al. 2011; Pietroski et al. 2011; Tomaszewicz 2011)

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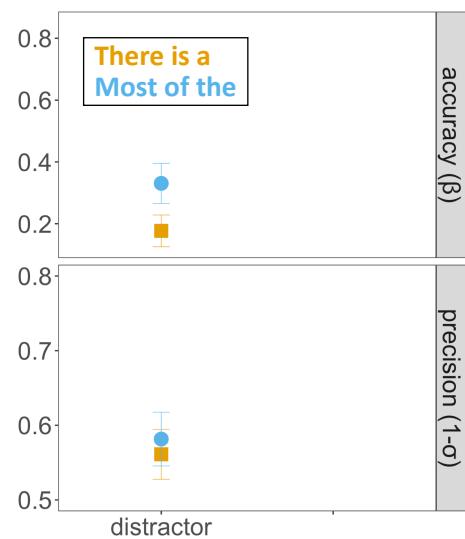
Most vs. Exists (1sec viewing time)

n=12

There is a **big dot that's blue**
Most of the **big dots are blue**



How many **small** dots were there?

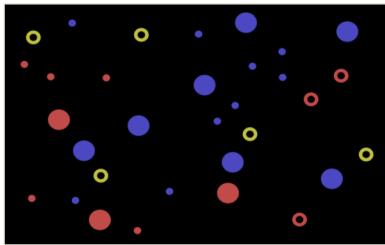


24

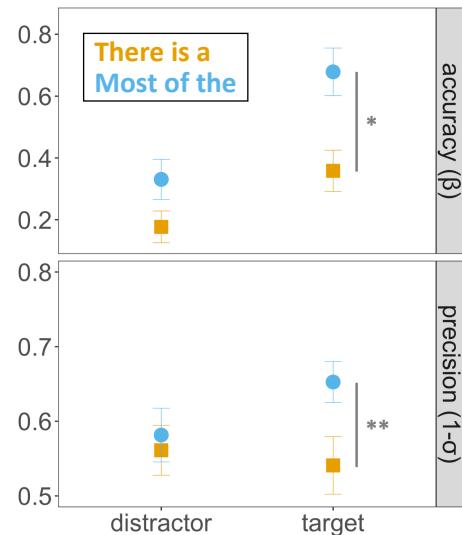
n=12

Most vs. Exists (1sec viewing time)

There is a **big** dot that's blue
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How many **big** dots were there?

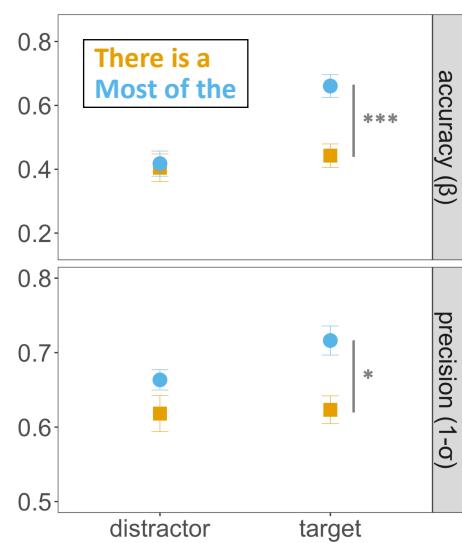
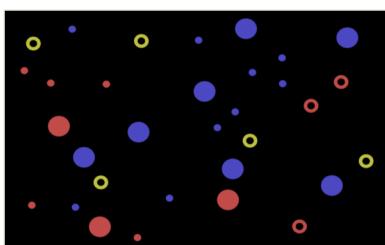


25

n=12

Most vs. Exists (unlimited viewing time)

There is a **big** dot that's blue
Most of the **big** dots are blue

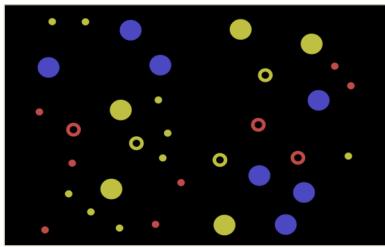


26

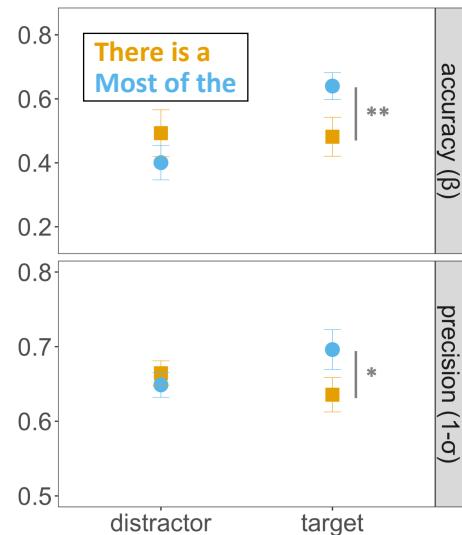
n=24

Most vs. Exists (_{FALSE} trials only)

There is a big dot that's red
Most of the big dots are red



- Each big dot must be considered!

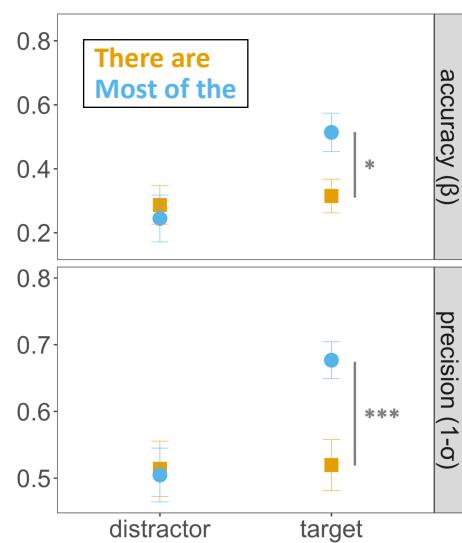
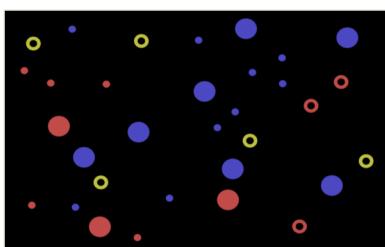


27

n=12

Most vs. Exists (w/ plural NPs)

There are big dots that are blue
Most of the big dots are blue



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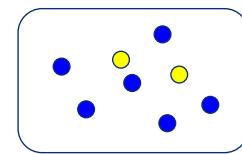
29

Linking hypothesis

Pietroski et al. 2009; Lidz et al. 2011; Pietroski et al. 2011

A **first-order** format will invite representation of **individuals**

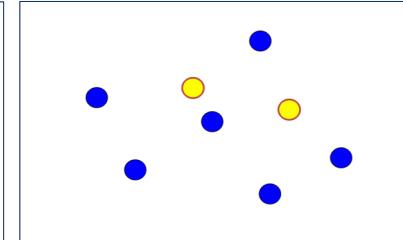
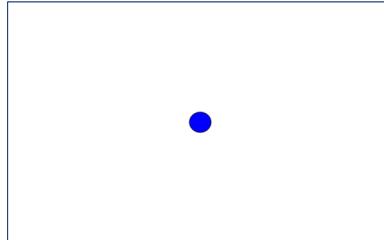
A **second-order** format will invite representation of **groups** → leading to knowledge of group properties



Each/Every circle is blue

$\forall x: Circle(x)[Blue(x)]$

CIRCLE \subseteq BLUE

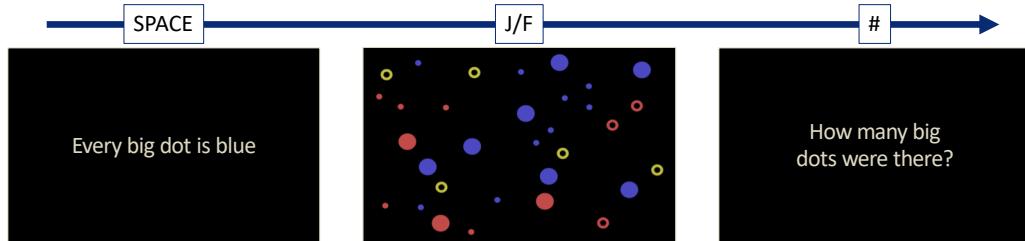


Good estimate of summary statistics
(**number**, avg. size, center of mass, ...)
Ariely 2001; Cohen & Treisman 2003; Feigenson et al.
2004; Burr & Ross 2008; Alvarez 2011; ao

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Testing the universals

- Only the quantifier differs between blocks



Each (of the) {big/medium/small} dot(s) is(/are)...
Every {big/medium/small} dot is...
All of the {big/medium/small} dots are...
{blue/red/yellow}

How many
{big/medium/small/blue/red/yellow/total}
dots were there?

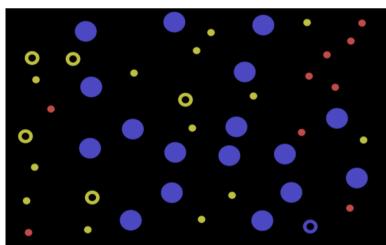
31

Each of the v. All of the

Intuitively, *each* might be first-order

Each of the big dots are blue

All of the big dots are blue



- Unfriendliness to genericity

(Gil 1991; Beghelli & Stowell 1997)

- (1) a. All birds lay eggs
- b. #Each bird lays eggs

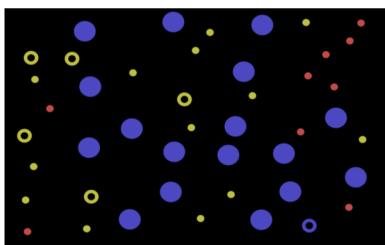
32

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(Gil 1991; Beghelli & Stowell 1997)

- (1) a. All birds lay eggs
b. #Each bird lays eggs

- Resistance to collective predicates

(Vendler 1962; Dowty 1987)

- (2) a. All of the students gathered
b. *Each (of the) student(s) gathered

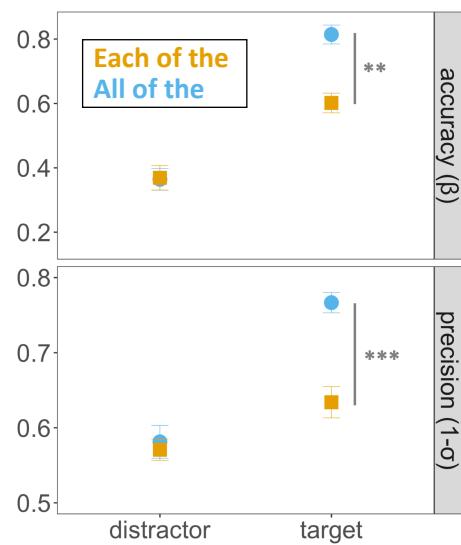
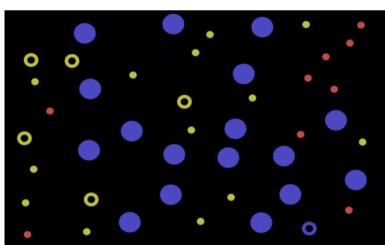
33

Each of the v. All of the

n=12

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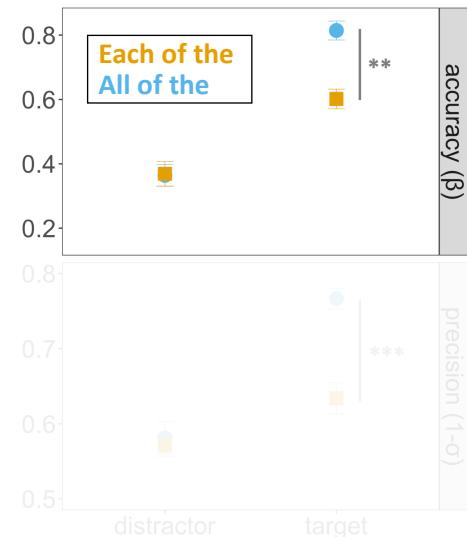
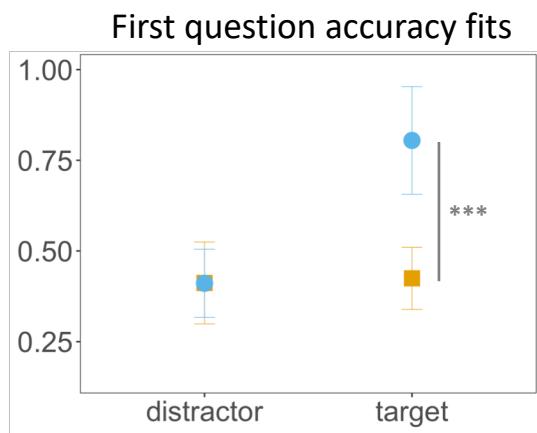
All of the big dots are blue



34

Each of the v. All of the

n=12



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Each v. Every

Every sometimes patterns with *all*...

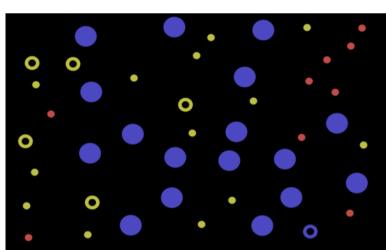
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(Gil 1991; Beghelli & Stowell 1997)

- a. All birds lay eggs
- b. Every bird lays eggs
- c. #Each bird lays eggs

Each big dot is blue

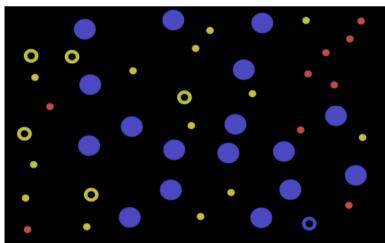
Every big dot is blue



36

Each v. Every

Each big dot is blue
Every big dot is blue



Every sometimes patterns with *all*...

- Friendliness to genericity

(Gil 1991; Beghelli & Stowell 1997)

- (1) a. All birds lay eggs
b. Every bird lays eggs
c. #Each bird lays eggs

...and sometimes patterns with *each*

- Resistance to collective predicates

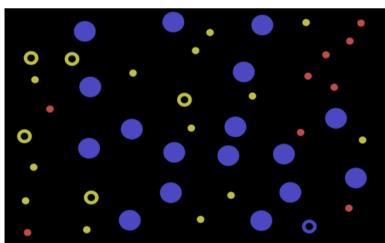
(Vendler 1962; Dowty 1987)

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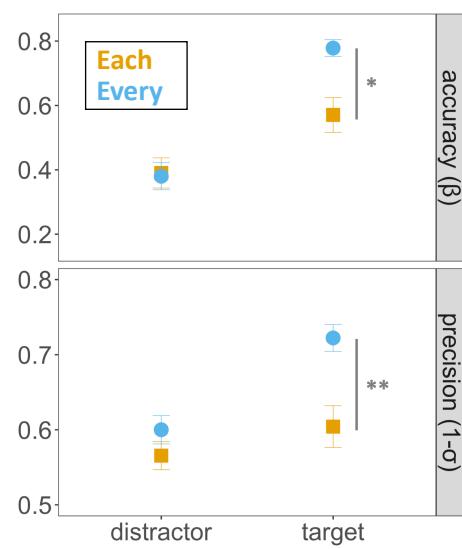
37

Each v. Every

Each big dot is blue
Every big dot is blue

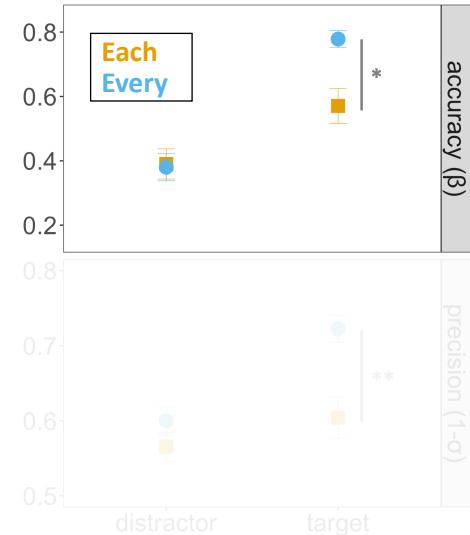
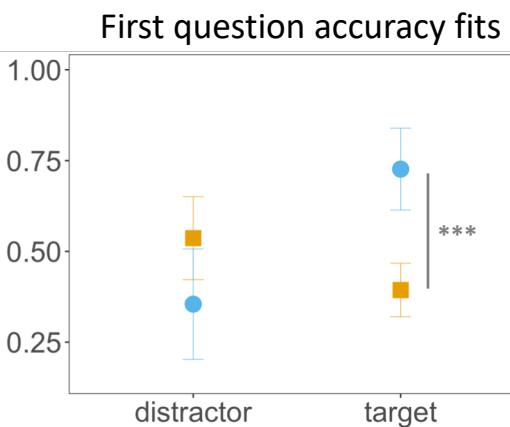


n=12



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Each v. Every

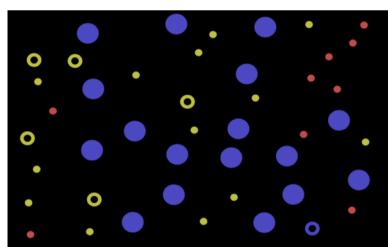


39

Every v. All of the

Every big dot is blue

All of the big dots are blue

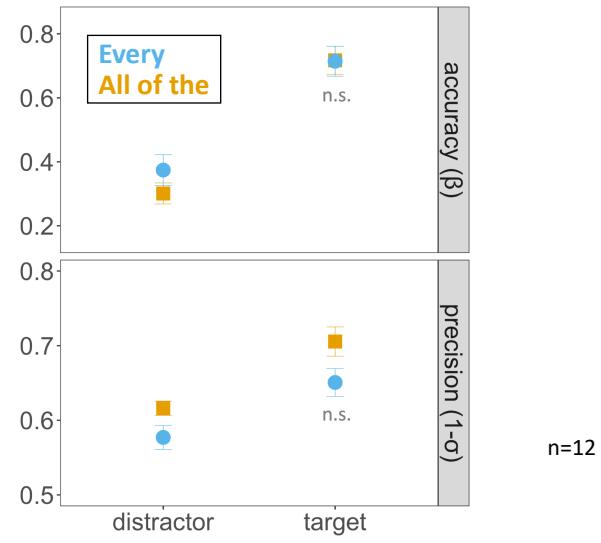
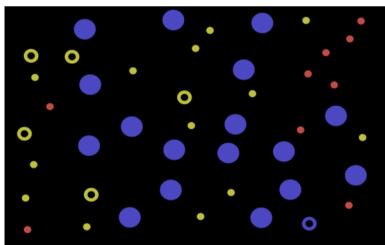


All > Each
Every > Each
Every = All

40

Every v. All of the

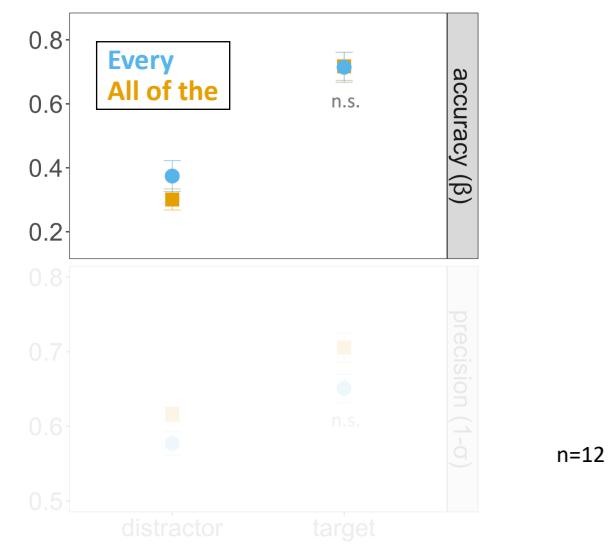
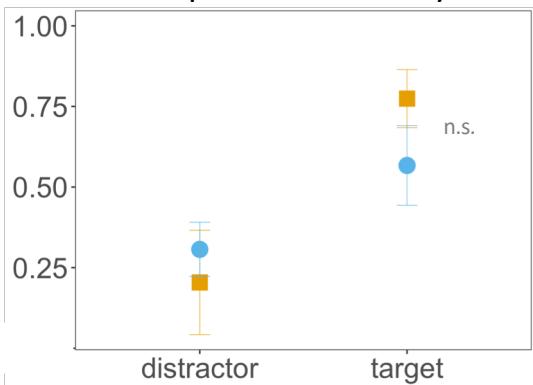
Every big dot is blue
All of the big dots are blue



41

Every v. All of the

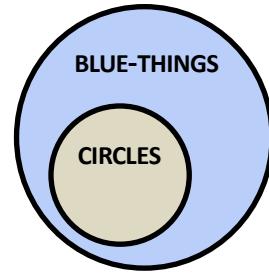
First question accuracy fits



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Conclusion

- Despite being TC=, the universal quantifiers differ in how they're represented:



First-order

$$\left[\begin{array}{l} \forall x : \text{circle}(x)[\text{blue}(x)] \\ \neg \exists x : \text{circle}(x)[\neg \text{blue}(x)] \end{array} \right]$$

each

Second-order

$$\left[\begin{array}{l} \text{CIRCLES} \subseteq \text{BLUE-THINGS} \\ \text{CIRCLES} = \text{CIRCLES} \cap \text{BLUE-THINGS} \end{array} \right]$$

every/all

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Conclusion

- Despite being TC=, the universal quantifiers differ in how they're represented:

First-order

You **can** ask whether people have the same meaning for some word (at least for the logical vocabulary)

Pietroski et al. 2009; Lidz et al. 2011;

Pietroski et al. 2011; Knowlton et al. *under review*

Second-order

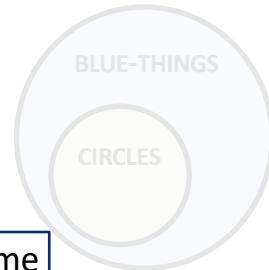
$$\left[\begin{array}{l} \text{CIRCLES} \subseteq \text{BLUE-THINGS} \\ \text{CIRCLES} = \text{CIRCLES} \cap \text{BLUE-THINGS} \end{array} \right]$$

every/all

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