

The psycho-logic of universal quantifiers

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Slides available at:

tylerknowlton.com/talks/USC.pdf

Meaning in mental grammar



- ➔ What sorts of instructions do meanings provide to cognition?
- ➔ What can that relationship tell us about semantic representations?

Methodological strategy

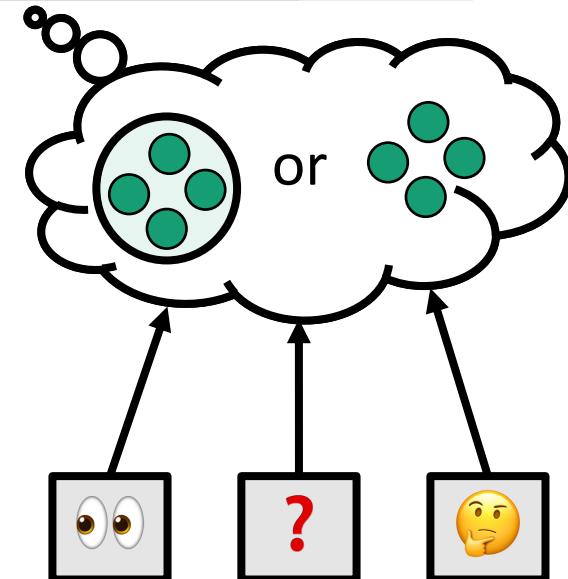
Ask which cognitive systems are engaged

→ e.g., systems for group vs. individual representation

Isolate the contribution of the semantic representation

“Every circle is green”

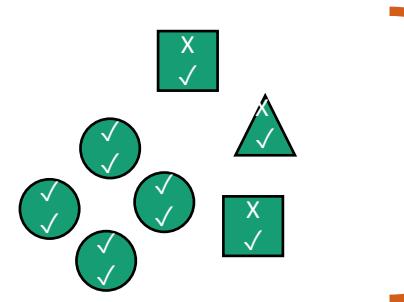
*The circles are such
that they are all green*



Many ways to describe “every circle is green”

For each individual_x, if it_x is a circle, it_x is green

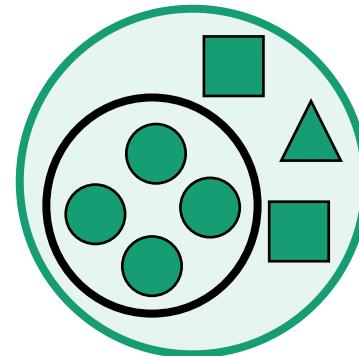
UNIVERSE: $\forall x[\text{Circle}(x) \rightarrow \text{Green}(x)]$



**Unrestricted
First-order**

The circles_x are included in the green-things_y

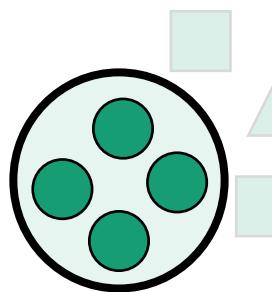
UNIVERSE: $\text{TheX:Circle}(X) \subseteq \text{TheY:Green}(Y)$



**Unrestricted
Second-order**

The circles_x are such that all of them_x are green

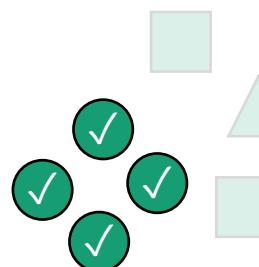
CIRCLES: $\forall X[\text{Green}(X)]$



**Restricted
Second-order**

The individual circles_x are s.t. each one_x is green

CIRCLES: $\forall x[\text{Green}(x)]$



**Restricted
First-order**

How are universal quantifiers mentally represented?

Relational vs. Restricted (*every*)

- ➔ Number cognition as a probe into which arguments are represented
- ➔ Explanation of the “conservativity” universal

First- vs. Second-order (*each* vs. *every*)

- ➔ Object-files vs. Ensembles as a probe into how arguments are encoded

Consequences for language acquisition (*each* vs. *every*)

- ➔ Precursors of quantification in infants
- ➔ Differences in child-directed speech

Different representations

“Every circle is green”

Relational (unrestricted)

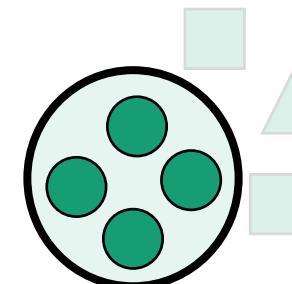
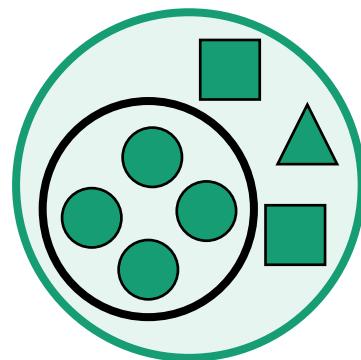
$\text{TheX:Circle}(X) \subseteq \text{TheY:Green}(Y)$

\approx The circles_X are included in
the green-things_Y

Restricted

CIRCLES: $\forall X[\text{Green}(X)]$

\approx The circles_X are such that
all of them_X are green



Different behavioral predictions

Linking hypothesis: people are biased toward verification strategies that directly compute the relations & operations expressed by the semantic representation under evaluation

Relational (unrestricted)

$\text{TheX:Circle}(X) \subseteq \text{TheY:Green}(Y)$

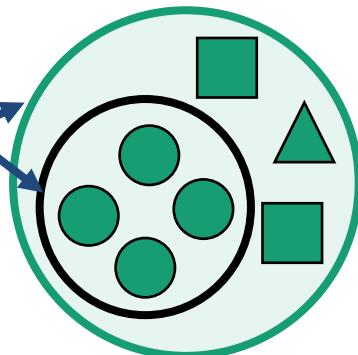
\approx The circles_X are included in
the green-things_Y

Restricted

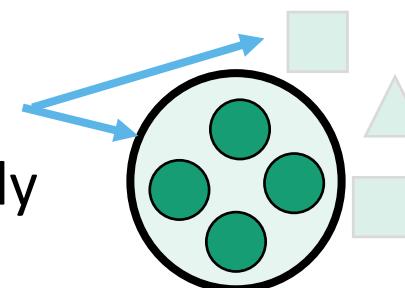
CIRCLES: $\forall X[\text{Green}(X)]$

\approx The circles_X are such that
all of them_X are green

Represent &
compare both
arguments



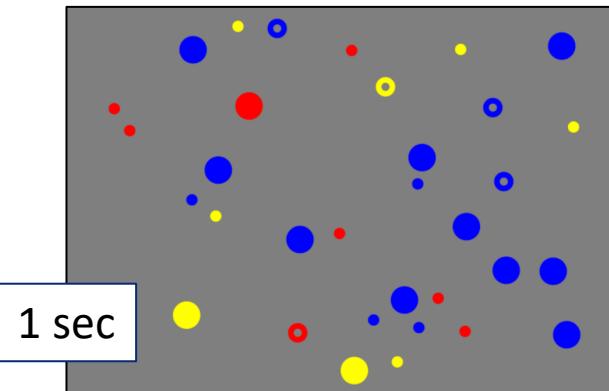
Treat
arguments
asymmetrically



Every big circle is blue

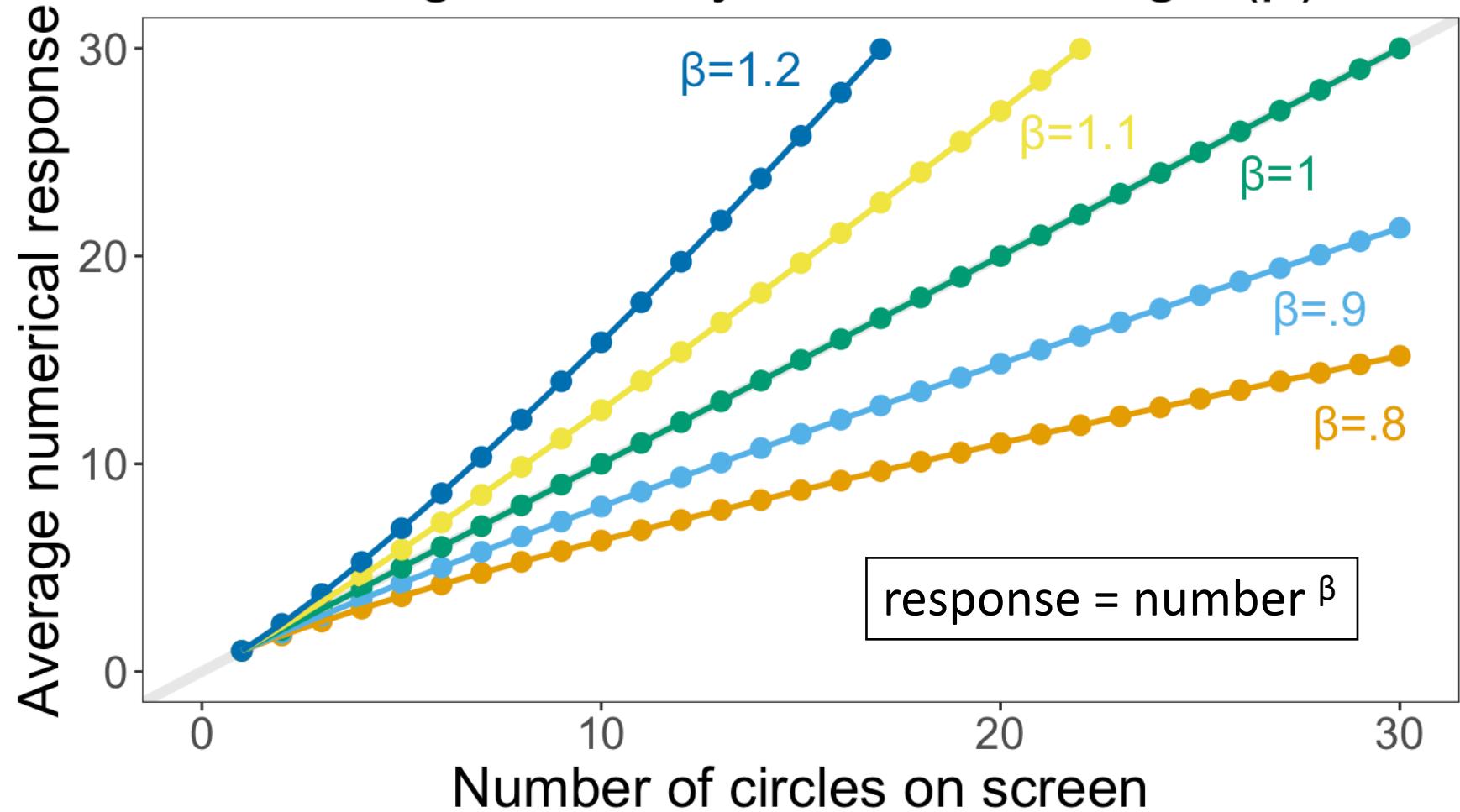
TRUE

FALSE



How many big circles
were there?

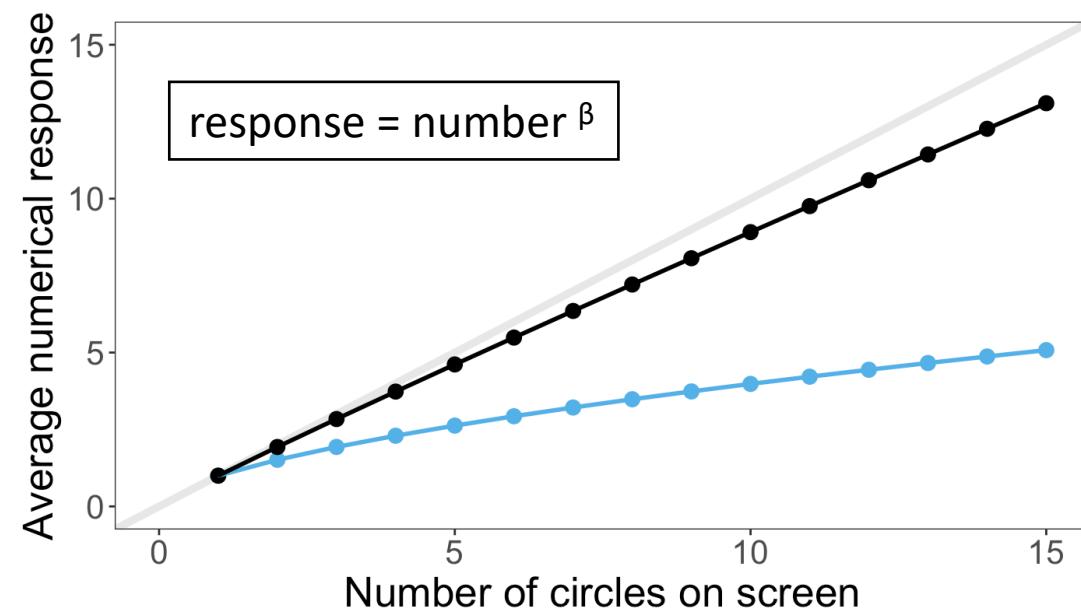
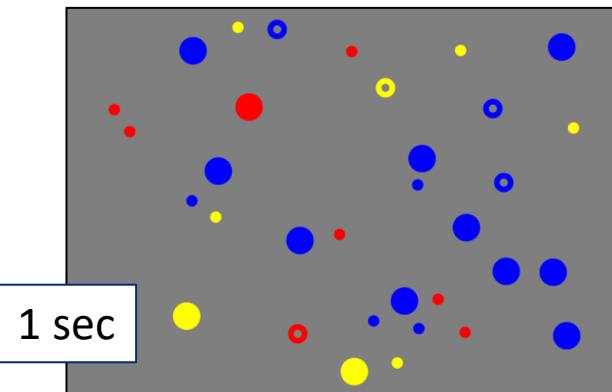
Measuring accuracy of #‐knowledge (β)



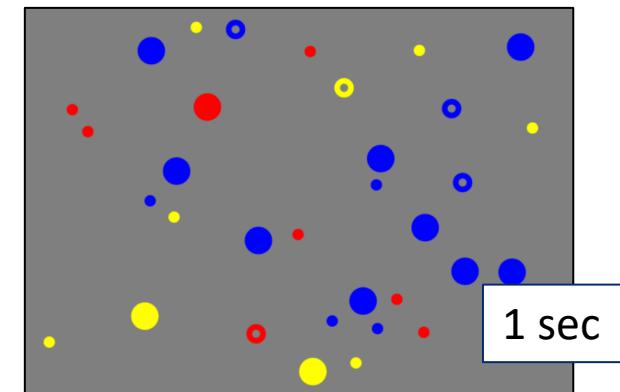
Every big circle is blue

TRUE

FALSE



How many big circles
are there?

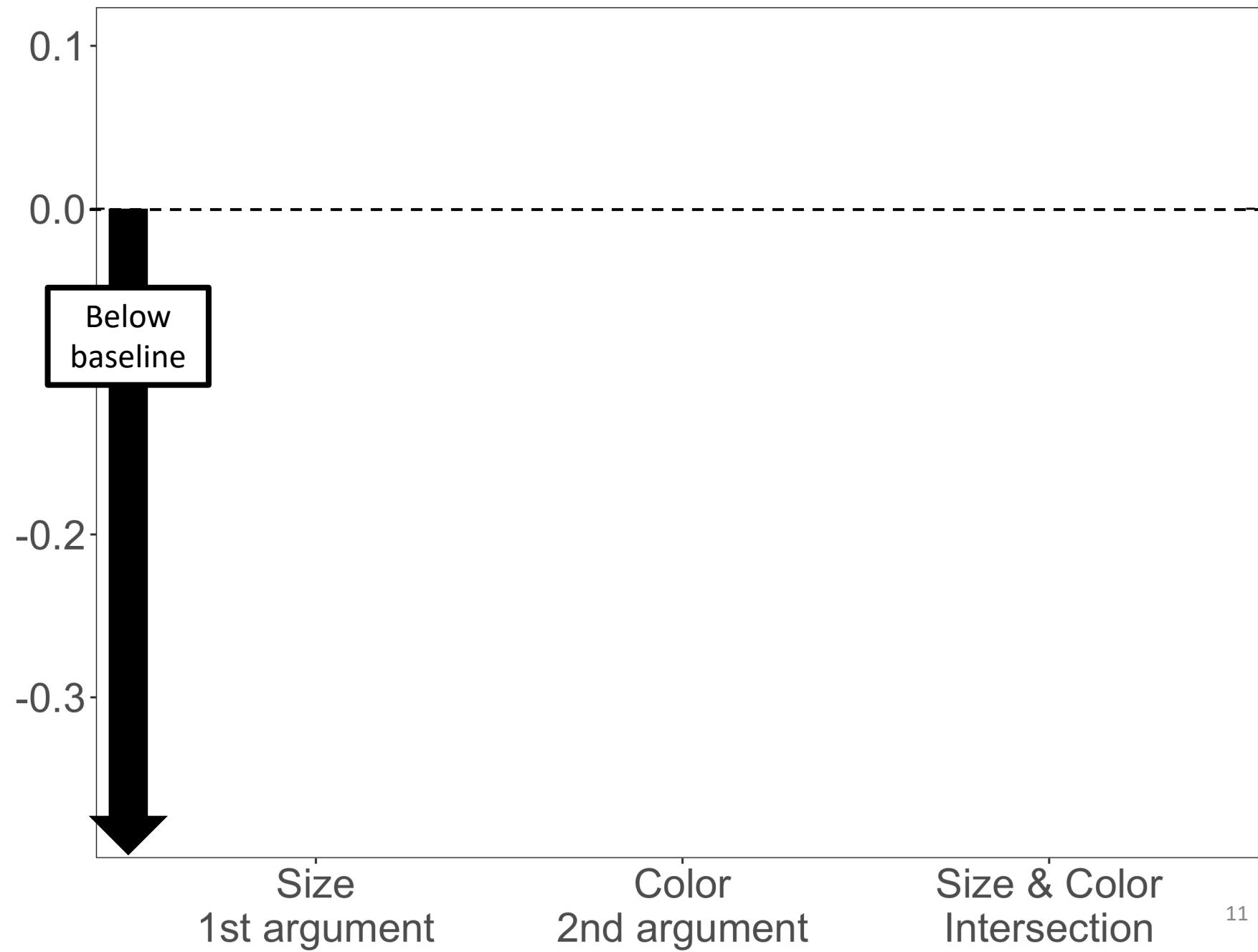


How many big circles
were there?

#-knowledge following *every*
vs.
#-knowledge **baseline**

How many big circles
were there?

Every big circle is blue

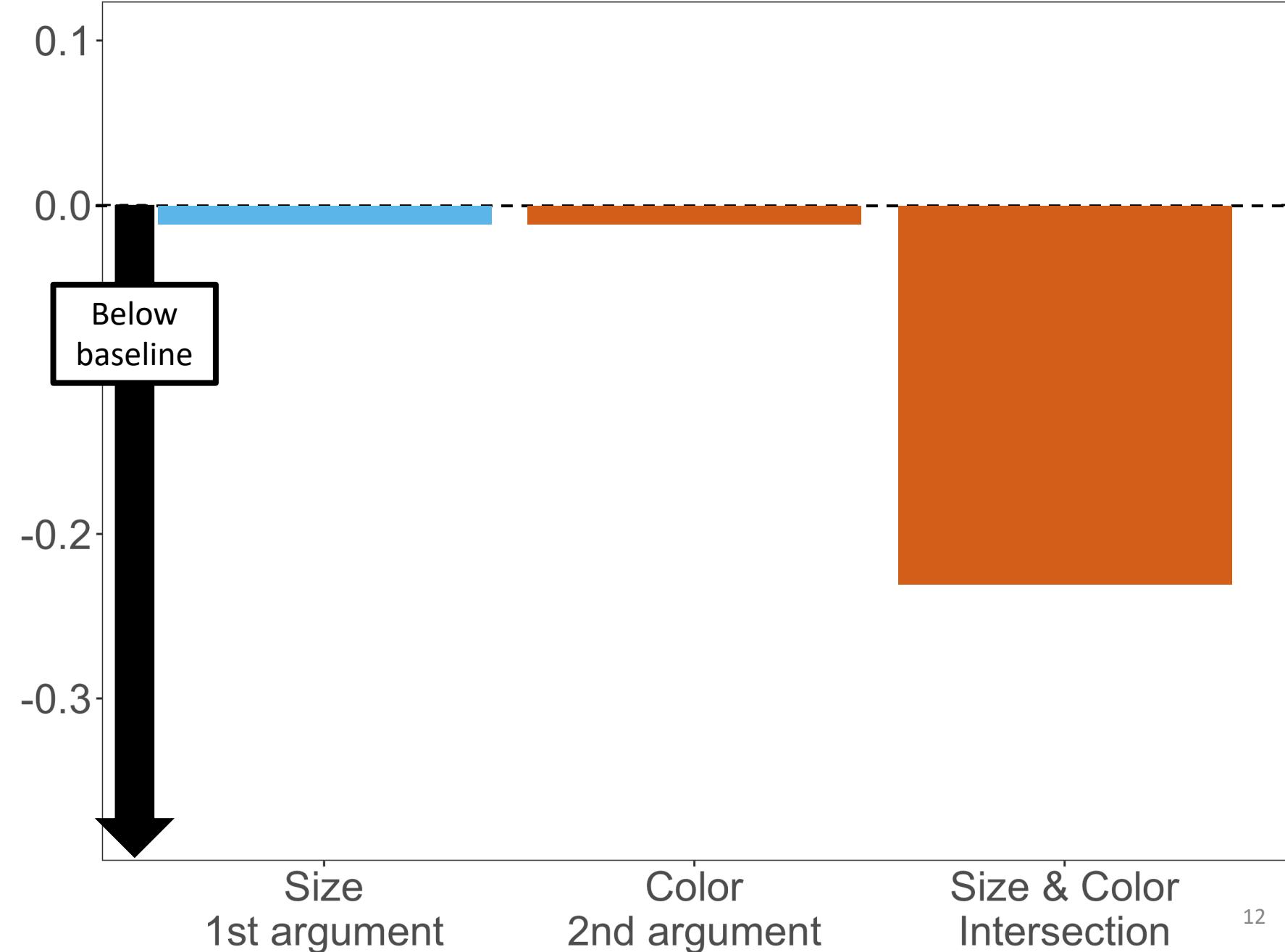


Every big circle is blue

Relational

The big-circles_X are a subset of the blue-circles_Y

Represent both arguments



Every big circle is blue

Relational

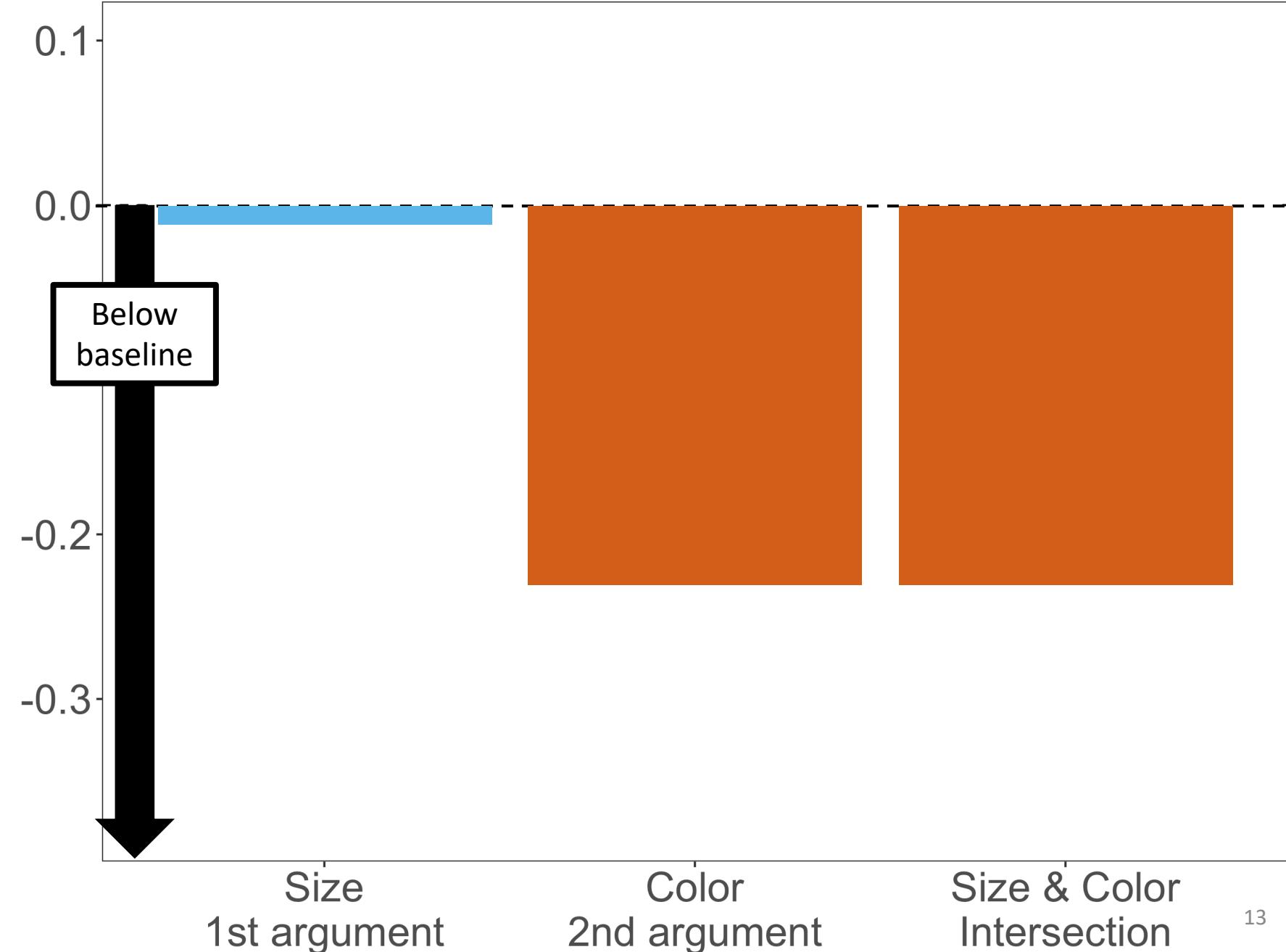
The big-circles_X are a subset of the blue-circles_Y

Represent both arguments

Restricted

The big-circles_X are such that all of them_X are blue

Treat arguments asymmetrically



Every big circle is blue

Relational

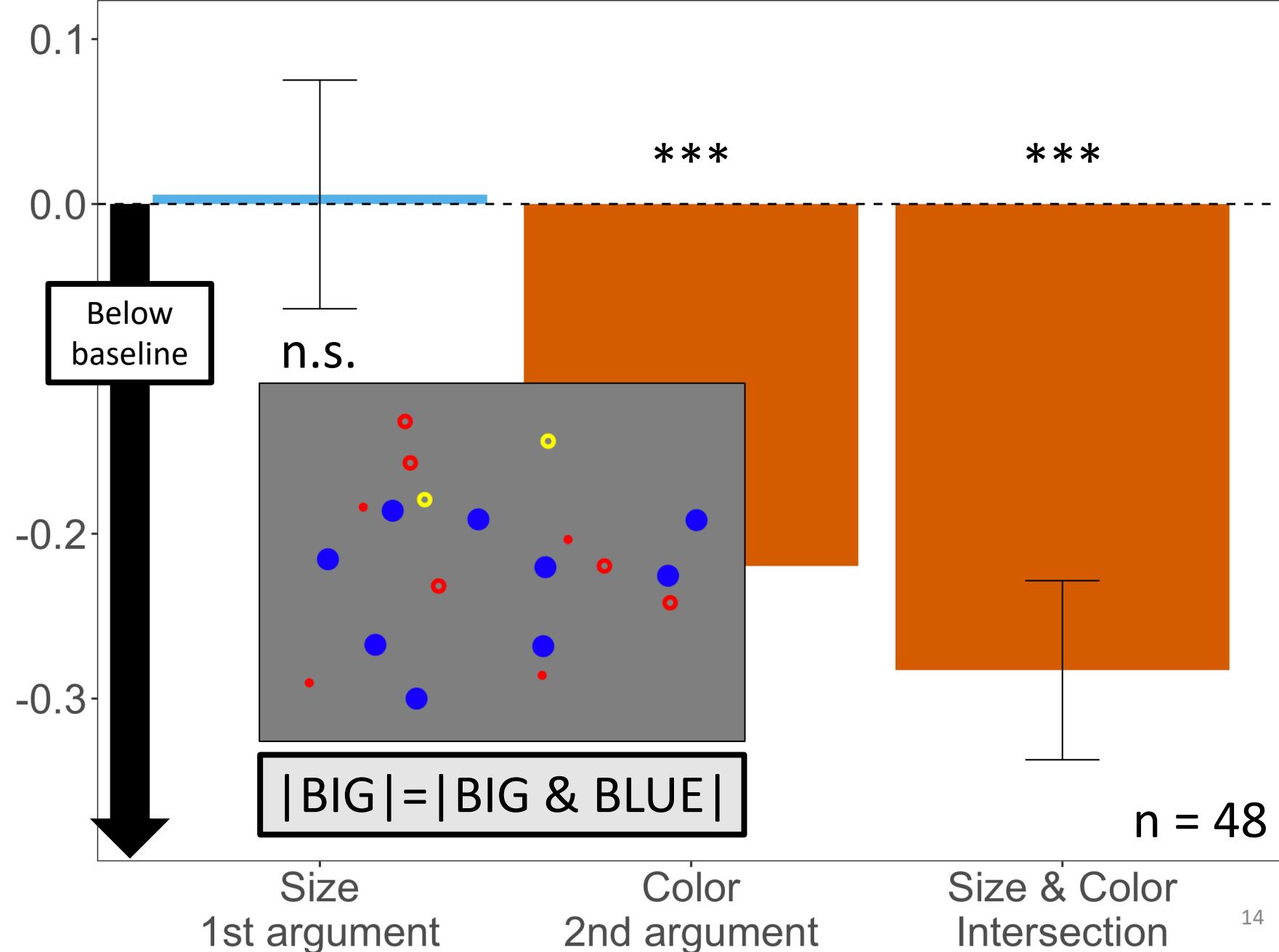
The big-circles_X are a subset of the blue-circles_Y

Represent both arguments

Restricted

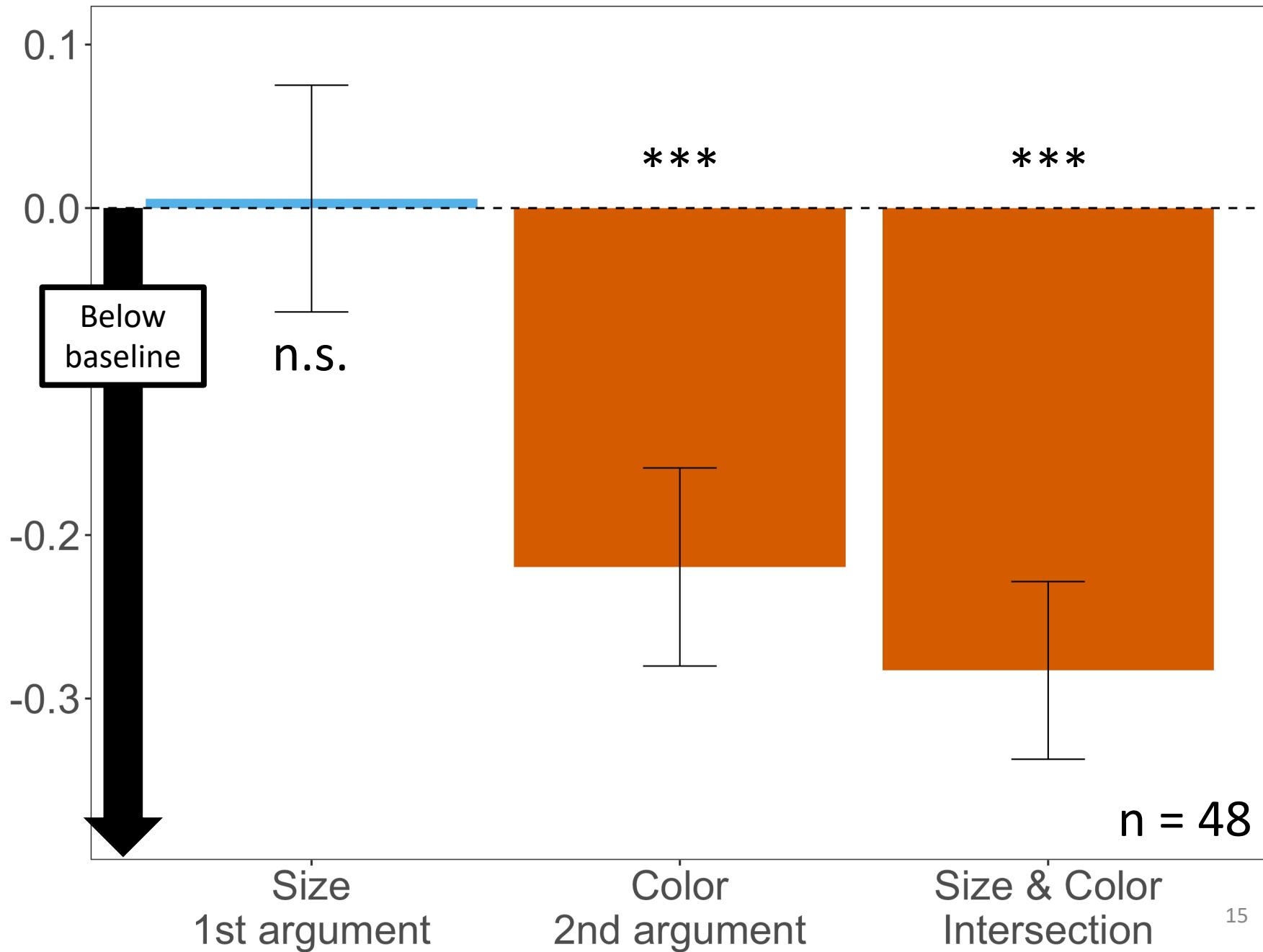
The big-circles_X are such that all of them_X are blue

Treat arguments asymmetrically



Every big circle is blue

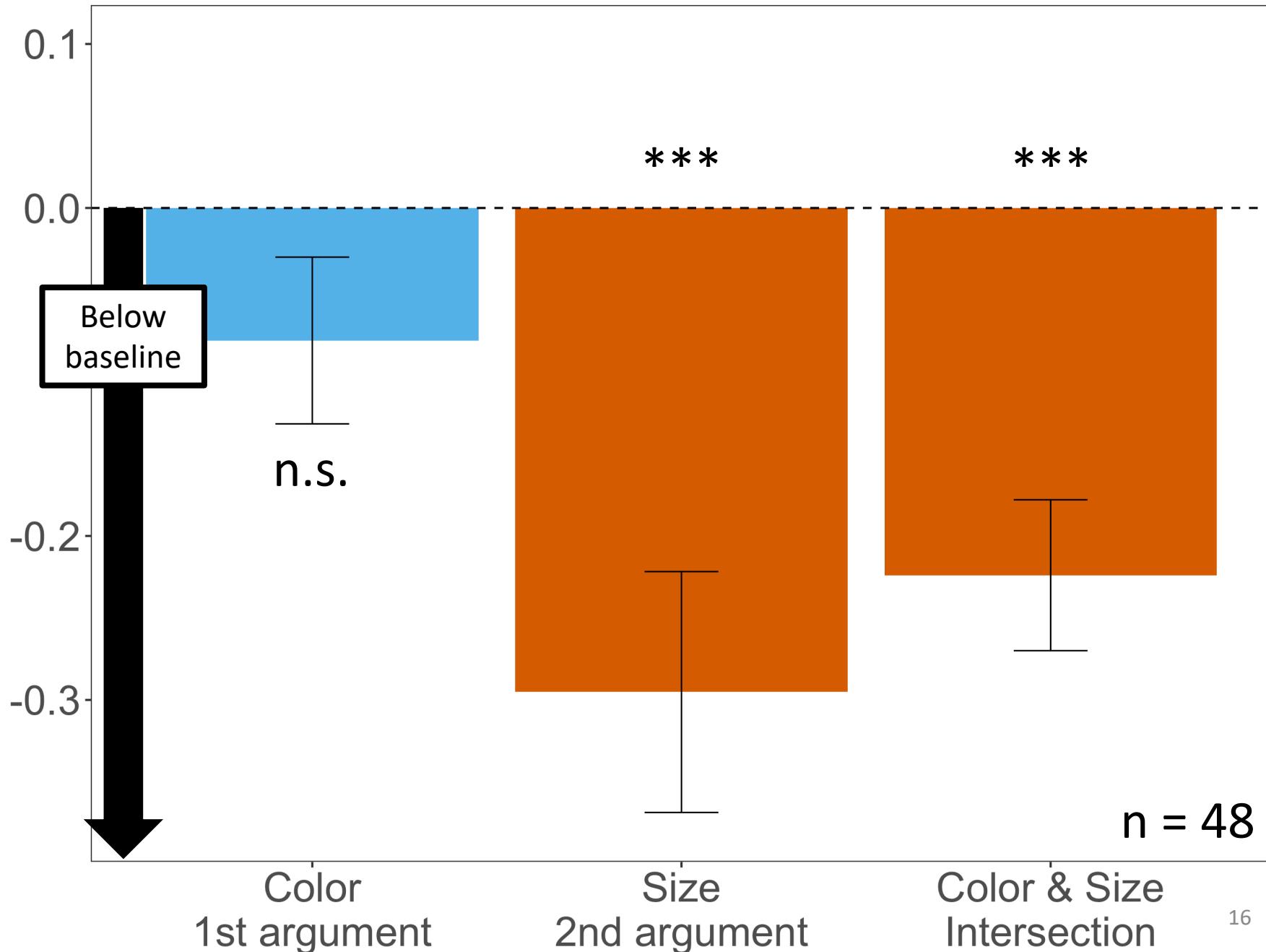
Is there something
special about size?



Every blue circle is big

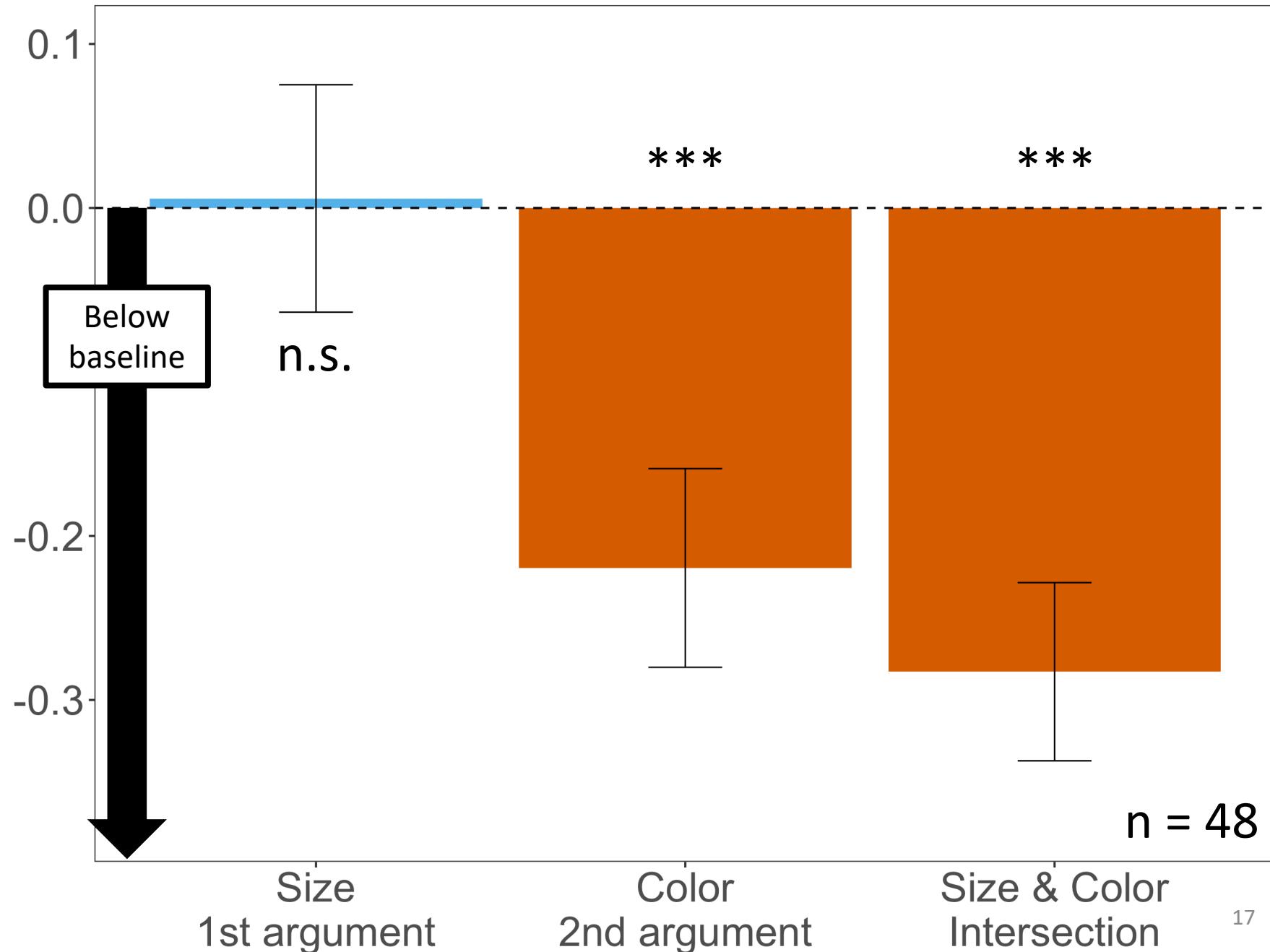
Is there something special about size?

No:
swap arguments,
same result!



Every big circle is blue

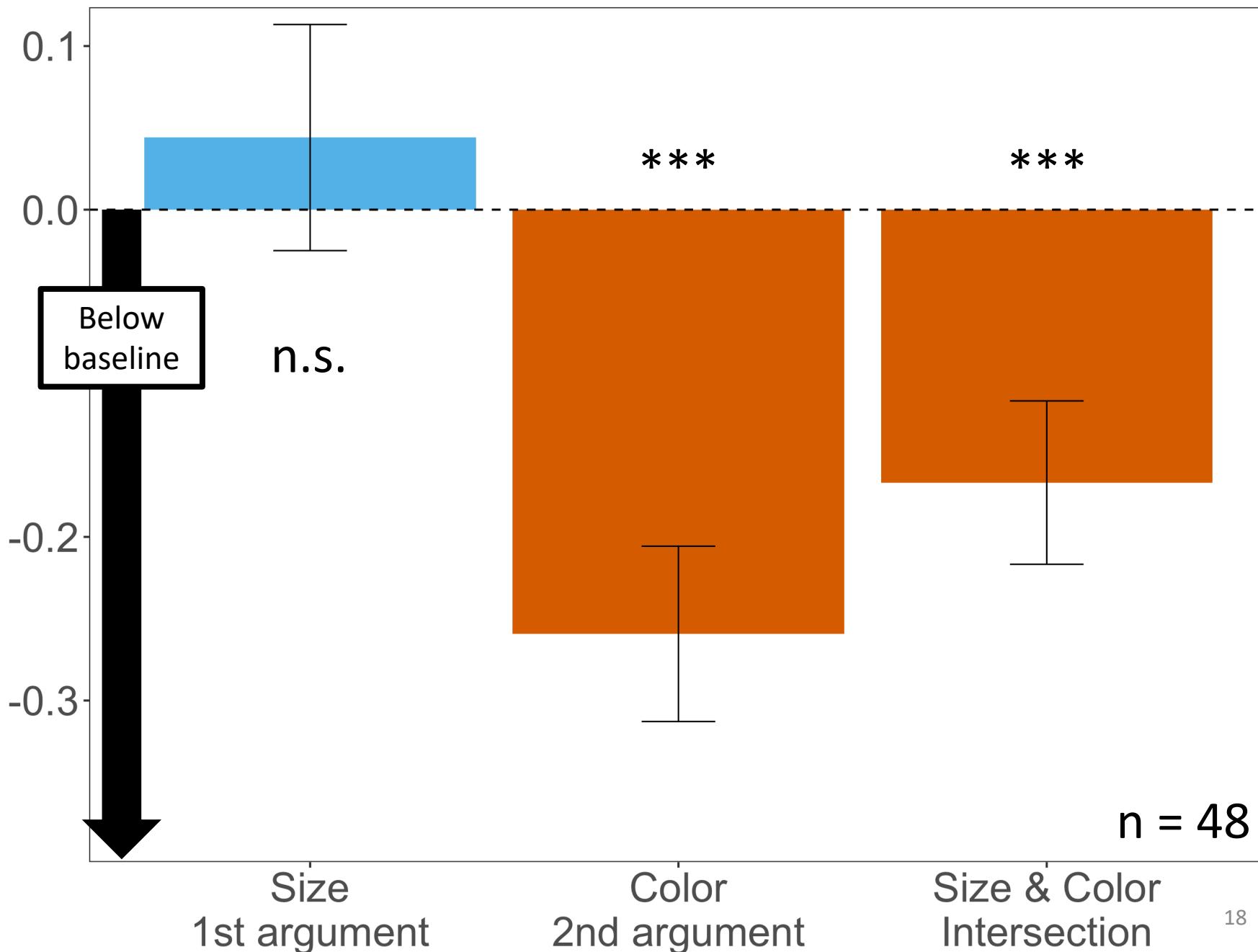
Is the problem that
the predicates are
introduced in two
different ways?



Every circle that is big is blue

Is the problem that
the predicates are
introduced in two
different ways?

No:
both can be
introduced with *is*!



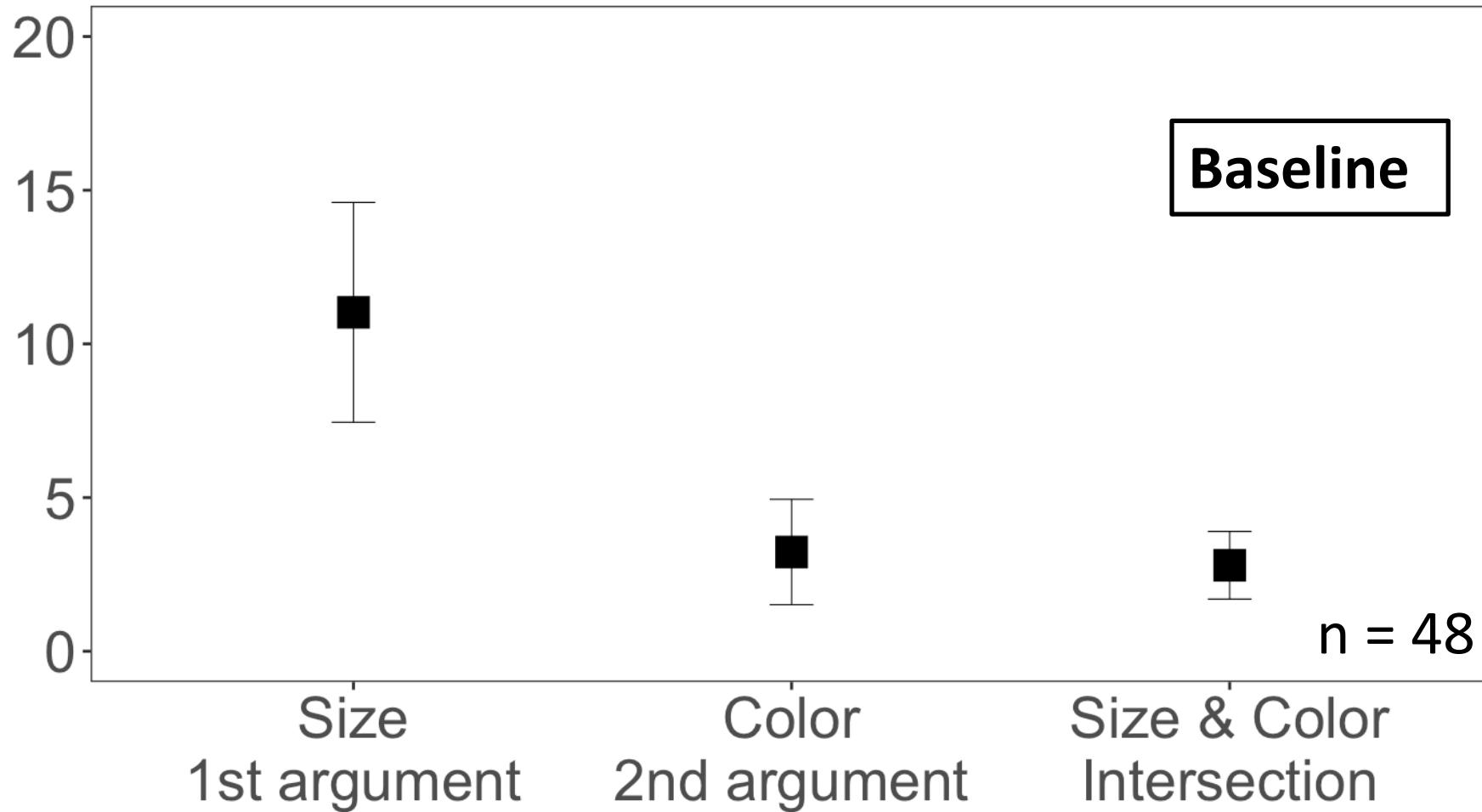
Is there another
signature of the
asymmetry?

Yes:
rate of opting not
to answer

How many {big/blue} circles
were there?

I don't know!

Average % pressing IDK button



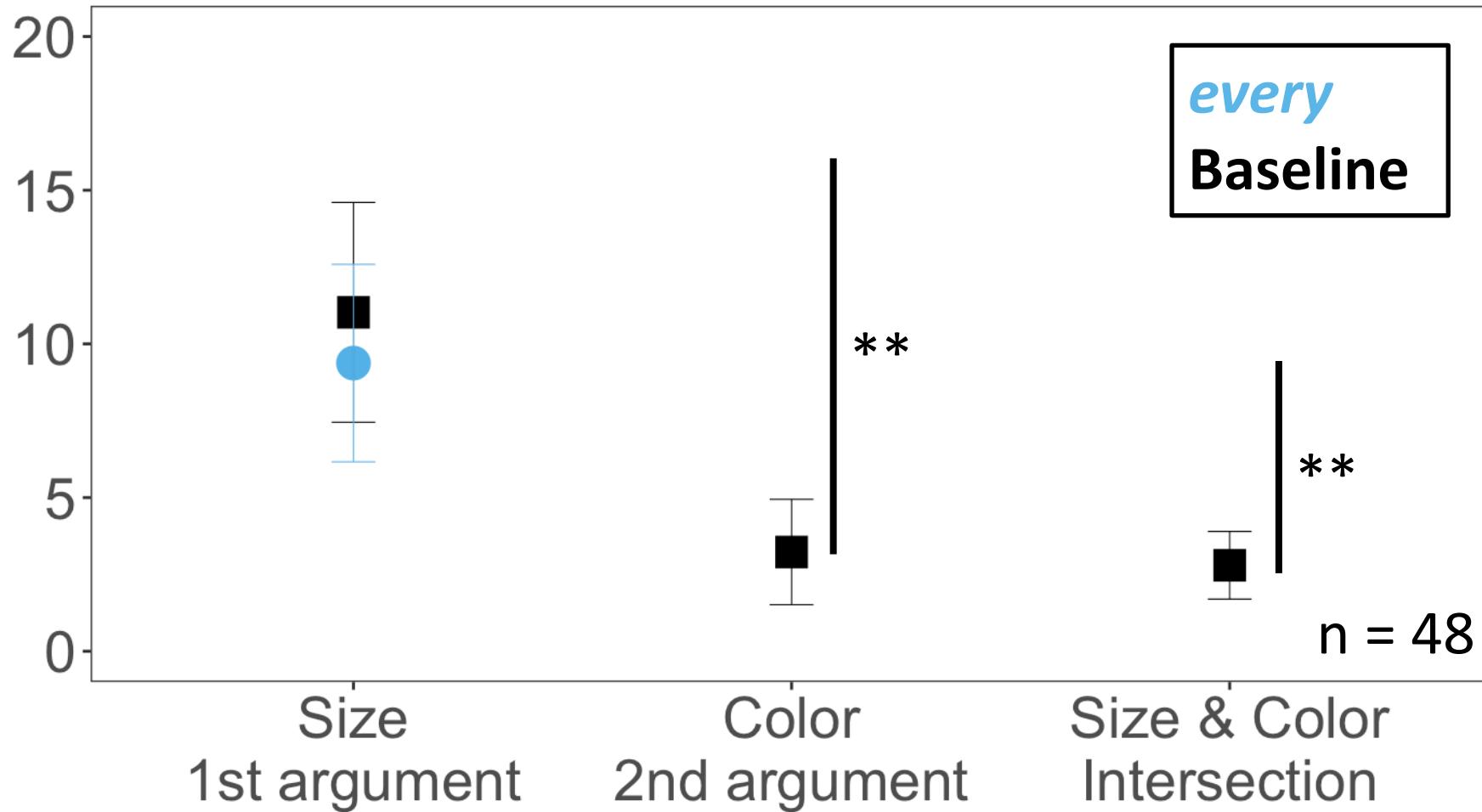
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How many {big/blue} circles
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I don't know!

Average % pressing IDK button



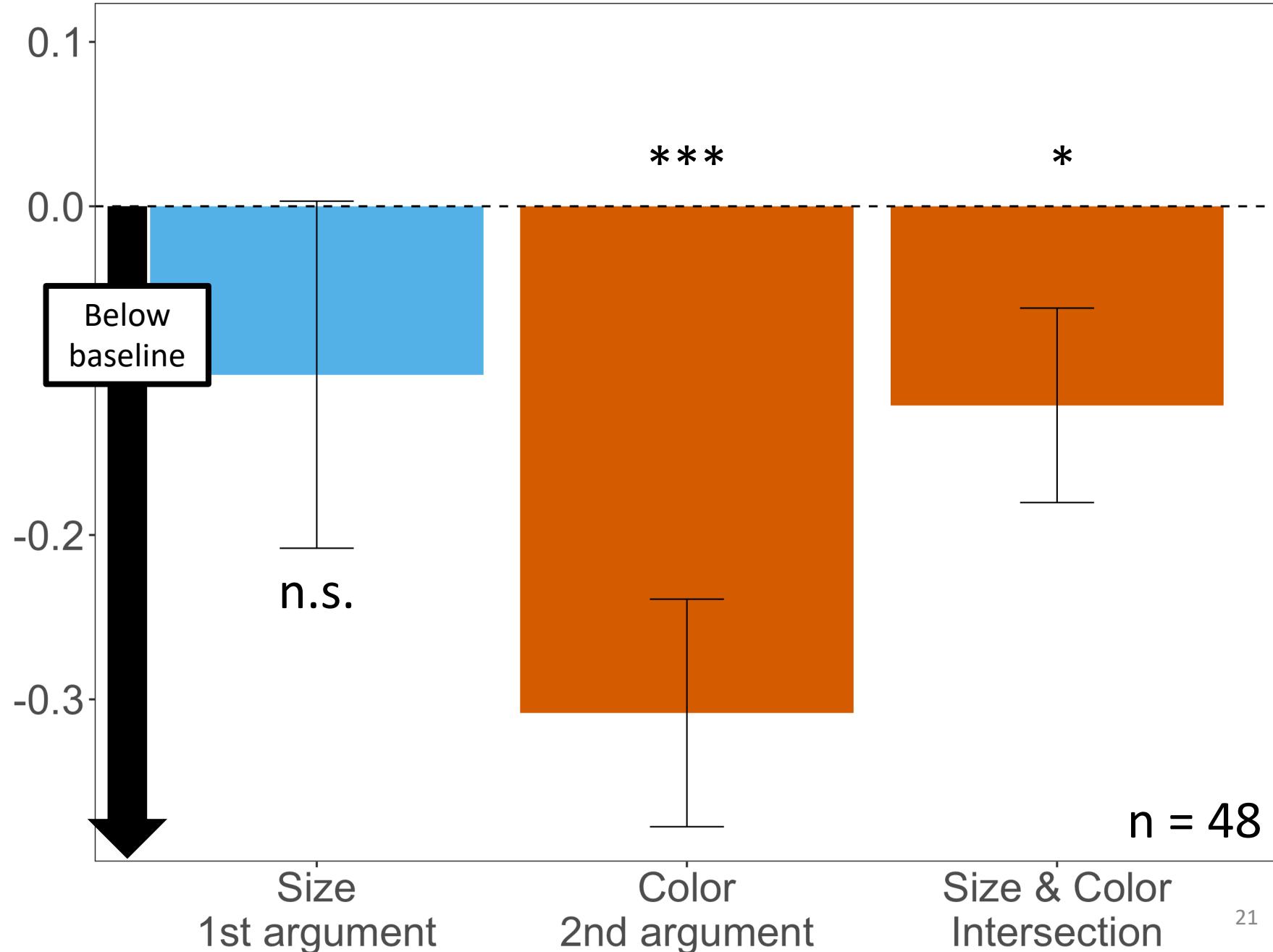
Every big circle is blue (not pressing IDK)

Are participants
fully conscious of
not knowing #?

No:
same result when
they did respond

How many {big/blue} circles
were there?

I don't know!



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- ➡ Explanation of the “conservativity” universal

First- vs. Second-order (*each* vs. *every*)

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Consequences for language acquisition (*each* vs. *every*)

- ➡ Precursors of quantification in infants
- ➡ Differences in child-directed speech

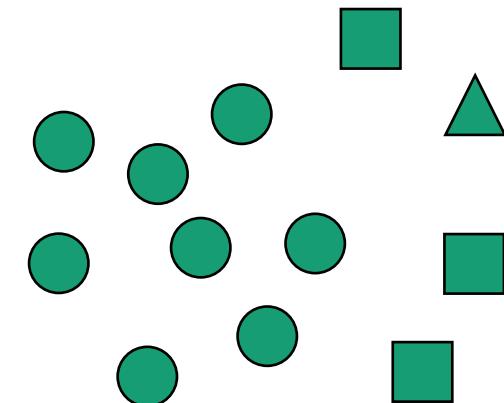
Natural language determiners are “conservative”

A determiner **DET** is conservative iff

- (1) $[[\text{DET } \text{NP}] \text{ PRED}] =$
- (2) $[[\text{DET } \text{NP}] [\text{be NP that PRED}]]$

every circle is green (TRUE) =

every circle is a circle that is green (TRUE)



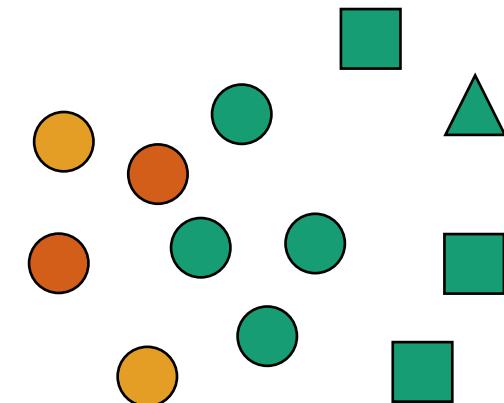
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- (2) $[[\text{DET } \text{NP}] [\text{be NP that PRED}]]$

every circle is green (FALSE) =

every circle is a circle that is green (FALSE)



Natural language determiners are “conservative”

A determiner **DET** is conservative iff

- (1) $[[\text{DET } \text{NP}] \text{ PRED}] =$
- (2) $[[\text{DET } \text{NP}] [\text{be } \text{NP} \text{ that PRED}]]$

- Cross-linguistically, all determiners are conservative
- 5year-olds can learn novel conservative determiners
but not novel non-conservative ones!

“Conservativity” is puzzling on the relational view

What rules out all the non-conservative relations?

$$|\text{CIRCLES} \cap \text{GREEN}| > |\text{CIRCLES} - \text{GREEN}|$$

≈ most circles are green

$$\text{CIRCLES} \subseteq \text{GREEN}$$

≈ every circle is green

$$|\text{CIRCLES}| = |\text{GREEN}|$$

$$\text{CIRCLES} \supseteq \text{GREEN}$$

“Conservativity” is entailed on the restricted view

Relative to the circles, *is green* applies to

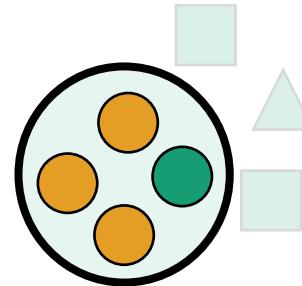
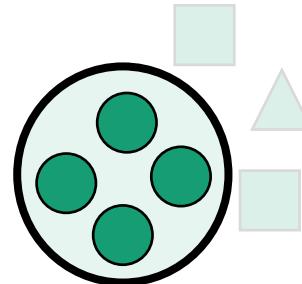
...**all** things

...**most** things

...**at least 2 & at most 4** things

...**???** things

(intended: $|\text{CIRCLES}| = |\text{GREEN}|$)



→ Non-conservative meanings are not stateable if the first argument restricts the domain of quantification

How are universal quantifiers mentally represented?

Relational vs. Restricted (*every*)

- ✓ Number cognition as a probe into which arguments are represented
- ✓ Explanation of the “conservativity” universal

First- vs. Second-order (*each* vs. *every*)

- ➔ Object-files vs. Ensembles as a probe into how arguments are encoded

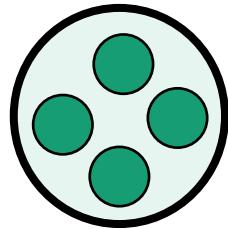
Consequences for language acquisition (*each* vs. *every*)

- ➔ Precursors of quantification in infants
- ➔ Differences in child-directed speech

Different representations

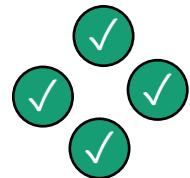
Second-order representations (*every*)

CIRCLES: $\forall X[\text{Green}(X)] \approx \text{The circles}_X \text{ are such that all of them}_X \text{ are green}$



First-order representations (*each*)

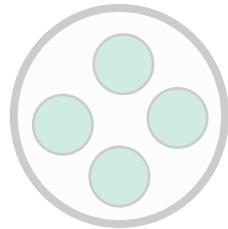
CIRCLES: $\forall x[\text{Green}(x)] \approx \text{The individual circles}_x \text{ are s.t. each one}_x \text{ is green}$



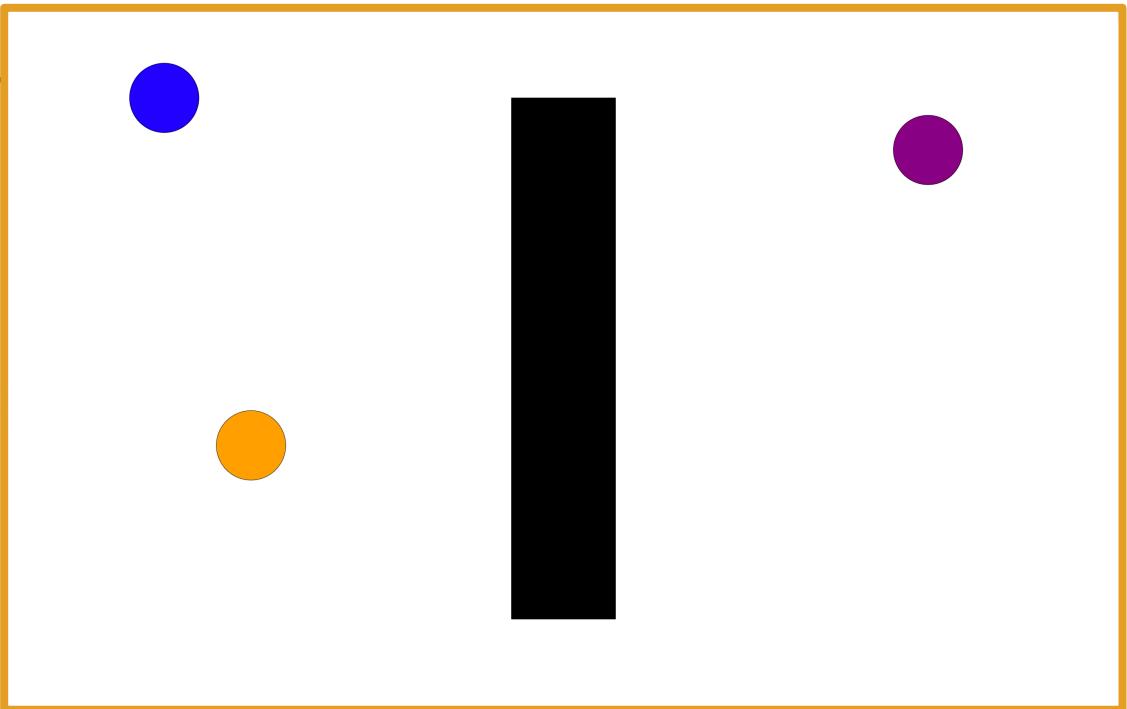
Different underlying cognitive representations

Second-order representations (every)

CIRCLES: $\forall X[\text{Green}(X)] \approx \text{The circles}_x \text{ are such that every } X \text{ is green}$



take on ≥ 1
value at a time



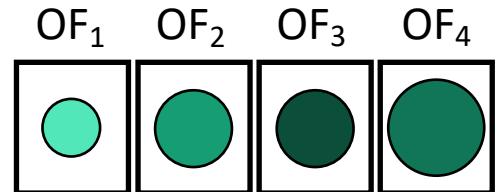
First-order representations (each) rely on object-files

CIRCLES: $\forall x[\text{Green}(x)] \approx \text{The individual circles}_x \text{ are s.t. each one}_x \text{ is green}$



take on only 1
value at a time

Objects individuated
individual properties encoded

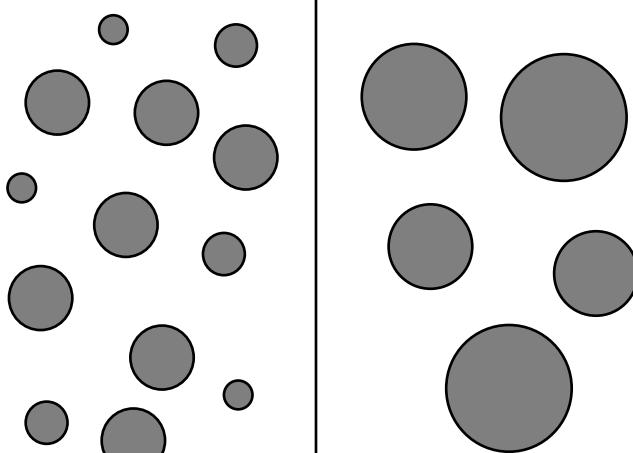


Different underlying cognitive systems

Second-order representations (every) rely on ensembles

CIRCLES: $\forall x [Green(x)] \approx$ The circles_x are such that all of them_x are green

Which side has **more** circles?



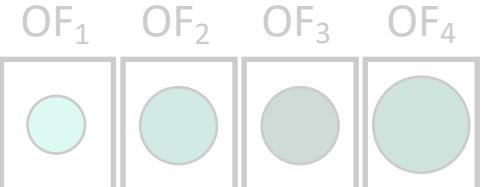
Objects abstracted away from
summary statistics encoded

Ensemble₁

Center: (x,y)
Cardinality: 4
Avg. Size:

each) rely on **object-files**
Individual circles_x are s.t. each one_x is green

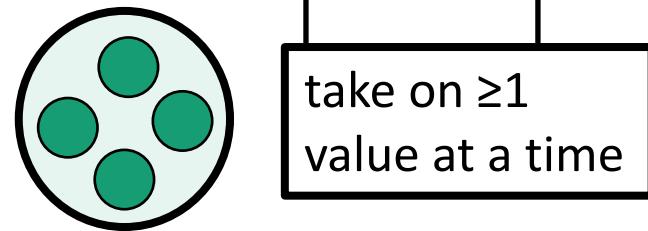
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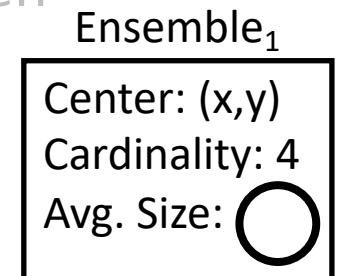
Different representations & cognitive systems

Second-order representations (every) rely on ensembles

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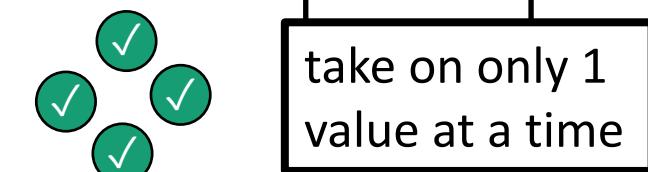


Objects abstracted away from
summary statistics encoded

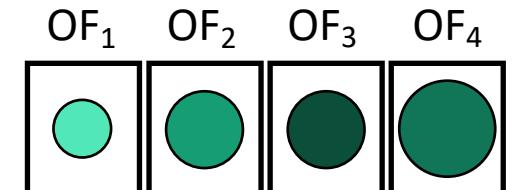


First-order representations (each) rely on object-files

CIRCLES: $\forall x[\text{Green}(x)] \approx \text{The individual circles}_x \text{ are s.t. each one}_x \text{ is green}$

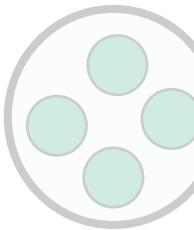


Objects individuated
individual properties encoded



Different representations & cognitive systems

Second
CIRCLES



First-or
CIRCLES



This representational distinction isn't the whole story

e.g., Distributivity

*each student {gathered/surrounded the teacher/is similar}

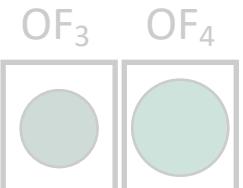
?every student {gathered/surrounded the teacher/is similar}

Determine whether {each
every} student has a copy of Aspects

Student₁ does; Student₂ does; but Student₃ doesn't (✓ Pair-list)

No, only two of them do (✗ Pair-list)

ensemble₁
center: (x,y)
density: 4
Size:



Different behavioral predictions

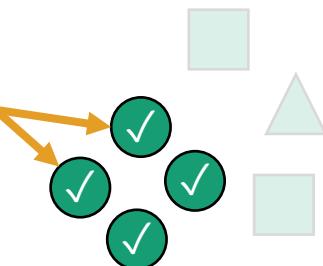
Linking hypothesis: people are biased toward verification strategies that directly compute the relations & operations expressed by the semantic representation under evaluation

First-order (each)

CIRCLES: $\forall x[\text{Green}(x)]$

\approx The individual circles_x are such that each one_x is green

Encode individual properties (e.g., color)

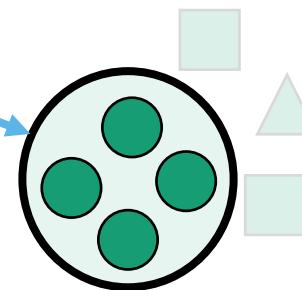


Second-order (every)

CIRCLES: $\forall X[\text{Green}(X)]$

\approx The circles_X are such that all of them_X are green

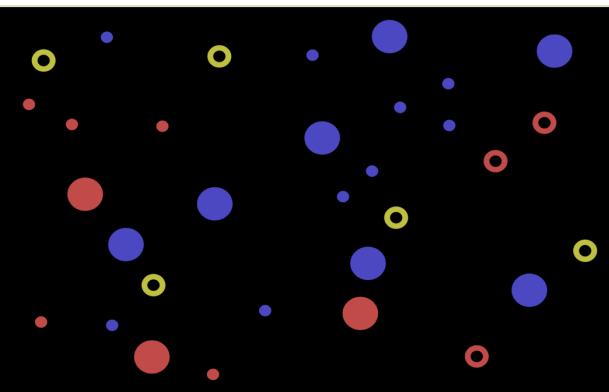
Encode summary statistics (e.g., #)



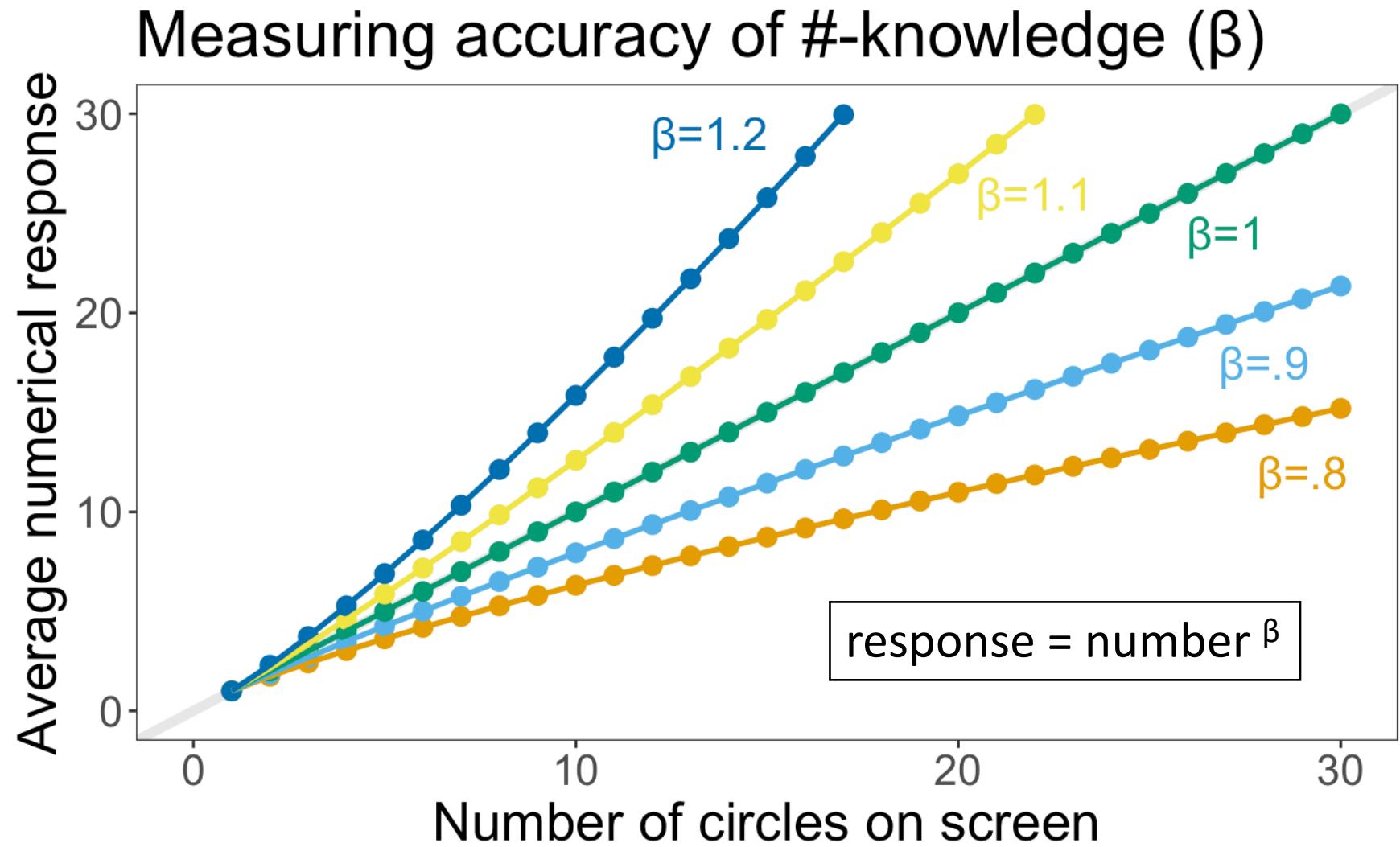
Abstract away from individual properties

{Each/Every} big circle is blue

TRUE FALSE



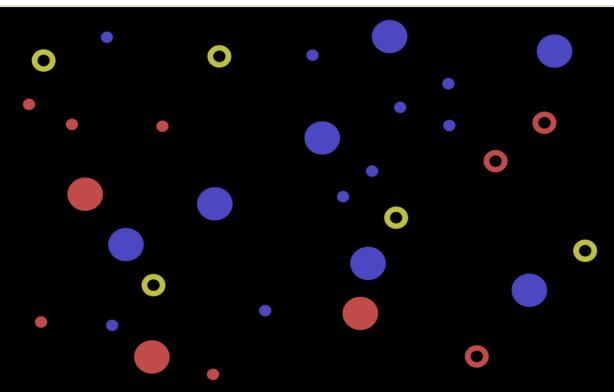
How many
{big/medium/small}
circles were there?



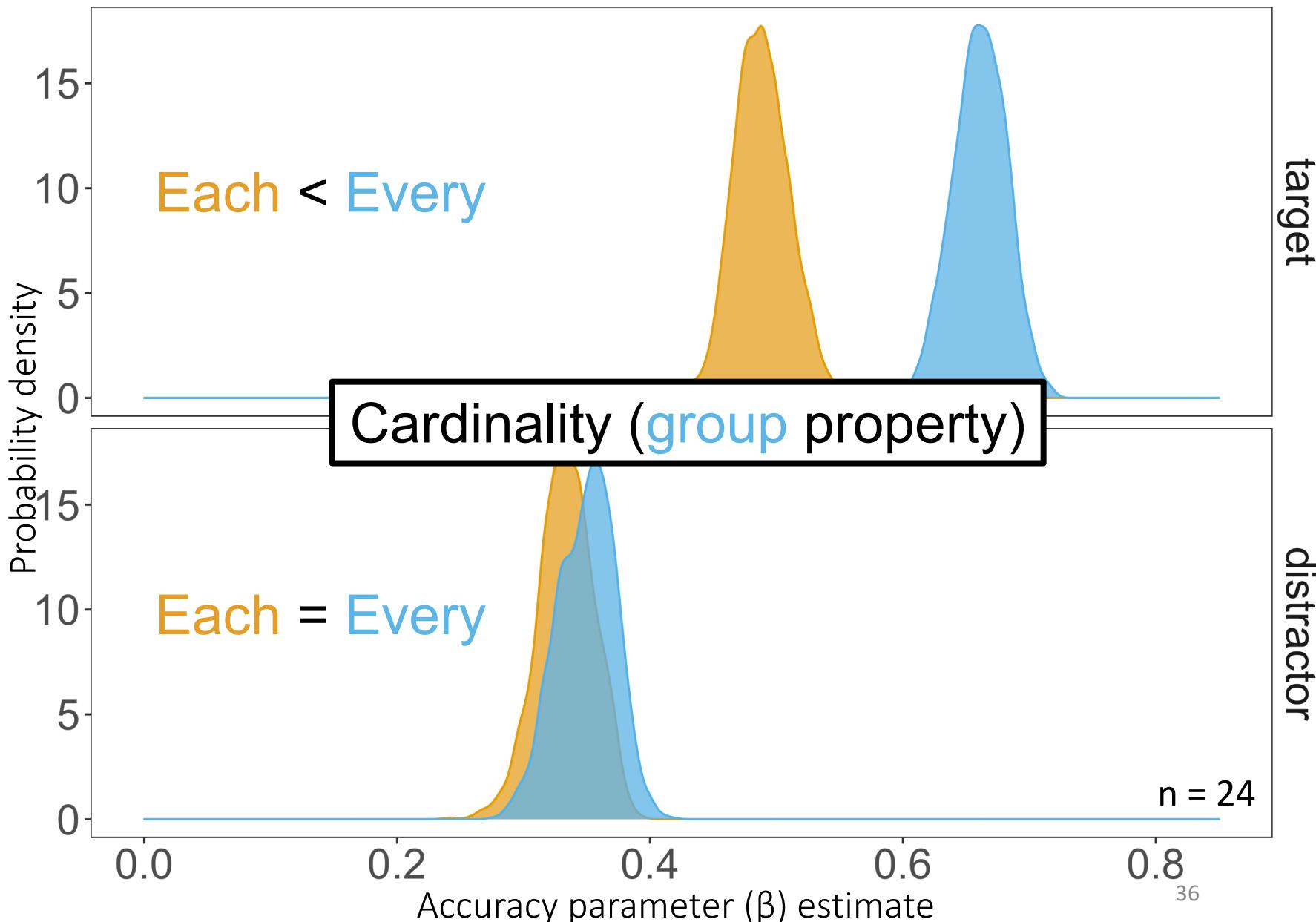
#-knowledge accuracy - Bayesian estimates

{Each/Every} big circle is blue

TRUE FALSE



How many
{big/medium/small}
circles were there?



{Each/Every}
circle is green



TRUE

FALSE

300 ms

One circle
changed its color



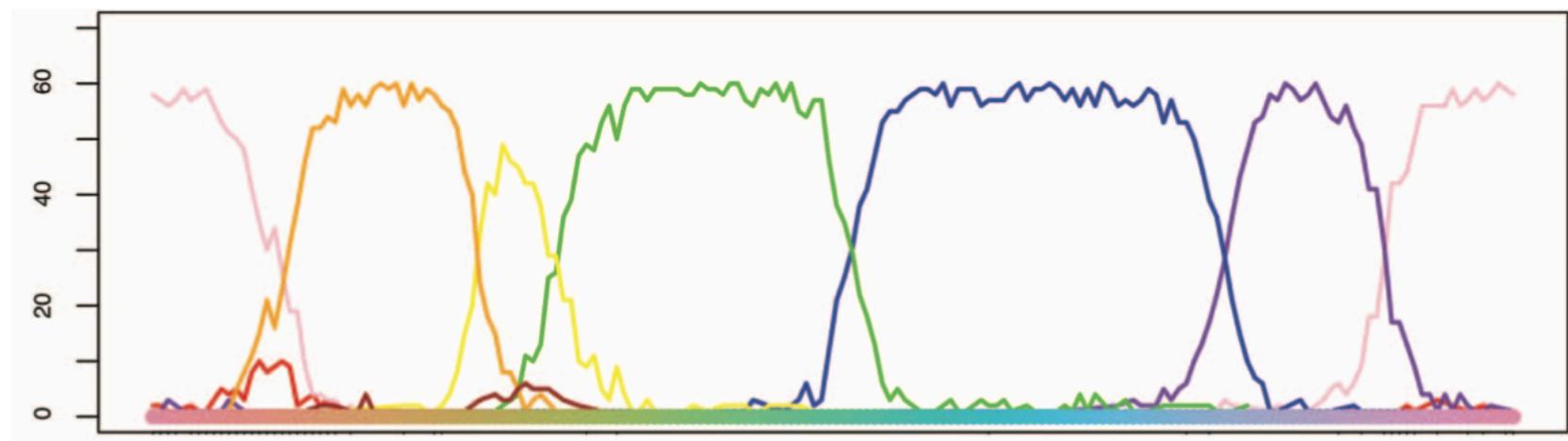
TRUE

FALSE

Color (individual property)



Color category naming task



{Each/Every}
circle is green



TRUE

FALSE

300 ms

One circle
changed its color

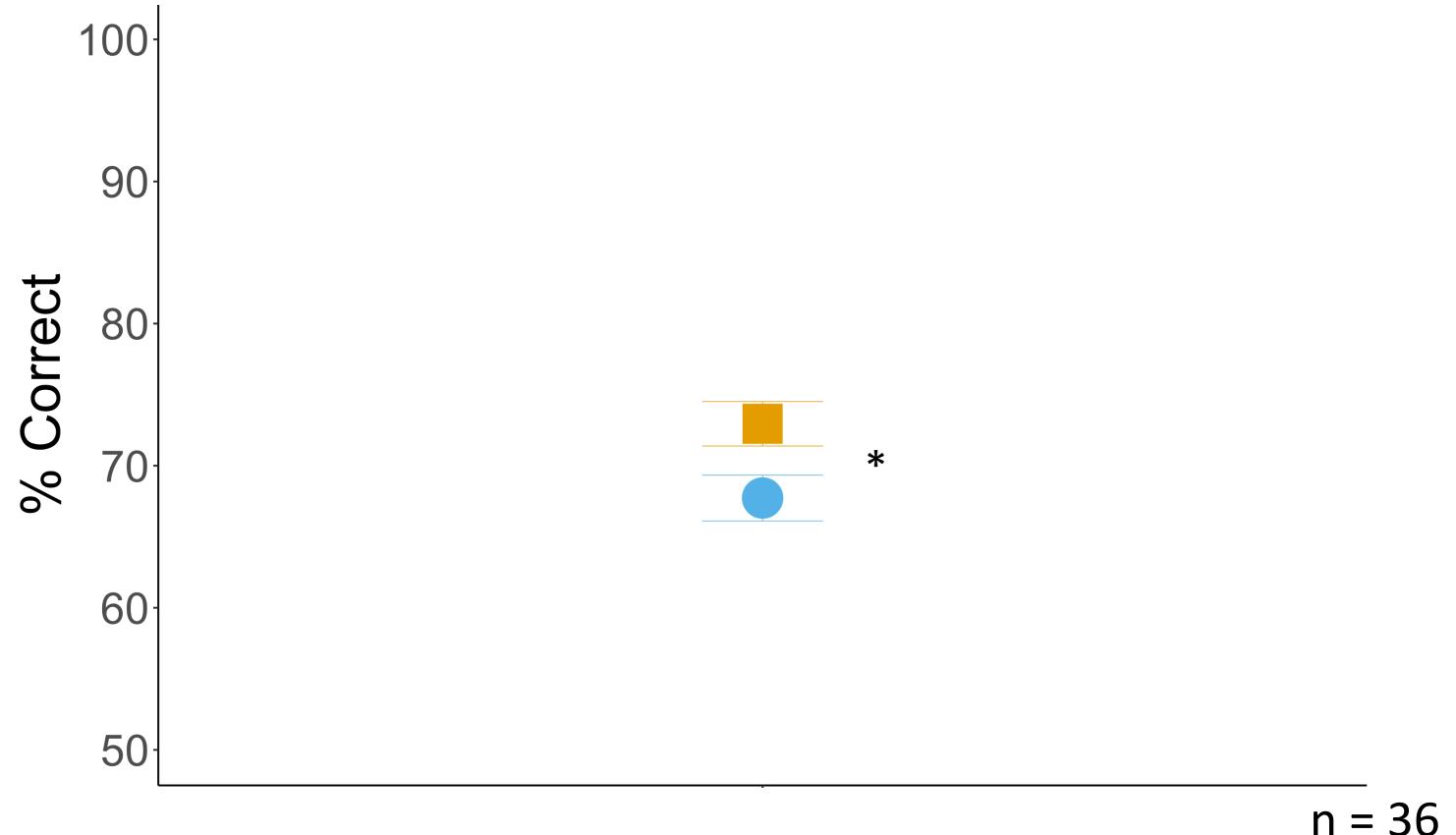


TRUE

FALSE

Color (**individual** property)

Change detection accuracy



{Each/Every}
circle is green



TRUE

FALSE



One circle
changed its color

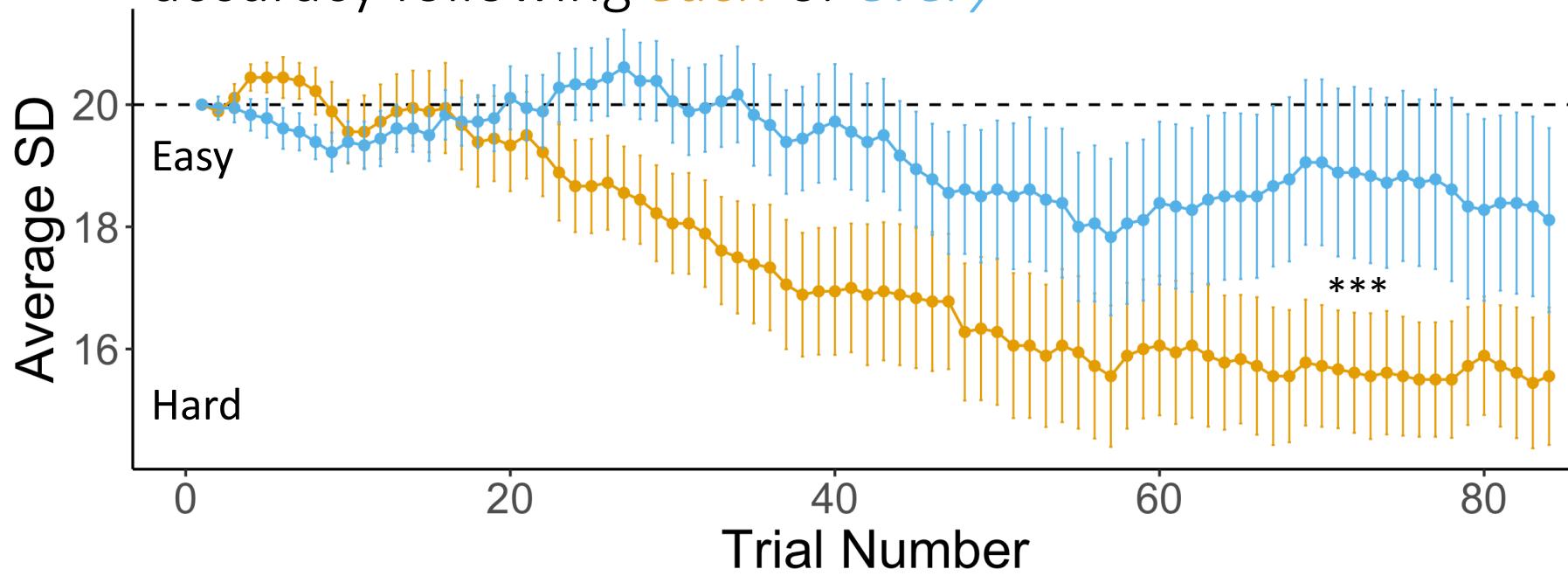


TRUE

FALSE

Color (individual property)

Color change detection: difficulty required for 70% accuracy following *each* or *every*



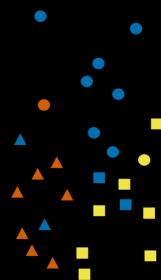
n = 36

Center of Mass (group property)

Is {each/every} circle blue?

“Yes”

“No”



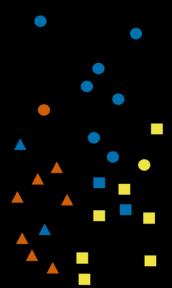
How young can we observe the effect?

Where was the middle
of the circles?

Is {each/every} circle blue?

“Yes”

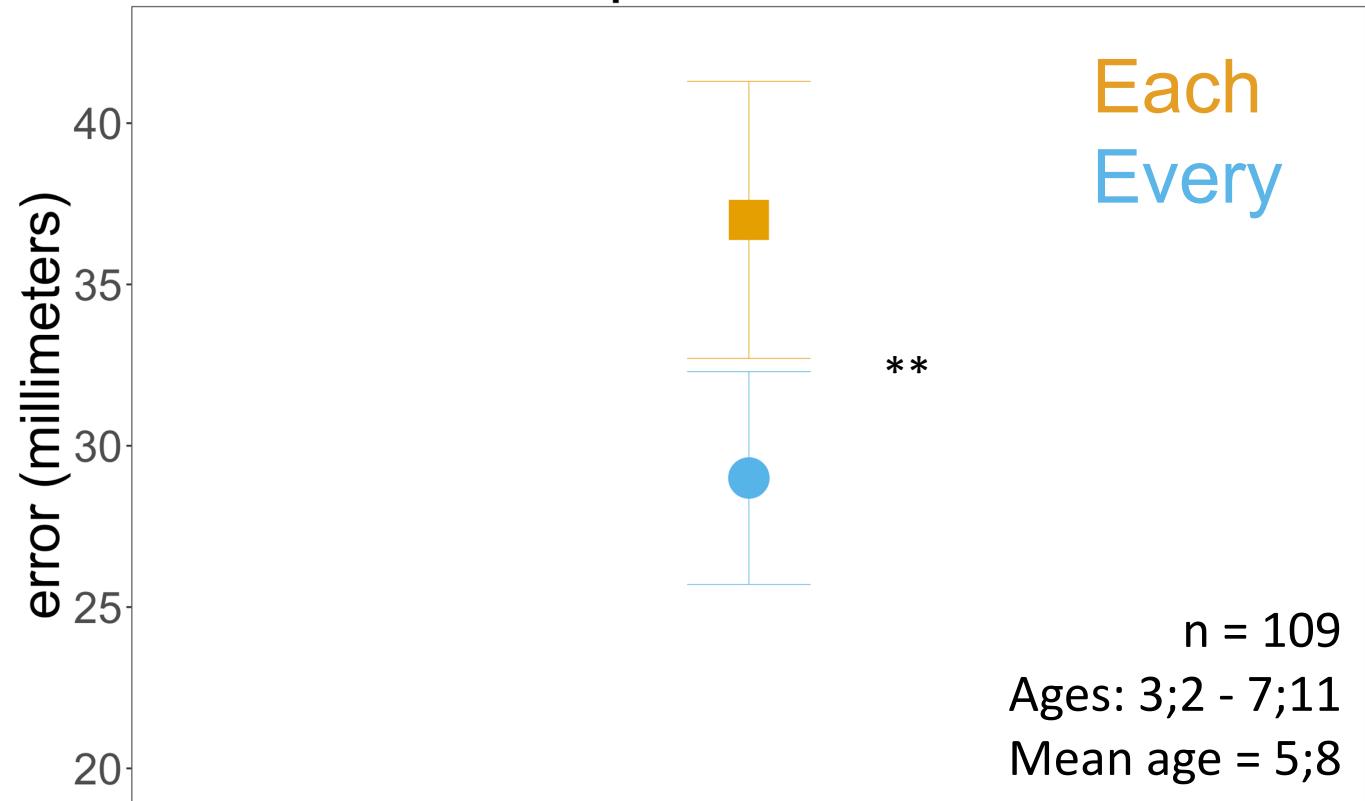
“No”



Where was the middle
of the circles?

Center of Mass (group property)

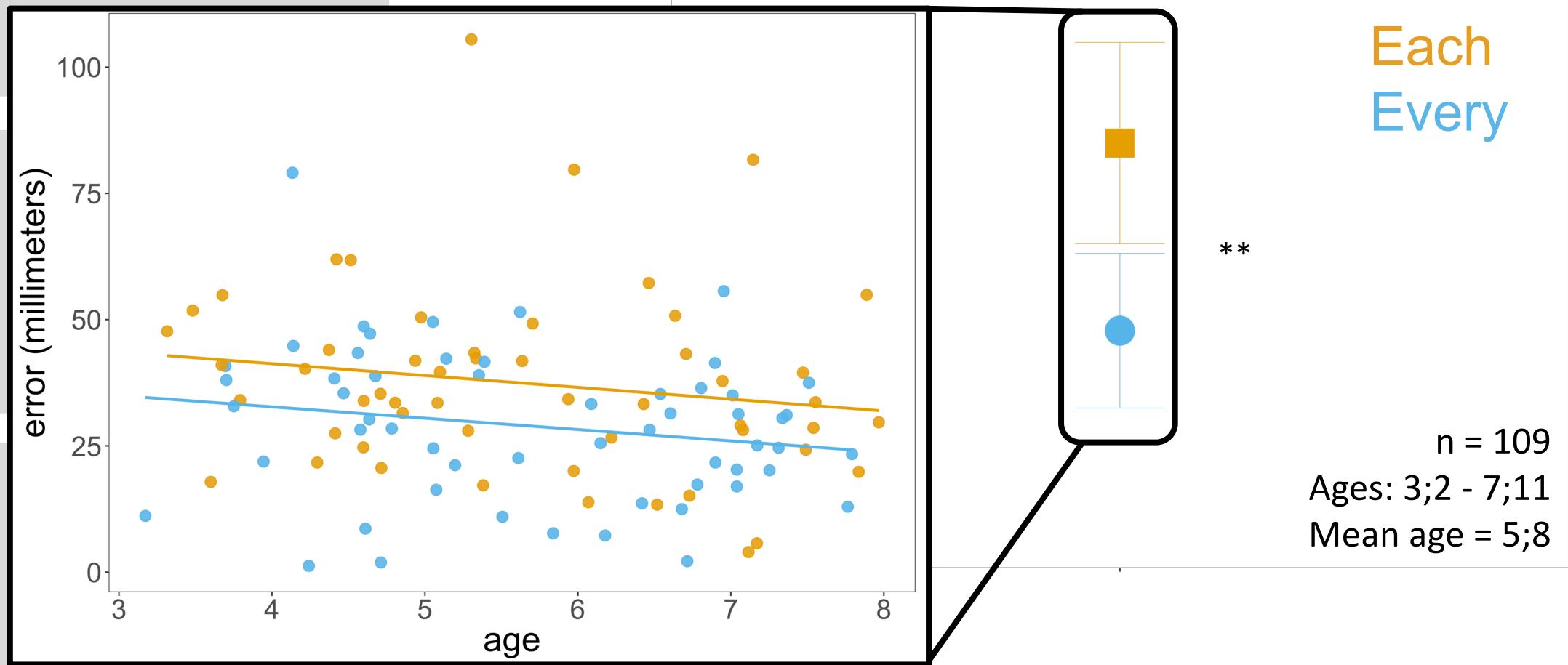
Distance from tap to actual set center



Center of Mass (group property)

Is {each/every} circle blue?

Distance from tap to actual set center



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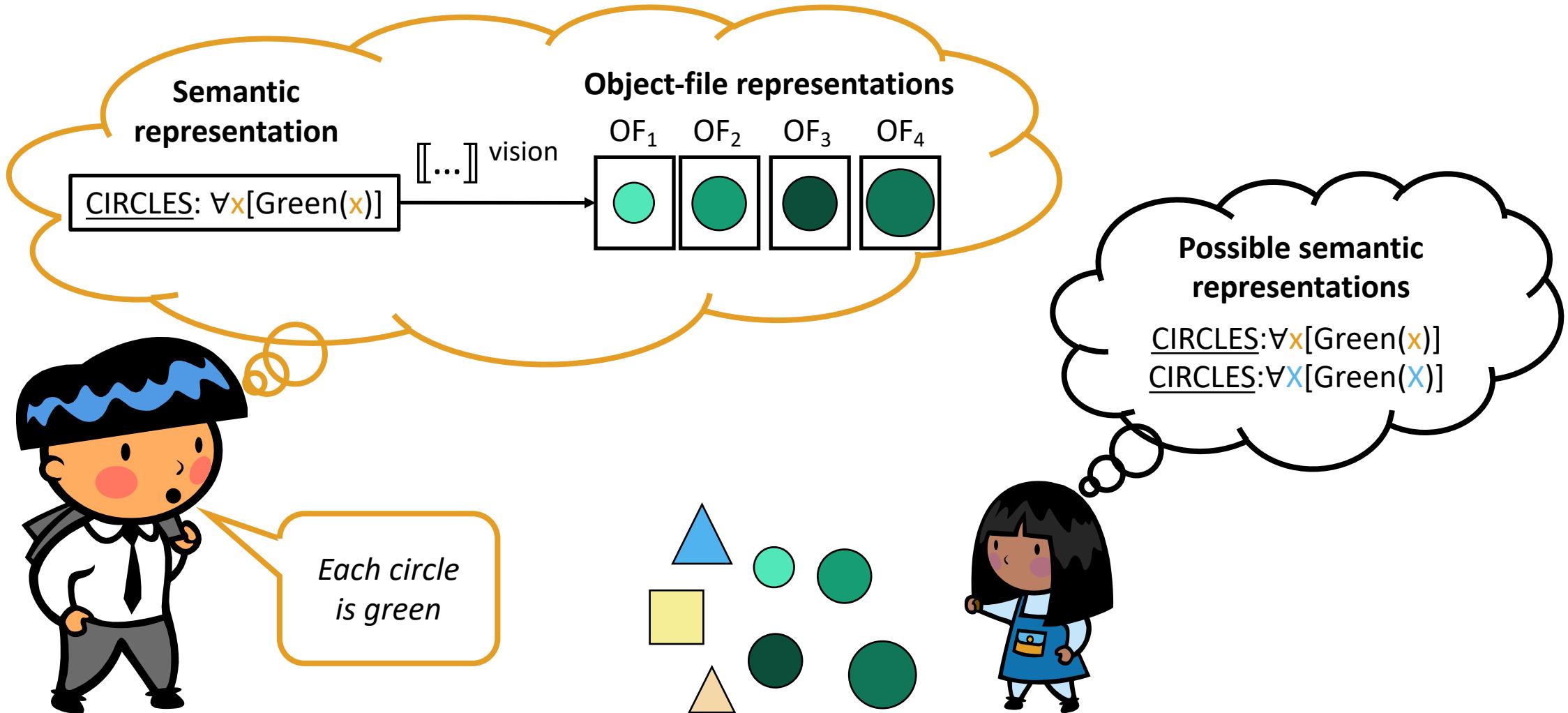
First- vs. Second-order (*each* vs. *every*)

- ✓ Object-files vs. Ensembles as a probe into how arguments are encoded

Consequences for language acquisition (*each* vs. *every*)

- ➔ Precursors of quantification in infants
- ➔ Differences in child-directed speech

Acquiring each and every



Sketching a learning story for *each* & *every*

Semantic representations?

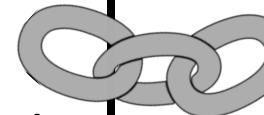
First-order universal concept

Second-order universal concept

Supporting cognitive systems

Object-file representations

Ensemble representations

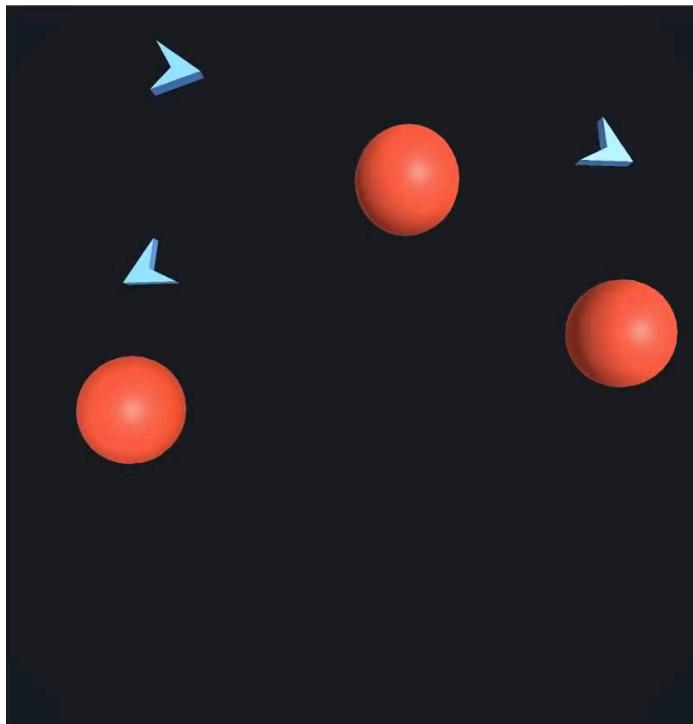


Concepts

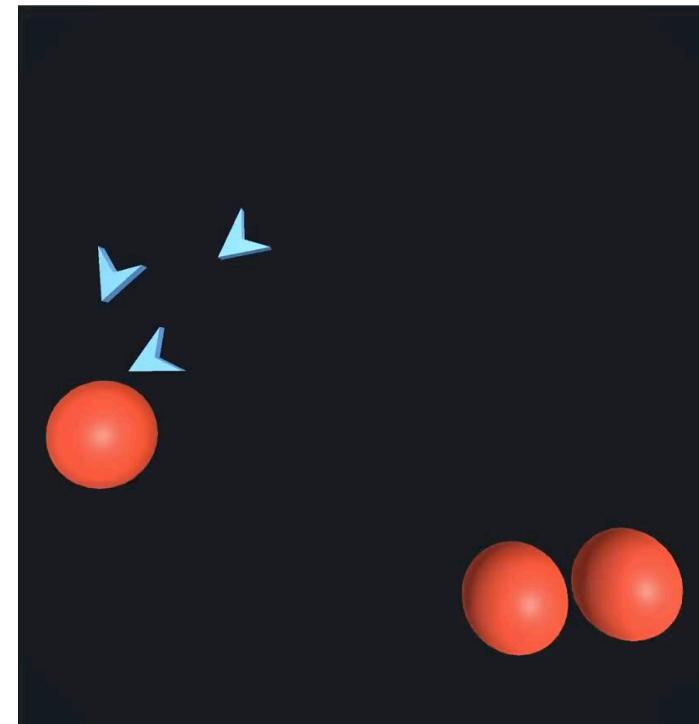
How early are the relevant concepts available?

- Exploring precursors of quantificational thought in 10-month-olds

Individual-chasing

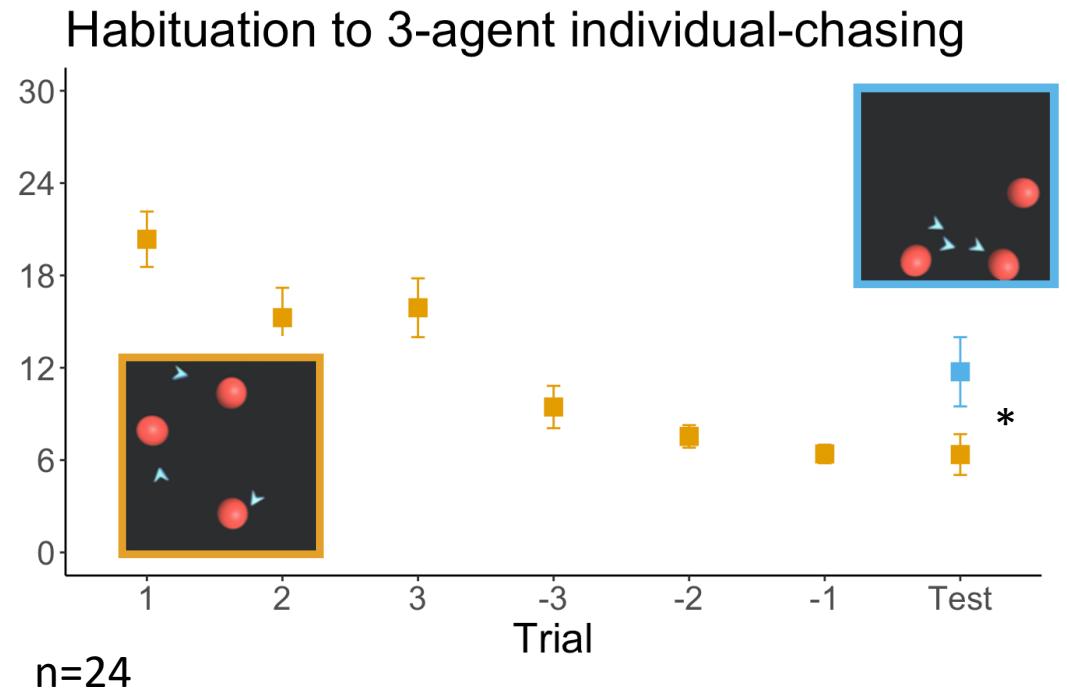
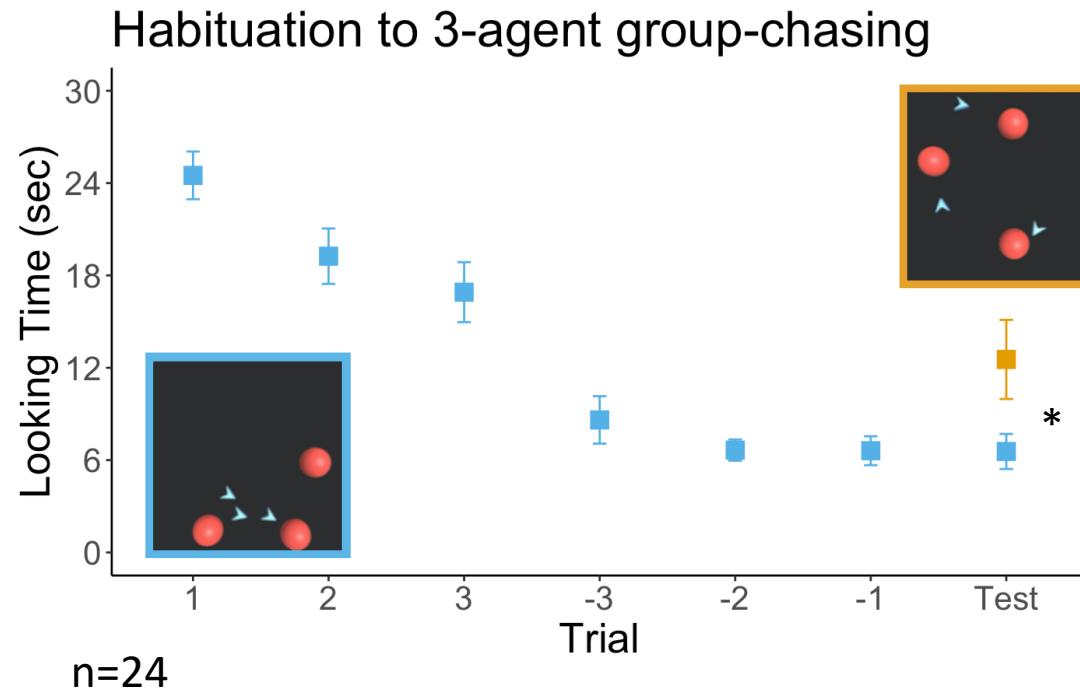


Group-chasing

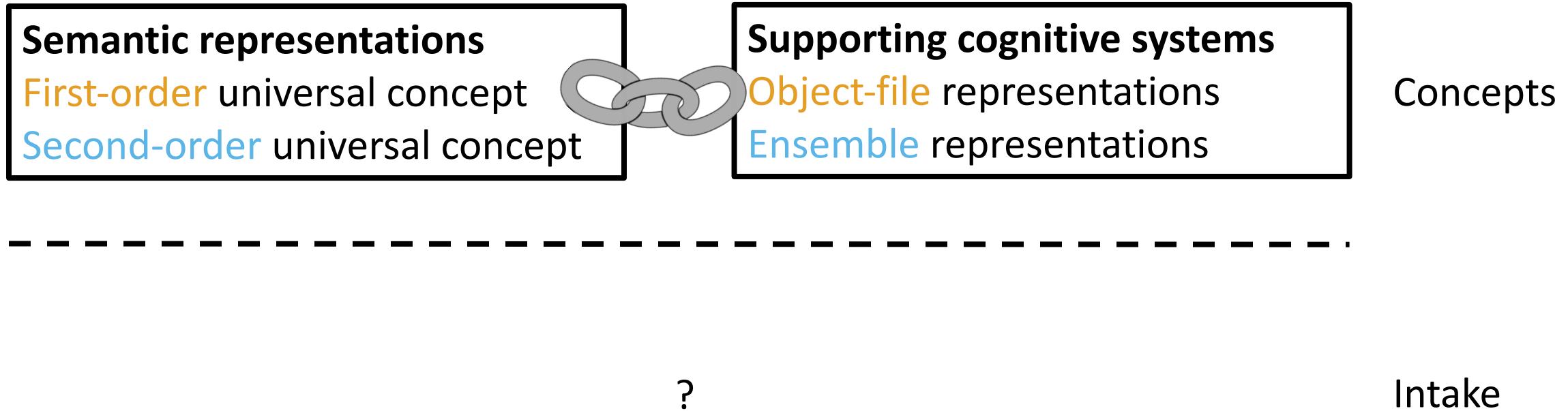


How early are the relevant concepts available?

- 10-month-olds differentiate **group**- and **individual**-chasing events



Sketching a learning story for *each* & *every*



Each and *every* in child-directed speech

→ Could the pair-list asymmetry signal a difference? (*each*: ✓ PL; *every*: X PL)

Out of over 1.7 million utterances:

WH-question & *each*: 11

With possible PL-responses: 1

Dad: What do you think each animal is about to do?

Child (3;04): Clean up that mess

WH-question & *every*: 19

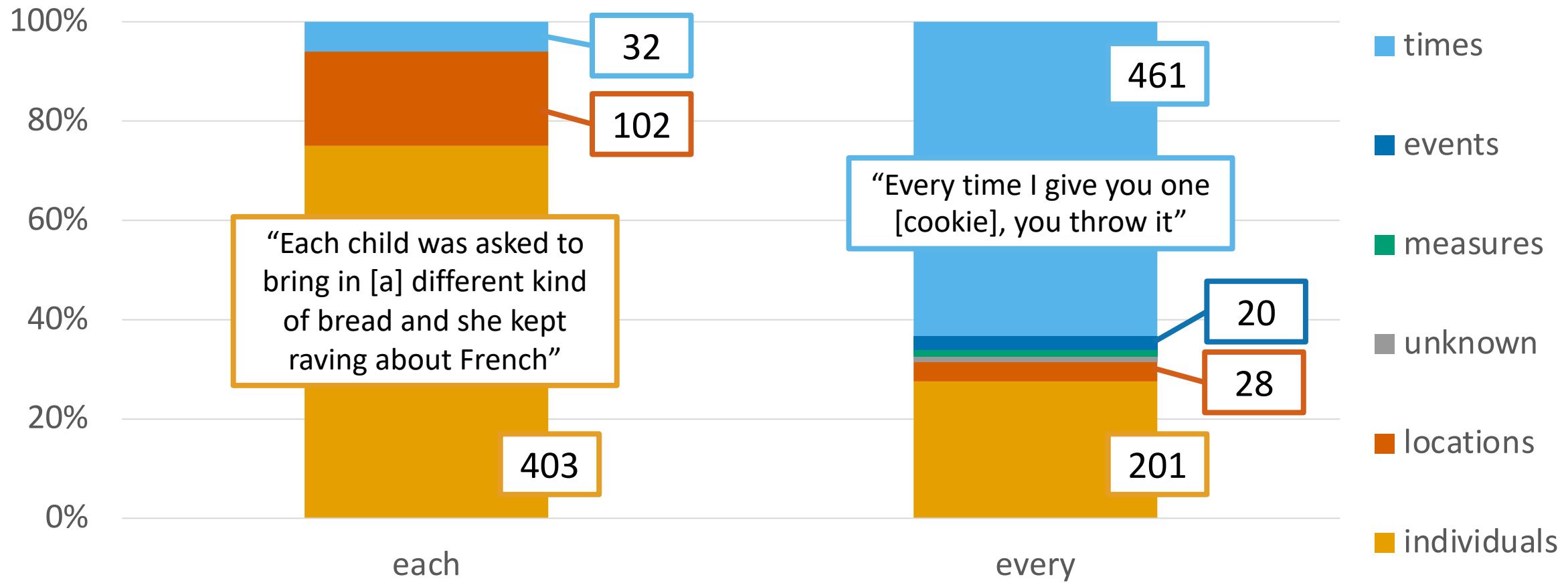
With possible PL-responses: 1

Mom: What did you play every day while you were there?

Child (4;11): ...the water game

Each and *every* in child-directed speech

What's being quantified over?



Each and *every* in child-directed speech

Low-level differences, including:

- Quantifying over **individuals** vs. **times**
- Being in **argument** vs. **adjunct** position
- Relative clause modification: **rarely** vs. **sometimes**

“You have to ring up **each** thing”

“Could you put a flower on **each** plate?”

“Put sugar in **each** coffee”

“We’ll put one finger on **each** thing we count”

“We **each** have three”

“**Every** time I ask a question, you say you don’t know”

“**Every** time I see ya, ya got something in your mouth”

“You turn into a wild man **every** time we get out”

“Your birthday’s always at the same time **every** year”

“She watches **every** movie they make”

Generalize over local domain

Project beyond local domain

Projecting beyond the local domain

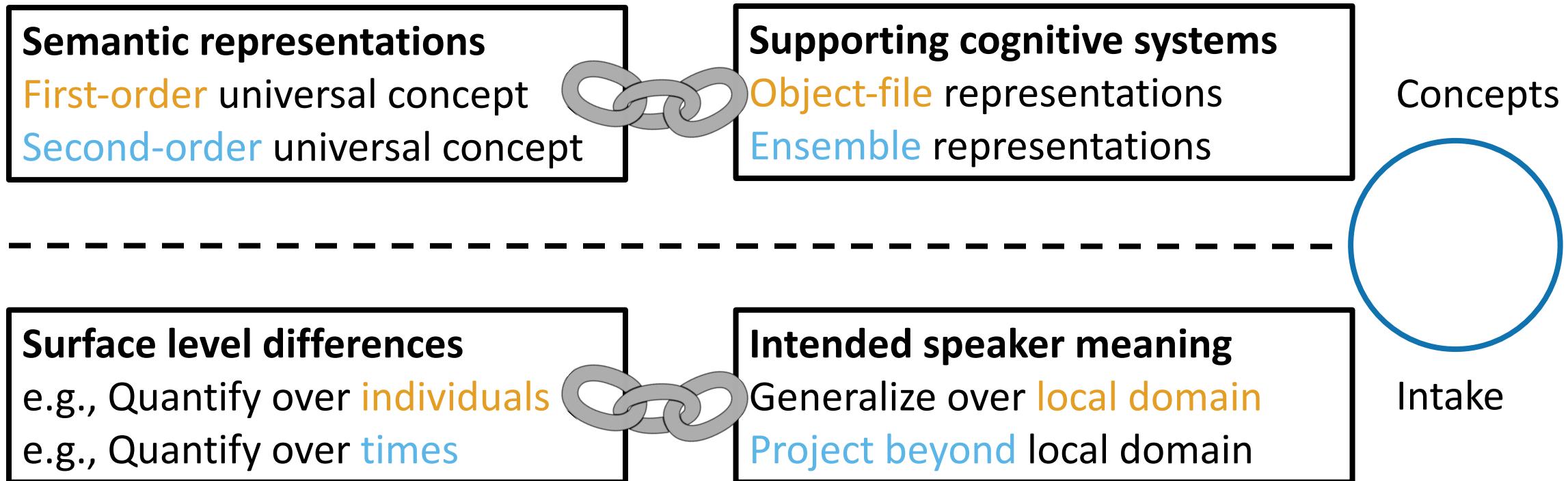
After a lifetime of investigation, Suzie came to a striking discovery:

$\left\{ \begin{matrix} \text{\#Each} \\ \text{Every} \end{matrix} \right\}$ language has over 20 color words

Suzie just discovered 4 new languages and interestingly,

$\left\{ \begin{matrix} \text{Each} \\ \text{\#Every} \end{matrix} \right\}$ language has over 20 color words

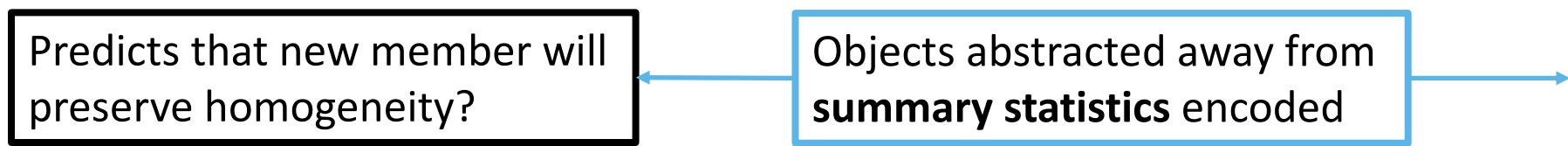
Sketching a learning story for *each* & *every*



Projecting beyond the local domain

Second-order representations (every) rely on ensembles

CIRCLES: $\forall X[\text{Green}(X)] \approx \text{The circles}_x \text{ are such that all of them}_x \text{ are green}$

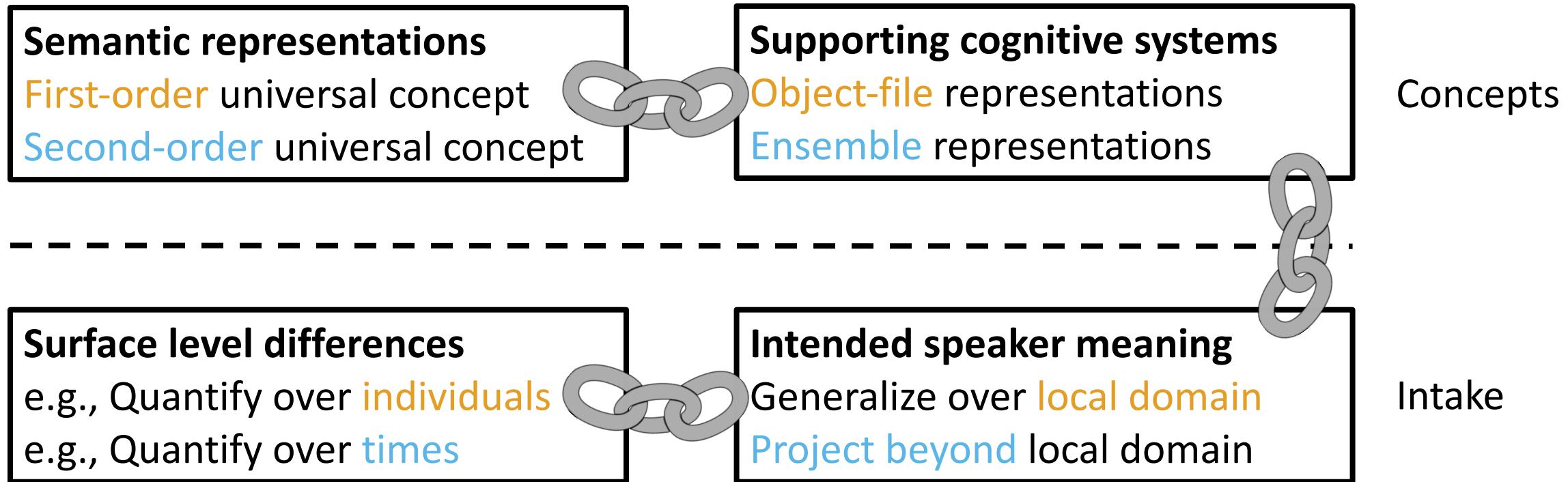


First-order representations (each) rely on object-files

CIRCLES: $\forall x[\text{Green}(x)] \approx \text{The individual circles}_x \text{ are s.t. each one}_x \text{ is green}$



Sketching a learning story for *each* & *every*

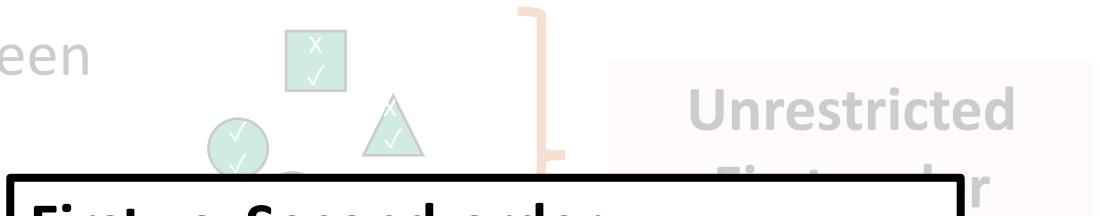


For each individual_x, if it_x is a circle, it_x is green

UNIVERSE: $\forall x[\text{Circle}(x) \rightarrow \text{Green}(x)]$

Restricted quantification:

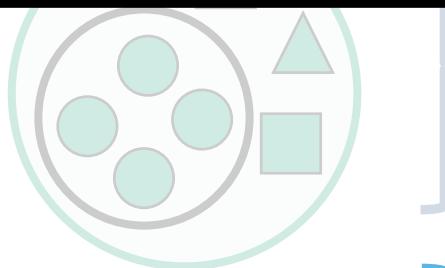
- Only 1st argument represented
- Explains “conservativity”



First vs. Second-order:

- Encode **individuals** vs. **groups**
- Consequences for acquisition

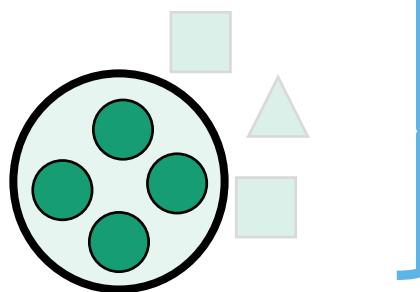
UNIVERSE: $\text{TheX:Circle}(X) \subseteq \text{TheY:Green}(Y)$



Unrestricted
Second-order

The circles_x are such that all of them_x are green

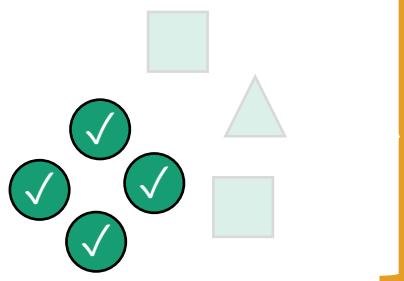
CIRCLES: $\forall X[\text{Green}(X)]$



Restricted
Second-order

The individual circles_x are s.t. each one_x is green

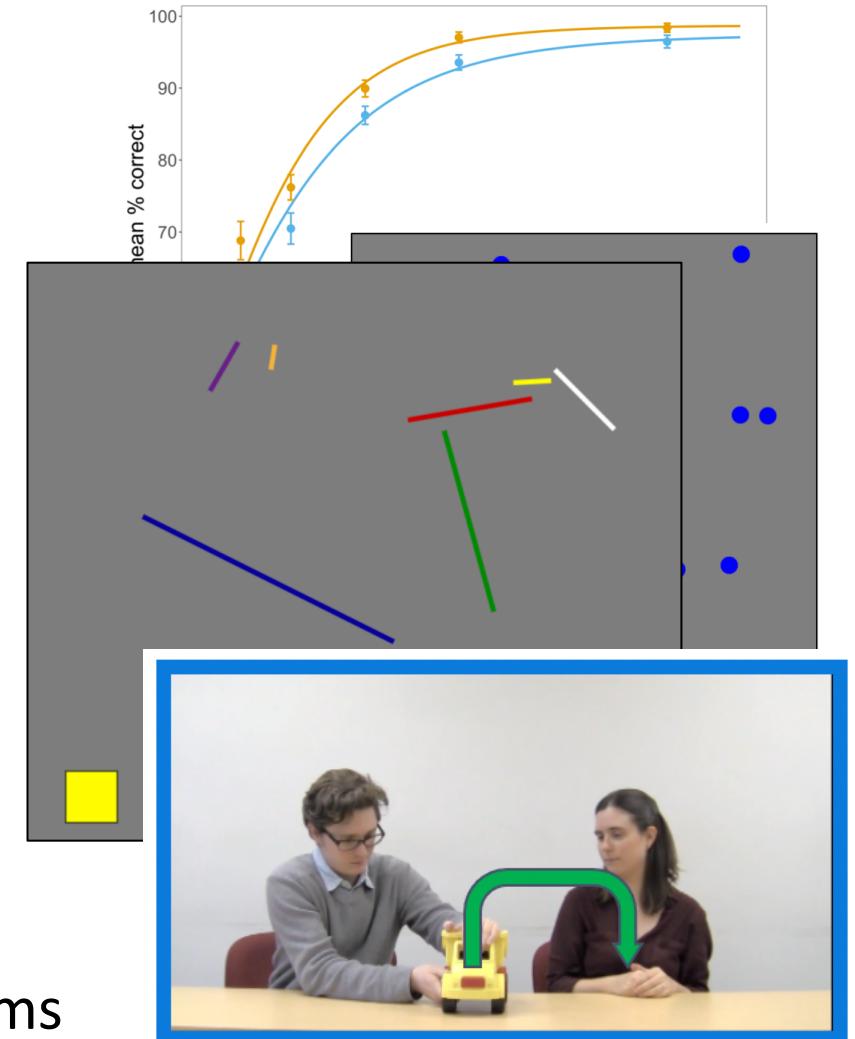
CIRCLES: $\forall x[\text{Green}(x)]$



Restricted
First-order

What do semantic representations look like?

- *More* vs. *Most* in English and Cantonese
- Superlatives in language and cognition
- Event concepts and verb learning
- Anytime we have:
 - ➔ Specifications of competing representations
 - ➔ Understanding of the supporting cognitive systems



Thanks!

Slides available at: tylerknowlton.com/talks/USC.pdf

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