### Hypothesis Testing In Regression

### Agenda

- 1. Sample Vs. Population
- 2. Regression Coefficients
- 3. Hypothesis Tests with Regression Coefficients

Parameter - Numerical Characteristic of a Population

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- Mean  $\mu$
- Standard Deviation σ

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Statistic - Numerical Characteristic of a Sample

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- Mean X
- Standard Deviation s

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- Standard Deviation s

Statistics are **ESTIMATES** of Parameters

Car Example

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2020 Cars Data (418 cars) Means					
Sample N MPG Wt (7					
Population (µ)	418	24.17	4.408		

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Sample 2 ( $\overline{\mathbf{X}}_2$ )	100	23.91	4.42				
Sample 3 ( $\overline{\mathbf{X}}_3$ )	100	24.67	4.39				

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What about the relationship between MPG & Weight?

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$
$$y = MPG$$
$$x_1 = Weight$$

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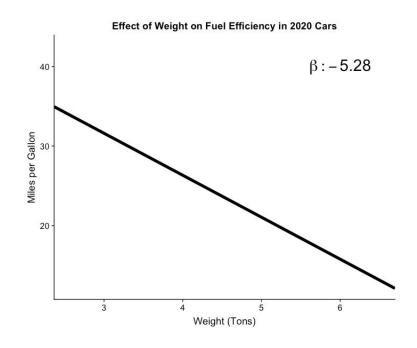
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2020 Cars Data (418 cars) Means				
Sample N MPG Wt β				
Population (μ)	418	24.17	4.408	-5.28

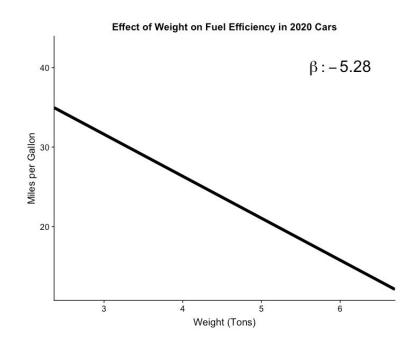
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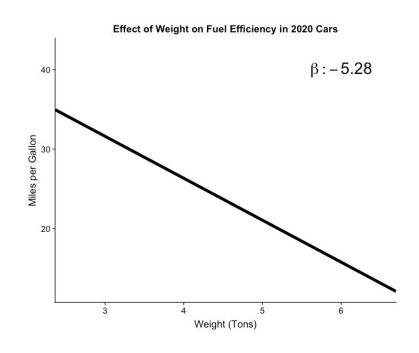
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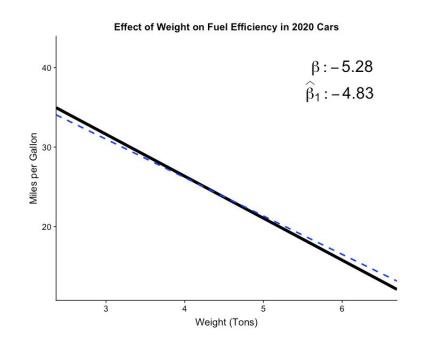
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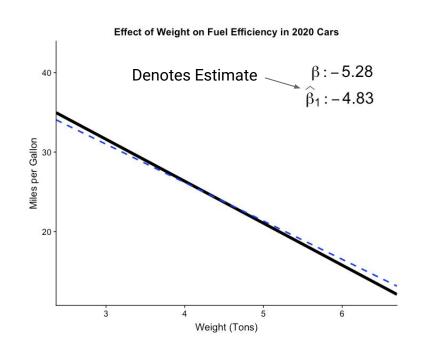
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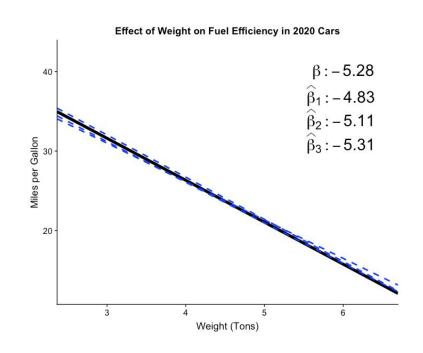
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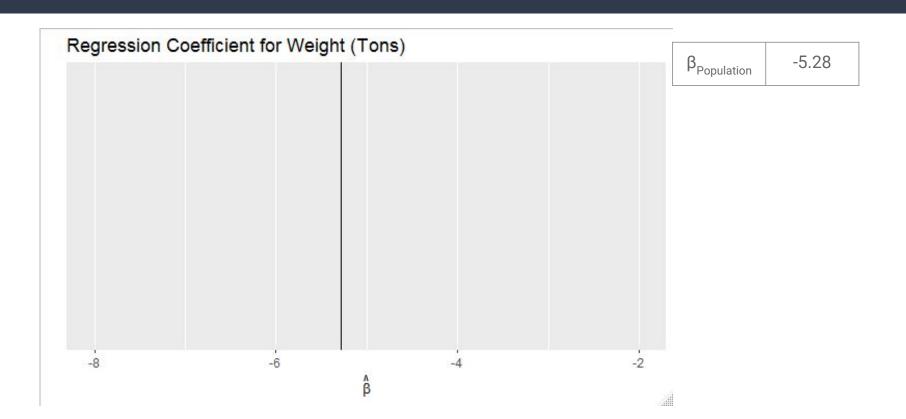
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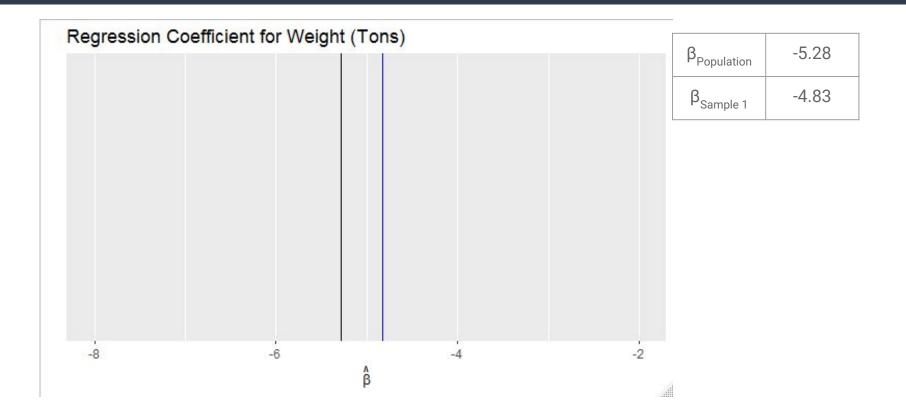


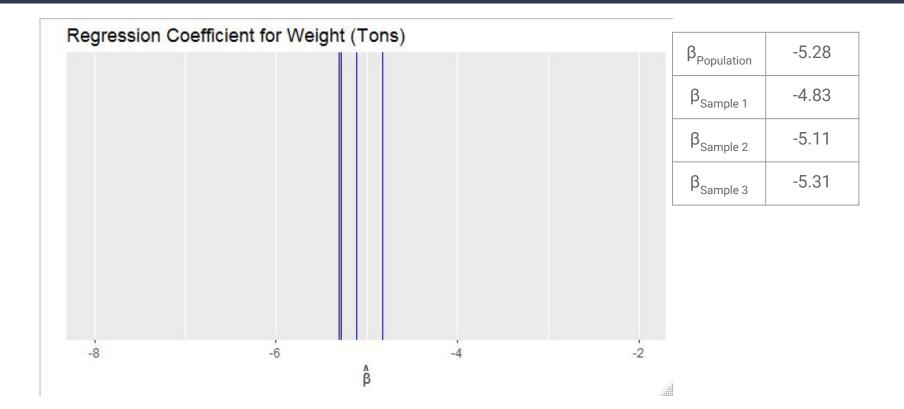
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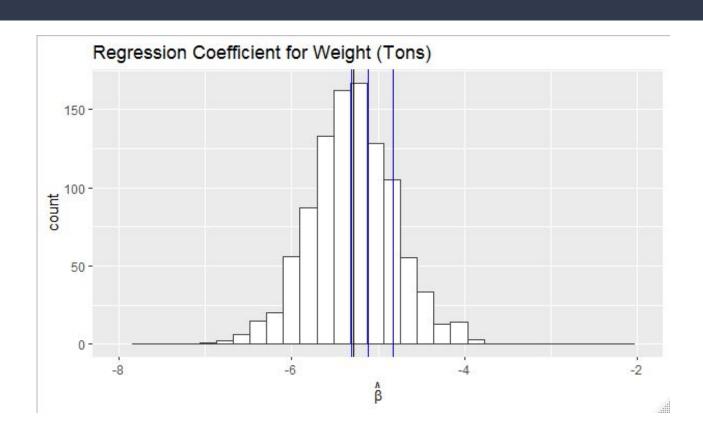
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Sample 1	100	23.84	4.48	-4.83	
Sample 2	100	23.91	4.42	-5.11	
Sample 3	100	24.67	4.39	-5.31	

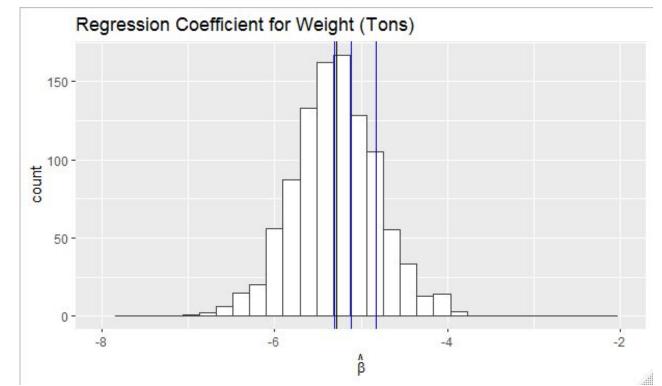




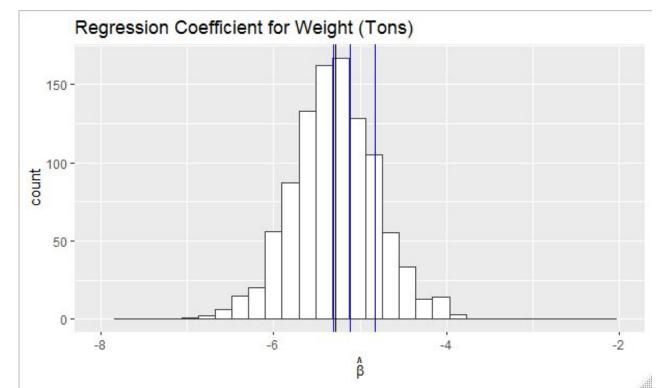






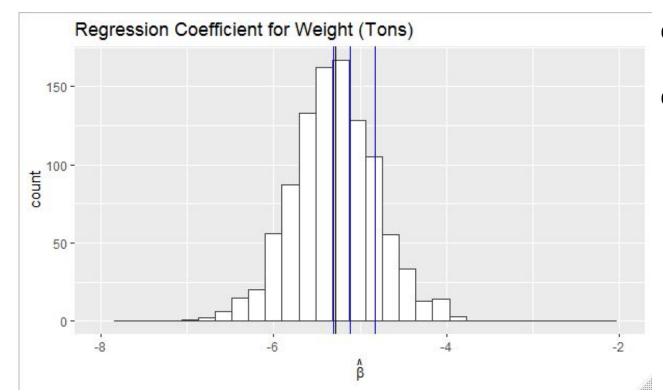


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**Standard Error!** 

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- Used for Hypothesis Testing as well!

Null Hypothesis (H<sub>0</sub>)

Alternative Hypothesis (H<sub>A</sub>)

Null Hypothesis (H<sub>0</sub>)

Alternative Hypothesis (H<sub>A</sub>)

$$\beta = 0$$

$$\beta$$
 <,>,  $\neq$  0

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 Regression Coefficients Follow a T - Distribution (Generally speaking)

$$t = (\beta_{\text{Estimate}} - \beta_{\text{Hypothesis}}) / SE_{\beta}$$

$$t = (\beta_{\text{Estimate}} - 0)/SE_{\beta}$$

$$t = (\beta_{\text{Estimate}} - 0)/SE_{\beta}$$

Compare this t to your Student's t distribution

Example:

Example: Sample 1

	N	MPG	Wt	β <sub>1</sub>	
Sample 1	100	23.84	4.48	-4.83	

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	N	MPG	Wt	$\beta_1$	$SE_{\pmb{eta}}$	t
Sample 1	100	23.84	4.48	-4.83	0.53	-9.11

Critical t-value (a = .99):

Example: Sample 1

	N	MPG	Wt	$\boldsymbol{\beta}_1$	$SE_{\pmb{eta}}$	t
Sample 1	100	23.84	4.48	-4.83	0.53	-9.11

Critical *t*-value (a = .01): ~ ± 2.626

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Sample 1	100	23.84	4.48	-4.83	0.53	-9.11

Critical *t*-value (a = .01):  $\sim \pm 2.626$ 

Reject the Null!!