PHYS 161: Homework 4

Due on Thursday February 12, 2015

Professor Landee 11:00am

Zhuoming Tan

February 12, 2015

Problem 1

(2.34) Extremum of ϕ... (problem omitted)

For point charge $\phi = \frac{Q}{4\pi\epsilon_0 r}$. For the line $(0, y, 0)$, superposition of contribution from each point charge is then

$$\phi(0, y, 0) = 2\frac{-C}{4\pi\epsilon_0\sqrt{1+(1-y)^2}} + \frac{2C}{4\pi\epsilon_0\sqrt{y^2}} = \frac{2C}{4\pi\epsilon_0}\left(\frac{1}{\sqrt{y^2}} - \frac{1}{\sqrt{1+(1-y)^2}}\right)$$

so take derivative respect to y

$$\phi'(0, y, 0) = \frac{2C}{4\pi\epsilon_0}\left(\frac{y-1}{(y^2-2y+2)^{3/2}} - \frac{y}{(y^2)^{3/2}}\right)$$

Let $\phi'(0, y, 0) = 0$, and compute numeric root with computer. $y \approx 1.62072$. Calculating value of $\phi'(0, y, 0)$ besides this point could reveal that $y \approx 1.62072$ is a minimum of ϕ.
Because $\mathbf{E} = -\nabla\phi$, at minimum or maximum of ϕ, $\mathbf{E} = 0$.

Problem 2

(2.50) Dividing the charge... (problem omitted)

For one of the spheres with charge q, the electric field $E = \frac{q}{4\pi\epsilon_0 r^2}$. When $r < R$, $u = 0$; when $r > R$,

$$u = \frac{\epsilon_0}{2}\int_0^{\pi}\int_0^{2\pi}\int_{R_1}^{\infty} E^2\, dV = \frac{q^2}{8\pi\epsilon_0 R_1}$$

So for the other one with charge $Q - q$, $u = \frac{(Q-q)^2}{8\pi\epsilon_0 R_2}$. Derivative of total potential is then

$$\frac{q(R_1+R_2) - QR_1}{4\pi\epsilon_0 R_1 R_2}$$

Because their separation is much larger than their radii, let $R_1 = R_2$. Then $q = Q/2$. So each of the two spheres have the same charge $Q/2$.

Problem 3

(2.58) Energy of a shell... (problem omitted)

$$U = \frac{1}{2}\int \rho\phi\, dv \tag{2.32}$$

So

$$U = \frac{1}{2}\int \rho\phi\, dv = \frac{1}{2}\int_0^{2\pi} d\theta \int_0^{\pi} \frac{Q^2R^2\sin\phi}{16\pi^2\epsilon_0 R}\, d\phi = \frac{Q^2}{8\pi\epsilon_0 R}$$

Problem 4

(3.9) Grounding a shell... (problem omitted)

Even if the outer shell is grounded, because from Gauss's Law, this is the lowest energy configuration, the amount of charge would remain $-Q$ on the outer shell. This could be further explained by taking the Gaussian surface at R within the outer shell, and because the outer shell is a conductor, the field inside the shell must be zero. Therefore $-Q$ charge must reside on the inner surface of the outer shell to make this possible.
However when the inner shell is grounded, case changes. The potential inside the outer shell due to the outer shell is $V = \frac{-Q}{4\pi\epsilon_0 R_2}$, but the inner shell is at $V = 0 = \frac{-Q}{4\pi\epsilon_0 R_2} + \frac{x}{4\pi\epsilon_0 R_1}$. The charge on inner shell is then $x = \frac{R_1}{R_2}Q$.

Problem 5

(3.13) Image charge for a grounded spherical shell... (problem omitted)

(a) Consider only on the xy-plane. Potential of one point charge is $\phi = \frac{-q}{4\pi\epsilon_0 r}$, where $r = \sqrt{(x-a)^2+y^2}$; the other one is $\phi = \frac{Q}{4\pi\epsilon_0 R}$, where $R = \sqrt{(x-A)^2+y^2}$. So if $\phi = 0$,

$$\frac{q}{\sqrt{(x-a)^2+y^2}} = \frac{Q}{\sqrt{(x-A)^2+y^2}}$$

which could be rewritten as

$$\left(x - \frac{Q^2a - q^2A}{Q^2-q^2}\right)^2 + y^2 = \left(\frac{Q^2a-q^2A}{Q^2-q^2}\right)^2 + \frac{q^2A^2 - Q^2a^2}{Q^2-q^2}$$

which is indeed a circle in the xy-plane. Hence it could also be shown as a sphere in space by adding z axis into the equations.

(b) As shown above the x of center is $\frac{Qa-qA}{Q-q}$. Let that equal zero. $Q^2a = q^2A$.

(c) It would be $R = \sqrt{\frac{qA^2-Qa^2}{Q-q}} = \sqrt{Aa}$.

(d) From $R = \sqrt{Aa}$, $a = R^2/a$. From $Q^2a = q^2A$, $q = QR/A$.

(e) Same logic as the previous question.

Problem 6

(3.59) Coaxial capacitor... (problem omitted)

By Gauss's Law

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

so

$$\Delta V = \int_a^b E\,dr = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{a}{b}$$

so for a length L, $Q = \lambda L$

$$C = \frac{Q}{\Delta V} = \frac{2\pi\epsilon_0 L}{\ln\frac{a}{b}}$$

If $a-b$ is small, $2\pi L$ could be approximated as a flat area A. $\ln\frac{a}{b} = \ln a - \ln b \approx a - b = d$. So $C \approx \frac{A\epsilon_0}{d}$.