

## Lecture 7: September 15

Lecturer: Vijay Garg

Scribe: Tong Zhou

## 7.1 Outline

The lecture covered the following topics:

- Review of Lamport's Fast Mutex Algorithm
- Introduction to OpenMP
- Two new ways to find maximum from N numbers

## 7.2 Lamport's Fast Mutex Algorithm

Lamport's fast mutex algorithm uses two shared variables  $X$  and  $Y$  and a shared single-writer-multiple-reader registers  $flag[i]$ . A process  $P_i$  can enter the critical section either along a *fast* path or along a *slow* path. The algorithm is shown below.

The variable  $flag[i]$  is set to value *up* if  $P_i$  is actively contending for mutual exclusion using the fast path. When  $Y$  is -1, the door is open. A process  $P_i$  closes the door by updating  $Y$  with  $i$ . Lamport's Fast Mutex Algorithm is deadlock-free but allows starvation of individual processes.

In the algorithm, process  $i$  first sets  $X$  to  $i$ , and then checks the value of  $Y$ . When it finds  $Y = -1$ , it sets  $Y$  to  $i$  and then checks the value of  $X$ . If  $X = i$  process  $i$  enters its critical section along the *fast* path. At most one process can enter its critical section along *fast* path. If a process finds  $X \neq i$  then it delays itself by looping until it sees that all the values in the array  $flag$  are *down*. Checking these  $n$  values in array  $flag$  plays two roles:

1. Say that process  $i$  enters its critical section along *slow* path, if it finds  $Y = i$  after exiting the for-loop. A consequence of having observed all the values in the array  $flag$  to be *down* is that the value of  $Y$  will not be changed thereafter until process  $i$  leaves the critical section. This follows because every other contending process either reads  $Y \neq -1$  and waits at  $waitUntil(Y == -1)$ , or reads  $Y = 0$  and then finished the assignment  $Y := i$  before setting  $flag[i]$  to *down*. Hence, once process  $i$  finds  $Y = i$  after the loop, no other process can change the value of  $Y$  until process  $i$  sets  $Y$  to -1 in its exit code. It follows that at most one process can enter along *slow* path.

2. The for-loop ensures that if a process enters the critical section along *fast* path, then any other process is prevented from entering along *slow* path. To see this, observe that when a process  $i$  enters its critical section along *fast* path, its  $flag[i]$  remains *up*. Thus, if another process tries to enter along *slow* path it will find that  $flag[i] == up$  and will have to wait until process  $i$  exits its critical section and set  $flag[i]$  to *down*.

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**Algorithm 1** Lamport's Fast Mutex Algorithm

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```
1: var
2:   X, Y: int initially -1;
3:   flag: array[1..n] of {down, up};

4: acquire(int i)
5: {
6:   while(true):
7:     flag[i] = up;
8:     X = i;
9:     if(Y != -1): // splitter's left
10:      flag[i] = down;
11:      waitUntil(Y == -1);
12:      continue;
13:   else:
14:     Y = i;
15:     if(X == i): return; // fast path
16:     else: // splitter's right
17:      flag[i] = down;
18:      waitUntil( $\forall j : \text{flag}[j] == \text{down}$ );
19:      if(Y == i): return; // slow path
20:      else:
21:        waitUntil(Y == -1);
22:        continue;
23: }

24: release(int i)
25: {
26:   Y = -1;
27:   flag[i] = down;
28: }
```

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## 7.3 OpenMP

OpenMP (Open Multi-Processing) is an application programming interface that supports multi-platform shared memory multiprocessing programming in C, C++, and Fortran, on most platforms, processor architectures and operating systems.

Fork-Join Parallelism:

- Master thread spawns a team of threads as needed.
- Parallelism added incrementally until performance goals are met. The sequential program evolves into a parallel program.

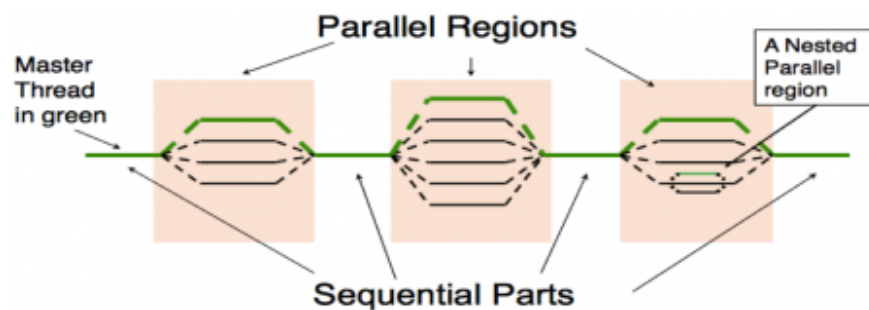


Figure 7.1: Fork-Join Parallelism

We create threads in OpenMP with the parallel construct. For example, to create a 4 thread Parallel region. Each thread calls  $foo(ID, A)$  for  $ID = 0$  to 3.

```
double A[1000];
omp_set_num_threads(4);
#pragma omp parallel
{
    int ID = omp_get_thread_num();
    foo(ID, A);
}
```

Thread creation: parallel regions

### 7.3.1 Synchronization

Synchronization is used to impose order constraints and to protect access to shared data. In OpenMP, we have 4 high level synchronization:

- critical
- atomic
- barrier
- ordered

### 7.3.1.1 Synchronization: critical

Mutual exclusion: only one thread at a time can enter a *critical* region. Example code:

```
float result;
#pragma omp parallel
{
    float B; int i, id, nthrds;
    id = omp_get_thread_num();
    nthrds = omp_get_num_threads();
    for(i=id; i<N; i = i+nthrds){
        B = foo(i); // expensive computation
        #pragma omp critical
            consume(B, result)
    }
}
```

### 7.3.1.2 Synchronization: atomic

*Atomic* provides mutual exclusion but only applies to the update of a memory location. In the following example code, *foo's* are run in parallel but *r* updated atomically.

```
int n, r;
#pragma omp parallel shared(n, r)
{
    for(i=0; i<n; i = i++){
        #pragma omp atomic
            r += foo(i); // expensive computation
    }
}
```

### 7.3.1.3 Synchronization: barrier

*Barrier*: Each thread waits until all threads arrive. Example code:

```
#pragma omp parallel shared (A, B, C) private(id)
{
    id=omp_get_thread_num();
    A[id] = big_calc1(id);
    #pragma omp barrier
    #pragma omp for
    for(i=0; i<N; i++){ C[i]=big_calc3(i, A); }
    #pragma omp for nowait
    for(i=0; i<N; i++){ B[i]=big_calc2(C, i); }
    A[id] = big_calc4(id);
}
```

### 7.3.1.4 Synchronization: ordered

The *ordered* region executes in the sequential. In the following example code, array *a* updated in any order but printed in sequential order

```
#pragma omp parallel for
{
    for(i=0; i<n; i = i++){
        tid = omp_get_thread_num();
        printf("Thread_%d_updates_a[%d]\n", tid, i);
        a[i] += i;
        #pragma omp ordered
        {
            printf("Thread_%d_prints_value_of_a[%d]=%d\n",
                    tid, i, a[i]);
        }
    }
}
```

### 7.3.1.5 Synchronization: locks

We also have low level synchronization *locks* (both simple and nested).

- Simple Lock routines: A simple lock is available if it is unset.
  - omp\_init\_lock()
  - omp\_set\_lock()
  - omp\_unset\_lock()
  - omp\_test\_lock()
  - omp\_destroy\_lock()
- Nested Locks: A nested lock is available if it is unset or if it is set but owned by the thread executing the nested lock function.
  - omp\_init\_nest\_lock()
  - omp\_set\_nest\_lock()
  - omp\_unset\_nest\_lock()
  - omp\_test\_nest\_lock()
  - omp\_destroy\_nest\_lock()

## 7.3.2 Data Environment and Data Attributes

All variables declared outside parallel for pragma are shared by default, except for loop index. *for* index variable is private. One can selectively change storage attributes for constructs using the following clauses.

- shared
- private
- firstprivate

### 7.3.2.1 Private Clause

`private(var)` creates a new local copy of `var` for each thread. The value is uninitialized and is undefined after the region.

### 7.3.2.2 firstprivate Clause

*firstprivate* is a special case of *private*. Initializes each private copy with the corresponding value from the master thread. In the following example, All copies have value of `tmp` initialized as 0.

```
void Foo()
{
    int tmp = 0;
    #pragma omp for firstprivate(tmp)
    for (int j = 0; j < 1000; ++j) {
        tmp += j;
    }
}
```

### 7.3.2.3 lastprivate Clause

*lastprivate* passes the value of a private from the last iteration to a global variable. In the following example, All copies have value of `tmp` initialized as 0. After the for loop, the variable `tmp` has the value from the last iteration (i.e. `j=99`).

```
void Foo()
{
    int tmp = 0;
    #pragma omp for firstprivate(tmp) lastprivate(tmp)
    for (int j = 0; j < 100; ++j)
        tmp += j;
    printf("%d\n", tmp);
}
```

## 7.4 Find Maximum from N Numbers

In the first lecture, we mentioned four ways to find maximum from  $N$  numbers.

	time complexity	space complexity
Sequential	$O(N)$	$O(N)$
Binary Tree	$O(\log(N))$	$O(N)$
All-pair	$O(1)$	$O(N^2)$
Comparison	$O(1)$	$O(N^{3/2})$

### 7.4.1 DoublyLog Algorithm

As we mentioned in the All-pair Algorithm,  $N$  numbers can be divided into  $\sqrt{N}$  groups. In each group, there are  $\sqrt{N}$  numbers. In DoublyLog Algorithm, we further divide every  $\sqrt{N}$  group into  $\sqrt{\sqrt{N}}$  sub-groups. We keep doing so until the size of every group is 1 or 2. The height of this tree structure is  $\log(\log(N))$ . For each layer, the computing time complexity is  $O(N)$ . The total time complexity for DoublyLog Algorithm is  $O(N\log(\log(N)))$ .

### 7.4.2 Cascaded Algorithm

In Cascaded Algorithm, we combine Sequential Algorithm and DoublyLog Algorithm to reduce the number of processors that we need.

Divide  $N$  numbers into  $N/\log(\log(N))$  groups. Then, in every group, there are  $\log(\log(N))$  numbers. We use Sequential Algorithm to get  $N/\log(\log(N))$  maximum candidates in every group. For these maximum candidates, we use DoublyLog Algorithm.

For Sequential Algorithm, the work complexity is  $O(N)$  and time complexity is  $\log(\log(N))$ . To select maximum from candidates with DoublyLog Algorithm, the time complexity is  $O(\log(\log(N)))$  and work complexity is  $O(N)$  (There are  $N/(\log(\log(N)))$  groups. Every group has  $\log(\log(N))$  time complexity).

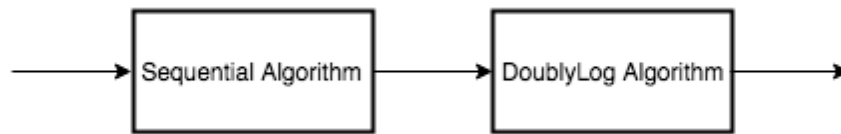


Figure 7.2: Cascaded Algorithm Model

## References

- [1] LESLIE LAMPORT, A Fast Mutual Exclusion Algorithm(1986).
- [2] MICHAEL MERRITT, GADI TAUBENFELD, Speeding Lamport's Fast Mutual Exclusion Algorithm(1991).
- [3] TIM MATTSO, A "Hands-on" Introduction to OpenMP (2008).
- [4] <https://github.com/vijaygarg1/UT-Garg-EE382C-EE361C-Multicore>