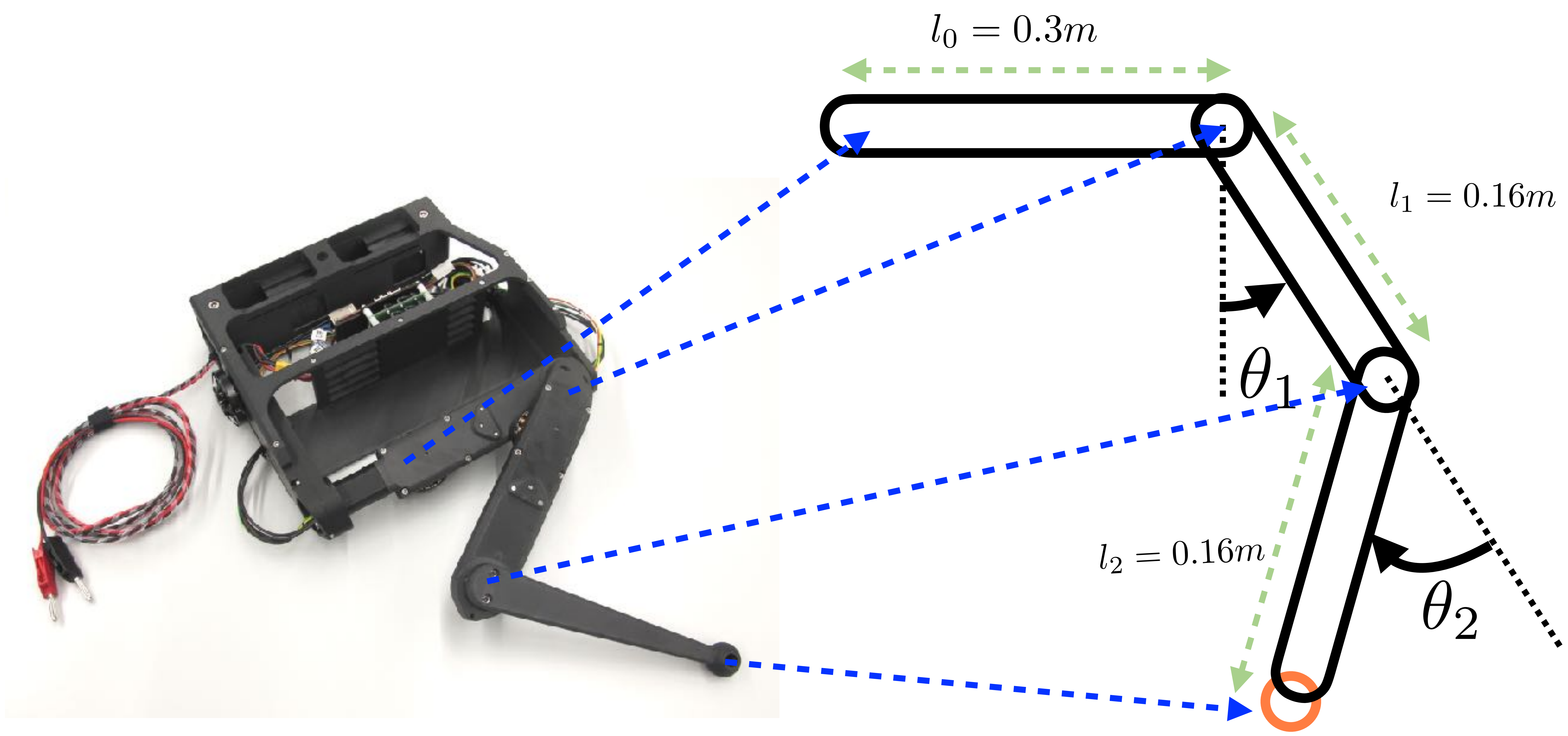
ROB-UY 2004

Robotic Manipulation & Locomotion

Lecture 04A - Transformations



Agenda

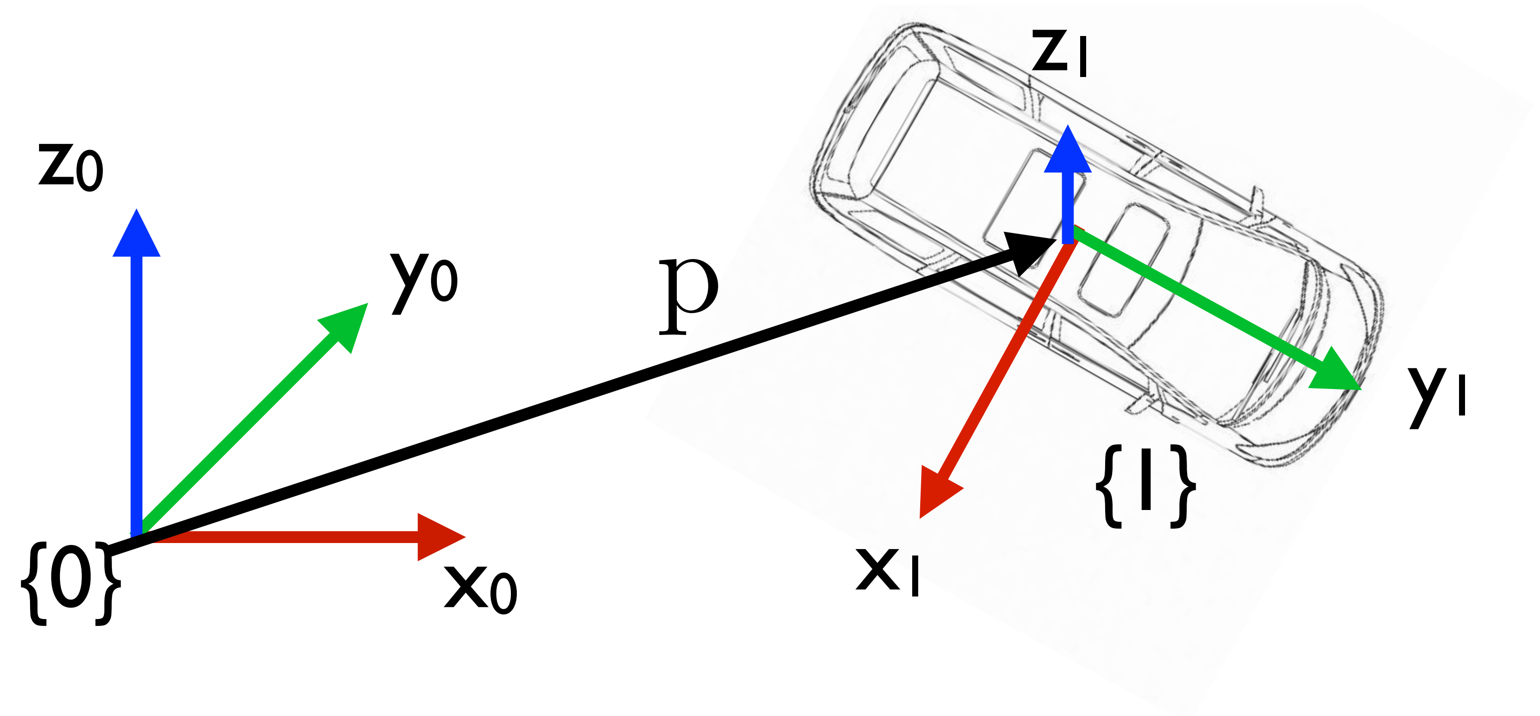
1. Homogenous Transformations
2. Three Uses of Transformations
3. Properties of Transformations
4. Forward Kinematics

Agenda

1. **Homogenous Transformations**
2. Three Uses of Transformations
3. Properties of Transformations
4. Forward Kinematics

Rigid Body Transformations

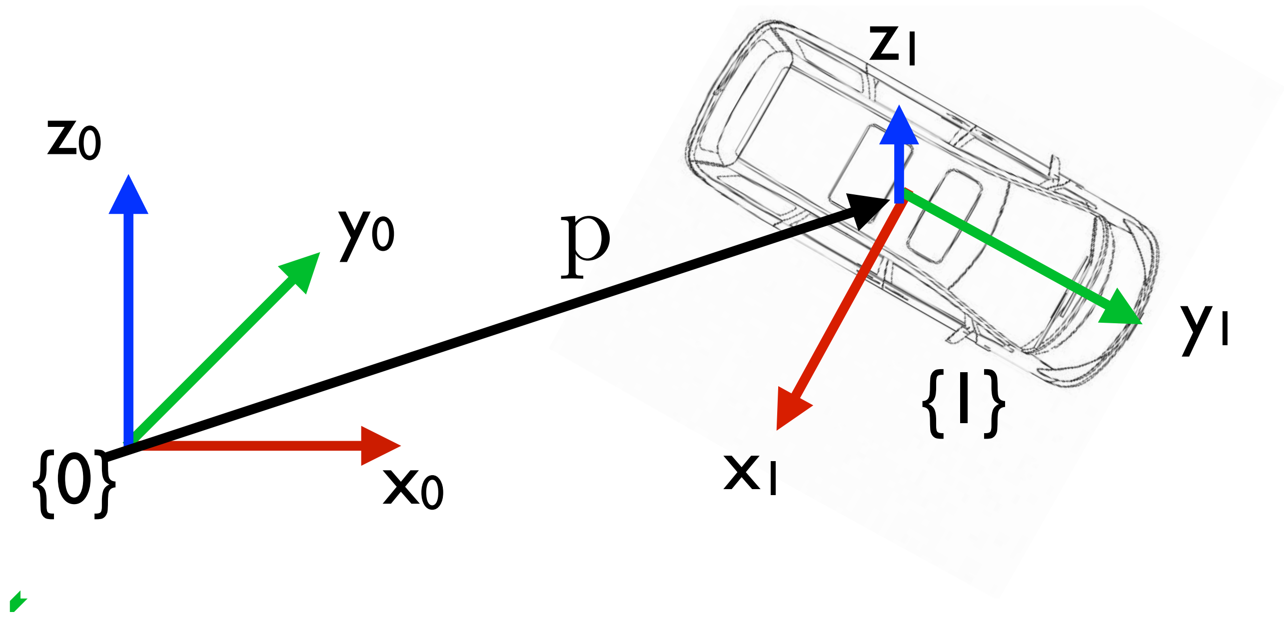
- Any **rigid body transformation** can be described by a translation p and a rotation R



Homogeneous Transforms

Rigid body transformations can be conveniently described by homogeneous matrices, which summarizes the position p_{01} and orientation R_{01}

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$



Homogeneous Transforms

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

Homogeneous Transforms

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

Homogeneous Transforms

$$T_{01} = \begin{bmatrix} R_{01} & \boxed{p_{01}} \\ 0 & 1 \end{bmatrix}$$

Homogeneous Transforms

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

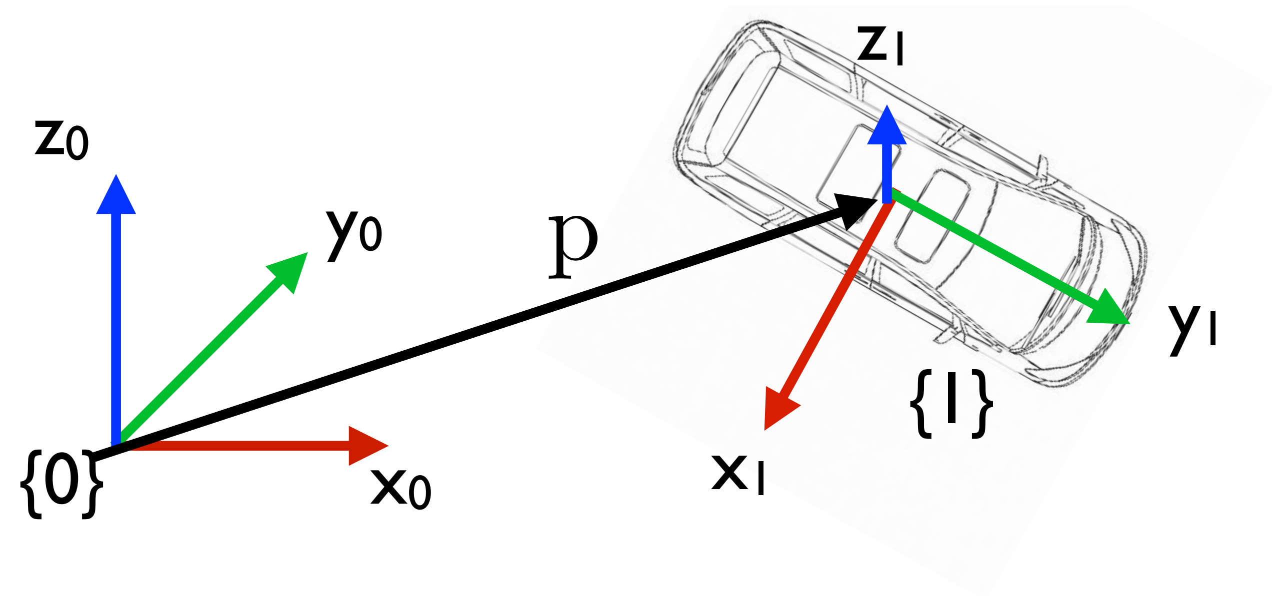
Homogeneous Transforms

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

Homogeneous Transforms

A point p with 3D (or 2D) coordinates p is then described with 4D (or 3D) coordinates as

$$\bar{p} = \begin{pmatrix} p \\ 1 \end{pmatrix}$$



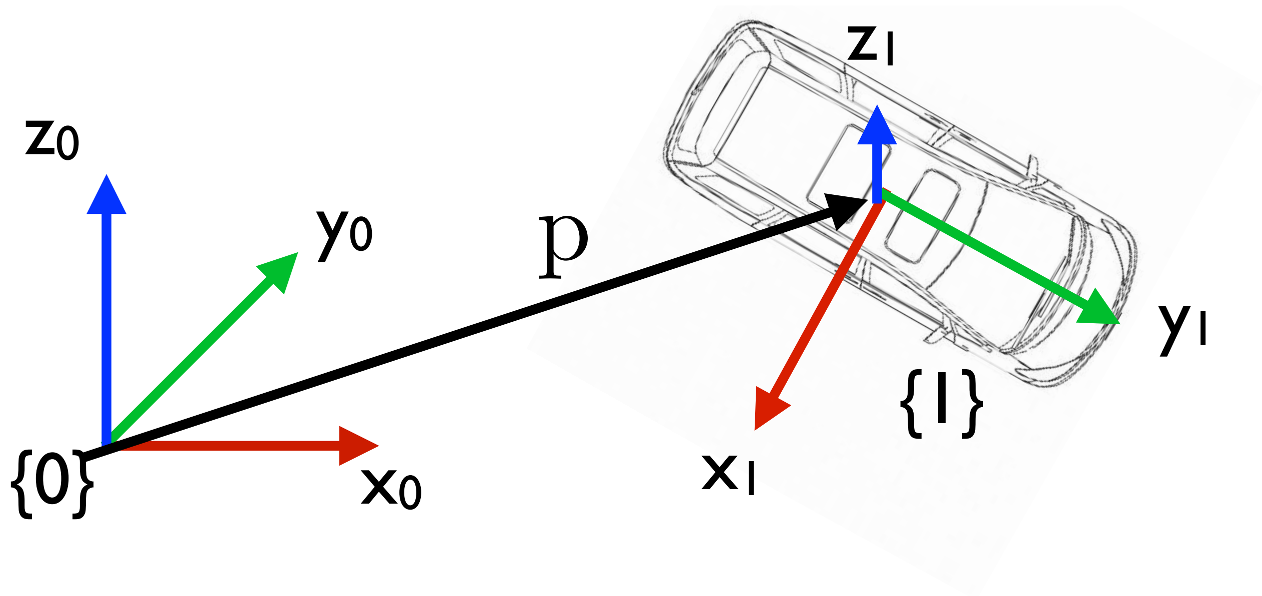
Homogeneous Transforms

A point p with 3D (or 2D) coordinates p is then described with 4D (or 3D) coordinates as

$$\bar{p} = \begin{pmatrix} p \\ 1 \end{pmatrix}$$

A vector v with 3D (or 2D) coordinates v is then described as a 4D (or 3D) coordinates as

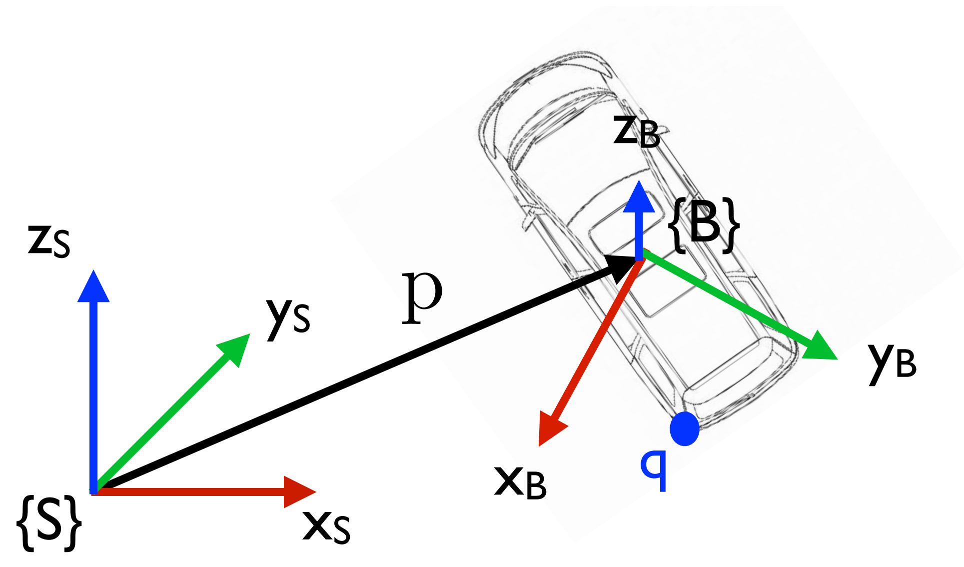
$$\bar{v} = \begin{pmatrix} v \\ 0 \end{pmatrix}$$



Homogeneous Transforms

We extend the coordinates of a point in frame B with a 1 as

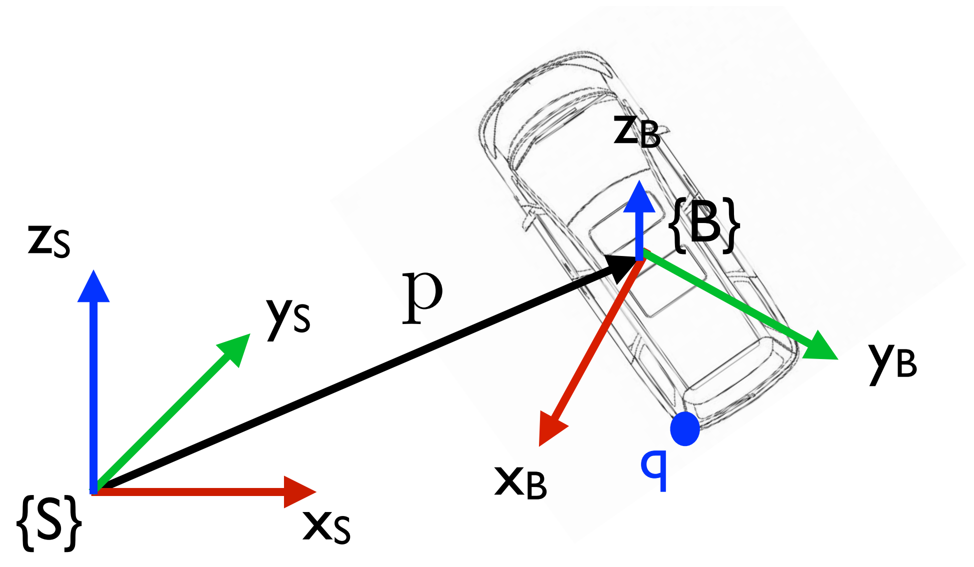
$$\bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$



Homogeneous Transforms

We extend the coordinates of a point in frame B with a 1 as

$$\bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$



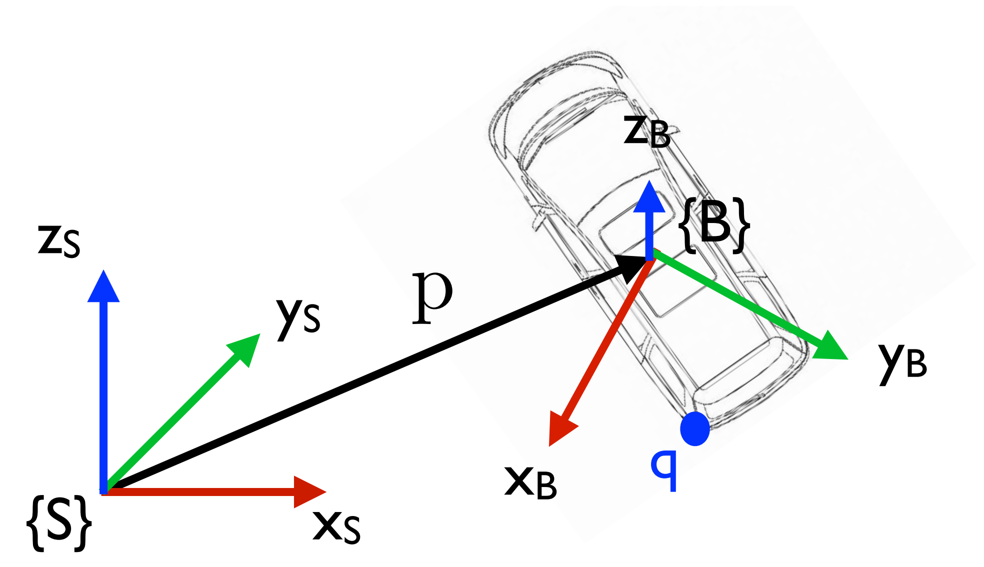
so its coordinates in frame S are

$$\bar{q}_S = T_{SB}\bar{q}_B$$

Homogeneous Transforms

We extend the coordinates of a point in frame B with a 1 as

$$\bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$



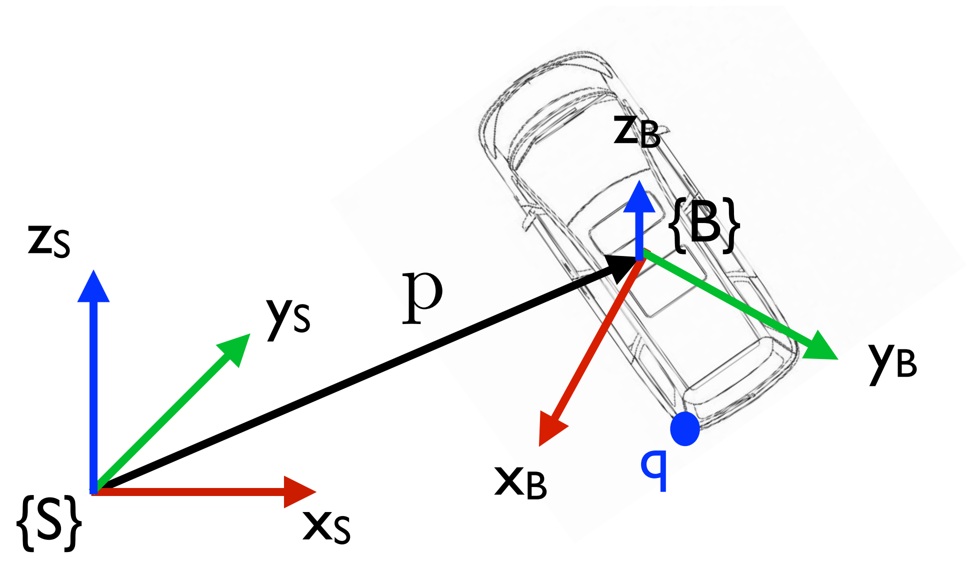
so its coordinates in frame S are

$$\bar{q}_S = T_{SB}\bar{q}_B = \begin{bmatrix} R_{SB} & p_{SB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$

Homogeneous Transforms

We extend the coordinates of a point in frame B with a 1 as

$$\bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$

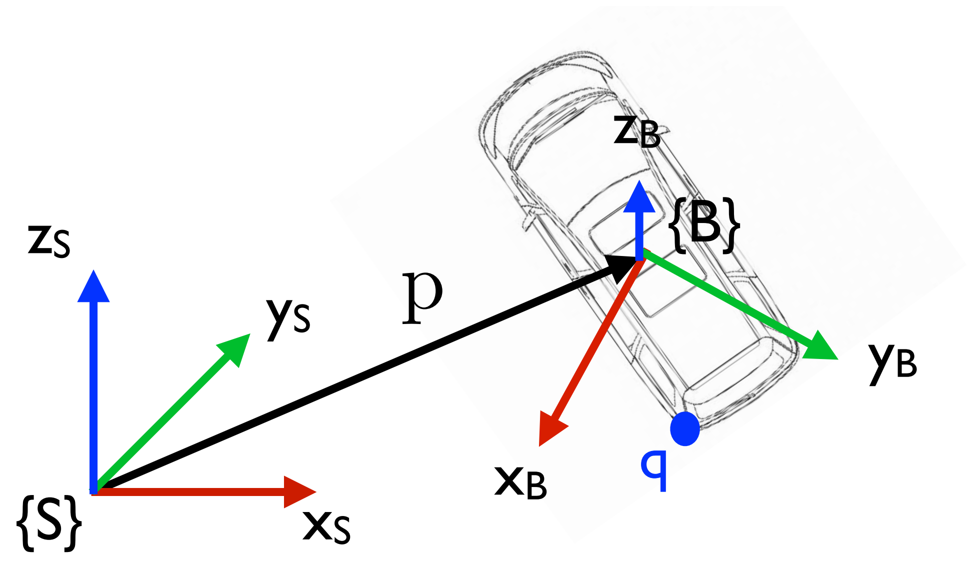


so its coordinates in frame S are

$$\bar{q}_S = T_{SB}\bar{q}_B = \begin{bmatrix} R_{SB} & p_{SB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$

Homogeneous Transforms

We extend the coordinates of a point in frame B with a 1 as



$$\bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$

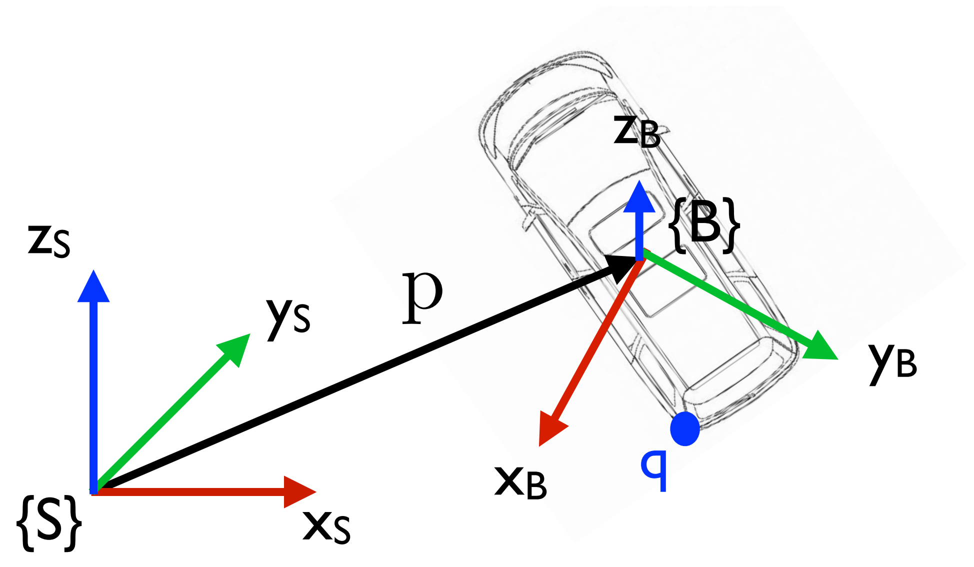
so its coordinates in frame S are

$$\bar{q}_S = T_{SB}\bar{q}_B = \begin{bmatrix} R_{SB} & p_{SB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$

Homogeneous Transforms

We extend the coordinates of a point in frame B with a 1 as

$$\bar{q}_B = \begin{bmatrix} q_B \\ 1 \end{bmatrix}$$



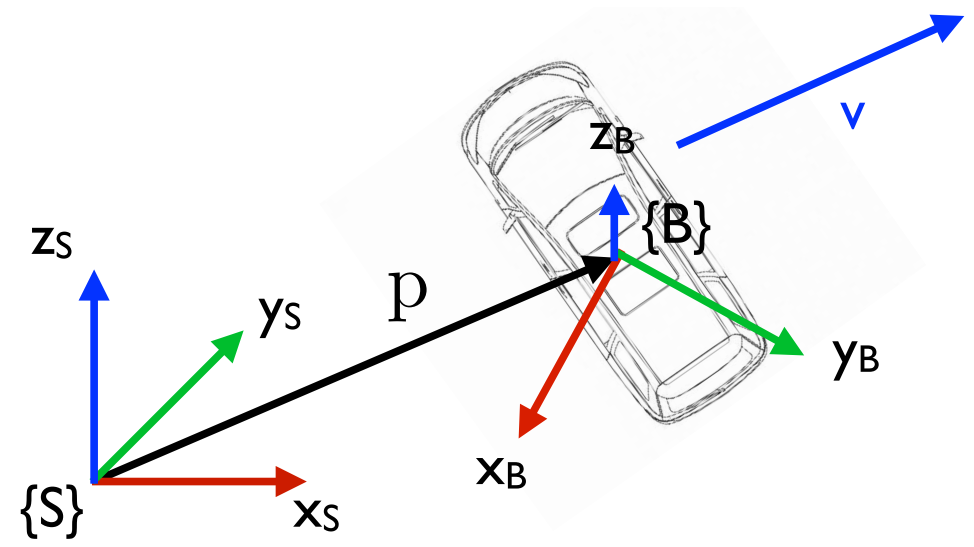
so its coordinates in frame S are

$$\bar{q}_S = T_{SB}\bar{q}_B = \begin{bmatrix} R_{SB} & p_{SB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_B \\ 1 \end{bmatrix} = \begin{bmatrix} R_{SB}q_B + p_{SB} \\ 1 \end{bmatrix}$$

Homogeneous Transforms

We extend the coordinates of a vector in frame B with a 0 as

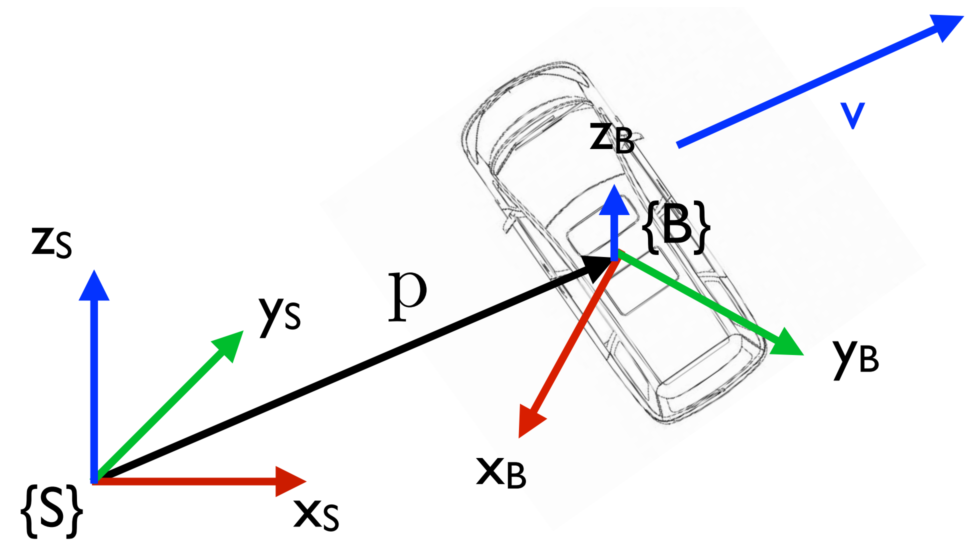
$$\bar{v}_B = \begin{bmatrix} v_B \\ 0 \end{bmatrix}$$



Homogeneous Transforms

We extend the coordinates of a vector in frame B with a 0 as

$$\bar{v}_B = \begin{bmatrix} v_B \\ 0 \end{bmatrix}$$



so its coordinates in frame S are

$$\bar{v}_S = T_{SB}\bar{v}_B$$

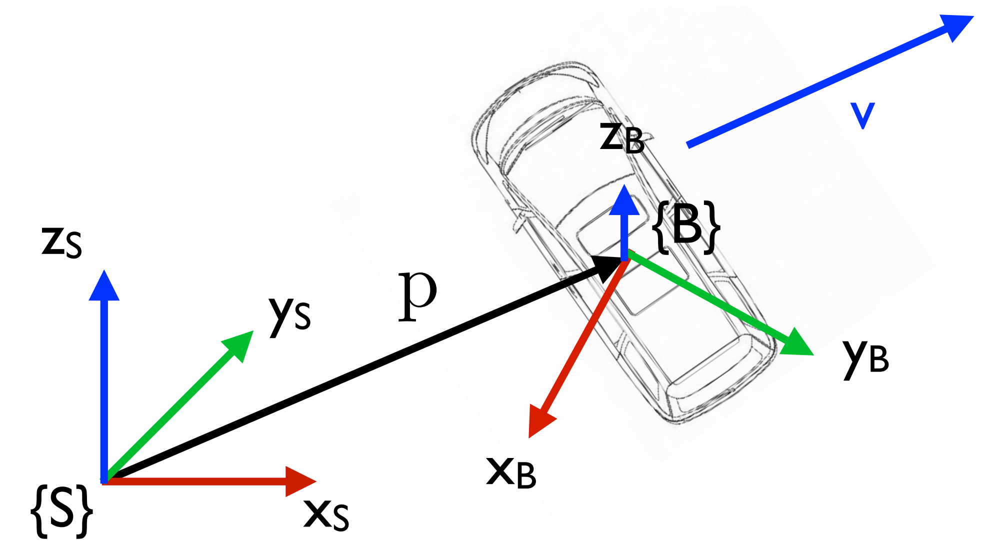
Homogeneous Transforms

We extend the coordinates of a vector in frame B with a 0 as

$$\bar{v}_B = \begin{bmatrix} v_B \\ 0 \end{bmatrix}$$

so its coordinates in frame S are

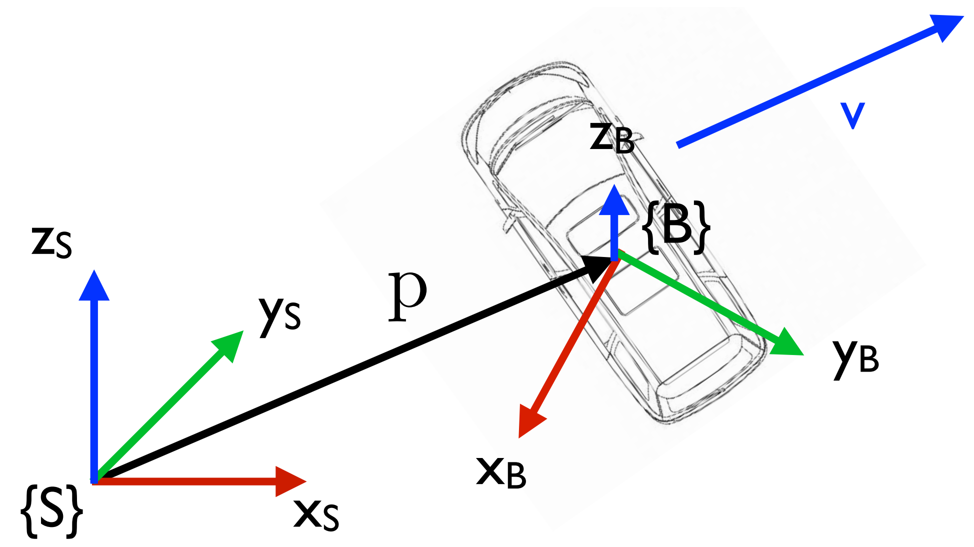
$$\bar{v}_S = T_{SB}\bar{v}_B = \begin{bmatrix} R_{SB} & p_{SB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_B \\ 0 \end{bmatrix}$$



Homogeneous Transforms

We extend the coordinates of a vector in frame B with a 0 as

$$\bar{v}_B = \begin{bmatrix} v_B \\ 0 \end{bmatrix}$$

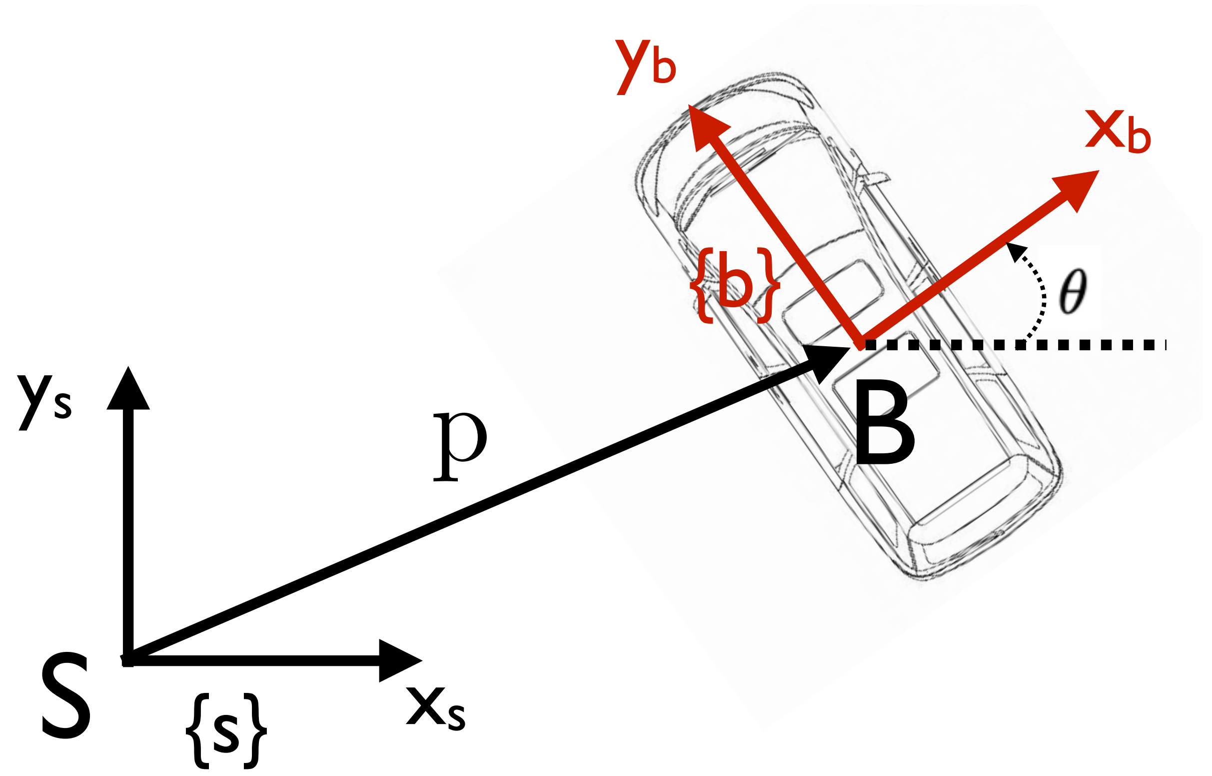


so its coordinates in frame S are

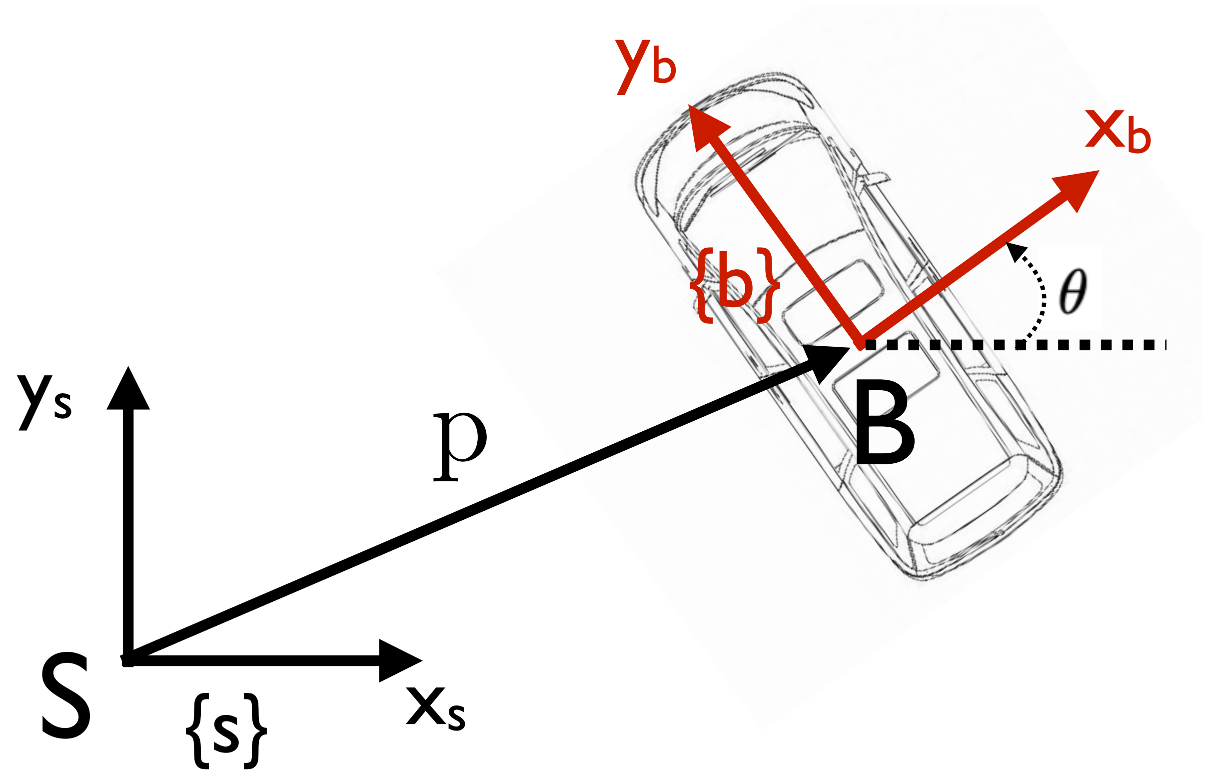
$$\bar{v}_S = T_{SB}\bar{v}_B = \begin{bmatrix} R_{SB} & p_{SB} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_B \\ 0 \end{bmatrix} = \begin{bmatrix} R_{SB}\boldsymbol{v}_B \\ 0 \end{bmatrix}$$

Agenda

1. Homogenous Transformations
2. **Three Uses of Transformations**
3. Properties of Transformations
4. Forward Kinematics

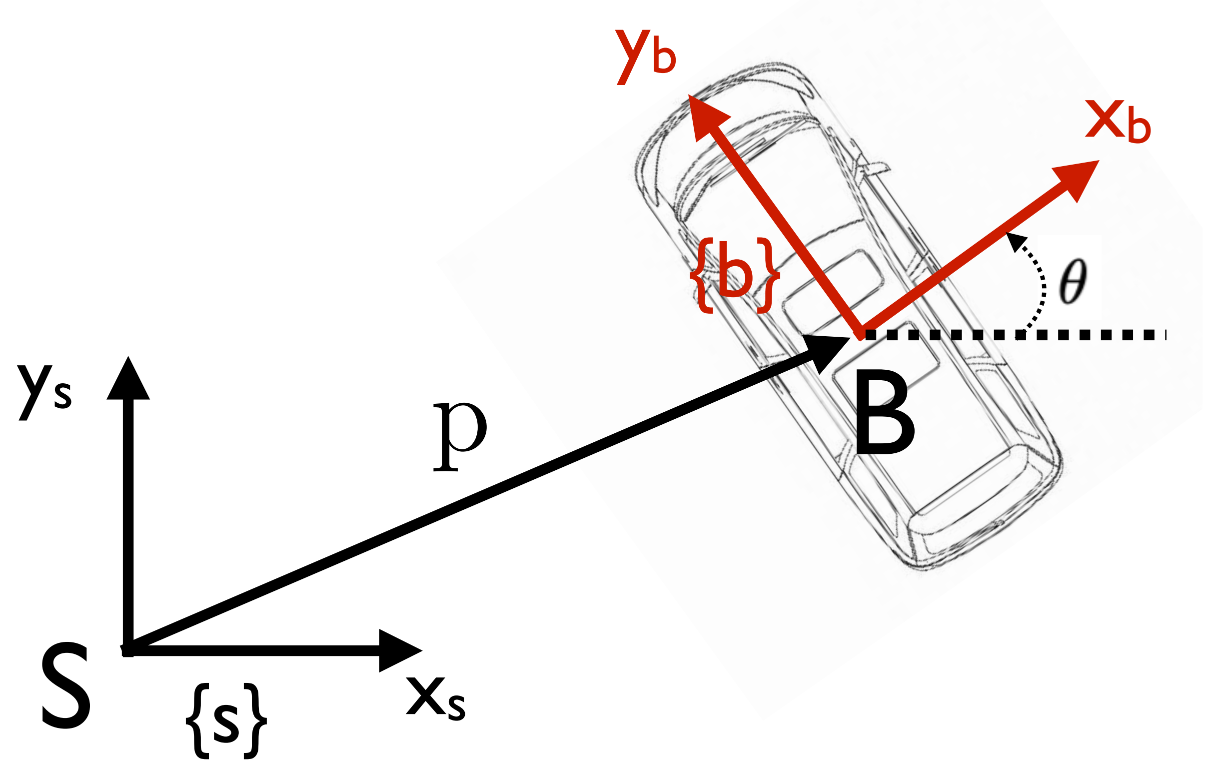


1) describe the position and orientation of an object in a given frame with (p,R)



1) describe the position and orientation of an object in a given frame with (p, R)

2) do a coordinate transform:
if frame B has position p_{SB} and orientation R_{SB} with respect to frame S, then a point with coordinates q_B in frame B has coordinates $q_S = R_{SB} q_B + p_{SB}$ in frame S



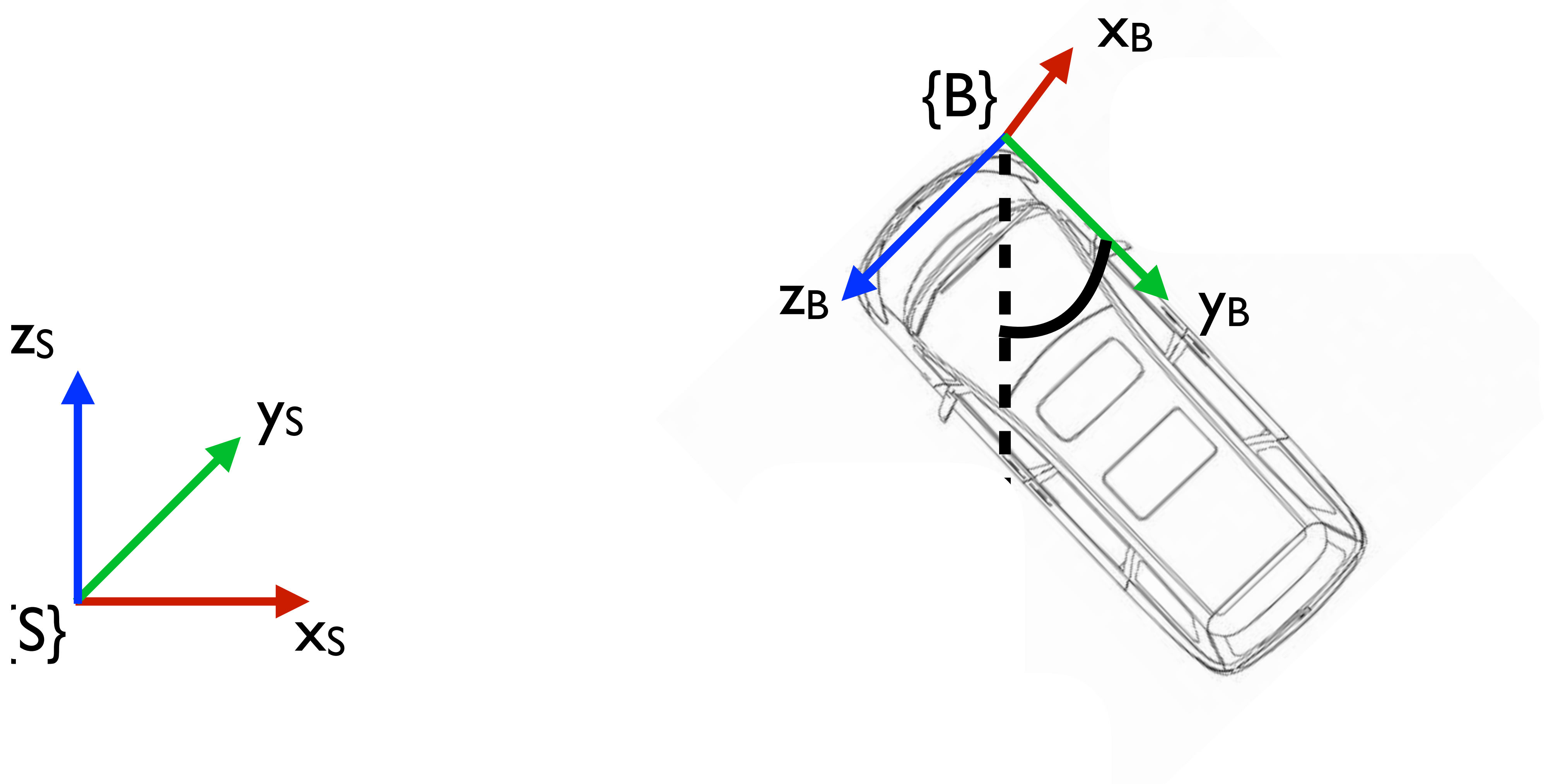
1) describe the position and orientation of an object in a given frame with (p, R)

2) do a coordinate transform:
   if frame B has position p_{SB} and orientation R_{SB} with respect to frame S, then a point with coordinates q_B in frame B has coordinates $q_S = R_{SB} q_B + p_{SB}$ in frame S

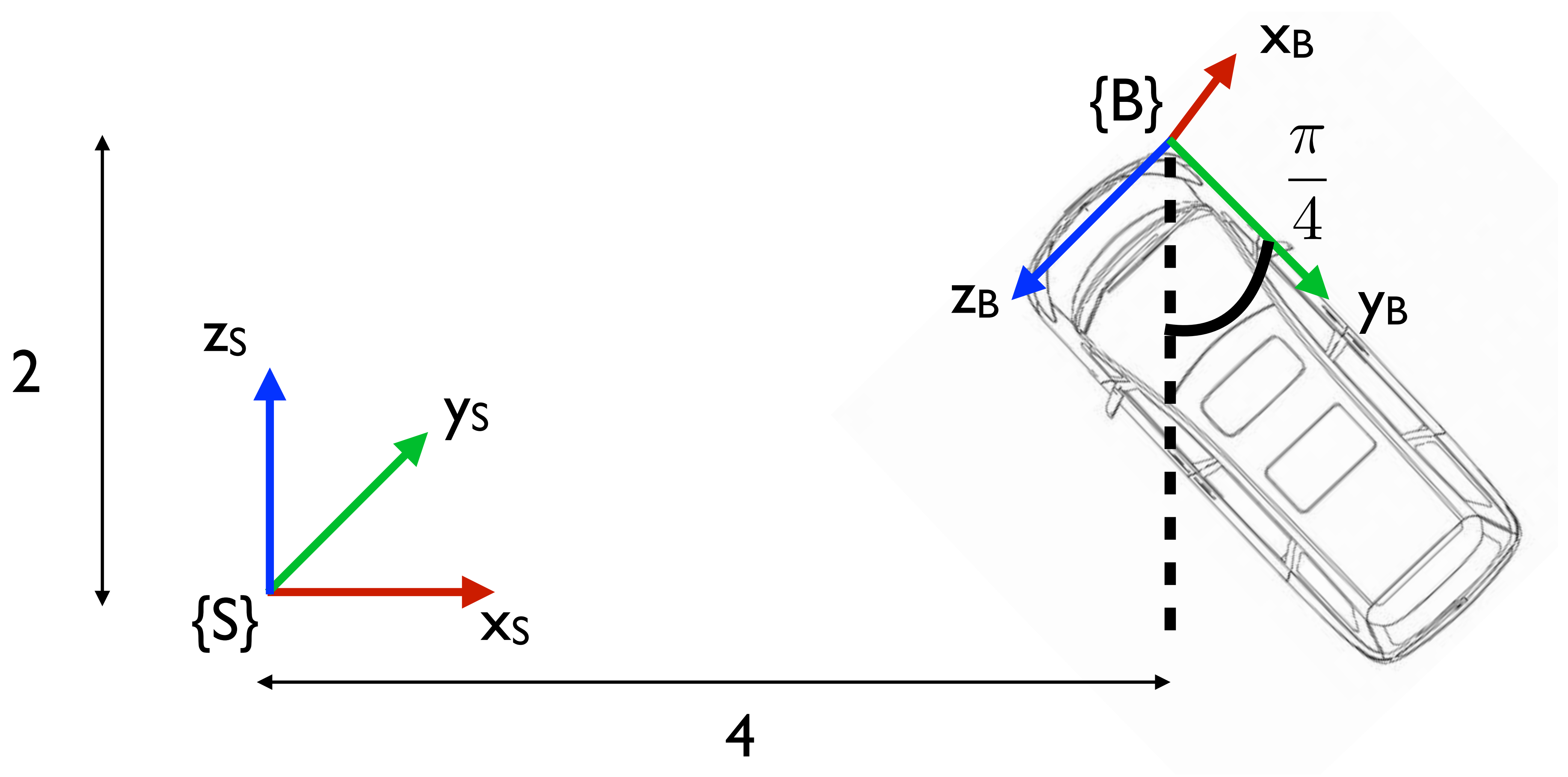
3) move an object in space:
   if an object has position and orientation (p_{SB}, R_{SB}), then we can move it by p_{BE} and rotate it by R_{BE} with respect to its own frame. The resulting position and orientation of the object in frame S are $p_{SB} + R_{SB} p_{BE}$ and $R_{SB} R_{BE}$

1) Describe Position & Orientation of an Object (aka it's "Pose")

x_B points inside the screen



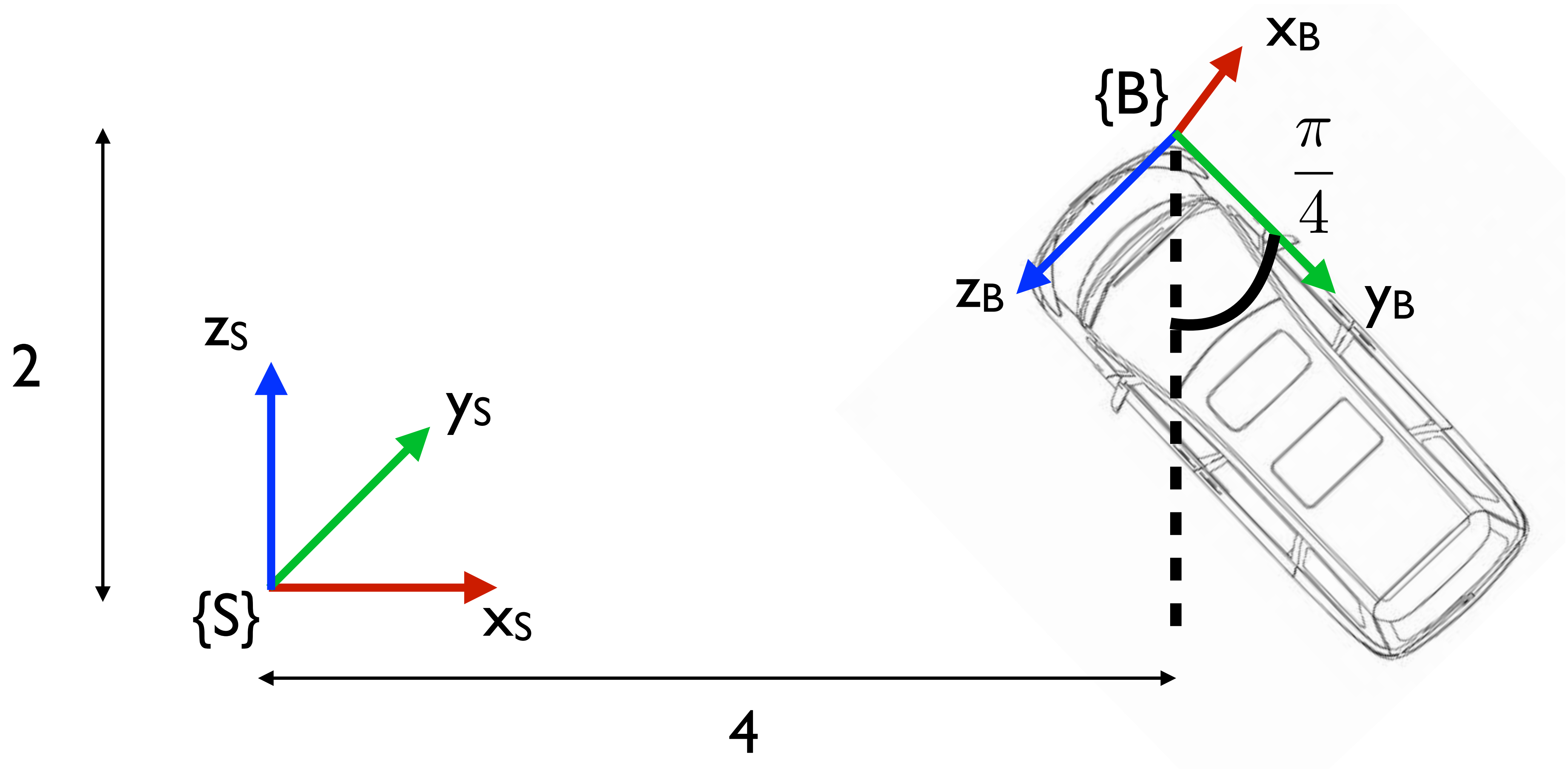
1) Describe Position & Orientation of an Object (aka it's "Pose")



$$R_{SB} = R_z(\pi/2)R_x(-3\pi/4)$$

$$p_{SB} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

1) Describe Position & Orientation of an Object (aka it's "Pose")

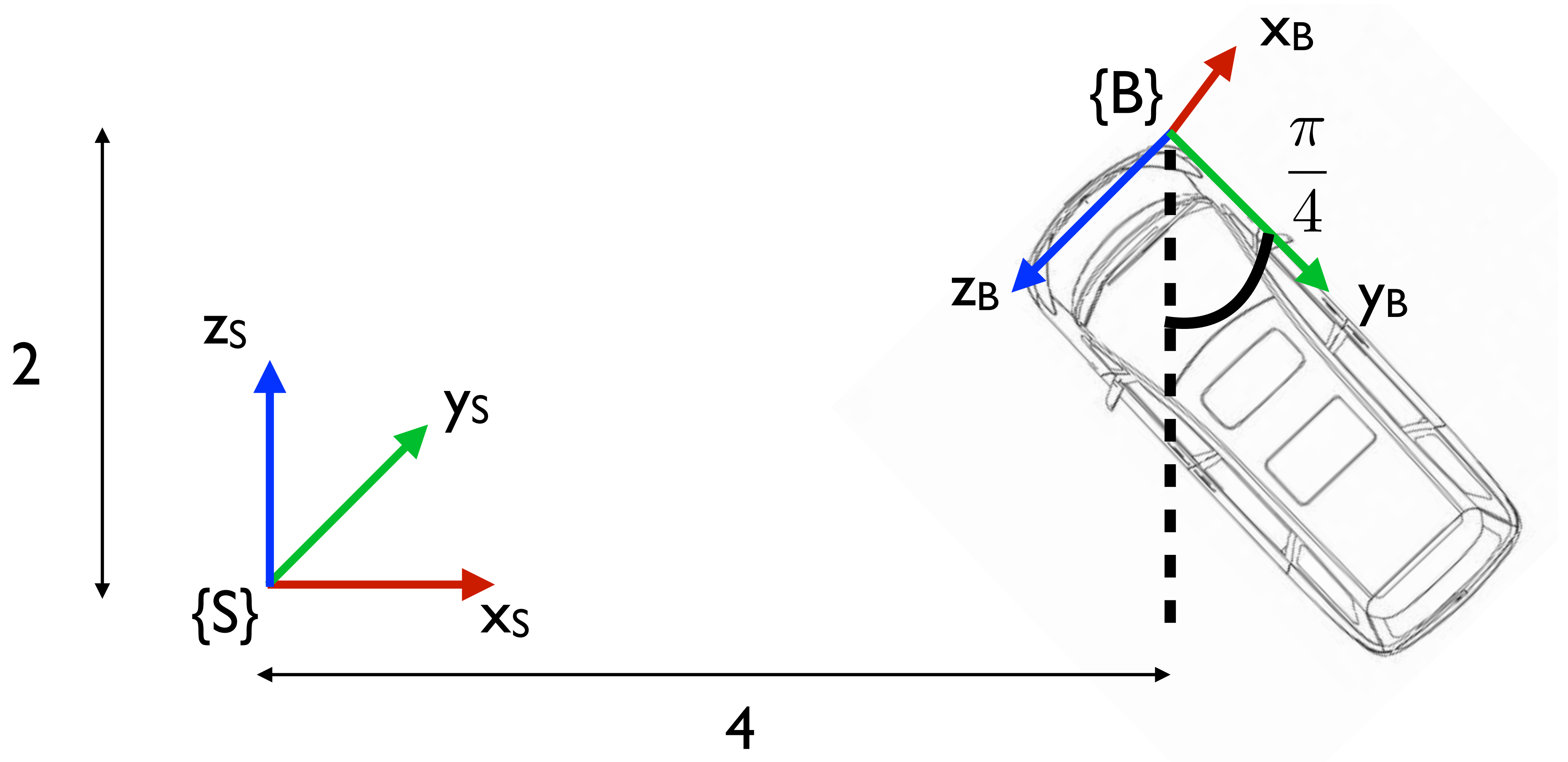


$$R_{SB} = R_z(\pi/2)R_x(-3\pi/4)$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$p_{SB} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

1) Describe Position & Orientation of an Object (aka it's "Pose")



$$R_{SB} = R_z(\pi/2)R_x(-3\pi/4)$$

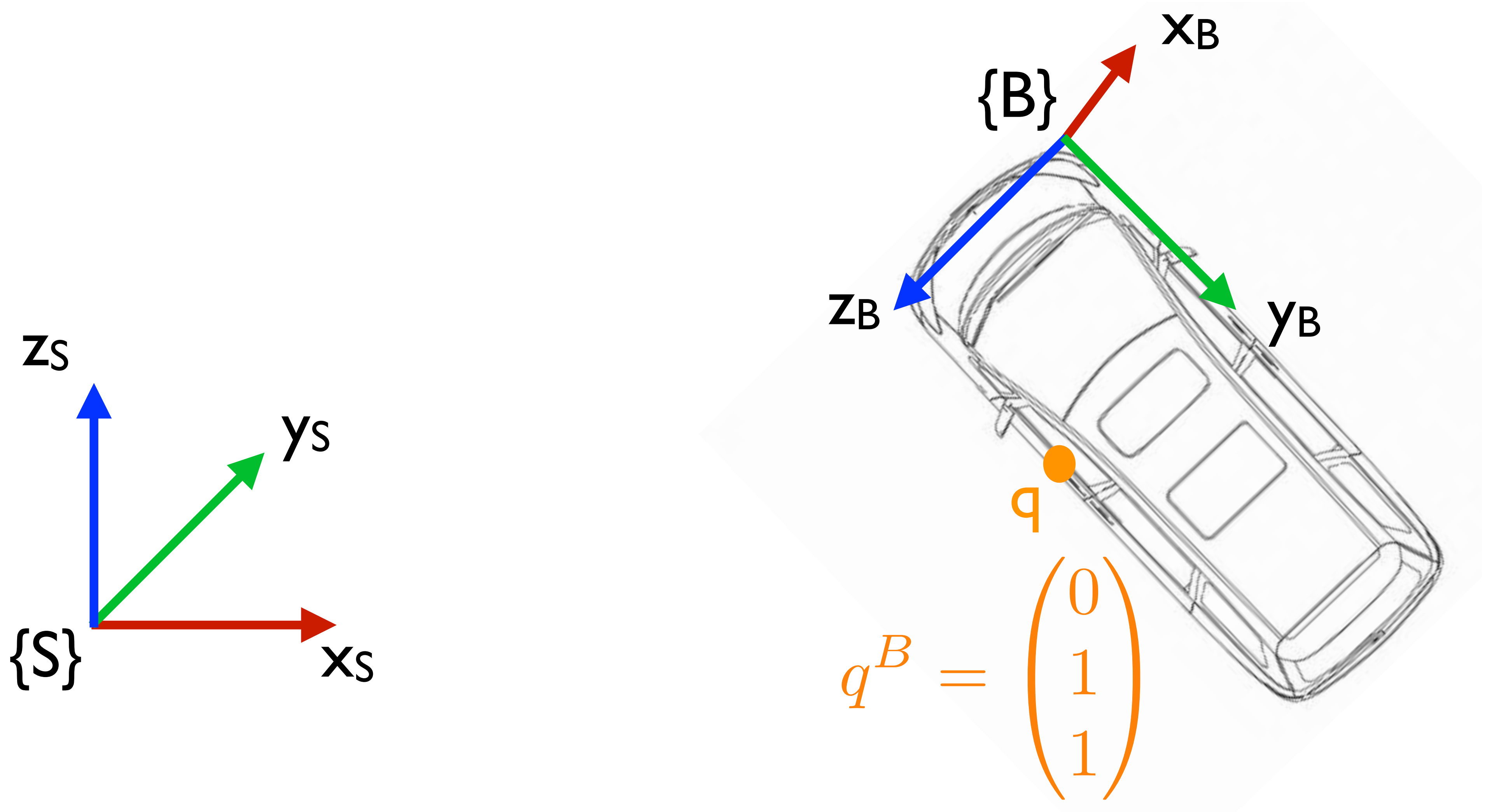
$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 1 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix}$$

$$p_{SB} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$T_{SB} = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

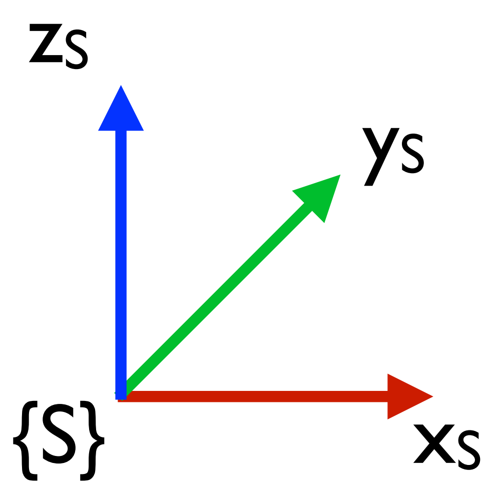
2) Coordinate Transform (e.g. find point q^S in S from point q^B)

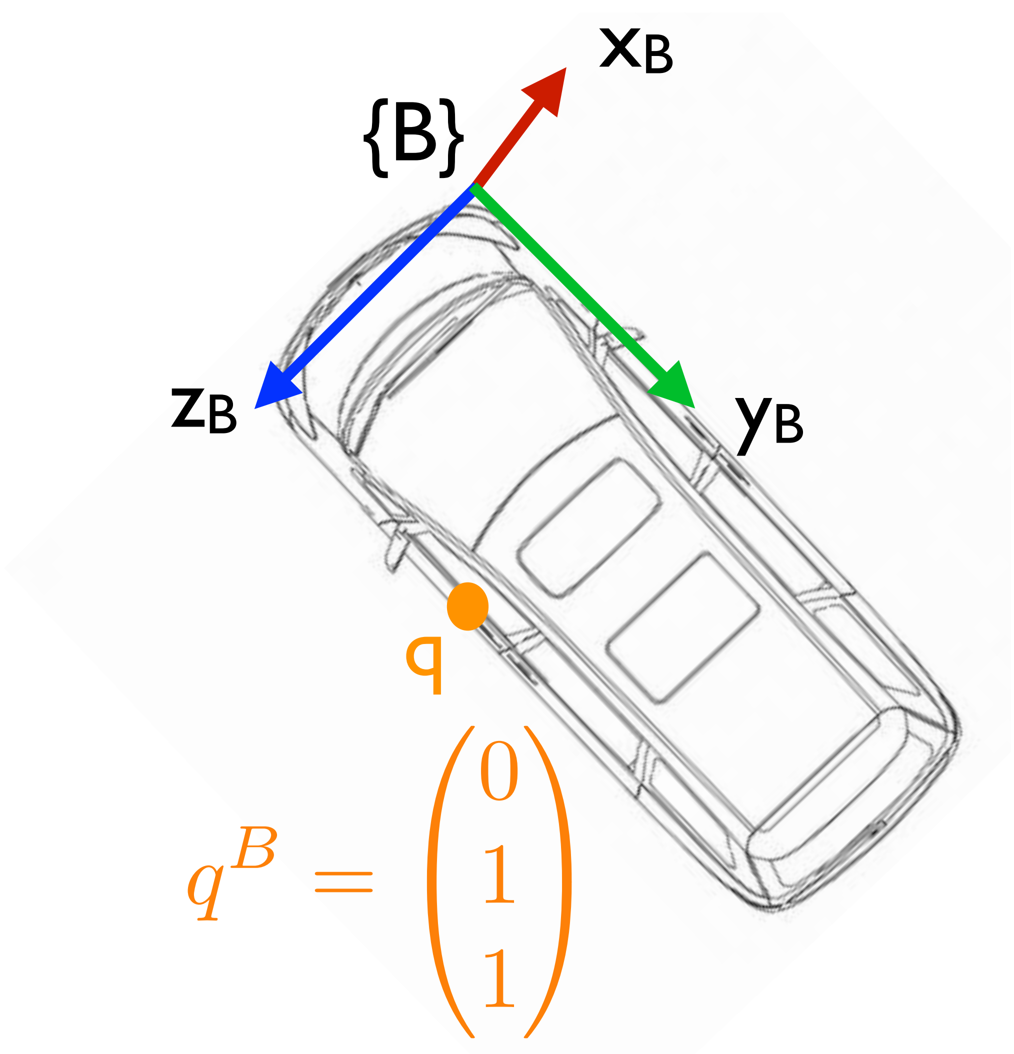
$$q^S =$$



2) Coordinate Transform (e.g. find point q^S in S from point q^B)

$$q^S = T_{SB} \cdot q^B$$

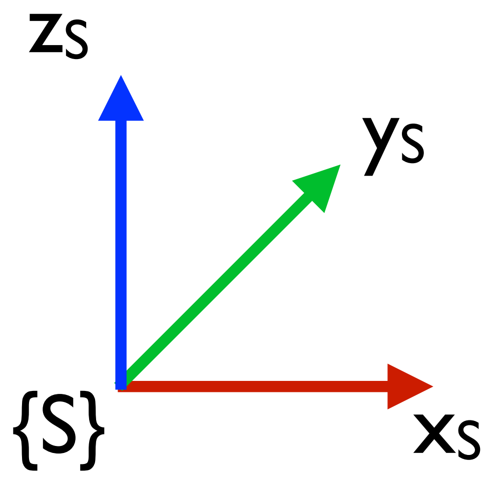


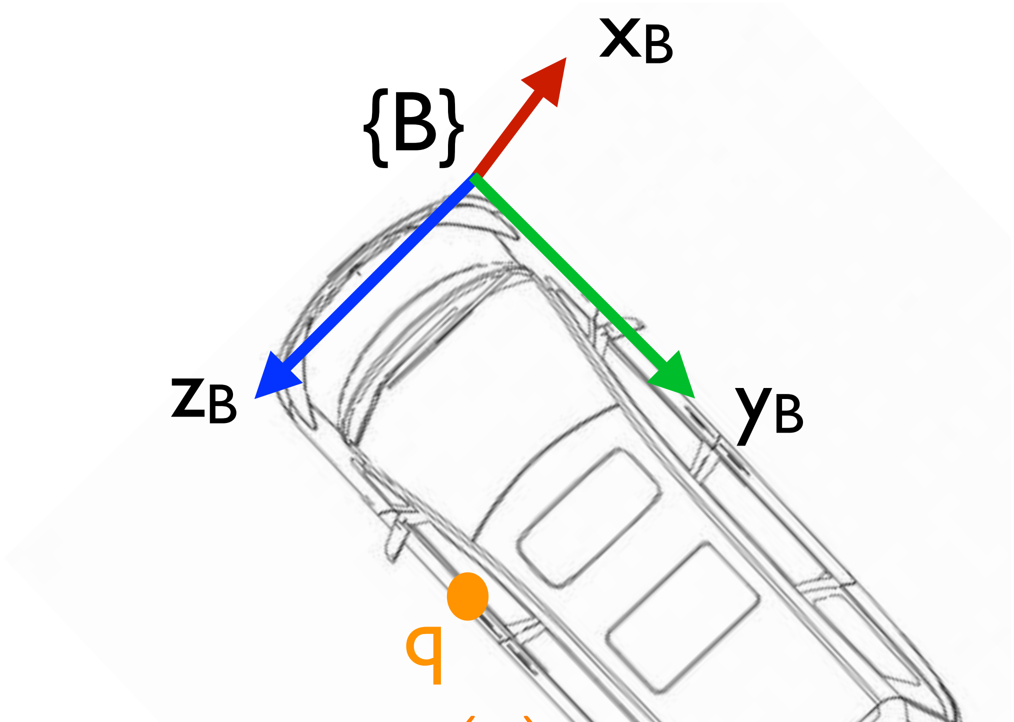


2) Coordinate Transform (e.g. find point q^S in S from point q^B)

$$q^S = T_{SB} \cdot q^B$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$





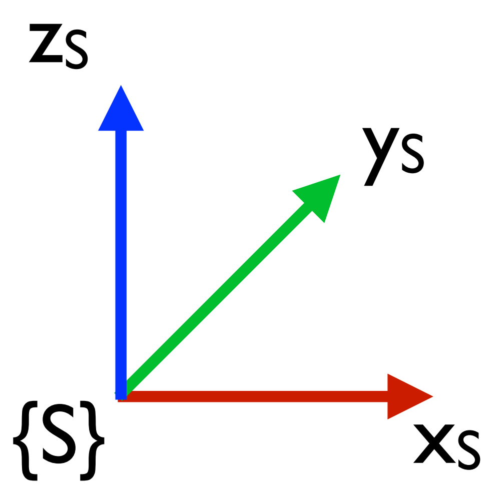
$$q^B = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

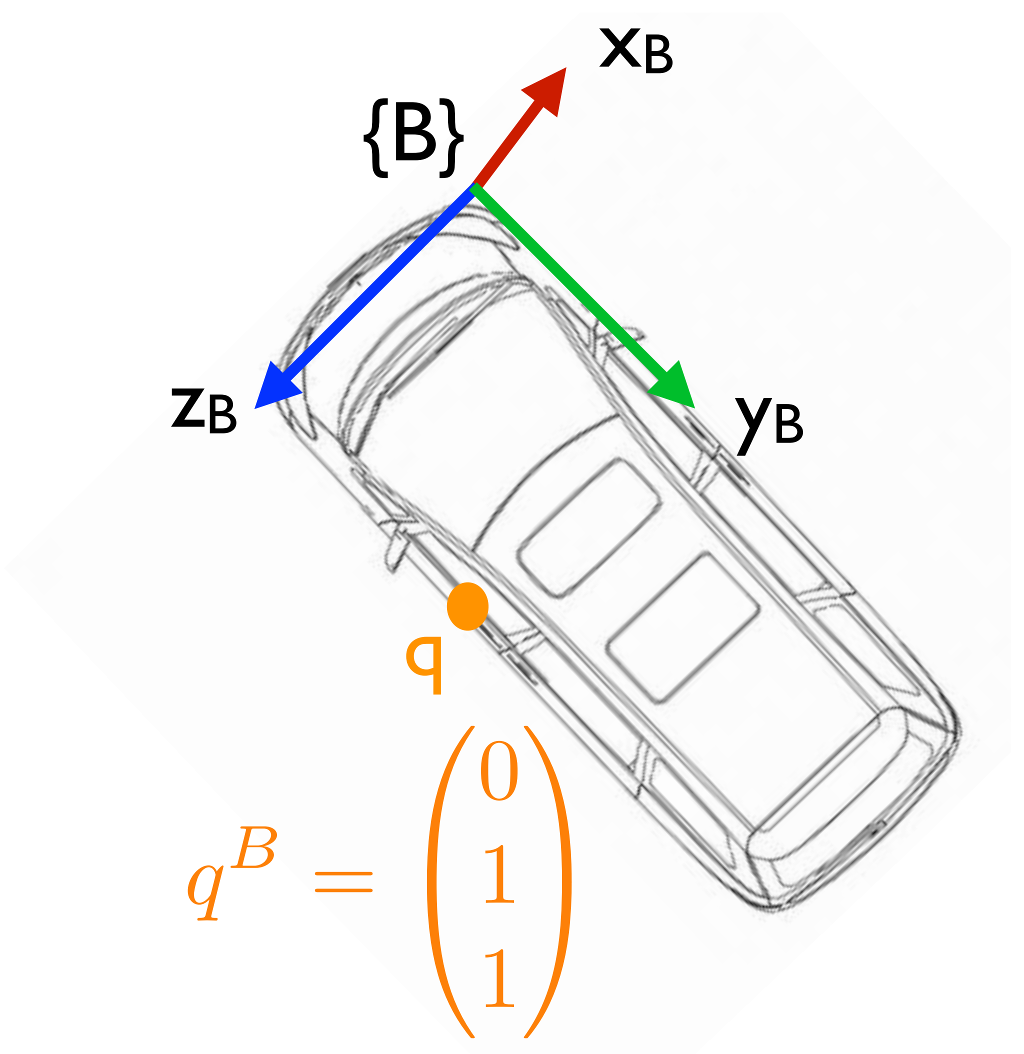
2) Coordinate Transform (e.g. find point q^S in S from point q^B)

$$q^S = T_{SB} \cdot q^B$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 4 \\ 0 \\ 2-\sqrt{2} \\ 1 \end{pmatrix}$$

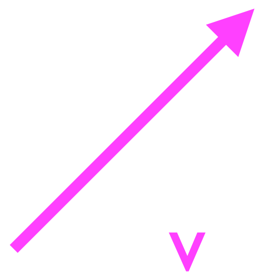


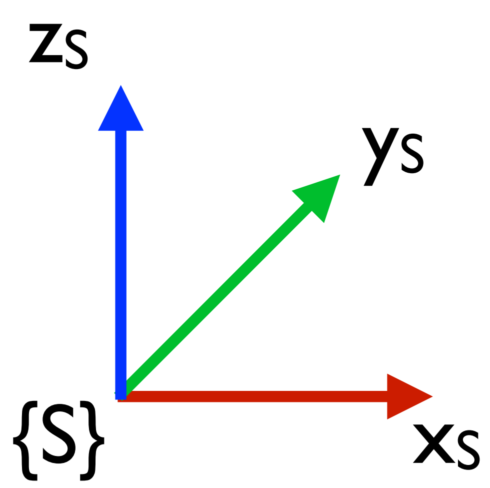


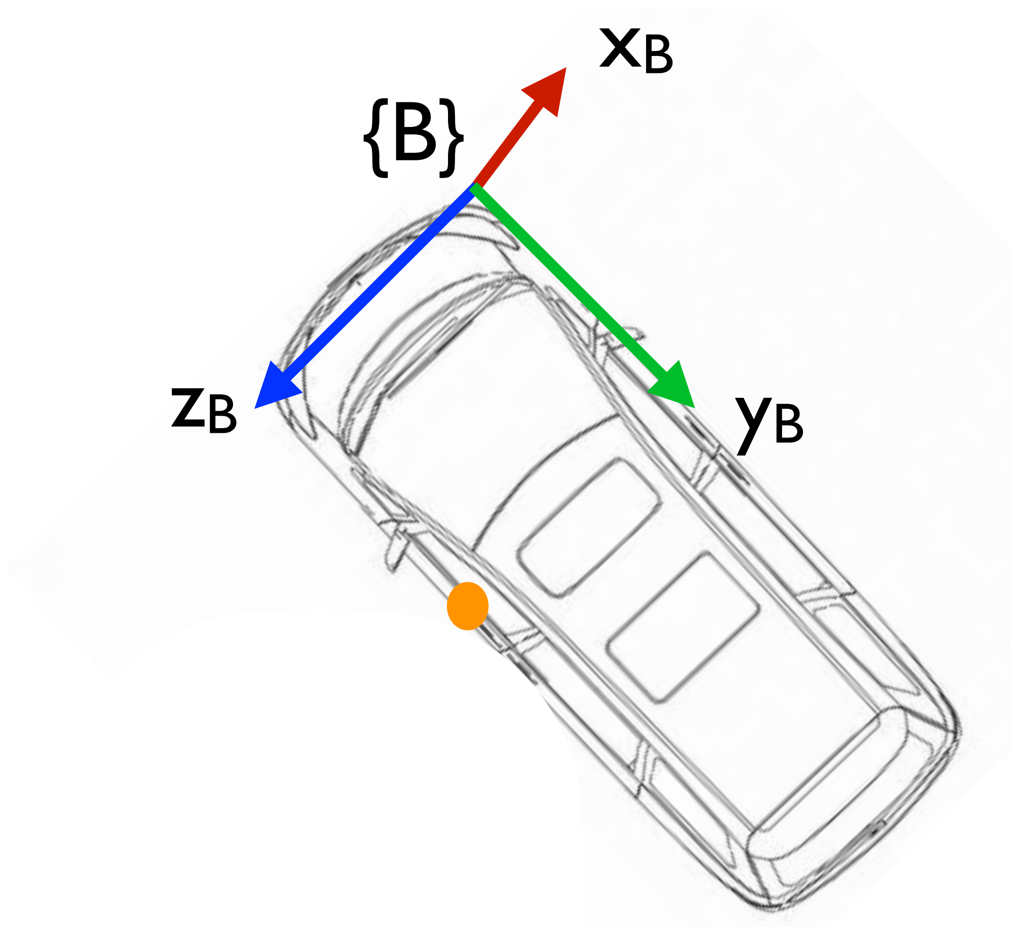
2) Coordinate Transform (e.g. find vector v^S in S from vector v^B)

$$v^S =$$

$$v^B = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$



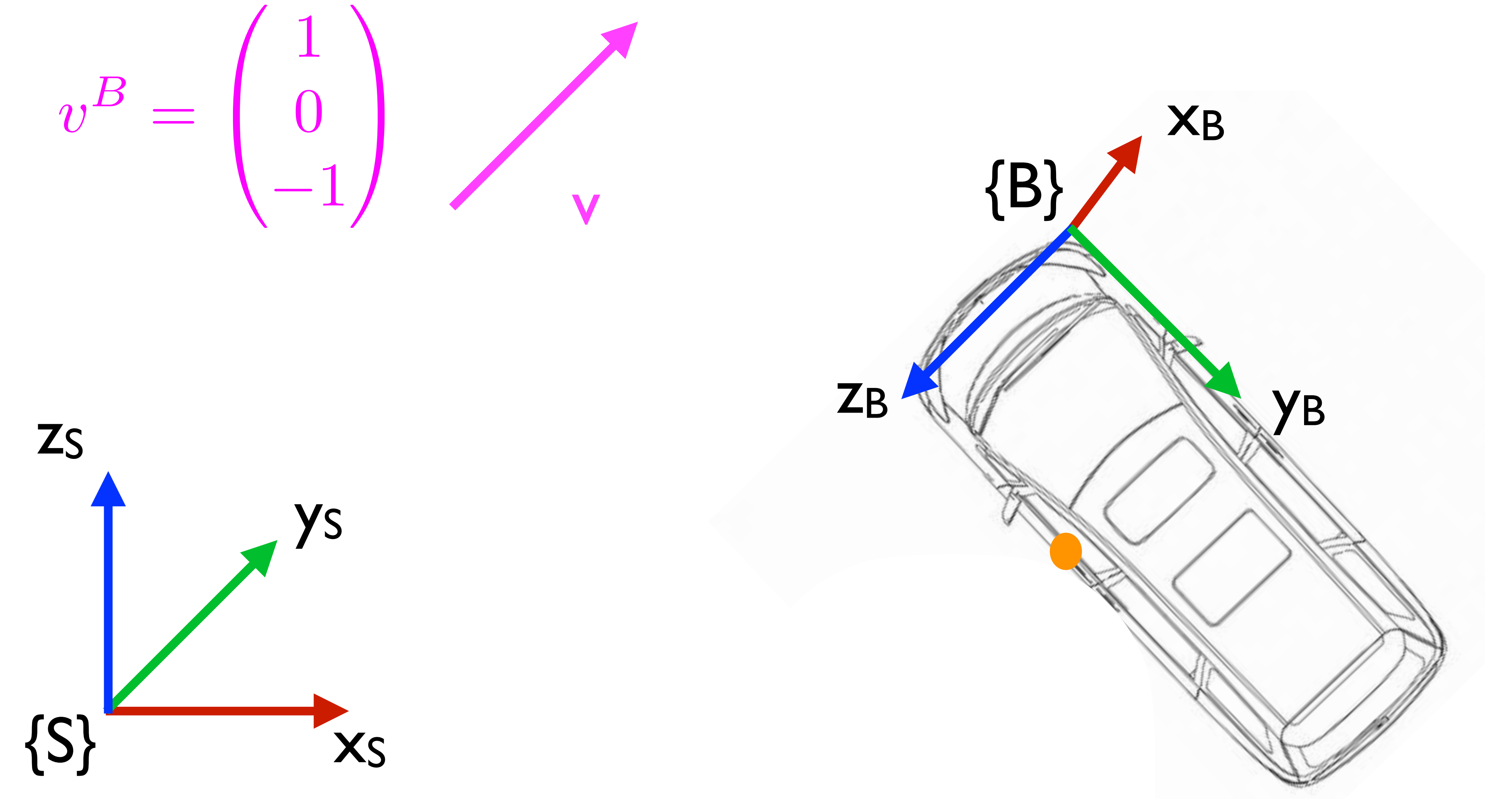




2) Coordinate Transform (e.g. find vector v^S in S from vector v^B)

$$v^S = T_{SB} \cdot v^B$$

$$v^B = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

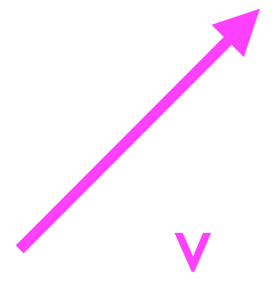


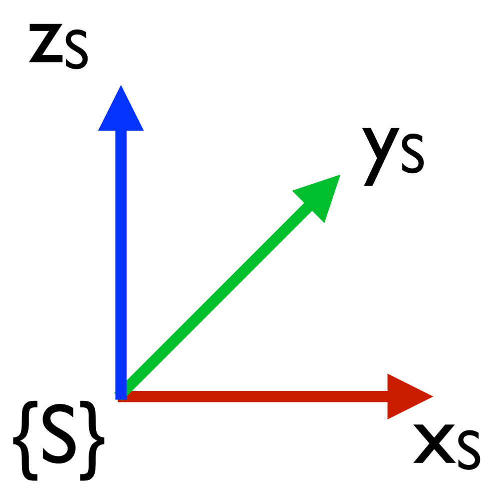
2) Coordinate Transform (e.g. find vector v^S in S from vector v^B)

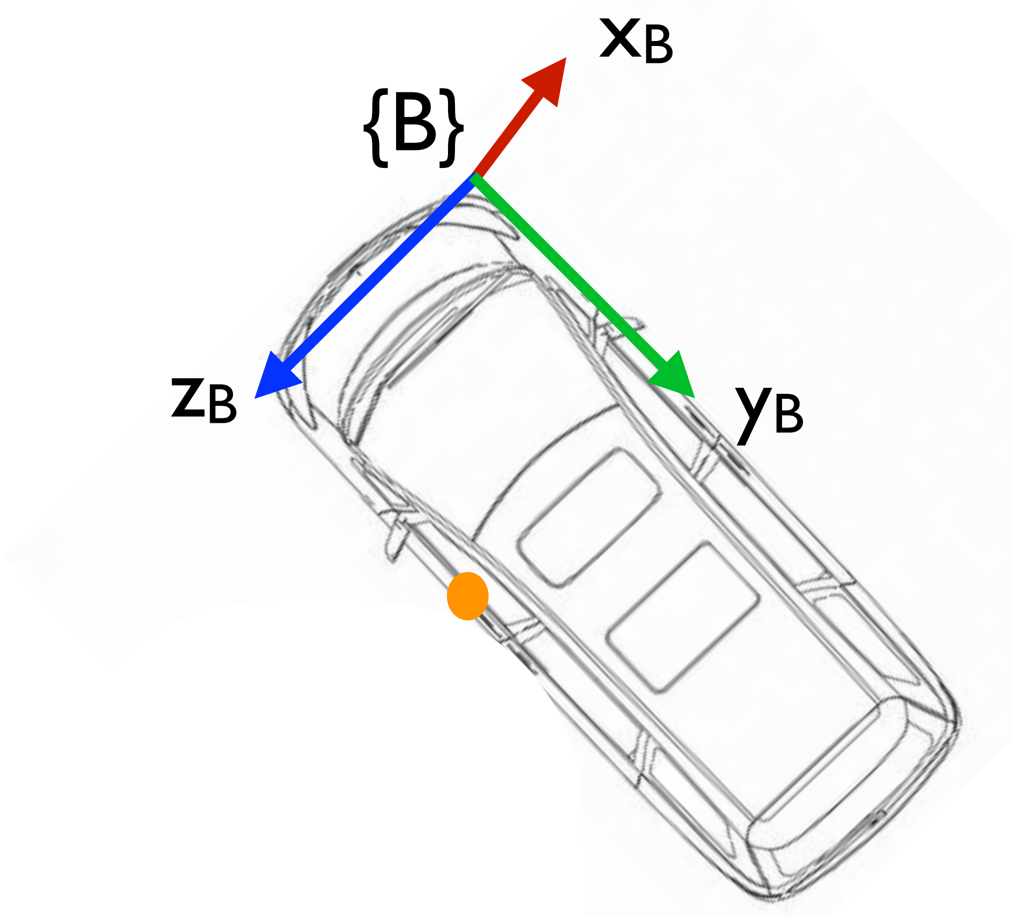
$$v^S = T_{SB} \cdot v^B$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$v^B = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$



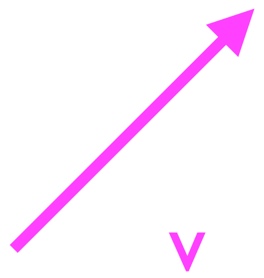


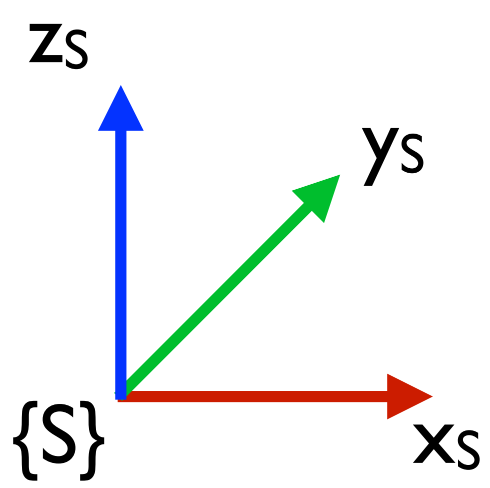


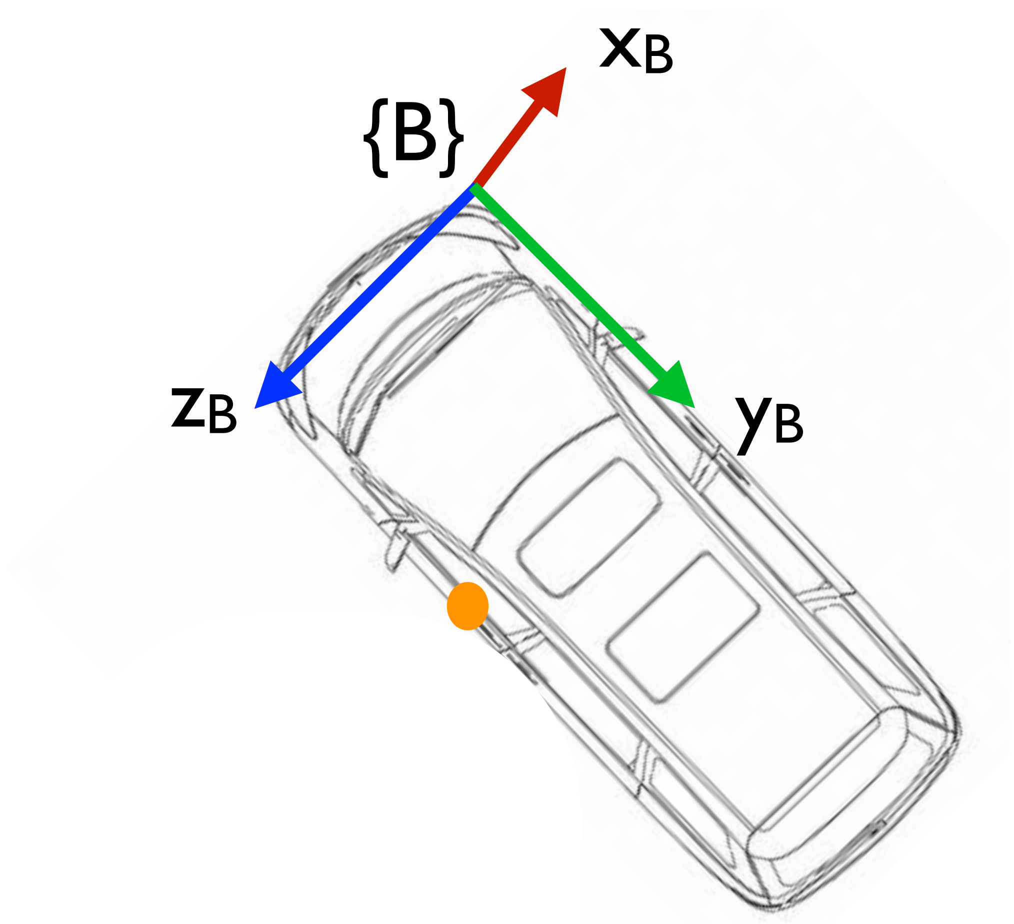
2) Coordinate Transform (e.g. find vector v^S in S from vector v^B)

$$
\begin{aligned}
v^S &= T_{SB} \cdot v^B \\
&= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \\
&= \begin{pmatrix} \frac{\sqrt{2}}{2} \\ 1 \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}
\end{aligned}
$$

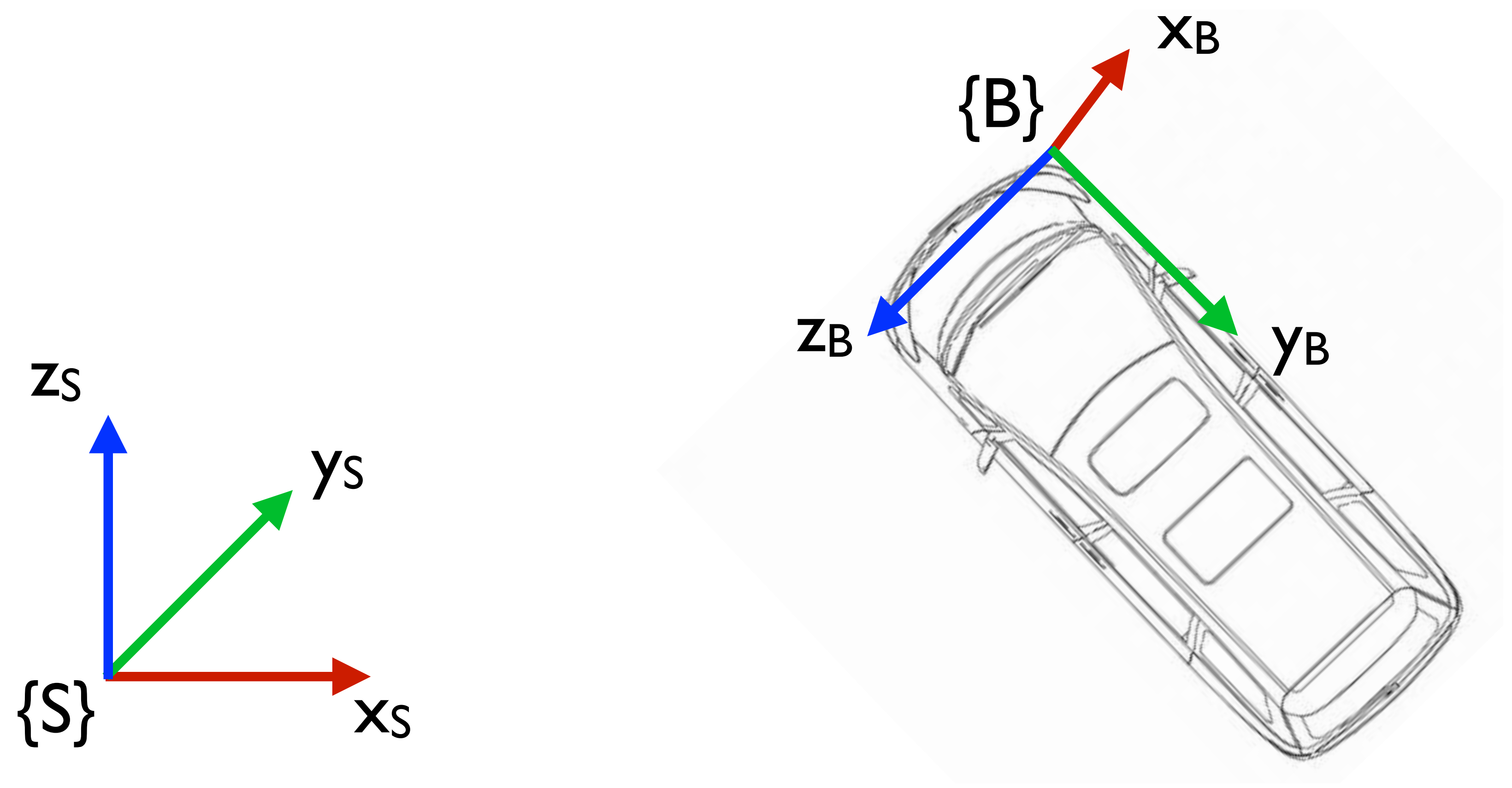
$$v^B = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$



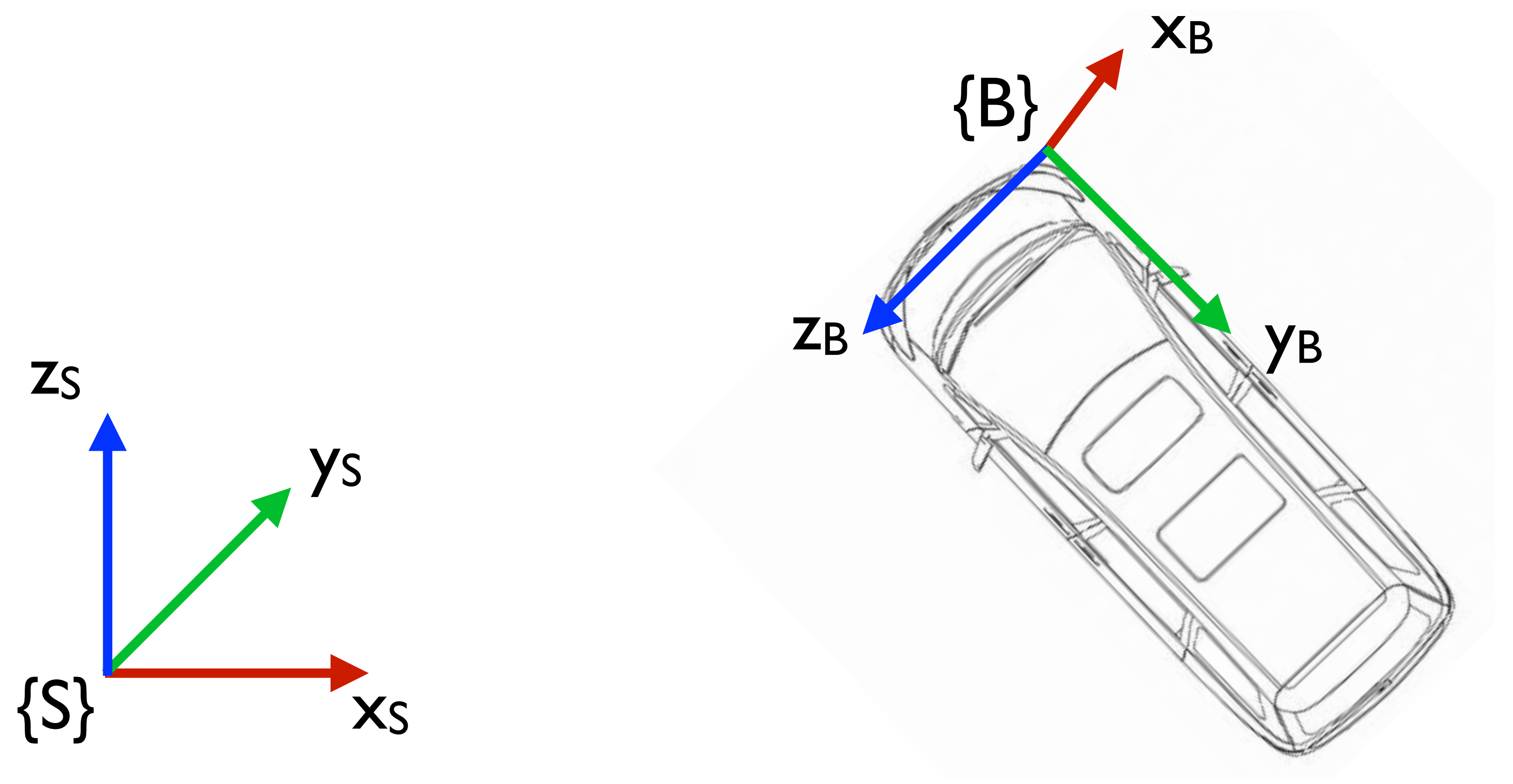




3) Move an Object in Space



3) Move an Object in Space



$$\underbrace{T_{rotz}(\pi) = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Transformation}} \qquad \underbrace{T_{SB} = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{\text{Pose of Object}}$$

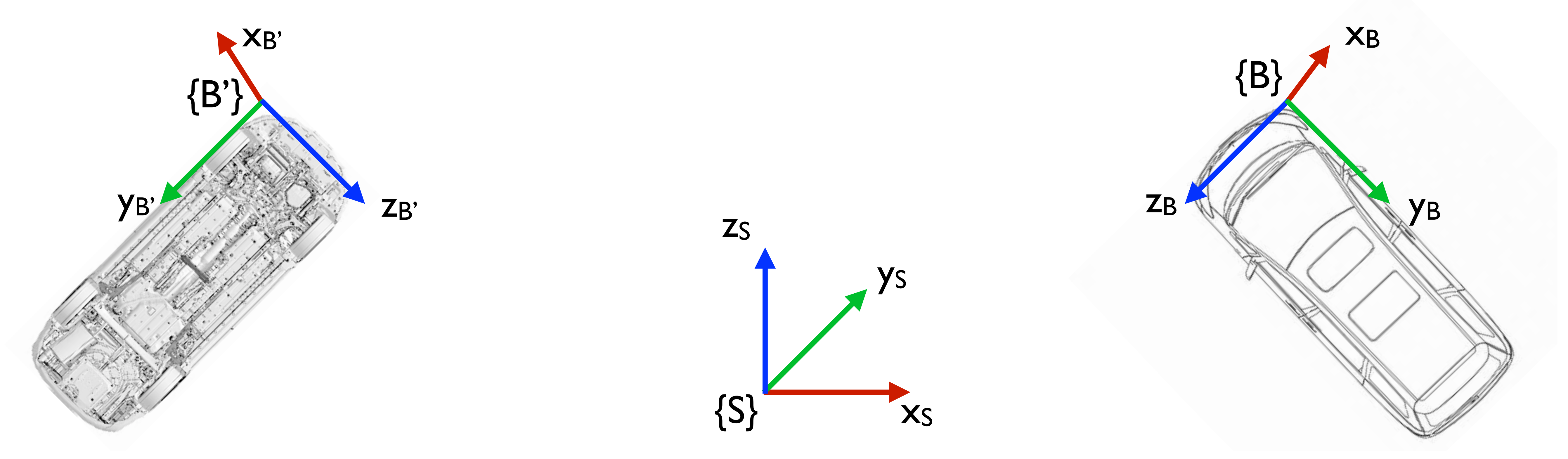
3) Move an Object in Space

Given T_{SB}, we can rotate the car around:

1) The z axis of the S frame (spatial transform)
   or
2) The z axis of the B frame (body transform)

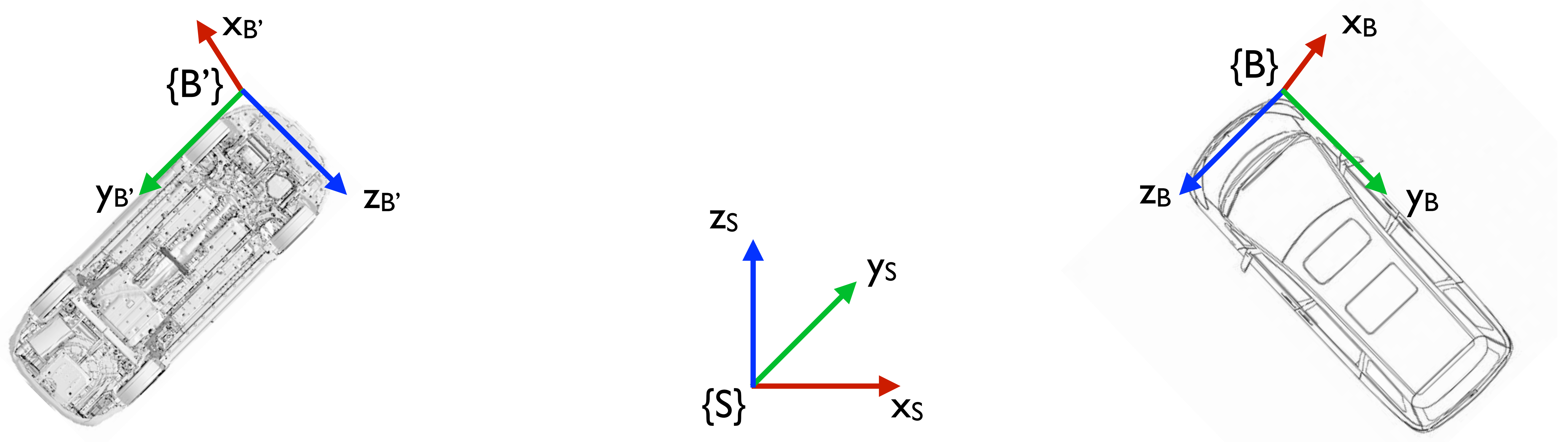
3) Move an Object in Space, i.e. Spatial Transform wrt S

Rotate along z_S by π



3) Move an Object in Space, i.e. Spatial Transform wrt S

Rotate along z_S by π

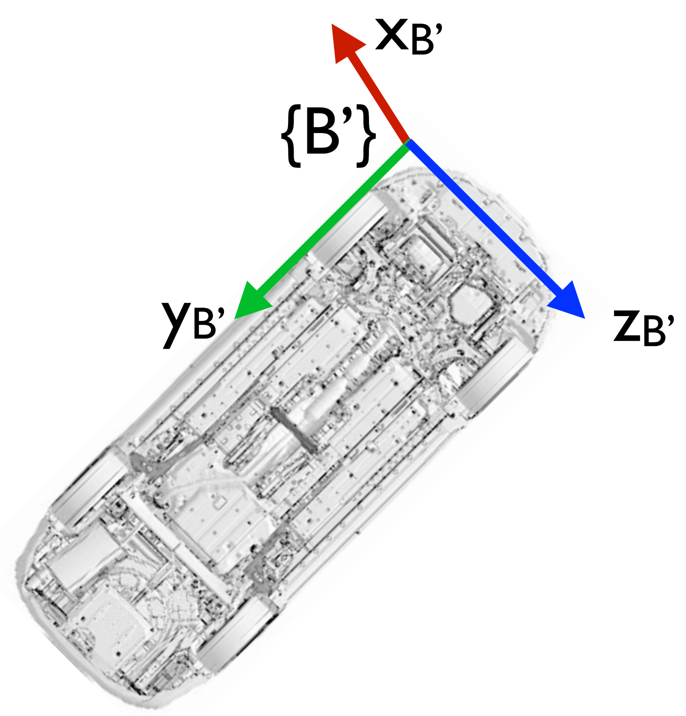


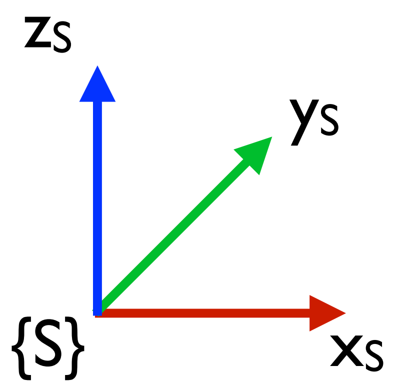
To move the object with respect to the S frame, we left multiply

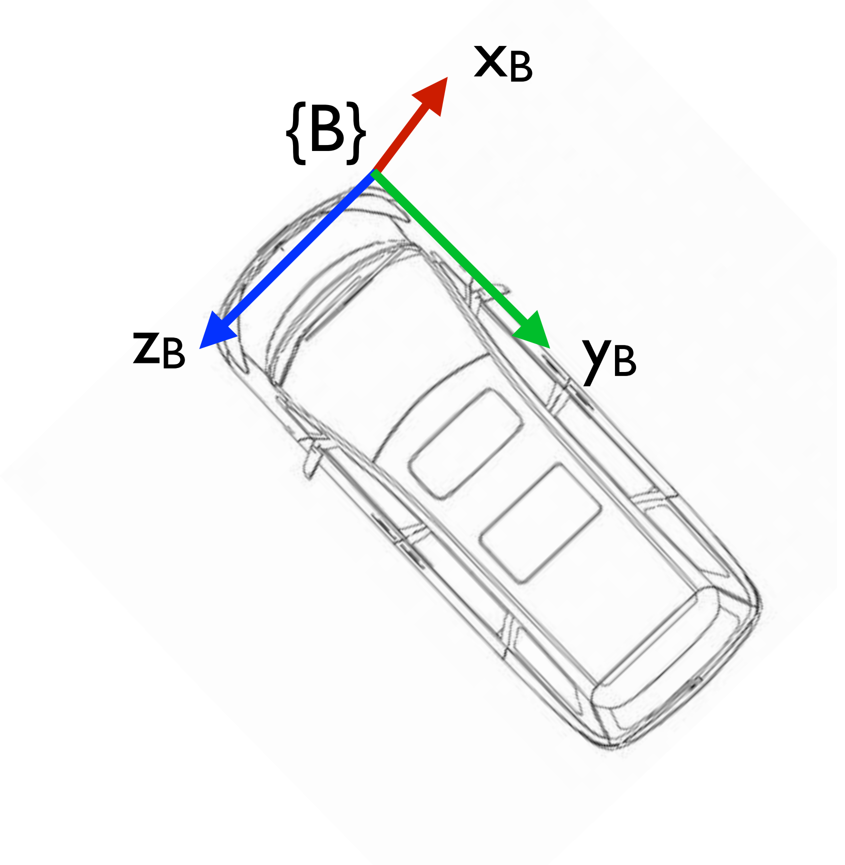
$$T_{SB'} = T_{rotz}(\pi) \cdot T_{SB}$$

3) Move an Object in Space, i.e. Spatial Transform wrt S

Rotate along z_S by π





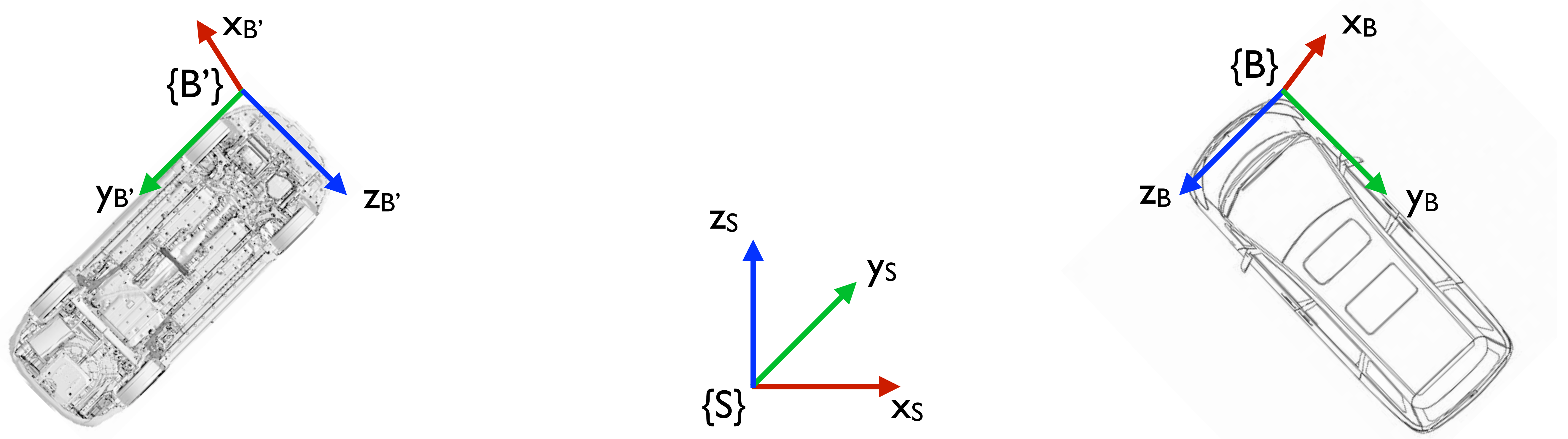


To move the object with respect to the S frame, we <u>left multiply</u>

$$T_{SB'} = T_{rotz}(\pi) \cdot T_{SB} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Move an Object in Space, i.e. Spatial Transform wrt S

Rotate along z_S by π

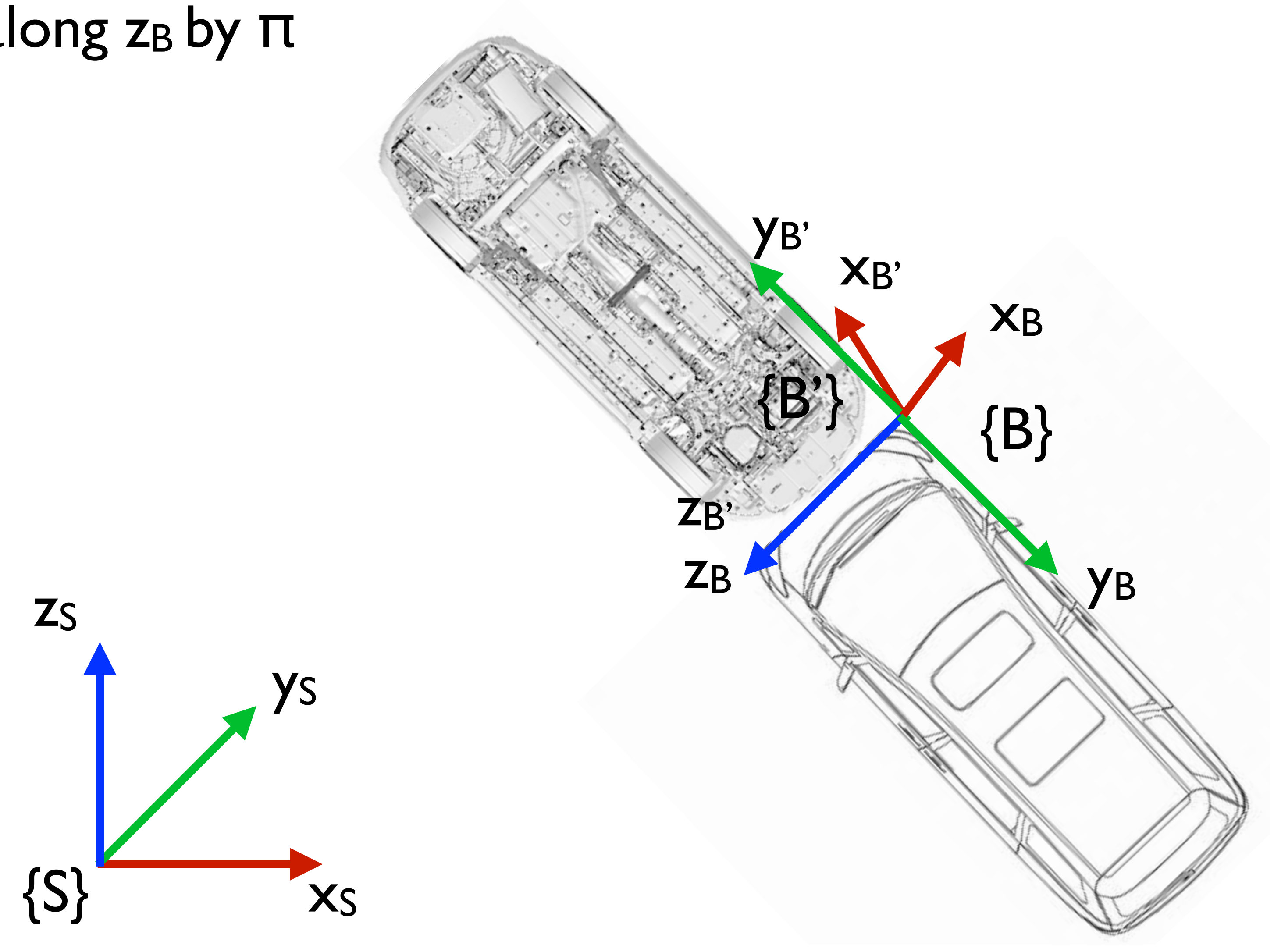


To move the object with respect to the S frame, we left multiply

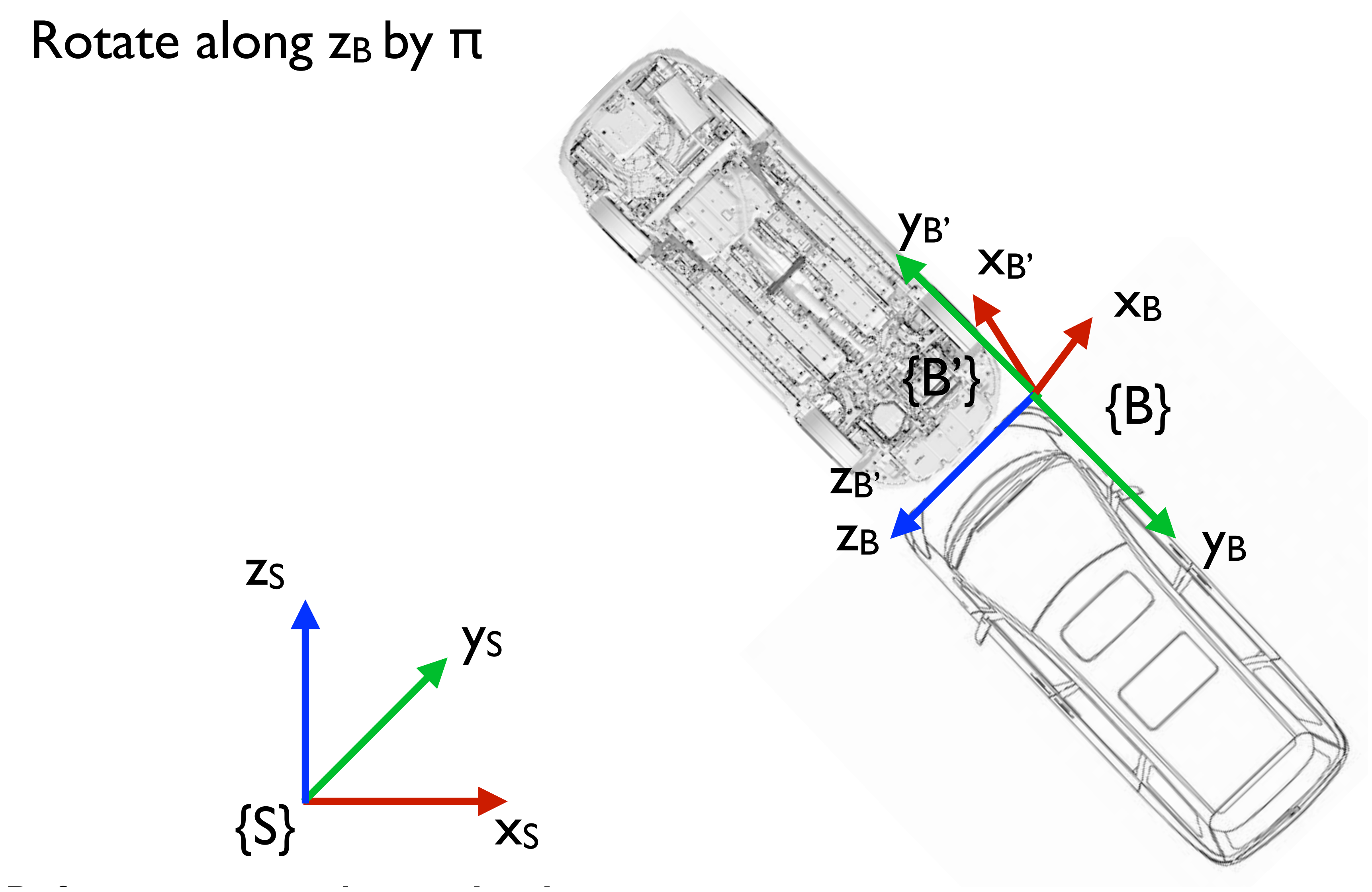
$$T_{SB'} = T_{rotz}(\pi) \cdot T_{SB} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & -4 \\ -1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Move an Object in Space, i.e. Body transform wrt B

Rotate along z_B by π



3) Move an Object in Space, i.e. Body transform wrt B

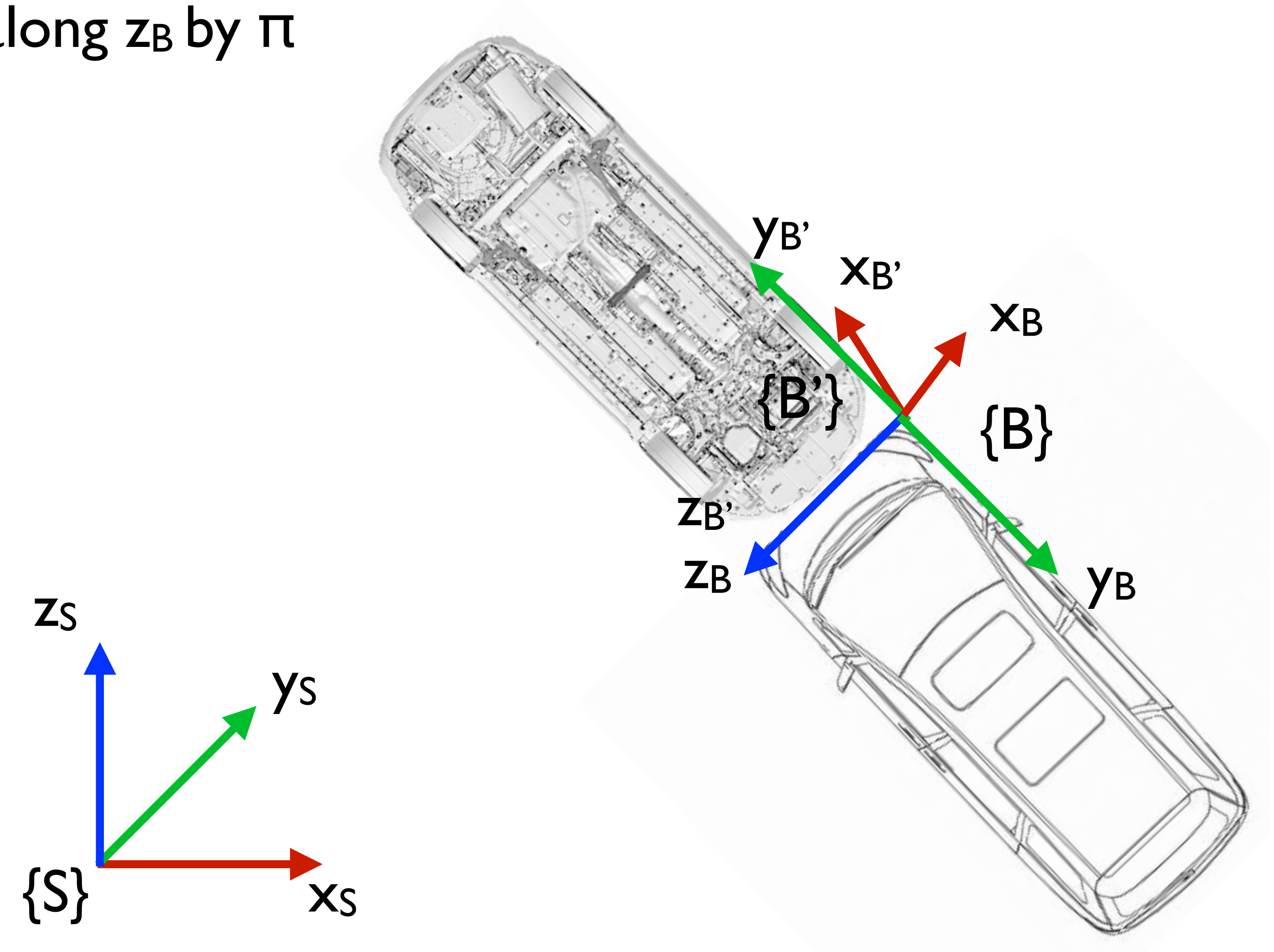


To move the object with respect to the B frame, we right multiply

$$T_{SB'} = T_{SB}\, T_{rotz}(\pi)$$

3) Move an Object in Space, i.e. Body transform wrt B

Rotate along z_B by π

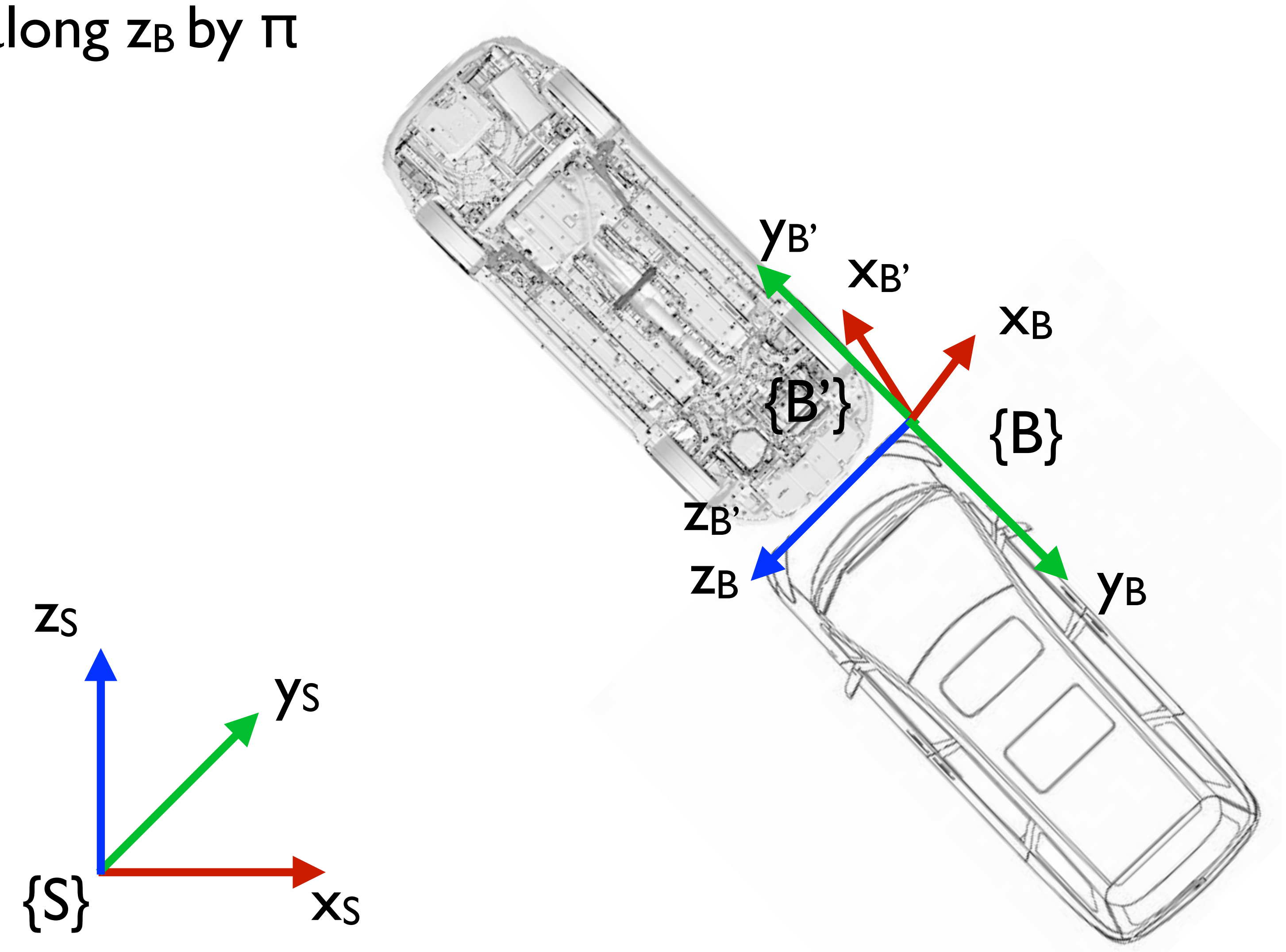


To move the object with respect to the B frame, we <u>right multiply</u>

$$T_{SB'} = T_{SB}\, T_{rotz}(\pi) = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Move an Object in Space, i.e. Body transform wrt B

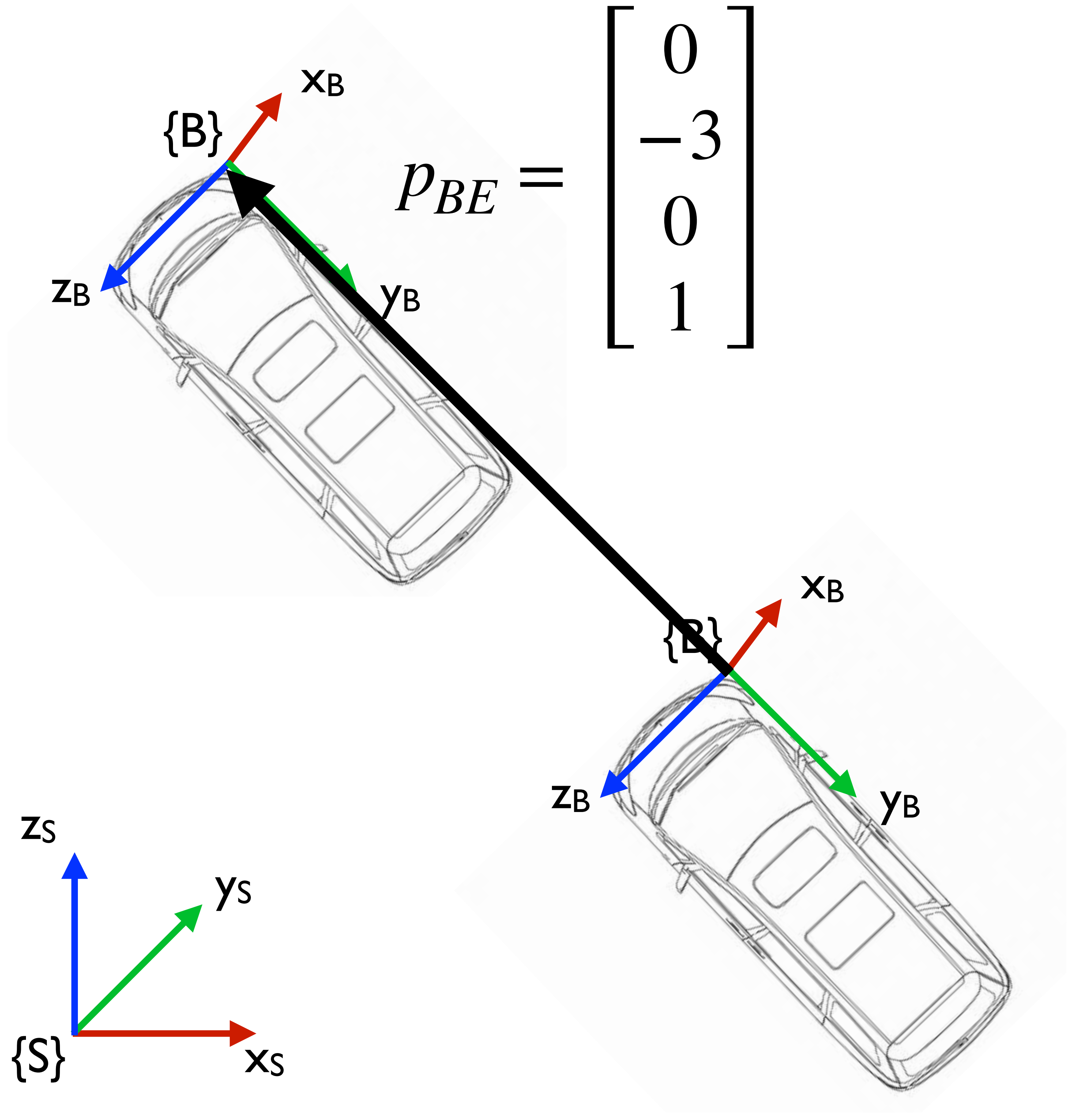
Rotate along z_B by π



To move the object with respect to the B frame, we right multiply

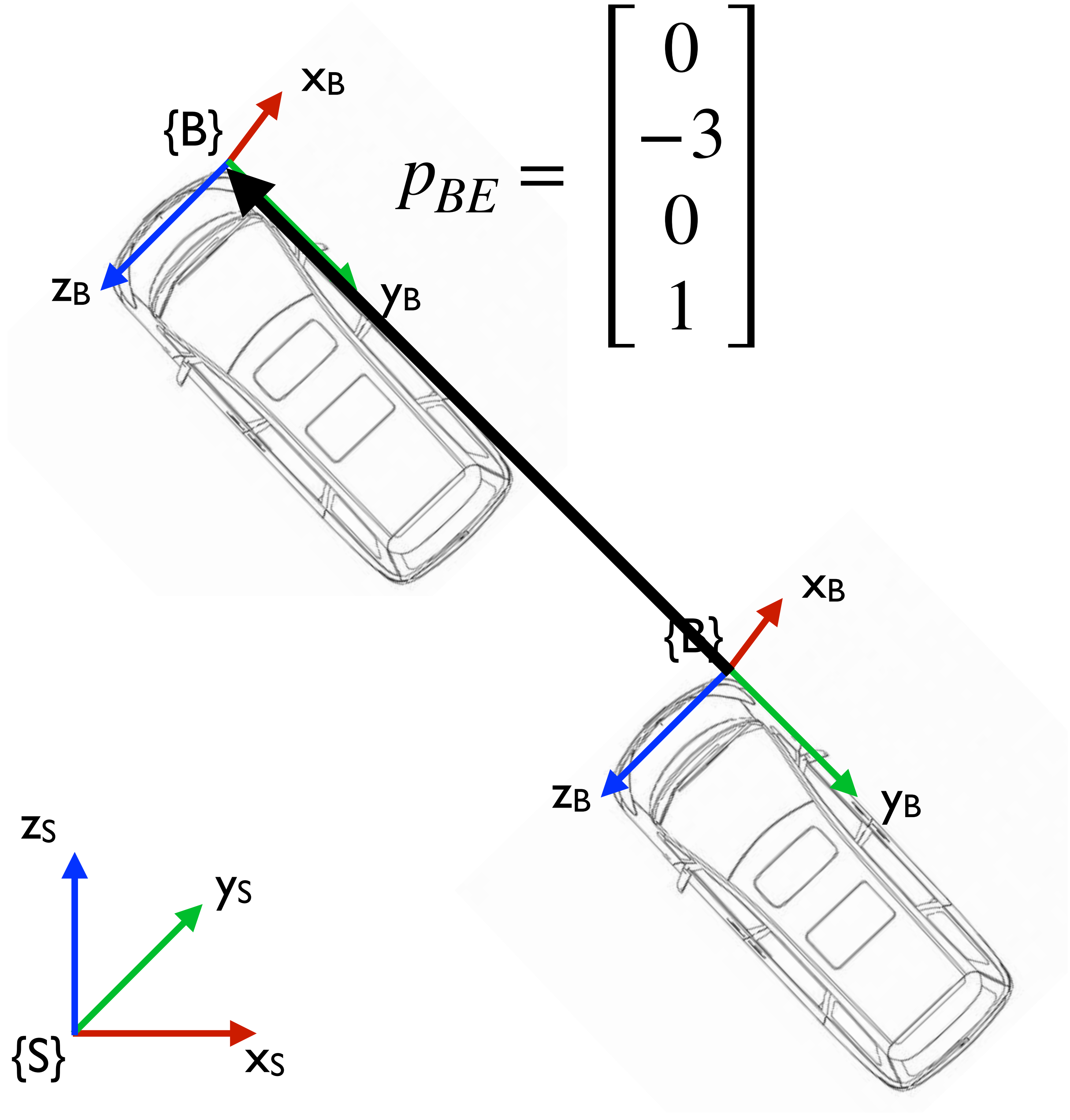
$$T_{SB'} = T_{SB}\, T_{rotz}(\pi) = \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ -1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Move an Object in Space, i.e. Body transform wrt B



$$p_{BE} = \begin{bmatrix} 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

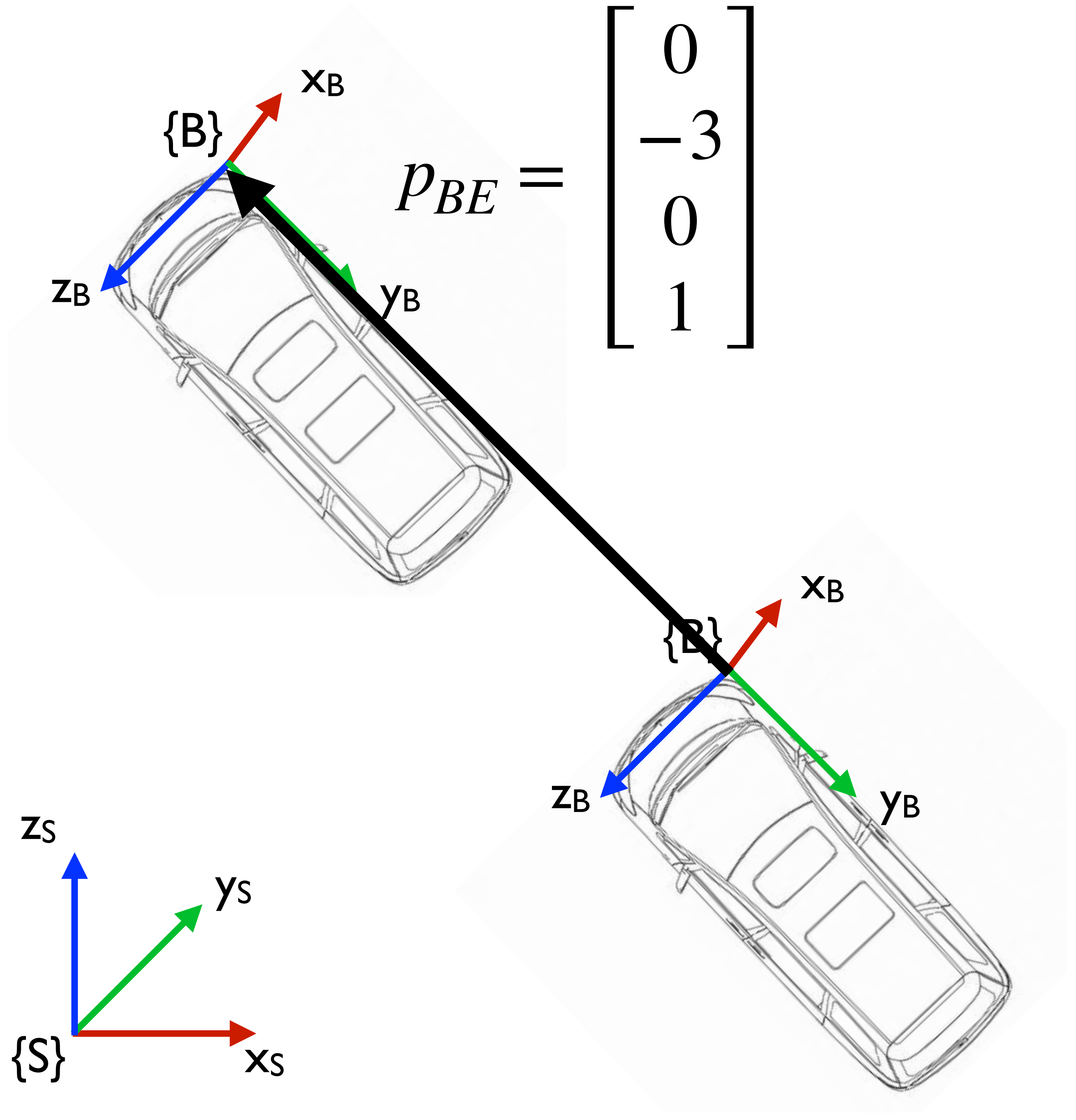
3) Move an Object in Space, i.e. Body transform wrt B



$$p_{BE} = \begin{bmatrix} 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$T_{SE} = T_{SB} T_{BE}$$

3) Move an Object in Space, i.e. Body transform wrt B

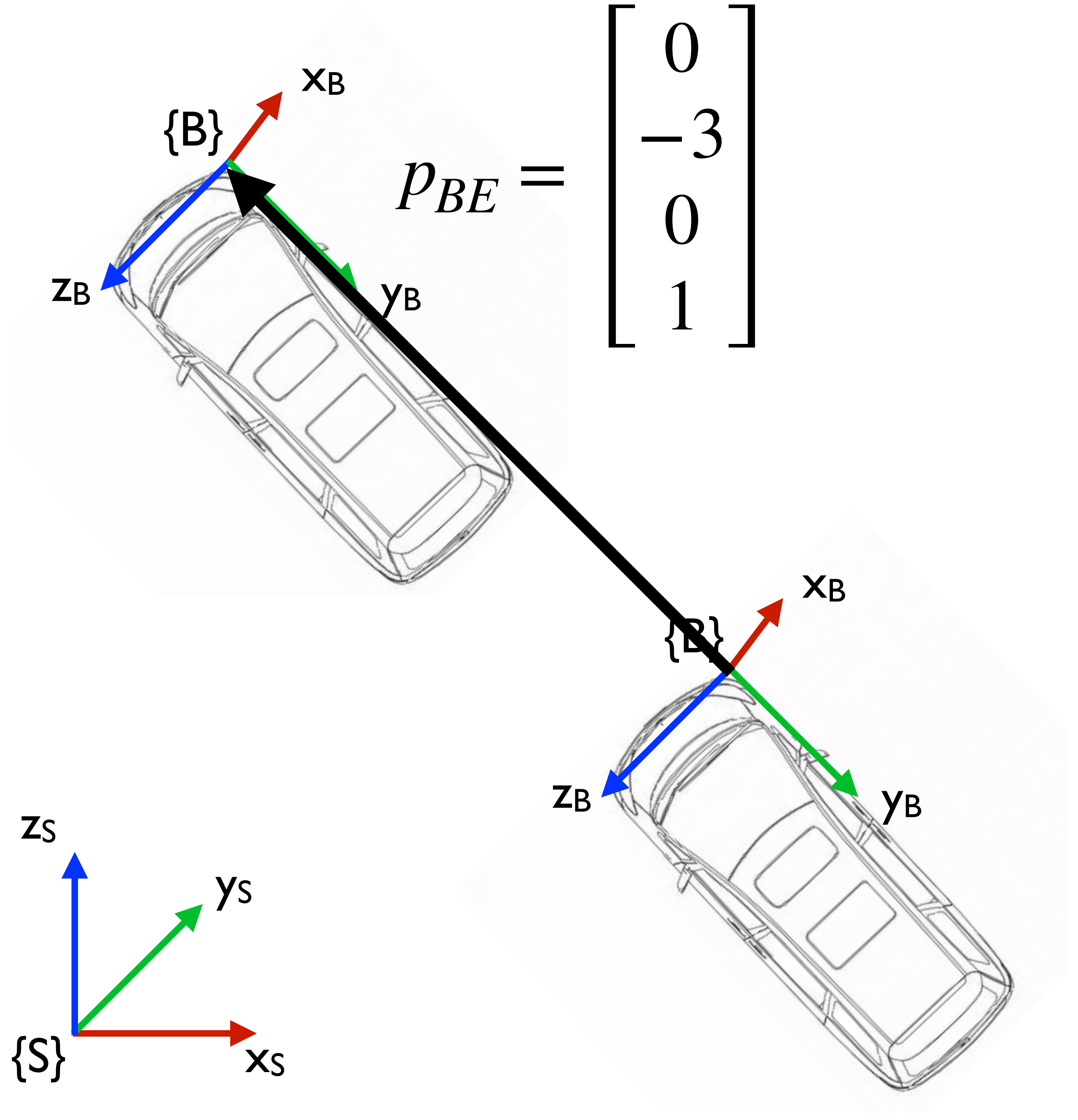


$$p_{BE} = \begin{bmatrix} 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$T_{SE} = T_{SB} T_{BE}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Move an Object in Space, i.e. Body transform wrt B



$$p_{BE} = \begin{bmatrix} 0 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

$$T_{SE} = T_{SB} T_{BE}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} + 4 \\ 1 & 0 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\frac{3\sqrt{2}}{2} + 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Agenda

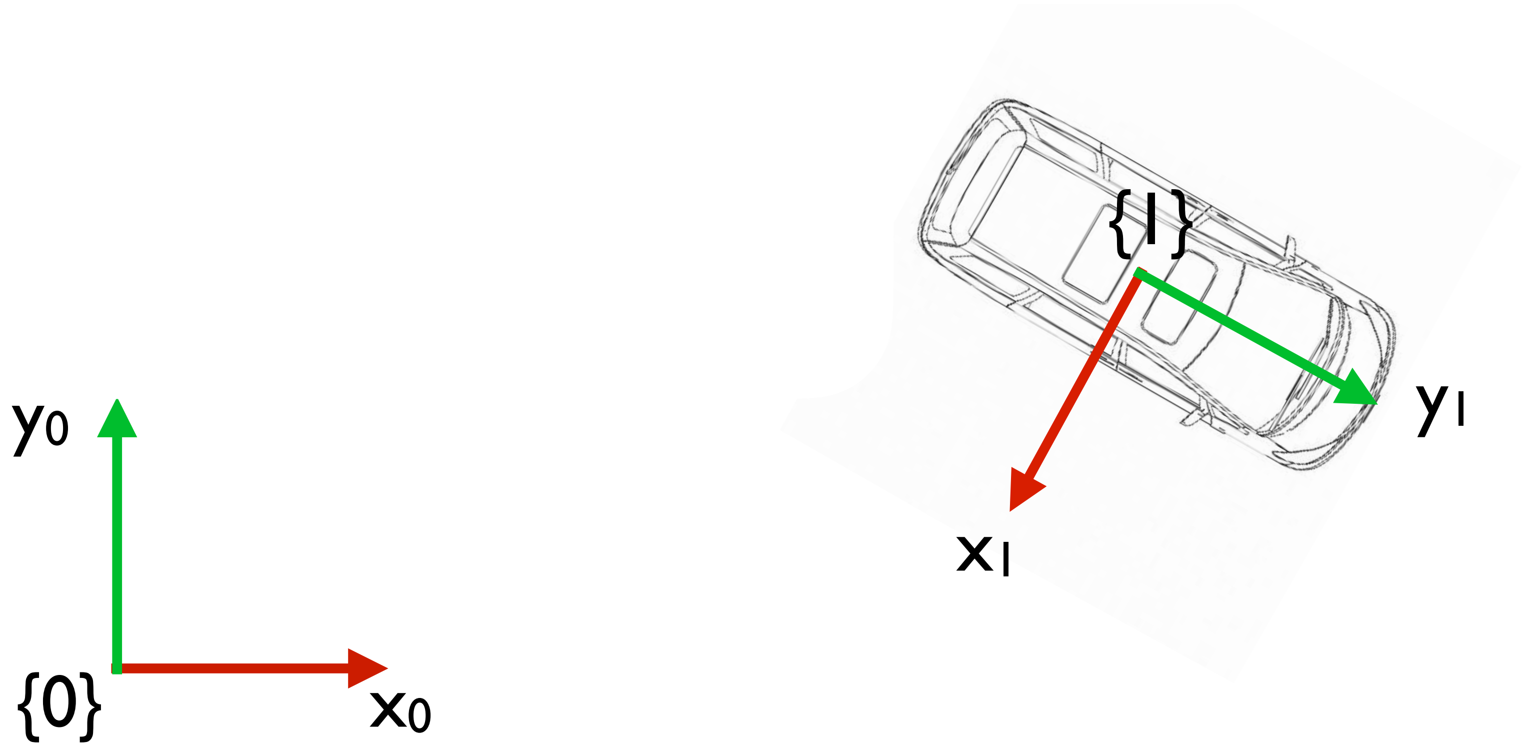
1. Homogenous Transformations
2. Three Uses of Transformations
3. **Properties of Transformations**
4. Forward Kinematics

Homogeneous Transforms

The set of all possible 3D rigid body transform, (i.e. the set of all possible 3D homogeneous transforms) is called the Special Euclidian Group of dimension 3

$$SE(3) = \left\{ T \in \mathbb{R}^{4\times 4} \;\middle|\; T = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix},\; R \in SO(3),\; p \in \mathbb{R}^3 \right\}$$

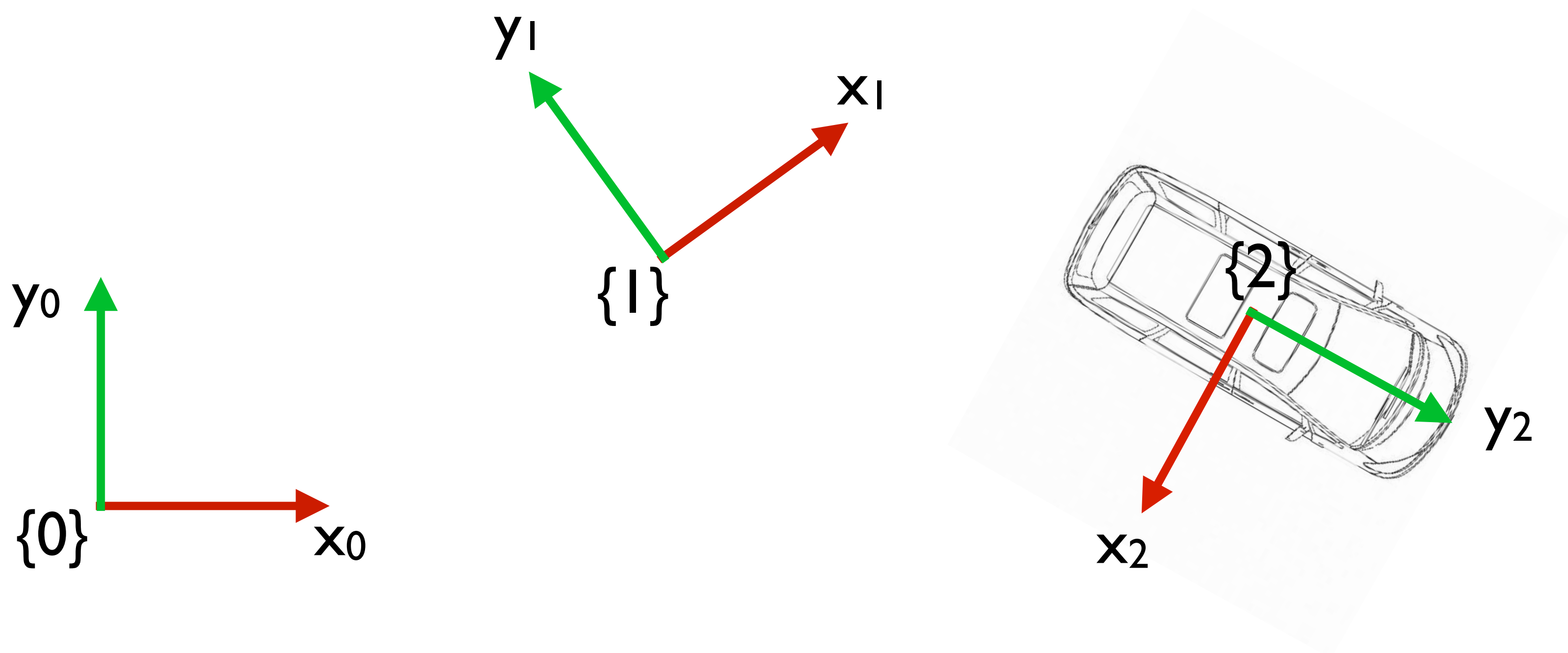
Inverse of Homogeneous Transforms



If frame 1 has pose $T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$ in frame 0, what is the pose of frame 0 in frame 1?

$$T_{10} = \begin{bmatrix} R_{01}^T & -R_{01}^T p_{01} \\ 0 & 1 \end{bmatrix}$$

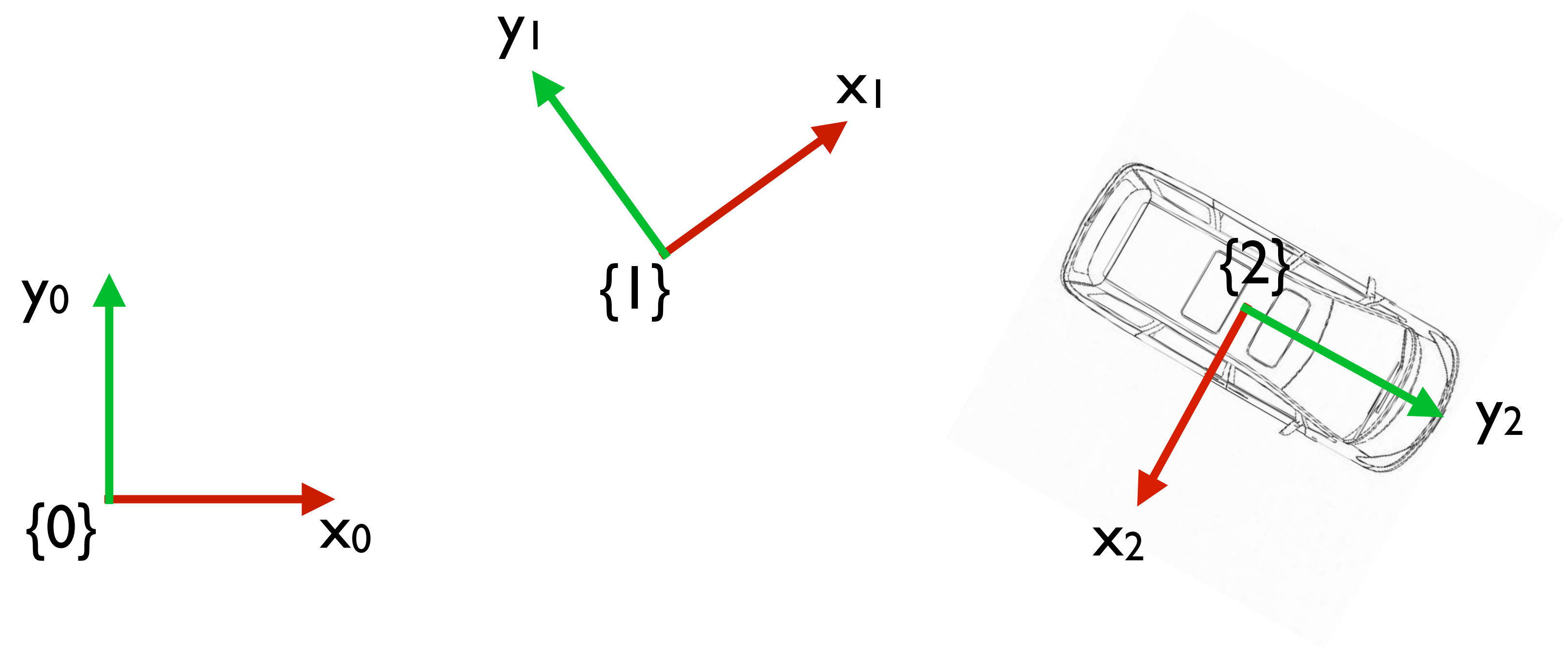
Composition of Homogeneous Transforms



If frame 1 has pose (p_{01}, R_{01}) in frame 0 and frame 2 has pose (p_{12}, R_{12}) in frame 1, what is the pose of frame 2 in frame 0?

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

Composition of Homogeneous Transforms

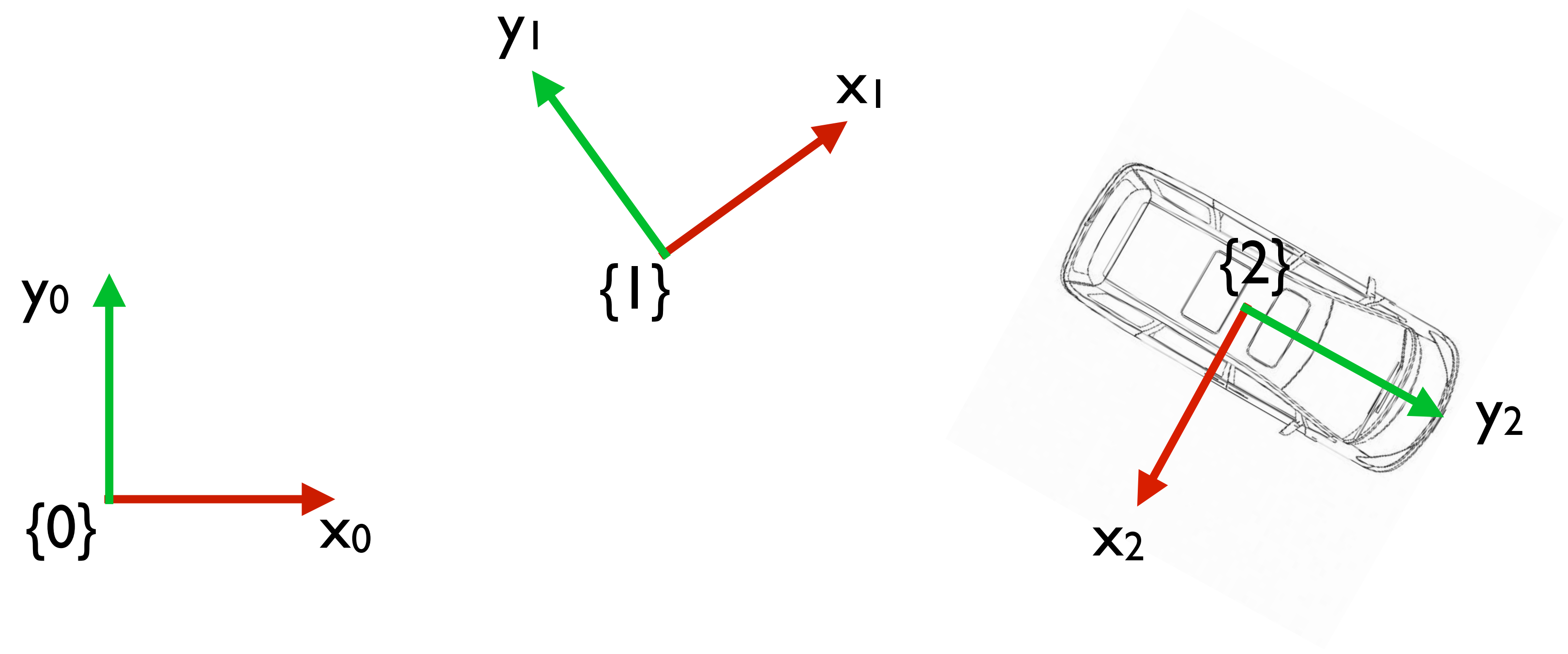


If frame 1 has pose (p_{01}, R_{01}) in frame 0 and frame 2 has pose (p_{12}, R_{12}) in frame 1, what is the pose of frame 2 in frame 0?

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} R_{12} & p_{12} \\ 0 & 1 \end{bmatrix}$$

Composition of Homogeneous Transforms



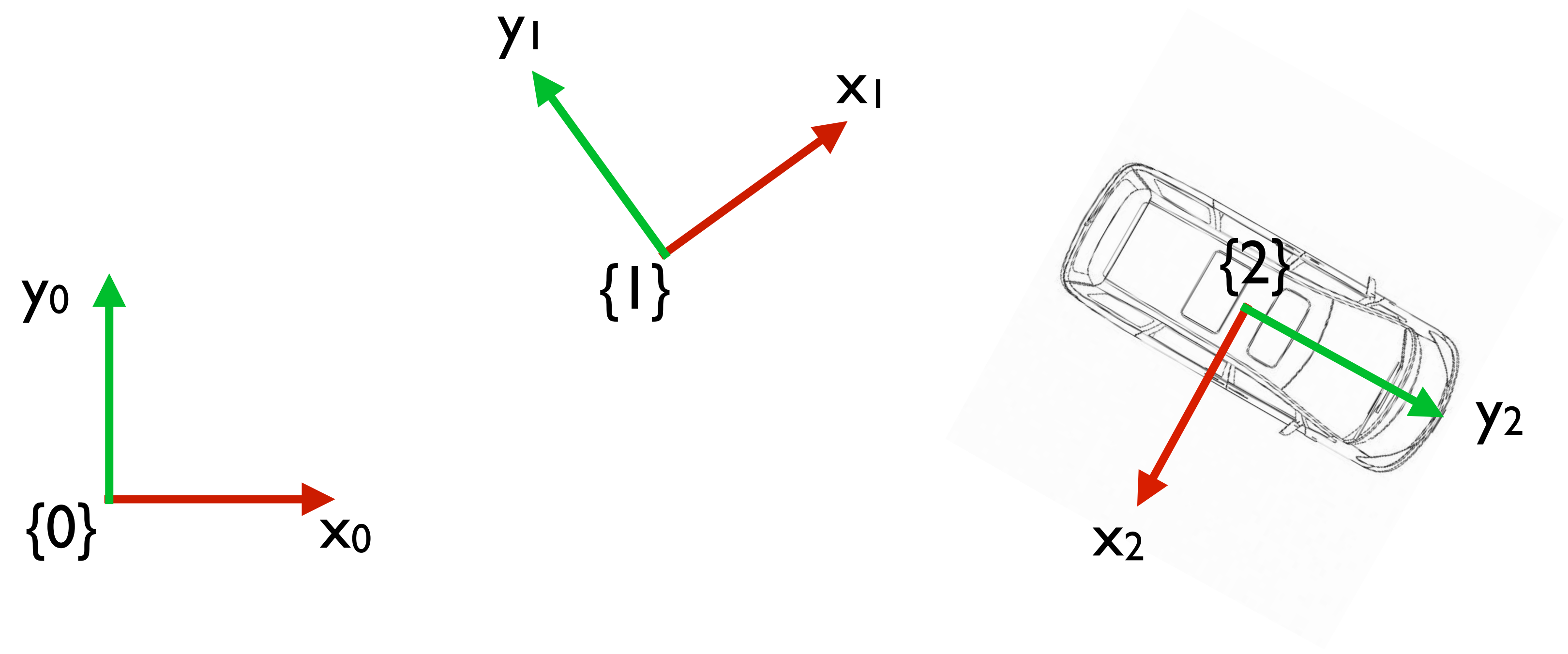
If frame 1 has pose (p_{01}, R_{01}) in frame 0 and frame 2 has pose (p_{12}, R_{12}) in frame 1, what is the pose of frame 2 in frame 0?

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} R_{12} & p_{12} \\ 0 & 1 \end{bmatrix}$$

$$T_{02} = T_{01} T_{12}$$

Composition of Homogeneous Transforms



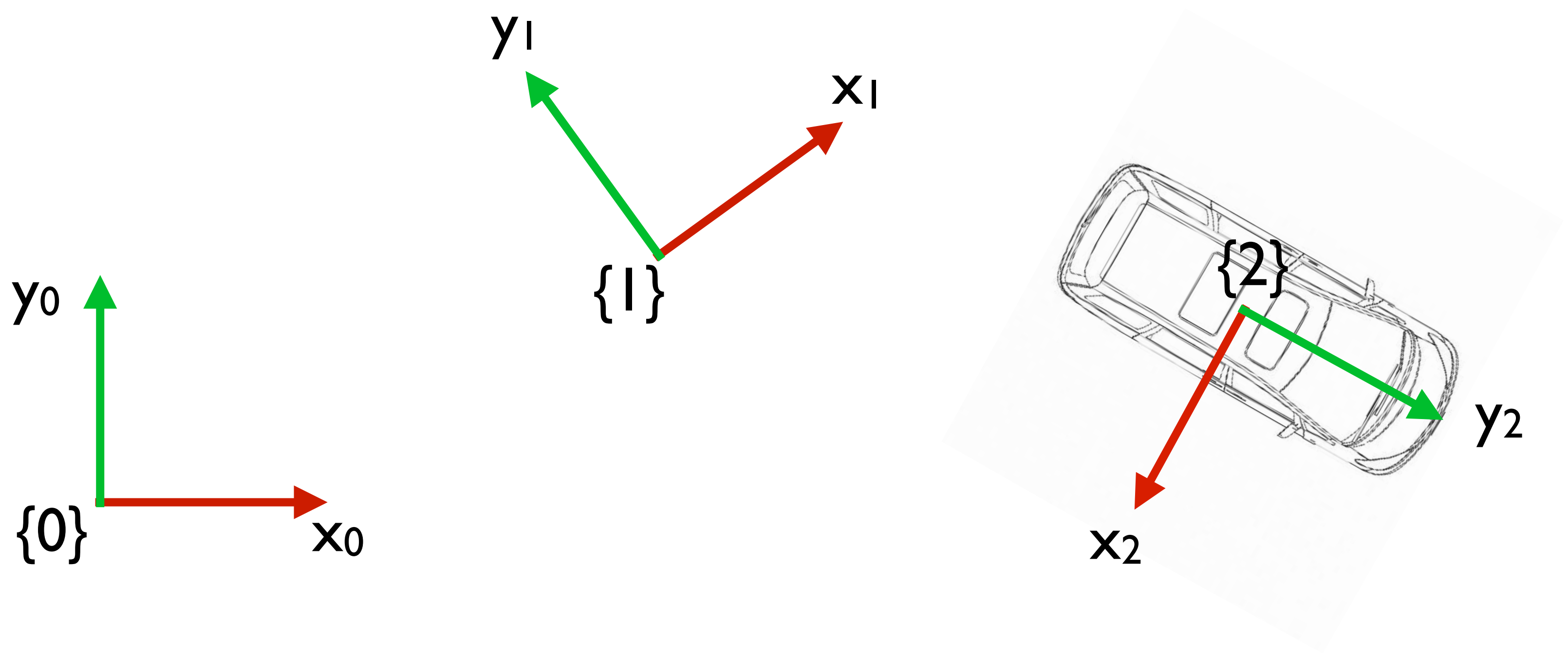
If frame 1 has pose (p_{01}, R_{01}) in frame 0 and frame 2 has pose (p_{12}, R_{12}) in frame 1, what is the pose of frame 2 in frame 0?

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} R_{12} & p_{12} \\ 0 & 1 \end{bmatrix}$$

$$T_{02} = T_{01} T_{12} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{12} & p_{12} \\ 0 & 1 \end{bmatrix}$$

Composition of Homogeneous Transforms



If frame 1 has pose (p_{01}, R_{01}) in frame 0 and frame 2 has pose (p_{12}, R_{12}) in frame 1, what is the pose of frame 2 in frame 0?

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix}$$

$$T_{12} = \begin{bmatrix} R_{12} & p_{12} \\ 0 & 1 \end{bmatrix}$$

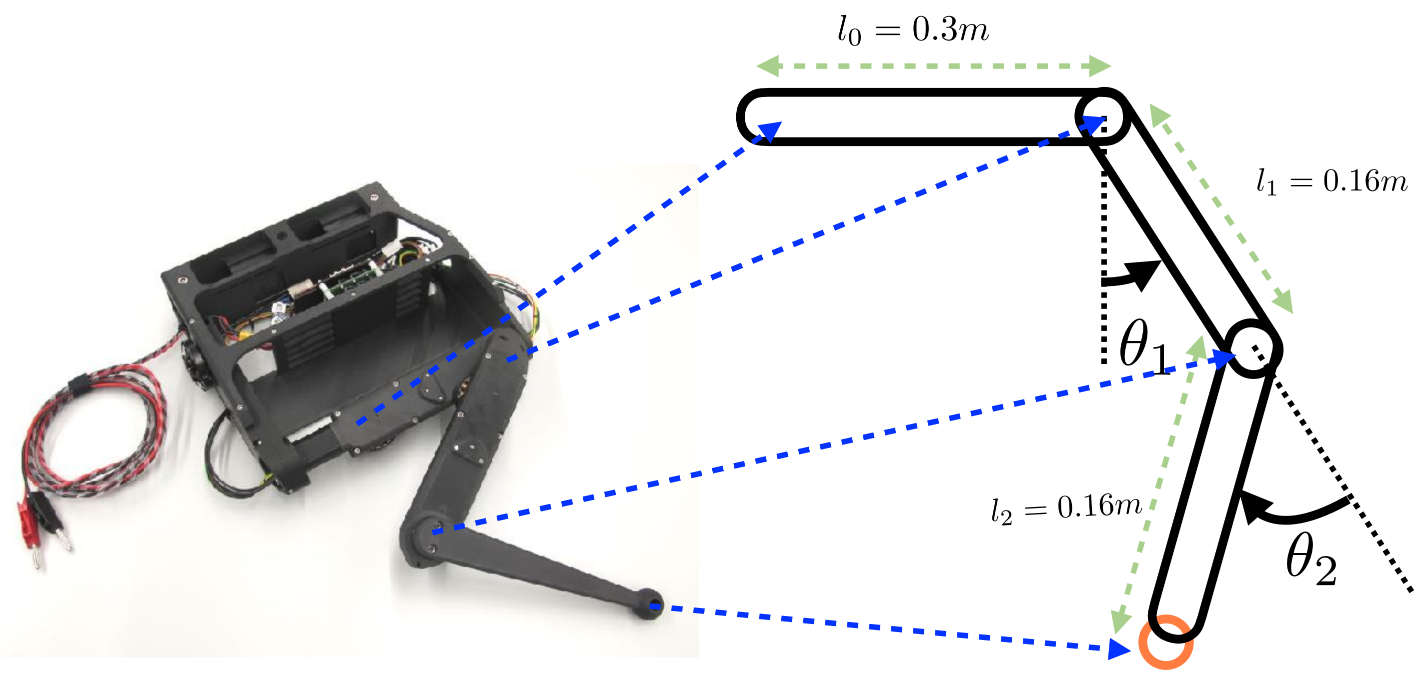
$$T_{02} = T_{01} T_{12} = \begin{bmatrix} R_{01} & p_{01} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_{12} & p_{12} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_{01} R_{12} & R_{01} p_{12} + p_{01} \\ 0 & 1 \end{bmatrix}$$

Agenda

1. Homogenous Transformations
2. Three Uses of Transformations
3. Properties of Transformations
4. **Forward Kinematics**

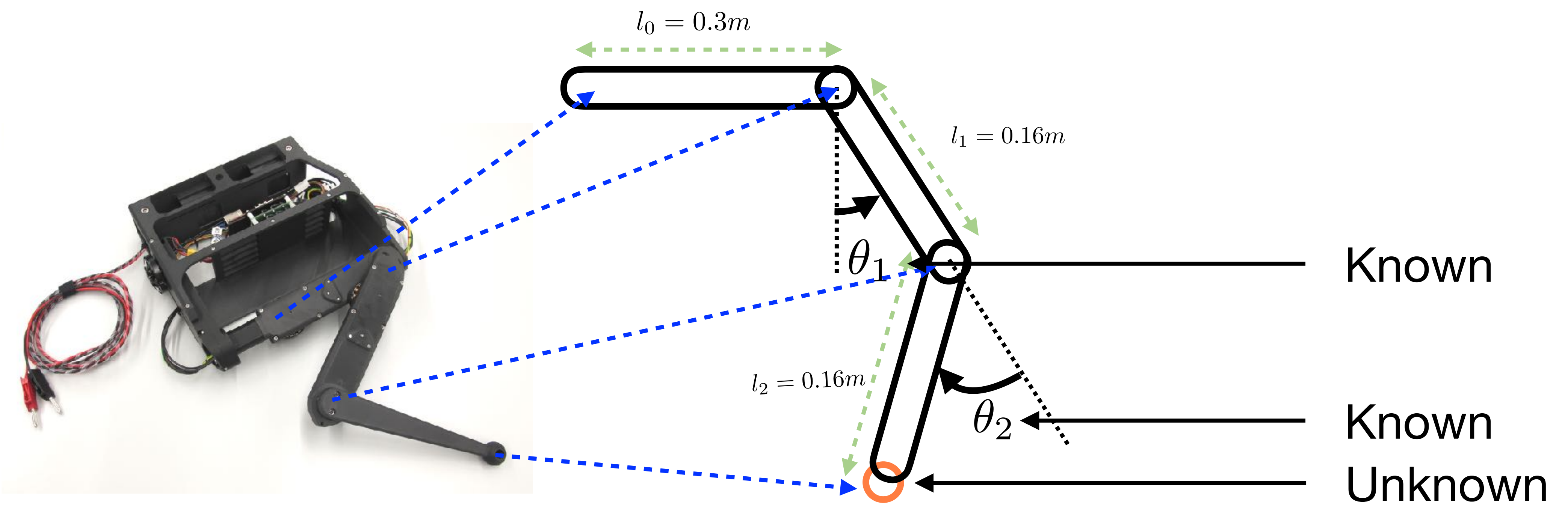
Forward Kinematics

- Determine the pose of a robot's end-effector, given the robot's joint angles.



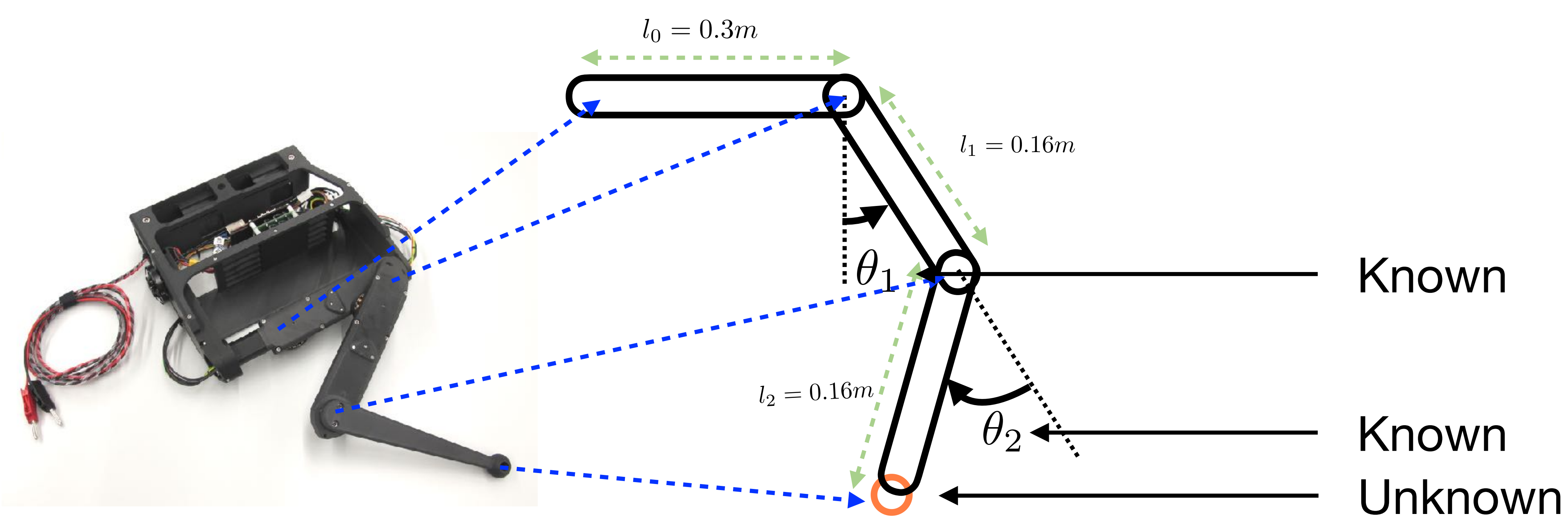
Forward Kinematics

- Determine the pose of a robot's end-effector, given the robot's joint angles.



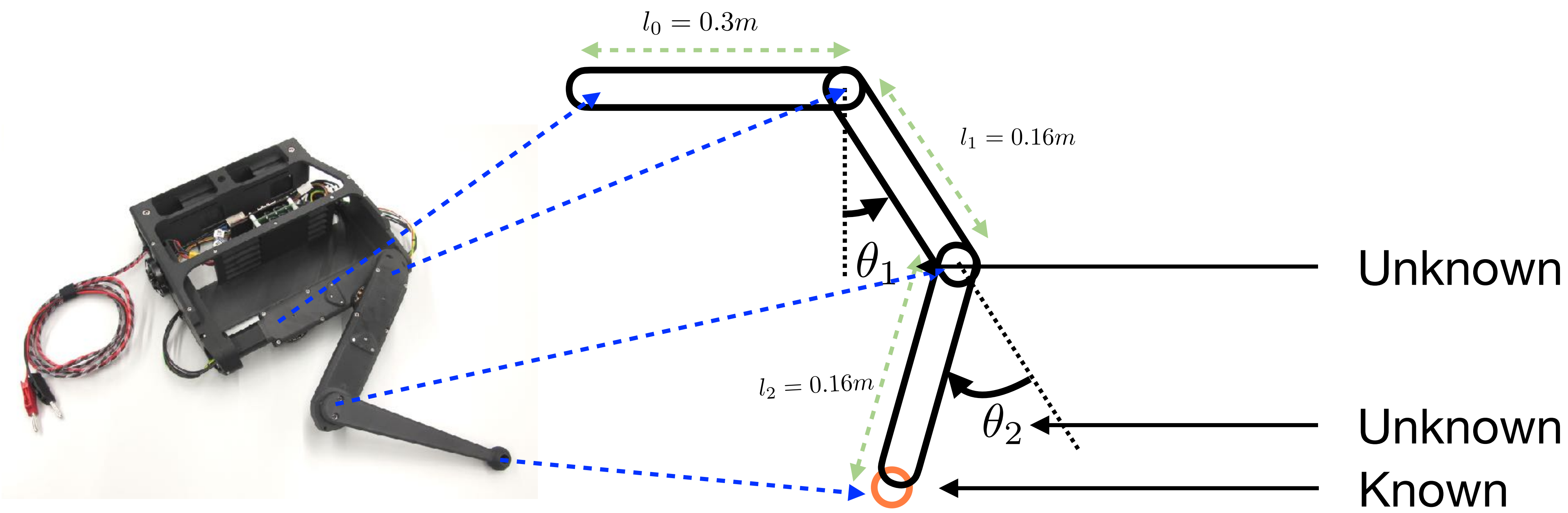
Forward Kinematics (more precise)

- Determine the pose of the end-effector (or some specific part of the robot) with respect to a stationary base frame, given the joint angles of the robot's revolute joints, (and/or lengths of the robot's prismatic joints).

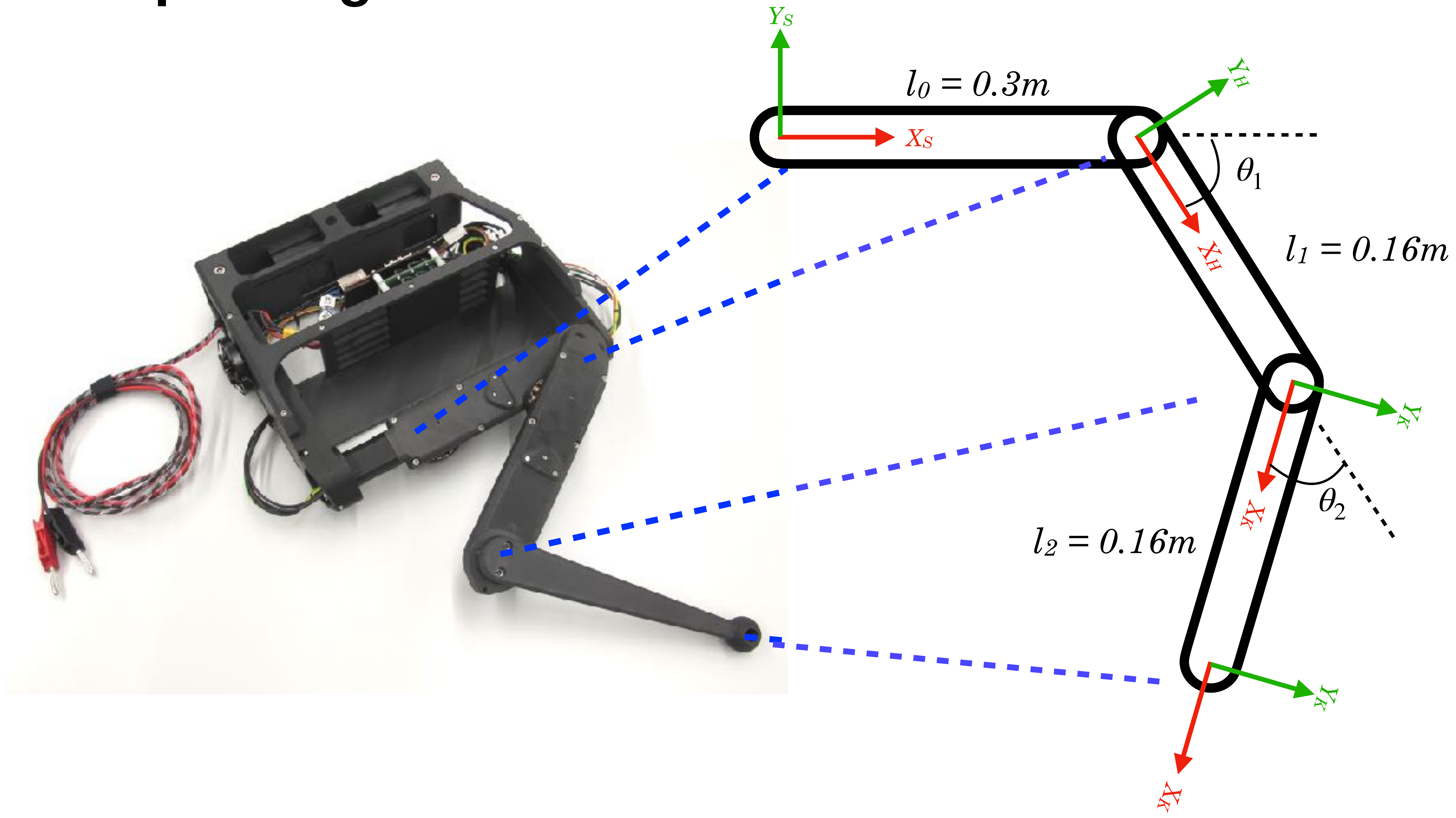


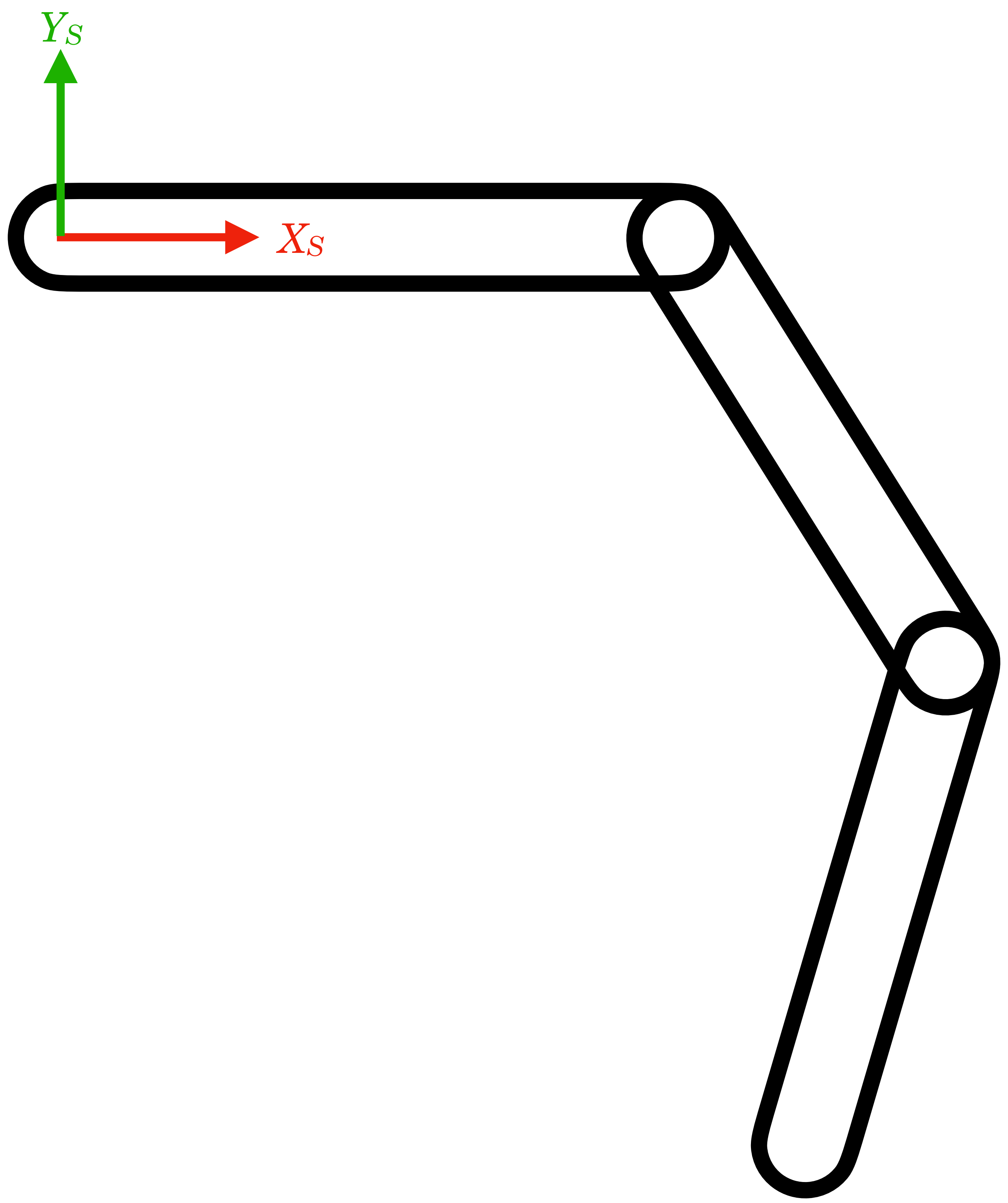
Inverse Kinematics

- Given the pose of the end-effector (or some specific part of the robot) with respect to a stationary base frame, determine the joint angles of the robot's revolute joints, (and/or lengths of the robot's prismatic joints).

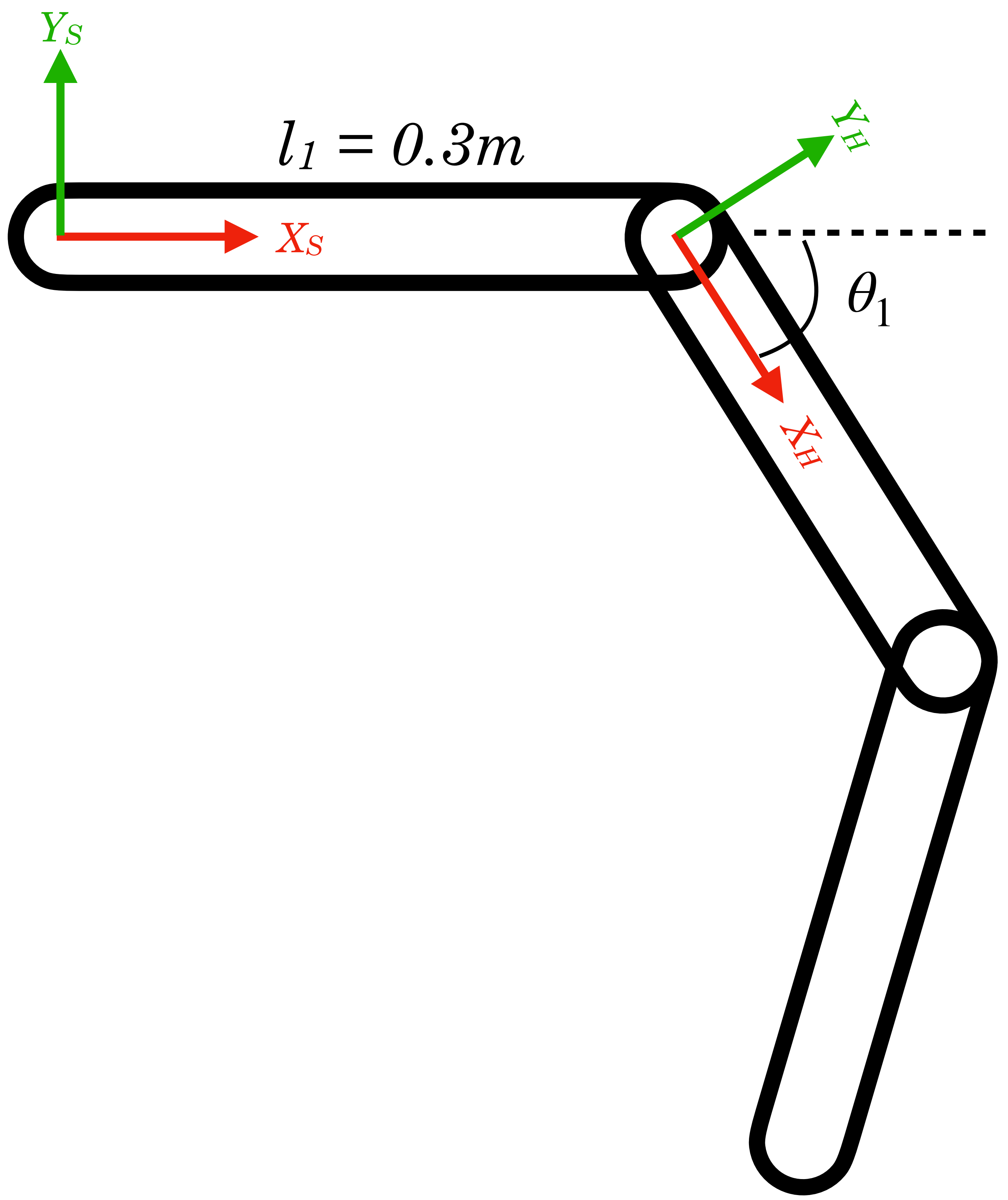


Example - Quadraped Leg

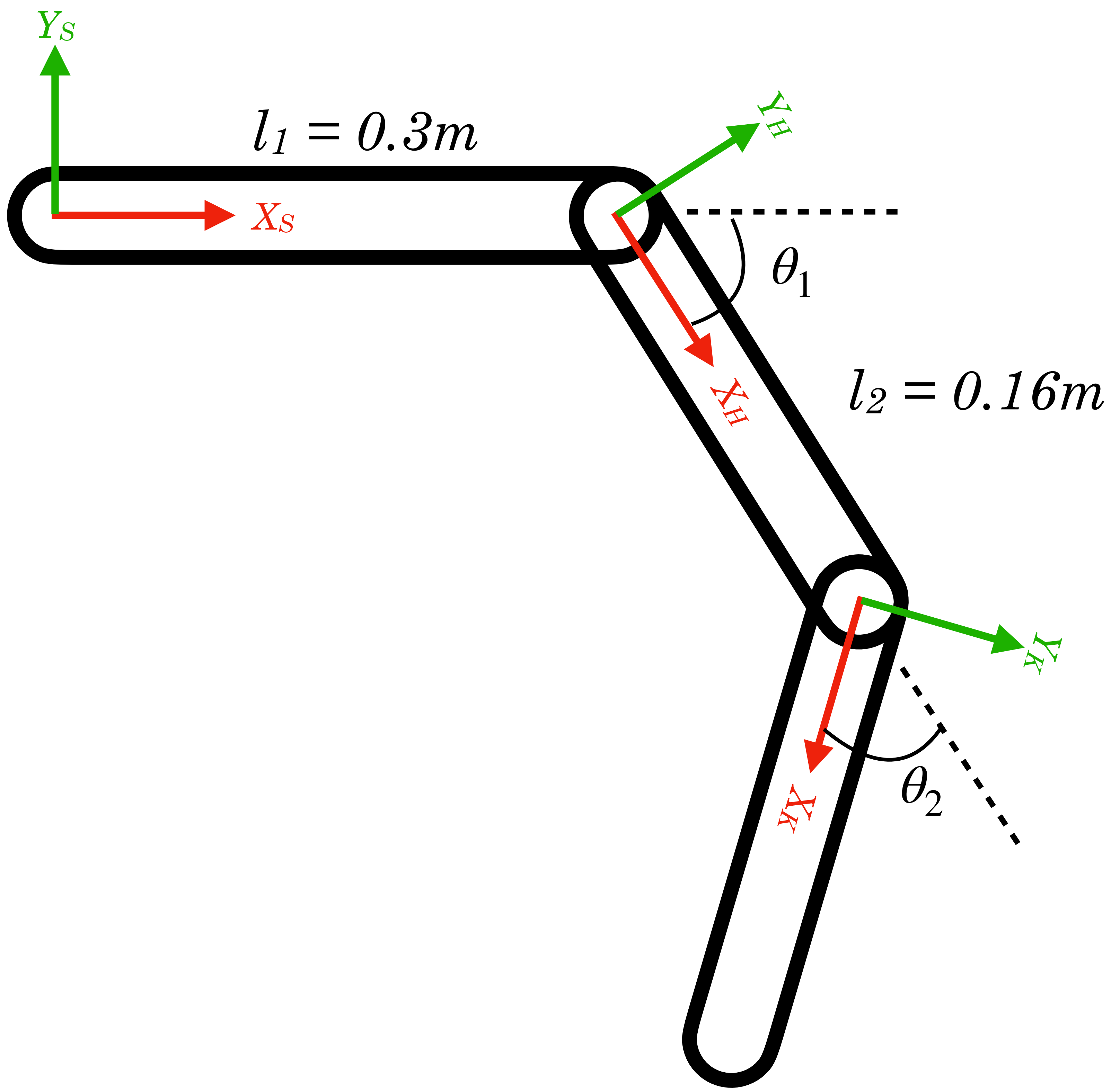




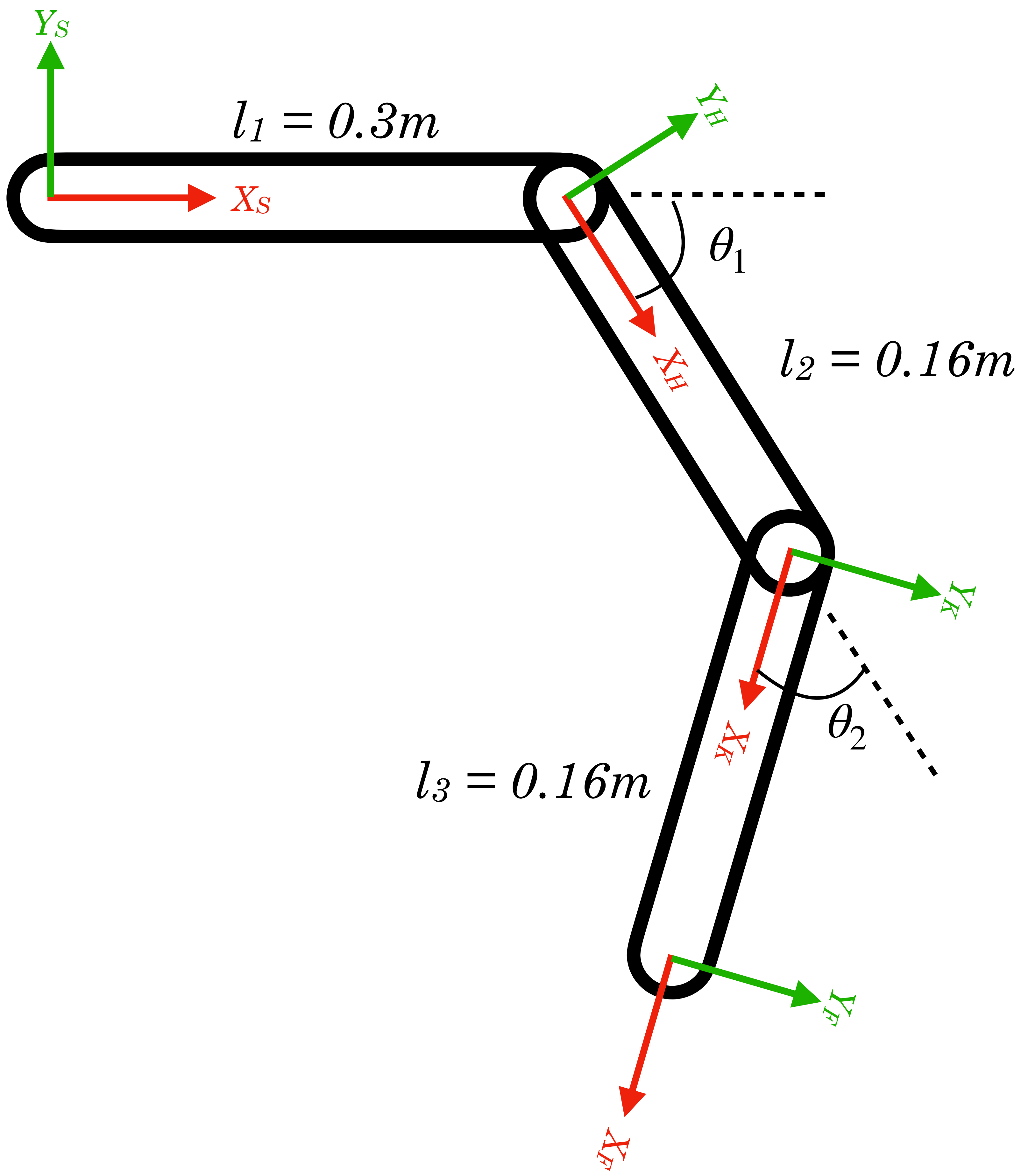
- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame



- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame
- The hip frame $\{H\}$ is translated by l_1 and rotated by θ_1 with respect to frame $\{S\}$.



- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame
- The hip frame $\{H\}$ is translated by l_1 and rotated by θ_1 with respect to frame $\{S\}$.
- The knee frame $\{K\}$ is translated by l_2 and rotated by θ_2 with respect to frame $\{H\}$.



- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame
- The hip frame $\{H\}$ is translated by l_1 and rotated by θ_1 with respect to frame $\{S\}$.
- The knee frame $\{K\}$ is translated by l_2 and rotated by θ_2 with respect to frame $\{H\}$.
- The foot frame $\{F\}$ is translated by l_3 with respect to frame $\{K\}$.

Take-aways

- **Homogeneous Transformations** are matrices comprised of rotations and translations
- They are useful for:
  - Describing a pose
  - Coordinate transformations
  - Moving an object
  - Forward Kinematics