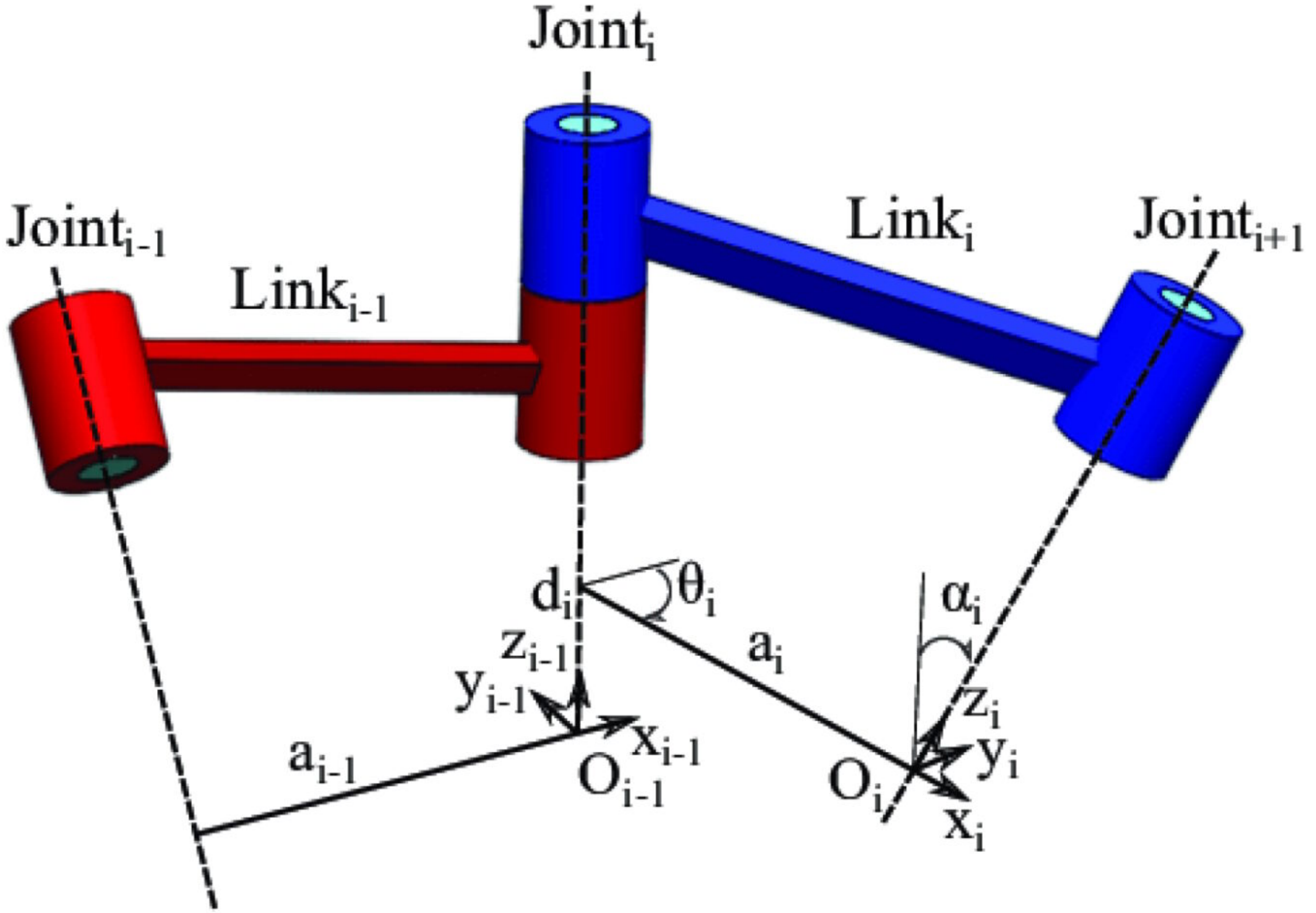
ROB-UY 2004

Robotic Manipulation & Locomotion

Lecture 05A - Forward Kinematics



MDPI Sensors

Agenda

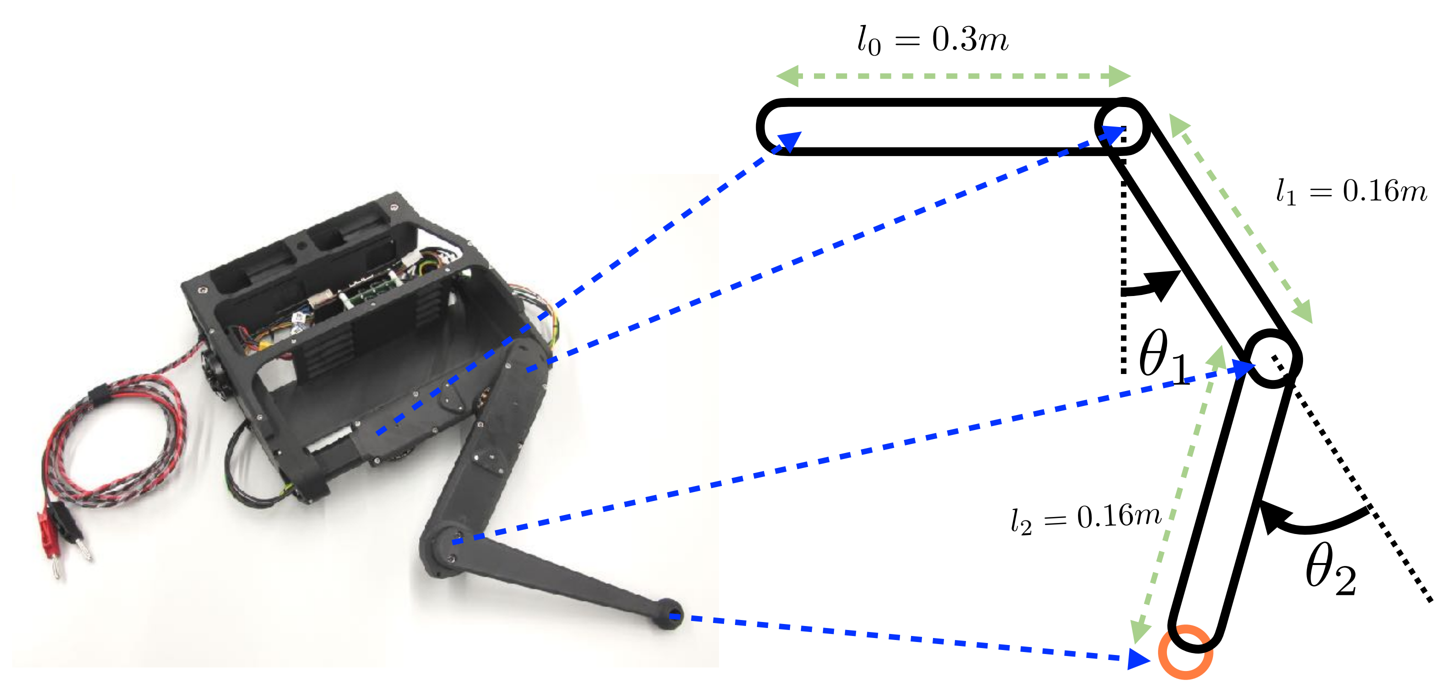
1. Forward Kinematics Introduction
2. DH Convention
3. Systematic Approach to FK
4. URDF
5. Kinematic Trees

Agenda

1. **Forward Kinematics Introduction**
2. DH Convention
3. Systematic Approach to FK
4. URDF
5. Kinematic Trees

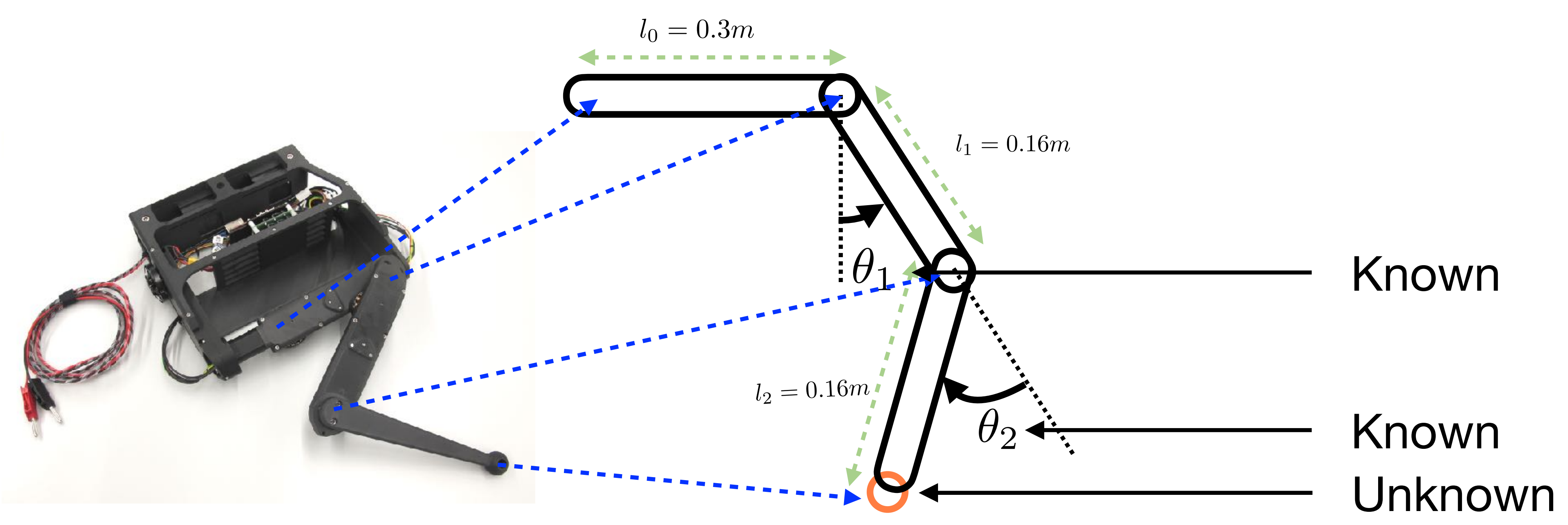
Forward Kinematics

- Determine the pose of a robot’s end-effector, given the robot’s joint angles.



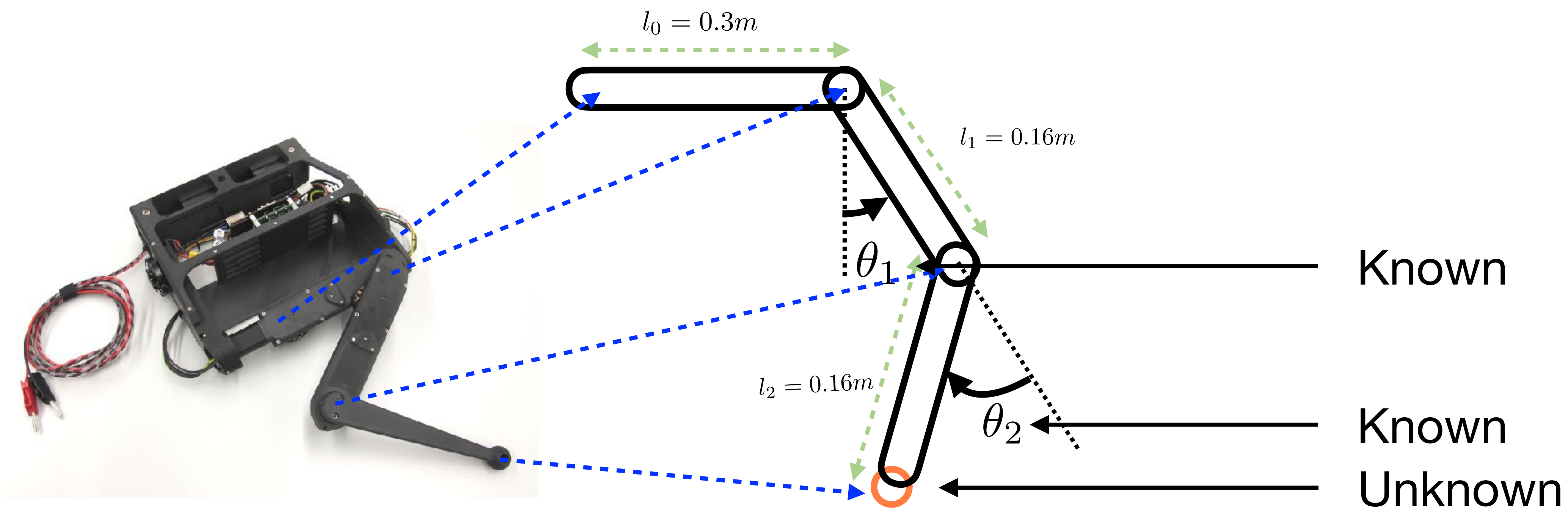
Forward Kinematics

- Determine the pose of a robot's end-effector, given the robot's joint angles.



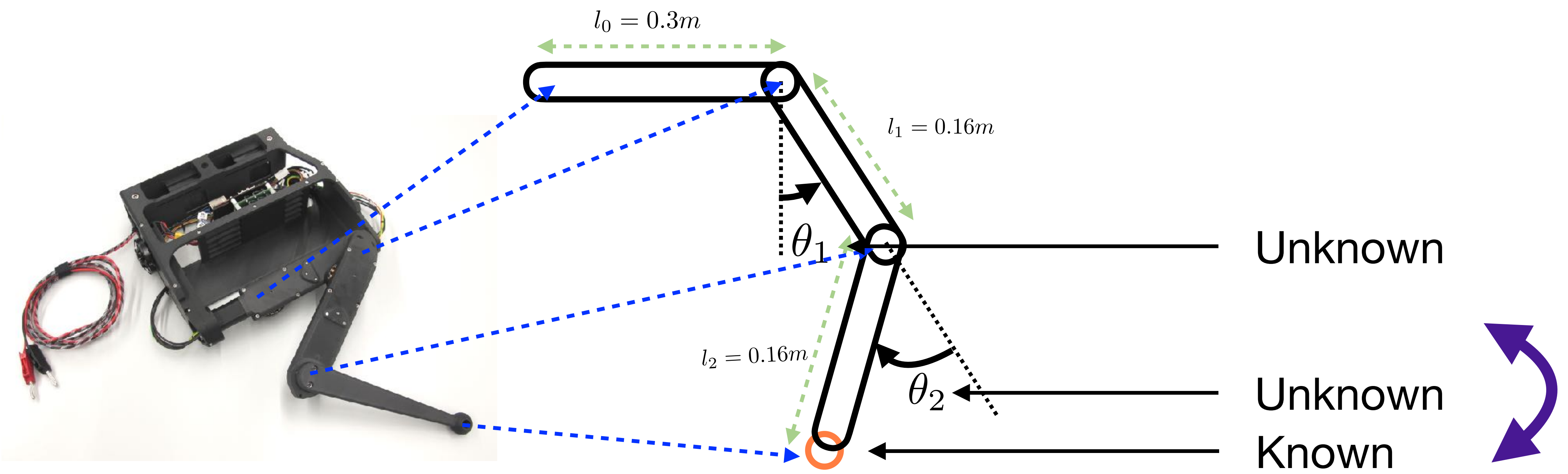
Forward Kinematics (more precise)

- Determine the pose of the end-effector (or some specific part of the robot) with respect to a stationary base frame, given the joint angles of the robot's revolute joints, (and/or lengths of the robot's prismatic joints).

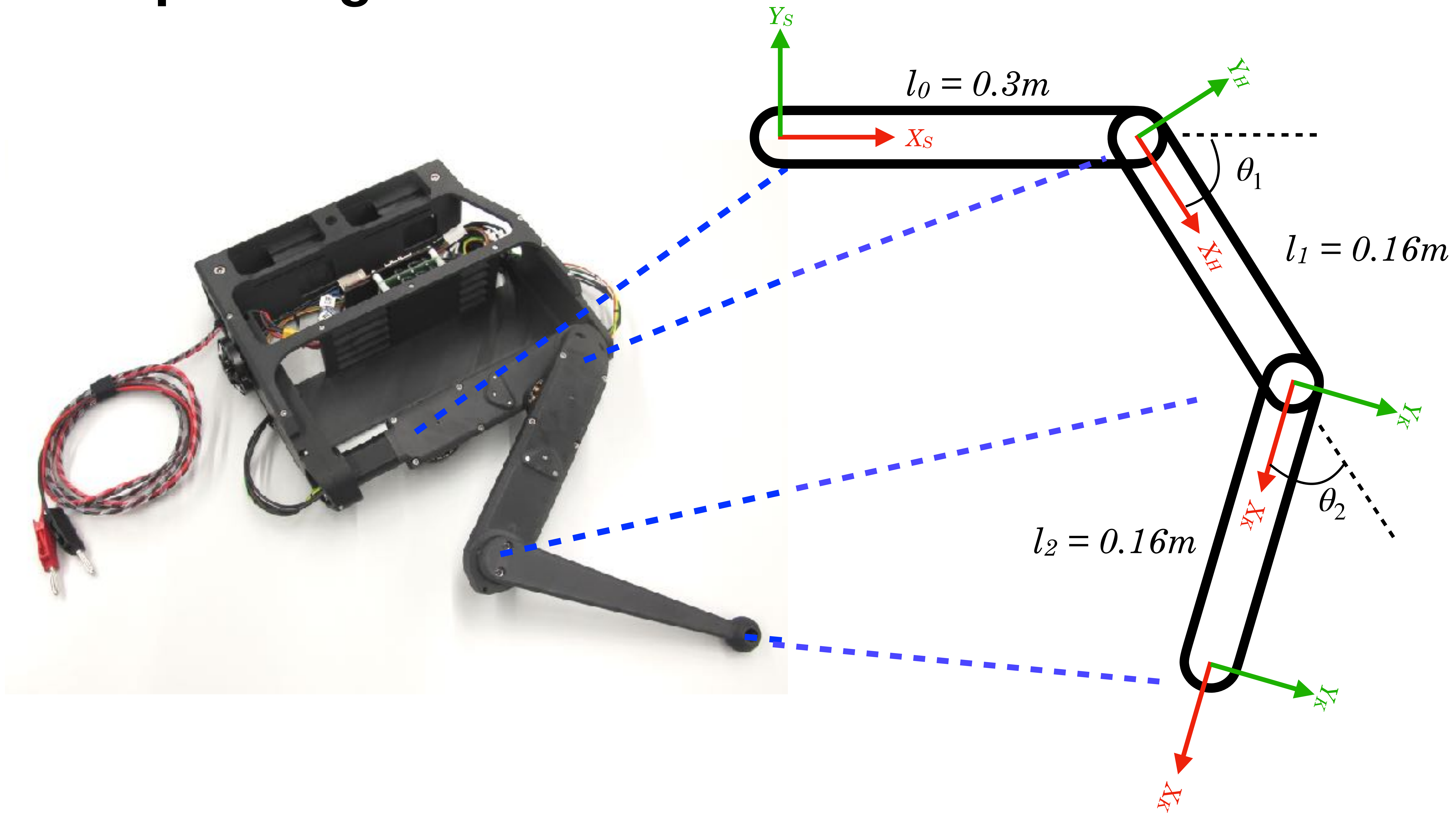


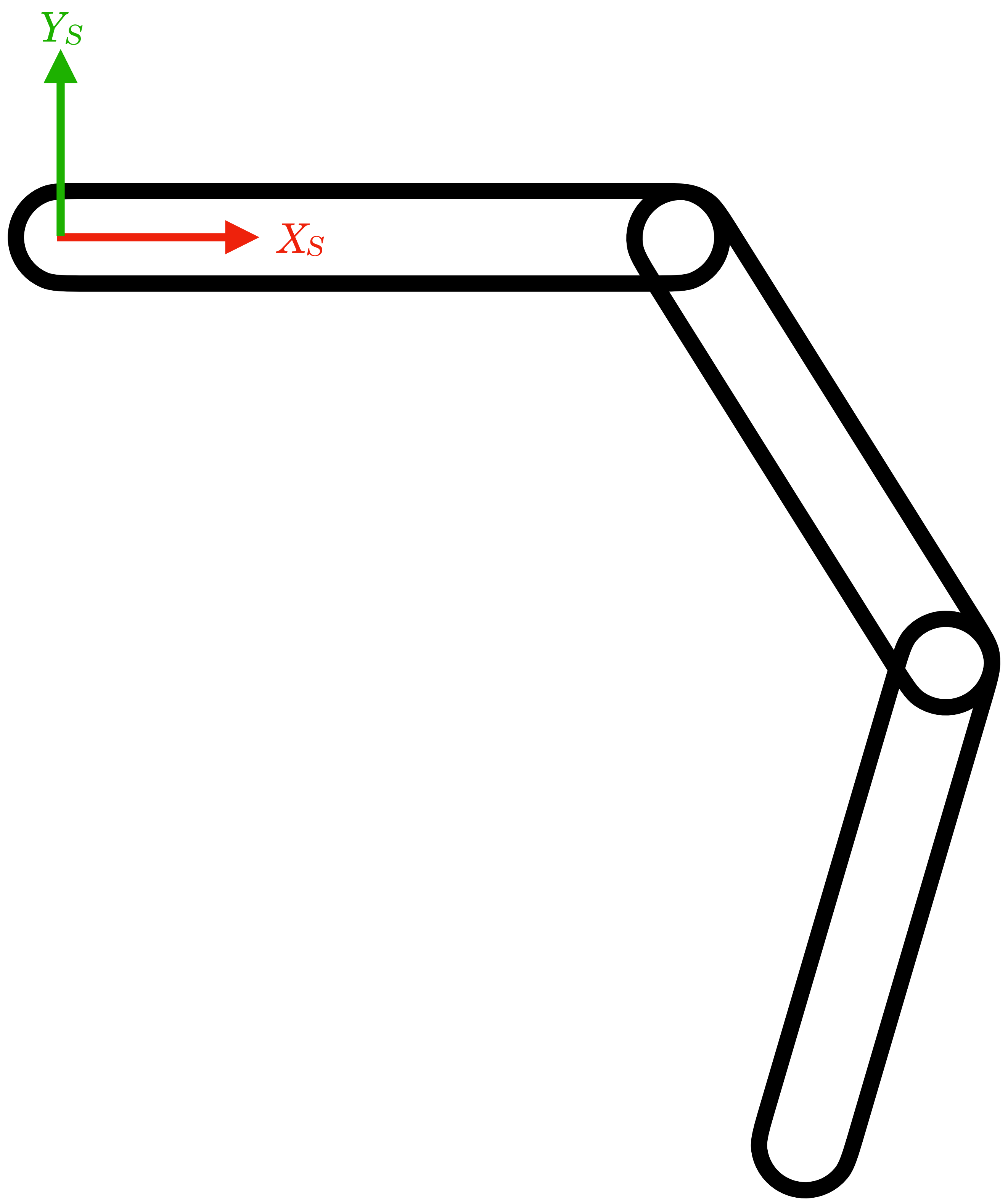
Inverse Kinematics

- Given the pose of the end-effector (or some specific part of the robot) with respect to a stationary base frame, determine the joint angles of the robot's revolute joints, (and/or lengths of the robot's prismatic joints).

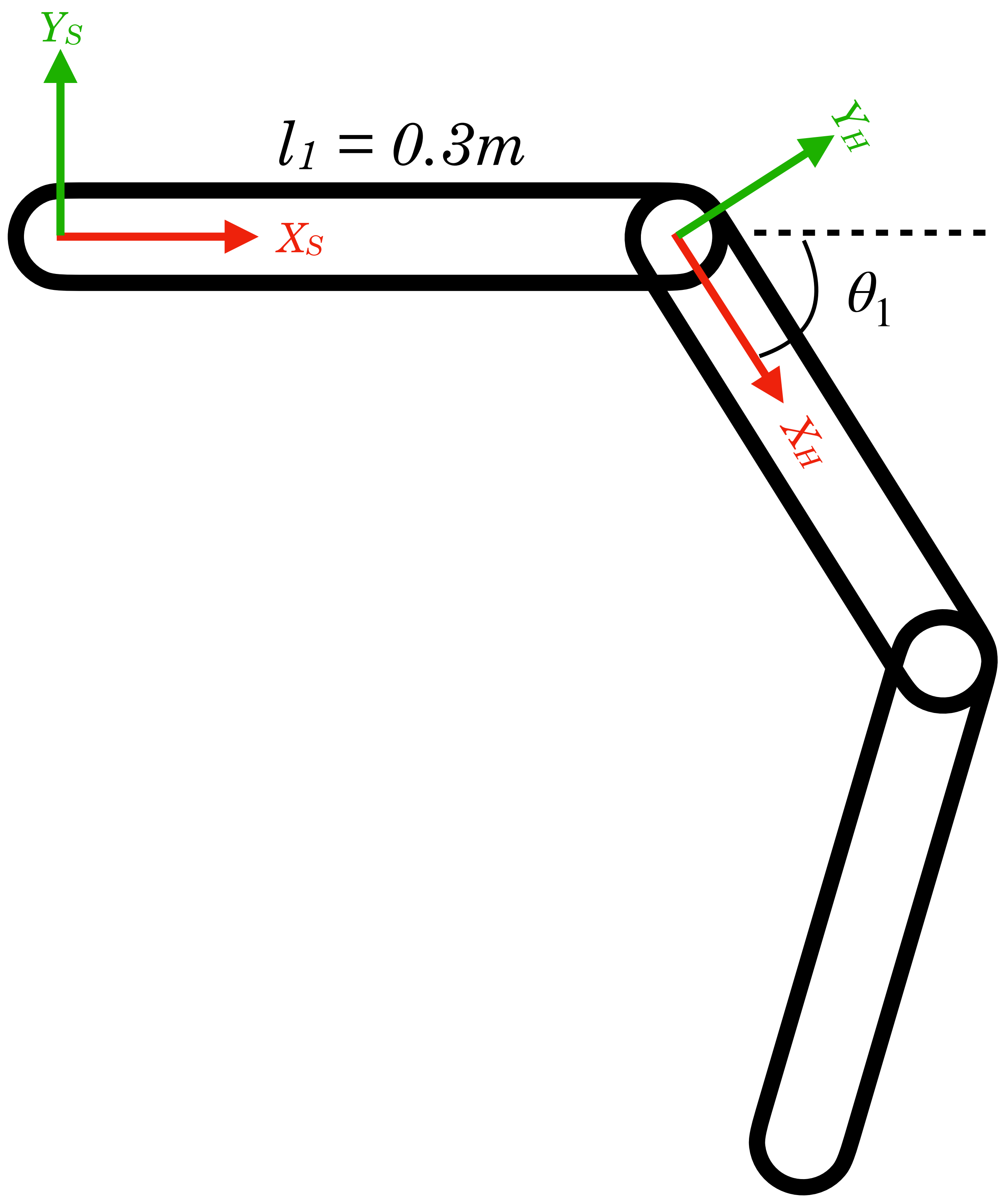


Example - Quadraped Leg

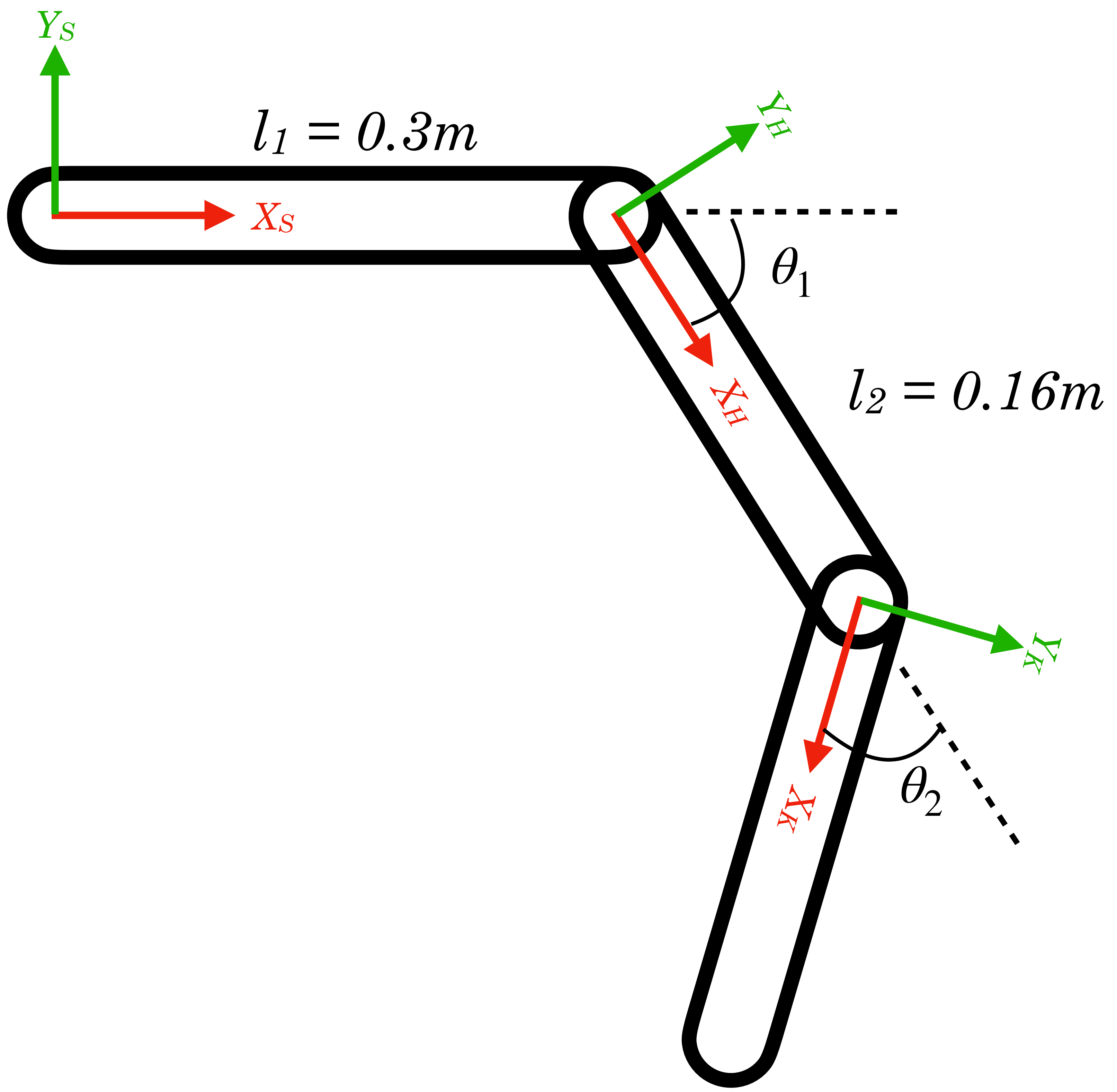




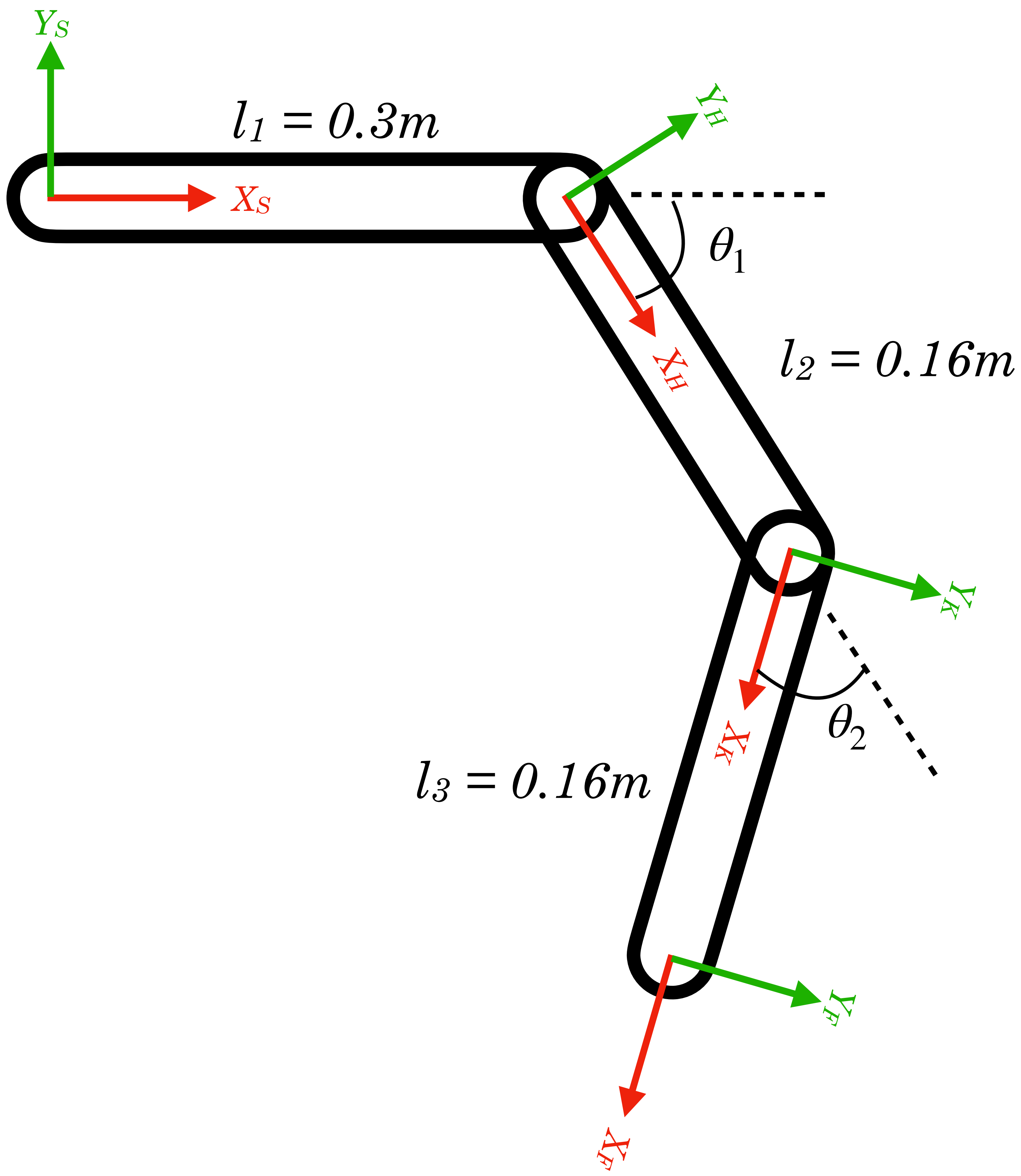
- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame



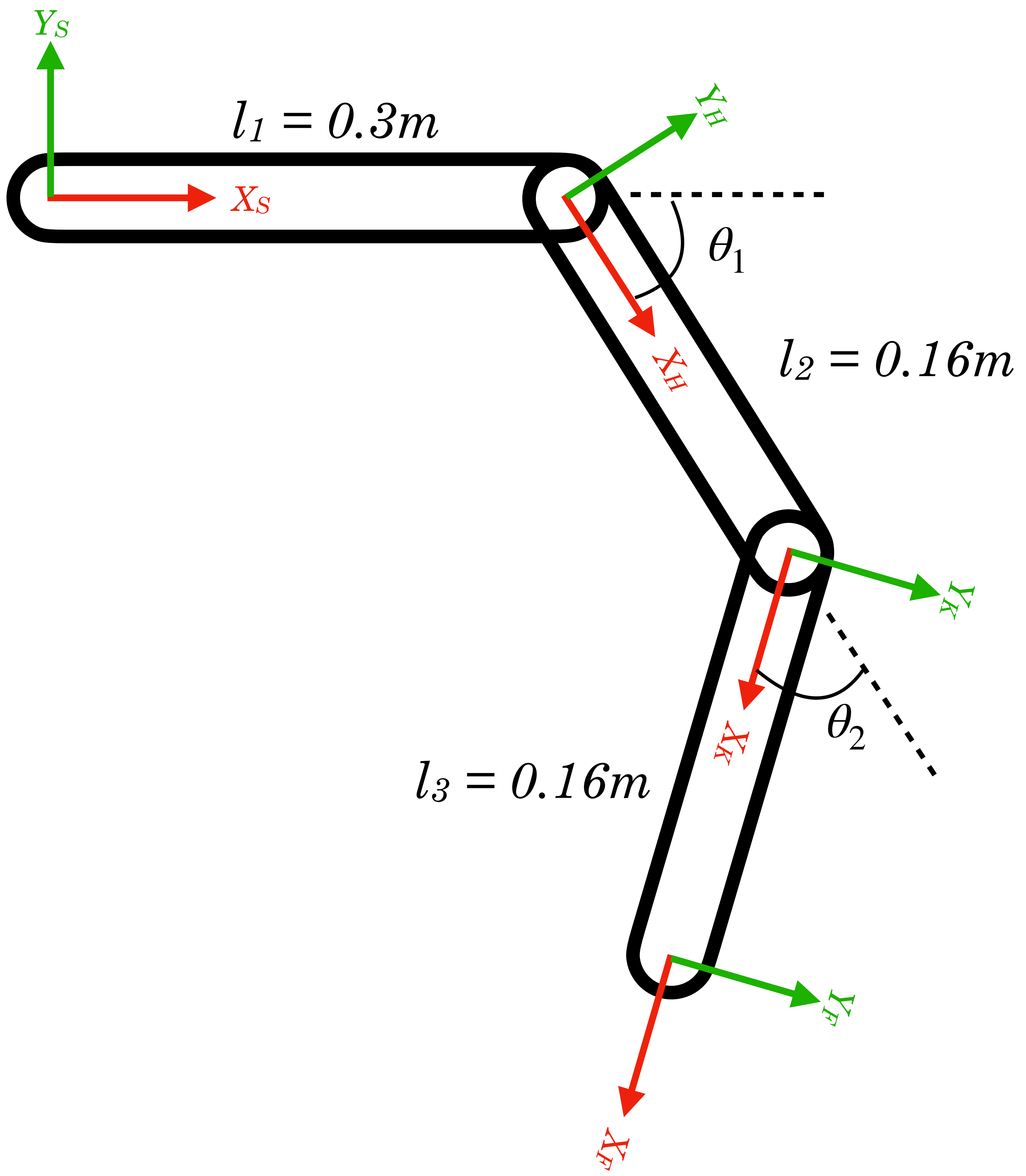
- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame
- The hip frame $\{H\}$ is translated by l_1 and rotated by θ_1 with respect to frame $\{S\}$.

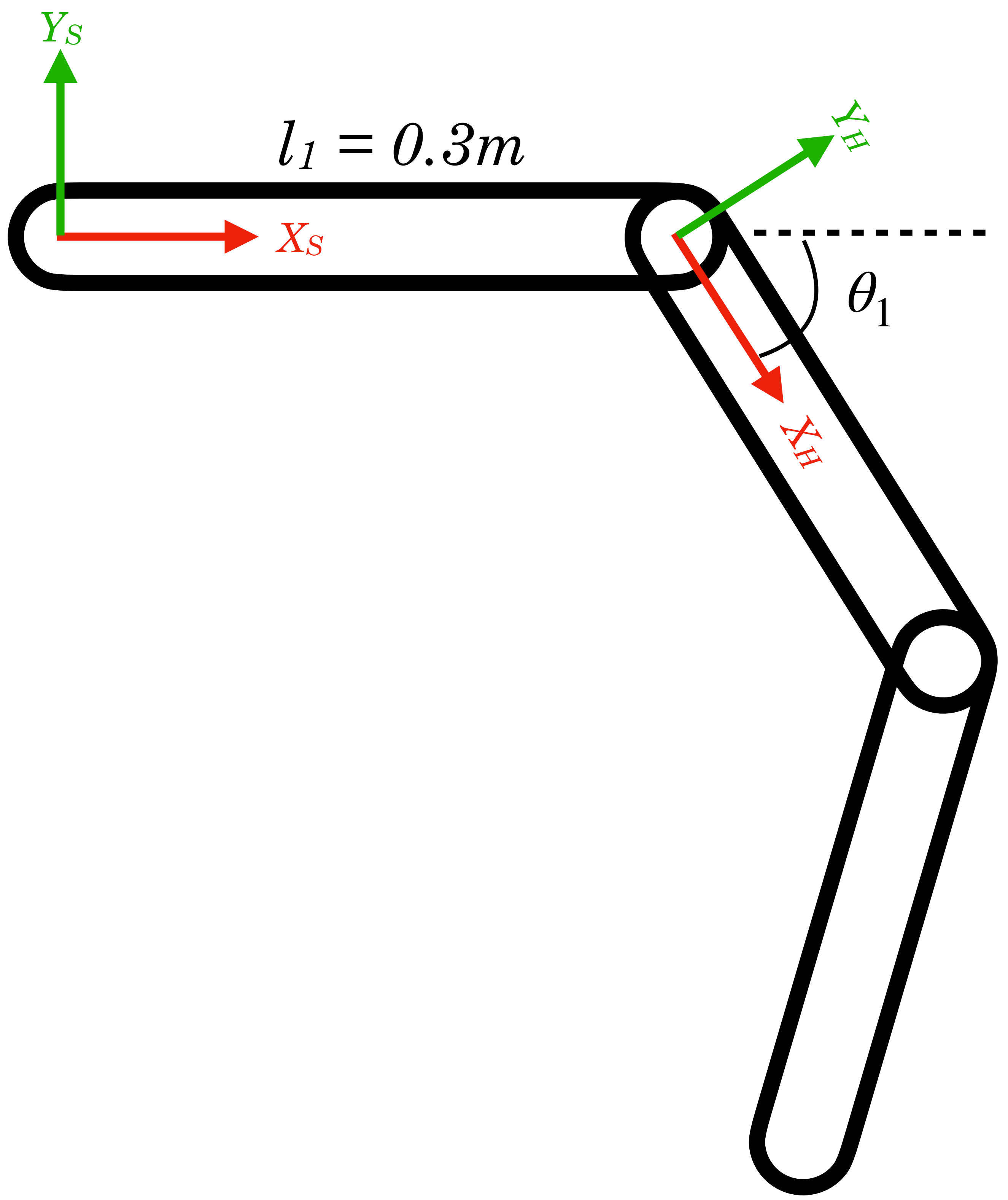


- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame
- The hip frame $\{H\}$ is translated by l_1 and rotated by θ_1 with respect to frame $\{S\}$.
- The knee frame $\{K\}$ is translated by l_2 and rotated by θ_2 with respect to frame $\{H\}$.



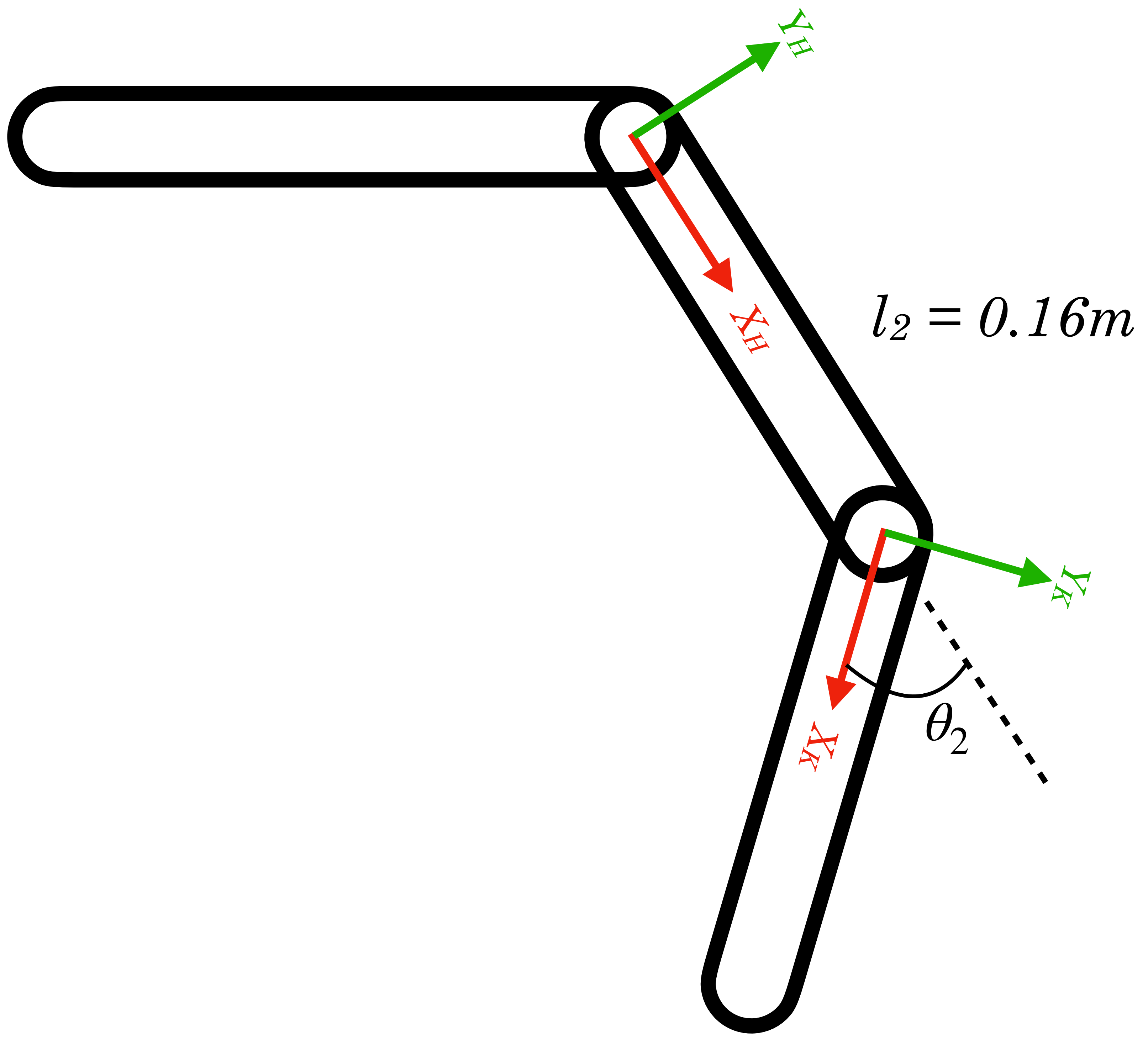
- Frame $\{S\}$ is our fixed frame, i.e. the spatial frame
- The hip frame $\{H\}$ is translated by l_1 and rotated by θ_1 with respect to frame $\{S\}$.
- The knee frame $\{K\}$ is translated by l_2 and rotated by θ_2 with respect to frame $\{H\}$.
- The foot frame $\{F\}$ is translated by l_3 with respect to frame $\{K\}$.


Y_S
X_S
$l_1 = 0.3m$
Y_H
X_H
θ_1
$l_2 = 0.16m$
Y_K
X_K
θ_2
$l_3 = 0.16m$
Y_F
X_F



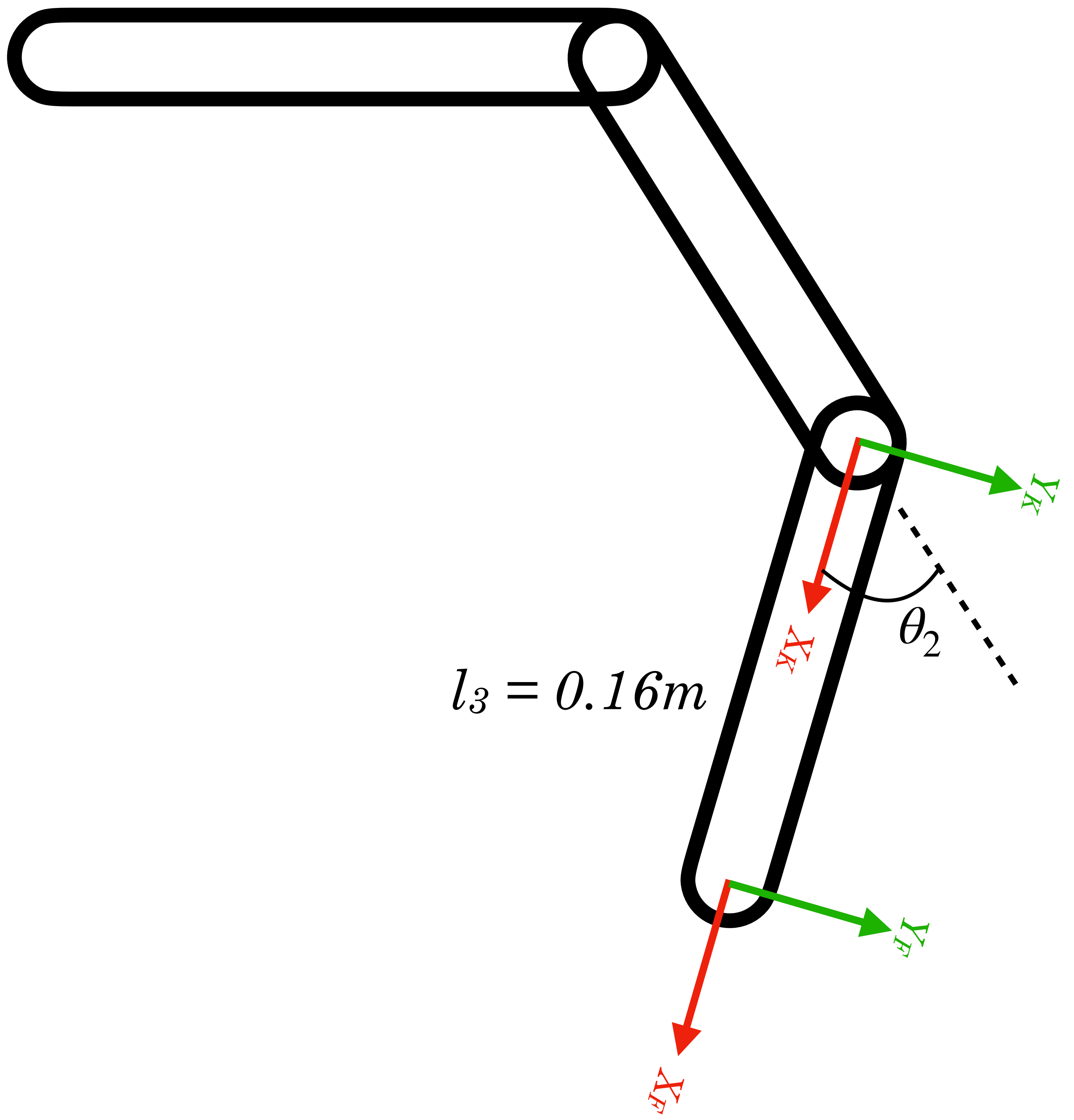
- The frame $\{H\}$, relative to $\{S\}$, is translated by l_1 and then rotated around its z axis by θ_1

$$T_{SH} = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & l_1 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



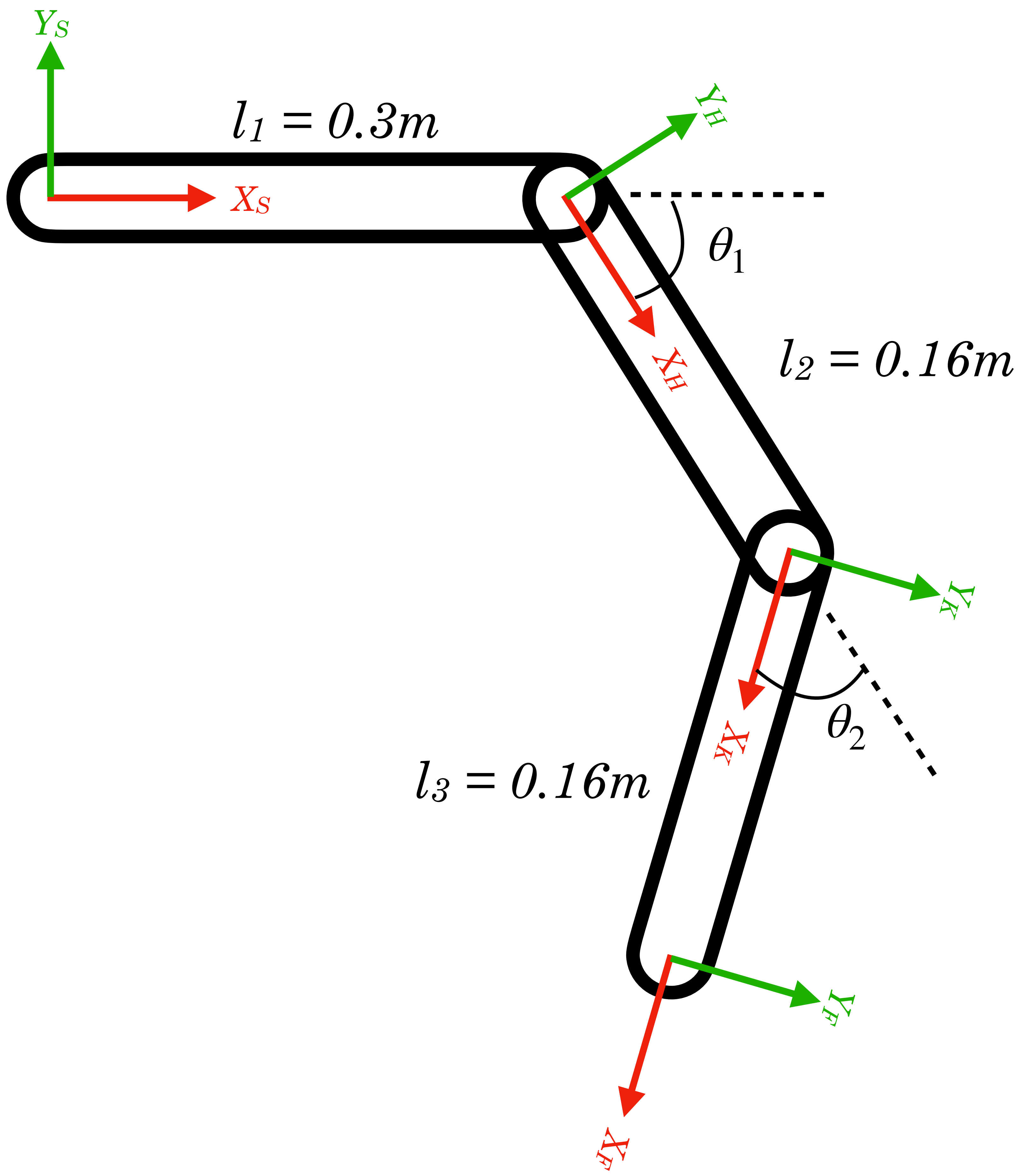
- The frame $\{K\}$, relative to $\{H\}$, translated by l_2 and then rotated around its z axis by θ_2

$$T_{HK} = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & l_2 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- The frame $\{F\}$, relative to $\{K\}$, is only translated by l_3

$$T_{KF} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$T_{SH} = \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & l_1 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{HK} = \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & l_2 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{KF} = \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- The pose of the foot in frame $\{S\}$ is:

$$T_{SF} = T_{SH} T_{HK} T_{KF}$$

- The pose of the foot in frame $\{S\}$ is:

$$T_{SF} = T_{SH}(l_1, \theta_1)T_{HK}(l_2, \theta_2)T_{KF}(l_3)$$

- The pose of the foot in frame $\{S\}$ is:

$$T_{SF} = T_{SH}(l_1, \theta_1) T_{HK}(l_2, \theta_2) T_{KF}(l_3)$$

Only some variables will change as a function of time!

$$T_{SF} = T_{SH}T_{HK}T_{KF}$$

$$= \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & l_1 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & l_2 \\ sin\theta_2 & cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & l_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta_1 & -sin\theta_1 & 0 & l_1 \\ sin\theta_1 & cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta_2 & -sin\theta_2 & 0 & l_3cos\theta_2 + l_2 \\ sin\theta_2 & cos\theta_2 & 0 & l_3sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3cos\theta_2 + l_2)cos\theta_1 - l_3sin\theta_1sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3cos\theta_2 + l_2)sin\theta_1 + l_3cos\theta_1sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{SF} = \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2) cos\theta_1 - l_3 sin\theta_1 sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2) sin\theta_1 + l_3 cos\theta_1 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{SF} = \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)cos\theta_1 - l_3 sin\theta_1 sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)sin\theta_1 + l_3 cos\theta_1 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{SF} = \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)cos\theta_1 - l_3 sin\theta_1 sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)sin\theta_1 + l_3 cos\theta_1 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A homogeneous transformation matrix!

$$T_{SF} = \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)cos\theta_1 - l_3 sin\theta_1 sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)sin\theta_1 + l_3 cos\theta_1 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{SF} = \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)cos\theta_1 - l_3 sin\theta_1 sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)sin\theta_1 + l_3 cos\theta_1 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A rotation of $\theta_1 + \theta_2$ about the z axis!

$$T_{SF} = \begin{bmatrix} cos(\theta_1 + \theta_2) & -sin(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)cos\theta_1 - l_3 sin\theta_1 sin\theta_2 + l_1 \\ sin(\theta_1 + \theta_2) & cos(\theta_1 + \theta_2) & 0 & (l_3 cos\theta_2 + l_2)sin\theta_1 + l_3 cos\theta_1 sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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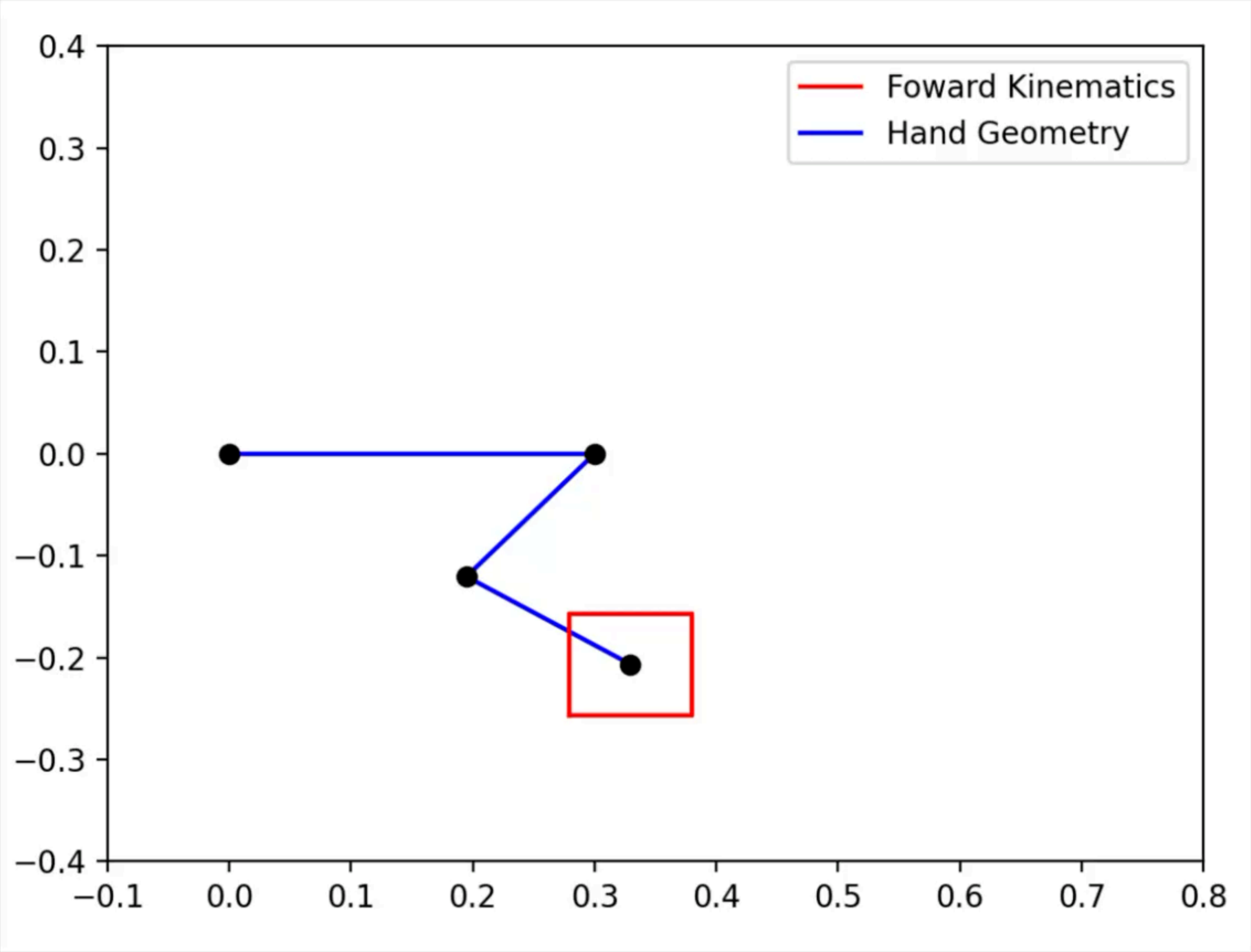
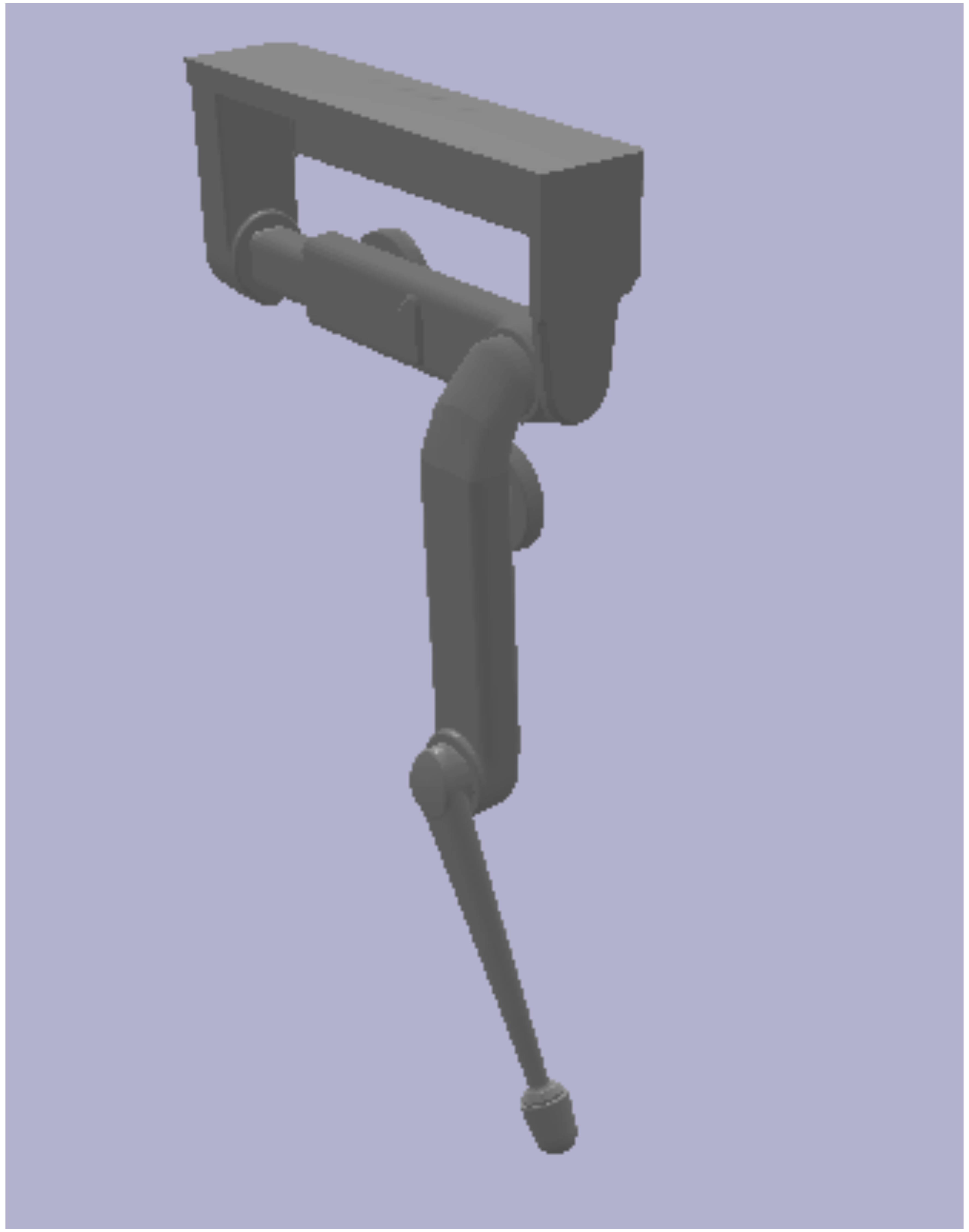
The position of the foot in the frame $\{S\}$!

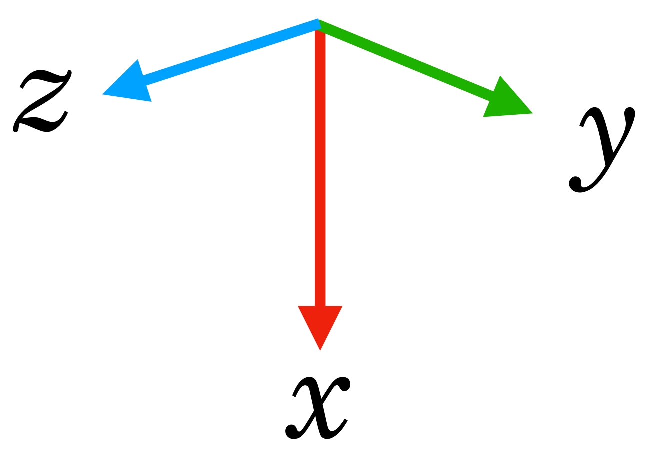

Foward Kinematics
Hand Geometry
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

Forward Kinematics Is not 2 Dimensional!

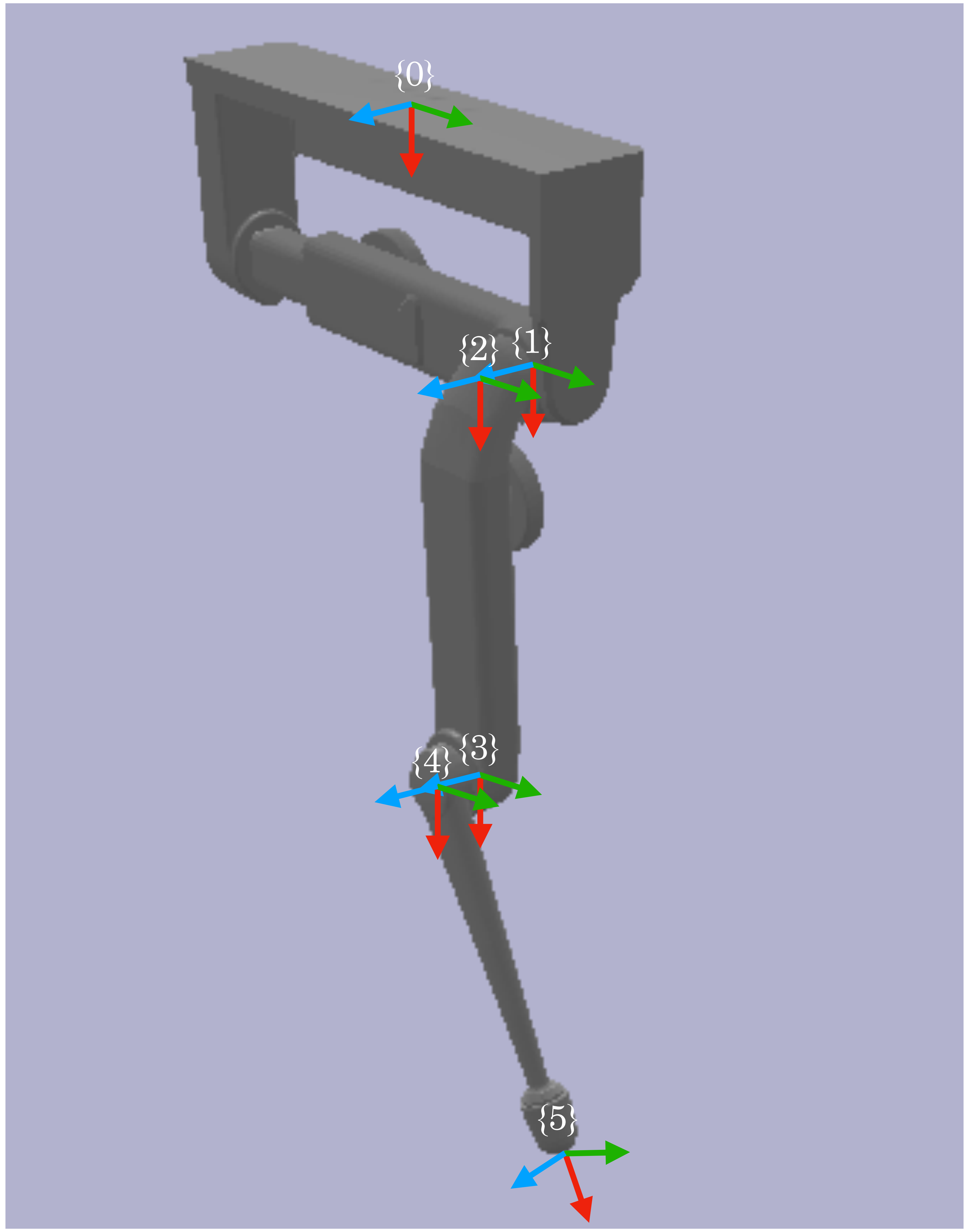


Forward Kinematics Is not 2 Dimensional!

- It is sometimes easier to add extra coordinate frames



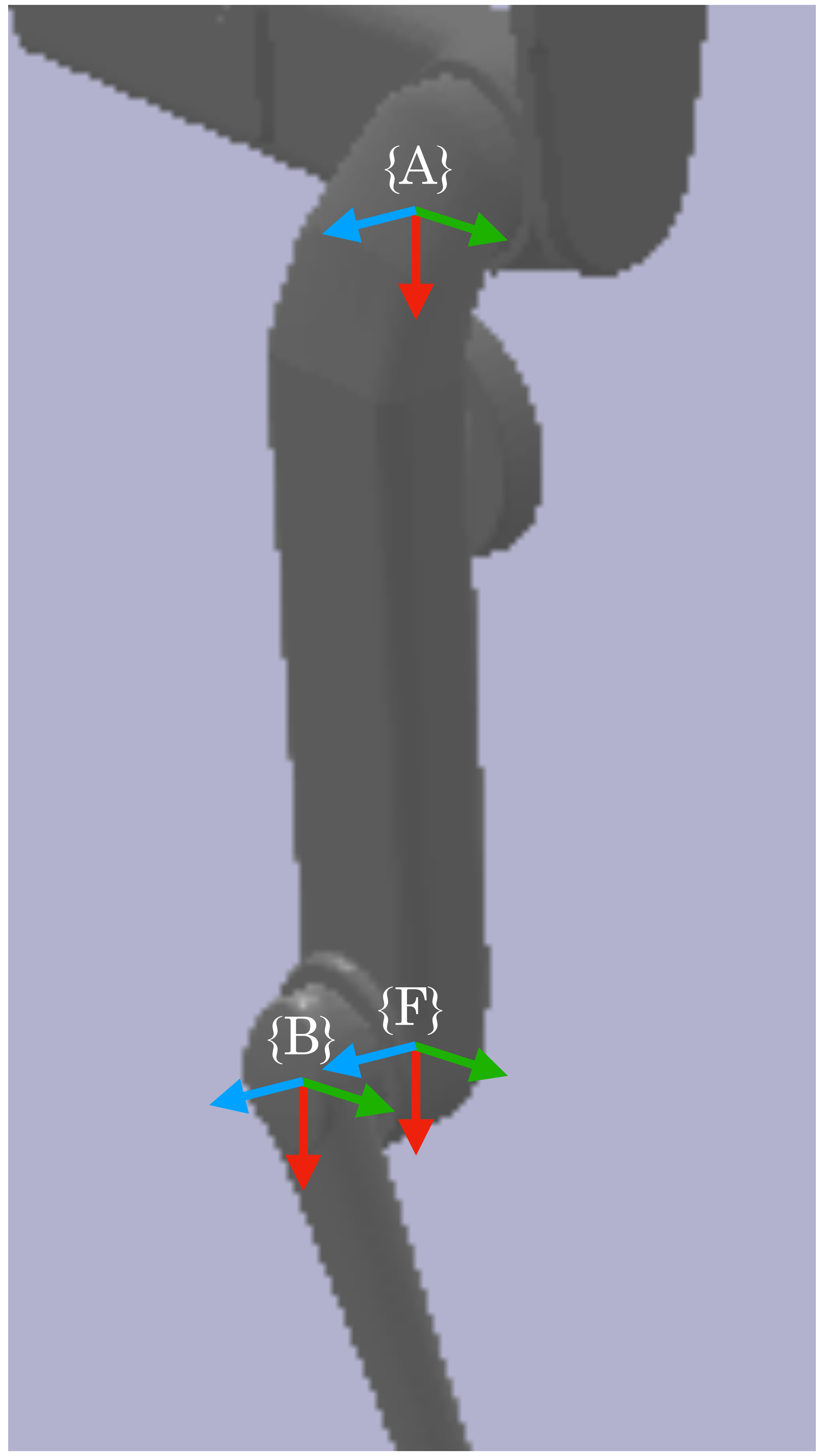
$$T_{05}(l_{0x}, l_{0y}, l_{1z}, \theta_2, l_{3z}, \theta_4, l_{4x})$$
$$= T_{01}(l_{0x}, l_{0y})T_{12}(l_{1z})T_{23}(\theta_2)T_{34}(l_{3z})T_{45}(\theta_4, l_{4x})$$

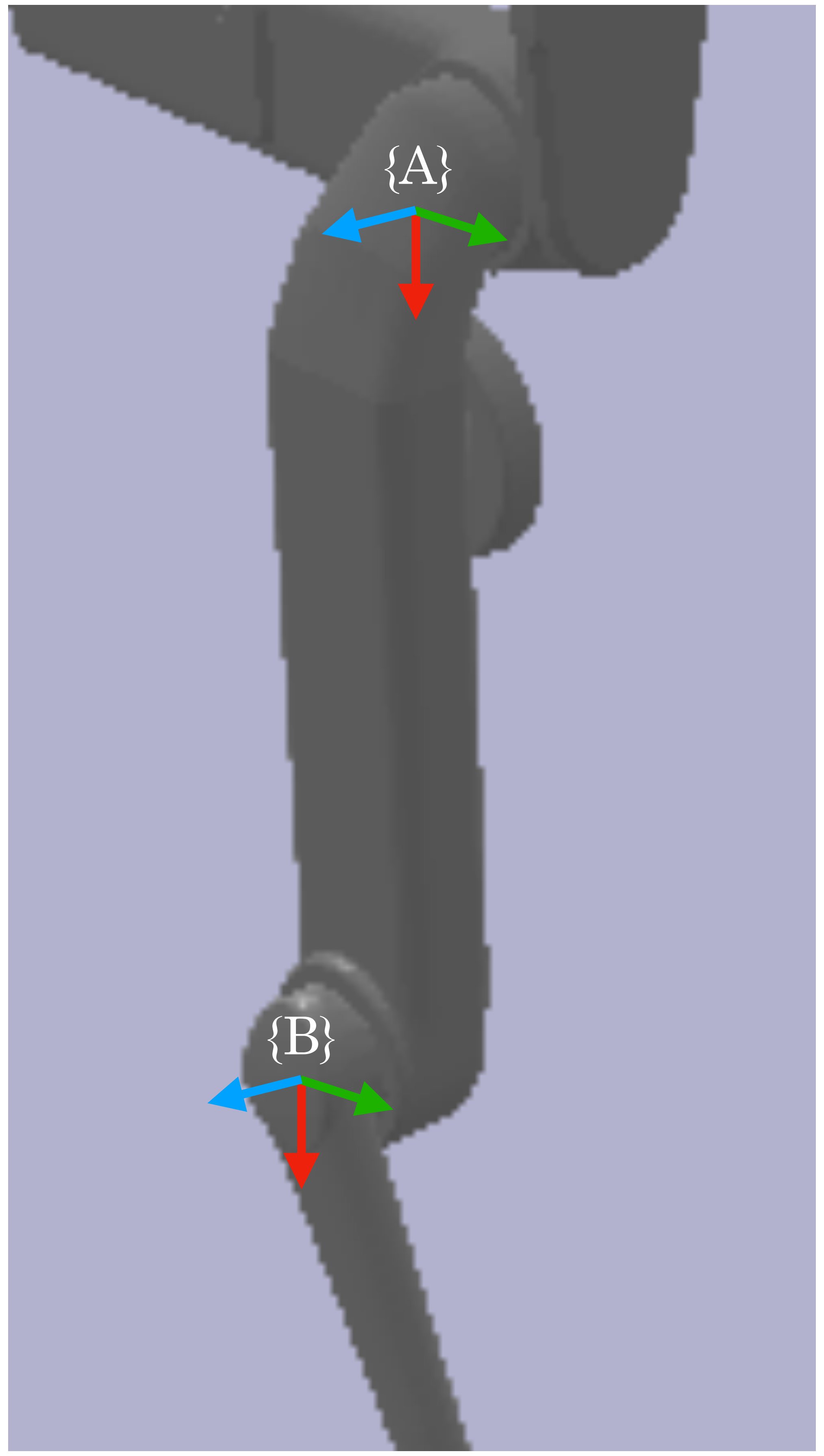


Forward Kinematics Is not 2 Dimensional!

- The FK will be the same, if between any two frames $\{A\}$ and $\{B\}$, you
  - Add an extra coordinate frame, e.g. $\{F\}$ that is co-axial with $\{A\}$ in one direction, and co-axial with $\{B\}$ in another direction

    OR
  - Include the same rotations and translations between $\{A\}$ and $\{B\}$





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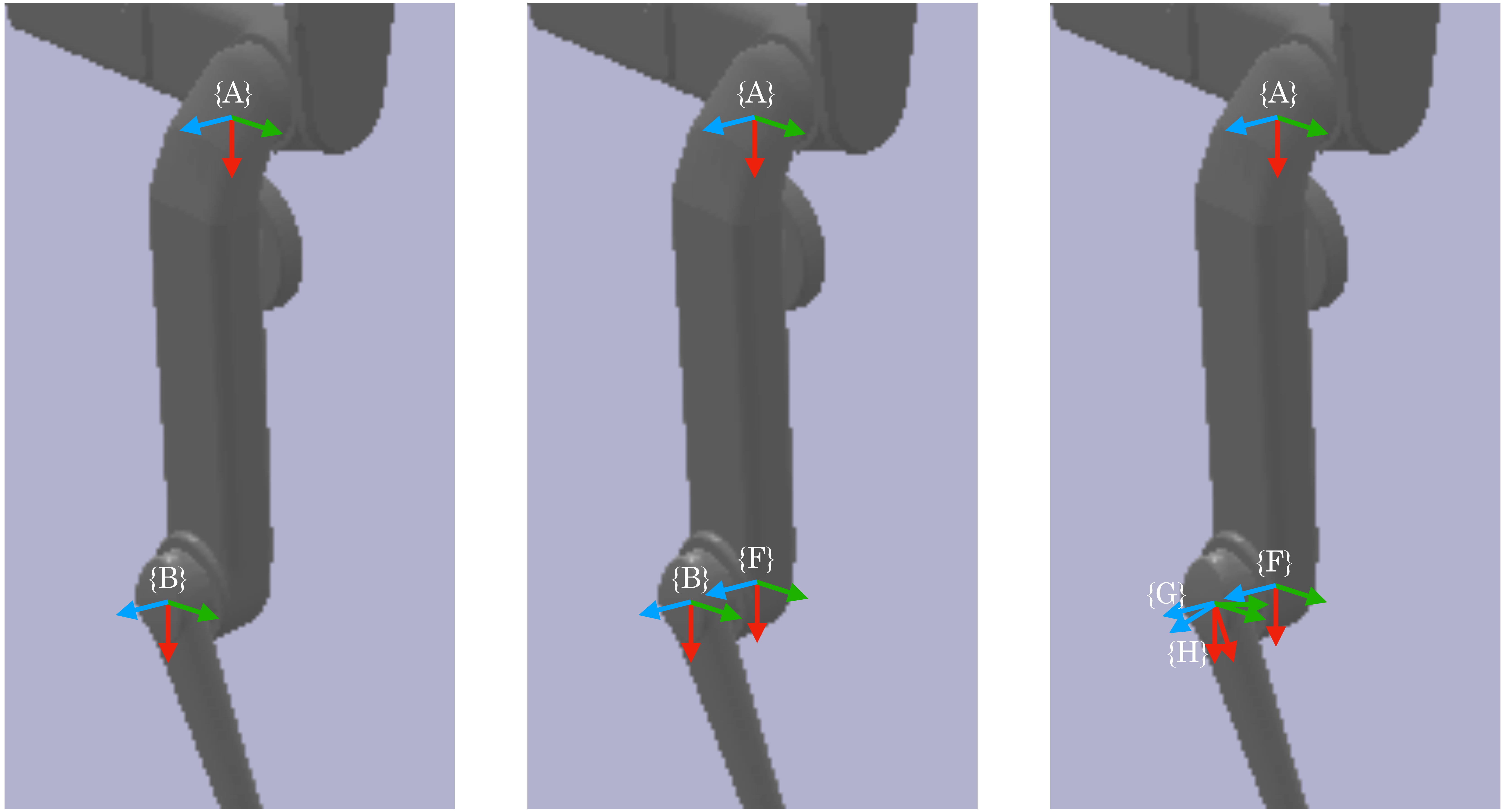
Robotic Manipulation & Locomotion



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2. **DH Convention**
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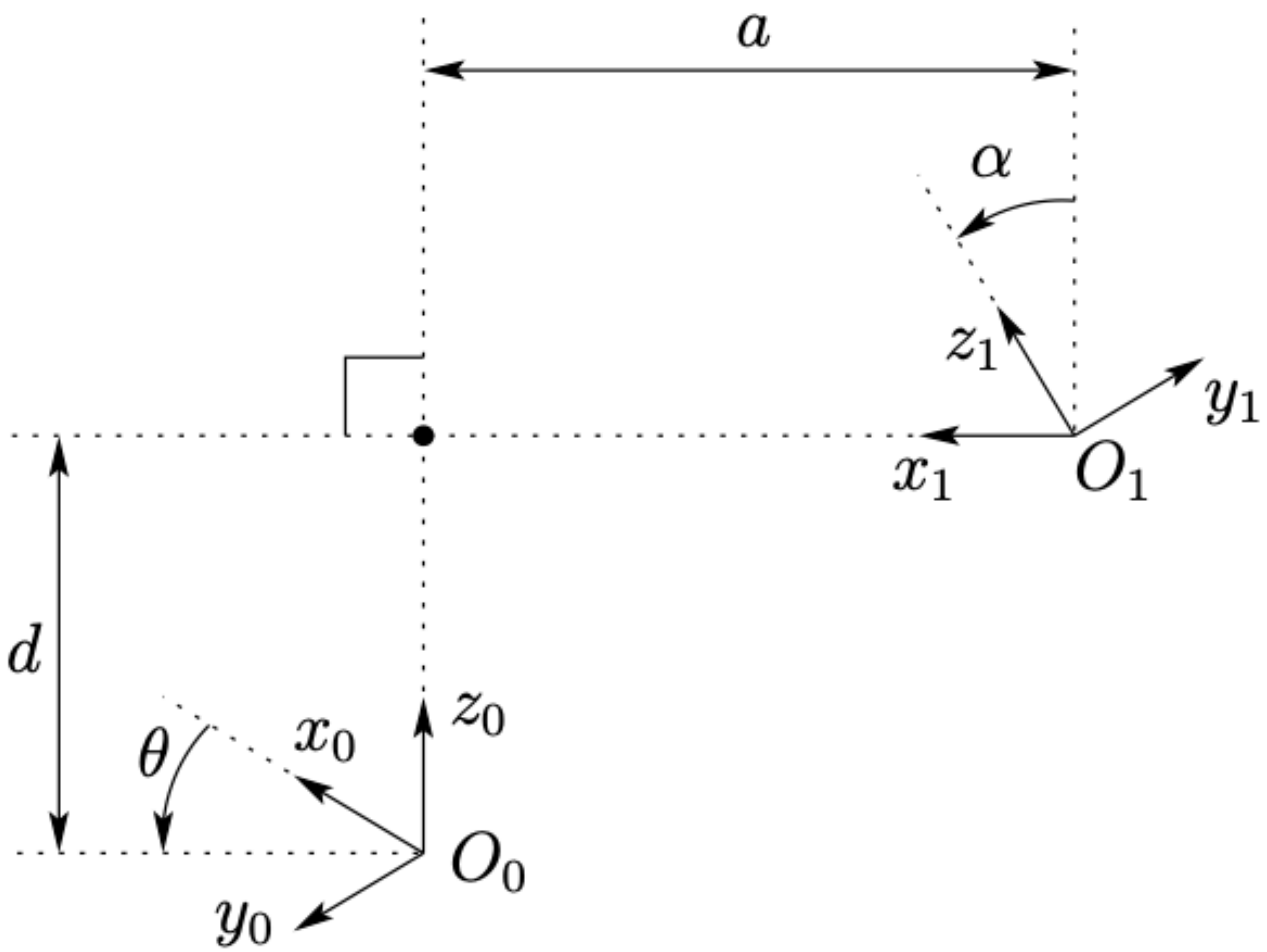
Chapter 3

FORWARD KINEMATICS: THE DENAVIT-HARTENBERG CONVENTION

https://users.cs.duke.edu/~brd/Teaching/Bio/asmb/current/Papers/chap3-forward-kinematics.pdf

D-H Convention

1. From one frame to the next, there are 4 possible transformations that come from variables
   1. Link length a_i
   2. Link twist α_i
   3. Link offset d_i
   4. Joint angle θ_i



$$
\begin{aligned}
A_i &= R_{z,\theta_i}\mathrm{Trans}_{z,d_i}\mathrm{Trans}_{x,a_i}R_{x,\alpha_i} \qquad (3.10)\\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i} & 0 & 0 \\ s_{\theta_i} & c_{\theta_i} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{\alpha_i} & -s_{\alpha_i} & 0 \\ 0 & s_{\alpha_i} & c_{\alpha_i} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\\
&= \begin{bmatrix} c_{\theta_i} & -s_{\theta_i}c_{\alpha_i} & s_{\theta_i}s_{\alpha_i} & a_i c_{\theta_i} \\ s_{\theta_i} & c_{\theta_i}c_{\alpha_i} & -c_{\theta_i}s_{\alpha_i} & a_i s_{\theta_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

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Forward Kinematics

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1. Describe the kinematic chain (links and joints)

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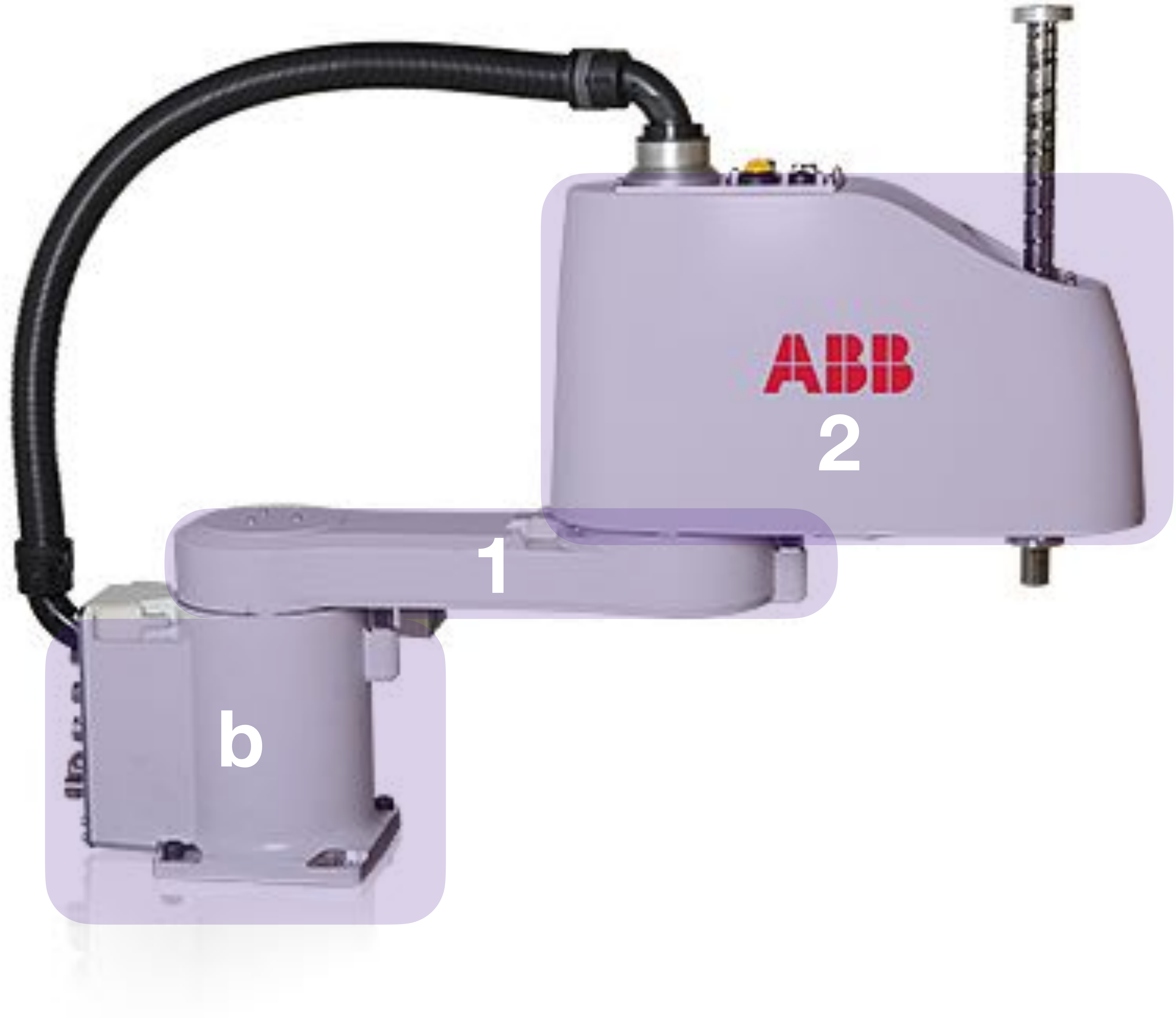
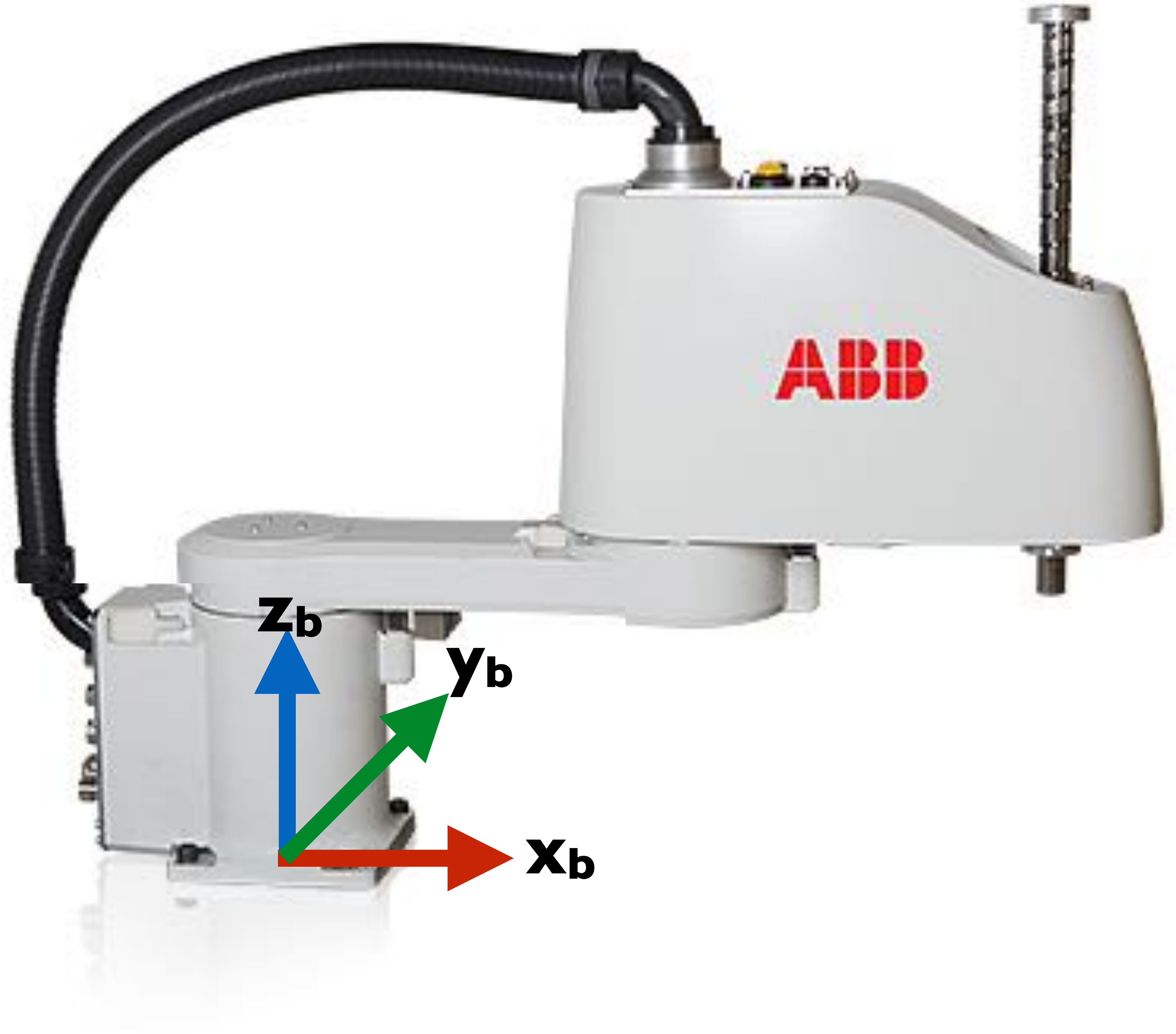


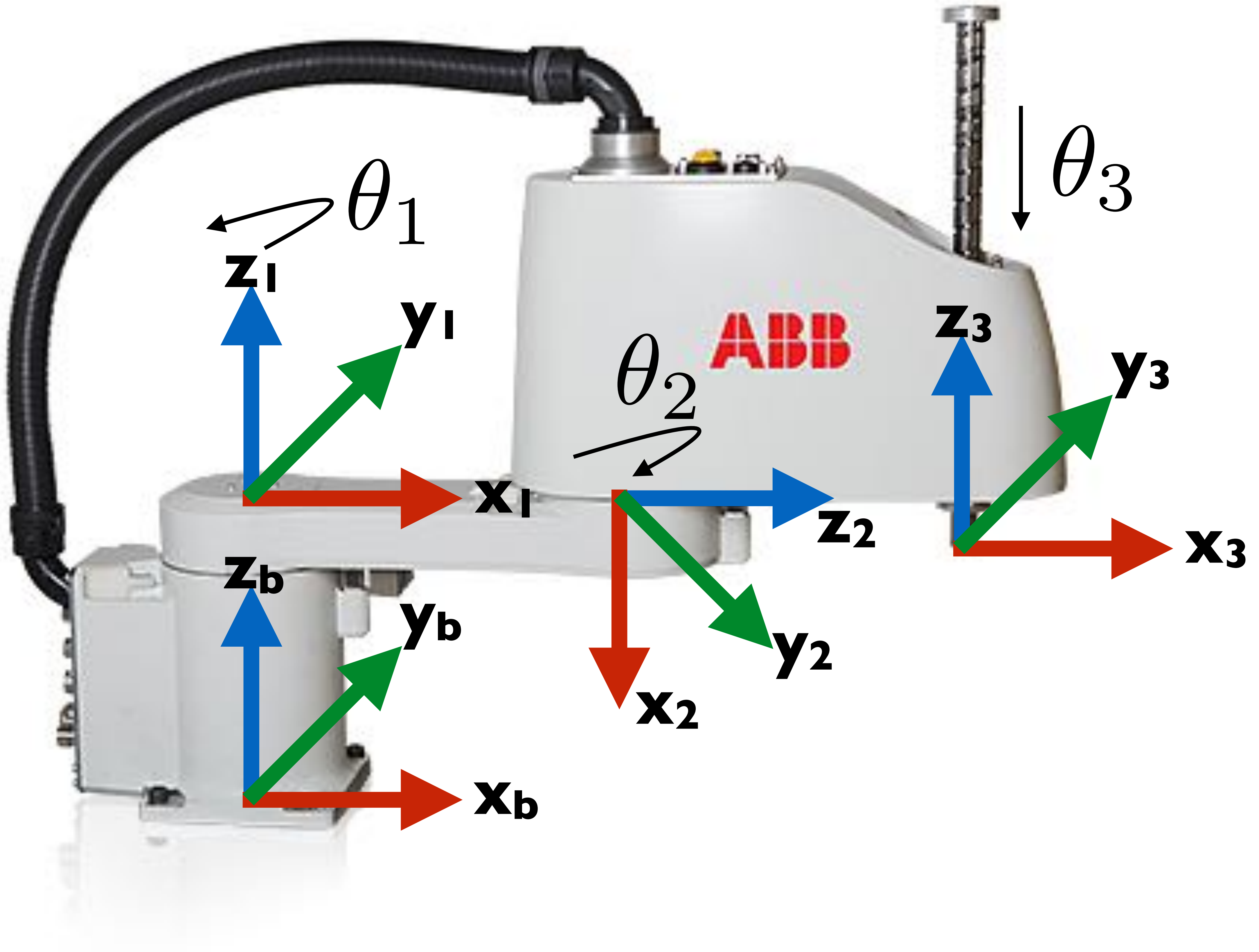
Forward Kinematics

1. Describe the kinematic chain (links and joints)
2. Choose a base frame



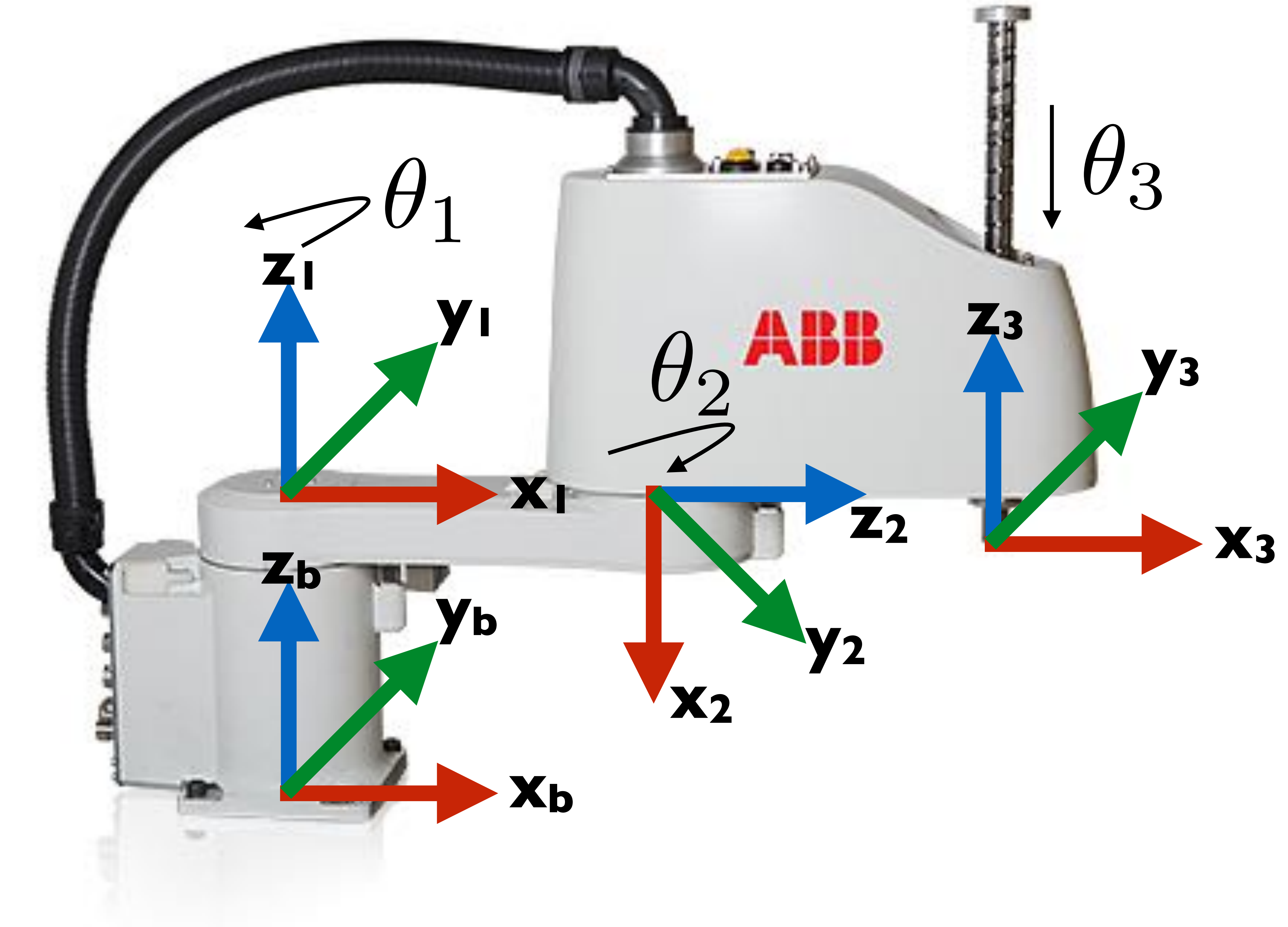
Forward Kinematics

1. Describe the kinematic chain (links and joints)
2. Choose a base frame
3. Attach a frame at each link (choose the attachment at the point of connection with the previous link to simplify computations)



Forward Kinematics

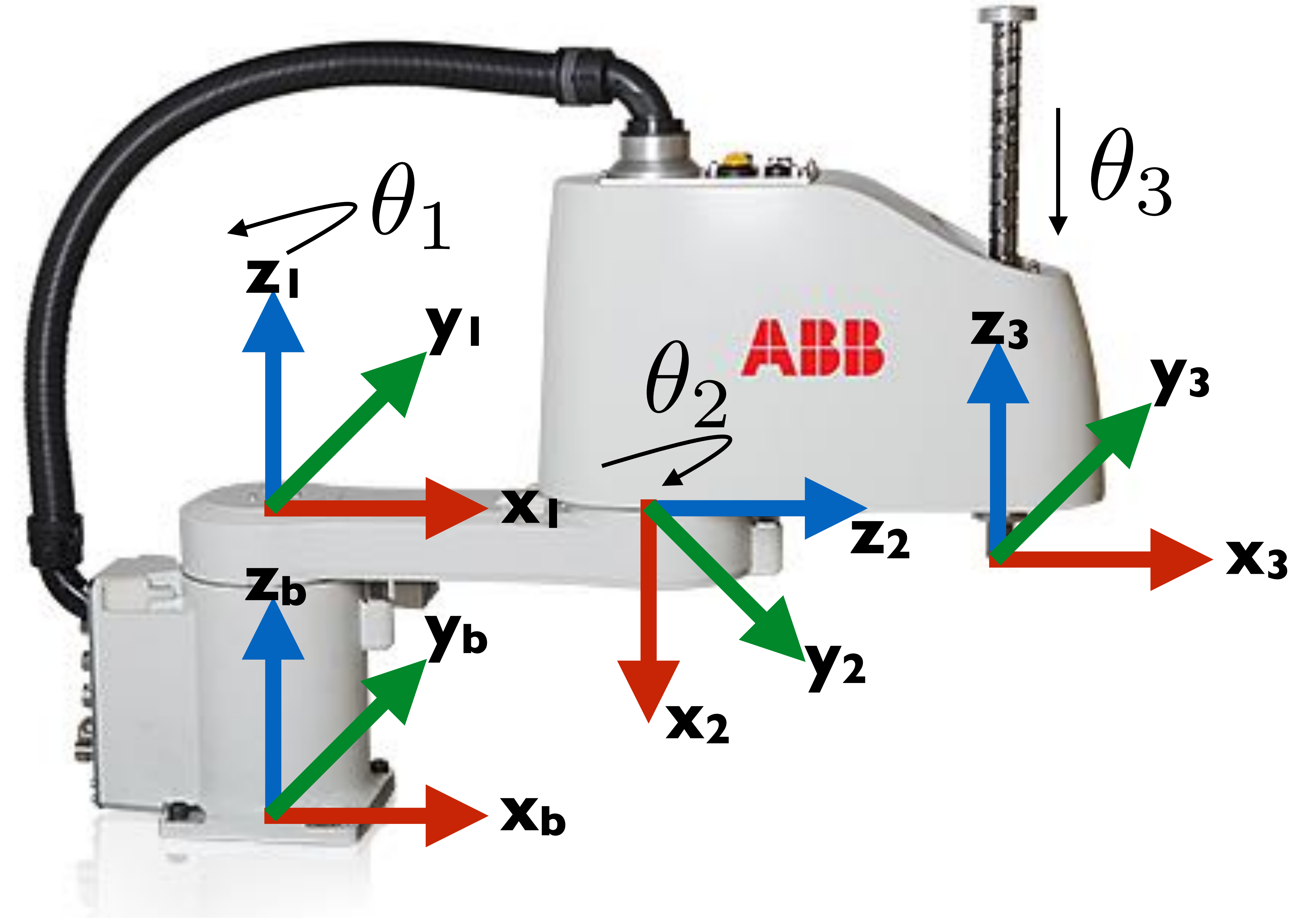
1. Describe the kinematic chain (links and joints)
2. Choose a base frame
3. Attach a frame at each link (choose the attachment at the point of connection with the previous link to simplify computations)
4. Compute the relative transform of each link relative to its predecessor - the transform will be a function of the joint state (angle or linear displacement)



$$T_{b1}(\theta_1) = \dots \quad T_{12}(\theta_2) = \dots \quad T_{23}(\theta_3) = \dots$$

Forward Kinematics

1. Describe the kinematic chain (links and joints)
2. Choose a base frame
3. Attach a frame at each link (choose the attachment at the point of connection with the previous link to simplify computations)
4. Compute the relative transform of each link relative to its predecessor - the transform will be a function of the joint state (angle or linear displacement)
5. Forward kinematics is found by multiplication of all the frames down the chain



$$FK(\theta_1, \theta_2, \theta_3) = T_{b1}(\theta_1)T_{12}(\theta_2)T_{23}(\theta_3)$$

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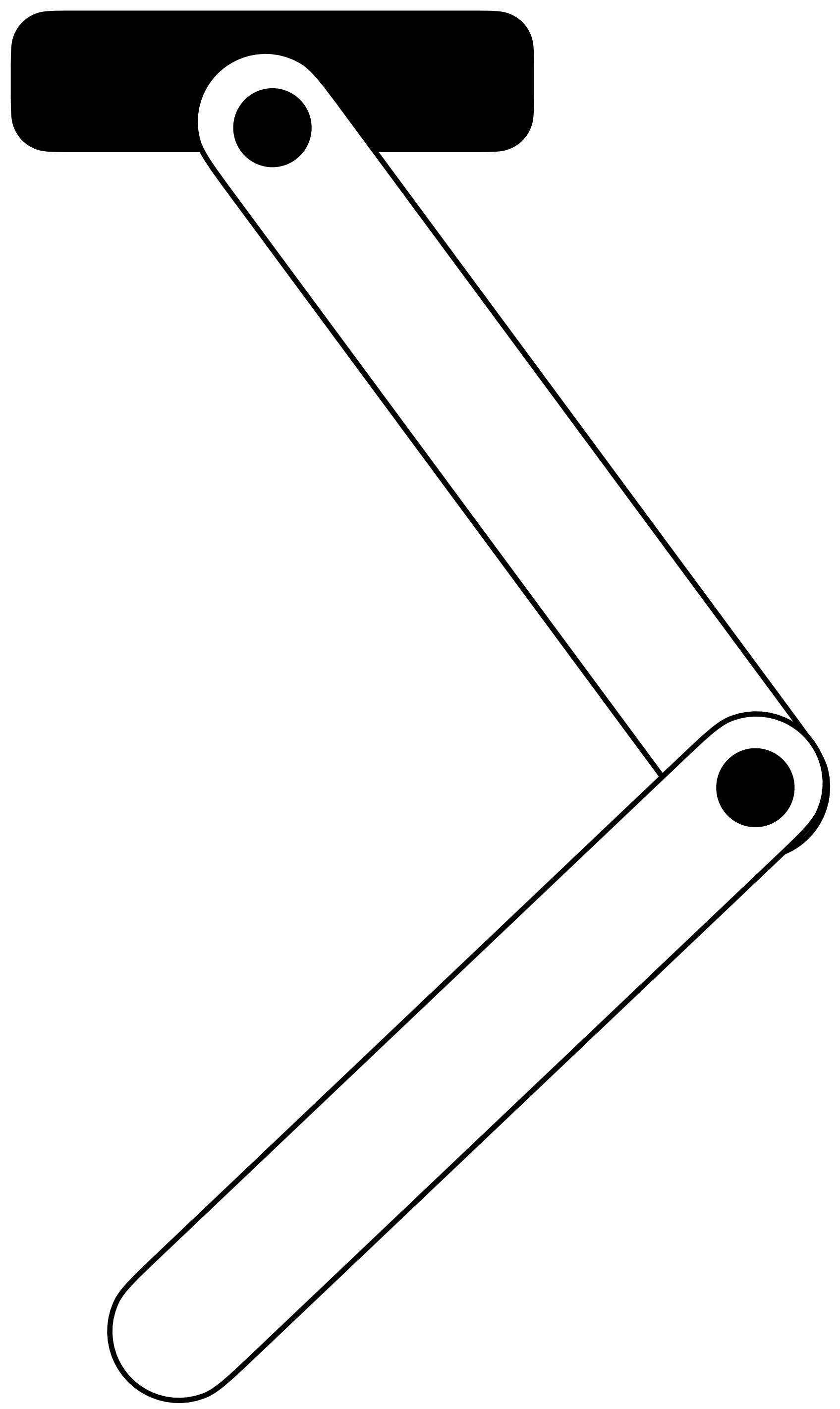
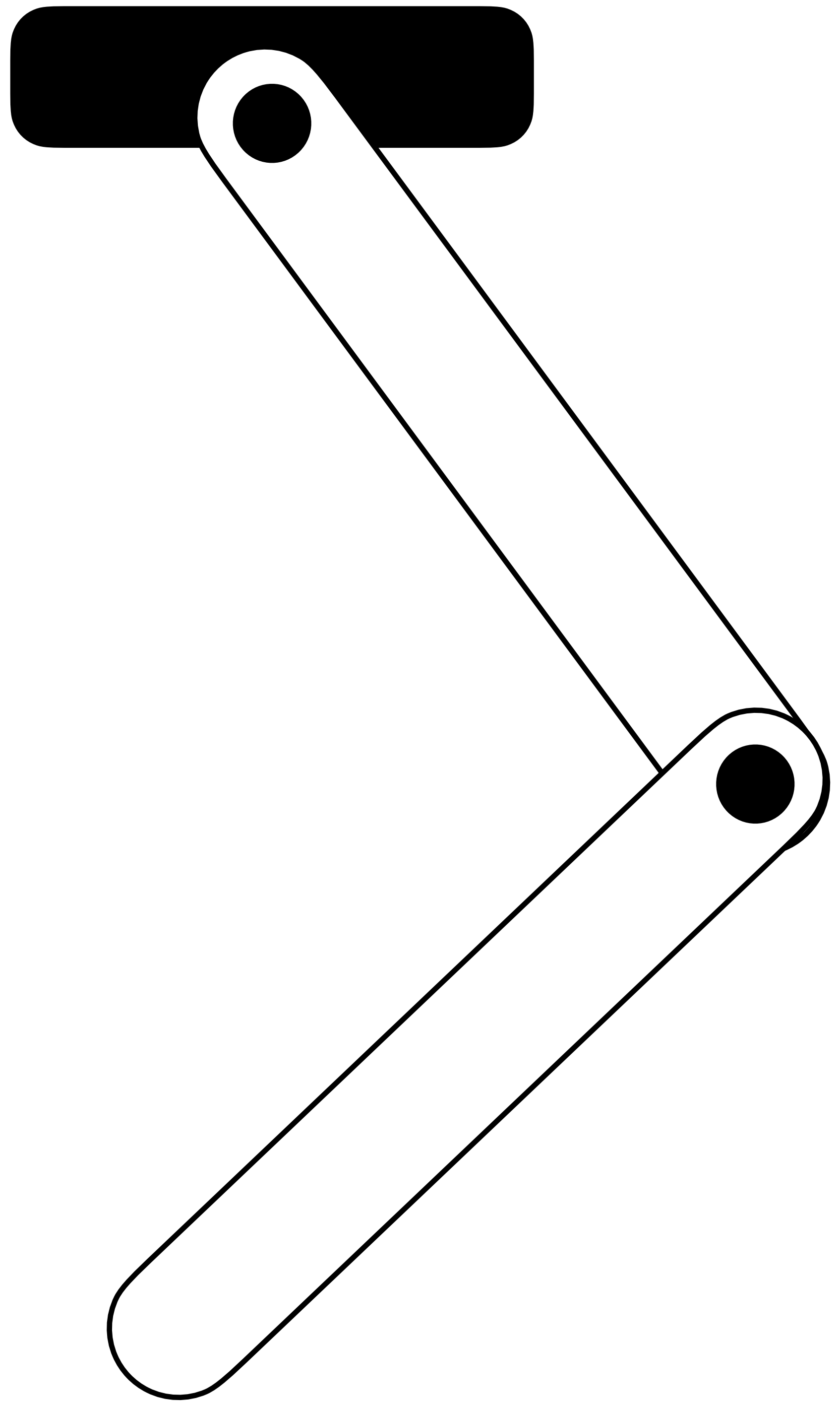
URDF - Unified Robotics Description Format

1. Is a geometrical model of a multi-body system
2. XML specification
3. Key model elements are
    1. Robots
    2. Links
    3. Joints
4. Elements are related to each other via a tree structure of:
    1. Parents
    2. Children



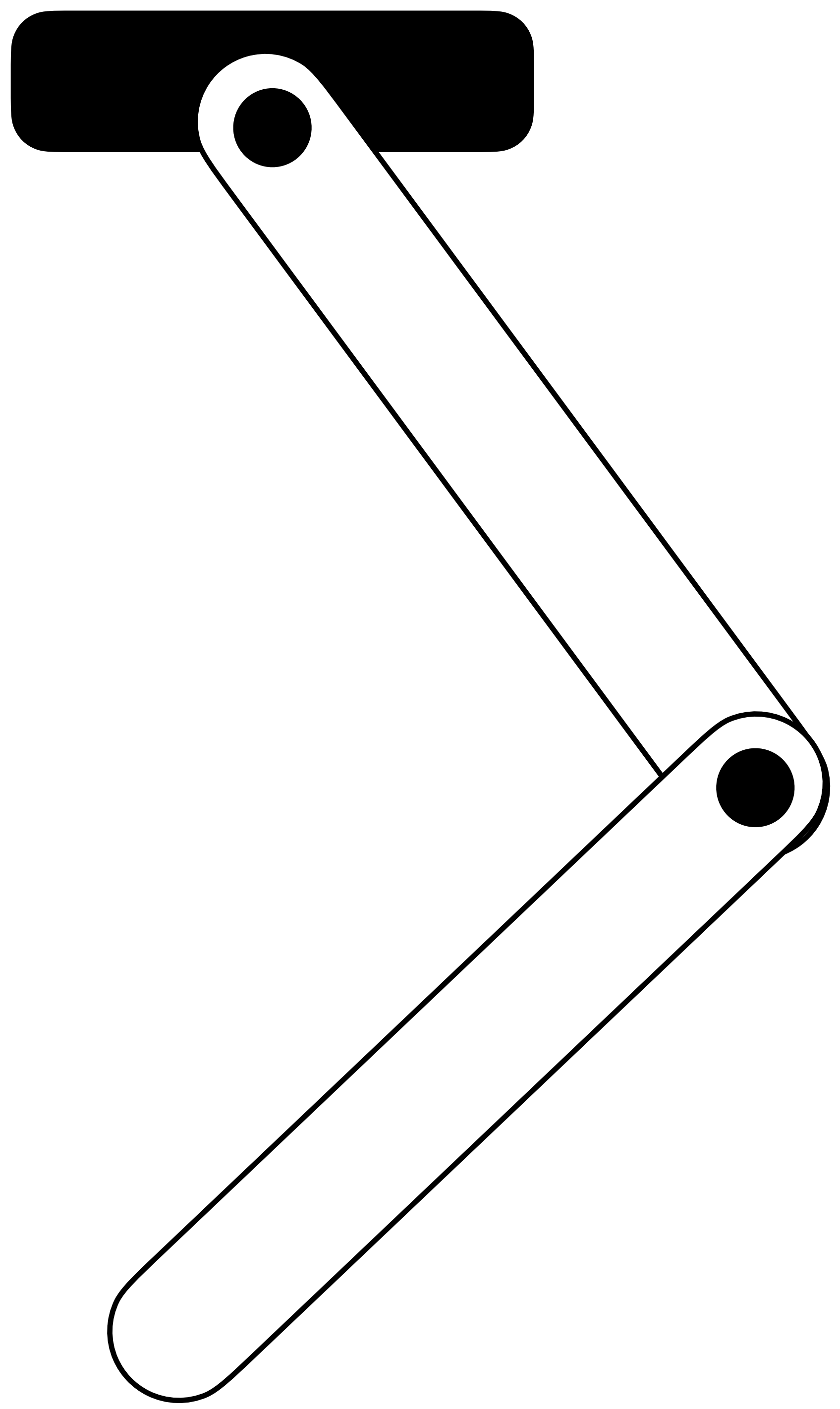
URDF - Unified Robotics Description Format

```
<robot>
        <link>
                ...
        </link>
        <link>
                ...
        </link>
        <joint>
                ...
        </joint>
</robot>
```



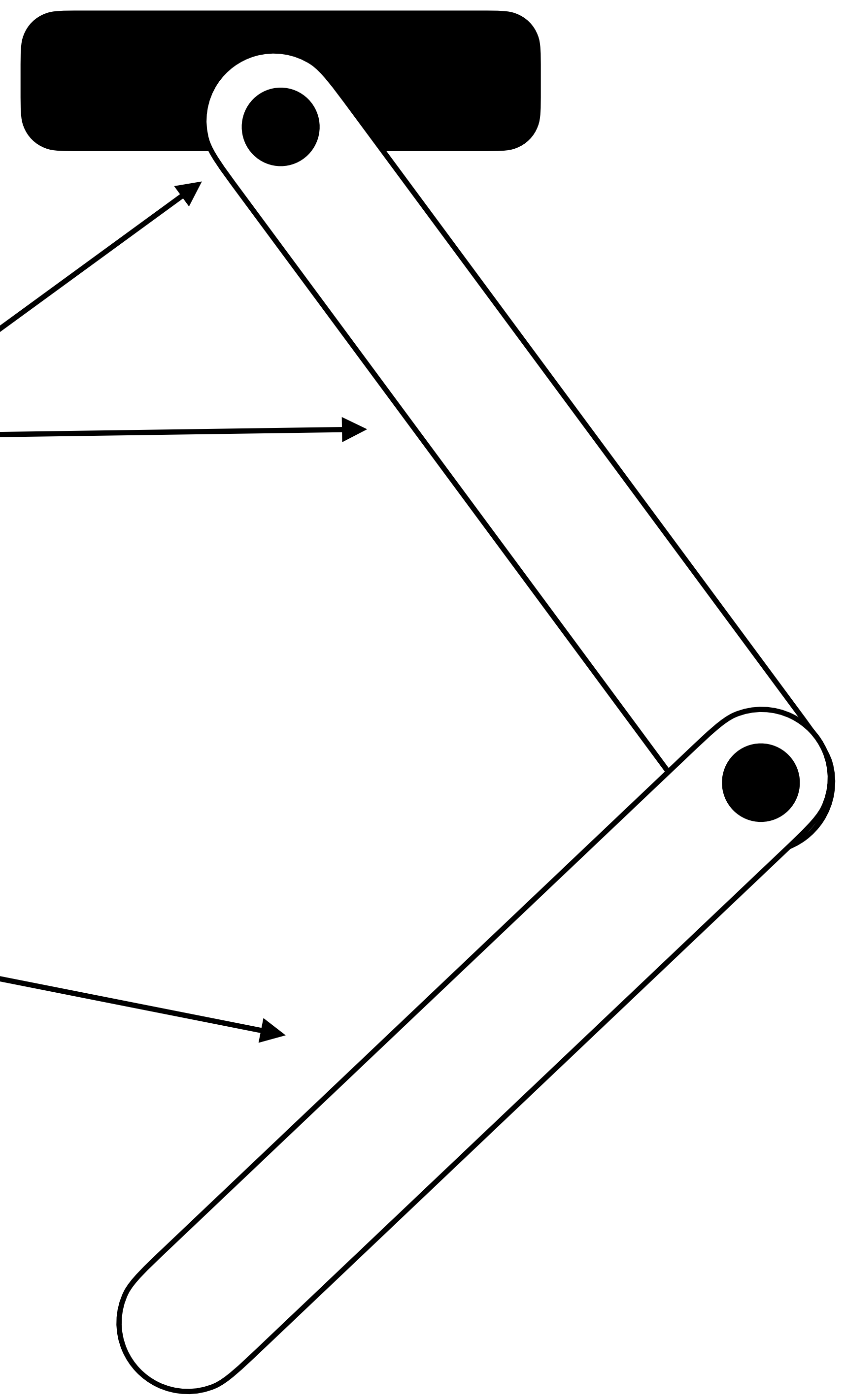
URDF - Unified Robotics Description Format

```
<robot>
    <link>
        ...
    </link>
    <link>
        ...
    </link>
    <joint>
        ...
    </joint>
</robot>
```



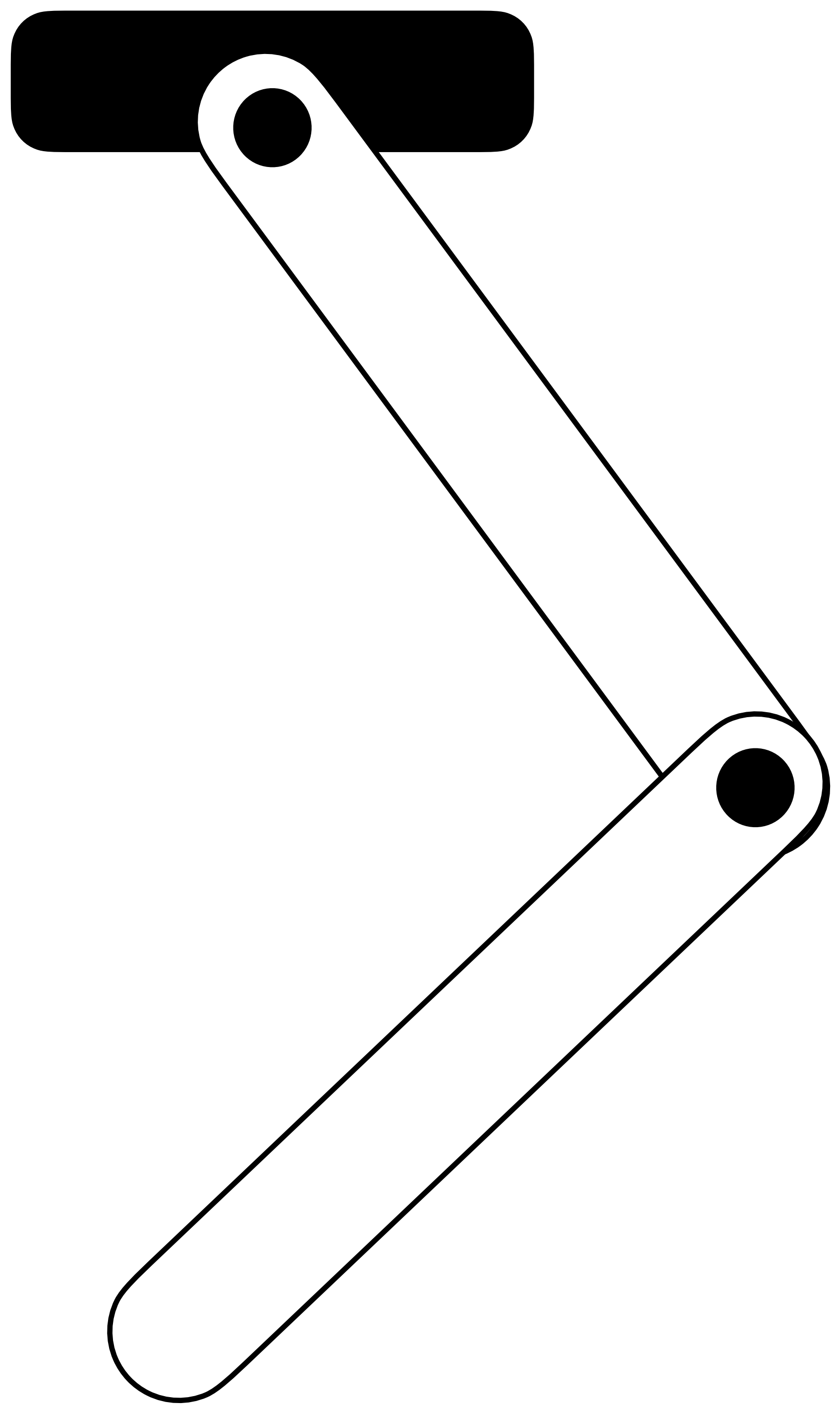
URDF - Unified Robotics Description Format

```
<robot>
    <link>
        ...
    </link>
    <link>
        ...
    </link>
    <joint>
        ...
    </joint>
</robot>
```

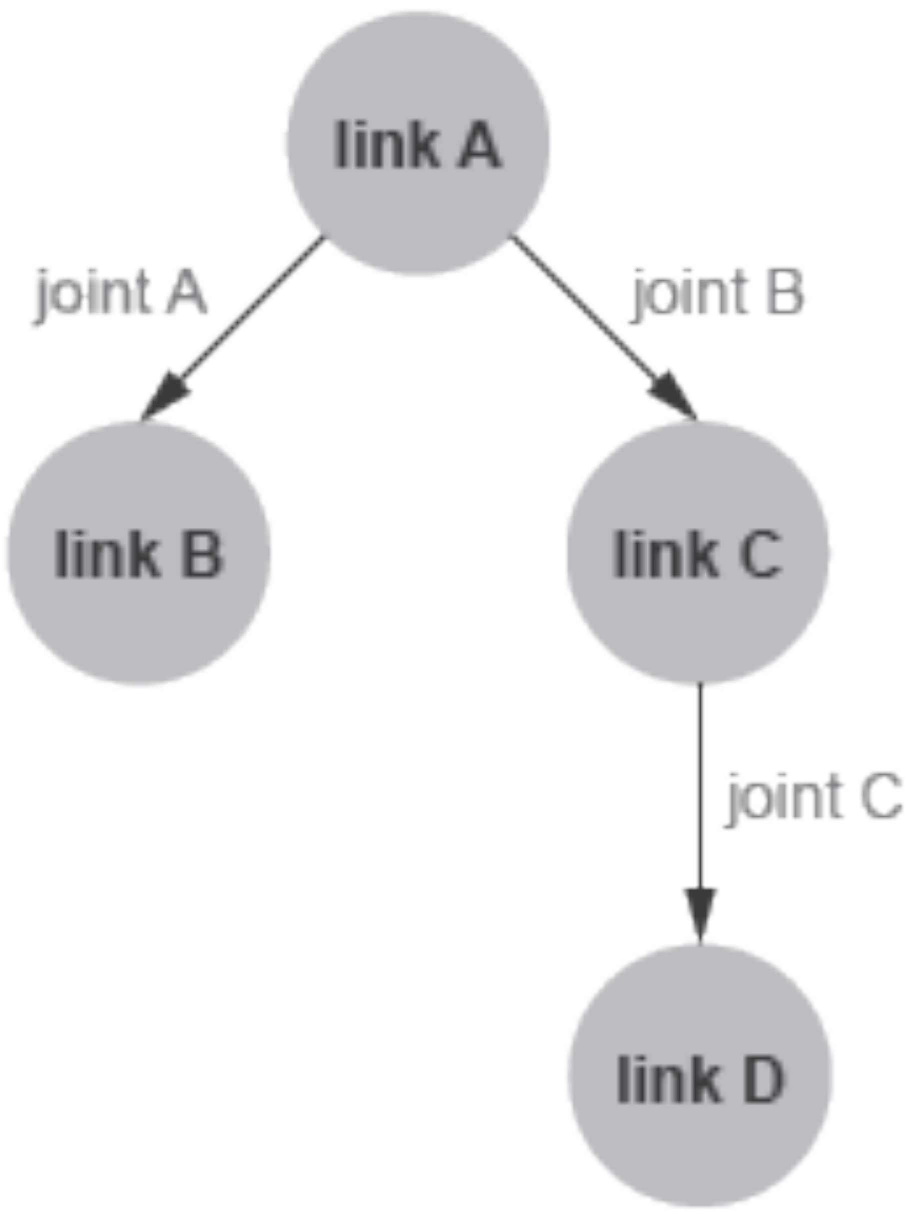


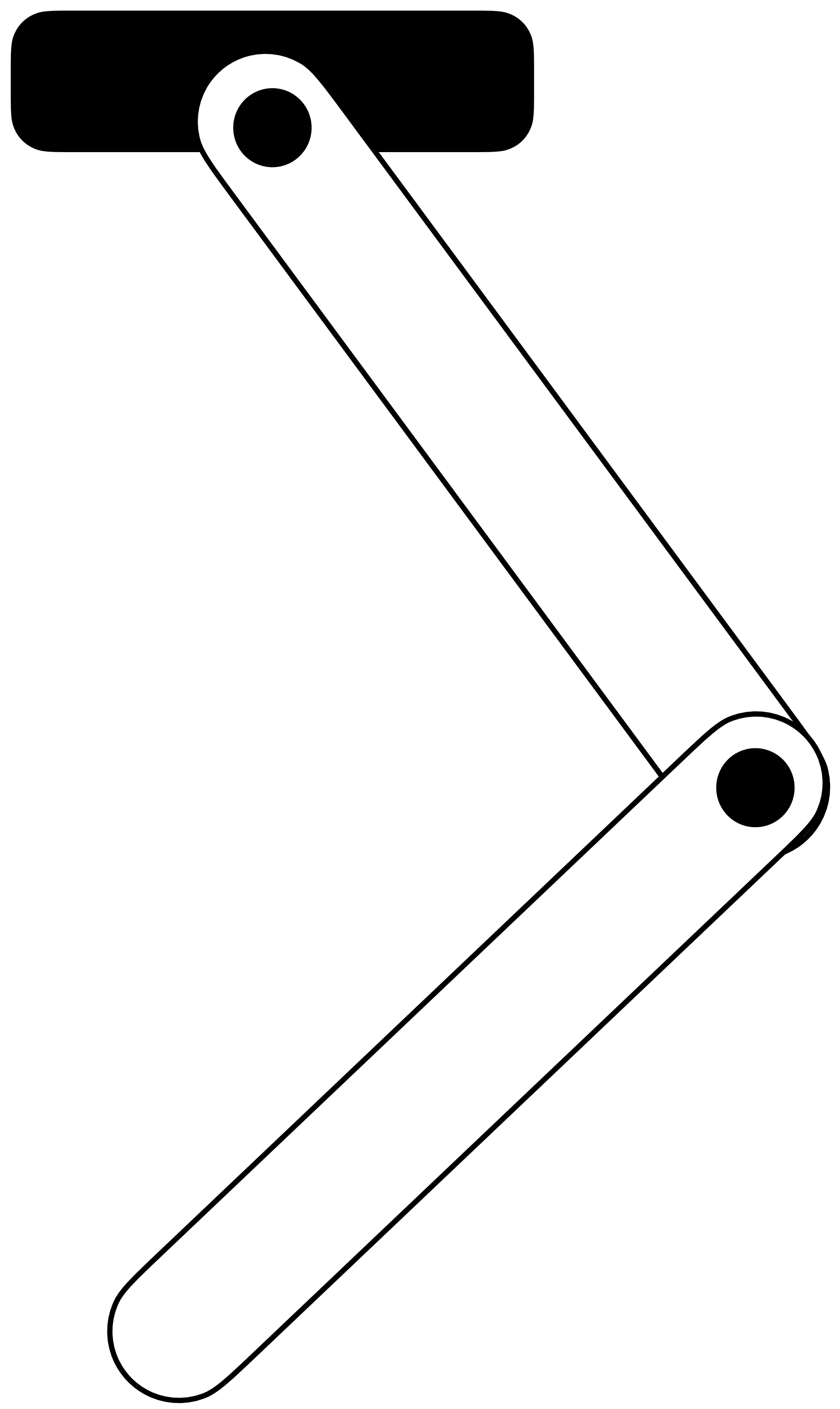
URDF - Unified Robotics Description Format

```
<robot>
        <link>
                <inertial>
                        ...
                </inertial>
                <visual>
                        <geometry>
                                ...
                        </geometry>
                        <material>
                                <color />
                        </material>
                </visual>
        </link>
        ...
</robot>
```



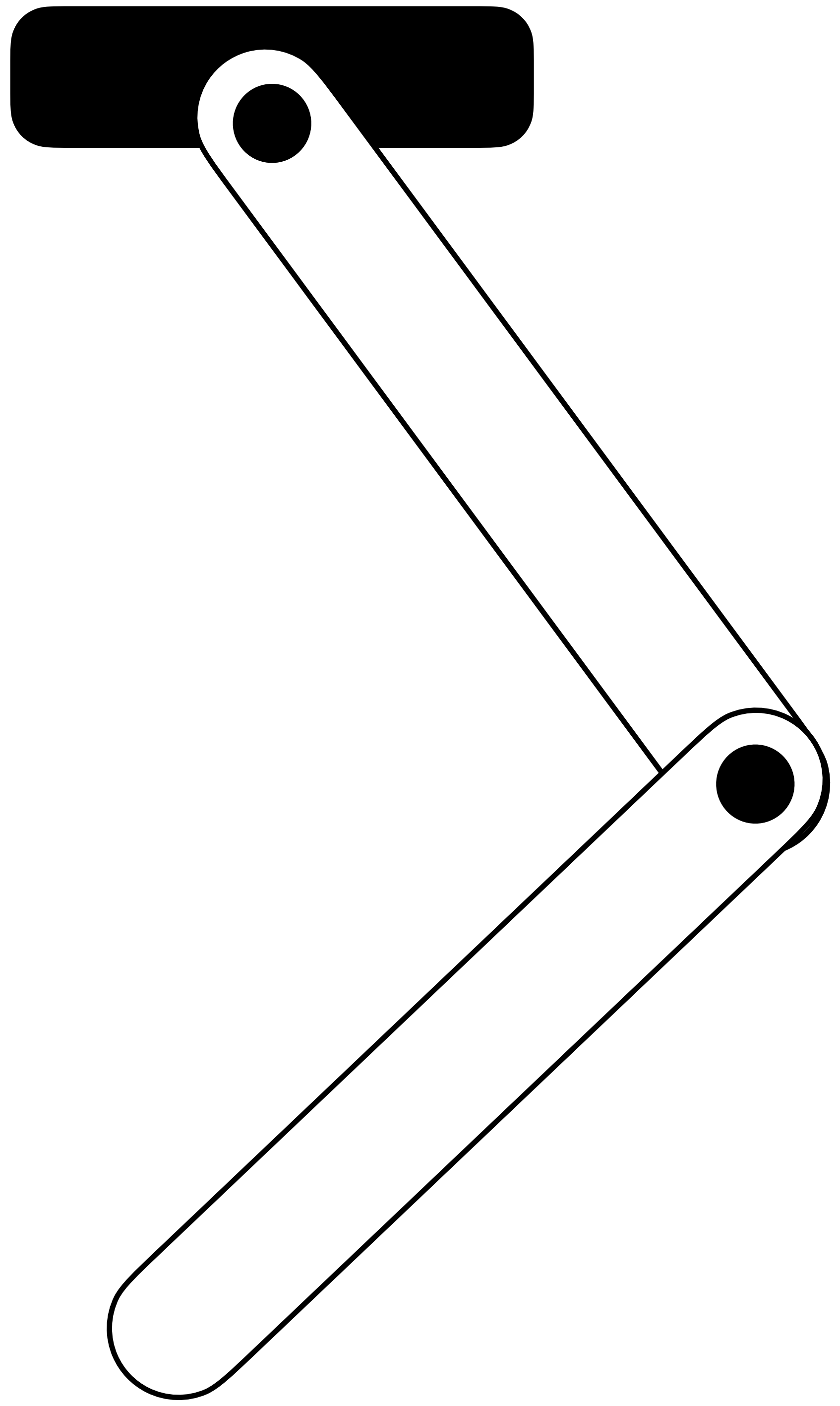
URDF - Unified Robotics Description Format





URDF - Unified Robotics Description Format

```
<robot name = "linkage">
        <joint name = "joint A ... >
                <parent link = "link A" />
                <child link = "link B" />
        </joint>
        <joint name = "joint B ... >
                <parent link = "link A" />
                <child link = "link C" />
        </joint>
        <joint name = "joint C ... >
                <parent link = "link C" />
                <child link = "link D" />
        </joint>
</robot>
```



URDF - Unified Robotics Description Format

URDF file

```
<robot name = "linkage">
        <joint name = "joint A ... >
                <parent link = "link A" />
                <child link = "link B" />
        </joint>
        <joint name = "joint B ... >
                <parent link = "link A" />
                <child link = "link C" />
        </joint>
        <joint name = "joint C ... >
                <parent link = "link C" />
                <child link = "link D" />
        </joint>
</robot>
```

→

URDFpy
Or
Pinocchio

→

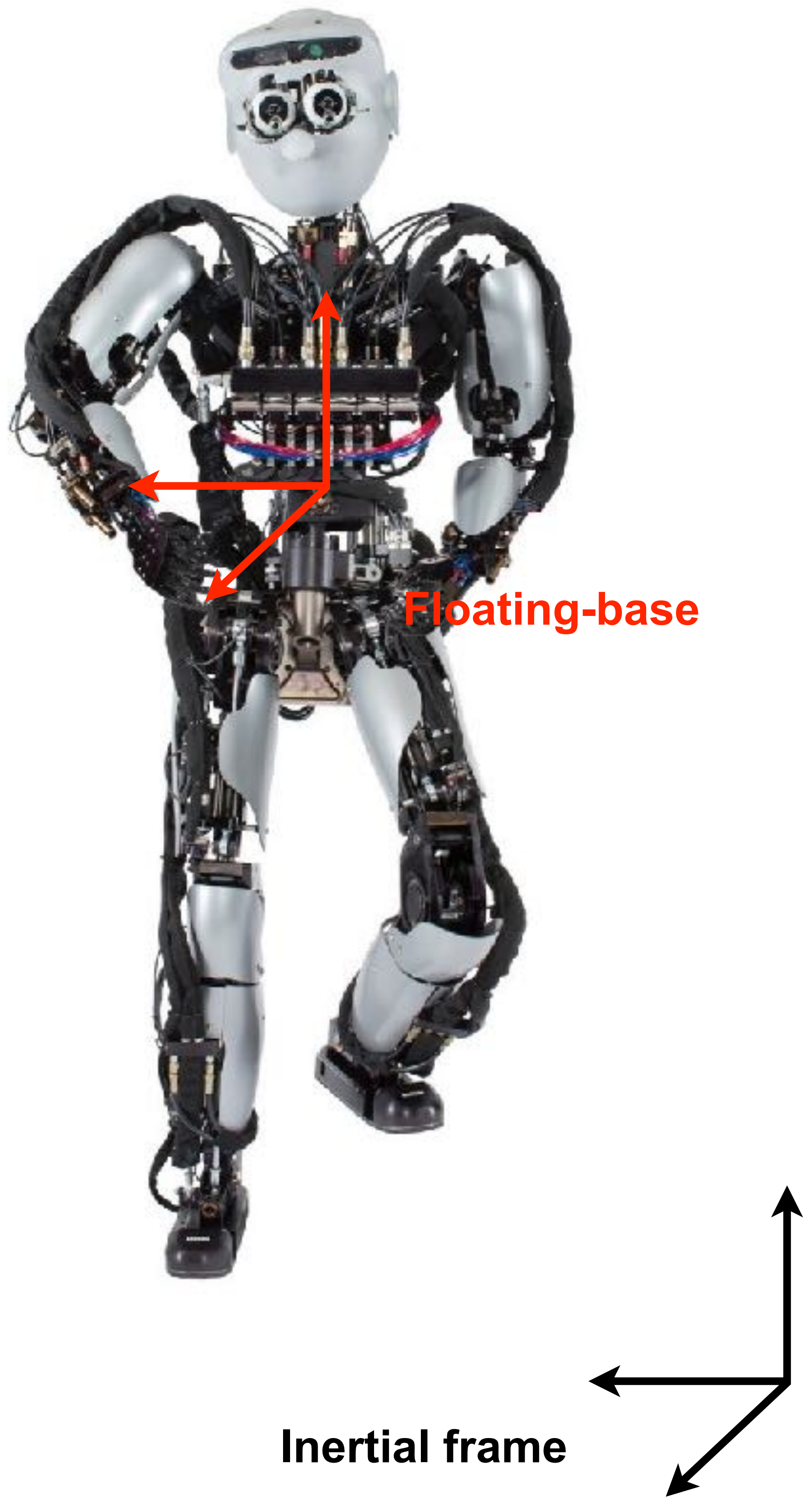
$$FK(\theta_1, \theta_2, \theta_3) = T_{b1}(\theta_1)T_{12}(\theta_2)T_{23}(\theta_3)$$

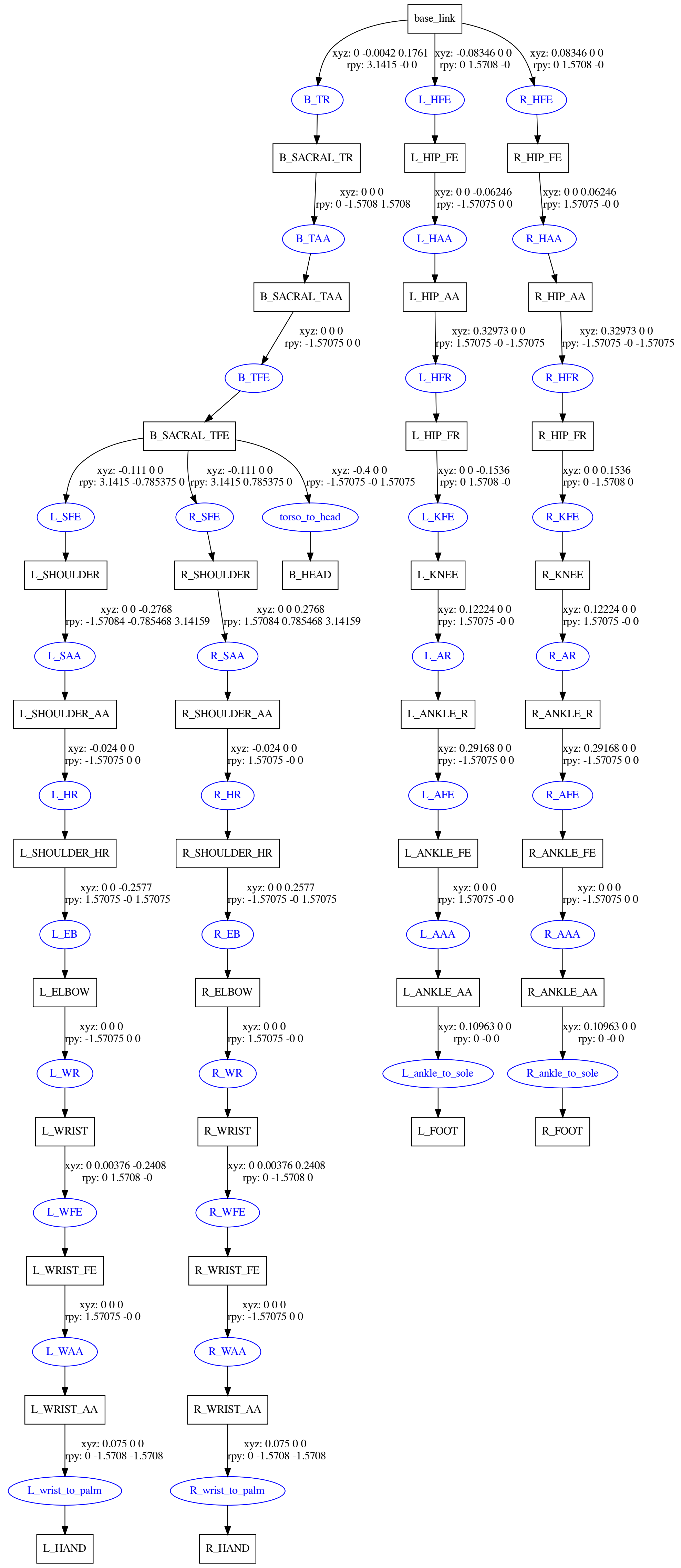
Agenda

1. Forward Kinematics Introduction
2. DH Convention
3. Systematic Approach to FK
4. URDF
5. **Kinematic Trees**

ROB-UY 2004

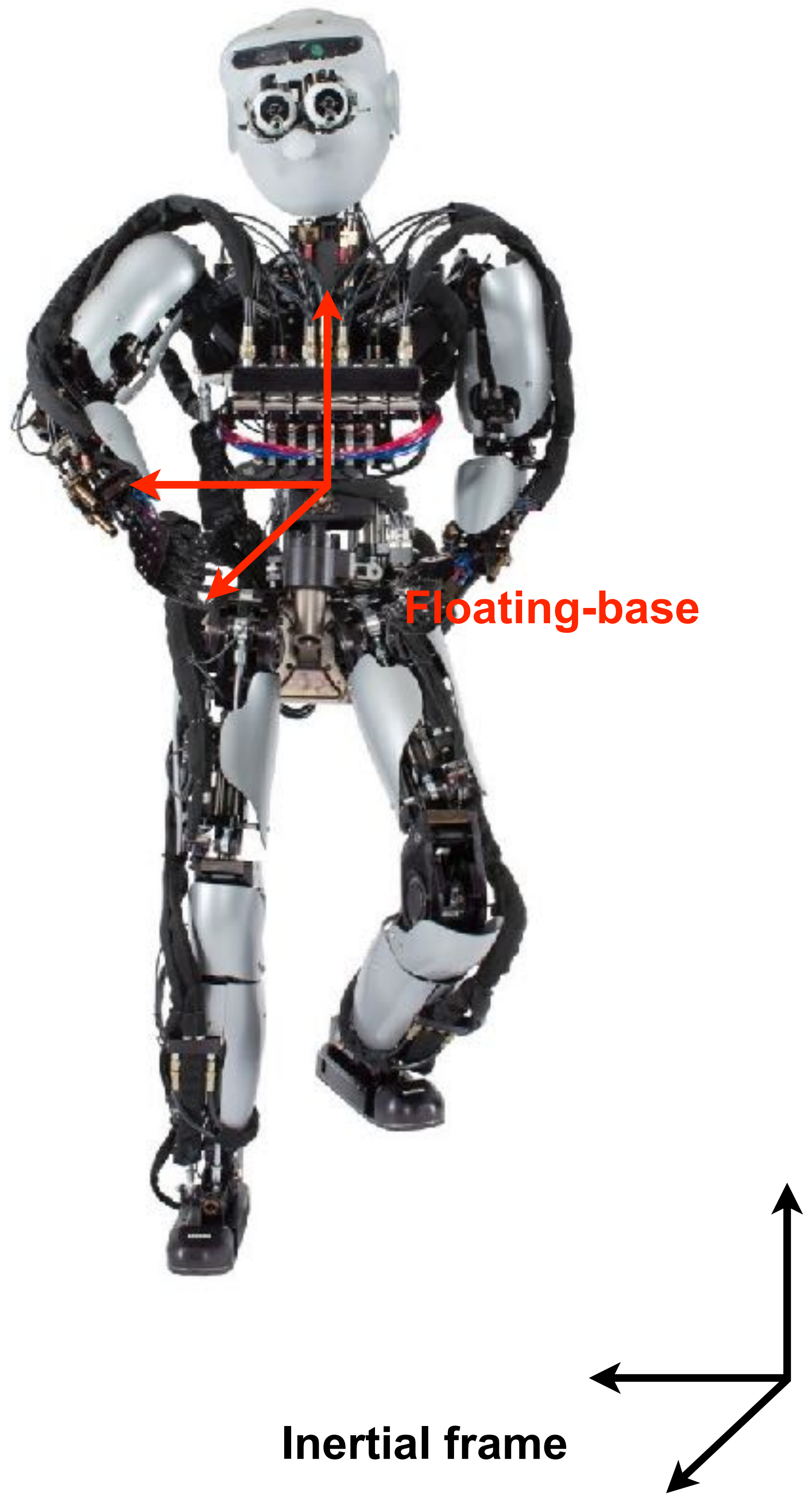
Robotic Manipulation & Locomotion

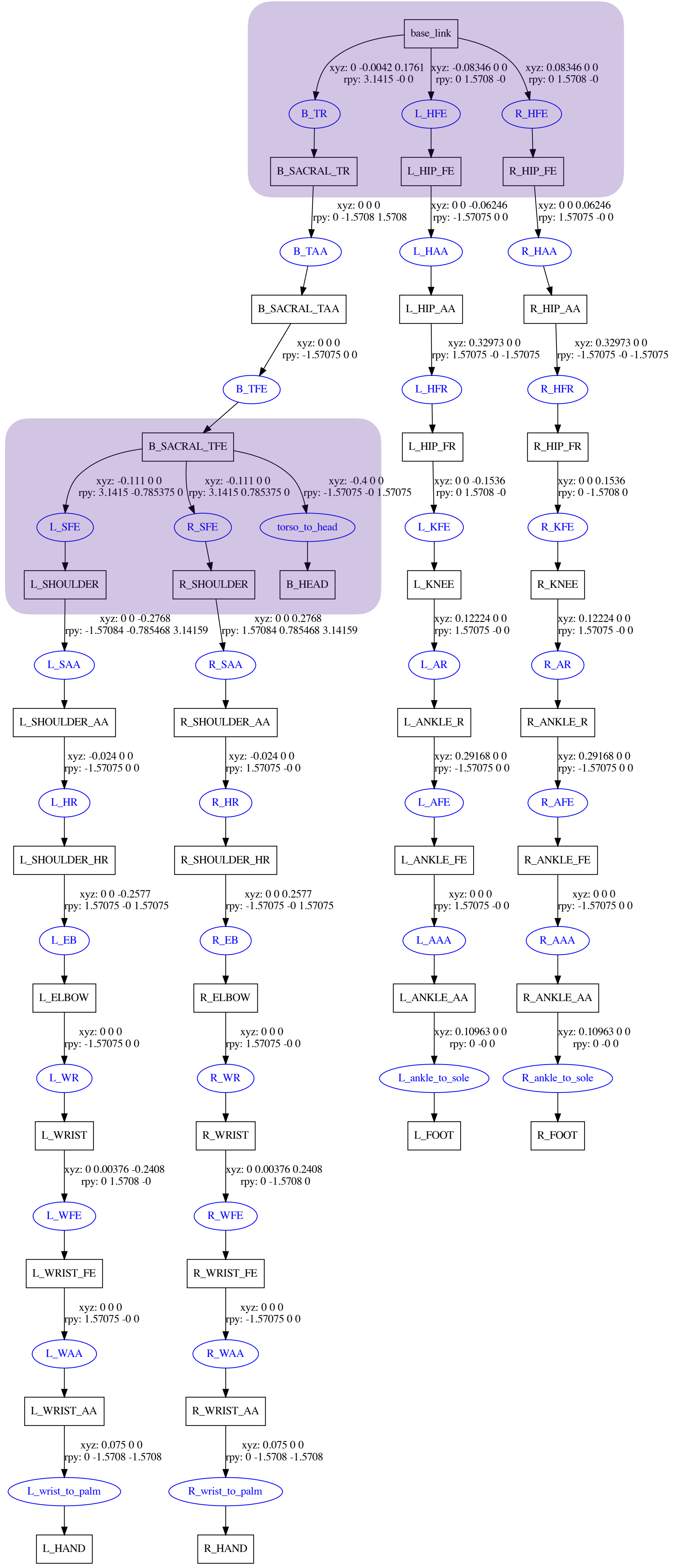


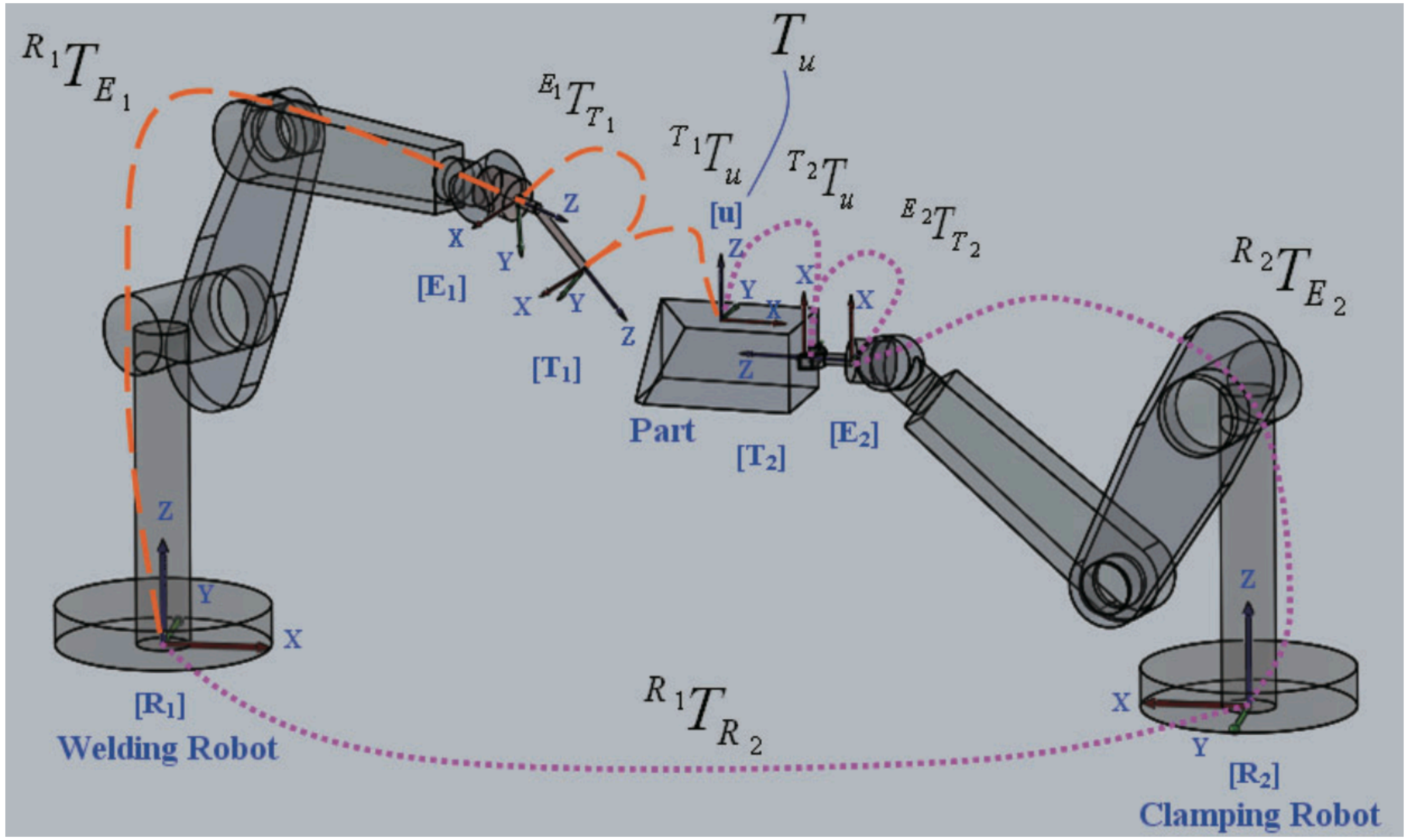


ROB-UY 2004

Robotic Manipulation & Locomotion



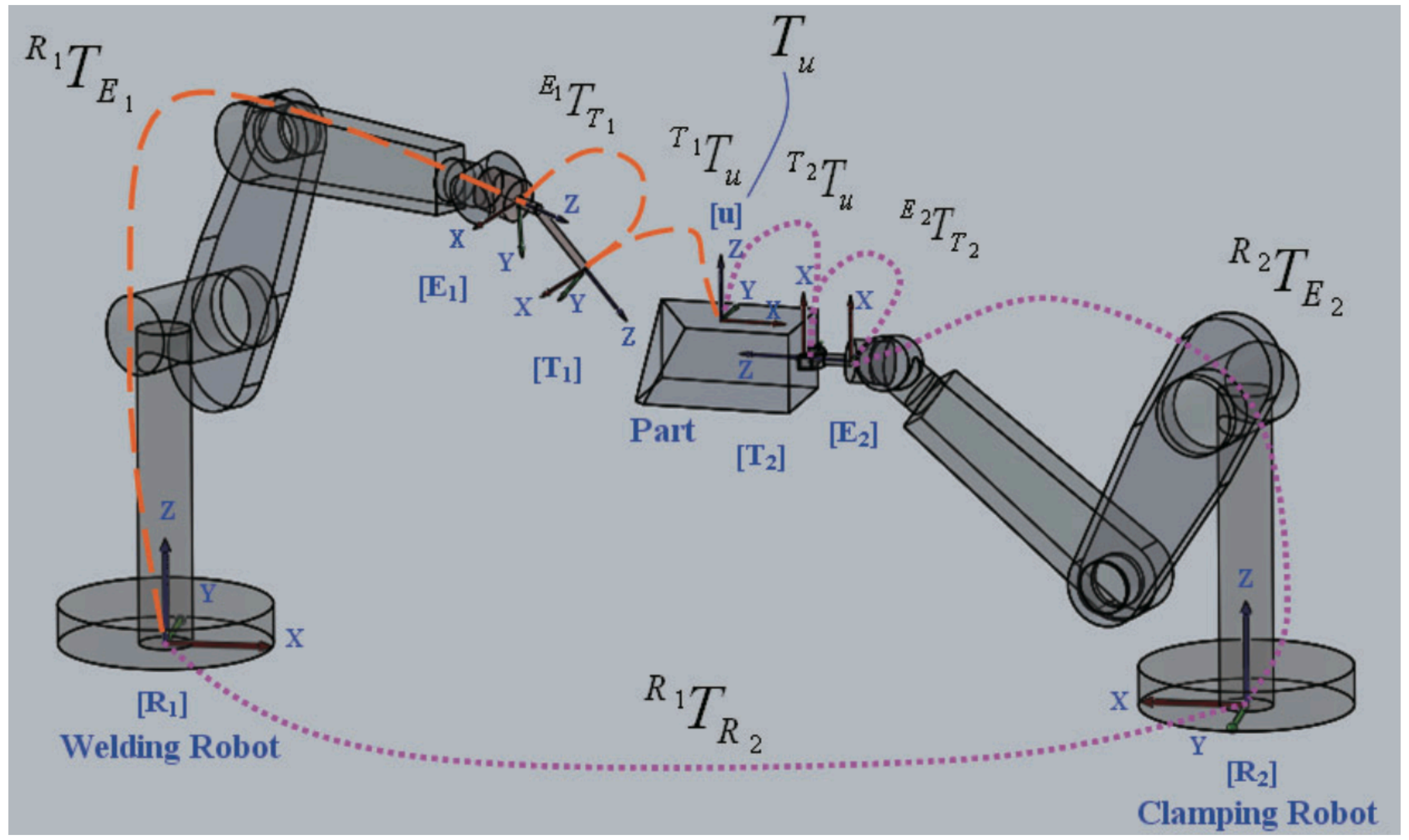




Ouyang et. Al. , Offline Kinematics Analysis and Path Planning of Two-Robot Coordination in Exhaust Manifold Welding, 2012

Take-aways

- **Homogeneous Transformations** are matrices comprised of rotations and translations
- They are useful for:
  - Describing a pose
  - Coordinate transformations
  - Moving an object
  - Forward Kinematics



Take-aways

- **Forward Kinematics** are homogeneous transformations
- They are useful for:
  - Reachability calculations (workspace mapping)
  - Collision checking
  - Motion planning
  - Jacobian computation