

---

## Contents

Expression for formula of integration by parts . . . . .	2
Derivation for expression of formula of integration by parts . . . . .	2
Expression for formula of integration of product of e raised to the power ax with cosine of bx . . . . .	2
Derivation for expression of formula of integration of product of e raised to the power ax with cosine of bx . . . . .	2
Expression for formula of integration of product of e raised to the power of ax with sine of bx . . . . .	3
Derivation for expression for formula of integration of square root of sum of square of x and square of a . . . . .	3
Expression for formula of integration of square root of sum of square of x and square of a . . . . .	4
Expression for formula of integration of square root of difference of square of x and square of a . . . . .	4
Expression for formula of integration of square root of difference of square of a and square of x . . . . .	4
Expression for integration of product of $mx + e$ with square root of quadratic function in terms of p and q . . . . .	4
Expression of p in resolution of integration of square root of quadratic functions	4
Expression for q in resolution of integration of square root of quadratic functions	4

---

### Expression for formula of integration by parts

$$\int (uv)dx = u \int vdx - \int \left(\frac{du}{dx} \int vdx\right)dx$$

### Derivation for expression of formula of integration by parts

•

$$\frac{d}{dx}(uv_1) = \frac{du}{dx}v_1 + u\frac{dv_1}{dx}$$

•

Integrating both sides,

•

$$uv_1 = \int \left(\frac{du}{dx}v_1\right)dx + \int \left(u\frac{dv_1}{dx}\right)dx$$

•

$$\int \left(u\frac{dv_1}{dx}\right)dx = uv_1 - \int \left(\frac{du}{dx}v_1\right)dx$$

•

$$v = \frac{dv_1}{dx}$$

•

$$v_1 = \int vdx$$

•

$$\int (uv)dx = u \int vdx - \int \left(\frac{du}{dx} \int vdx\right)dx$$

### Expression for formula of integration of product of e raised to the power ax with cosine of bx

•

$$\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$$

### Derivation for expression of formula of integration of product of e raised to the power ax with cosine of bx

•

$$\int e^{ax} \cos(bx) = e^{ax} \int \cos(bx)dx - \int \left(\frac{de^{ax}}{dx} \int \cos bx dx\right)dx$$

---

•

$$= \frac{e^{ax} \sin bx}{b} - \frac{a}{b} \int e^{ax} \sin bx dx$$

•

$$= \frac{e^{ax} \sin bx}{b} - \frac{a}{b} [e^{ax} \int \sin bx dx - \int (\frac{de^{ax}}{dx} \int \sin bx dx) dx]$$

•

$$= \frac{e^{ax} \sin bx}{b} + \frac{a}{b^2} e^{ax} \cos bx - \frac{a^2}{b^2} \int e^{ax} \cos bx dx$$

•

$$\frac{a^2 + b^2}{b^2} \int e^{ax} \cos bx dx = e^{ax} \frac{(b \sin bx + a \cos bx)}{b^2}$$

•

$$\int e^{ax} \cos bx dx = \frac{e^{ax} (a \cos bx + b \sin bx)}{a^2 + b^2}$$

**Expression for formula of integration of product of e raised to the power of ax with sine of bx**

•

$$\int e^{ax} \sin bx dx = \frac{e^{ax} (a \sin bx - b \cos bx)}{a^2 + b^2}$$

**Derivation for expression for formula of integration of square root of sum of square of x and square of a**

•

$$\int \sqrt{x^2 + a^2} dx = \sqrt{x^2 + a^2} \int 1 dx - \int (\frac{d\sqrt{x^2 + a^2}}{dx} \int 1 dx) dx$$

•

$$= x\sqrt{x^2 + a^2} - \int \frac{2x}{2\sqrt{x^2 + a^2}} x dx$$

•

$$= x\sqrt{x^2 + a^2} - \int \frac{(x^2 + a^2) - a^2}{\sqrt{x^2 + a^2}} dx$$

•

$$= x\sqrt{x^2 + a^2} - \int \sqrt{x^2 + a^2} dx + a^2 \int \frac{dx}{\sqrt{x^2 + a^2}}$$

•

$$2 \int \sqrt{x^2 + a^2} dx = x\sqrt{x^2 + a^2} + a^2 \log(x + \sqrt{x^2 + a^2})$$

•

$$\int \sqrt{x^2 + a^2} = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2})$$

---

**Expression for formula of integration of square root of sum of square of x and square of a**

.

$$\int \sqrt{x^2 + a^2} = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2})$$

**Expression for formula of integration of square root of difference of square of x and square of a**

$$\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2})$$

**Expression for formula of integration of square root of difference of square of a and square of x**

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

**Expression for integration of product of mx + e with square root of quadratic function in terms of p and q**

$$\int (mx + e) \sqrt{ax^2 + bx + c} dx = p \int (2ax + b) \sqrt{ax^2 + bx + c} dx + q \int \sqrt{ax^2 + bx + c} dx$$

**Expression of p in resolution of integration of square root of quadratic functions**

$$p = \frac{m}{2a}$$

**Expression for q in resolution of integration of square root of quadratic functions**

$$q = e - bp$$