## Contents



## Simple Harmonic Motion

**Objective:** *Define simple harmonic motion and state it's equation.*

## Background

- The motion which repeats itself after a *time* interval is **periodic motion**.
- Oscillation, vibration, rotation, are *examples* of periodic motion.

## Simple Harmonic Motion

- The motion in which the *restoring force* is directed towards a fixed point and is **proportional** to the *displacement* is **simple harmonic motion.**
- A body in *simple harmonic motion* is called **harmonic oscillator.**

## Equation for simple harmonic motion

- The restoring force $F$ is directly proportional to displacement $x$ .

$$
\begin{aligned}
F_x &= -kx \\
F_x &= ma_x \\
F_x &= m\frac{dv_x}{dt} \\
F_x &= m\frac{d^2x}{dt^2} \\
\frac{d^2x}{dt^2} &= \frac{-k}{m}x
\end{aligned}
$$

- The quantity $\frac{k}{m}$ can be expressed as $\omega^2$
- The differential equation for a body exhibiting **simple harmonic motion** is:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

## Acceleration in $S.H.M.$

- The equation expressing acceleration in $S.H.M.$ is:

$$a = -\omega^2 x$$

**Velocity in** $S.H.M.$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$
$$\frac{dv}{dt} = -\omega^2 x$$
$$\frac{dv}{dx} \times \frac{dx}{dt} = -\omega^2 x$$
$$vdv = -\omega^2 xdx$$
$$\int vdv = -\omega^2 \int xdx$$
$$\frac{v^2}{2} = -\omega^2 \frac{x^2}{2} + C$$

- At maximum displacement $x = A$ and $v = 0$.

$$C = \omega^2 \frac{A^2}{2}$$
$$\frac{v^2}{2} = -\omega^2 \frac{x^2}{2} + \omega^2 \frac{A^2}{2}$$
$$v^2 = \omega^2 (A^2 - x^2)$$

- The expression for **velocity** is given by:

$$v = \omega \sqrt{A^2 - x^2}$$

**Displacement in** $S.H.M.$

$$
\begin{aligned}
v &= \omega\sqrt{A^2 - x^2} \\
\frac{dx}{dt} &= \omega\sqrt{A^2 - x^2} \\
\frac{dx}{\sqrt{A^2 - x^2}} &= \omega dt \\
\int \frac{dx}{\sqrt{A^2 - x^2}} &= \int \omega dt \\
x &= A\sin\theta \\
\frac{dx}{d\theta} &= A\cos\theta \\
dx &= A\cos\theta d\theta \\
\int \frac{A\cos\theta d\theta}{\sqrt{A^2 - x^2}} &= \omega t + \phi \\
\int \frac{A\cos\theta d\theta}{\sqrt{A^2(1 - \sin^2\theta)}} &= \omega t + \phi \\
\int \frac{A\cos\theta d\theta}{A\cos\theta} &= \omega t + \phi \\
\theta = \omega t + \phi
\end{aligned}
$$

- The equation for **displacement** $x$ is expressed as:

$$x = A\sin(\omega t + \phi)$$

- The equation for **velocity** can also be expressed as:

$$v = \omega A\cos(\omega t + \phi)$$

- The equation for **acceleration** can also be expressed as:

$$a = -\omega^2 A\sin(\omega t + \phi)$$

## Energy in $S.H.M$

**Objective:** *Derive the expressions for energy in simple harmonic motion.*

- Energy in **simple harmonic motion** is exhibited in the form of both *kinetic and potential* energy.

## Potential Energy in $S.H.M.$

- The expression for force in relation with displacement is $F = -kx$.
- $x$ and $F$ are opposite in direction.

$$\begin{aligned} dU &= -dW \\ dU &= kxdx \\ \int dU &= \int kxdx \\ U &= k\frac{x^2}{2} \\ U &= \frac{1}{2}kA^2 \sin^2(\omega t + \phi) \end{aligned}$$

- The relation for **potential energy** in $S.H.M.$ is expressed as:

$$U = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

## Kinetic Energy in $S.H.M.$

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ K &= \frac{1}{2}m\omega\sqrt{A^2 - x^2}^2 \\ K &= \frac{1}{2}m(\omega A \cos(\omega t + \phi))^2 \end{aligned}$$

- The relation for **kinetic energy** in $S.H.M.$ is expressed as:

$$K = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$$

## Total Energy in $S.H.M.$

$$\begin{aligned} &E = K + U \\ &E = \frac{1}{2}\omega^2 A^2 \cos^2(\omega t + \phi) + \frac{1}{2}\omega^2 A^2 \sin^2(\omega t + \phi) \\ &E = \frac{1}{2}m\omega^2 A^2 \end{aligned}$$

- The total energy in $S.H.M.$ is expressed as:

$$E = \frac{1}{2}kA^2$$

## Period

**Objective:** *Derive the expressions for period for vertical oscillation of a mass suspended from coiled spring.*

### Period in vertical oscillation

- The restoring **force** acting on a body of mass $m$ due to **gravity** is proportional to the extension $e$ .

$$\begin{aligned} W &= mg \\ W &= -ke \\ mg &= -ke \end{aligned}$$

- After further *extension* $y$ ,

$$\begin{aligned} F_1 &= -k(e+y) \\ F &= F_1 - W \\ F &= -k(e+y) + ke \\ F &= -ke - ky + ke \\ F &= -ky \\ a &= \frac{-k}{m}y \\ a &= -\omega^2 y \\ \omega^2 &= \frac{k}{m} \\ \omega &= 2\pi f \\ \omega &= \sqrt{\frac{k}{m}} \\ f &= \frac{\omega}{2\pi} \\ f &= \frac{1}{2\pi}\sqrt{\frac{k}{m}} \\ T &= \frac{1}{f} \end{aligned}$$

- The relation expressing *time period* in **vertical oscillation** is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

## Angular S.H.M.

**Objective:** *Describe angular simple harmonic motion and find it's period.*

### Angular Simple Harmonic Motion

- The displacement is proportional **torque** $\tau$ acting on a body as :

$$\tau \alpha \theta$$

- The displacement and **torque** are opposite to each other.
- The *restoring* **torque** can be expressed as:

$$\tau = -k\theta$$

**Period in Angular $S.H.M.$**

$$\tau = -k\theta$$
$$\tau = I\alpha$$
$$I\alpha = -k\theta$$
$$\alpha = \frac{-k}{I}\theta$$
$$a = -\omega^2 x$$
$$\frac{-k}{I}\theta r = -\omega^2 x$$
$$\theta = \frac{x}{r}$$
$$\omega^2 = \frac{k}{I}$$
$$\omega = \sqrt{\frac{k}{I}}$$
$$f = \frac{\omega}{2\pi}$$
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{I}}$$
$$T = \frac{1}{f}$$

- The **time period** $T$ for **angular** $S.H.M.$ can be expressed as:

$$T = 2\pi\sqrt{\frac{I}{k}}$$

## Pendulum

**Objective:** *Derive expression for period of simple pendulum.*

### Simple Pendulum

- A bob of **mass** $m$ is taken.
- The mass $m$ is suspended at a string of length $l$ .
- Force acting on the *bob* at a **distance** $x$.
- Angle $\theta$ is made at that distance.

- The restoring force can be expressed in **two** components.
- The **force** on *string* is balanced by the $cos$ component.

$$\tau = mg\cos\theta$$

- The **restoring** force for a *pendulum* is given by:

$$F = -mg\sin\theta$$

### Time Period for Simple Pendulum

$$\begin{aligned}
\sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} \\
F &= -mg\theta \\
\theta &= \frac{x}{l} \\
a &= -\omega^2 x \\
-\omega^2 x &= -g\frac{x}{l} \\
\omega &= \sqrt{\frac{g}{l}} \\
f &= \frac{\omega}{2\pi} \\
f &= \frac{1}{2\pi}\sqrt{\frac{g}{l}} \\
f &= \frac{1}{T}
\end{aligned}$$

- The relation expressing *time period* for simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

## Damped Oscillation

**Objective:** *Explain damped oscillation.*

- The gradual *decrement* of amplitude of a body in **simple harmonic motion** due to the *action* of resistive forces in it is **damping.**
- The **oscillation** in damping is called **damped oscillation**

## Forced Oscillation

**Objective:** *Describe forced oscillation and resonance with suitable examples.*

### Forced Oscillation

- **Forced oscillation** is the phenomenon of setting a body in *oscillation* with large **periodic** force.

### Resonance

- **Resonance** is the phenomenon of setting a body to *oscillate* in it's *natural frequency* by applying a **periodic** force of **same** magnitude of natural *frequency* of the body.

#### Examples of resonance

- **Resonance** is used in tuning *circuits* in radios to communicate in a particular **frequency.**
- **Tachometers** detect the $rpm$ of motors by resonating *metal* strips.
- *Sodium chloride* **crystals** resonate in *oscillating* electric field.
- **Bridge** being marched by soldiers may collapse it **the marching** matches the **natural frequency** of the *bridge* .