Contents

Types of Vectors

- **Null** vectors are those that have:
  - zero magnitude
  - any arbitary direction
- **Polar** vectors :

  - Start from a point and end at another point.
  - Force, Velocity , Acceleration are examples
- **Axial** vectors:
  - Have no Starting point
  - Torque , Angular momentum are examples

Addition of Two Vectors

Triangle Law

- When two vectors taken in order are represented by two sides of a triangle taken in order then the third side of the triangle taken in opposite order represents the resultant of the two vectors in magnitude and direction.

Parallelogram Law

- When two placed tail to tail are represented by adjacent sides of a parallelogram then the resultant of the two vectors is given by the diagonal of the parallelogram.

Magnitude of Resultant

- The magnitude is give by:

$$R = \sqrt{A^2 + 2AB\cos\theta + B^2}$$

- The direction of vector A with vector R is:

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

- The direction of vector B with vector R is:

$$\tan\beta = \frac{A\sin\theta}{A + B\cos\theta}$$

- The magnitude of direction of a vector with resultant comes separately .

Special Cases for addition of two vectors

- The **maximum** resultant is given by:

$$R_{max} = A + B$$

- The **minimum** resultant is given by:

$$R_{min} = A - B$$

- If the direction of vectors is not mentioned the resultant lies between R_{max} and R_{min} .

Special Cases for angles

Equal Magnitude

The resultant of **vectors** in the case of two vectors A **equal magnitude** *in particular* angles are:

- Case $60°$:

$$R = \sqrt{3}A$$

- Case $90°$:

$$R = \sqrt{2}A$$

- Case $120°$:

$$R = A$$

Number of Vectors

Resultant Zero

The number of vectors required to have the resultant zero in:

- a plane of same magnitude is: **2**
- a plane is: **3**
- not in a plane is: **4**

Equilibrium on 3 vectors

- If resultant of any two vectors is equal to third vector in magnitude and opposite in direction.
- The net resultant of 3 vectors is zero.
- The case where this condition is not satisfied, the net resultant is non zero.

Equilibrium on 4 vectors

- If resultant of any three vectors is equal to fourth vector in magnitude and opposite in direction.
- The net resultant of 4 vectors is zero.
- The case where this condition is not satisfied, the net resultant is non zero.

Application of equilibrium

- If the vectors are initially in equilibrium.
- One of them is removed.
- The change is body is equivalent to the removed vector.
- If the resultant is perpendicular to either vector, the angle between vectors is obtuse.

Method of solving problem related to vectors

1. Draw the figure for condition.
2. Apply parallelogram law for composing the figure of the condition.
3. Check for evaluation with Pythagorean theorem.
4. Check for evaluation with angle made by the vector with the resultant.
    - This step is used often in conditions of two cases.

Vector myths

- Polygonal law can never give the:
  - magnitude of resultant of many vectors
  - direction of resultant of many vectors

Resolution of vectors

- When $\vec{A_1}, \vec{A_2}.....\vec{A_n}$ act on a body at an angle $\theta_1, \theta_2.....\theta_n$ then, Total components of vector along the direction of $\hat{i}$ and $\hat{j}$ is given by:

$$\vec{R_x} = A_1 \cos\theta_1 \times \hat{i} + A_2 \cos\theta_2 \times \hat{i} + ... + A_n \cos\theta_n \times \hat{i}$$
$$\vec{R_y} = A_1 \sin\theta_1 \times \hat{i} + A_2 \sin\theta_2 \times \hat{i} + ... + A_n \sin\theta_n \times \hat{i}$$

The expression for resultant by resolving into components is given by:

$$R = \sqrt{R_x^2 + R_y^2}$$

The angle α with the axis along $\hat{i}$ is given by:

$$\tan\alpha = \frac{R_y}{R_x}$$

Convention of vector notation

- East = $\hat{i}$
- West = $\hat{-i}$
- North = $\hat{j}$
- South = $\hat{-j}$
- Upward = $\hat{k}$
- Downward = $\hat{-k}$

Convention of angle

The term representing the angle of vectors is expressed as:

- Where the angle is pointing from which side.
  - 30° North of East
    - Oriented in north direction originating from east.
  - 40° West of North
    - Oriented in west originating from north.
  - 60° South of West
    - Oriented in west originating from south.
  - 45° South of East
    - Oriented in east originating from south.