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Formula for derivatives

$$\begin{aligned}\frac{d}{dx}x^n &= nx^{n-1} \\ \frac{d}{dx}(ax+b)^n &= n(ax+b)^{n-1}a \\ \frac{d}{dx}\sin x &= \cos x \\ \frac{d}{dx}\cos x &= -\sin x \\ \frac{d}{dx}\tan x &= \sec^2 x \\ \frac{d}{dx}\sec x &= \sec x \tan x \\ \frac{d}{dx}\csc x &= \csc x \cot x \\ \frac{d}{dx}\cot x &= -\csc^2 x \\ \frac{d}{dx}e^x &= e^x \\ \frac{d}{dx}e^{ax} &= ae^{ax} \\ \frac{d}{dx}\log_e x &= \frac{1}{x} \\ \frac{d}{dx}\log_e(ax+b) &= \frac{1}{ax+b}\end{aligned}$$

Formulae for anti derivatives

$$\begin{aligned}\int x^n dx &= \frac{x^{n+1}}{n+1} \\ \int (ax+b)^n dx &= \frac{(ax+b)^{n+1}}{(n+1)a} + C \\ \int \cos x dx &= \sin x + C \\ \int \sin x dx &= -\cos x + C \\ \int \sec^2 x dx &= \tan x + C \\ \int \csc^2 x dx &= -\cot x + C \\ \int \sec x \tan x dx &= \sec x + C \\ \int \csc x \cot x dx &= -\csc x + C \\ \int \csc^2 x dx &= -\cot x + C \\ \int e^x dx &= e^x + C \\ \int e^{ax} dx &= \frac{e^{ax}}{a} + C \\ \int \frac{1}{x} dx &= \log_e x + C \\ \int \frac{1}{ax+b} dx &= \frac{\log_e(ax+b)}{a} + C\end{aligned}$$

Formulae for substitution

$$\begin{aligned}x^2 + a^2 &\implies x = a \tan \theta \\ a^2 - x^2 &\implies x = a \sin \theta \\ x^2 - a^2 &\implies x = a \sec \theta\end{aligned}$$

Formulae for standard integrals

$$\begin{aligned}\int \frac{1}{x^2 + a^2} dx &= \frac{1}{a} \tan^{-1} \frac{x}{a} + C \\ \int \frac{1}{x^2 - a^2} dx &= \frac{1}{2a} \log_e \left(\frac{x-a}{x+a} \right) + C \\ \int \frac{1}{a^2 - x^2} dx &= \frac{1}{2a} \log_e \left(\frac{a+x}{a-x} \right) + C \\ \int \frac{1}{\sqrt{x^2 + a^2}} dx &= \log_e (x + \sqrt{x^2 + a^2}) + C \\ \int \frac{1}{\sqrt{x^2 - a^2}} dx &= \log_e (x + \sqrt{x^2 - a^2}) + C \\ \int \frac{1}{\sqrt{a^2 - x^2}} dx &= \sin^{-1} \left(\frac{x}{a} \right) + C\end{aligned}$$

Concept of Integration

- Indefinite integral is the collection of family of curves.
- Indefinite integral is the curve formed by composing the contact points of all possible tangents in a curve.
- The constant C is a family of possibilities.

$$\frac{d}{dx} \left[x^2, x^2 + 6, x^2 + 100, x^2 + \frac{12}{9087}, x^2 + \sqrt{31212} \times \frac{2617}{2893} \right] = 2x$$

$$\int 2x \cdot dx = x^2 + C$$

Integration of exponential function

Integral structure of function in base of euler's number

For a function $f(x)$:

$$\begin{aligned}\frac{d}{dx} [e^{f(x)}] &= e^{f(x)} \times \frac{d}{dx} f(x) \\ \int e^{f(x)} \cdot f'(x) dx &= e^{f(x)} + C\end{aligned}$$

- The integral of functions $e^{f(x)} \times f'(x)$ is given by:

$$\int e^{f(x)} \cdot f'(x) dx = e^{f(x)} + C$$

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- If the function can be expressed as $f'(x)$ forms only after operating with a constant in it should be added.

Integral structure of function in base of a constant a

For a function $f(x)$ and a constant a :

$$\begin{aligned}\frac{d}{dx}[a^x] &= a^x \cdot \ln(a) \\ \frac{d}{dx}[a^{f(x)}] &= a^{f(x)} \cdot \frac{d}{dx}f(x) \\ \int a^{f(x)} \cdot f'(x) &= a^{f(x)} \cdot \frac{1}{\ln(a)} + C\end{aligned}$$

The integral of functions $a^{f(x) \times f'(x)}$ is given by:

$$\int a^{f(x)} \cdot f'(x) = \frac{a^{f(x)}}{\ln(a)} + C$$

If the function can be expressed as $f'(x)$ forms only after operating with a constant in it should be added. - The **trigonometric** functions are related to each other.

- **Cosine** functions: $\sin x$, $\cos x$
- **Cosecant** functions: $\csc x$, $\sec x$
- **Cotangent** functions: $\cot x$, $\tan x$
- The **relation** of antiderivative by parts is:

$$\int u v dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

- The v and u functions are chosen by **ILLATEC** rule.
 - I = **Inverse Circular**
 - L = **Logarithmic**
 - A = **Algebraic**
 - T = **Trigonometric**

- E = **Exponential**

- C = **Constant**

- The **antiderivative** of $\sec^3 x$ is:

$$\int \sec^3 x dx = \frac{1}{2} \left(\frac{d}{dx} \sec x + \int \sec x \right) + C$$

- The **antiderivative** of functions of the type $x^n f(x)$ is evaluated by:

- $f(x)$ is **easily** integrated.
- **expression** can be evaluated using integration by **parts** rule.

$$\int x^n f(x) dx = x^n \int f(x)$$

$$\int x^n f(x) dx = \frac{d}{dx} x^n \int f(x) - \frac{d^2}{dx^2} x^n \int \int f(x) + \frac{d^3}{dx^3} x^n \int \int \int f(x)$$

$$\int x^n f(x) dx = \int u v dx$$

- The **antiderivative** is calculated by **differentiation** of u and **integration** of v with **alternate** signs.

$$\int u v dx = \frac{d}{dx} u \int v - \frac{d^2}{dx^2} u \int \int v + \frac{d^3}{dx^3} u \int \int \int v$$

De -Dt rule for integration

- The integration of the function of form $e^{ax} \sin(bx) dx$ can be done in:

$$\int e^{ax} \sin bx dx = \frac{1}{a^2 + b^2} \left(\frac{d}{dx} e^{ax} \times \sin bx - e^{ax} \frac{d}{dx} \sin bx \right) + C$$