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## Simple Harmonic Motion

**Objective:** Define simple harmonic motion and state its equation.

### Background

- The motion which repeats itself after a *time* interval is **periodic motion**.
- Oscillation, vibration, rotation, are *examples* of periodic motion.

### Simple Harmonic Motion

- The motion in which the *restoring force* is directed towards a fixed point and is **proportional** to the *displacement* is **simple harmonic motion**.
- A body in *simple harmonic motion* is called **harmonic oscillator**.

### Equation for simple harmonic motion

- The restoring force  $F$  is directly proportional to displacement  $x$ .

$$F_x = -kx$$

$$F_x = ma_x$$

$$F_x = m \frac{dv_x}{dt}$$

$$F_x = m \frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{-k}{m}x$$

- The quantity  $\frac{k}{m}$  can be expressed as  $\omega^2$
- The differential equation for a body exhibiting **simple harmonic motion** is:

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

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### Acceleration in *S.H.M.*

- The equation expressing acceleration in *S.H.M.* is:

$$a = -\omega^2 x$$

### Velocity in *S.H.M.*

$$\begin{aligned}\frac{d^2 x}{dt^2} &= -\omega^2 x \\ \frac{dv}{dt} &= -\omega^2 x \\ \frac{dv}{dx} \times \frac{dx}{dt} &= -\omega^2 x \\ v dv &= -\omega^2 x dx \\ \int v dv &= -\omega^2 \int x dx \\ \frac{v^2}{2} &= -\omega^2 \frac{x^2}{2} + C\end{aligned}$$

- At maximum displacement  $x = A$  and  $v = 0$ .

$$\begin{aligned}C &= \omega^2 \frac{A^2}{2} \\ \frac{v^2}{2} &= -\omega^2 \frac{x^2}{2} + \omega^2 \frac{A^2}{2} \\ v^2 &= \omega^2 (A^2 - x^2)\end{aligned}$$

- The expression for **velocity** is given by:

$$v = \omega \sqrt{A^2 - x^2}$$

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## Displacement in *S.H.M.*

$$\begin{aligned}v &= \omega \sqrt{A^2 - x^2} \\ \frac{dx}{dt} &= \omega \sqrt{A^2 - x^2} \\ \frac{dx}{\sqrt{A^2 - x^2}} &= \omega dt \\ \int \frac{dx}{\sqrt{A^2 - x^2}} &= \int \omega dt \\ x &= A \sin \theta \\ \frac{dx}{d\theta} &= A \cos \theta \\ dx &= A \cos \theta d\theta \\ \int \frac{A \cos \theta d\theta}{\sqrt{A^2 - x^2}} &= \omega t + \phi \\ \int \frac{A \cos \theta d\theta}{\sqrt{A^2(1 - \sin^2 \theta)}} &= \omega t + \phi \\ \int \frac{A \cos \theta d\theta}{A \cos \theta} &= \omega t + \phi \\ \theta &= \omega t + \phi\end{aligned}$$

- The equation for **displacement**  $x$  is expressed as:

$$x = A \sin(\omega t + \phi)$$

- The equation for **velocity** can also be expressed as:

$$v = \omega A \cos(\omega t + \phi)$$

- The equation for **acceleration** can also be expressed as:

$$a = -\omega^2 A \sin(\omega t + \phi)$$

## Energy in *S.H.M*

**Objective:** *Derive the expressions for energy in simple harmonic motion.*

- Energy in **simple harmonic motion** is exhibited in the form of both *kinetic and potential* energy.

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### Potential Energy in *S.H.M.*

- The expression for force in relation with displacement is  $F = -kx$ .
- $x$  and  $F$  are opposite in direction.

$$\begin{aligned}dU &= -dW \\dU &= kx dx \\ \int dU &= \int kx dx \\ U &= k \frac{x^2}{2} \\ U &= \frac{1}{2}kA^2 \sin^2(\omega t + \phi)\end{aligned}$$

- The relation for **potential energy** in *S.H.M.* is expressed as:

$$U = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$$

### Kinetic Energy in *S.H.M.*

$$\begin{aligned}K &= \frac{1}{2}mv^2 \\ K &= \frac{1}{2}m\omega \sqrt{A^2 - x^2}^2 \\ K &= \frac{1}{2}m(\omega A \cos(\omega t + \phi))^2\end{aligned}$$

- The relation for **kinetic energy** in *S.H.M.* is expressed as:

$$K = \frac{1}{2}m\omega^2 A^2 \cos^2(\omega t + \phi)$$

### Total Energy in *S.H.M.*

$$\begin{aligned}E &= K + U \\ E &= \frac{1}{2}\omega^2 A^2 \cos^2(\omega t + \phi) + \frac{1}{2}\omega^2 A^2 \sin^2(\omega t + \phi) \\ E &= \frac{1}{2}m\omega^2 A^2\end{aligned}$$

- The total energy in *S.H.M.* is expressed as:

$$E = \frac{1}{2}kA^2$$

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## Period

**Objective:** Derive the expressions for period for vertical oscillation of a mass suspended from coiled spring.

### Period in vertical oscillation

- The restoring **force** acting on a body of mass  $m$  due to **gravity** is proportional to the extension  $e$ .

$$W = mg$$

$$W = -ke$$

$$mg = -ke$$

- After further extension  $y$ ,

$$F_1 = -k(e + y)$$

$$F = F_1 - W$$

$$F = -k(e + y) + ke$$

$$F = -ke - ky + ke$$

$$F = -ky$$

$$a = \frac{-k}{m}y$$

$$a = -\omega^2 y$$

$$\omega^2 = \frac{k}{m}$$

$$\omega = 2\pi f$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$T = \frac{1}{f}$$

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- The relation expressing *time period* in **vertical oscillation** is:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

## Angular S.H.M.

**Objective:** Describe angular simple harmonic motion and find its period.

### Angular Simple Harmonic Motion

- The displacement is proportional **torque**  $\tau$  acting on a body as :

$$\tau \propto \theta$$

- The displacement and **torque** are opposite to each other.
- The *restoring torque* can be expressed as:

$$\tau = -k\theta$$

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## Period in Angular *S.H.M.*

$$\tau = -k\theta$$

$$\tau = I\alpha$$

$$I\alpha = -k\theta$$

$$\alpha = \frac{-k}{I}\theta$$

$$a = -\omega^2 x$$

$$\frac{-k}{I}\theta r = -\omega^2 x$$

$$\theta = \frac{x}{r}$$

$$\omega^2 = \frac{k}{I}$$

$$\omega = \sqrt{\frac{k}{I}}$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{I}}$$

$$T = \frac{1}{f}$$

- The **time period**  $T$  for **angular** *S.H.M.* can be expressed as:

$$T = 2\pi \sqrt{\frac{I}{k}}$$

## Pendulum

**Objective:** *Derive expression for period of simple pendulum.*

### Simple Pendulum

- A bob of **mass**  $m$  is taken.
- The mass  $m$  is suspended at a string of length  $l$ .
- Force acting on the *bob* at a **distance**  $x$ .
- Angle  $\theta$  is made at that distance.



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- The restoring force can be expressed in **two** components.
  - The **force** on *string* is balanced by the *cos* component.

$$\tau = mg \cos \theta$$

- The **restoring** force for a *pendulum* is given by:

$$F = -mg \sin \theta$$

### Time Period for Simple Pendulum

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!}$$

$$F = -mg\theta$$

$$\theta = \frac{x}{l}$$

$$a = -\omega^2 x$$

$$-\omega^2 x = -g \frac{x}{l}$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$f = \frac{\omega}{2\pi}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

$$f = \frac{1}{T}$$

- The relation expressing *time period* for simple pendulum is given by:

$$T = 2\pi \sqrt{\frac{l}{g}}$$

### Damped Oscillation

**Objective:** Explain damped oscillation.

- The gradual *decrement* of amplitude of a body in **simple harmonic motion** due to the *action* of resistive forces in it is **damping**.
- The **oscillation** in damping is called **damped oscillation**

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## Forced Oscillation

**Objective:** *Describe forced oscillation and resonance with suitable examples.*

### Forced Oscillation

- **Forced oscillation** is the phenomenon of setting a body in *oscillation* with large **periodic** force.

### Resonance

- **Resonance** is the phenomenon of setting a body to *oscillate* in its *natural frequency* by applying a **periodic** force of **same** magnitude of *natural frequency* of the body.

### Examples of resonance

- **Resonance** is used in tuning *circuits* in radios to communicate in a particular **frequency**.
- **Tachometers** detect the *rpm* of motors by resonating *metal* strips.
- *Sodium chloride* **crystals** resonate in *oscillating* electric field.
- **Bridge** being marched by soldiers may collapse if **the marching** matches the **natural frequency** of the *bridge*.