Background

- **Solenoid** is a **coiled** conductor.
- The **magnetic field** of **solenoid** is as that of **bar magnet** .

Magnetic field at axis

- The **magnetic field** is to be calculated at P .
- A small length segment dl is taken.
- Number of turns in dl = n

$$
\begin{aligned}
dB &= \frac{\mu_0 N I a \sin\theta}{2r^2} \\
N &= ndl \\
dB &= \frac{\mu_0 n I a dl \sin\theta}{2r^2} \\
BC &= dl \sin\theta \\
BC &= rd\theta \\
dl \sin\theta &= rd\theta \\
dB &= \frac{\mu_0 n I a r d\theta}{2r^2} \\
dB &= \frac{\mu_0 n I a d\theta}{2r} \\
\frac{a}{r} &= \sin\theta \\
dB &= \frac{\mu_0 n I \sin\theta d\theta}{2} \\
B &= \int dB \\
B &= \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 n I \sin\theta d\theta}{2r} \\
B &= \frac{\mu_0 n I}{2} \int_{\alpha_1}^{\alpha_2} \sin\theta d\theta \\
B &= \frac{\mu_0 n I}{2} [\cos\alpha_1 - \cos\alpha 2]
\end{aligned}
$$

- The general **expression** for magnetic field at the axis of a **current** carrying **solenoid** is:

$$B = \frac{\mu_0 n I}{2} [\cos\alpha_1 - \cos\alpha_2]$$

Magnetic Field for long solenoid

- α_1 = 0
- α_2 = π
- The **expression** for **magnetic field** at axis for **long solenoid** is given by:

$$B = \mu_0 n I$$

$$\begin{aligned} dB &= \frac{u_0 I dl \sin\theta}{4\pi r^2} \\ dl \sin\theta &= AN \\ d\phi &= \frac{AN}{r} \\ AN &= d\phi r \\ dB &= \frac{u_0 I r d\phi}{4\pi r^2} \\ \cos\phi &= \frac{a}{r} \\ r &= \frac{a}{\cos\phi} \\ dB &= \frac{u_0 I \cos\phi d\phi}{4\pi a} \\ B &= \int_{\phi_1}^{-\phi_2} dB \\ B &= \frac{u_0 I}{4\pi a}\int_{\phi_1}^{-\phi_2} \cos\phi d\phi \\ B &= \frac{u_0 I}{4\pi a}[\sin\phi_1 + \sin\phi_2] \end{aligned}$$

- The general expression for **magnetic field** due to a **long straight** conductor is given by:

$$B = \frac{u_0 I}{4\pi a}[\sin\phi_1 + \sin\phi_2]$$

Infinite long conductor

$$\begin{aligned} \phi_1 = \phi_2 &= \frac{\pi}{2} \\ B &= \frac{u_0 I}{4\pi a} \times 2 \end{aligned}$$

- The expression for **magnetic field** due to **infinitely** long magnetic field is given by:

$$B = \frac{u_0 I}{2\pi a}$$

Application of Ampere’s law

Solenoid

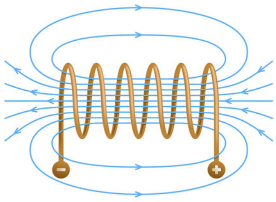

Figure 1: Structure of a solenoid

Application of ampere’s law in solenoid.

- Ampere’s law is valid only under closed path.
- A closed path is formed in a solenoid in the form of a rectangle.

Derivation for the expression of magnetic field at the axis of solenoid from ampere's law

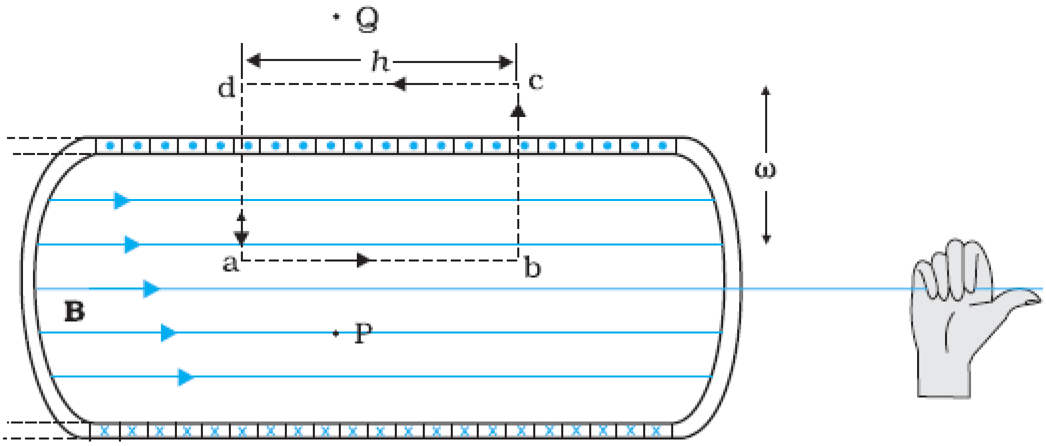

Figure 2: Magnetic field of closed loop in a solenoid

- The solenoid has turns.
- The turns of solenoid are ring shaped.

Line integral in amperes law

$$\oint \vec{B}.\vec{dl} = \oint_a^b \vec{B}.\vec{dl} + \oint_b^c \vec{B}.\vec{dl} + \oint_c^d \vec{B}.\vec{dl} + \oint_d^a \vec{B}.\vec{dl}$$

Magnitude of angle of parallel sides with the axis of magnetic field

The angle between side of rectangle parallel to the axis of magnetic field with the magnetic field is

- $$0$$

Evaluation of expression for side parallel ab to the magnetic field

$$
\begin{aligned}
&\oint_a^b Bdl\cos\theta \\
&= \oint_a^b Bdl\cos(0) \\
&= \oint_a^b Bdl \\
&= B\oint_a^b \\
&= B[l]_a^b \\
&= B(b-a) \\
&= Bl
\end{aligned}
$$

Evaluation of expression for side parallel cd to the magnetic field

- The side cd doesnot lie on the axis of magnetic field.
- The magnetic field present across cd is negligible compared to the field present inside.

Magnitude of angle of perpendicular sides with the axis of magnetic field

The angle between side of rectangle perpendicular to the axis of magnetic field with the magnetic field is

- $$90$$

Evaluation of expression for the side bc perpendicular to the magnetic field

$$
\begin{aligned}
\oint_b^c \vec{B}.dl &= \oint_b^c Bdl\cos\theta \\
&= \oint_b^c Bdl\cos(90) \\
&= 0
\end{aligned}
$$

Evaluation of expression for the side da perpendicular to the magnetic field

$$\begin{aligned}\oint_d^a \vec{B}.dl &= \oint_d^a Bdl\cos\theta \\ &= \oint_d^a Bdl\cos(90) \\ &= 0\end{aligned}$$

Sum of line integral of magnetic field across the closed loop

$$\oint \vec{B}\vec{dl} = Bl$$

Equality of amperes law in solenoid

The expression of amperes law is

$$\oint \vec{B}\vec{dl} = \mu_0 I$$

The expression for N number of turns is

$$\oint \vec{B}\vec{dl} = \mu_0 NI$$

$$\begin{aligned}\oint \vec{B}\vec{dl} &= \mu_0 NI \\ \oint \vec{B}\vec{dl} &= Bl \\ Bl &= \mu_0 NI \\ B &= \mu_0 I\frac{N}{l} \\ B &= \mu_0 nI\end{aligned}$$

Expression for magnetic field at the axis of solenoid from ampere's law

The expression for magnetic field at the axis of solenoid from ampere's law is

$$B = \mu_0 nI$$

Toroid

Structure of toroid

- Toroid is a circular structure.
- The circular structure is a form of bent solenoid.
- Toroid is a bent solenoid.

Magnetic field at the axis of toroid

- The magnetic field at the axis of toroid is circular .
- The axis is distributed over the circle.
- The length of the distribution of the axis is the circumference of circle.
- The length of the distribution of axis is $2\pi r$.

Derivation for the expression of magnetic field at the axis of a toroidal coil from ampere's law

$$\begin{aligned}
\oint_0^{2\pi r} \vec{B}.\vec{dl} = \oint_0^{2\pi r} Bdl\cos\theta \\
= \oint_0^{2\pi r} Bdl\cos(0) \\
= \oint_0^{2\pi r} Bdl \\
= B\oint_0^{2\pi r} dl \\
= B[l]_0^{2\pi r} \\
= B(2\pi r - 0) \\
= B(2\pi r)
\end{aligned}$$

Expression of ampere's law in toroid

$$
\begin{aligned}
\oint_0^{2\pi r} \vec{B}.\vec{dl} &= B2\pi r \\
\oint_0^{2\pi r} \vec{B}.\vec{dl} &= \mu_0 NI \\
B2\pi r &= \mu_0 NI \\
B &= \frac{\mu_0 NI}{2\pi r}
\end{aligned}
$$

Expression for the magnetic field at the axis of a toroidal coil from ampere's law

The expression for the magnetic field at the axis of a toroidal coil from ampere's law is

$$B = \frac{\mu_0 NI}{2\pi r}$$