Contents

Formula for derivatives

$$\frac{d}{dx}x^n = nx^{n-1}$$
$$\frac{d}{dx}(ax+b)^n = n(ax+b)^{n-1}a$$
$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\tan x = \sec^2 x$$
$$\frac{d}{dx}\sec x = \sec x \tan x$$
$$\frac{d}{dx}\csc x = \csc x \cot x$$
$$\frac{d}{dx}\cot x = -\csc^2 x$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}e^{ax} = ae^{ax}$$
$$\frac{d}{dx}\log_e x = \frac{1}{x}$$
$$\frac{d}{dx}\log_e(ax+b) = \frac{1}{ax+b}$$

Formulae for anti derivatives

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$
$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{(n+1)a} + C$$
$$\int \cos x dx = \sin x + C$$
$$\int \sin x dx = -\cos x + C$$
$$\int \sec^2 x dx = \tan x + C$$
$$\int \csc^2 x dx = -\cot x + C$$
$$\int \sec x \tan x dx = \sec x + C$$
$$\int \csc x \cot x dx = -\csc x + C$$
$$\int \csc^2 x dx = -\cot x + C$$
$$\int e^x dx = e^x + C$$
$$\int e^{ax} dx = \frac{e^{ax}}{a} + C$$
$$\int \frac{1}{x} dx = \log_e x + C$$
$$\int \frac{1}{ax+b} dx = \frac{\log_e(ax+b)}{a} + C$$

Formulae for substitution

$$x^2 + a^2 \implies x = a\tan\theta$$
$$a^2 - x^2 \implies x = a\sin\theta$$
$$x^2 - a^2 \implies x = a\sec\theta$$

Formulae for standard integrals

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$
$$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\log_e(\frac{x-a}{x+a}) + C$$
$$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\log_e(\frac{a+x}{a-x}) + C$$
$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \log_e(x+\sqrt{x^2+a^2}) + C$$
$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \log_e(x+\sqrt{x^2-a^2}) + C$$
$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}(\frac{x}{a}) + C$$

Concept of Integration

- Indefinite integral is the collection of family of curves.
- Indefinite integral is the curve formed by composing the contact points of all possible tangents in a curve.
- The constant C is a family of possibilities.

$$\frac{d}{dx}[x^2, x^2+6, x^2+100, x^2+\frac{12}{9087}, x^2+\sqrt{31212}\times\frac{2617}{2893}] = 2x$$

$$\int 2x.dx = x^2 + C$$

Integration of exponential function

Integral structure of function in base of euler's number

For a function $f(x)$:

$$\frac{d}{dx}[e^{f(x)}] = e^{f(x)} \times \frac{d}{dx}f(x)$$
$$\int e^{f(x)}.f^{'}(x)dx = e^{f(x)} + C$$

- The integral of functions $e^{f(x)} \times f^{'}(x)$ is given by:

$$\int e^{f(x)}.f^{'}(x)dx = e^{f(x)} + C$$

- If the function can be expressed as $f^{'}(x)$ forms only after operating with a constant in it should be added.

Integral structure of function in base of a constant a

For a function $f(x)$ and a constant a :

$$\frac{d}{dx}[a^x] = a^x.1.\log_e(a)$$
$$\frac{d}{dx}[a^{f(x)}] = a^{f(x)}.\frac{d}{dx}f(x)$$
$$\int a^{f(x)}.f^{'}(x) = a^{f(x)}.\frac{1}{\log_e(a)} + C$$

The integral of functions $a^{f(x)\times f^{'}(x)}$ is given by:

$$\int a^{f(x)}.f^{'}(x) = \frac{a^{f(x)}}{\log_e(a)} + C$$

If the function can be expressed as $f^{'}(x)$ forms only after operating with a constant in it should be added. - The **trigonometric** functions are related to each other.

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-   **Cosine** functions: $\sin x , \cos x$

-   **Cosecant** functions: $\csc x , \sec x$

-   **Cotangent** functions: $\cot x , \tan x$
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- The **relation** of antiderivative by parts is:

$$\int uvdx = u\int vdx - \int (\frac{du}{dx}\int vdx)dx$$

- The v and u functions are chosen by **ILLATEC** rule.
  - I = **Inverse Circular**
  - L = **Logarithmic**
  - A = **Algaebraic**
  - T = **Trigonometric**

- E = **Exponential**
- C = **Constant**

- The **antiderivative** of $\sec^3 x$ is:

$$\int \sec^3 x dx = \frac{1}{2}(\frac{d}{dx}\sec x + \int \sec x) + C$$

- The **antiderivative** of functions of the type $x^n f(x)$ is evaluated by:
  - f(x) is **easily** integrated.
  - **expression** can be evaluated using integration by **parts** rule.

$$\int x^n f(x)dx = x^n \int f(x)$$
$$\int x^n f(x)dx = \frac{d}{dx}x^n \int f(x) - \frac{d^2}{dx^2}x^n \int\int f(x) + \frac{d^3}{dx^3}\int\int\int f(x)$$

$$\int x^n f(x)dx = \int uvdx$$

- The **antiderivative** is calculated by **differentiation** of u and **integration** of v with **alternate** signs.

$$\int uvdx = \frac{d}{dx}u\int v - \frac{d^2}{dx^2}u\int\int v + \frac{d^3}{dx^3}u\int\int\int v$$

De -Dt rule for integration

- The integration of the function of form $e^{ax}\sin(bx)dx$ can be done in:

$$\int e^{ax}\sin bx dx = \frac{1}{a^2+b^2}(\frac{d}{dx}e^{ax} \times \sin bx - e^{ax}\frac{d}{dx}\sin bx) + C$$