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$$\int cosecxdx = \log(\tan\frac{x}{2}) + C$$

Derivation for expression of integration of cosec x

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- $$= \int \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$
- $$= \frac{1}{2}\int \frac{1}{\sin\frac{x}{2}\cos\frac{x}{2}}\frac{\sec\frac{x}{2}}{\sec^2\frac{x}{2}}dx$$
- $$\frac{1}{2}\int \frac{\sec^2\frac{x}{2}}{\tan\frac{x}{2}}dx$$
- $$cosecxdx = \log(\frac{x}{2}) + C$$

Expression for integration of sec x

$$\int \sec xdx = \log(\tan(\frac{\pi}{4} + \frac{x}{2})) + C$$

Derivation for expression of integration of sec x

- $$\int \sec xdx = \int \frac{dx}{\cos x}$$
- $$= \int \frac{1}{\sin(\frac{\pi}{2} + x)}$$
- $$= \frac{1}{2}\frac{dx}{\sin(\frac{\pi}{4} + \frac{x}{2})\cos(\frac{\pi}{4} + \frac{x}{2})}$$
- $$= \frac{1}{2}\int \frac{\sec^2\frac{\pi}{4} + \frac{x}{2}}{\tan(\frac{\pi}{4} + \frac{x}{2})}$$
- $$\int \sec xdx = \log(\tan(\frac{\pi}{4} + \frac{x}{2})) + C$$

Derivation for general expression of integration of (a + b cosx) in denominator in unresolved cases

- $$\int \frac{dx}{a + b\cos(x)}$$
- $$= \int \frac{dx}{a\left(\cos^2 \frac{1}{2}x + \sin^2 \frac{1}{2}x\right) + b\left(\cos^2 \frac{1}{2}x - \sin^2 \frac{1}{2}x\right)}$$
- $$= \int \frac{1}{(a+b)\cos^2 \frac{1}{2}x + (a-b)\sin^2 \frac{1}{2}x} \times \frac{\sec^2 \frac{1}{2}x}{\sec^2 \frac{1}{2}x} dx$$
- $$= \int \frac{\sec^2 \frac{1}{2}x}{(a+b) + (a-b)\tan^2 \frac{1}{2}x}$$

Derivation for expression of integration of (a + b cosx) in denominator when a > b in standard integrals of special trigonometric functions

- $$\int \frac{\sec^2 \frac{1}{2}x}{(a+b) + \sqrt{(a-b)}\tan \frac{1}{2}x}^{2}$$
- $$y = \sqrt{a-b}\tan \frac{1}{2}x$$
- $$\frac{dy}{dx} = \frac{1}{2}\sqrt{a-b}\sec^2 \frac{1}{2}x$$
- $$\sec^2 \frac{1}{2}x = \frac{2dy}{\sqrt{a-b}}$$
- $$I = \frac{2}{\sqrt{a-b}} \int \frac{dy}{(\sqrt{(a+b)})^2 + y^2}$$
- $$I = \frac{2}{\sqrt{a-b}} \times \frac{1}{\sqrt{a+b}} \tan^{-1}(\frac{y}{\sqrt{a+b}})$$
- $$I = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1}(\sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2})$$

Derivation for expression of integration of (a + b cosx) in denominator when b > a in standard integrals of special trigonometric functions

- $$\int \frac{\sec^2 \frac{1}{2}x}{(a+b) - \sqrt{(b-a)}\tan\frac{1}{2}x}^2$$
- $$\text{Let } y = \sqrt{b-a}\tan\frac{1}{2}x$$
- $$\frac{dy}{dx} = \frac{1}{2}\sqrt{b-a}\sec^2\frac{1}{2}x$$
- $$\sec^2\frac{1}{2}x = \frac{2dy}{\sqrt{b-a}}$$
- $$I = \frac{2}{\sqrt{b-a}}\int\frac{dy}{(\sqrt{(a+b)})^2 - y^2}$$
- $$I = \frac{2}{\sqrt{b-a}} \times \frac{1}{2\sqrt{a+b}}\log_e(\frac{\sqrt{a+b}+y}{\sqrt{a+b}-y})$$
- $$I = \frac{1}{\sqrt{b^2-a^2}}\log_e(\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{1}{2}x}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{1}{2}x})$$

Derivation for expression of integration of (a + b cosx) in denominator when a = b in standard integrals of special trigonometric functions

- $$I = \int\frac{dx}{a(1+\cos(x))}$$
- $$= \frac{1}{a}\int\frac{dx}{2\cos^2(\frac{x}{2})}dx$$
- $$= \frac{1}{2a}\int\sec^2\frac{x}{2}dx$$
- $$= \frac{1}{2a}\frac{\tan\frac{x}{2}}{\frac{1}{2}}$$
- $$= \frac{1}{a}\tan\frac{x}{2}$$

Expression for integration of special trigonometric function in the terms of (a + b cosx) as denominator when a > b

$$\int \frac{dx}{a+b\cos x} = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2})$$

Expression for integration of special trigonometric function in the terms of (a + b cosx) as denominator when b > a

$$\int \frac{dx}{a+b\cos x} = \frac{1}{\sqrt{b^2-a^2}}\log_e(\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{1}{2}x}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{1}{2}x})$$

Expression for integration of special trigonometric function in the terms of (a + b cosx) as denominator when a = b

$$\int \frac{dx}{a+b\cos x} = \frac{1}{a}\tan\frac{x}{2}$$

Derivation of integral of special trigonometric function in terms of (a + b sinx) as denominator when a > b

- $$I = \int \frac{dx}{a+b\sin(x)}$$
- $$I = \int \frac{dx}{a\sin^2\frac{x}{2}+a\cos^2\frac{x}{2}+b2\sin(x)\cos(x)}$$
- $$I = \int \frac{\sec^2\frac{x}{2}}{a\tan^2\frac{x}{2}+a+2b\tan(\frac{x}{2})}dx$$
- $$\text{Let} y = \tan\frac{x}{2}$$
- $$\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$
- $$\sec^2\frac{x}{2}dx = 2dy$$
- $$I = \int \frac{2dy}{ay^2+2by+a}$$

- $$I = 2\int \frac{dy}{(\sqrt{a}y)^2 + 2\sqrt{a}y\frac{b}{\sqrt{a}} + (\frac{b}{\sqrt{a}})^2 + a - (\frac{b}{\sqrt{a}})^2}$$

- $$I = 2\int \frac{dy}{(\sqrt{a}y + fracb\sqrt{a})^2 + (\sqrt{\frac{a^2-b^2}{a}})^2}$$

- $$I = \frac{2}{a\sqrt{a^2-b^2}}\tan^{-1}(\frac{\sqrt{a}\tan\frac{x}{2} + \frac{b}{\sqrt{a}}}{\frac{\sqrt{a^2-b^2}}{\sqrt{a}}})$$

- $$I = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}(\frac{a\tan(\frac{x}{2}) + b}{\sqrt{a^2-b^2}})$$

Expression for integration of special trigonometric function in the terms of (a + b sinx) as denominator where a > b

$$\int \frac{dx}{a + b\sin(x)} = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}(\frac{a\tan(\frac{x}{2}) + b}{\sqrt{a^2-b^2}})$$

Derivation of integral of special trigonometric function in terms of (a + b sinx) as denominator when b > a

- $$I = \int \frac{dx}{a + b\sin(x)}$$

- $$I = \int \frac{dx}{a\sin^2\frac{x}{2} + a\cos^2\frac{x}{2} + b2\sin(x)\cos(x)}$$

- $$I = \int \frac{\sec^2\frac{x}{2}}{a\tan^2\frac{x}{2} + a + 2b\tan(\frac{x}{2})}dx$$

- $$\text{Let} y = \tan\frac{x}{2}$$

- $$\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$

- $$\sec^2\frac{x}{2}dx = 2dy$$

- $$I = \int \frac{2dy}{ay^2 + 2by + a}$$

- $$I = 2\int \frac{dy}{(\sqrt{a}y)^2 + 2\sqrt{a}y\frac{b}{\sqrt{a}} + (\frac{b}{\sqrt{a}})^2 + a - (\frac{b}{\sqrt{a}})^2}$$
- $$I = 2\int \frac{dy}{(\sqrt{a}y + \frac{b}{\sqrt{a}})^2 - (\sqrt{(\frac{b^2-a^2}{a})})^2}$$
- $$I = \frac{1}{\sqrt{b^2-a^2}} \log_e \frac{a\tan\frac{x}{2} + b - \sqrt{b^2-a^2}}{a\tan\frac{x}{2} + b + \sqrt{b^2-a^2}}$$

Expression for integration of special trigonometric function in the terms of (a + b sinx) as denominator when b > a

$$I = \frac{1}{\sqrt{b^2-a^2}} \log_e \frac{a\tan\frac{x}{2} + b - \sqrt{b^2-a^2}}{a\tan\frac{x}{2} + b + \sqrt{b^2-a^2}}$$

Derivation for the integration of special trigonometric function having (a + b sinx) in denominator when a = b

- $$\int \frac{dx}{a + b\sin x}$$
- $$\int \frac{dx}{a + a\sin x}$$
- $$\frac{1}{a}\int \frac{dx}{1 + \sin x}$$
- $$\frac{1}{a}\int \frac{dx}{\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}$$
- $$\frac{1}{a}\int \frac{\sec^2\frac{x}{2}dx}{\tan^2\frac{x}{2} + 2\tan\frac{x}{2} + 1}$$
- $$y = \tan\frac{x}{2}$$
- $$\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$
- $$2dy = \sec^2\frac{x}{2}dx$$

- $$\frac{2}{a}\int\frac{dy}{(y+1)^2}$$
- $$\frac{2}{a}\frac{(y+1)^{-2+1}}{-1}$$
- $$\frac{-2}{a(y+1)}$$
- $$\frac{-2}{a(\tan\frac{x}{2}+1)}$$

Expression for integration of (a + b sinx) in denominator when a = b in special trigonometric function

- $$\frac{-2}{a(\tan\frac{x}{2}+1)}$$

Derivation for expression of integration of (a sinx + b cosx) in denominator in special trigonometric functions

- $$a = r\cos\alpha$$
- $$b = r\sin\alpha$$
- $$r = \sqrt{a^2+b^2}$$
- $$\alpha = \tan^{-1}(\frac{b}{a})$$
- $$\int\frac{dx}{a\sin x+b\cos x} = \frac{1}{r}\int\frac{dx}{\cos\alpha\sin x+\sin\alpha\cos x}$$
- $$\frac{1}{r}\int\frac{dx}{\sin(x+\alpha)}$$
- $$\frac{1}{r}\int cosec(x+\alpha)dx$$

- $$\frac{1}{r}\log(\tan\frac{x+\alpha}{2})$$

- $$\int\frac{dx}{a\sin x+b\cos x}=\frac{1}{\sqrt{a^2+b^2}}\log(\tan(\frac{x+\tan^{-1}\frac{b}{a}}{2}))$$

Expression for expression of integration of (a sinx + b cosx) in denominator in special trigonometric functions

- $$\int\frac{dx}{a\sin x+b\cos x}=\frac{1}{\sqrt{a^2+b^2}}\log(\tan(\frac{x+\tan^{-1}\frac{b}{a}}{2}))$$