Contents

Standard Integrals

Standard Integrals of Special Trigonometrical Functions

Expression of special trigonometric function as $a+b\cos(x)$ in denominator The expression for special trigonometric function is

$$\int \frac{dx}{a+b\cos x}$$

Derivation of Integral

$$\begin{aligned}
&\int \frac{dx}{a+b\cos(x)} \\
&= \int \frac{dx}{a\left(\cos^2\frac{1}{2}x+\sin^2\frac{1}{2}x\right)+b\left(\cos^2\frac{1}{2}x-\sin^2\frac{1}{2}x\right)} \\
&= \int \frac{1}{(a+b)\cos^2\frac{1}{2}x+(a-b)\sin^2\frac{1}{2}x}\times\frac{\sec^2\frac{1}{2}x}{\sec^2\frac{1}{2}x}dx \\
&= \int \frac{\sec^2\frac{1}{2}x}{(a+b)+(a-b)\tan^2\frac{1}{2}x}
\end{aligned}$$

Derivation of condition $a > b$

$$\begin{aligned}
&\int \frac{\sec^2\frac{1}{2}x}{(a+b)+\sqrt{(a-b)}\tan\frac{1}{2}x}^2 \\
&y = \sqrt{a-b}\tan\frac{1}{2}x \\
&\frac{dy}{dx} = \frac{1}{2}\sqrt{a-b}\sec^2\frac{1}{2}x \\
&\sec^2\frac{1}{2}x = \frac{2dy}{\sqrt{a-b}} \\
&I = \frac{2}{\sqrt{a-b}}\int \frac{dy}{(\sqrt{(a+b)})^2+y^2} \\
&I = \frac{2}{\sqrt{a-b}}\times\frac{1}{\sqrt{a+b}}\tan^{-1}(\frac{y}{\sqrt{a+b}}) \\
&I = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2})
\end{aligned}$$

Derivation of condition $b > a$

$$\int \frac{\sec^2 \frac{1}{2}x}{(a+b) - \sqrt{(b-a)}\tan\frac{1}{2}x}^2$$

$$\text{Let } y = \sqrt{b-a}\tan\frac{1}{2}x$$

$$\frac{dy}{dx} = \frac{1}{2}\sqrt{b-a}\sec^2\frac{1}{2}x$$

$$\sec^2\frac{1}{2}x = \frac{2dy}{\sqrt{b-a}}$$

$$I = \frac{2}{\sqrt{b-a}}\int \frac{dy}{(\sqrt{(a+b)})^2 - y^2}$$

$$I = \frac{2}{\sqrt{b-a}} \times \frac{1}{2\sqrt{a+b}}\log_e(\frac{\sqrt{a+b}+y}{\sqrt{a+b}-y})$$

$$I = \frac{1}{\sqrt{b^2-a^2}}\log_e(\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{1}{2}x}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{1}{2}x})$$

Derivation of condition $a = b$

$$I = \int \frac{dx}{a(1+\cos(x))}$$

$$= \frac{1}{a}\int \frac{dx}{2\cos^2(\frac{x}{2})}dx$$

$$= \frac{1}{2a}\int \sec^2\frac{x}{2}dx$$

$$= \frac{1}{2a}\frac{\tan\frac{x}{2}}{\frac{1}{2}}$$

$$= \frac{1}{a}\tan\frac{x}{2}$$

Expression for special trigonometric function in the teems of $a + b\cos(x)$ **as denominator in** $b > a$

$$\int \frac{dx}{a+b\cos x} = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2})$$

Expression for special trigonometric function in the teems of $a + b\cos(x)$ **as denominator in** $a > b$

$$\int \frac{dx}{a+b\cos x} = \frac{1}{\sqrt{b^2-a^2}}\log_e(\frac{\sqrt{b+a}+\sqrt{b-a}\tan\frac{1}{2}x}{\sqrt{b+a}-\sqrt{b-a}\tan\frac{1}{2}x})$$

Expression for special trigonometric function in the teems of $a + b\cos(x)$ as denominator in $a = b$

$$\int \frac{dx}{a + b\cos x} = \frac{1}{a}\tan\frac{x}{2}$$

Expression for special trigonometric function as $a + b\sin(x)$ in denomminator

The expression for special trigonometric function as $a + b\sin(x)$ in deominator is

$$\int \frac{dx}{a + b\sin(x)}$$

Derivation of integral of special trigonometric function in terms of $a + b\sin(x)$ as denominator wher $a > b$

$$I = \int \frac{dx}{a + b\sin(x)}$$

$$I = \int \frac{dx}{a\sin^2\frac{x}{2} + a\cos^2\frac{x}{2} + b2\sin(x)\cos(x)}$$

$$I = \int \frac{\sec^2\frac{x}{2}}{a\tan^2\frac{x}{2} + a + 2b\tan(\frac{x}{2})}dx$$

$$\text{Let} y = \tan\frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$

$$\sec^2\frac{x}{2}dx = 2dy$$

$$I = \int \frac{2dy}{ay^2 + 2by + a}$$

$$I = 2\int \frac{dy}{(\sqrt{a}y)^2 + 2\sqrt{a}y\frac{b}{\sqrt{a}} + (\frac{b}{\sqrt{a}})^2 + a - (\frac{b}{\sqrt{a}})^2}$$

$$I = 2\int \frac{dy}{(\sqrt{a}y + fracb\sqrt{a})^2 + (\sqrt{\frac{a^2-b^2}{a}})^2}$$

$$I = \frac{2}{a\sqrt{a^2-b^2}}\tan^{-1}(\frac{\sqrt{a}\tan\frac{x}{2} + \frac{b}{\sqrt{a}}}{\frac{\sqrt{a^2-b^2}}{\sqrt{a}}})$$

$$I = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}(\frac{a\tan(\frac{x}{2}) + b}{\sqrt{a^2-b^2}})$$

Expression for integration of special trigonometric function in the terms of $a + b\sin(x)$ as denominator where $a > b$

The expression for special trigonometric functions in the terms of $a + b\sin(x)$ as denominator is

$$\int \frac{dx}{a + b\sin(x)} = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1}(\frac{a\tan(\frac{x}{2}) + b}{\sqrt{a^2 - b^2}})$$

Derivation of integral of special trigonometric function in terms of $a + b\sin(x)$ as denominator wher $b > a$

$$I = \int \frac{dx}{a + b\sin(x)}$$

$$I = \int \frac{dx}{a\sin^2\frac{x}{2} + a\cos^2\frac{x}{2} + b2\sin(x)\cos(x)}$$

$$I = \int \frac{\sec^2\frac{x}{2}}{a\tan^2\frac{x}{2} + a + 2b\tan(\frac{x}{2})} dx$$

$$\text{Let} y = \tan\frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$$

$$\sec^2\frac{x}{2} dx = 2dy$$

$$I = \int \frac{2dy}{ay^2 + 2by + a}$$

$$I = 2\int \frac{dy}{(\sqrt{a}y)^2 + 2\sqrt{a}y\frac{b}{\sqrt{a}} + (\frac{b}{\sqrt{a}})^2 + a - (\frac{b}{\sqrt{a}})^2}$$

$$I = 2\int \frac{dy}{(\sqrt{a}y + \frac{b}{\sqrt{a}})^2 - (\sqrt{(\frac{b^2 - a^2}{a}})^2}$$

$$I = \frac{1}{\sqrt{b^2 - a^2}} \log_e \frac{a\tan\frac{x}{2} + b - \sqrt{b^2 - a^2}}{a\tan\frac{x}{2} + b + \sqrt{b^2 - a^2}}$$

Expression for integration of special trigonometric function in the terms of $a + b\sin(x)$ as denominator where $b > a$

The expression for special trigonometric functions in the terms of $a + b\sin(x)$ as denominator is

$$I = \frac{1}{\sqrt{b^2 - a^2}} \log_e \frac{a \tan \frac{x}{2} + b - \sqrt{b^2 - a^2}}{a \tan \frac{x}{2} + b + \sqrt{b^2 - a^2}}$$

Derivation for the integration of special trigonometric function $a + b \sin(x)$ where $a = b$ in terms of denominator