What is the correct relationship between credence and belief?

I   Introduction

Recently, epistemologists have disagreed on what counts as a belief. Jackson (2020a, p.5077) postulates that in a lottery Lara cannot (rationally) believe that her ticket is going to lose. In contrast, Hawthorne et al. (2016, p.1397) argue that Lara's belief is perfectly 'reasonable', even though she cannot *know* that her ticket will lose. Tentatively, I shall define belief as an all-or-nothing propositional attitude: either I disbelieve proposition p, withhold belief on p or believe p. On the other hand, credences should be understood as numbers on the interval $[0, 1]$ that allow us to capture different levels of confidence among propositions: the higher the credence, the stronger my confidence in p.

I argue for three claims about beliefs and credences. First, depending on our definitions, we can reach radically different conclusions about their relationship. In particular, we have to decide whether to include *precision* and *qualitative confidence* in our definitions of credences and beliefs, respectively. The *precision* property requires credences to be precise numbers, e.g., credences must be exact decimals such as $0.4936(71)$ and not just approximate numbers such as ≈ 0.5. The *qualitative confidence* property allows us to qualitatively rank our confidence in different beliefs without appealing to credences. Second, I suggest that irrespectively of whether credences are allowed to be imprecise, *qualitative confidence* is a reasonable assumption on our notion of belief. Third, given *qualitative confidence*, I conclude that while credences cannot be eliminated from our mental lives, beliefs are more fundamental, i.e., I argue for a belief-first view.

To support this position, drawing on Jackson (2020*b*), I begin with a framework for thinking about belief and credence (Section II). Next, I introduce *qualitative confidence* and *precision* and explore the implications of different definition (Section III). Lastly, I justify why my belief-first view is the most sensible position (Section IV).

II   Framework

It is instructive to distinguish several questions before examining the relationship between belief and credence. First, should we approach belief and credence from the standpoint of *rational* humans or of *actual* humans (Buchak, 2014, pp.287-288)? The latter is a descriptive question about how people actually respond to evidence whereas the former is a normative question about how a rational person ought to respond. Second, the *elimitavist question* asks whether we can eliminate either credences or beliefs from our taxonomy of mental states (Jackson, 2020*b*, p.2). There are three potential answers: (i) belief-eliminativism which claims that we can eliminate beliefs completely, (ii) credence-eliminativism which claims that we can eliminate credences completely and (iii) elimitavist dualism which claims that neither credences, nor beliefs can be eliminated. Third, assuming we accept (iii), there is the *reductivist question* (Jackson, 2020*b*, pp.2-3). Can our beliefs be reduced to credences or can our credences be reduced to beliefs? Once again, we can answer by adopting one of three positions: (1) a belief-first view, on which credences can be reduced to beliefs, (2) a credence-first view, on which beliefs can be reduced to credences, and (3) reductive dualism, on which neither belief, nor credence is more fundamental. In this essay, I understand the claim that a reduces to b to require that b is a necessary condition for a where we might also need other conditions in addition to b to get a, i.e. b can be insufficient on its own for a. While we would ideally be more specific on our definition of reduction, I will not discuss any additional conditions due to space constraints.[1] Note that the *elimitavist question* is logically prior to the *reductivist question*, since the latter only arises if we accept elimitavist dualism.

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[1]Jackson (2020*b*, pp.3-4) discusses more options about how we may define reducing.

Without specifying further our definitions of credence and belief, we can conclusively answer only the first question. Following Buchak (2014, pp.287-288) and Jackson (2020a, p.5074), I focus on the normative question. This does not mean that the empirical literature will not be useful but only that we limit ourselves to cases, in which agents consciously and rationally respond to relevant evidence. Moreover, tackling the normative question does not require that we start from the perspective of a rational agent as defined formally in decision theory, e.g., an agent whose preferences satisfy transitivity and completeness and who uses expected utility maximisation to make choices under uncertainty (Mas-Colell et al., 1995, p.7). While taking the standpoint of rational *agents* might be an interesting question, our goal is to understand how rational *humans* should respond to evidence.

III   The importance of definitions

There are two ways to define credence and two ways to define belief that can provide different answers to the *reductivist question* and, arguably, the *elimitavist question*. First, consider the *precision* property that some impose on credences. For instance, Horgan (2017, p.1) require credences to be exact numbers on the interval $[0, 1]$, e.g., $cr(Liverpool\ wins\ EPL) = 0.4145(7)$ where EPL stands for the English Premier League which is the top-flight of English football. In contrast, Moon and Jackson (2020, p.654) think that '[c]redences need not be *precise*', e.g., $cr(Liverpool\ wins\ EPL) \approx 0.4$ is a valid credence.

Second, some authors think that by definition the notion of belief allows us to qualitatively rank our confidence in different beliefs (Horgan, 2017, p.4). For example, I might believe that (Manchester) City won the EPL in 2021 and that Newcastle won the EPL in 1927.[2] When asked which proposition I am more confident in, I might realise that I believe more strongly the claim about City. To capture this difference in confidence, we need to introduce modal operators of probability such as *likely* and *probably* into the propositions that form our beliefs (Yalcin, 2010). These operators allow us to turn our beliefs into modal beliefs:

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[2]Technically, the top-flight of English football was called First Division in 1927 but I call it EPL for brevity.

(1) it is *highly* likely that City won the EPL in 2021 and (2) it is likely that Newcastle won the EPL in 1927. We can see how modal belief (1) expresses greater qualitative confidence than modal belief (2). The difference in the confidence levels might be coming from the evidence that I can recall in support of each proposition: the evidence for (2) might be a subset of the evidence for (1), which implies that I am more confident in (1).[3] So, assuming qualitative confidence means that we allow different levels of confidence in our beliefs which can be linguistically captured via modal operators.

Some authors have either denied *qualitative confidence* or have not stressed its importance. If one accepts the Certainty View that believing p is $cr(p) = 1$, different levels of *qualitative confidence* cannot occur. Other authors who reject the Certainty View prefer to capture different degrees of confidence in propositions not via beliefs' *qualitative* confidence but instead via credences' *quantitative* confidence (Jackson, 2020*b*, p.2). So, we can either impose *qualitative confidence* on our notion of belief or reject it.

Thus, we have four sets of definitions to examine. Table 1 summarises the four options and suggests that they give raise to different answers to the *reductivist question*. Let us consider each case in more depth.

Property	*Qualitative confidence*	No *Qualitative confidence*
Precision	Belief-first view	Unclear
No *Precision*	Belief-first view	Reductive dualism

Table 1: Four ways to define belief and credence.

III.1   *Qualitative confidence* and *precision*

Definitions of credence and belief assuming both properties can be found in Horgan (2017). They imply that credences should be of the type $cr(Chelsea\ finishes\ in\ Top\ 4) = 0.7929(3)$ and beliefs are allowed to be of the type 'Chelsea will *most likely* finish in top 4'. In general,

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[3]See Section IV for a description about how this can occur.

forming exact numerical credences is not an easy task (Horgan, 2017, p.3). Most of the time, we cannot meaningfully distinguish $cr(Chelsea\ finishes\ in\ Top\ 4) = 0.7929(\mathbf{13})$ from $cr(Chelsea\ finishes\ in\ Top\ 4) = 0.7929(\mathbf{14})$, as both of these seem to capture our propositional attitude despite being different numerically. It is, thus, hard to imagine how we can attach precise credences to all propositional attitudes that we hold. Even if we use Ramsey's operational definition of credences[4] (Williamson, 2002, pp.1-2), we might still be indifferent between two payments when entering a bet about Chelsea's chances of finishing in Top 4.

In my view, we can only form precise credences in two relatively rare cases. First, there are cases of objective chance, where we know in advance all potential outcomes and their probabilities. When I am throwing a fair dice, I know that $cr(Get\ 6\ |\ fair\ dice) = 0.1(6)$. Second, we can conduct explicit updating based on a well-defined set of priors to arrive at precise credences. On Bayesian confirmation theory in philosophy of science (Talbott, 2016, Section 4), for example, we start with priors about the truthfulness of different hypotheses which we update given new evidence. Nevertheless, most often we do not conduct Bayesian updating explicitly. Most of our propositional attitudes are beliefs. I believe 'Chelsea will most likely finish in top 4' not because I have conducted Bayesian updating on the basis of all evidence I have seen, but because I can cite enough good reasons for my claim, e.g., they have a better squad than their competitors. Thus, under *qualitative confidence* and *precision*, elimitavist dualism sounds like the most reasonable positions: while precise credences are rare, they cannot be fully eliminated from our taxonomy.

What about the *reductivist question*? One can argue that we can explain objective chance propositions and Bayesian updated credences without introducing the notion of credences. We only need to introduce the notion of *probability*-beliefs which are defined as beliefs that involve a reference to an exact numerical probability and not just a modal operator. Objective chance propositions are equivalent to epistemic probability-beliefs. Based on my evidence that I am throwing a fair dice I believe that 'the probability of throwing a six is

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[4]My credence in C is the highest payment (divided by 100) that I am willing to make to enter a bet, in which I get £100, if C ends up true, and £0, if C ends up false.

1/6'. On the other hand, credences based on Bayesian updating can be reduced to logical arguments that only require believing each premises and probability-beliefs:

Prior beliefs: 'Chelsea finishes in Top 4 with probability p' and 'Chelsea does not finish in Top 4 with probability $1 - p$'

Evidence (Injuries): All of Chelsea's forwards get injured.

Updating rule: Use Bayes' rule to update the priors (following Talbott, 2016):

$$q = Pr(Chelsea\ in\ Top\ 4 | Injuries) = \frac{Pr(\{Chelsea\ in\ Top\ 4\} \cap \{Injuries\})}{Pr(\{Injuries\})}$$

where $Pr(\{Chelsea\ in\ Top\ 4\} \cap \{Injuries\})$ is the probability that both Chelsea finishes in Top 4 *and* their forwards get injured and $Pr(\{Injuries\})$ is the probability that Chelsea's forwards get injured.[5]

Result: Update prior belief to 'Chelsea finishes in Top 4 with probability q'.

The idea is that we use our belief in Bayes' rule to update our priors given new evidence. This allows us to form updated probability-beliefs: we replace the prior p with the updated q in our probability-belief. In that sense, the concept of belief allows us not only to explain standard non-probability beliefs, but also to explain objective-chance propositions and credences resulting from Bayesian updating. Thus, the belief-first view has greater explanatory power because we can reduce credences to probability-beliefs, i.e., beliefs are more fundamental than credences.[6]

Here a belief-sceptic can object that the notion of a probability-belief is equivalent to precise credence and it is a matter of personal taste which notion is used. A credence-sceptic will say that probability-beliefs are beliefs and a belief-sceptic will say that probability-beliefs are credences. So, we cannot conclusively eliminate credences from our taxonomy, even if we think that beliefs are more fundamental. Even if this is true, beliefs are more fundamental in the sense that we cannot form objective chance credences without the belief that we are throwing a fair dice and we cannot do updating without the belief that Bayes' rule is the

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[5]We can extend this by applying Bayes' Theorem to illustrate the connections with our prior p.

[6]This conclusion might also be taken to imply that we can *eliminate* credences, but I will not make this argument for reasons of space.

correct way to update my credences. Having beliefs is a necessary condition for credences based on objective chance or updating. This leads us to a belief-first view as the most plausible answer to the *reductivist question*.

III.2   *Qualitative confidence* and no *precision*

Following Moon and Jackson (2020), we can assume *qualitative confidence* and no *precision*. Our credences are allowed to be imprecise, e.g., $cr(Chelsea\ finishes\ in\ Top\ 4) \approx 0.8$, whereas our beliefs can state that 'Chelsea will most likely finish in top 4', i.e., we allow our beliefs to be modal. We can immediately see that these two propositional attitudes are quite similar, as they both express high confidence that Chelsea will finish in Top 4. With that in mind, I argue that (i) imprecise credences allow us to fine-grain modal operators (of possibility) but that (ii) ultimately imprecise credences are reducible to modal beliefs.

How can we support (i)? Let us assume that in our language there are only three legitimate modal operators: 'it is not very likely', 'it is likely' and 'it is very likely'. In contrast, even with imprecise credences with one decimal place (0.0, 0.1 ..., 1.0), we already have 11 modal operators. So, imprecise credences allow more flexibility in propositional attitudes. However, it is unclear if imprecise credences are more similar to modal beliefs or to precise credences. This distinction matters because if they are simply flexible modal beliefs, they can be interpreted as qualitatively-graded beliefs that simply use a different modal operator.

There are two reasons to think that imprecise credences are more similar to modal beliefs. First, certain procedures that can be applied to *precise* credences to yield insight can give very misleading results when applied to *imprecise* credences. For example, small changes in credences can yield big changes to posterior probabilities in Bayesian inference. I provide a simple numerical illustration in Appendix A. In the example, increasing one credence from 0.001 to 0.002 doubles the posterior probability of having cancer conditional on a certain symptom. So, using the precise formal tools of Bayesian inference with imprecise prior credences can lead to a very wide confidence interval of posterior credences which is not particularly informative. One might object that we simply have to live with such a wide

confidence interval, given the setup of the example. However, often we need to conduct several rounds of Bayesian updating, e.g., a person contains several symptoms which are correlated with cancer. If we have imprecise credences at each round, then the imprecisions accumulate which renders the final confidence interval wider and less helpful. This would be especially problematic in high-stakes scenarios such as deciding how likely it is that a patient has cancer. In any case, the issues ensuing from applying Bayesian updating to imprecise credences suggest that perhaps they are distinct from precise credences.

Second, we can distinguish different imprecise credences by appealing to other beliefs. Suppose I consider the proposition that *there is a 0.3 chance that Chelsea wins the FA Cup final*, in which Liverpool and Chelsea play. I then recall that the last time Chelsea played Liverpool in a cup final they had to play penalties after added extra time. So, I update my belief to *there is a 0.4 chance that Chelsea wins FA Cup*. In this case, the claim 'there is a p chance' simply reflects a numerical modal operator, not a probability. We managed to distinguish the two propositions by using another belief about the last final between Chelsea and Liverpool. There is no need to appeal to Bayesian updating to explain why we are more confident in one of them. Rather, we consider modal operator 0.4 as providing a better summary of our cumulative evidence than modal operator 0.3. Analogically, we can move from a modal belief containing 'likely' and to a model belief containing 'very likely' when we encounter a new piece of evidence without explicit updating.

Moreover, attaching numerical values to different confidence levels seems similar to the notion of utility. I can say that my utility from an Americano is 9 whereas my utility from an Oat Latte is 4. This reflects not only that I prefer an Americano over an Oat Latte but also by how much. Attaching a number to utility gives us more flexibility to express our preferences. Similarly, attaching imprecise credences instead of standard modal operators allows us more flexibility to express our propositional attitudes.[7] As long as we allow our beliefs to contain modal operators, i.e., qualitative grading, then imprecise credences reduce

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[7]We also know that utility functions are reducible to preferences under certain assumptions such as continuity and rationality (Mas-Colell et al., 1995, p.7 and p.47), so perhaps numerical credences can be reduced to modal beliefs too.

to beliefs with numerical modal operators.

So, imprecise credences can be reduced to modal beliefs which implies a belief-first view. As in Section III.1, we can reject credence-eliminativism since there exist situations, in which we can form *precise* credences that cannot be interpreted as flexible modal beliefs (unless we assume probability-beliefs).

III.3   No *Qualitative confidence* and *precision*

On these definitions, our credences are of the type $cr(Chelsea\ finishes\ in\ Top\ 4) = 0.7929(3)$ whereas our beliefs are constrained to be of the type 'Chelsea will finish in top 4'. This set of definitions is very restrictive, given how rarely we form precise credences (Section III.1). Moreover, it is not obvious how we can rank our confidence in different propositions unless they result from objective chance or explicit Bayesian updating. So, starting from these definitions is unlikely to illuminate the relationship between belief and credence.

III.4   No *Qualitative confidence* and no *precision*

Given these definitions which can be found in Buchak (2014), our credences are allowed to be imprecise, e.g., $cr(Chelsea\ finishes\ in\ Top\ 4) \approx 0.8$, whereas our beliefs are constrained to state that 'Chelsea will finish in top 4'. In my view, reductive dualism is the most reasonable position to hold here and I subscribe to Buchak's argument (2014):

A1: Credences exist and cannot be reduced to beliefs.

P1: If beliefs do not exist, they can always be (functionally) replaced by credences.

P2: Beliefs cannot always be replaced by credences.

C1: Thus, beliefs exist.

P3: Beliefs cannot be reduced to credences.

C2: Thus, beliefs and credences exist.

While Buchak decides not to justify explicitly A1 (2014, pp.288-289), without A1 there is no way to attach different confidence levels to our propositions since we assume beliefs

have no *qualitative confidence*. So, it is natural to justify A1 by arguing that we need it to capture different confidence levels. Next, if beliefs do not exist (credence-eliminativism), every role that they play in our mental lives can be performed by credences (P1) (Buchak, 2014, p.308). However, there exist certain reactive attitudes such as blaming, which assume the notion of belief in their most natural formulations and which cannot be reformulated using credences (Buchak, 2014, pp.296-301).[8] This means that beliefs cannot always be replaced by credences (P2). So, by *modus tollens* beliefs exist (C1): elimitavist dualism holds. Moreover, sometimes even if we have $cr(p) \approx 1$, we cannot form a belief in p (P3) (Buchak, 2014, pp.292). For example, if we participate in a lottery with one million tickets, we cannot rationally form the belief that our ticket will lose. Given C1, reductive dualism follows (C2), since beliefs are not reducible to credences. P1 and P3 are related, but P1 address the *elimitavist question* whereas P3 address the *reductivist question*, so they have distinct roles in Buchak's argument. Importantly, my representation of Buchak's argument does not follow the way, in which it is presented in her paper, since for me the *elimitavist question* is prior to the *reductivist question* (Section II).

While I do not object to Buchak's reasoning, her argument depends heavily on her definitions. If we assume that beliefs possess *qualitative confidence*, then reductive dualism is no longer the most sensible position. For this reason, I will now consider what lessons we can draw from emphasising the importance of definitions.

IV Discussion

From the preceding four cases, we can conclude that imprecise credences are reducible to modal beliefs and that precise credences are relatively rare but not fully eliminable from our taxonomy. Moreover, if *qualitative confidence* is indeed a property of beliefs, the belief-first view follows, irrespectively of how we define credences. However, reductive dualism follows if we reject *qualitative confidence* of beliefs. Thus, the key question is whether we can ascribe

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[8]For space constraints, I cannot discuss her argument about blaming in greater depth.

qualitative confidence to beliefs.

There are two reasons to accept *qualitative confidence*. First, our confidence in a belief depends on the number of sufficient reasons for holding it. Since we are looking from the standpoint of an epistemically rational person, having sufficient reason reduces to having sufficient evidence. The more evidence we have for a belief, the more confident we are. For example, recall that I am more confident that (i) City won EPL in 2021 than that (ii) Newcastle won the EPL in 1927. My evidence for both beliefs might be that I looked at the list of English football champions yesterday, but I additionally remember seeing on TV the ceremony, in which City was awarded the 2021 trophy. So, I have one additional piece of evidence for (i), meaning that I am more confident in it. Note that we can only rank our beliefs *qualitatively* (and not quantitatively), since we do not make a judgement on how to weight the two pieces of evidence for City's triumph. I am not saying how much more evidence there is for (i) but only that there is more evidence, i.e., the evidence for (ii) is a subset of the evidence for (i).[9]

Second, there is empirical support for *qualitative confidence*. In particular, memory stores information categorically and does not store degrees of confidence (Weisberg, 2020, pp.24-25). So, when memories are invoked, they function like beliefs. However, how well we remember memory m depends on various factors such as the quantity of background information associated with m (Koriat, 1993, p.636). The more information we recall about m, the greater our confidence in m. When recalling, our belief in m comes with a particular qualitative confidence which is a function of the background information. This provides evidence for the idea that beliefs inherently differ in their qualitative confidence. For example, I am more confident in my belief about what I ate this morning than about what I ate yesterday morning. The reason is that I can recall more background information about how my morning progressed today. Thus, empirical studies also support qualitative confidence which suggests that is a natural property to attach on our notion of belief, from which a

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[9]One might object that often the evidence for two beliefs is not comparable and our confidence in one of them is greater. Even if this is true, the purpose of the example is to show that it is at least possible to qualitatively rank our confidence in different beliefs.

belief-first view follows.

V Conclusion

Given the aforementioned, a belief-first view correctly captures the relationship between belief and credence. In other words, while we have both beliefs and credences, beliefs are the more fundamental propositional attitude. To support this position, I suggested that deciding whether to assume *precision* and *qualitative confidence* in our definitions of belief and credence matters considerably for how we should view their relationship (Table 1). For example, if we drop *precision* and accept *qualitative confidence*, a belief-first view follows. However, if we drop both *qualitative confidence* and *precision*, a *reductive dualism* falls.

More broadly, my argument about the importance of definitions suggested three things about credences: (i) imprecise credences are reducible to modal beliefs, (ii) precise credences are ineliminable from our mental lives and (iii) beliefs are more fundamental than precise credences. So, both precise and imprecise credences are less fundamental than beliefs. Next, I argued that *qualitative confidence* is a natural property to impose on our concept of belief because of empirical data and because there are cases, in which we can qualitatively rank our confidence in different beliefs on the basis of our evidence. Therefore, irrespectively of whether we impose *precision* on credences, beliefs are the more fundamental propositional attitude.

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VI   Appendix A

Here we show that imprecise credences combined with Bayesian updating can lead to strange results, even when we are slightly unsure about a credence. We are interested in the probability of having cancer conditional on having a certain symptom, i.e., $Pr(Cancer|Symptom)$, and we collect data on 1000 people.

Symptom	Yes	No	Row Share
Cancer			
Yes	1	0	1/1000
No	10	989	999/1000
Column Share	11/1000	989/1000	1

Table 2: Original data

We can calculate:

$$Pr(Cancer|Symptoms) = \frac{Pr(Cancer \cap Symptoms)}{Pr(Symptoms)} = \frac{1/1000}{11/1000} = 0.(09)$$

After doing the calculations, we realise that one person *might* have been misclassified as not having cancer despite having the symptom. We suspect a measurement error but we are not sure. We decide to examine what happens to $Pr(Cancer|Symptom)$ in case of measurement error. Our new data looks pretty much unchanged:

Symptom	Yes	No	Row Share
Cancer			
Yes	2	0	2/1000
No	9	989	998/1000
Column Share	11/1000	989/1000	1

Table 3: New data

The posterior probability is:

$$Pr(Cancer|Symptoms) = \frac{Pr(Cancer \cap Symptoms)}{Pr(Symptoms)} = \frac{2/1000}{11/1000} = 0.(18)$$

The conditional probability of cancer conditional on the symptom is twice as high now: our imprecise credence has dramatic consequences. Admittedly, this example is stylised, but it shows that we have to be careful when conducting Bayesian inference with imprecise credences.