

## ARRAYED\_HEAP+

## SCHEDULER+

### feature -- Attributes

array+: ARRAY[INTEGER]  
*-- Array representation of a heap*  
count+: INTEGER  
*-- Number of keys stored in the heap*  
max\_capacity+: INTEGER  
*-- Maximum number of keys that can be stored in the heap*

### feature -- Constructors

make+(keys: ARRAY[INTEGER]; n: INTEGER)  
*-- Creates a heap from array 'keys' and sets max\_capacity to n*  
**require**  
enough\_capacity: keys.count < n  
all\_positive:  $\forall i : 1 \leq i \leq \text{keys.count} : \text{keys}[i] > 0$   
no\_duplicates:  $\forall i : 1 \leq i \leq \text{keys.count} : \text{keys.occurrences}(\text{keys}[i]) = 1$   
**ensure**  
max\_capacity: array.count = max\_capacity  $\wedge$  max\_capacity = n  
heap\_size\_set: count = keys.count

### feature -- Commands

insert+(key: INTEGER)  
*-- Add 'key' into the heap, if it does not exist*  
**require**  
non\_existing\_key:  $\neg \text{key\_exists}(\text{key})$   
**ensure**  
size\_incremented: count = **old** count + 1  
same\_set\_of\_keys\_except\_the\_new\_key:  $\forall i : 1 \leq i \leq \text{count} : \text{array}[i] \neq \text{key} \Rightarrow \text{array}[i] \in (\text{old array.twin}) \vee \text{array}[i] = \text{key} \Rightarrow \text{True}$

### remove\_maximum+

*-- Remove the maximum of the heap, if it the heap is not empty*  
**require**  
non\_empty\_heap:  $\neg \text{is\_empty}$   
**ensure**  
size\_decremented: count = **old** count - 1  
same\_set\_of\_keys\_except\_the\_removed\_key:  $\forall i : 1 \leq i \leq \text{count} : (\text{old array}[1]) \in \text{array} \Rightarrow \text{False} \wedge (\text{old array.twin})[i] \in \text{array}$

### feature -- Queries

maximum+  
*-- Returns the current maximum key in the heap*  
**require**  
non\_empty:  $\neg \text{is\_empty}$   
**ensure**  
correct\_result:  $\forall i : 2 \leq i \leq \text{count} : \text{array}[1] > \text{array}[i]$

### is\_empty+: BOOLEAN

*-- Is the heap currently empty?*

### key\_exists+(a\_key: INTEGER): BOOLEAN

*-- Does 'a\_key' currently exist in the heap?*  
**ensure**  
correct\_result:  $\text{True} \Rightarrow \exists i : 1 \leq i \leq \text{count} : \text{array}[i] = \text{a\_key} \wedge \text{False} \Rightarrow \forall i : 1 \leq i \leq \text{count} : \text{array}[i] \neq \text{a\_key}$

### invariant

implementation\_indices: array.lower = 1 and array.upper = max\_capacity  
no\_heap\_overflow: count  $\leq$  max\_capacity  
no\_heap\_underflow: count  $\geq$  0  
contents\_of\_heap:  $\forall i : 1 \leq i \leq \text{count} : \text{array}[i] > 0 \wedge \forall j : \text{count} + 1 \leq j \leq \text{max\_capacity} : \text{array}[j] = 0$   
heap\_property:  $\forall i : 1 \leq i \leq \text{count}/2 : \text{array}[i] > \text{array}[2i] \wedge \text{array}[i] > \text{array}[2i+1] \wedge \forall i : \text{count}/2 + 1 \leq i \leq \text{count} : \text{True}$

### feature -- Attributes

tasks+: HASH\_TABLE[TASK, INTEGER]\*  
*-- Hash table storing tasks to be executed and their priority key*  
pq+: ARRAYED\_HEAP  
*-- A max heap to store priority keys of tasks*

### feature -- Constructors

make\_from\_array+(ntasks: ARRAY[TUPLE[task: TASK; priority: INTEGER]]; max\_capacity: INTEGER)  
*-- Creates a scheduler with [keys, task] pairs from ntasks*  
**require**  
enough\_capacity: ntasks.count  $\leq$  max\_capacity  
**ensure**  
scheduler\_size\_set: count = tasks.count  $\wedge$  count = pq.count

### feature -- Commands

add\_task+(new\_task: TUPLE[task: TASK; priority: INTEGER])  
*-- Add 'new\_task' to tasks and insert its priority key into pq*  
**require**  
non\_existing\_priority:  $\neg \text{priority\_exists}(\text{new\_task.priority})$

### next\_task\_to\_execute+: detachable TASK

*-- Returns the task with highest priority in the scheduler*  
**require**  
non\_empty\_tasks:  $\neg \text{is\_empty}$

### execute\_next\_task+

*-- Removes the task with highest priority in the scheduler*  
**require**  
non\_empty\_tasks:  $\neg \text{is\_empty}$

### invariant

consistent\_counts: count = tasks.count  $\wedge$  count = pq.count  
consistent\_priorities\_and\_keys:  $\forall x : x \in \text{tasks.current\_keys} \Rightarrow x \in \text{pq.array}$

pq

tasks

task

array

