MA323 : Lab 3 Report

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Q1

It is given to us that the PDF of the function is : $f(x) = 20x(1-x)^3$ for $0 < x < 1$

Now, in order to use the Acceptance-Rejection method, we first need to find out a suitable PDF $g(x)$ such that $f(x) \leq cg(x)$

We know that :

$$
\begin{aligned}
& 0 < x < 1 \\
\implies & 0 < x(1-x)^3 < (1-x)^3 \\
\implies & 0 < 20x(1-x)^3 < 20(1-x)^3 \\
\implies & 0 < f(x) < 20(1-x)^3
\end{aligned}
$$

Also that:

$$
\begin{aligned}
& 0 < (1-x)^3 < 1 \quad \because \; 0 < x < 1 \\
\implies & 0 < f(x) < 20
\end{aligned}
$$

We can take $g(x) = 1$ and $c = 20$
This means that $g(x) \equiv \text{Uniform}(0,1)$ distribution.

a

It is already proven that the probability of acceptance while using this algorithm is $\frac{1}{c}$. We can model the number of iterations as a random variable which with geometric distribution with the chance of success as $\frac{1}{c}$. We know that the expected value of a geometric random variable is $\frac{1}{\text{Chance of success}}$, which over here comes out to be equal to c.

b

$E[X] = \int_0^1 20x^2(1-x)^3 dx = \frac{1}{3}$

Experimental value : 0.33435

c

$$F(t) = \int_0^t 20x(1-x)^3 dx = -4t^5 + 15t^4 - 20t^3 + 10t^2$$

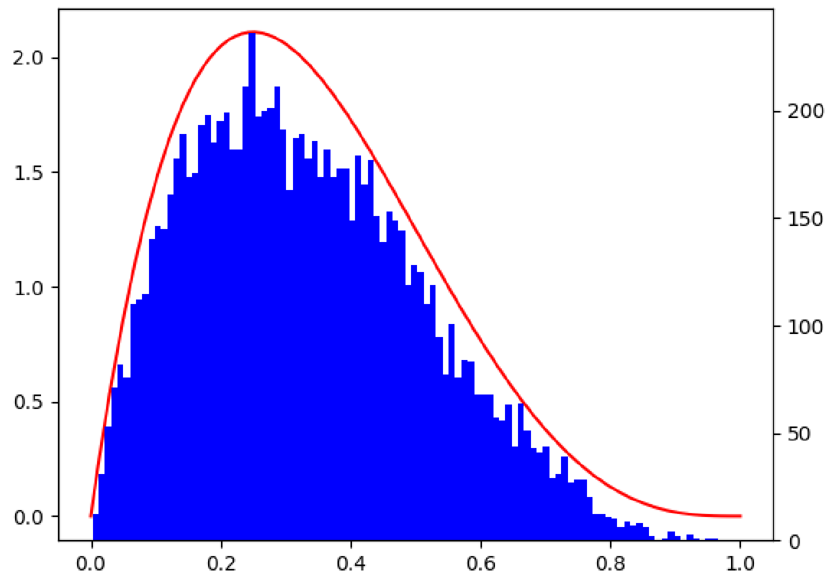
$P(0.25 \leq X \leq 0.75) = F(0.75) - F(0.25) = 0.6171875$
Experimental Probability : 0.6163

d

Average number of iterations (Experimental) : 20.0095

e

Left side axis : $f(x)$
Right side axis : Histogram counts



f

$c = 40$ was taken for this section. Obtained values :

- Average Iterations : 40.362
- $E[X]$: 0.33569
- $P(0.25 \leq X \leq 0.75)$: 0.6192

Q2

The given function resembles $Gamma(\alpha, 1)$. We first need to find out the constant which will make $f(x)$ a valid probability distribution function. Assume $f(x) = K_\alpha x^{\alpha-1}e^{-x}$ where K_α is a constant dependent on α.

$$\frac{1}{K_\alpha} = \int_0^1 x^{\alpha-1}e^{-x}dx = \Gamma(\alpha) - \Gamma(\alpha, 1)$$

So, $K_\alpha = \frac{1}{\Gamma(\alpha)-\Gamma(\alpha,1)}$

$\because 0 < x < 1 \implies \frac{1}{e} < e^{-x} < 1 \implies x^{\alpha-1}e^{-x} < x^{\alpha-1} \implies f(x) \leq \lceil K_\alpha \rceil x^{\alpha-1}$

So, I can take $c = \lceil K_\alpha \rceil$ and $g(x) = x^{\alpha-1}$

$$g(x) = x^{\alpha-1} \implies G(x) = \frac{x^\alpha}{\alpha} \implies G^{-1}(x) = (\alpha x)^{\frac{1}{\alpha}}$$

Obtained Results:

α	**0.7**	**3**	**3.7**
K_α	1.01208	6.2265	8.00461
Sample Mean	0.41735	0.41797	0.41651
Sample Var.	0.07934	0.07928	0.0788