MA323 : Lab 2 Report

Udit Jethva (220123067)

12 August 2024

Q1

First requirement of solving this question is finding out $F(x)$. It is given to us that:

$$f(x) = \begin{cases} 3(1-x)^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

We know that $f(x) = \frac{d}{dx}(F(x))$. So we can conclude that :

$$F(t) = \int_0^t 3(1-x)^2 dx$$

when $0 < t < 1$.

Solving we obtain : $F(t) = (t-1)^3 + 1 \implies \mathrm{F}^{-1}(x) = (x-1)^{\frac{1}{3}} + 1$

We can use this result to generate numbers from this distribution from Uniform distribution.

Calculating Mean and Variance

$E[X] = \int_0^1 3x(1-x)^2 dx = 0.25$

$E[X^2] = \int_0^1 3x^2(1-x)^2 dx = 0.10$

$\mu = E[X] = 0.25$

$\sigma^2 = E[X^2] - (E[X])^2 = 0.0375$

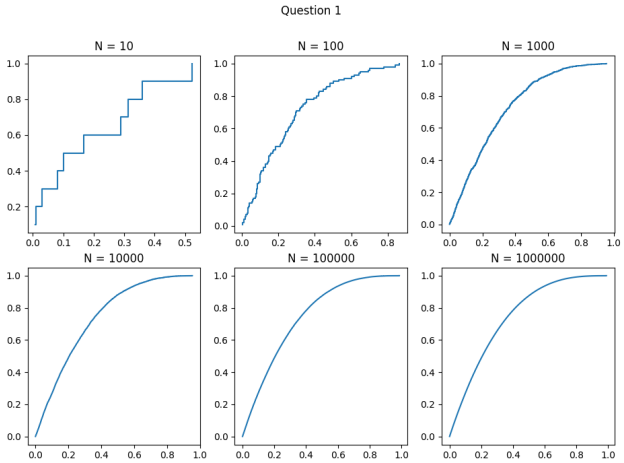
Obtained experimental values :

N	μ	σ^2
10	0.18825	0.02763
100	0.24606	0.04105
1000	0.25761	0.03873
10000	0.24874	0.03691
100000	0.25010	0.03771
1000000	0.24988	0.03740

We can observe that the experimental values of μ and σ^2 are converging to the theoretical values.

CDF Plots on next page

CDF Plots

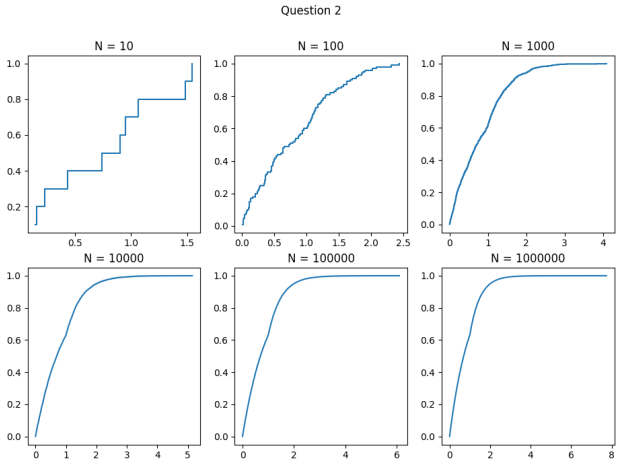


Q2

$F(x)$ is already given to us so we just need to find out the inverse function which goes like this:

$$F^{-1}(x) = \begin{cases} -\ln(1-x) & 0 < x \leq 1 - \frac{1}{e} \\ \frac{1}{2}(1 - \ln(1-x)) & 1 - \frac{1}{e} \leq x \end{cases}$$

The random variables were generated and the following plots of the CDF were obtained :



Calculating Mean and Variance

We know that $f(x) = \frac{d}{dx}F(x)$. Probability density function of the given distribution is given by:

$$f(x) = \begin{cases} e^{-x} & 0 < x \leq 1 \\ 2e^{-(2x-1)} & 1 < x \end{cases}$$

We use this to calculate the theoretical mean and variance which come out as :

$$\mu = 1 - \frac{1}{2e} \approx 0.81606 \ , \ \sigma^2 = -\frac{1}{4e^2} - \frac{3}{2e} + 1 \approx 0.41435$$

Observed values :

N	μ	σ^2
10	0.76712	0.24112
100	0.81685	0.35996
1000	0.82792	0.41040
10000	0.81767	0.41263
100000	0.81690	0.41449
1000000	0.81634	0.41473

Q3

The code generates the values and the frequencies are tabulated in `frequencies.csv`