MA323 : Lab 6 Report

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Q1

We know that we can find $I = E[g(X)]$ where X is a random variable with probability function $f(x)$ by :

$$I = \int_{-\infty}^{\infty} g(x)f(x)dx$$

We can estimate the value of I by utilising the Law of Large Numbers. Let I_M denote the value of I approximated by M sample values.

$$I_M = \frac{1}{M}\sum_{i=1}^{M} g(Y_i)$$

where $g(x) = e^{\sqrt{x}}$ and $\{Y_i\}$ is set of sample values generated from $U(0, 1)$ distribution.
The following values were obtained :

M	I_M
100	1.96843
1000	2.01607
10000	1.99884
100000	1.99952

We'd use the above formulation to find the value of I given in the question.

$$I = \int_0^1 e^{\sqrt{x}}dx = 2$$

Following on the lectures, if we want a 95% confidence interval, then

$$2\Phi(\Delta) - 1 = 0.95 \implies \Delta = \Phi^{-1}(0.975) \implies \Delta \approx 1.96$$

We know that the confidence interval is given by :

$$(\hat{\mu}_M - \frac{\Delta s_M}{\sqrt{M}}, \hat{\mu}_M + \frac{\Delta s_M}{\sqrt{M}})$$

where ${s_n}^2 = \frac{1}{M-1}\sum_{i=1}^{M}(Y_i - \bar{Y})^2$
These were the obtained confidence intervals for the given values of M:

M	Interval
100	(1.87996, 2.0569)
1000	(1.98891, 2.04323)
10000	(1.99025, 2.00744)
100000	(1.99678, 2.00226)