Submission Deadline: August 07, 2024, 4:30 PM

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1. Consider the probability density function

$$f(x) = \begin{cases} 3(1-x)^2 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Generate $X_1, X_2, \ldots, X_N$ from the above distribution for $N = 10, 100, 1000, 10000, 100000$.

   (b) For each value of N, plot the empirical CDF (definition is given below) of these generated values, and the actual distribution function (using the above formula).
   Defn: Let $x_1, x_2, \ldots, x_N$ be sample observations. Then the empirical CDF at $x \in \mathbb{R}$ is defined by

$$F_N(x) = \frac{\#\{i : x_i \leq x\}}{N}.$$

   (c) Provide the corresponding values of the sample mean and variance. Compare the values of sample mean and variance with their population counterpart to see if the desired convergences happen.

2. Consider the cumulative distribution function:

$$F(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 - e^{-x} & \text{if } 0 < x \leq 1 \\ 1 - e^{-(2x-1)} & \text{if } x > 1. \end{cases}$$

   (a) Generate $X_1, X_2, \ldots, X_N$ from the above distribution for $N = 10, 100, 1000, 10000, 100000$.

   (b) For each value of N, plot the empirical CDF of these generated values, and the actual distribution function (using the above formula).

   (c) Provide the corresponding values of the sample mean and variance.

3. Using the algorithm to generate random variables from a discrete distribution, generate 100000 random numbers from a discrete uniform distribution on $\{1, 3, 5, \ldots, 9999\}$. Tabulate the frequency of each observed values.
