MA323 : Lab 5 Report

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Q1

We know that :

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

We observe the given Σ matrix and it is deductible that $\sigma_1 = 3$, $\sigma_2 = 2$ and $\rho = a$
Given that the covariance matrix (Σ) is positive definite, we know that we can generate bivariate normal distribution (X_1, X_2) by using the following formulae:

$$\mathrm{X}_1 = \mu_1 + \sigma_1 Z_1$$
$$\mathrm{X}_2 = \mu_2 + \rho\sigma_2 Z_1 + \sqrt{1-\rho^2}\sigma_2 Z_2$$

where Z_1 and Z_2 are independent standard normal variables.

For the cases of $a = -1$ and $a = 1$, the covariance matrix is positive semi-definite. The expression $x^T\Sigma x$ expands to $(3x_1 - 2x_2)^2$ and $(3x_1 + 2x_2)^2$ for $a = -1$ and $a = 1$ respectively. This means that the join distribution function for the variables (X_1, X_2) won't exist.

However with a correlation value of ±1, there is complete dependence between the two variables, i.e. knowing one variable means that we know the second variable for sure.

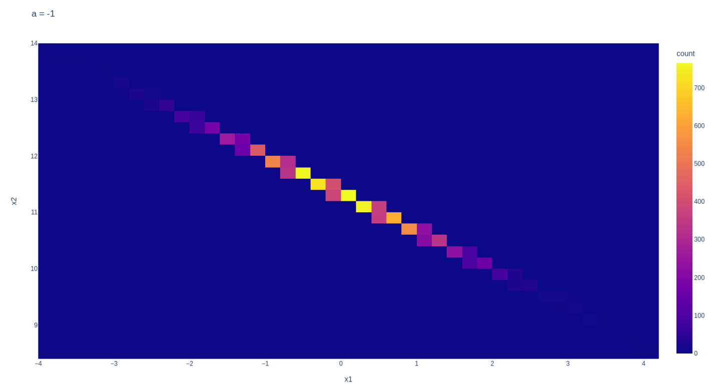
For perfect correlation ($\rho = \pm1$), we can generate X_2 from X_1 using :

$$\mathrm{x}_2 = \mu_2 + \rho\tfrac{\sigma_2}{\sigma_1}(x_1 - \mu_1)$$

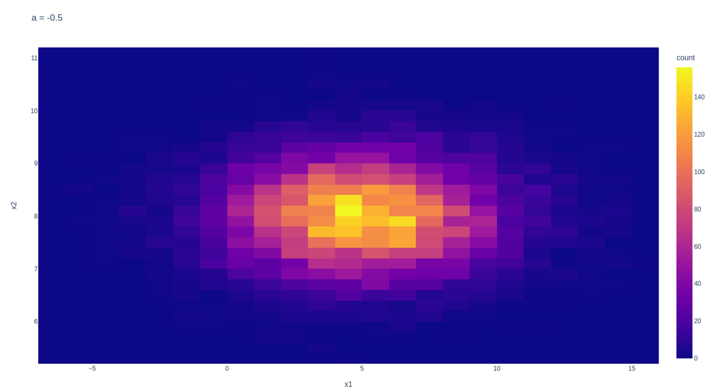
So, for these two cases, we can generate one of the variables normally and obtain the second one deterministically.

I used Box-Muller method to generate two independent standard normal variables in my code.
Here are the obtained 2D histograms or density heatmaps:

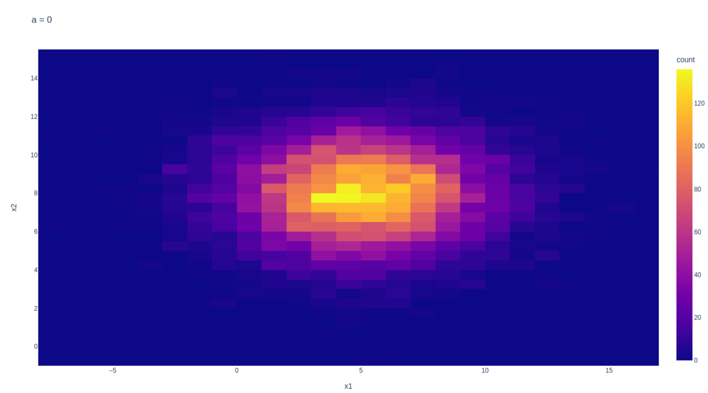
- $a = -1$



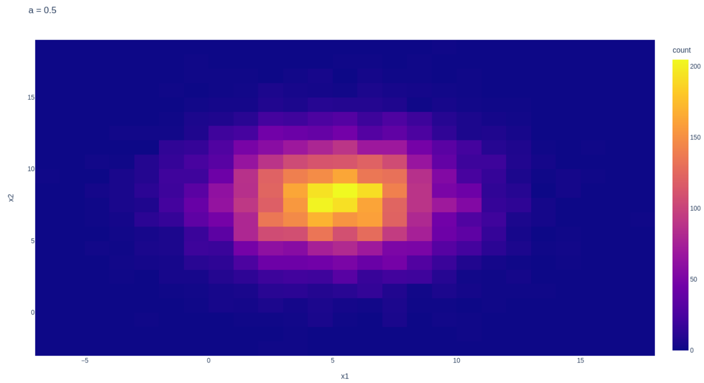
- $a = -0.5$



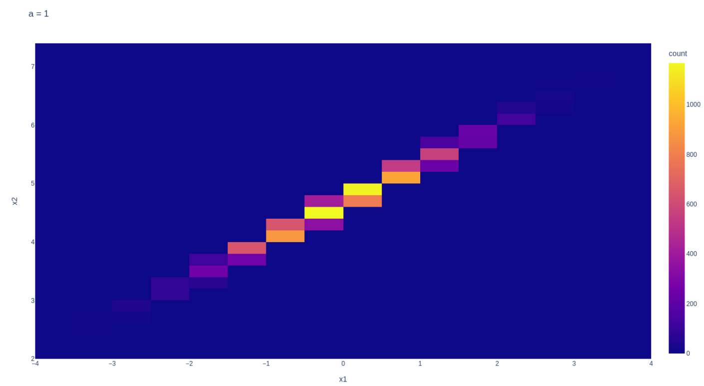
- $a = 0$



- $a = 0.5$

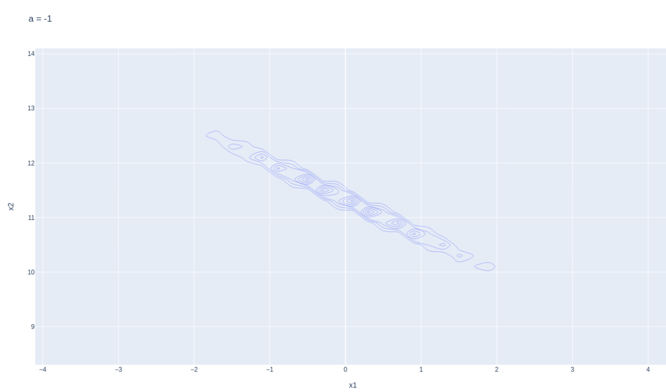


- $a = 1$

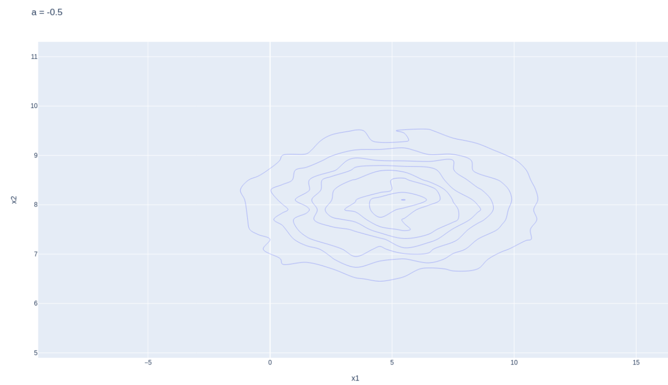


Here are the contour plots:

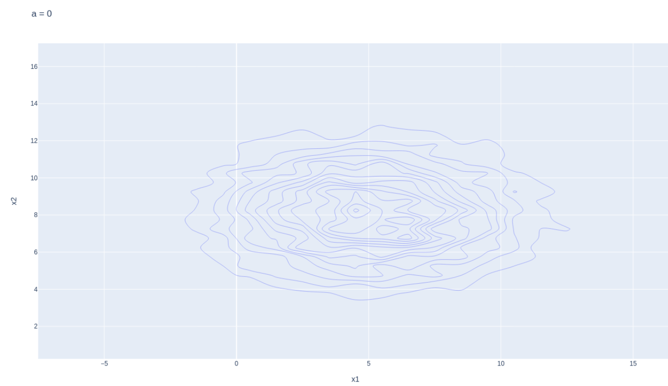
- $a = -1$



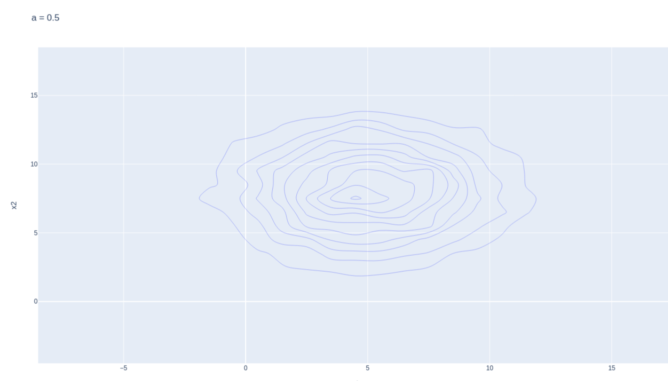
- $a = -0.5$



- $a = 0$



- $a = 0.5$



a = 1