LINEAR ALGEBRA (Fall 2016)　　　　　　　　姓名：________________________

Quiz #5　　班級：________________________學號：________________________

（請寫出<u>主要計算過程</u>，否則不計分）

Let $Y=(\mathbf{y}_1,\mathbf{y}_2,\mathbf{y}_3,\mathbf{y}_4)=\begin{bmatrix}1&1&0&3\\1&-1&2&1\\2&0&2&4\end{bmatrix}$.

1. Find the nullspace of Y. (20%)

2. Is $\mathbf{y}_3\in$ Span($\mathbf{y}_1$, $\mathbf{y}_2$) ? (a) Please answer Yes or No. (10%)
(b) Explain why. (10%)

3. Is the set { $\mathbf{y}_1$, $\mathbf{y}_2$, $\mathbf{y}_3$} a spanning set for $\mathbb{R}^3$? (a) Please answer Yes or No. (10%)　(b) Explain why (10%)

4. Does the set $\{(x_1,x_2,x_3)^T \mid x_1-x_2=1\}$ form a subspace of $\mathbb{R}^3$? (a) Please answer Yes or No. (10%)
(b) Explain why. (10%)

5. Let S be the set of all 2×2 symmetric matrices. Is the set S a subspace of $\mathbb{R}^{2\times2}$? (a) Please answer Yes or No. (10%)　(b) Explain why. (10%)

1.　① $N(Y)=\left\{\mathbf{x}\in R^4 \;\middle|\; \begin{bmatrix}1&1&0&3\\1&-1&2&1\\2&0&2&4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$ ② Reduce $\left[\begin{array}{cccc|c}1&1&0&3&0\\1&-1&2&1&0\\2&0&2&4&0\end{array}\right]$ to $\left[\begin{array}{cccc|c}1&0&1&2&0\\0&1&-1&1&0\\0&0&0&0&0\end{array}\right]$

③ Set $x_3=\alpha$, $x_4=\beta$.　④ Solution $\mathbf{x}=\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}-\alpha-2\beta\\\alpha-\beta\\\alpha\\\beta\end{bmatrix}=\alpha\begin{bmatrix}-1\\1\\1\\0\end{bmatrix}+\beta\begin{bmatrix}-2\\-1\\0\\1\end{bmatrix}$ ⑤ $N(Y)=\mathrm{Span}\left\{\begin{bmatrix}-1\\1\\1\\0\end{bmatrix},\begin{bmatrix}-2\\-1\\0\\1\end{bmatrix}\right\}$

2.　(a) Yes.　(b) Let $\mathbf{y}_3=\alpha\mathbf{y}_1+\beta\mathbf{y}_2=(\mathbf{y}_1,\mathbf{y}_2)\begin{bmatrix}\alpha\\\beta\end{bmatrix}$.　$\Rightarrow$　$\begin{bmatrix}1&1\\1&-1\\2&0\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix}=\begin{bmatrix}0\\2\\2\end{bmatrix}$

$\Rightarrow$　Reduce $\left[\begin{array}{cc|c}1&1&0\\1&-1&2\\2&0&2\end{array}\right]$ to $\left[\begin{array}{cc|c}1&0&1\\0&1&-1\\0&0&0\end{array}\right]$.　$\Rightarrow$　$\mathbf{y}_3=\mathbf{y}_1-\mathbf{y}_2$.

3.　(a) No.　(b) Solve $\begin{bmatrix}a\\b\\c\end{bmatrix}=\alpha_1\begin{bmatrix}1\\1\\2\end{bmatrix}+\alpha_2\begin{bmatrix}1\\-1\\0\end{bmatrix}+\alpha_3\begin{bmatrix}0\\2\\2\end{bmatrix}=\begin{bmatrix}1&1&0\\1&-1&2\\2&0&2\end{bmatrix}\begin{bmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{bmatrix}$

Reduce $\left[\begin{array}{ccc|c}1&1&0&a\\1&-1&2&b\\2&0&2&c\end{array}\right]$ to $\left[\begin{array}{ccc|c}1&1&0&a\\0&1&-1&\frac{a-b}{2}\\0&0&0&-a-b+c\end{array}\right]$　$\Rightarrow$　Inconsistence may occur.

4.　(a) No.　(b) Let $\mathbf{x}=(2,1,3)^T$. Then $\mathbf{x}$ belongs to the set.

Observe $2\mathbf{x}=(4,2,6)^T$ does not belong to the set, since $x_1-x_2=2\neq1$.

$\Rightarrow$　This set is not closed under scalar multiplication and does not form a subspace.

本題可任舉一例說明：沒有加法封閉性或沒有純量乘法封閉性即可。

5.　(a) Yes.　(b) Let $A=\begin{bmatrix}a&b\\b&c\end{bmatrix}$ and $B=\begin{bmatrix}d&e\\e&f\end{bmatrix}$.　⇨　$A\in S$ and $B\in S$. ⇨　$S\neq\phi$ (nonempty)

Since $\alpha A+\beta B=\begin{bmatrix}\alpha a+\beta d&\alpha b+\beta e\\\alpha b+\beta e&\alpha c+\beta f\end{bmatrix}\in S$, S is a subspace of $\mathbb{R}^{2\times2}$.