

Assigned: 11-1-21  
Due Date: 11-11-21

# CS 6210 Introduction to Scientific Computing and Data Computing

## Assignment 4

Note: Please use Matlab, or a public domain approximation to it in this assignment. The code must compile on one of the lab machines with your instructions. Document your code thoroughly!

### Instructions

1. Write a Newton's method code to solve the pair of equations given by

$$\begin{aligned}f_1(x_1, x_2) &= x_1^3 + x_2 - 1 \\f_2(x_1, x_2) &= x_2^3 - x_1 + 1\end{aligned}$$

Use appropriate criteria to measure convergence and to terminate the iteration. Use starting values of  $x_1 = 1.0, x_2 = 0.1$ . Perform a few experiments with different starting values and count the number of iterations it takes to get to convergence.

2. Repeat the process with the potentially difficult system of equations given by

$$\begin{aligned}f_1(x_1, x_2) &= x_1^2 + x_2^2 - 2 = 0 \\f_2(x_1, x_2) &= e^{(x_1-1)} - x_2^2 - 1.0 = 0\end{aligned}$$

Using the top equation to eliminate  $x_2$  from the bottom equation. Find the solution either by using Newton's method for a single equation or by guesswork. What happens if the last term in the second equation is -2.0 instead of -1.0?

Solve the full problem with values  $x_1 = 1.1, x_2 = 1.1$ ,  $x_1 = 2.0, x_2 = 0.5$ ,  $x_1 = 3.0, x_2 = 5.0$ ,  $x_1 = -0.7, x_2 = 1.14$  as starting values. In which cases do you get a solution?

Verify your results by using the top equation to eliminate  $x_2$  from the bottom equation. Find the solution either by using Newton's method for a single equation or by guesswork.

Does the matlab routine fsolve compute a solution in all these cases?

**3.** LORAN LOng RANGE Navigation calculates the position of a boat at sea using signals from fixed transmitters. From the time differences of the incoming signals the boat obtains differences of distances to the transmitters. This leads to two equations that represent the distances such as;

$$f_1(x_1, x_2) = \frac{x_1^2}{186^2} - \frac{x_2^2}{300^2 - 186^2} - 1$$

$$f_2(x_1, x_2) = \frac{(x_2 - 500)^2}{279^2} - \frac{(x_1 - 300)^2}{500^2 - 279^2} - 1$$

Implement a Newton solver for this system and define the Jacobian matrix.

Using (400,400) as a starting point and run the solution for 25 iterations. This is enough to get to convergence?. Repeat for all starting values from (401, 401) to (600,600) in increments of 1 (402,402) etc . Which values does the method converge to for which starting values and how many different solutions are there?

**4.** Use the simple gradient descent program provided in the lecture slides to solve Problem 8.21 in Holmes.

**5. (a)** Use the provided simple gradient descent program supplied to solve the Rosenbrock Problem. Use an accuracy requirement of  $1.0e-8$  and find how many iterations are needed to get the solution when starting from (0.5, 0.5) (0.1, 0.1) and (-0.5 0.5).

**5. (b)** Implement the Armijo algorithm on page 361-3 of Holmes. How much does this reduce the number of iterations needed?

**5 (c)** Implement the Adam method which is one of the most commonly used methods in machine learning and see if that helps. Notes on this method are on line as are many examples of its use with the Rosenbrock function.

## What to turn in

For these assignments, we expect both **SOURCE CODE** and a written **REPORT** be uploaded as a zip or tarball file to Canvas.

- Source code for all programs that you write, thoroughly documented.
  - Include a README file describing how to compile and run your code.
- Your report should be in PDF format and should stand on its own.
  - It should describe the methods used, explain your results and contain figures.
  - It should also answer any questions asked above.
  - It should cite any sources used for information, including source code.
  - It should list all of your collaborators.

This homework is due on November 11<sup>th</sup> by 11:59 pm. If you don't understand these directions, please send questions to me.