Chapter 9
Morphological Image Processing

9.1 Morphological Operations for Binary Images

-- Dilation and Erosion

-- Opening and Closing

-- Applications

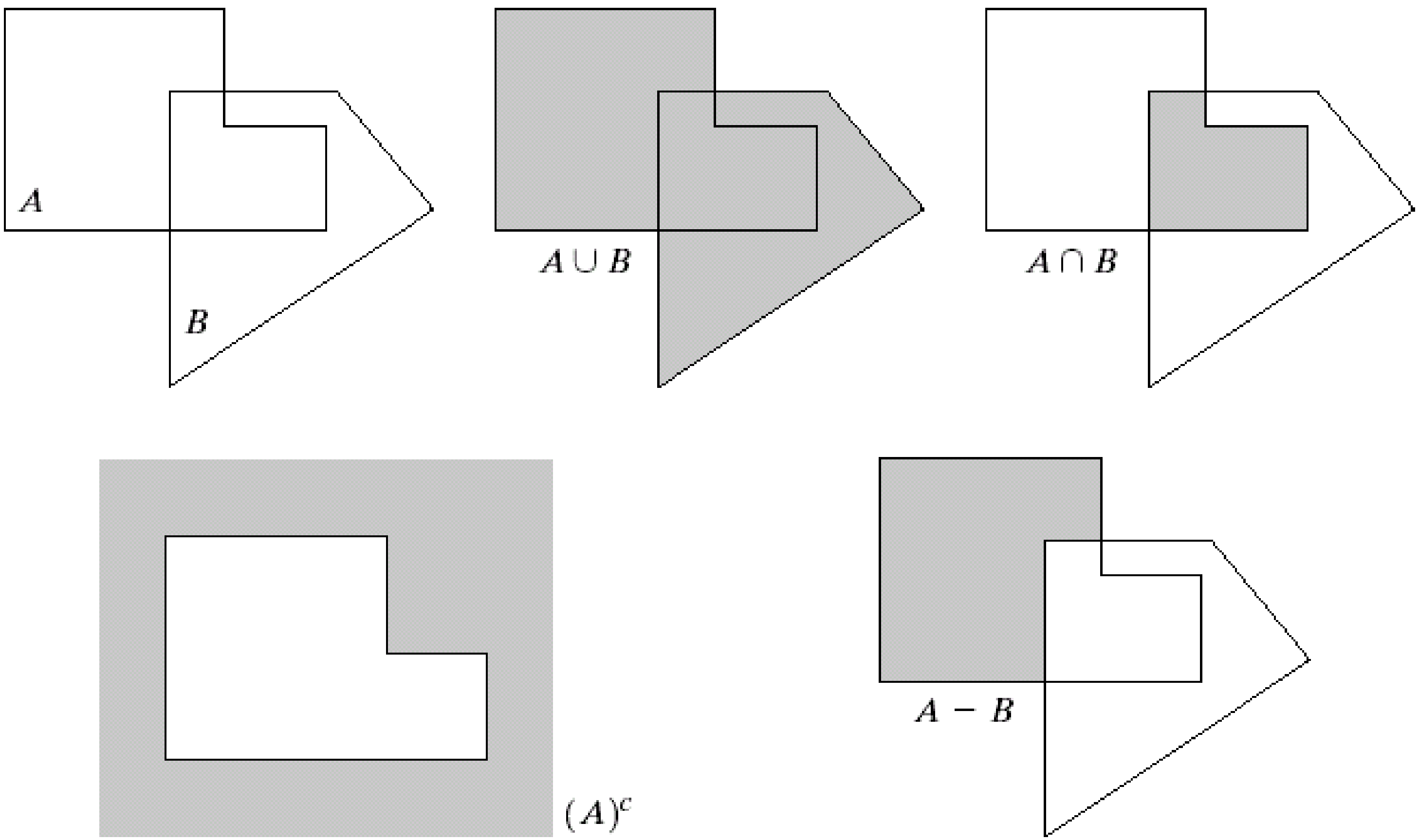
9.2 Morphological Operations for Gray-Scale Images

-- Dilation and Erosion

-- Opening and Closing

-- Applications

Some Basic Concepts from Set Theory

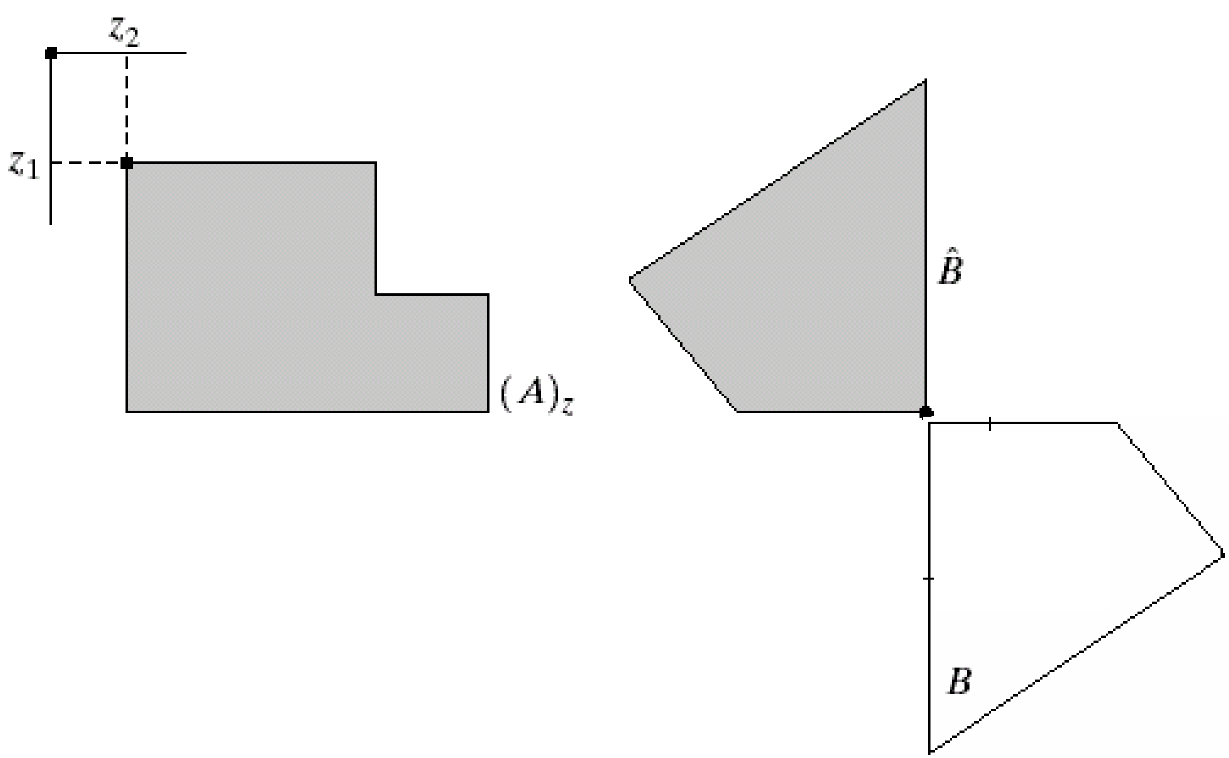


a b c
d e

FIGURE 9.1 (a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

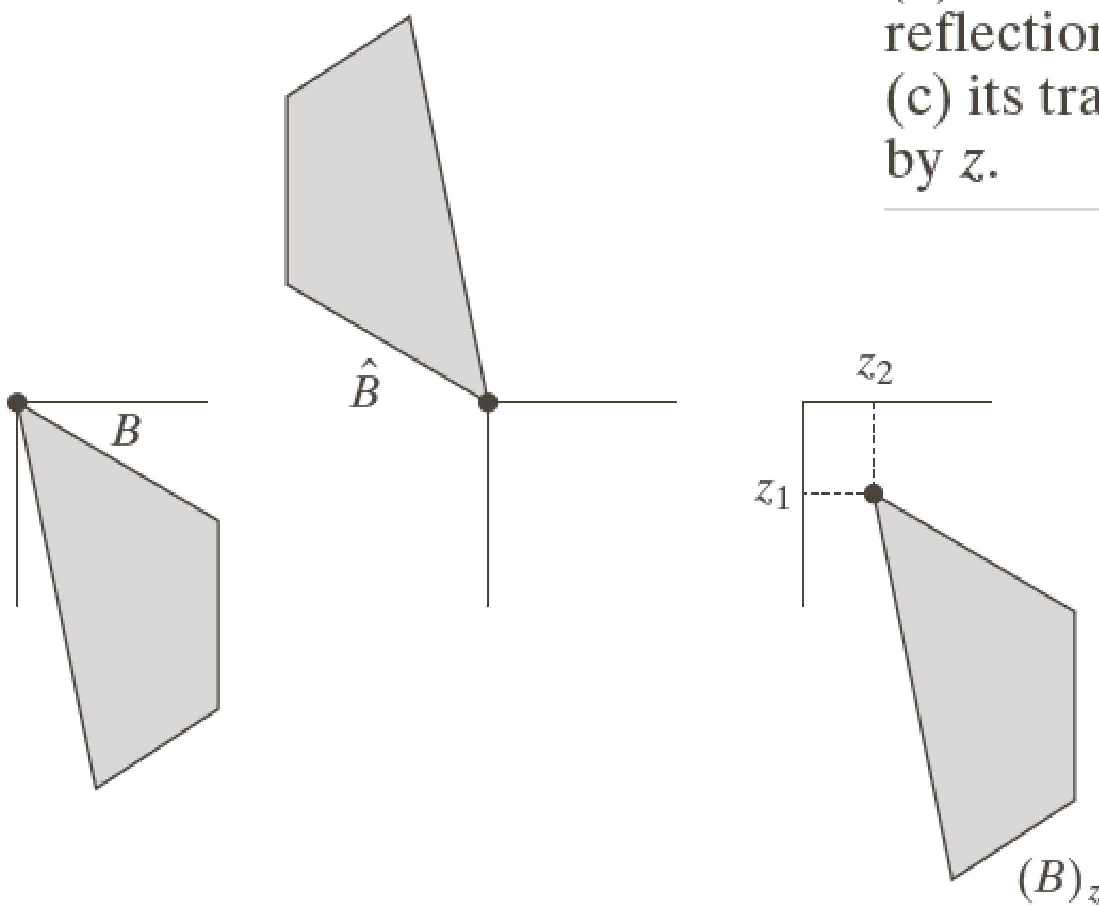
Some Basic Concepts from Set Theory

translation   reflection



a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.



a b c

FIGURE 9.1
(a) A set, (b) its reflection, and (c) its translation by z.

Three Basic Logical Operations

p	q	p **AND** q **(also** $p \cdot q$**)**	p **OR** q **(also** $p + q$**)**	**NOT (**p**) (also** $\bar{p}$**)**
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

Some Logic Operation between Binary Images

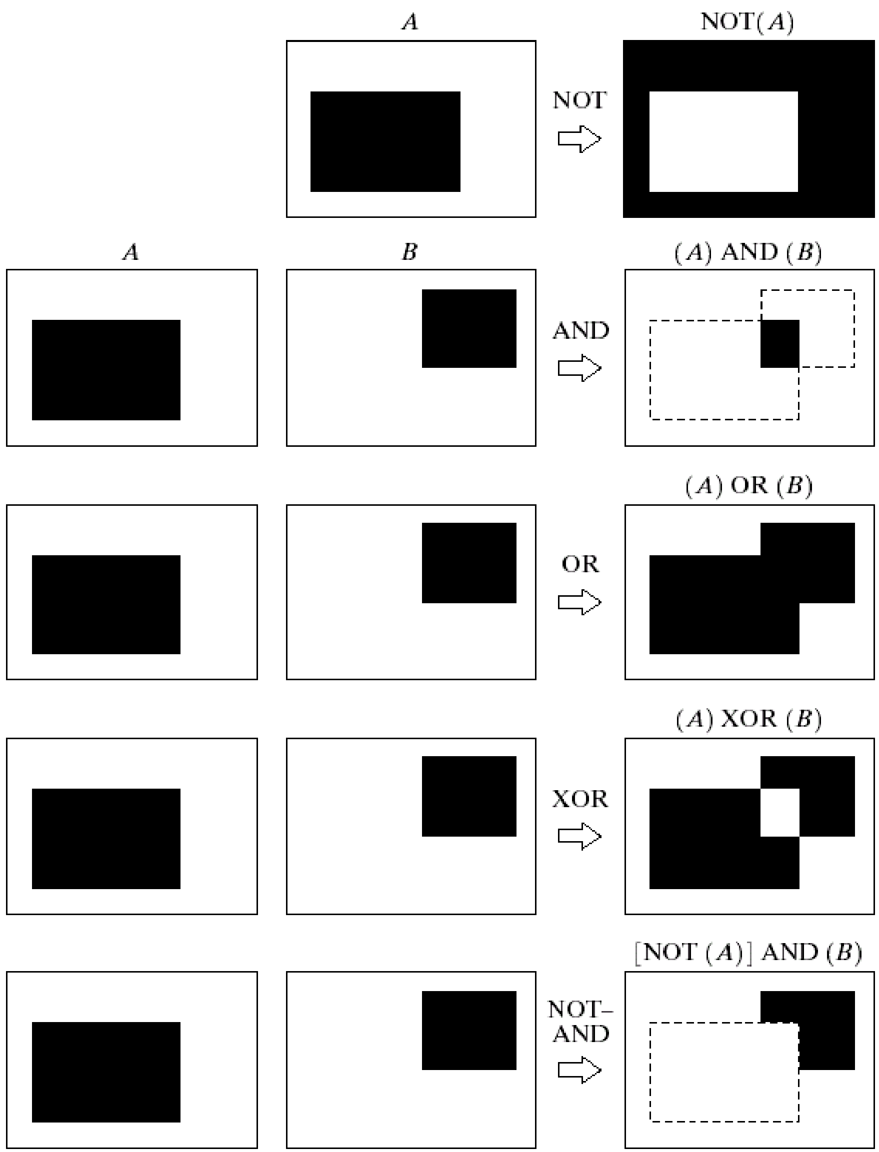
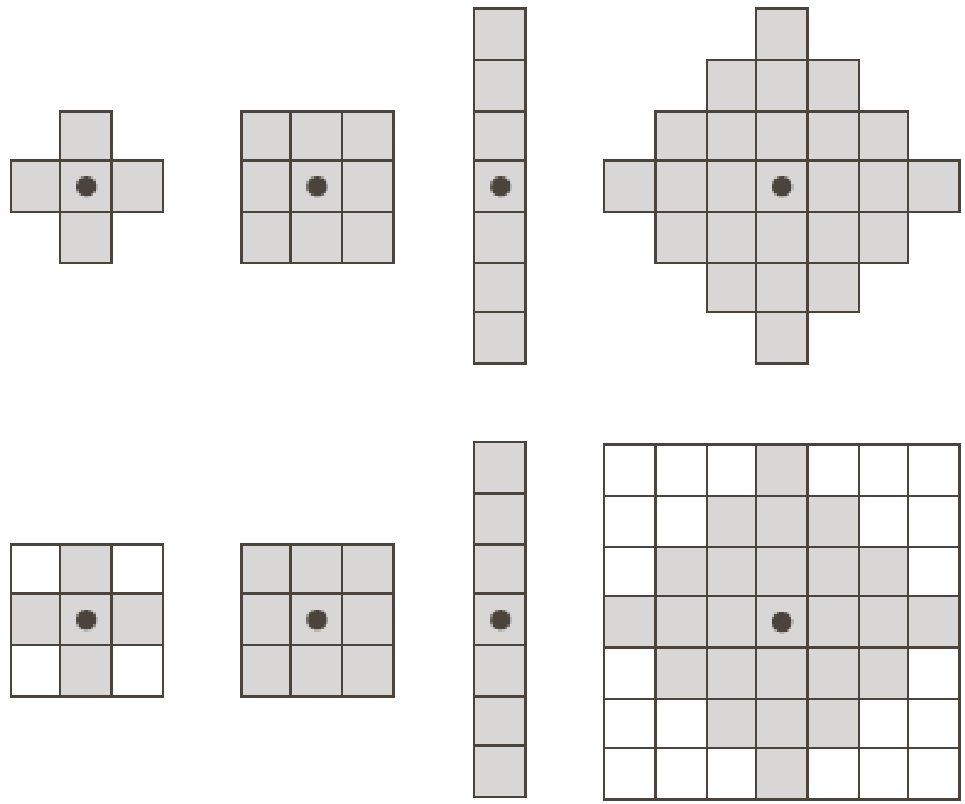


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.

FIGURE 9.2 First row: Examples of structuring elements. Second row: Structuring elements converted to rectangular arrays. The dots denote the centers of the SEs.



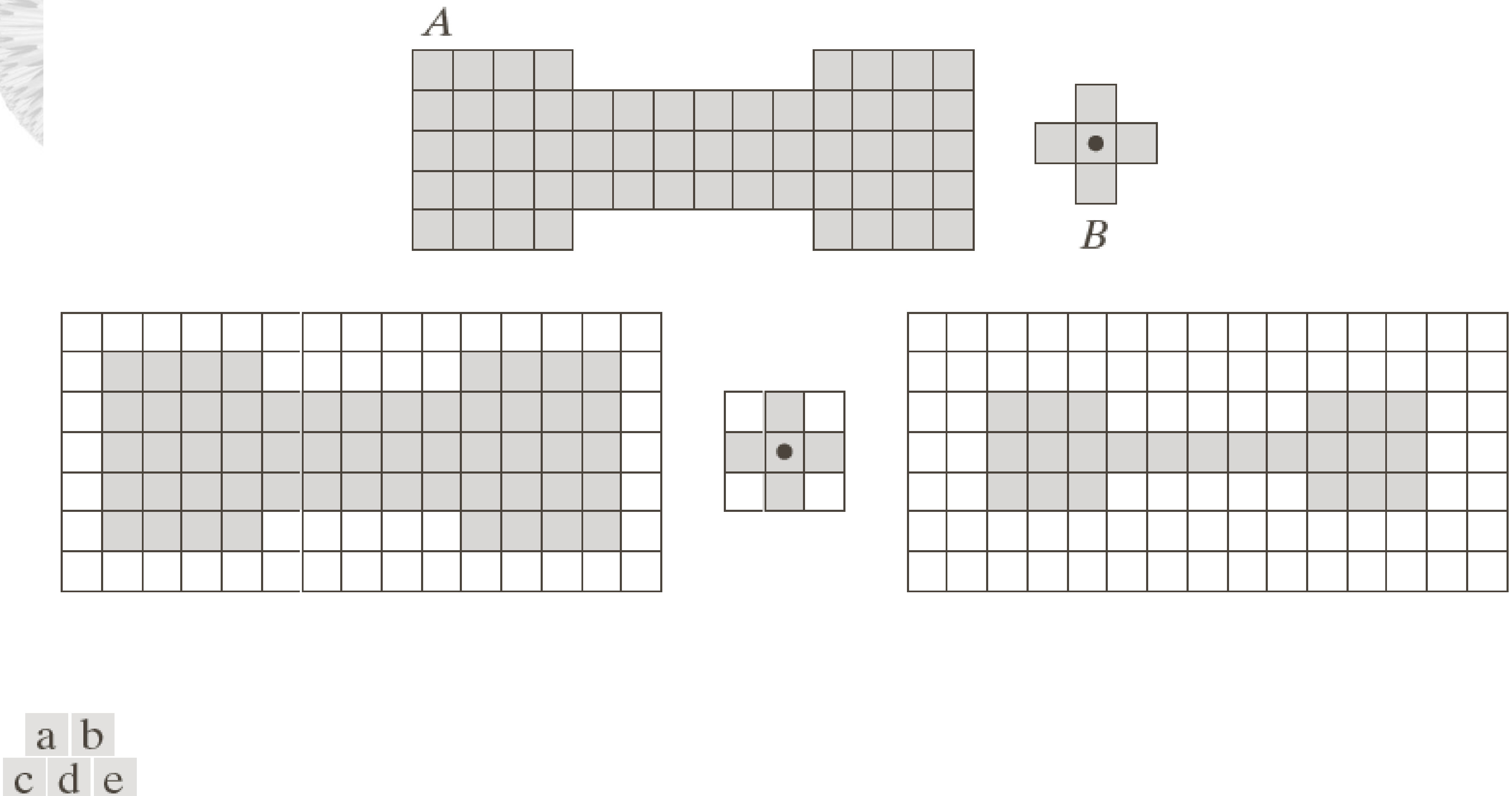


FIGURE 9.3 (a) A set (each shaded square is a member of the set). (b) A structuring element. (c) The set padded with background elements to form a rectangular array and provide a background border. (d) Structuring element as a rectangular array. (e) Set processed by the structuring element.

9.1 Morphological Operations for Binary Images

-- Dilation and Erosion

-- Opening and Closing

-- Applications

Dilation: Minkowski addition

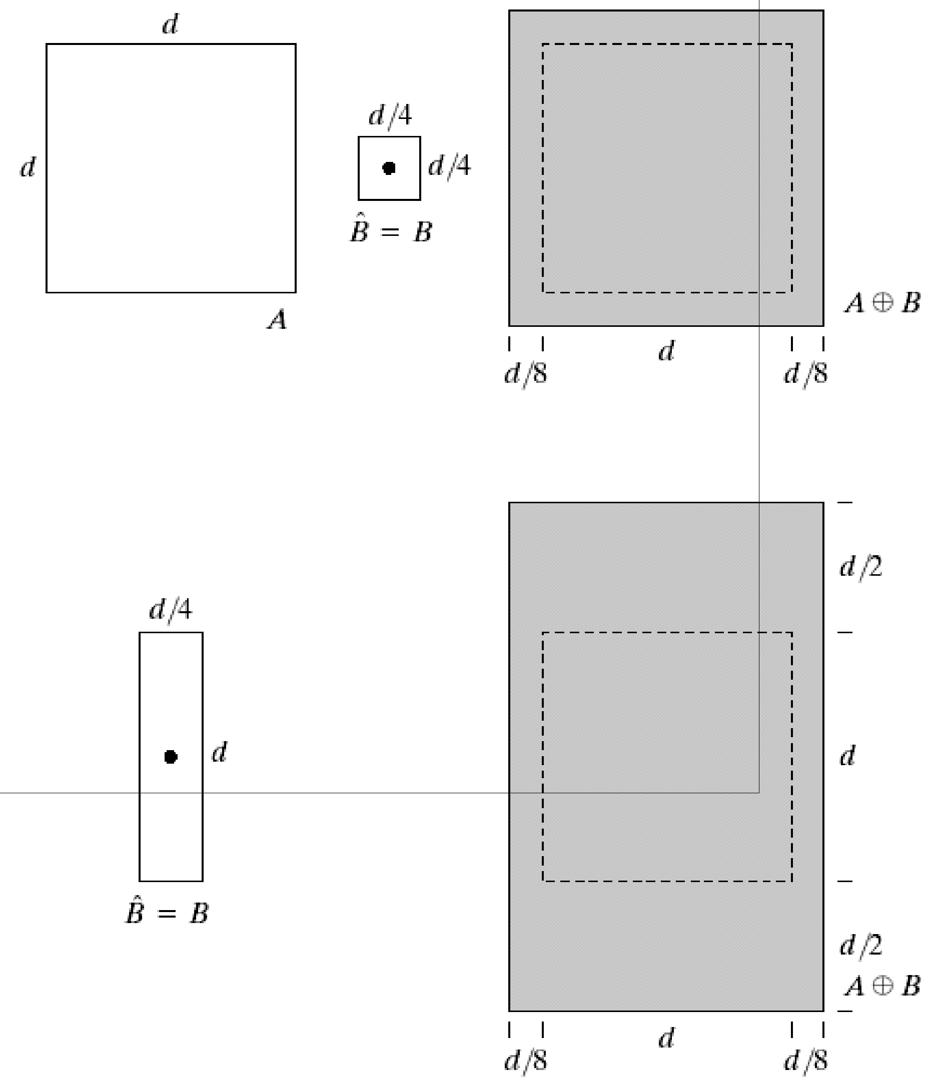
Erosion: Minkowski subtraction

-- see Haralick and Shapiro

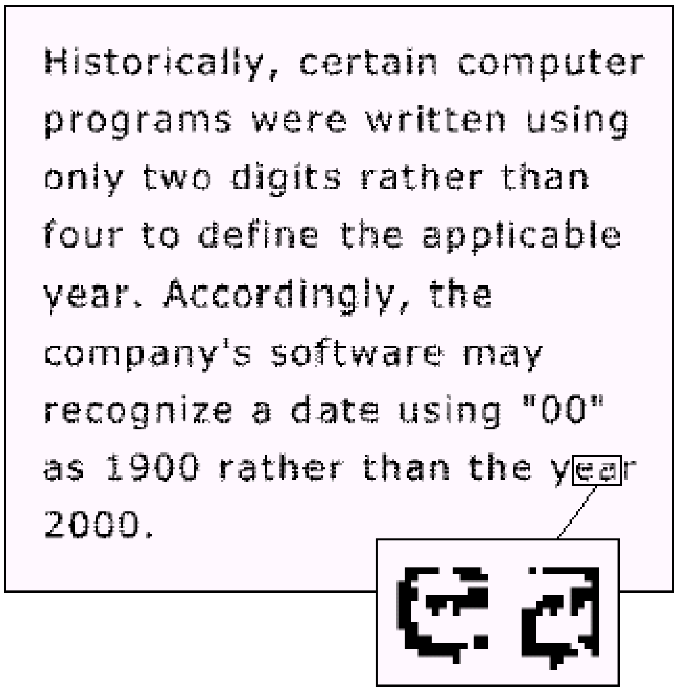
Dilation

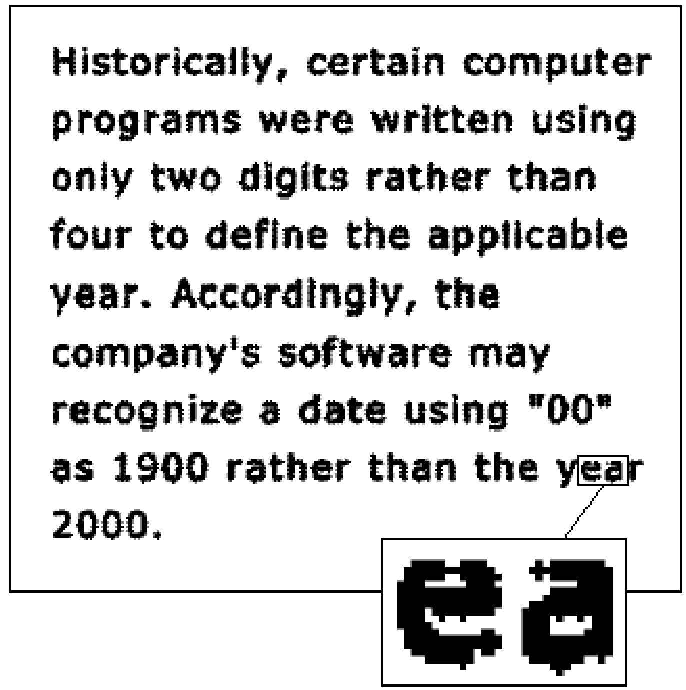
$$A \oplus B = \bigcup_{z \in A} (B)_z$$

$$A \oplus B = \{ z \mid (\hat{B})_z \cap A \neq \varnothing \}$$



Examples of Dilation



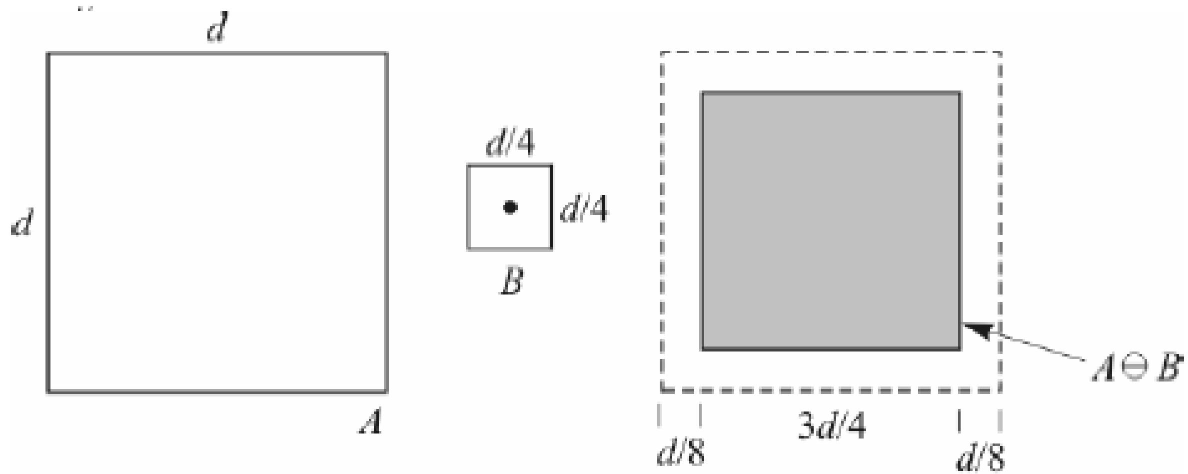


0	1	0
1	1	1
0	1	0

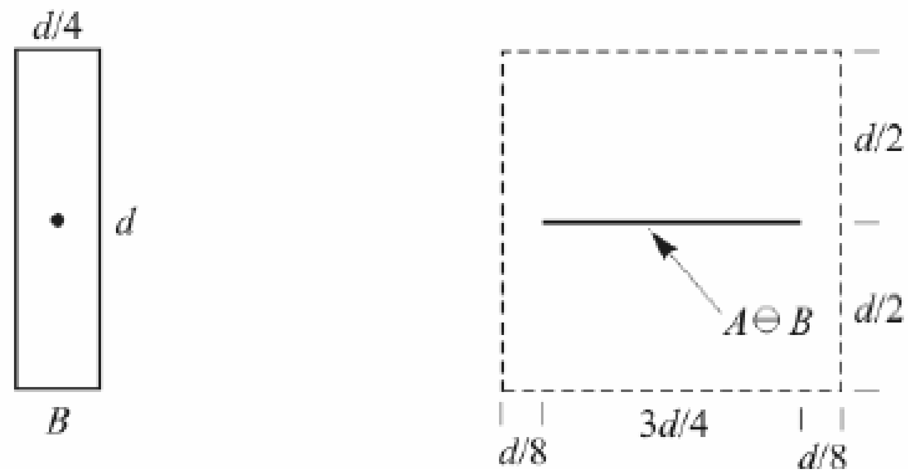
a c
b

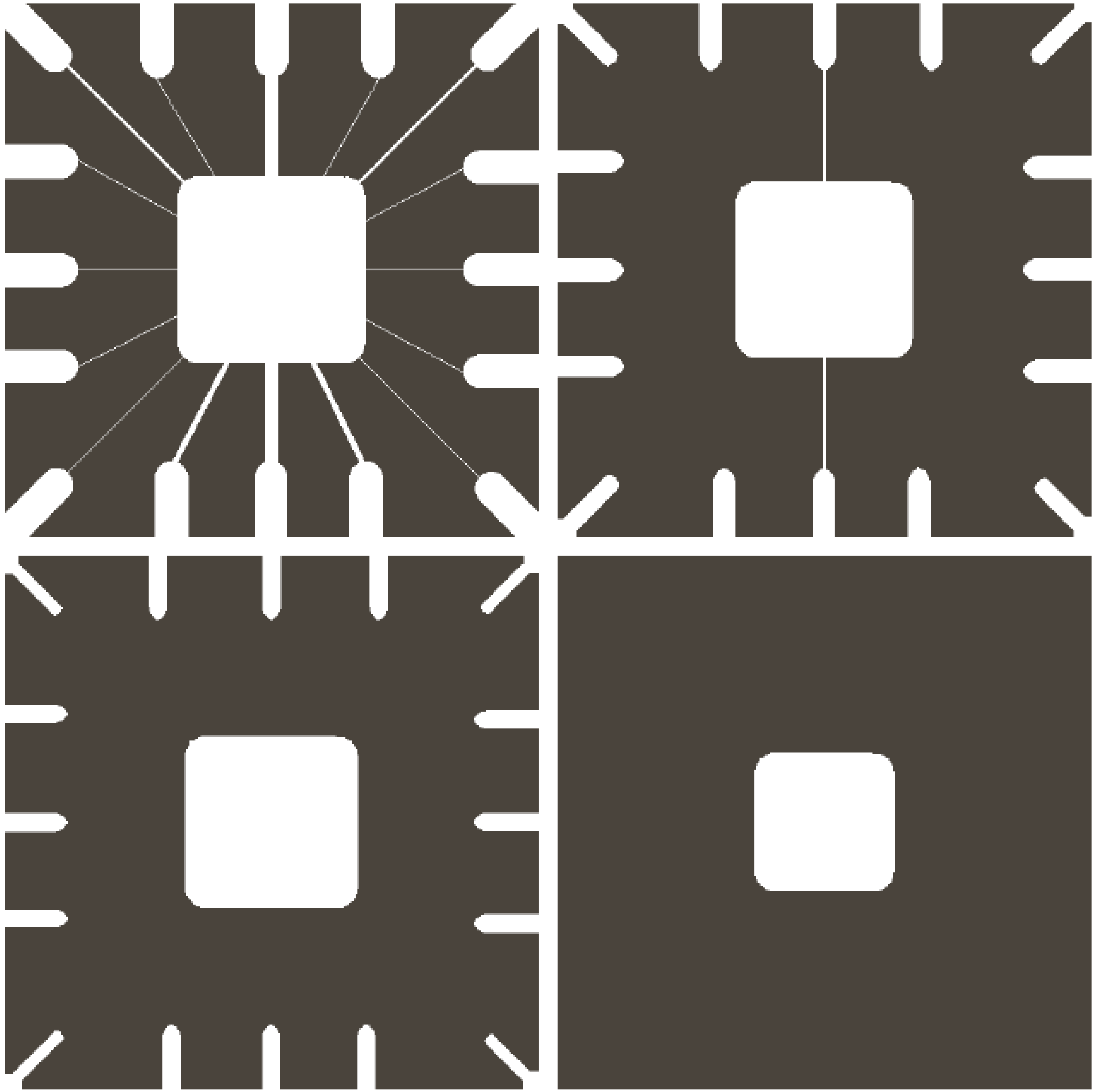
FIGURE 9.5 (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

Erosion



$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

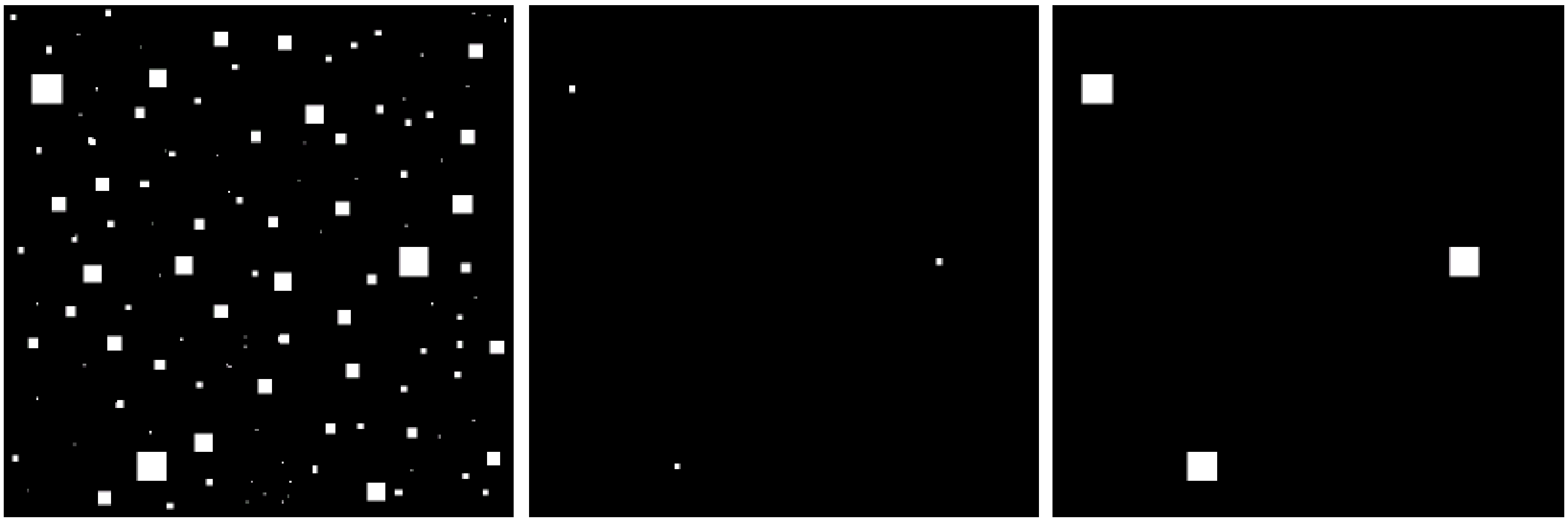




a b
c d

FIGURE 9.5 Using erosion to remove image components. (a) A 486 × 486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11 × 11, 15 × 15, and 45 × 45, respectively. The elements of the SEs were all 1s.

Example of Erosion



a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

Duality

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

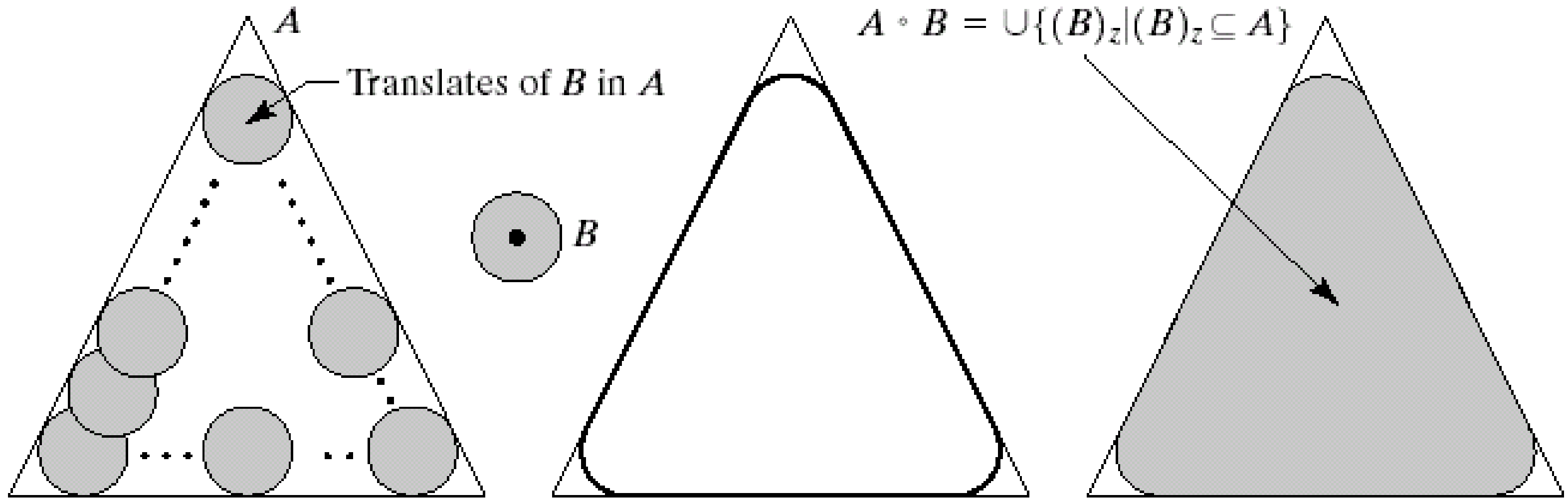
See textbook for proof

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

Opening

$$A \circ B = (A \ominus B) \oplus B$$

$$A \circ B = \bigcup \{(B)_z \mid (B)_z \subseteq A\}$$



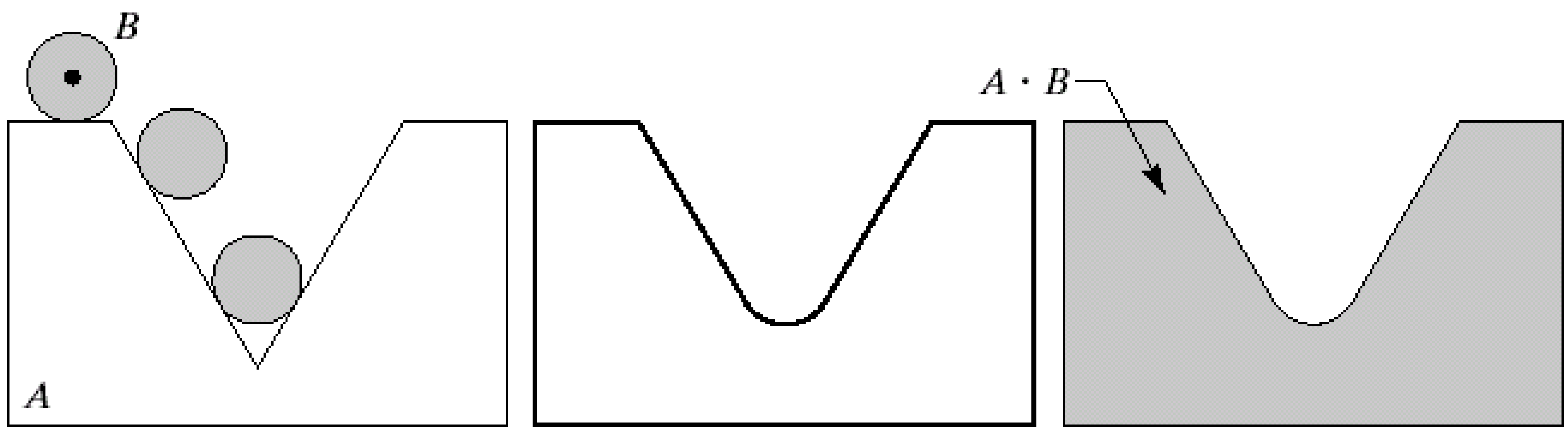
a b c d

FIGURE 9.8 (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

Closing

$$A \bullet B = (A \oplus B) \ominus B.$$

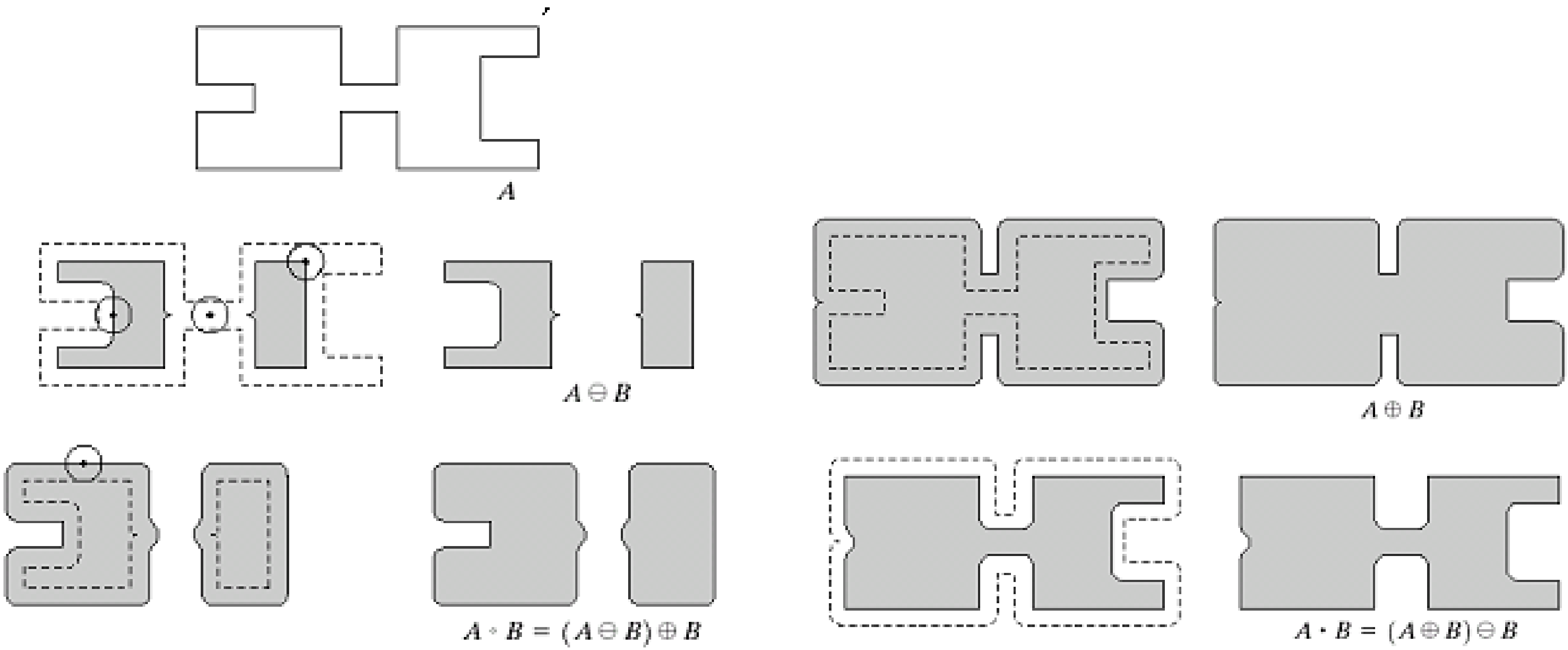
$$(A \bullet B)^c = (A^c \circ \hat{B}).$$



a | b | c

FIGURE 9.9 (a) Structuring element B "rolling" on the outer boundary of set A. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).

Example of Opening and Closing



Duality

$$(A \bullet B)^c = (A^c \circ \hat{B})$$

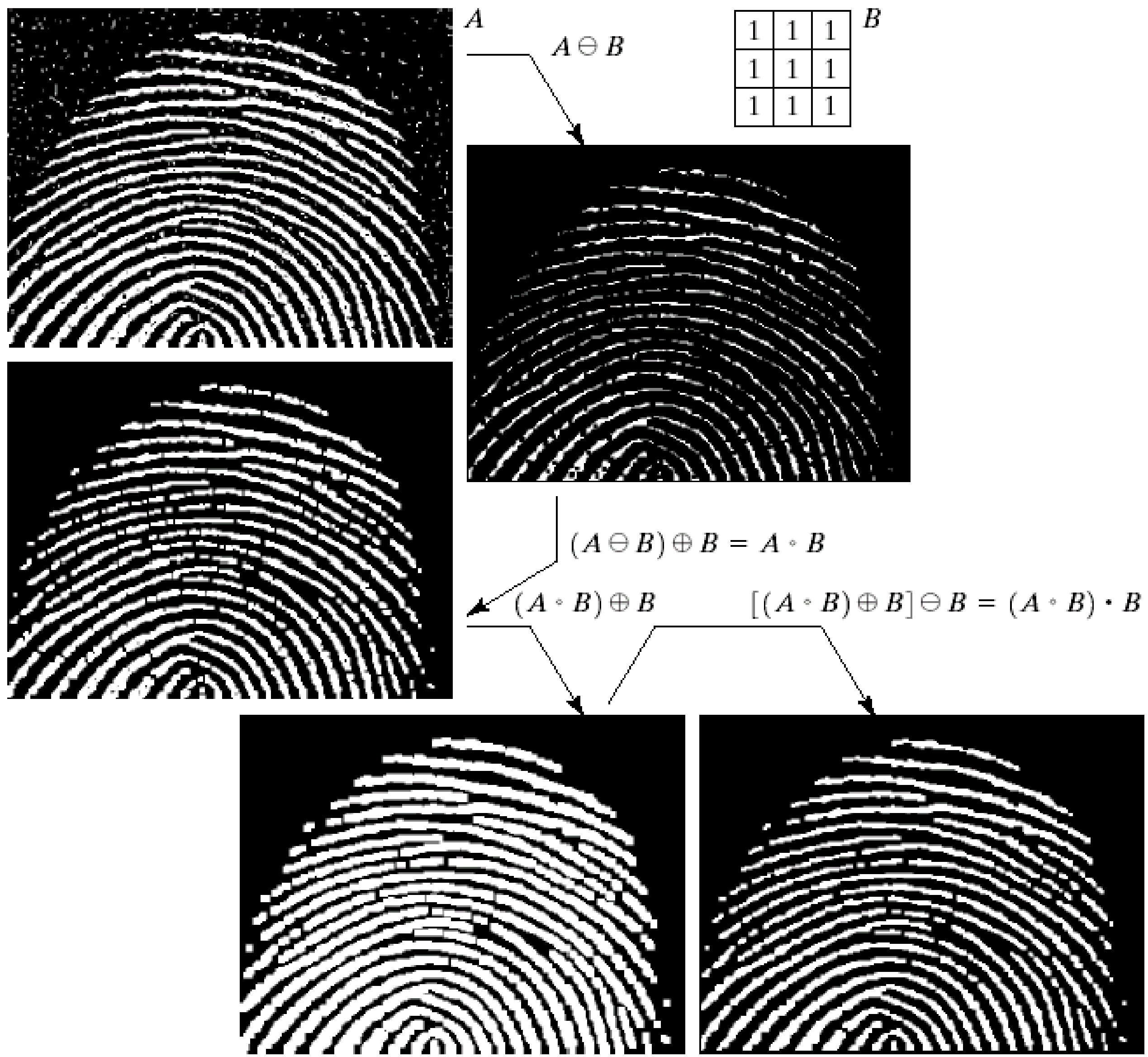
Opening

(i) $A \circ B$ is a subset (subimage) of A.
(ii) If C is a subset of D, then $C \circ B$ is a subset of $D \circ B$.
(iii) $(A \circ B) \circ B = A \circ B$.

Closing

(i) A is a subset (subimage) of $A \bullet B$.
(ii) If C is a subset of D, then $C \bullet B$ is a subset of $D \bullet B$.
(iii) $(A \bullet B) \bullet B = A \bullet B$.

Example of Opening and Closing



a b
c
d
e f

FIGURE 9.11
(a) Noisy image. (c) Eroded image. (d) Opening of A. (d) Dilation of the opening. (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

Applications

- Hit-or-Miss Transformation
- Boundary Extraction
- Region Filling
- Extraction of Connected Components
- Convex Hull
- Thinning and Thickening
- Skeleton
- Pruning

• Hit-or-Miss Transformation

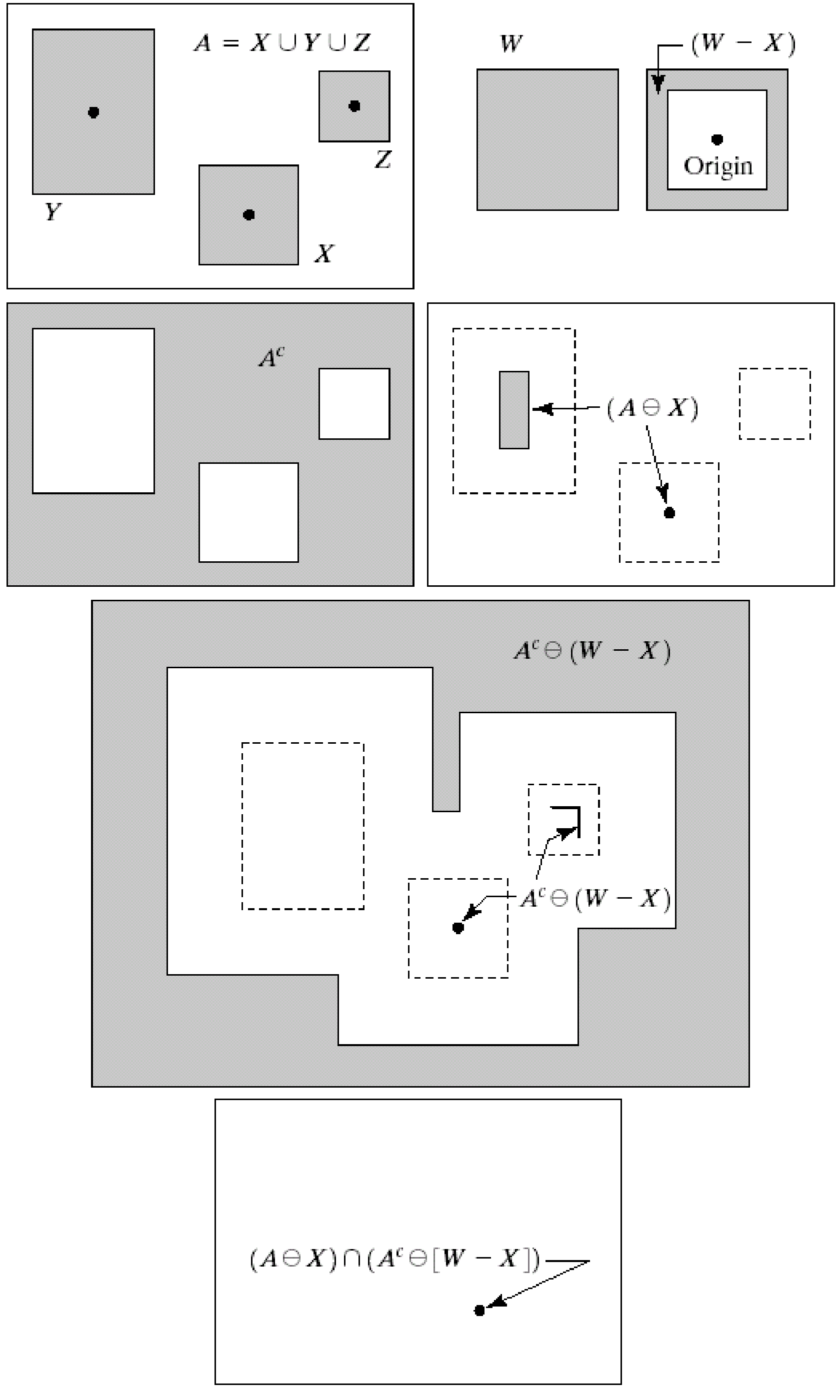
disjoint

Match of B in A

$$A \circledast B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

$$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$$

$$= (A \ominus B_1) - (A \oplus \hat{B}_2)$$

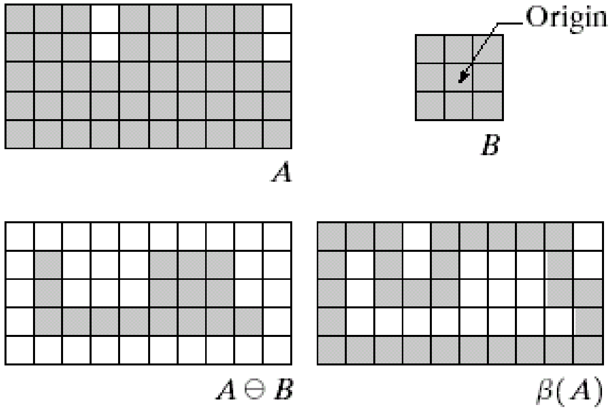


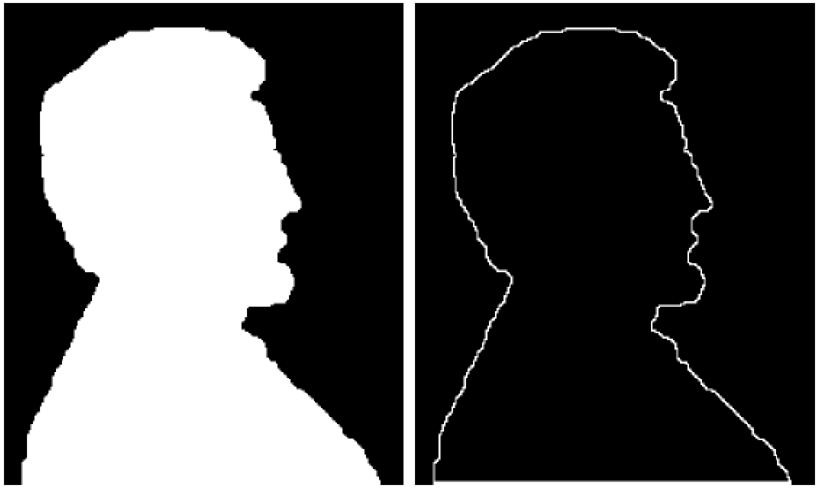
a b
c d
e
f

FIGURE 9.12 (a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$. (c) Complement of A. (d) Erosion of A by X. (e) Erosion of A^c by $(W - X)$. (f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

• Boundary Extraction

$$\beta(A) = A - (A \ominus B)$$





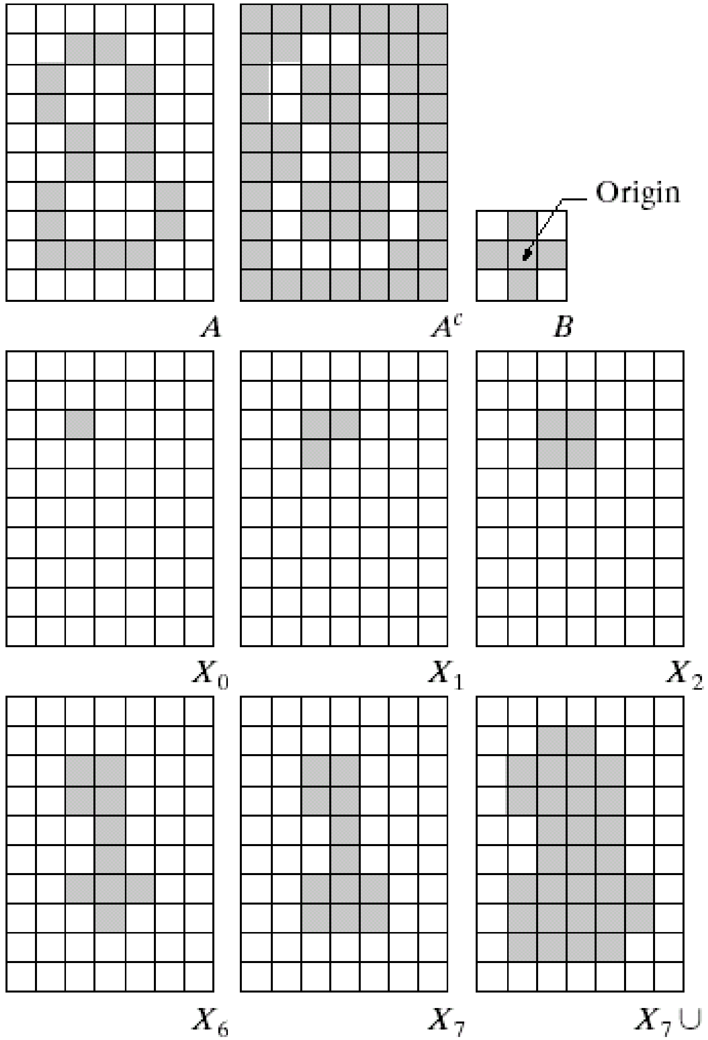
a b

FIGURE 9.14 (a) A simple binary image, with 1's represented in white. (b) Result of using Eq. (9.5-1) with the structuring element in Fig. 9.13(b).

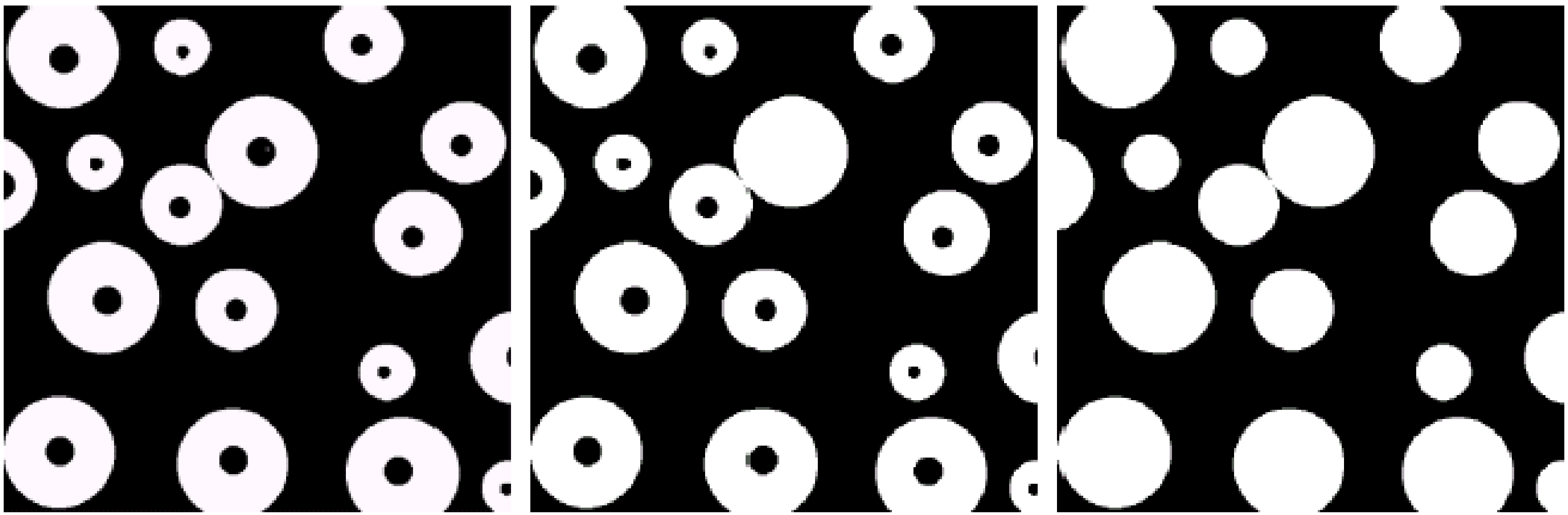
• Region Filling

$$X_k = (X_{k-1} \oplus B) \cap A^c$$

$$X_0 = p \text{ and } k = 1, 2, 3, \ldots$$



Example of Region Filling



a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

• Extraction of Connected Components

$$X_k = (X_{k-1} \oplus B) \cap A;$$

$$X_0 = p \text{ and } k = 1, 2, 3, \ldots$$

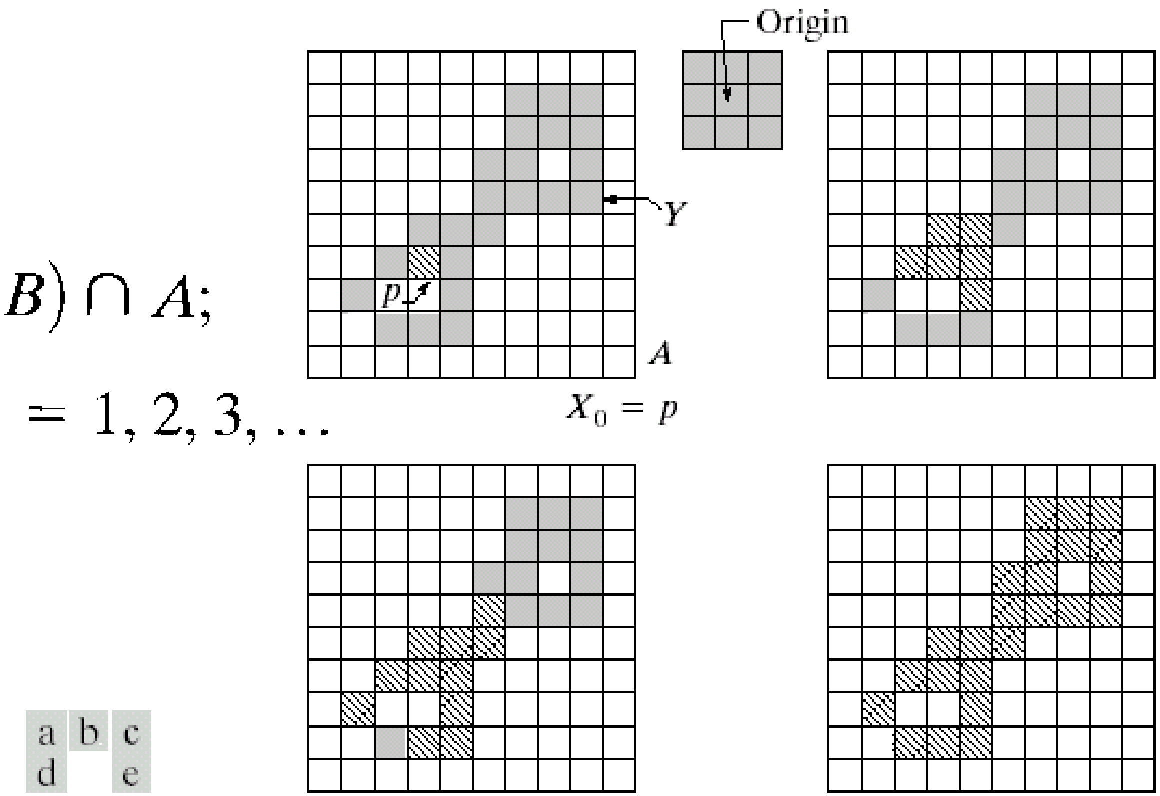


FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

Example of Connected Components Extraction

a
b
c d

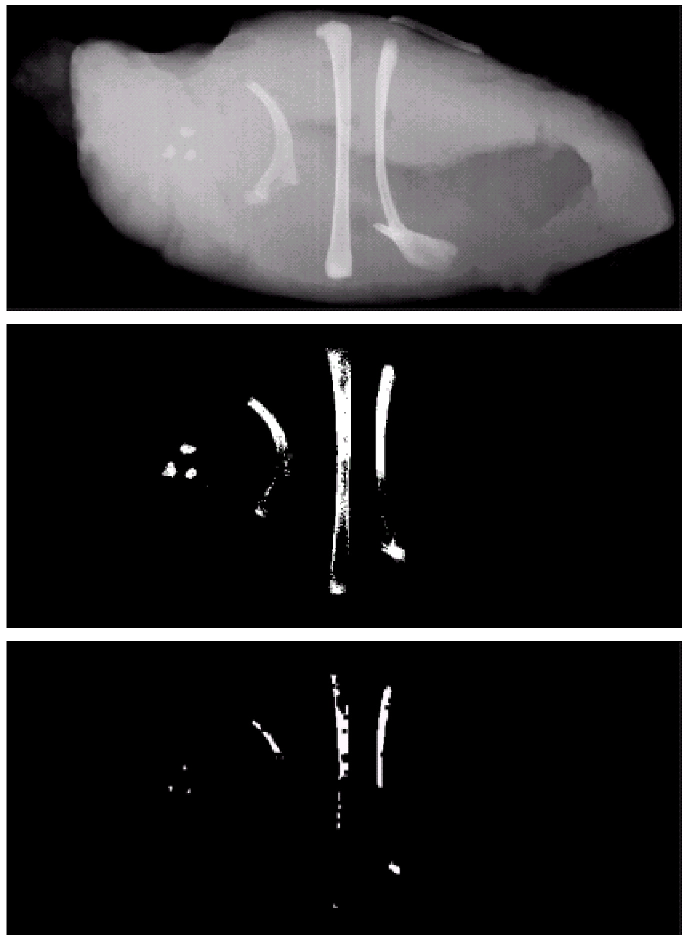


FIGURE 9.18 (a) X-ray image of chicken filet with bone fragments. (b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's. (d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

Five Basic Types of Structuring Elements

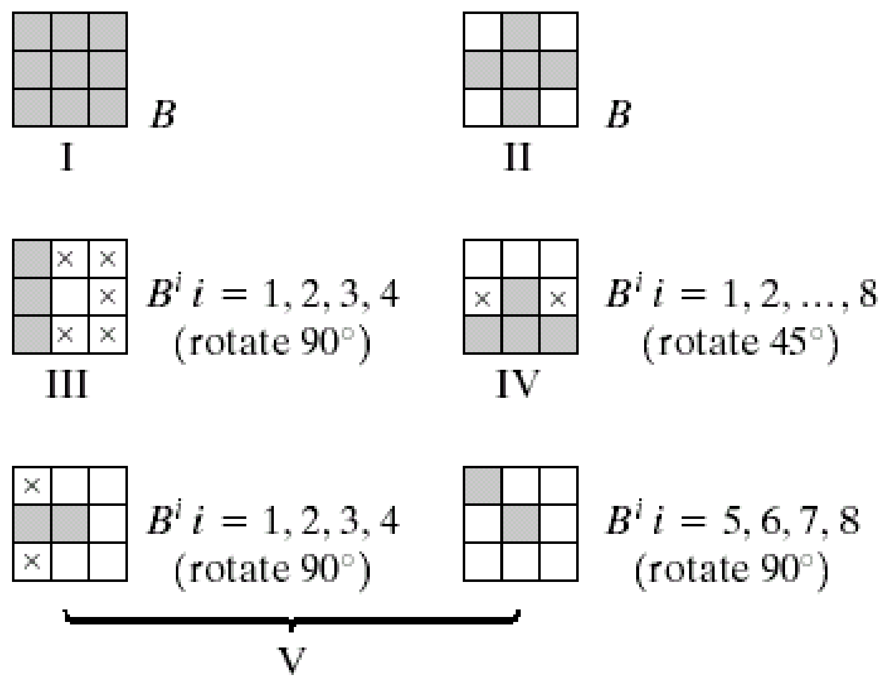


FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

• Convex Hull

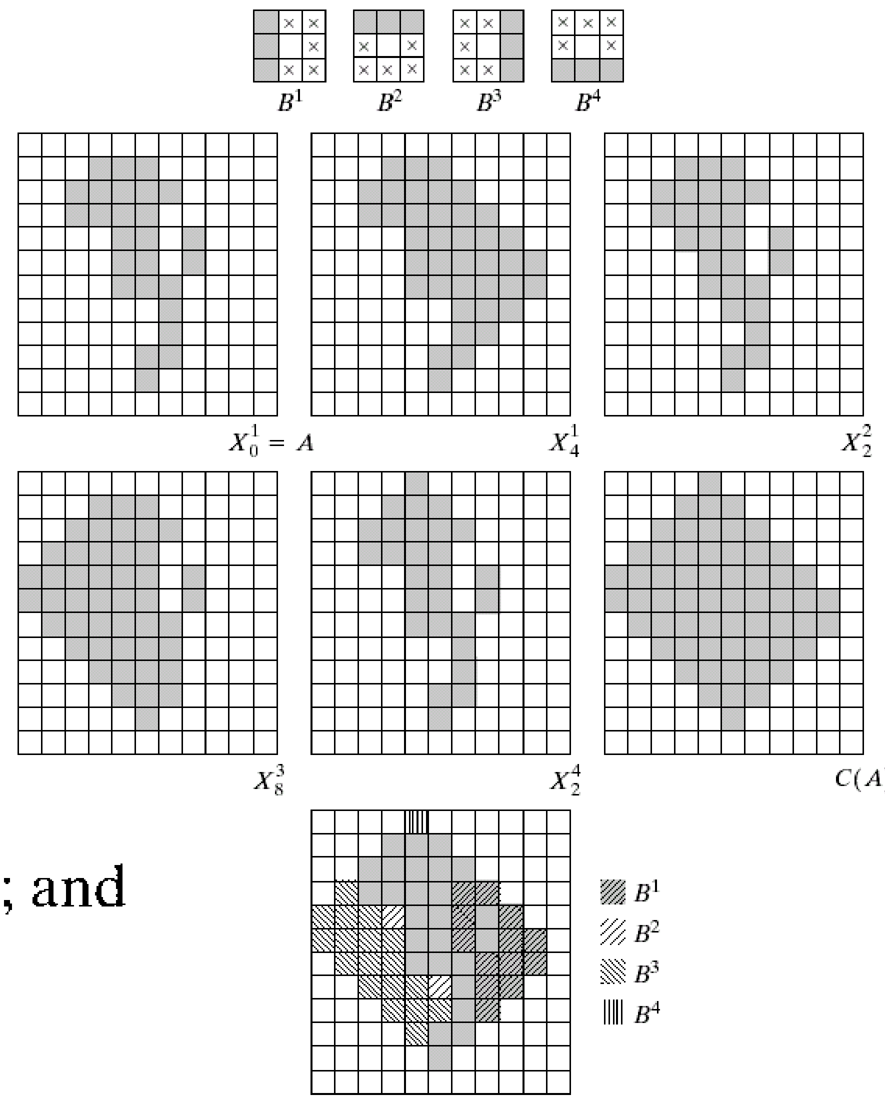


FIGURE 9.19 (a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.

$$X_k^i = (X_{k-1}^i \circledast B^i) \cup A; \; i = 1, 2, 3, 4;$$
$$k = 1, 2, 3, \ldots; \; X_0^i = A; \text{ and}$$
$$D^i = X_{\text{conv}}^i.$$

Approximate Convex Hull

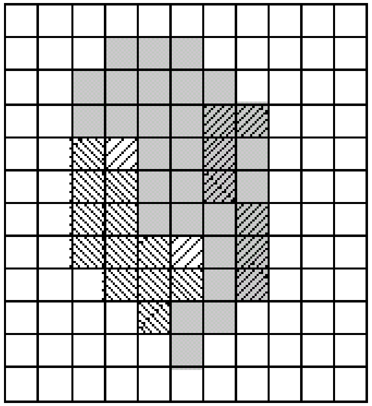


FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

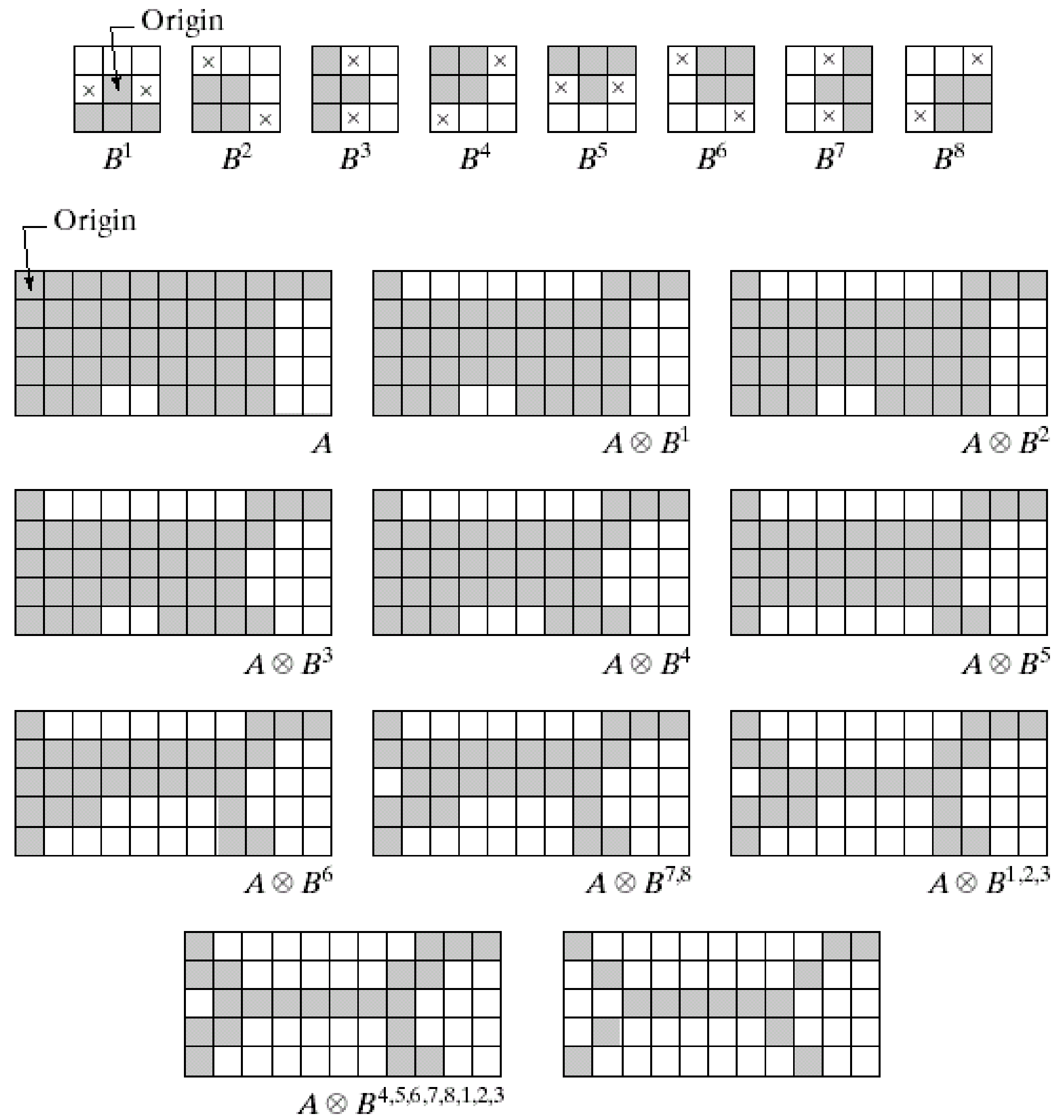
• Thinning and Thickening

Thinning of A by B

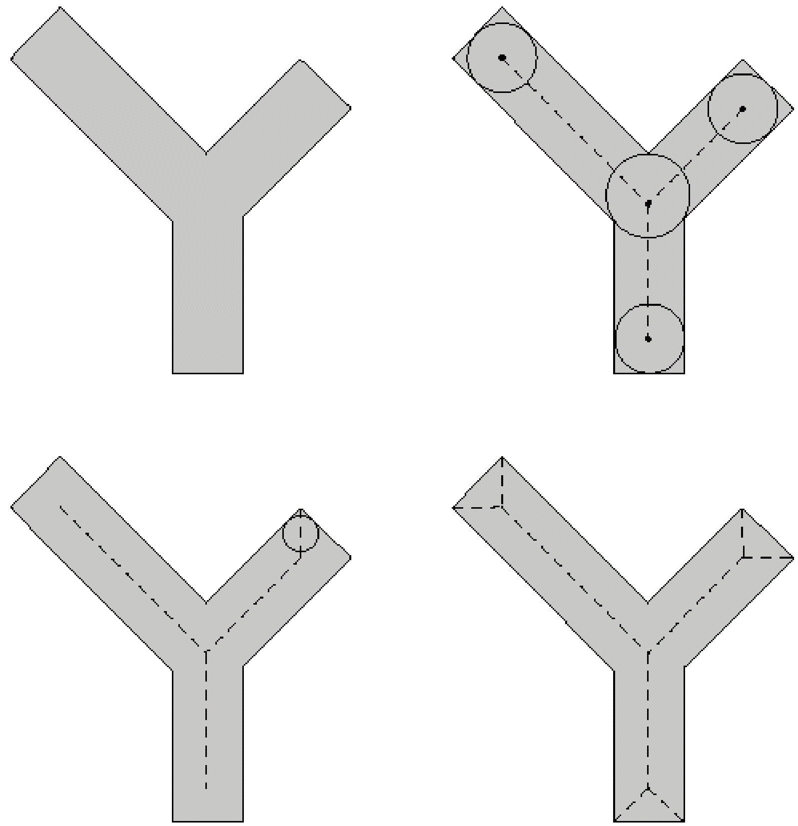
$$A \otimes B = A - (A \circledast B)$$
$$= A \cap (A \circledast B)^c$$

Thinning of A by {B}

$$A \otimes \{B\} =$$
$$((\ldots((A \otimes B^1) \otimes B^2) \ldots) \otimes B^n)$$
$$\{B\} = \{B^1, B^2, B^3, \ldots, B^n\}$$



• Skeletons



a b
c d

FIGURE 9.23 (a) Set A. (b) Various positions of maximum disks with centers on the skeleton of A. (c) Another maximum disk on a different segment of the skeleton of A. (d) Complete skeleton.

- Morphological Skeletons

$$S(A) = \bigcup_{k=0}^{K} S_k(A)$$

$$S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$$

Reconstruction of A:

$$A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$$

Example of Morphological Skeletons

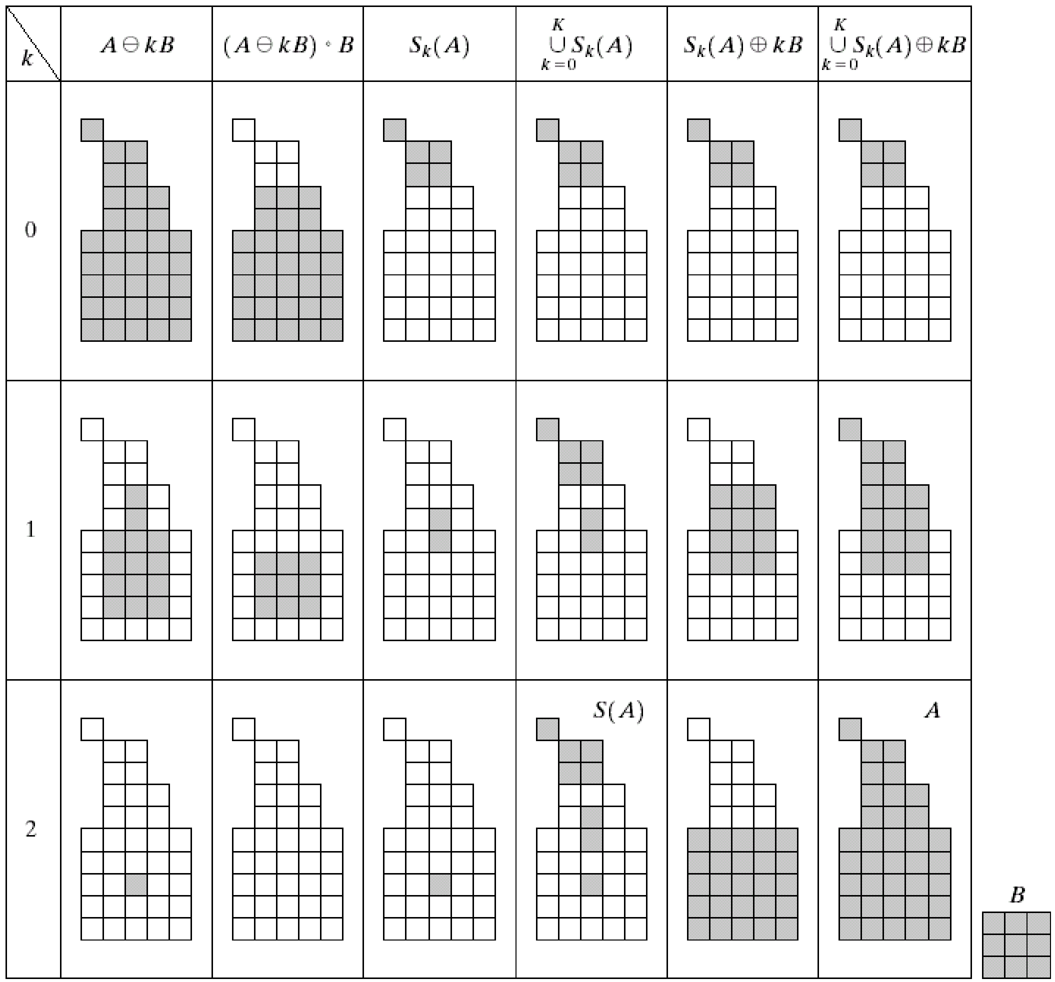


FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

• Pruning

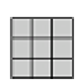
a b
c
d e
f g

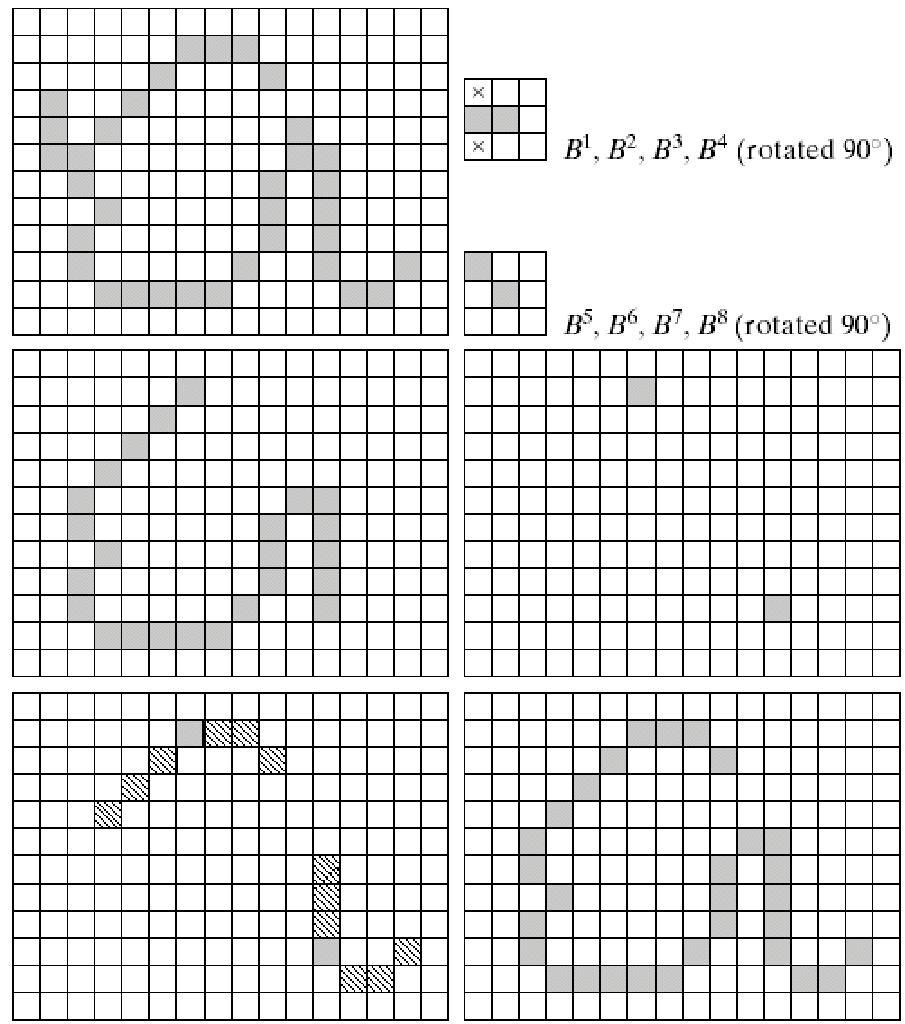
$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

$$X_4 = X_1 \cup X_3$$





9.2 Morphological Operations for Gray-Scale Images

-- Dilation and Erosion

-- Opening and Closing

-- Applications

Gray-Scale Dilation

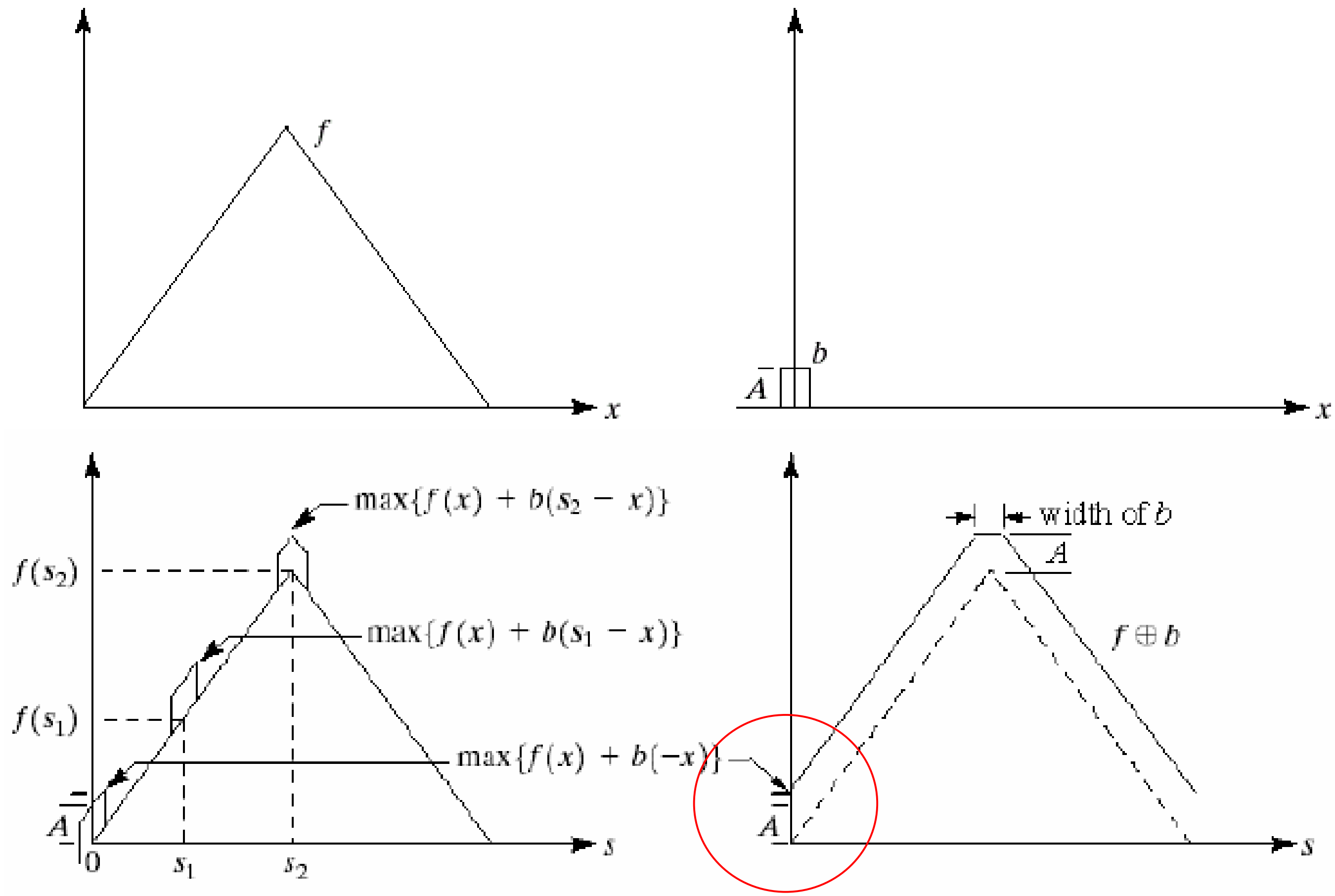
$$(f \oplus b)(s,t) =$$
$$\max\{f(s-x, t-y) + b(x,y) \mid (s-x),(t-y) \in D_f;\ (x,y) \in D_b\}$$

$$\left\{\begin{array}{l} A \oplus B = \left\{z \mid (\hat{B})_z \cap A \neq \varnothing\right\} \\ \\ f(x,y) * h(x,y) = \dfrac{1}{MN} \displaystyle\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) h(x-m, y-n) \end{array}\right.$$

Haralick: … can be defined by using Umbra and binary morphology

Simple 1D example

$$(f \oplus b)(s) = \max\{f(s-x) + b(x) \mid (s-x) \in D_f \quad \text{and} \quad x \in D_b\}$$



Gray-Scale Erosion

$$(f \ominus b)(s,t) =$$
$$\min\{f(s+x, t+y) - b(x,y) \mid (s+x), (t+y) \in D_f; (x,y) \in D_b\}$$

$$(f \ominus b)(s) = \min\{f(s+x) - b(x) \mid (s+x) \in D_f \text{ and } x \in D_b\}$$

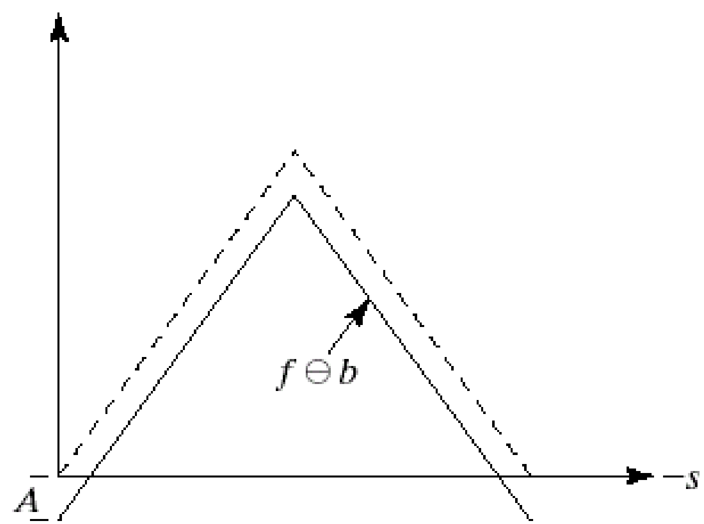
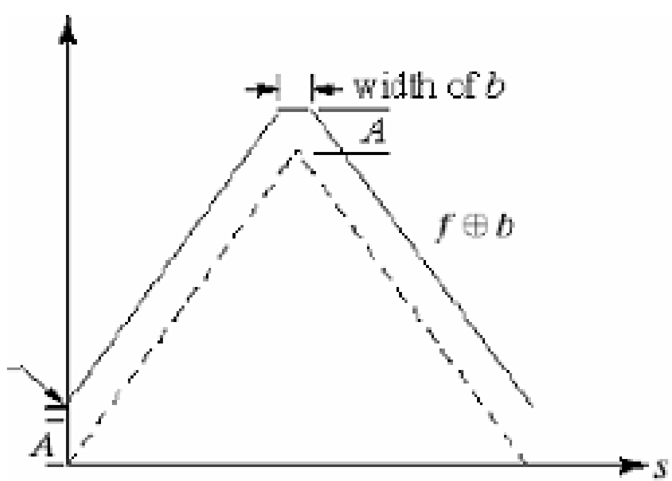
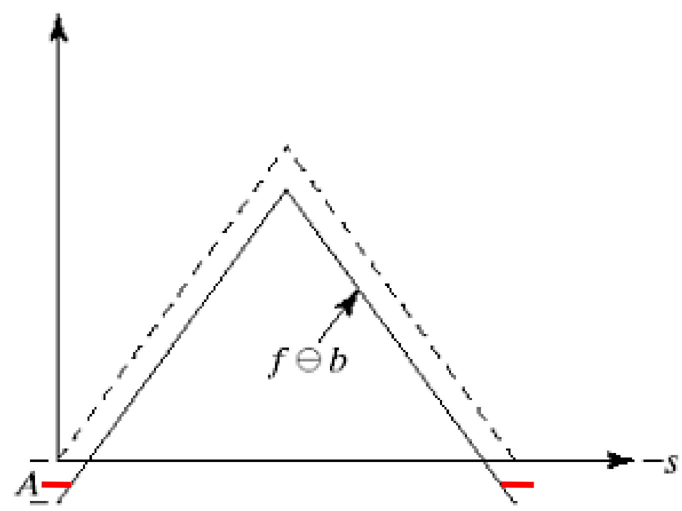


FIGURE 9.28 Erosion of the function shown in Fig. 9.27(a) by the structuring element shown in Fig. 9.27(b).

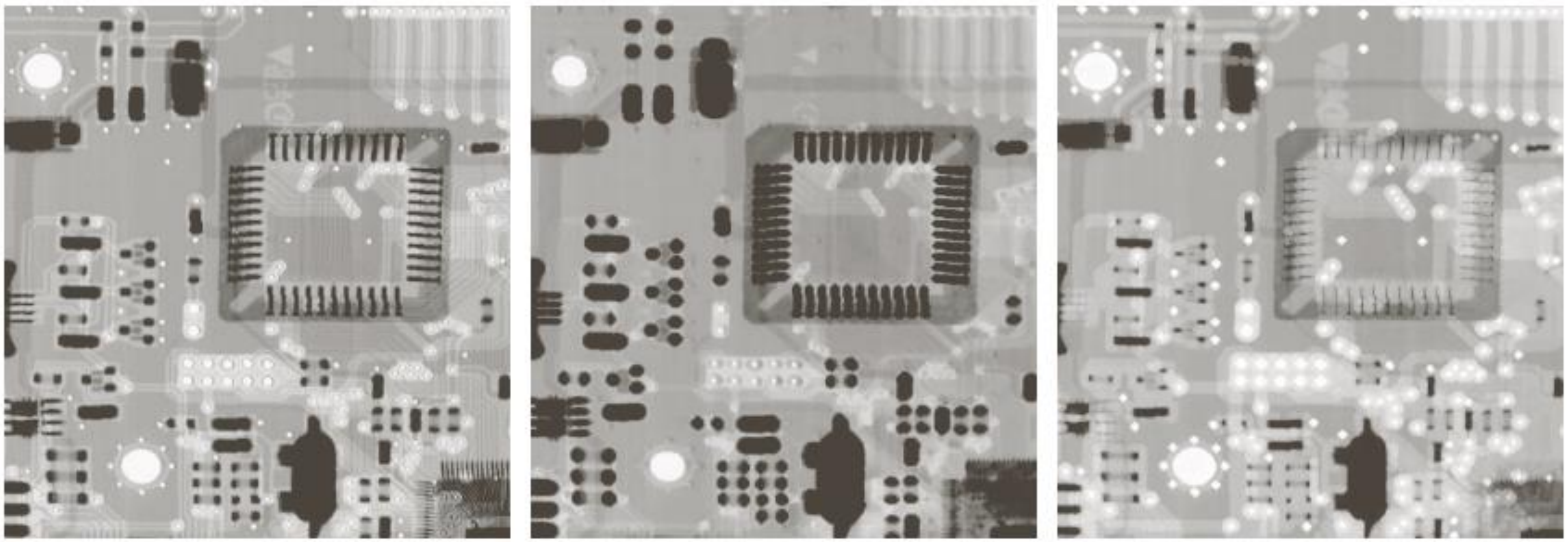
Duality

$$(f \ominus b)^c(s,t) = (f^c \oplus \hat{b})(s,t)$$





Example of Dilation and Erosion



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

Example of Dilation and Erosion

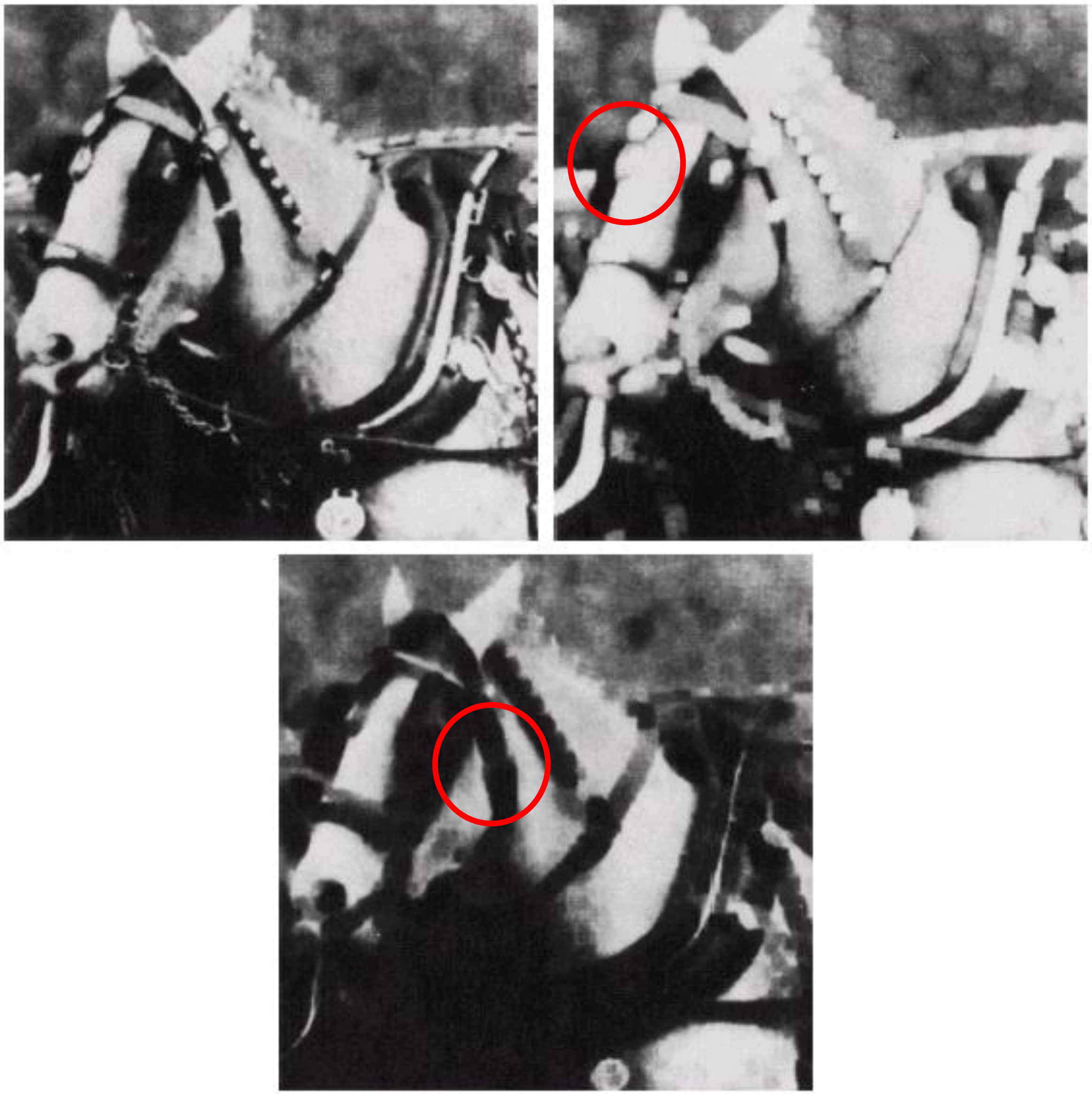


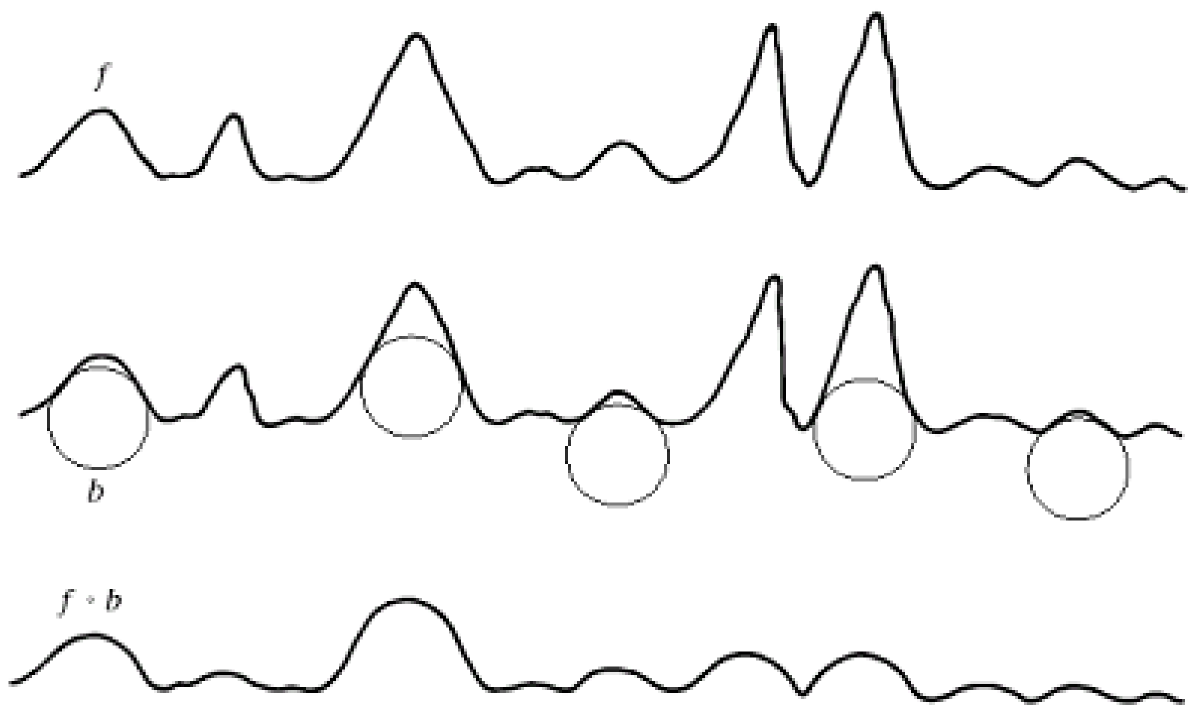
FIGURE 9.29 (a) Original image. (b) Result of dilation. (c) Result of erosion. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Opening and Closing

Opening

-- remove small light details, while leaving the overall gray levels and larger brighter features relatively undisturbed.

$$f \circ b = (f \ominus b) \oplus b.$$

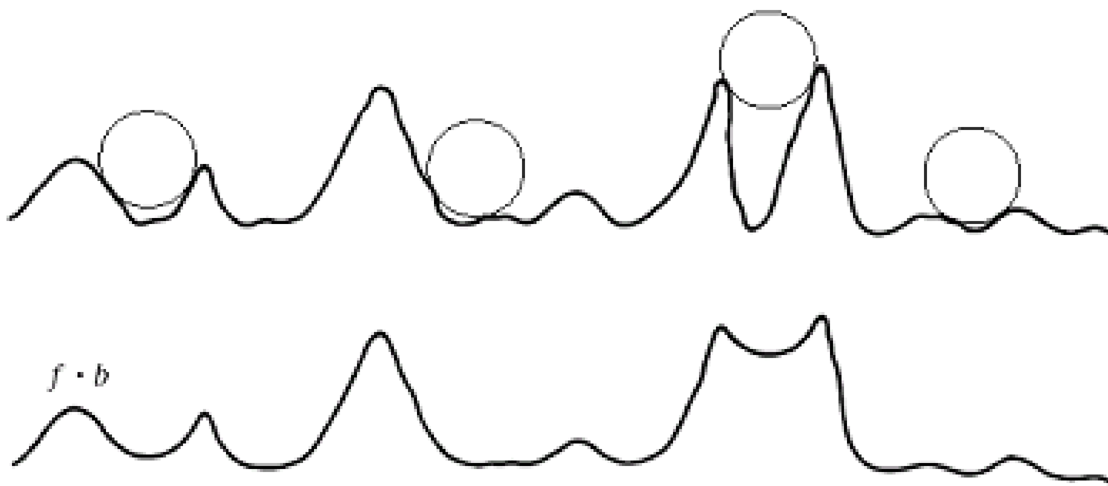


Opening and Closing

Closing

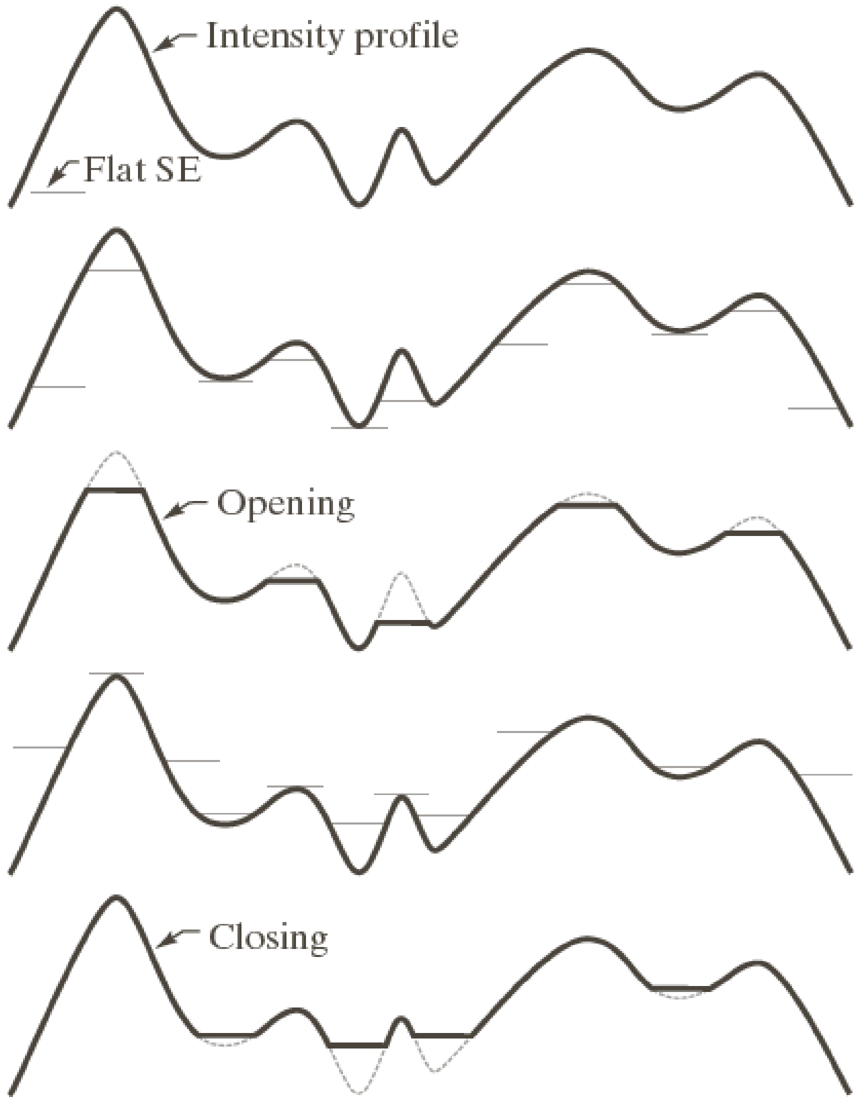
-- remove small dark details, while leaving the overall gray levels, bright features, and larger darker features relatively undisturbed.

$$f \bullet b = (f \oplus b) \ominus b$$



Duality: $$(f \bullet b)^c = f^c \circ \hat{b}$$

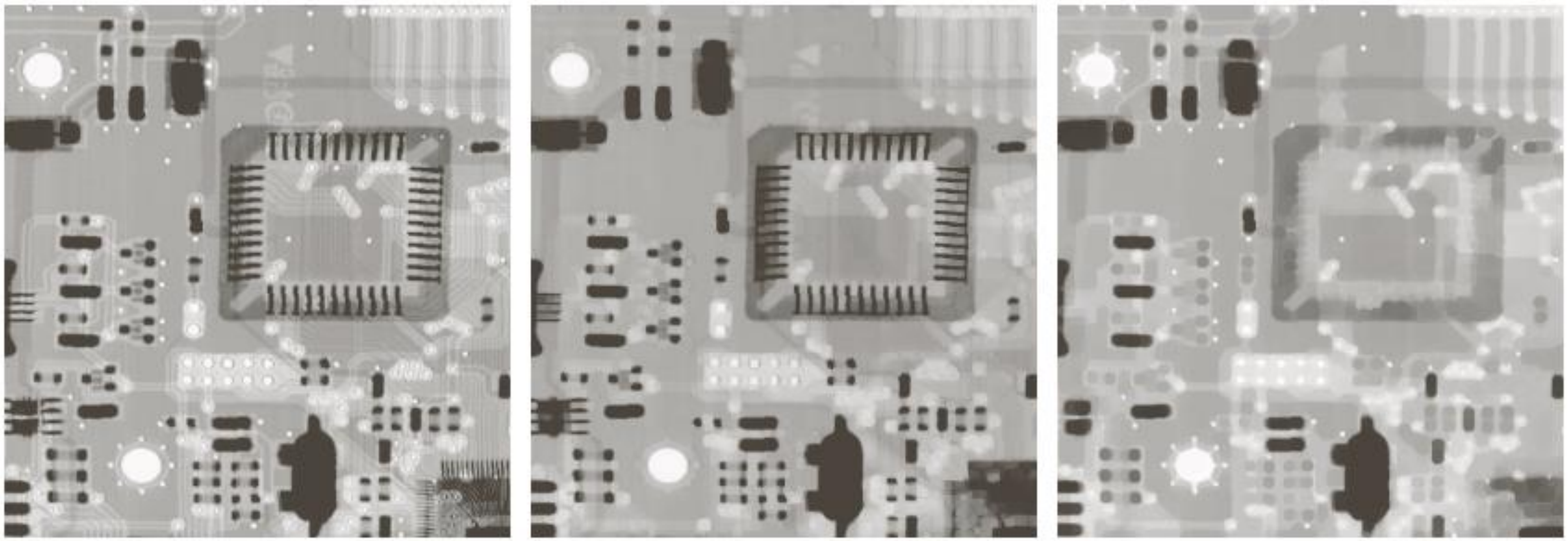
Opening and Closing



a
b
c
d
e

FIGURE 9.36 Opening and closing in one dimension. (a) Original 1-D signal. (b) Flat structuring element pushed up underneath the signal. (c) Opening. (d) Flat structuring element pushed down along the top of the signal. (e) Closing.

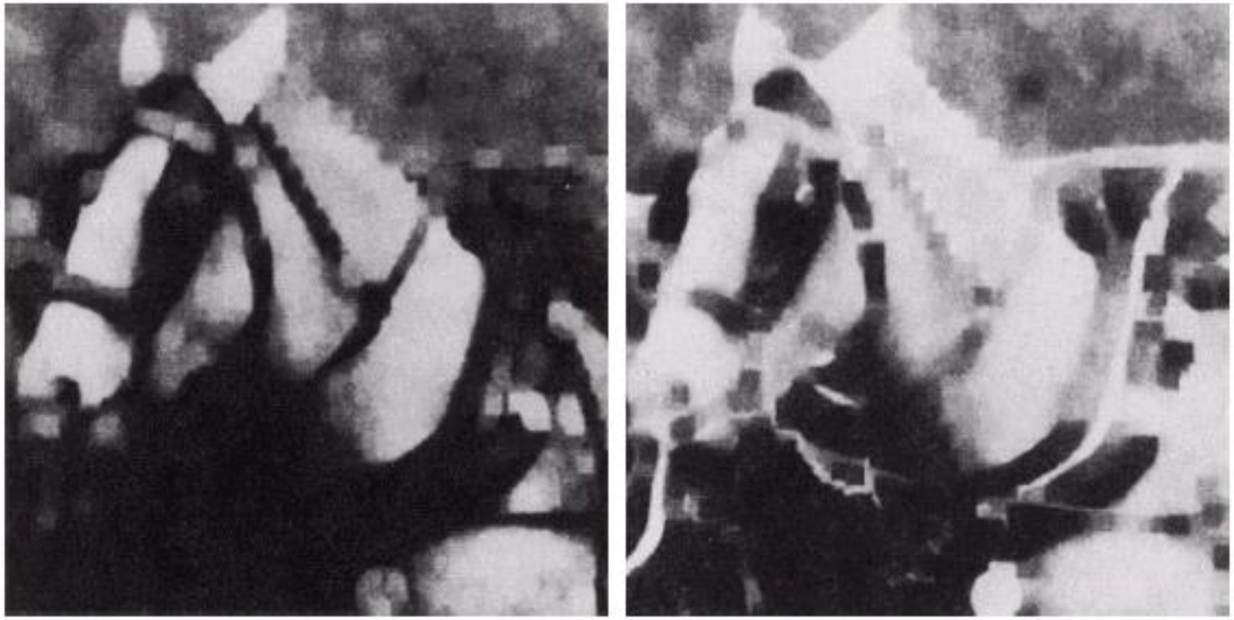
Example of Opening and Closing



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

Example of Opening and Closing



a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Applications

- Morphological Smoothing
- Morphological Gradient
- Top-hat Transformation
- Texture Segmentation
- Granulometry

Morphological Smoothing

- Morphological opening followed by a closing

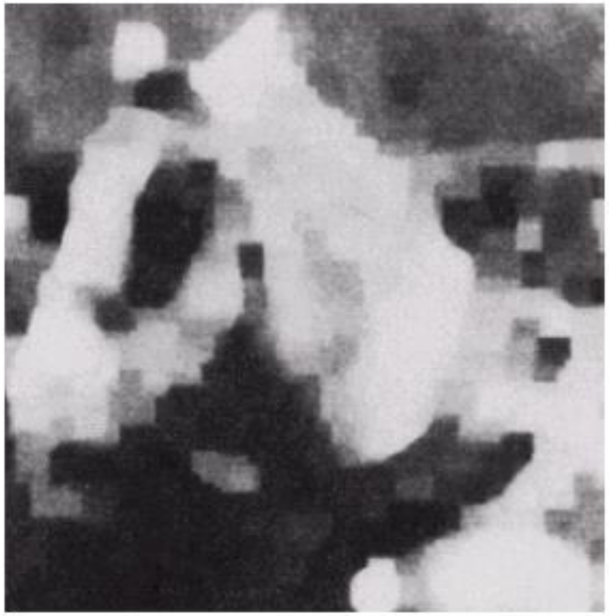


FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Morphological Gradient

$$g = (f \oplus b) - (f \ominus b).$$

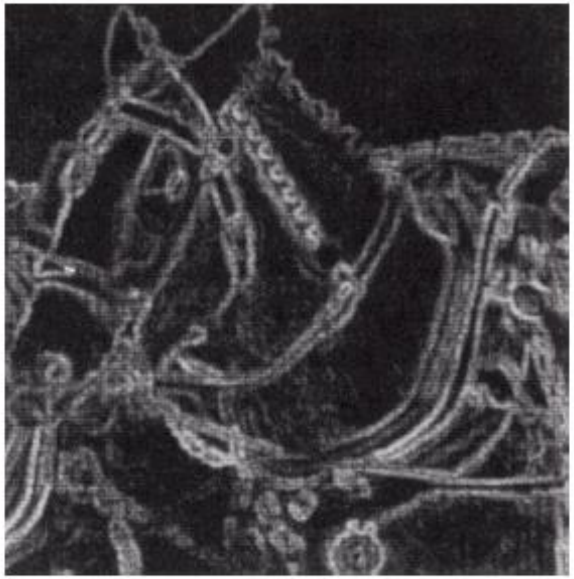


FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Top-hat Transformation

$$h = f - (f \circ b)$$

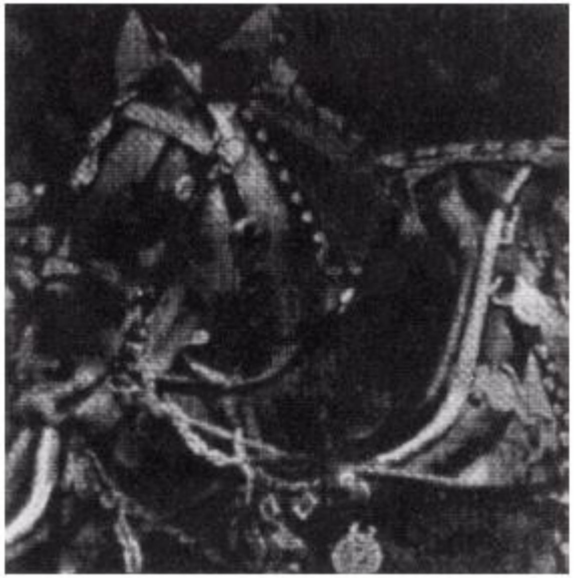
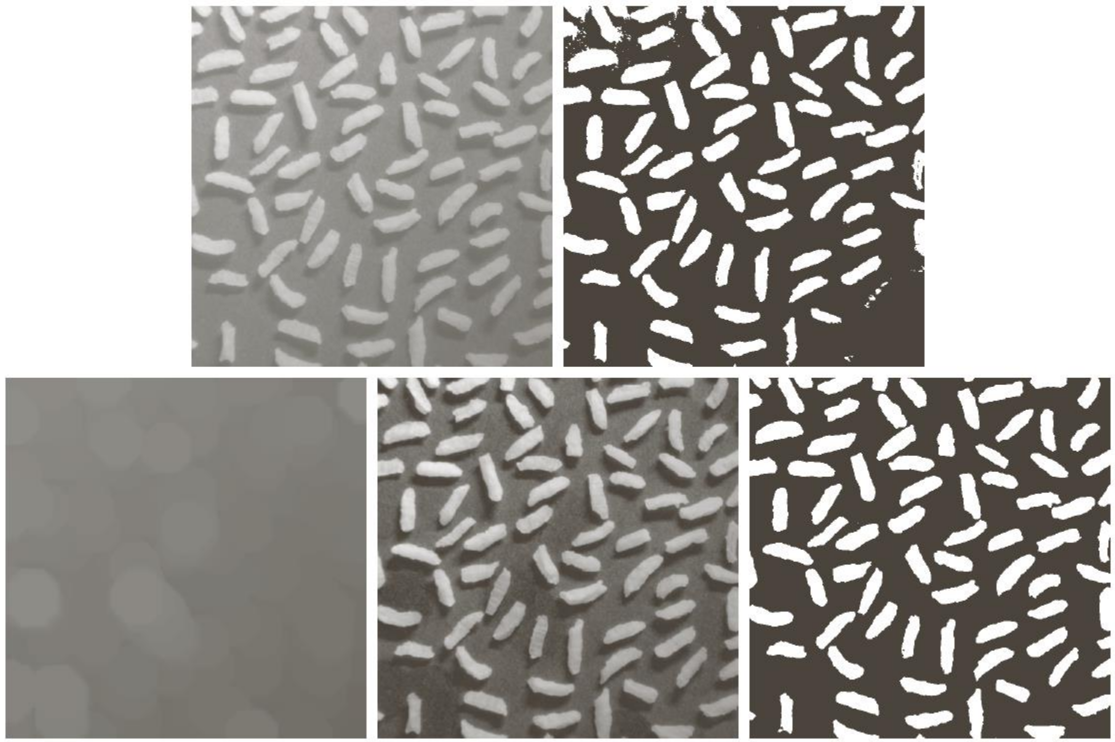


FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

Top-hat Transformation



a b
c d e

FIGURE 9.40 Using the top-hat transformation for *shading correction*. (a) Original image of size 600×600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

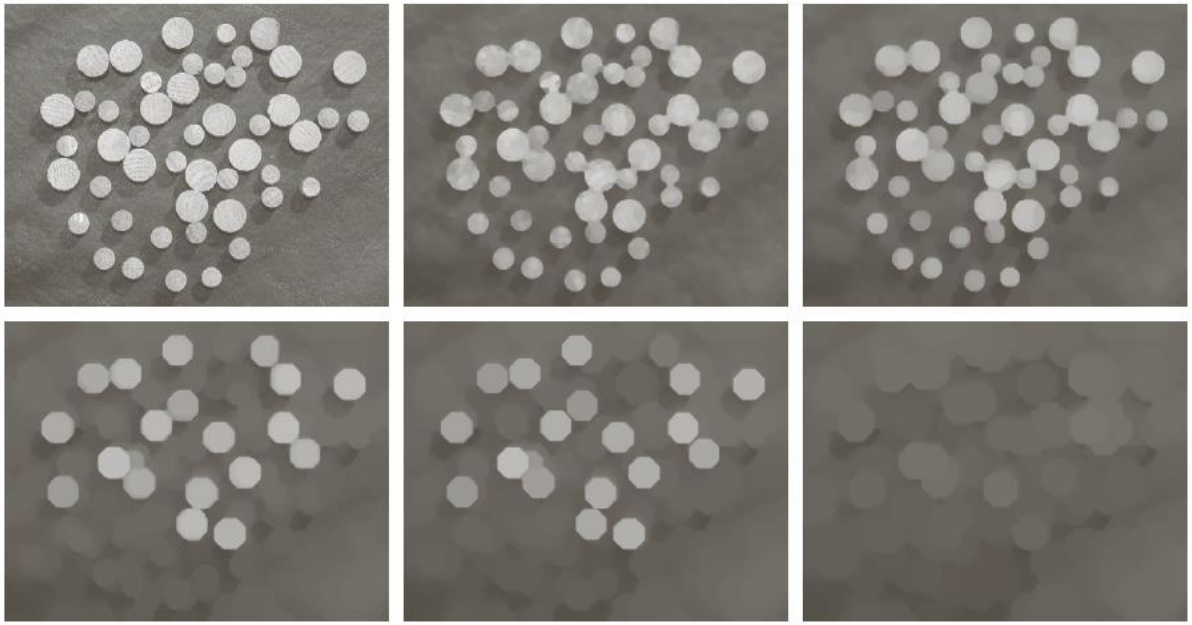
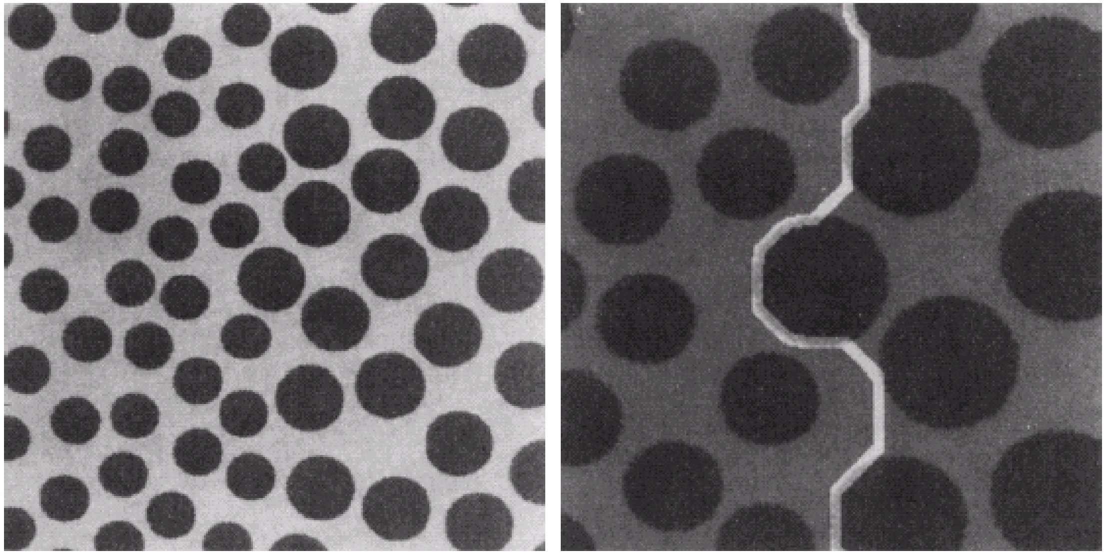


FIGURE 9.41 (a) 531 × 675 image of wood dowels. (b) Smoothed image. (c)–(f) Openings of (b) with disks of radii equal to 10, 20, 25, and 30 pixels, respectively. (Original image courtesy of Dr. Steve Eddins, The MathWorks, Inc.)

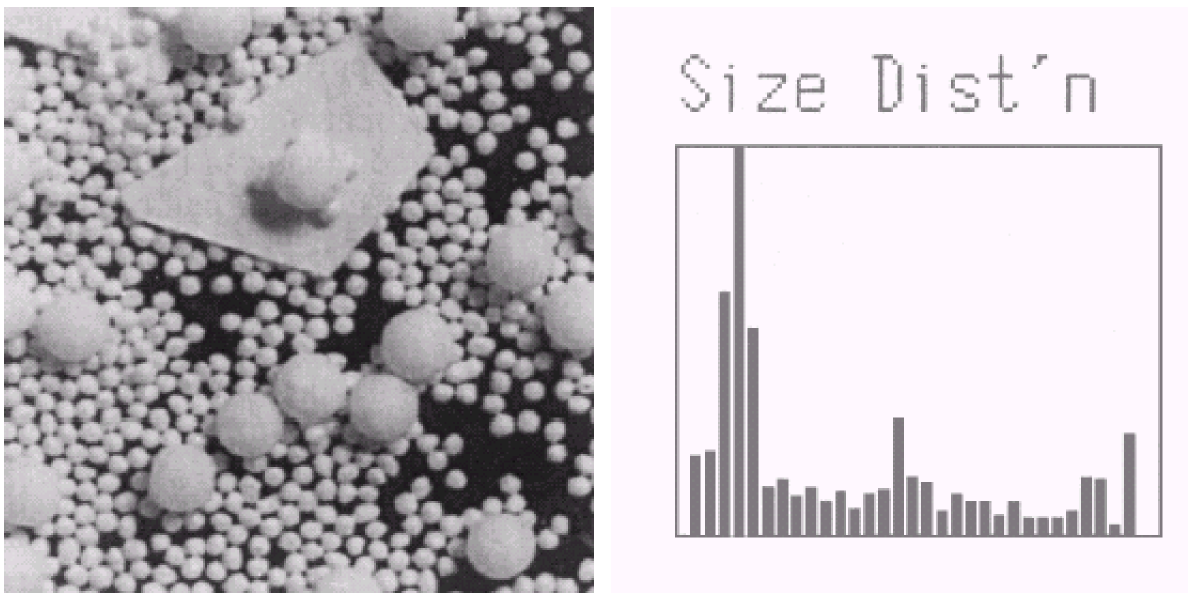
Texture Segmentation

a b

FIGURE 9.35 (a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)



Granulometry



a b

FIGURE 9.36
(a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)