Chapter 3 Intensity Transformations and Spatial Filtering

Image Enhancement in the Spatial Domain

- *No general theory* of image enhancement
- A certain amount of *trial and error* is usually required before a particular image enhancement approach is selected.

Outlines

3.1 Background

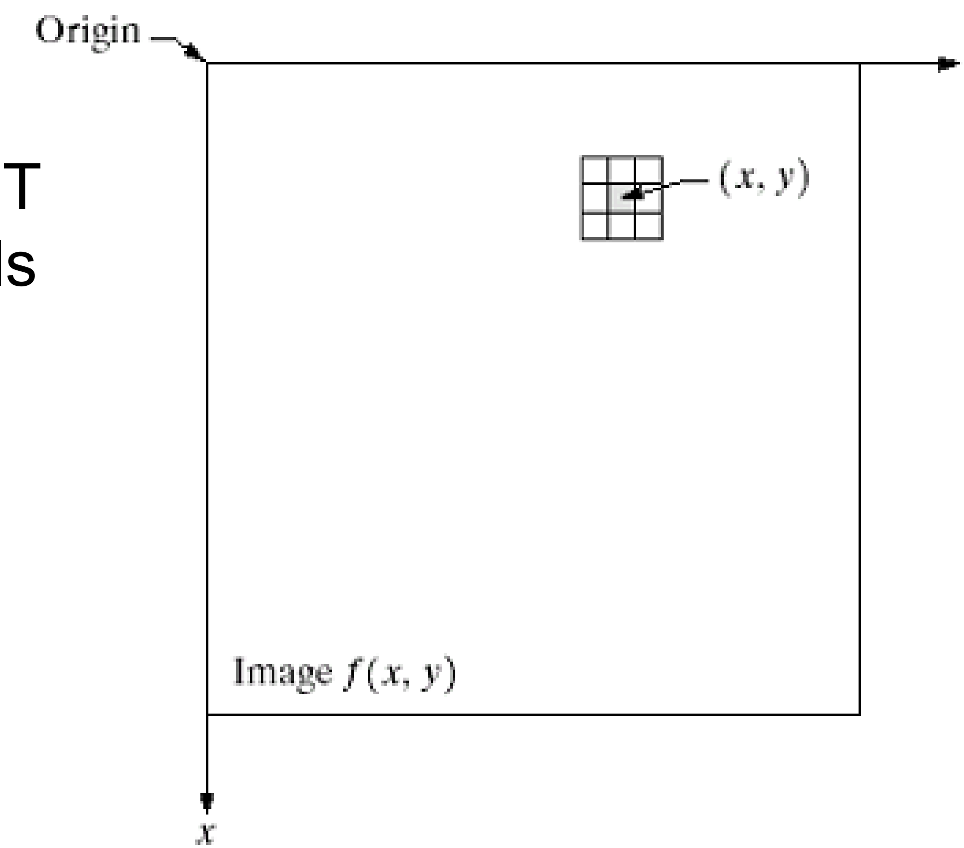
- Spatial Domain Methods
  - Procedures that operate directly on the *spatial domain* which refers to the aggregate of pixels composing an image.

$$g(x, y) = T[f(x, y)]$$

an operator on f,
defined over a **neighborhood** of (x,y)

Neighborhood

- For example, an operator T utilizes only the pixels in the area of the image spanned by the neighborhood, e.g., a 3x3 neighborhood.



Point Processing -- 1x1 neighborhood

只考慮自己不考慮鄰居

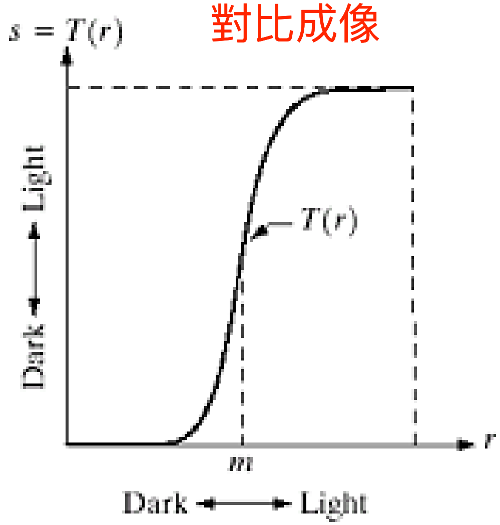
- Gray-Level Transformation Function

$$g(x, y) = T[f(x, y)] \longrightarrow s = T(r)$$

continuous <u>Contrast Stretching</u>

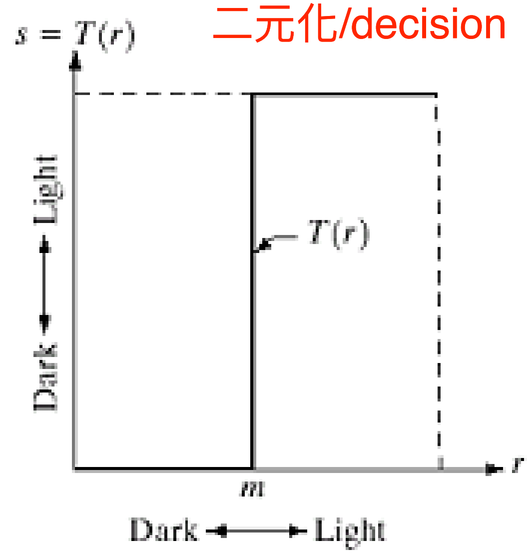
對比成像

把動態範圍留在想強化的地放，此圖為中間。



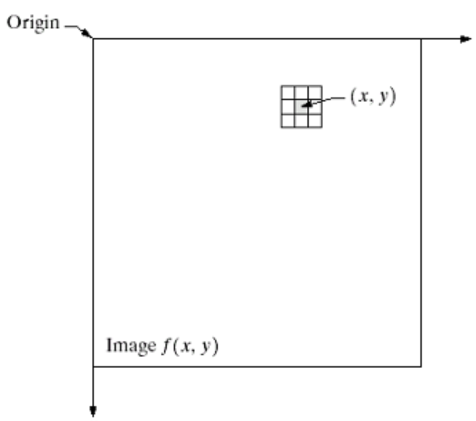
<u>Thresholding</u>

二元化/decision



Point Processing (3.2 - 3.4)
vs.
Mask Processing (3.5 – 3.7)

- Mask Processing (Filtering, Spatial Filtering)
  - e.g., smoothing (section 3.6),
    sharpening (section 3.7)
- Mask (filter, kernel, template, window)
  -- a small 2D array in which the values of the mask coefficients determine the nature of the process, e.g., smoothing or sharpening



放一些coefficients造成不同效果

Outlines

3.1 Background

3.2 Some Basic Intensity Transformation Functions

3.3 Histogram Processing

3.4 Fundamentals of Spatial Filtering

3.5 Smoothing Spatial Filters

3.6 Sharpening Spatial Filters

3.7 Combining Spatial Enhancement Methods

3.8 Using Fuzzy Techniques for Intensity Transformation and Spatial Filtering

3.2 Basics Gray-Level Transformation

- Implemented via table lookups
  -- 256 entries

3.2.1 Image Negatives

3.2.2 Log Transformations

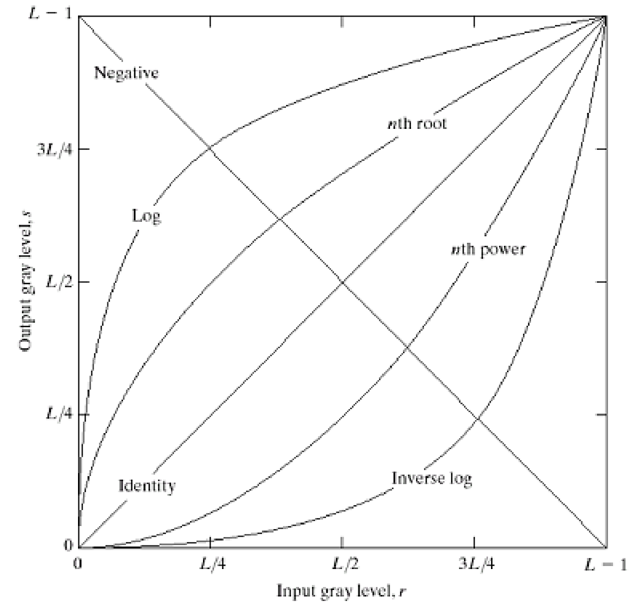
3.2.3 Power-Law Transformations

3.2.4 Piecewise-Linear Transformations

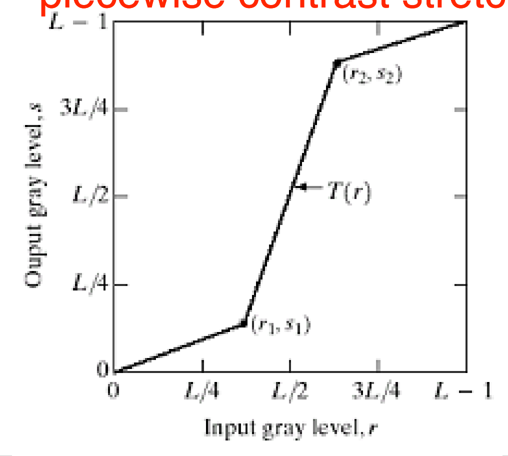
-- Contrast Stretching

-- Gray-Level Slicing

-- Bit-Plane Slicing

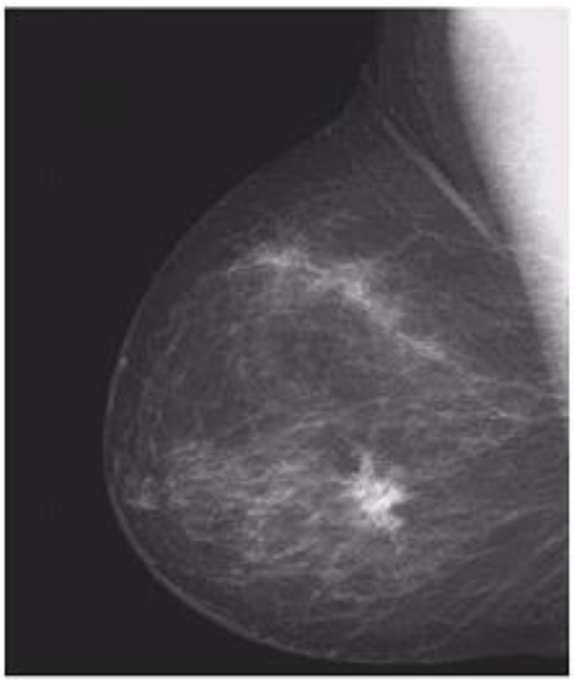


piecewise contrast stretch

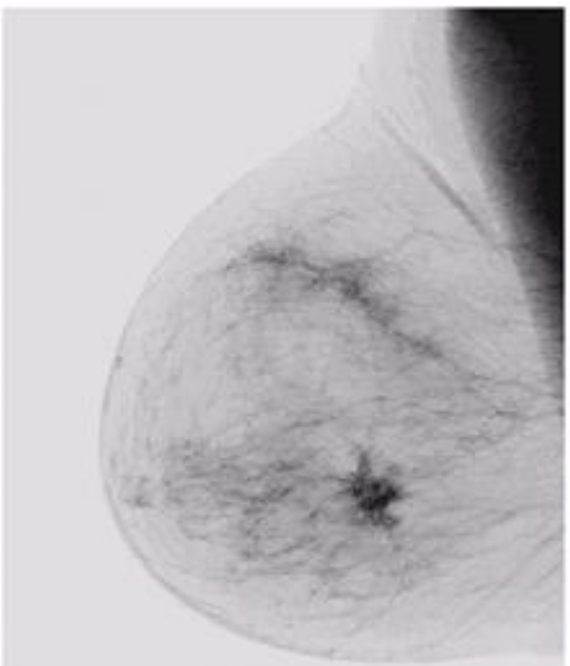


3.2.1 Image Negatives

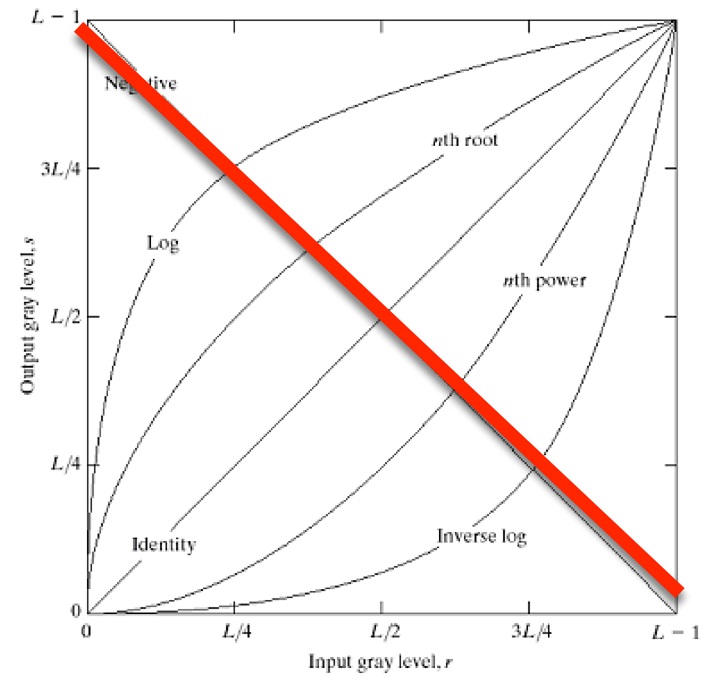
$$s = L - 1 - r = 255 - r$$



乳房X光攝影片



Negative Image



3.2.2 Log Transformations

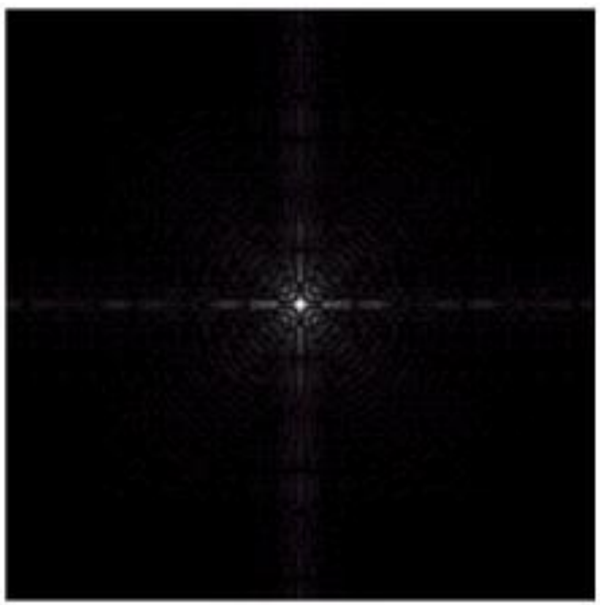
-- useful for compressing *large* dynamic range

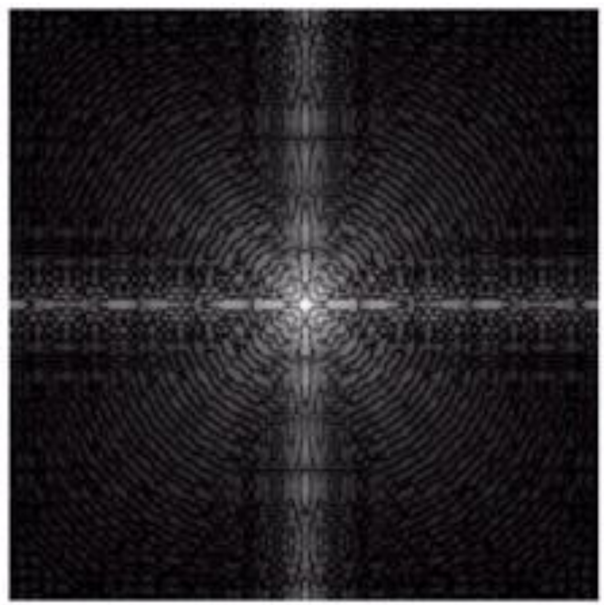
把不明顯的地方放大

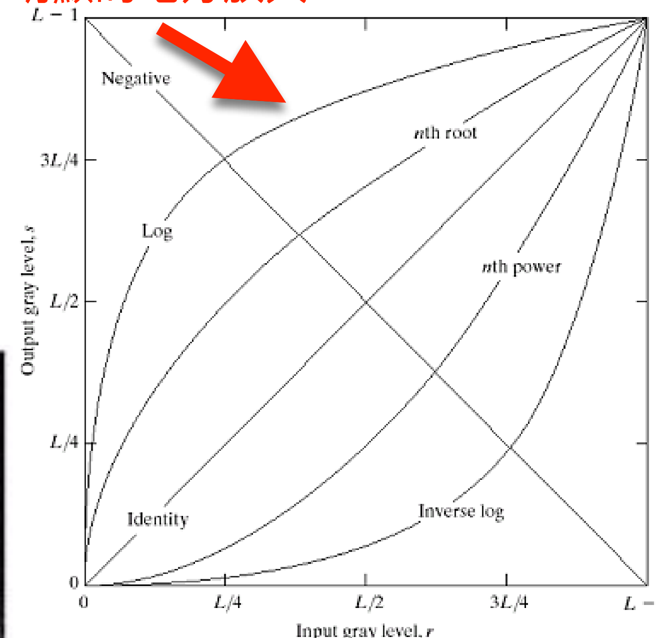
$$s = c \log (1 + r)$$

Fourier Spectrum

$0 \sim 1.5 \times 10^6$ $\xrightarrow{\text{LOG}}$ $0 \sim 6.2$







Not as versatile as *power-law*

Most of the Fourier Spectra seen in the IP publications have been scaled in this manner

指數
3.2.3 Power-Law Transformations

與log不同，多了參數gamma可以調整曲線

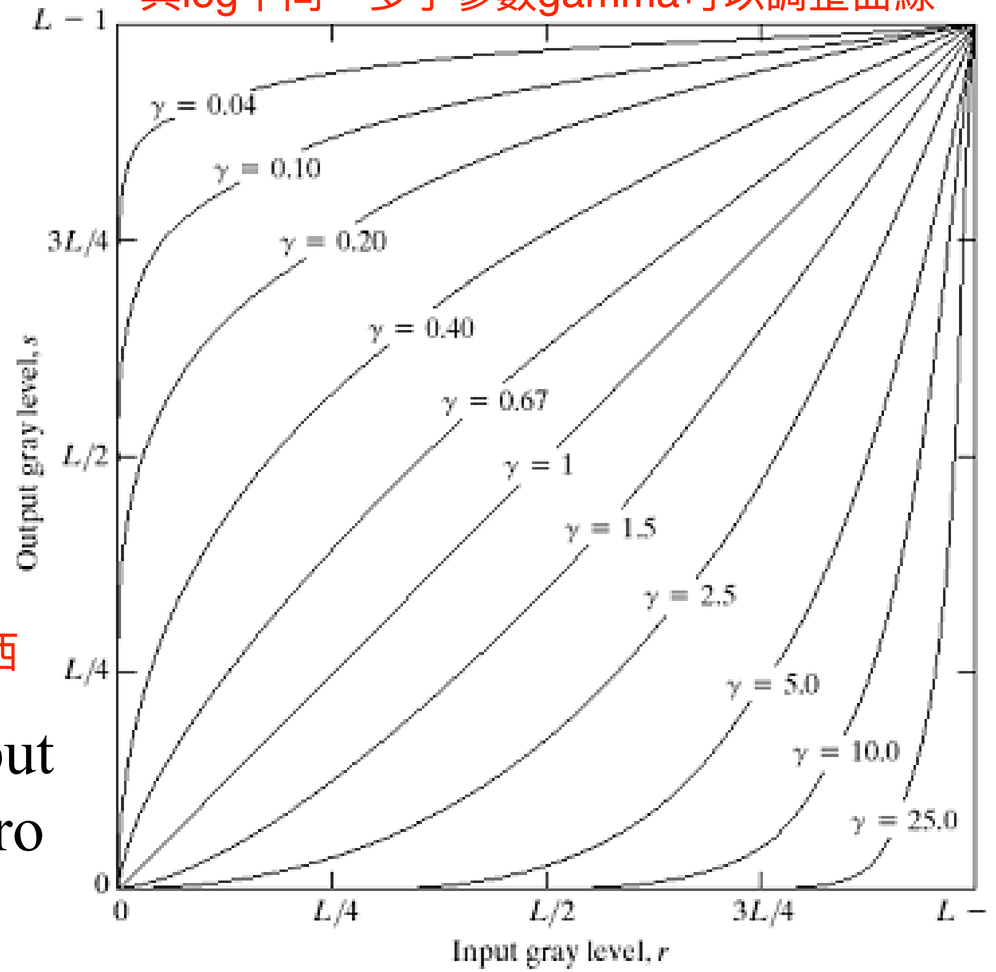
gamma

$$\boxed{s = c\, r^{\gamma}}$$

$$s = c\,(r+\varepsilon)^{\gamma}$$

螢幕沒開的時候還是有東西

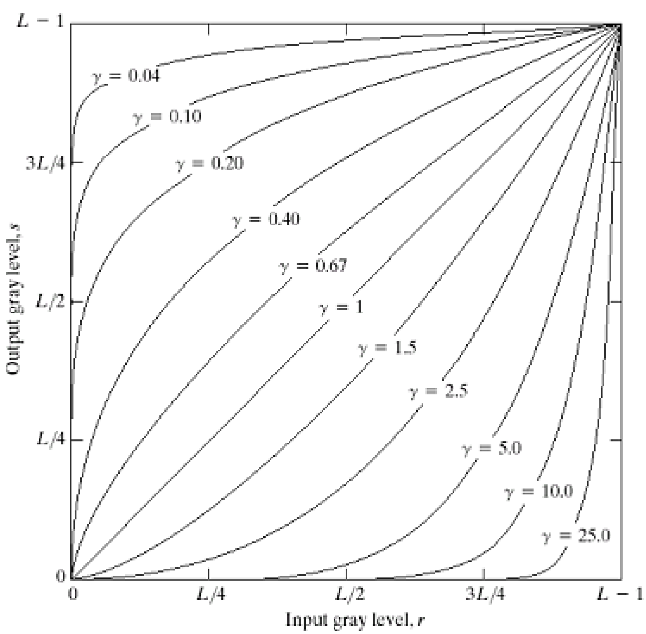
an offset: a measurable output when the input is zero

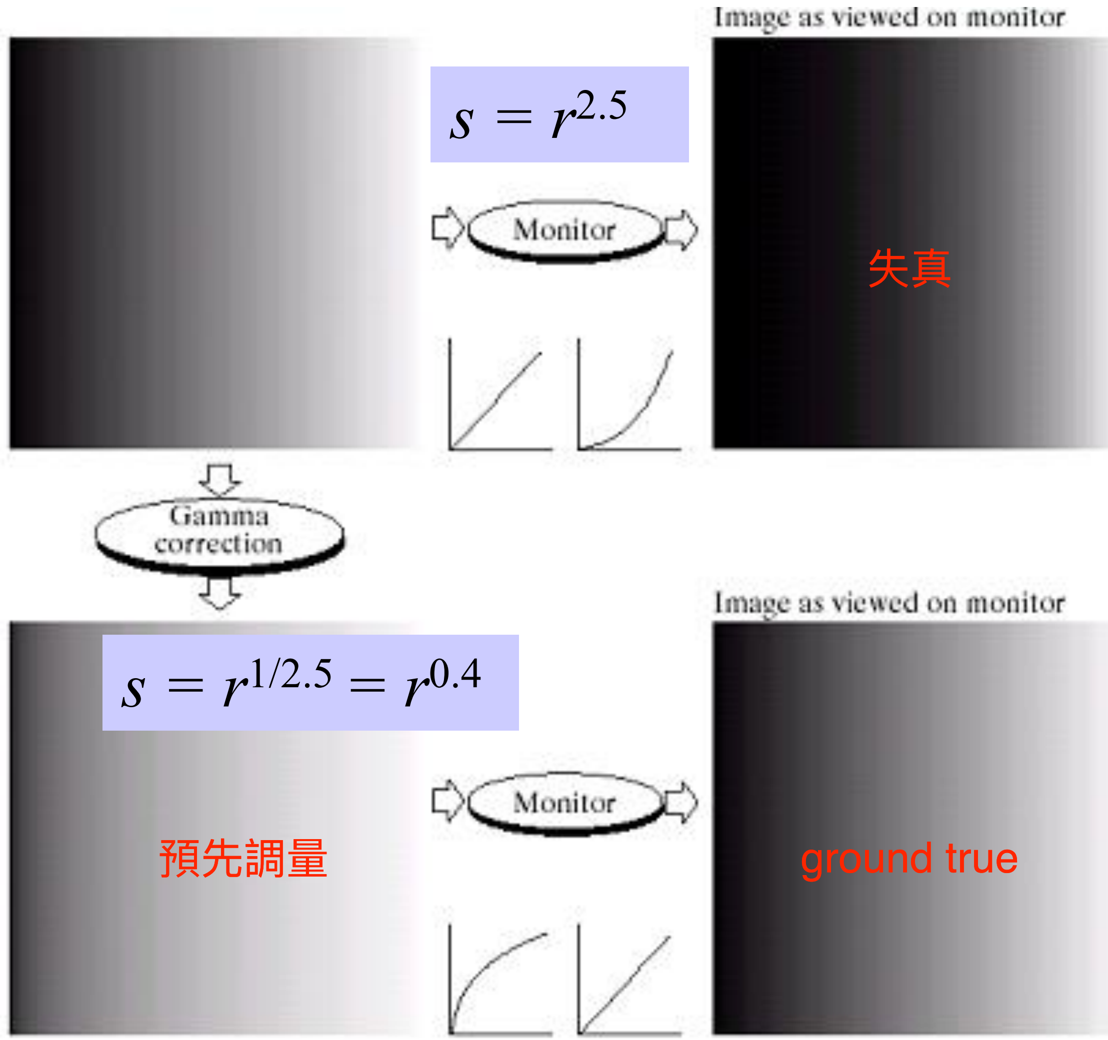


Gamma Correction

螢幕的gamma參數假設

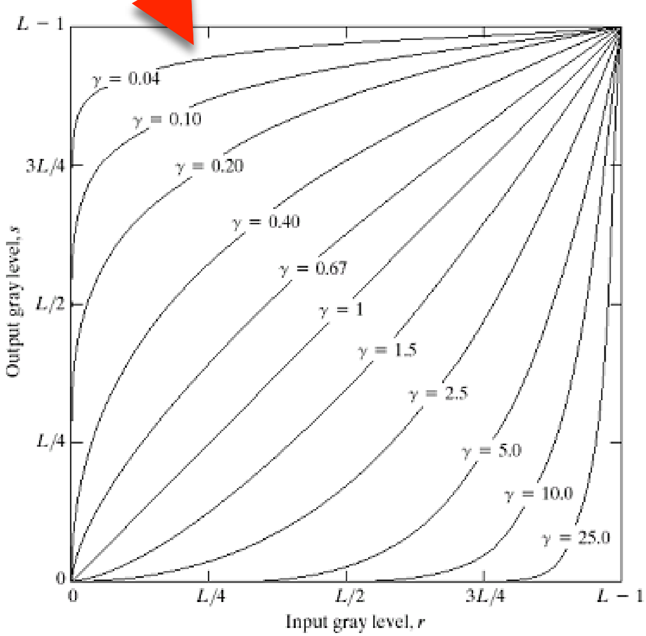
For example
$\gamma = 2.5$





Power-Law Transformation
for general-purpose contrast manipulation

萃取暗的地方的contrast



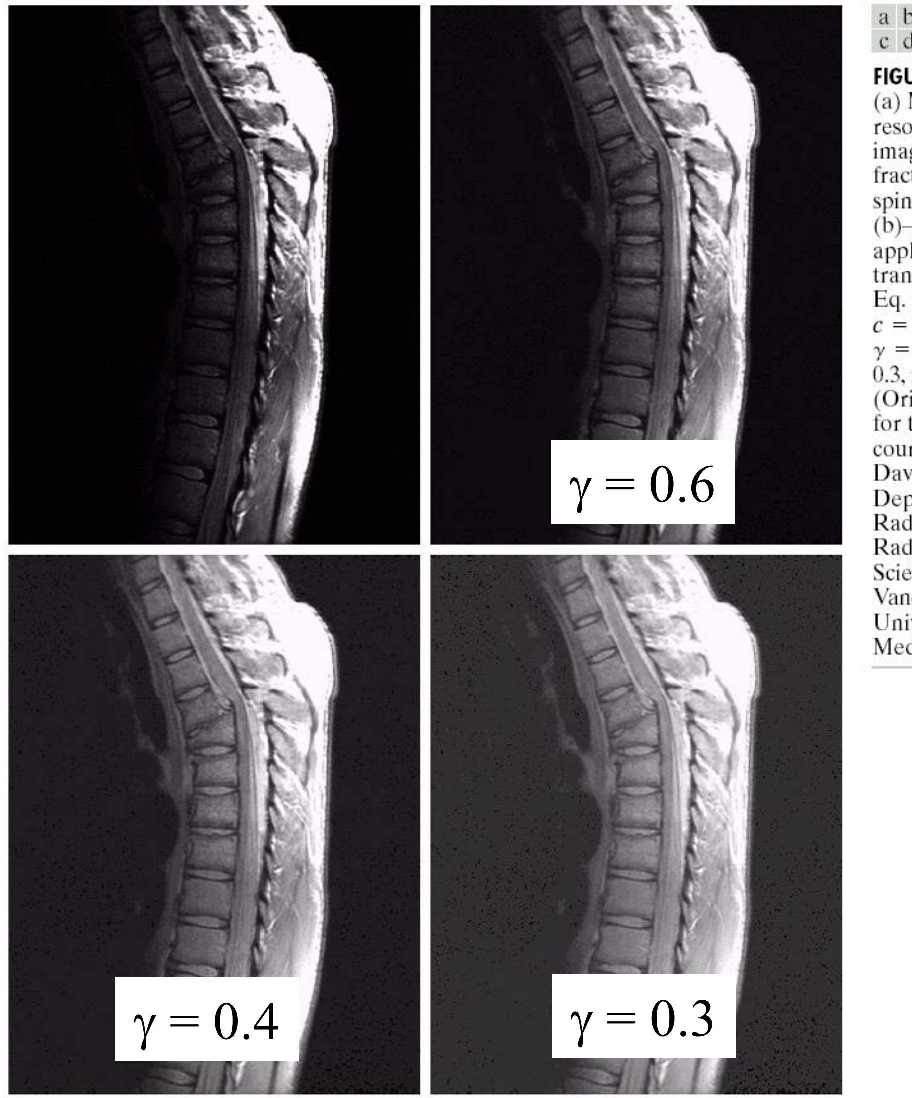
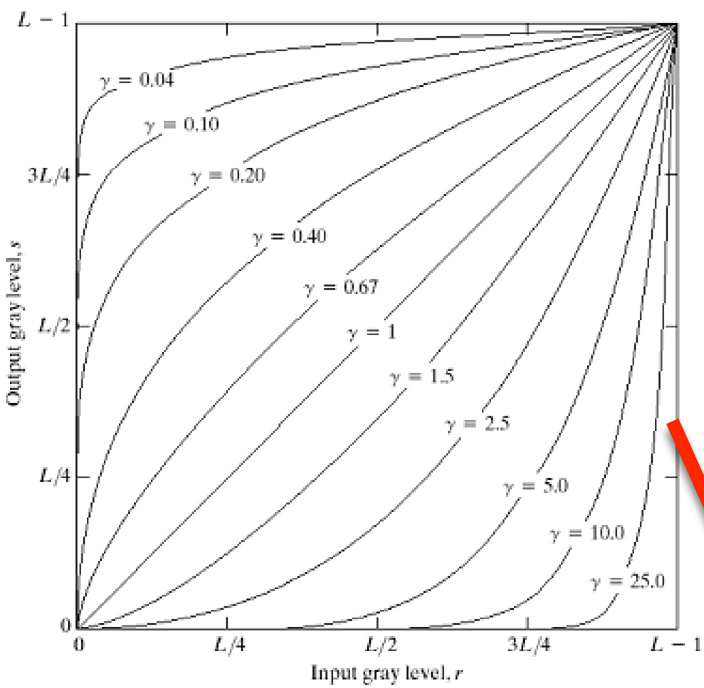
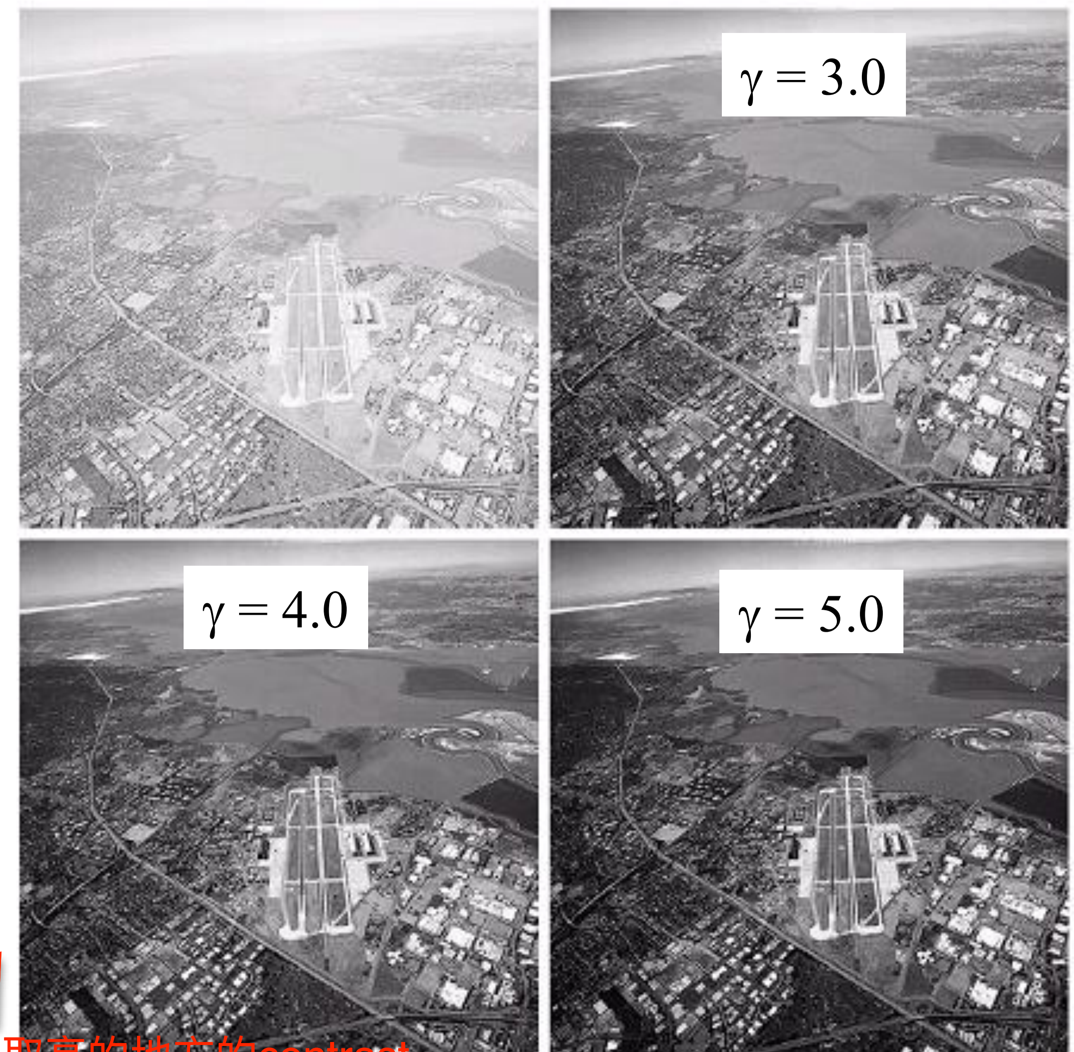


FIGURE 3.8 (a) Magnetic resonance (MR) image of a fractured human spine. (b)–(d) Results of applying the transformation in Eq. (3.2-3) with $c = 1$ and $\gamma = 0.6, 0.4,$ and 0.3, respectively. (Original image for this example courtesy of Dr. David R. Pickens, Department of Radiology and Radiological Sciences, Vanderbilt University Medical Center.)

Power-Law Transformation
for general-purpose contrast manipulation





萃取亮的地方的contrast

3.2.4 Piecewise-Linear Transformations

Contrast Stretching

Assume $r_2 \geq r_1$ and $s_2 \geq s_1$

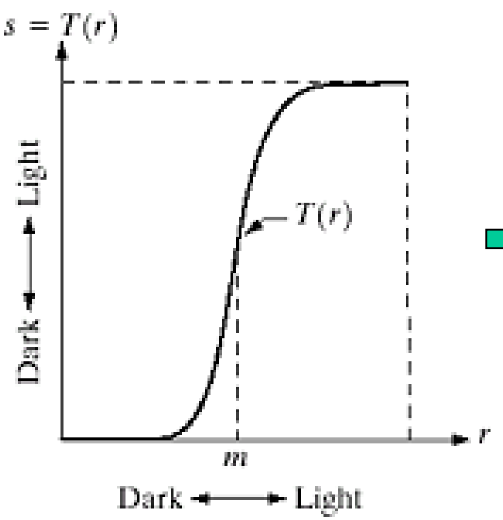
→ single-valued function, and monotonically increasing

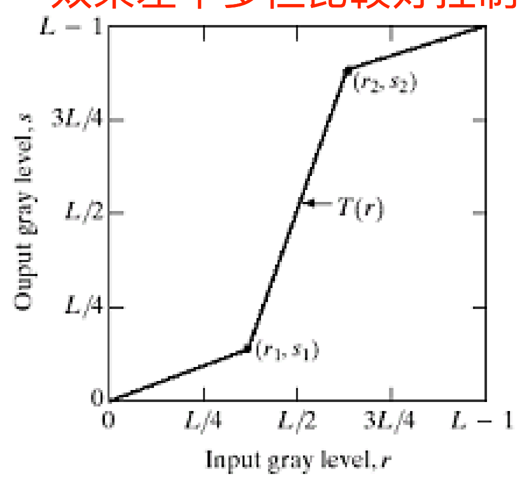
→ prevent intensity artifacts

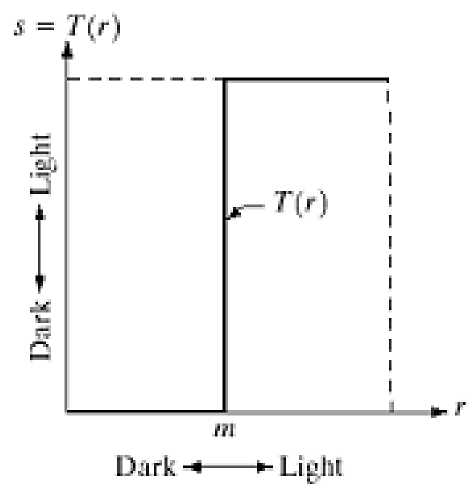
因為做影像處理而跑出來的東西

Linear or identity

效果差不多但比較好控制







Contrast Stretching

放大700倍的花粉

In digital video cameras, called AGC (automatic gain control).

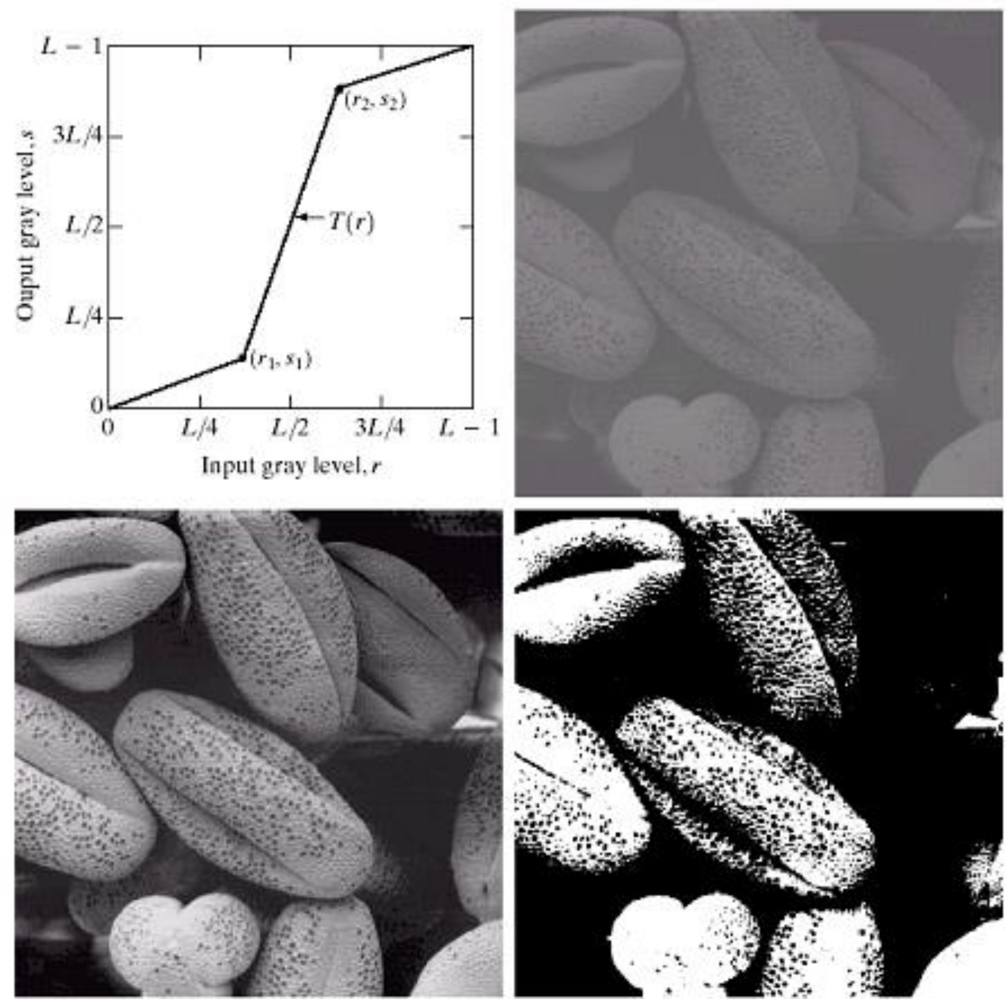
Full-Scale Histogram Stretch

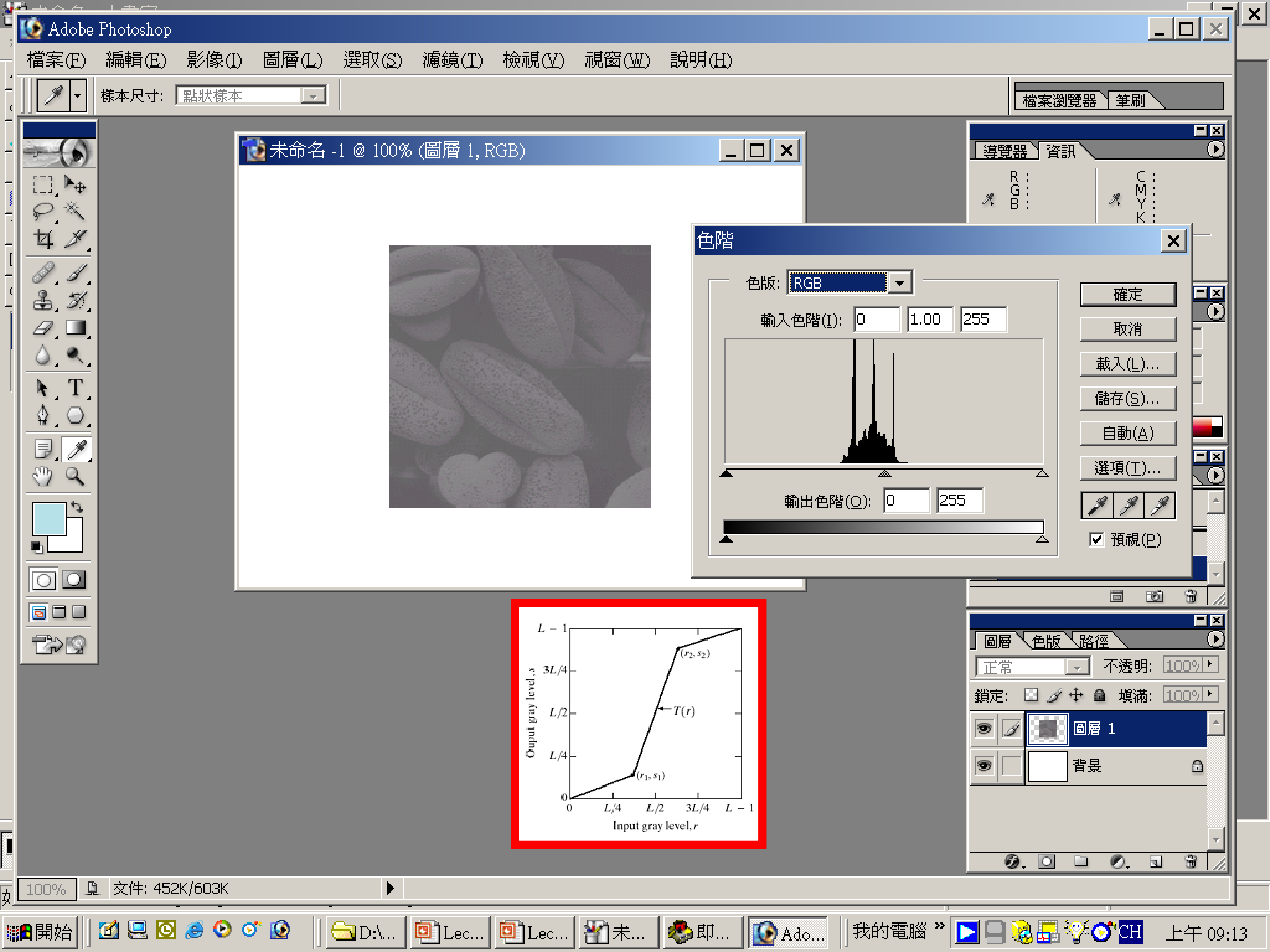
把最小的值拉倒0

把最大的值拉到255

$$(r_1, s_1) = (r_{min}, 0)$$

$$(r_2, s_2) = (r_{max}, L\text{-}1)$$




Adobe Photoshop
檔案(F) 編輯(E) 影像(I) 圖層(L) 選取(S) 濾鏡(T) 檢視(V) 視窗(W) 說明(H)
樣本尺寸: 點狀樣本
檔案瀏覽器 筆刷
未命名 -1 @ 100% (圖層 1, RGB)
導覽器 資訊
色階
色版: RGB
輸入色階(I): 0 1.00 255
輸出色階(O): 0 255
確定
取消
載入(L)...
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自動(A)...
選項(T)...
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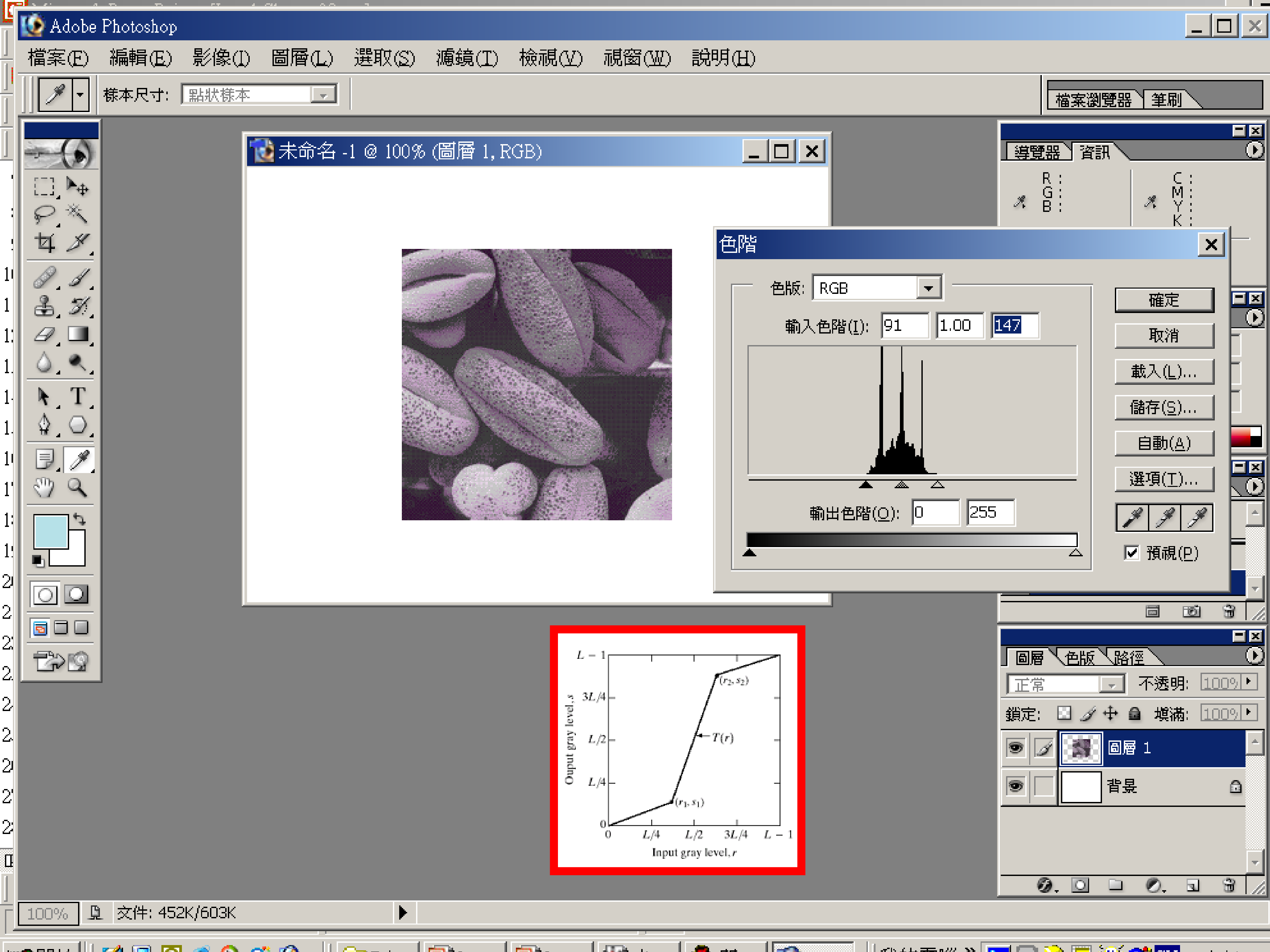
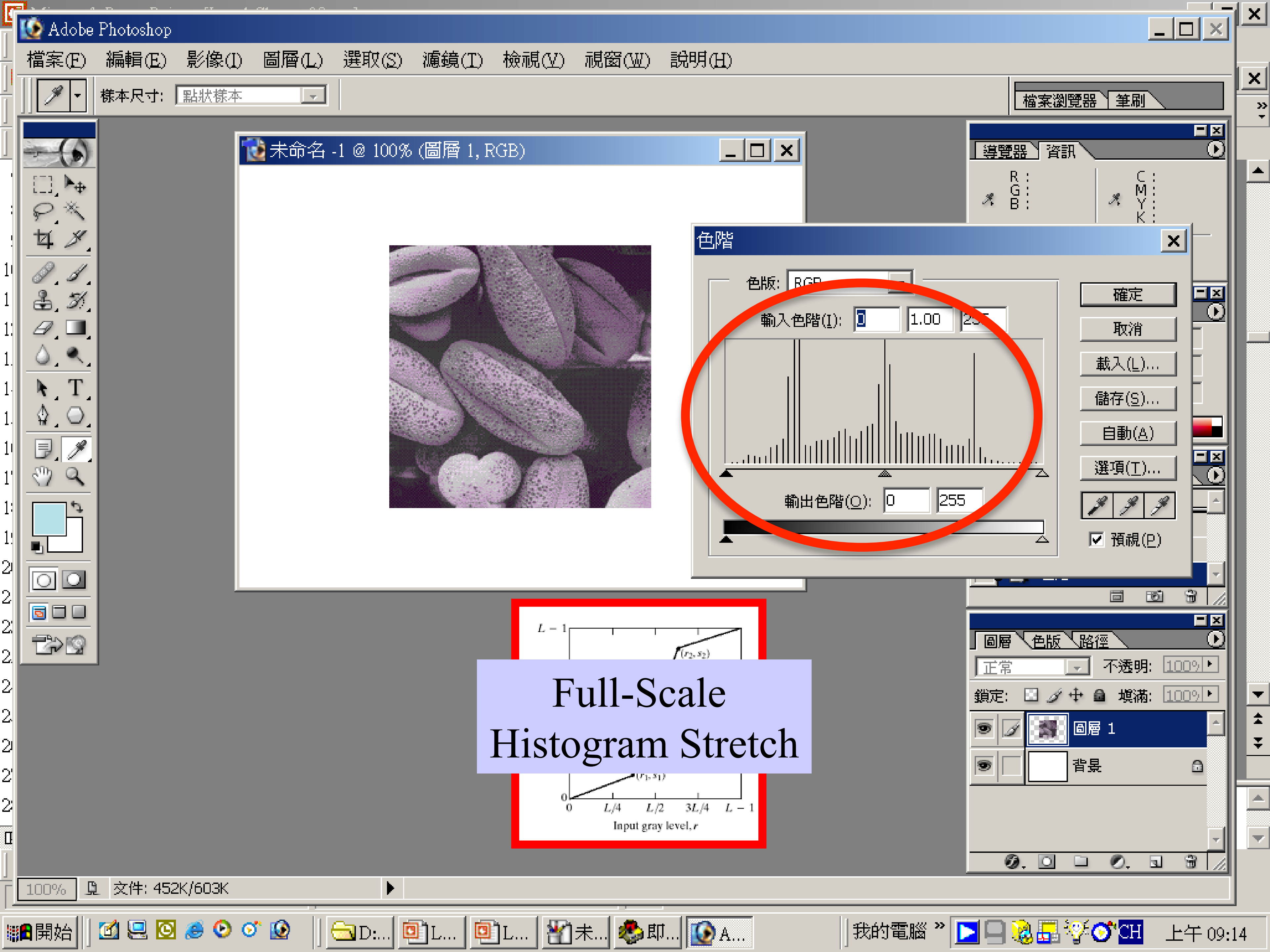
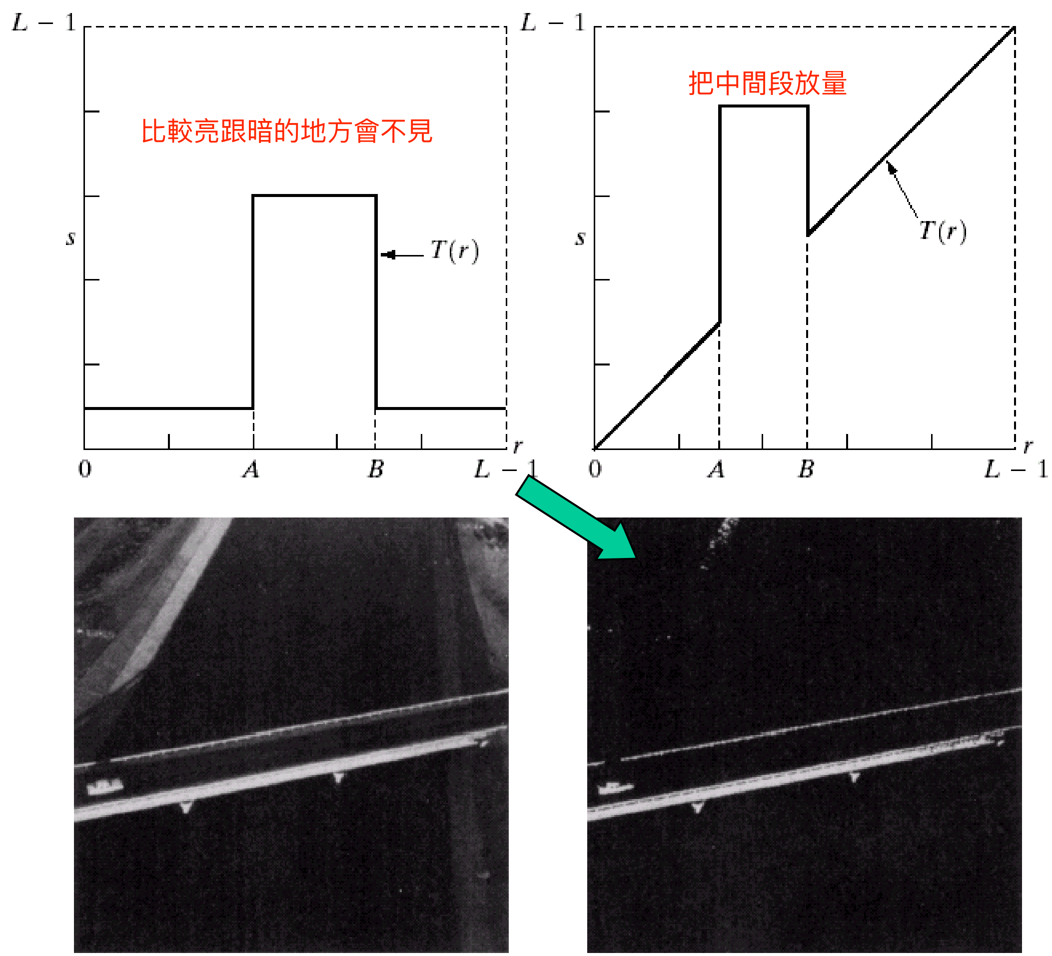
圖層 色版 路徑
正常 不透明: 100%
鎖定: 填滿: 100%
圖層 1
背景
100%
文件: 452K/603K
$L-1$
$3L/4$
$L/2$
$L/4$
0
Ouput gray level, s
(r_2, s_2)
$T(r)$
(r_1, s_1)
0 $L/4$ $L/2$ $3L/4$ $L-1$
Input gray level, r
開始
我的電腦
上午 09:13


Adobe Photoshop
檔案(F) 編輯(E) 影像(I) 圖層(L) 選取(S) 濾鏡(T) 檢視(V) 視窗(W) 說明(H)
樣本尺寸: 點狀樣本
檔案瀏覽器 筆刷
未命名 -1 @ 100% (圖層 1, RGB)
導覽器 資訊
R G B
C M Y K
色階
色版: RGB
輸入色階(I): 91 1.00 147
輸出色階(O): 0 255
確定
取消
載入(L)...
儲存(S)...
自動(A)
選項(T)...
預視(P)
圖層 色版 路徑
正常
不透明: 100%
鎖定:
填滿: 100%
圖層 1
背景
100%
文件: 452K/603K
L − 1
3L/4
L/2
L/4
0
(r_2, s_2)
$T(r)$
(r_1, s_1)
Ouput gray level, s
0 L/4 L/2 3L/4 L − 1
Input gray level, r


Adobe Photoshop
檔案(F) 編輯(E) 影像(I) 圖層(L) 選取(S) 濾鏡(T) 檢視(V) 視窗(W) 說明(H)
樣本尺寸: 點狀樣本
未命名 -1 @ 100% (圖層 1, RGB)
色階
色版: RGB
輸入色階(I): 0 1.00 255
輸出色階(O): 0 255
確定
取消
載入(L)...
儲存(S)...
自動(A)
選項(T)...
預視(P)
L − 1
(r_2, s_2)
(r_1, s_1)
0 L/4 L/2 3L/4 L − 1
Input gray level, r
Full-Scale Histogram Stretch
圖層 色版 路徑
正常 不透明: 100%
鎖定: 填滿: 100%
圖層 1
背景
100% 文件: 452K/603K
開始
我的電腦
上午 09:14

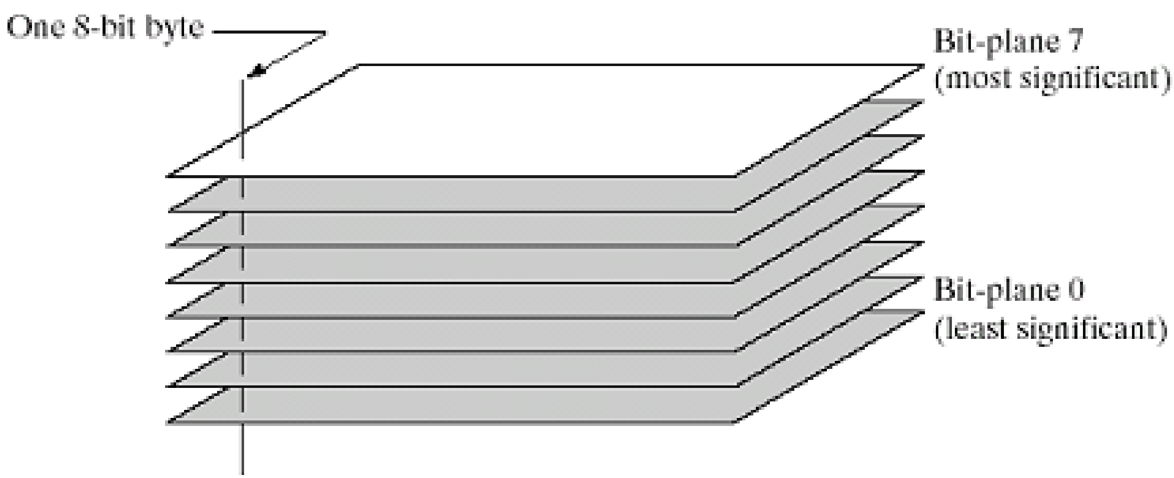
Gray-Level Slicing



a b
c d

FIGURE 3.11 (a) This transformation highlights range [A, B] of gray levels and reduces all others to a constant level. (b) This transformation highlights range [A, B] but preserves all other levels. (c) An image. (d) Result of using the transformation in (a).

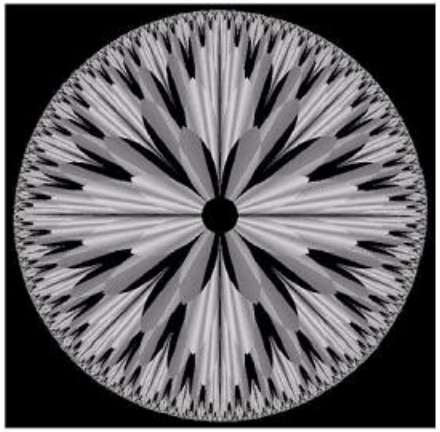
Bit-Plane Slicing



visually significant data

subtle details

Bit-Plane Slicing



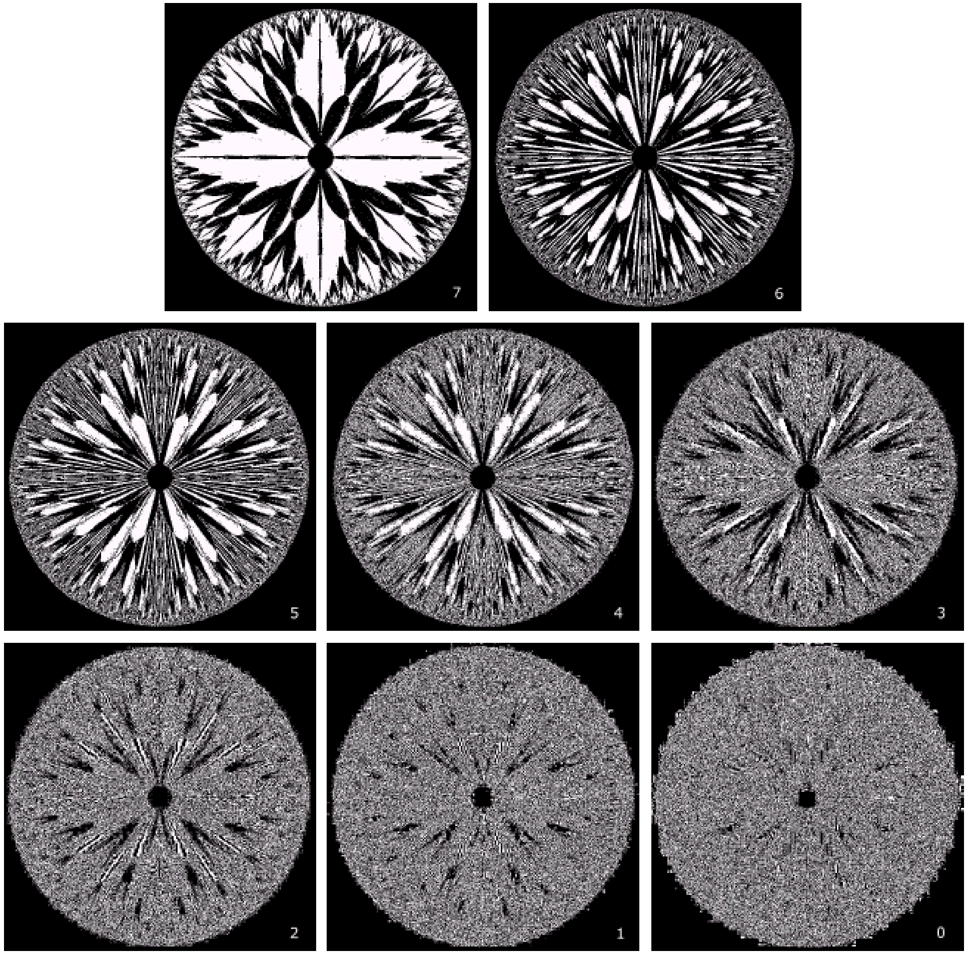


FIGURE 3.14 The eight bit planes of the image in Fig. 3.13. The number at the bottom, right of each image identifies the bit plane.

Outlines

3.1 Background

3.2 Some Basic Intensity Transformation Functions

3.3 Histogram Processing

3.4 Fundamentals of Spatial Filtering

3.5 Smoothing Spatial Filters

3.6 Sharpening Spatial Filters

3.7 Combining Spatial Enhancement Methods

3.8 Using Fuzzy Techniques for Intensity Transformation and Spatial Filtering

3.3 Histogram Processing

GLOBAL

等化器的概念

3.3.1 Histogram Equalization （直方圖等化、均衡化）

先處理直方圖後套在圖片上

3.3.2 Histogram Specification (Matching)

3.3.3 Local Enhancement （局部增強）

3.3.4 Histogram Analysis for Image Thresholding

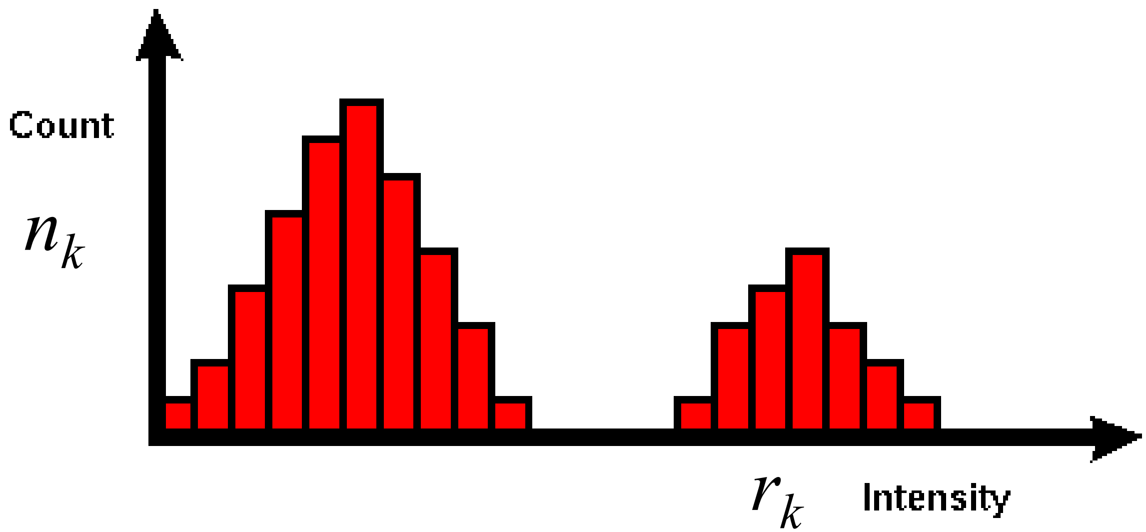
Histogram （直方圖）

- Histogram of a digital image is a distribution function

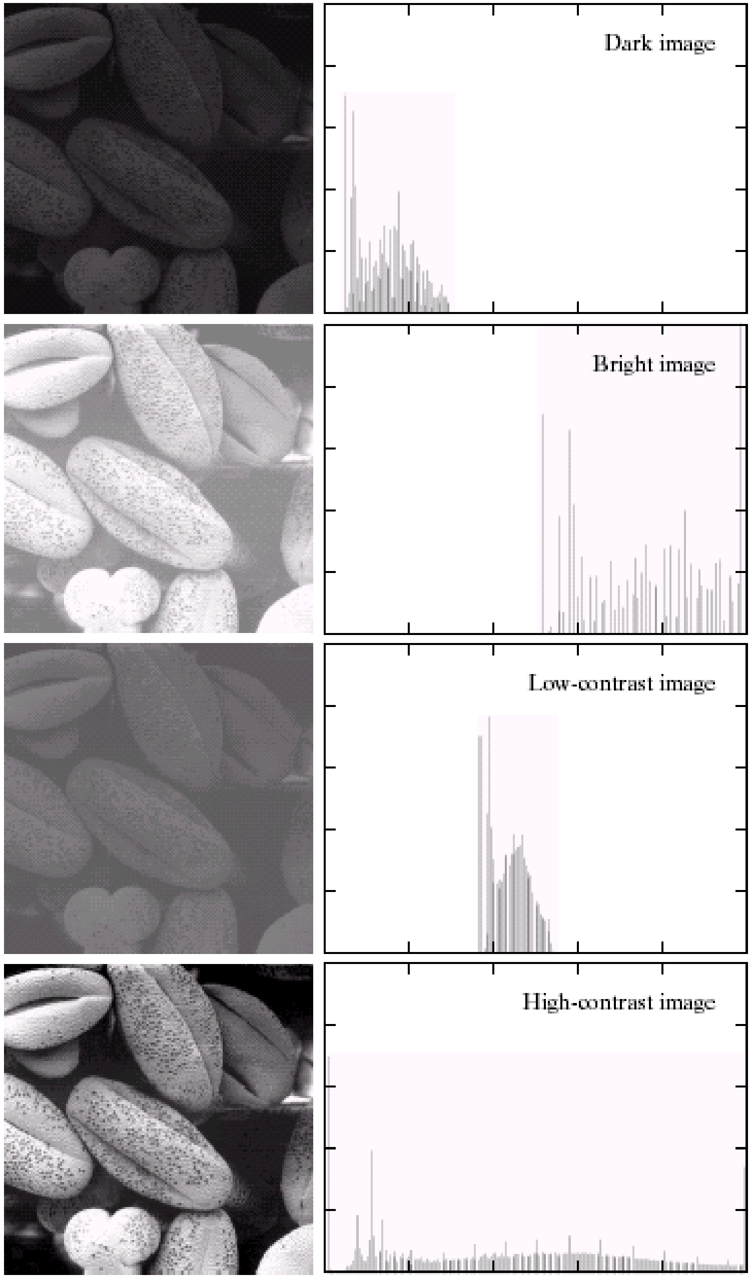
$$h(r_k) = n_k$$

  – where r_k is the kth gray level k : 0~255

  and n_k is the number of pixels having gray level r_k



Different types of histograms:



- Normalized histogram

n : # of pixels

$$p(r_k) = n_k / n \,, \quad k = 1, \ldots, L$$

p(r1)+….+p(rk) = 1

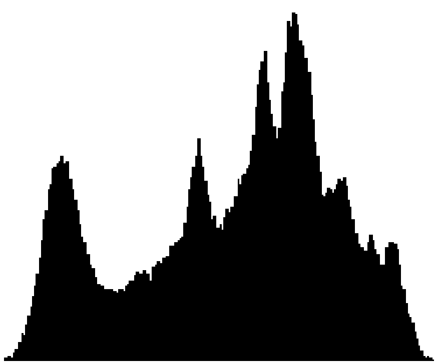
- Histogram is useful for
  - -- image enhancement
  - -- image compression
  - -- image segmentation
  - -- etc.

→ Want to have a more flat histogram !

3.3.1 Histogram Equalization (Flattening)

-- extension of full-scale histogram stretch

Image
Enhancement



Histogram
Equalization

開小視窗去掃average
會發現是uniform

Histogram Equalization

- Normalized histogram of an image f

$$p_f(k) = h_f(k) \;/\; n\,, \quad \text{for } k = 1, \ldots, L$$

$$\sum_{k=0}^{L-1} p_f(k) = 1$$

- Cumulative normalized histogram of f （累積正規化直方圖）

$$P_f(j) = \sum_{k=0}^{j} p_f(k); \quad j = 1, \cdots, L$$

$$P_f(j) = \Pr\{f(x, y) \le j\}$$

→ Empirical probability distribution function $P_f(j)$

-- monotonically non-decreasing, and in [0,1]

Histogram Equalization

1 ~ 256

discrete $$p_f(k) = P_f(k) - P_f(k-1)\ ; \quad k = 1, \cdots, L$$

- Suppose the cumulative normalized histogram $P_f(\,.)$ are a function of *continuous* variable x

continuous

$$p_f(x) = \frac{\mathrm{d}\,P_f(x)}{\mathrm{d}\,x}$$

Histogram Equalization (Proof)

找一個tranformation function T 使得f影像可轉成g影像使其為uniform distribution

Let $g(x,y) = P_f(\, f(x,y)\,)$, or $s = \mathrm{T}\,(\, r\,)$, where $\mathrm{T}\,(\,.)$ is $P_f(\,.)$

→ Consider g and f as random variables, then $g = P_f(\, f\,)$ cumulative function

Does the image g has a uniform (flat) histogram ?

$$P_g(u) = \Pr\{g \le u\} = \Pr\{P_f(\, f\,) \le u\}$$

$$= \Pr\{f \le P_f^{-1}(u)\} = P_f[P_f^{-1}(u)] = u, \qquad \text{for } 0 \le x \le 1$$

notation改變

→ $$p_g(u) = dP_g(u)/du = 1, \quad \text{for } 0 \le u \le 1$$

→ **g has a uniform (flat) histogram !**

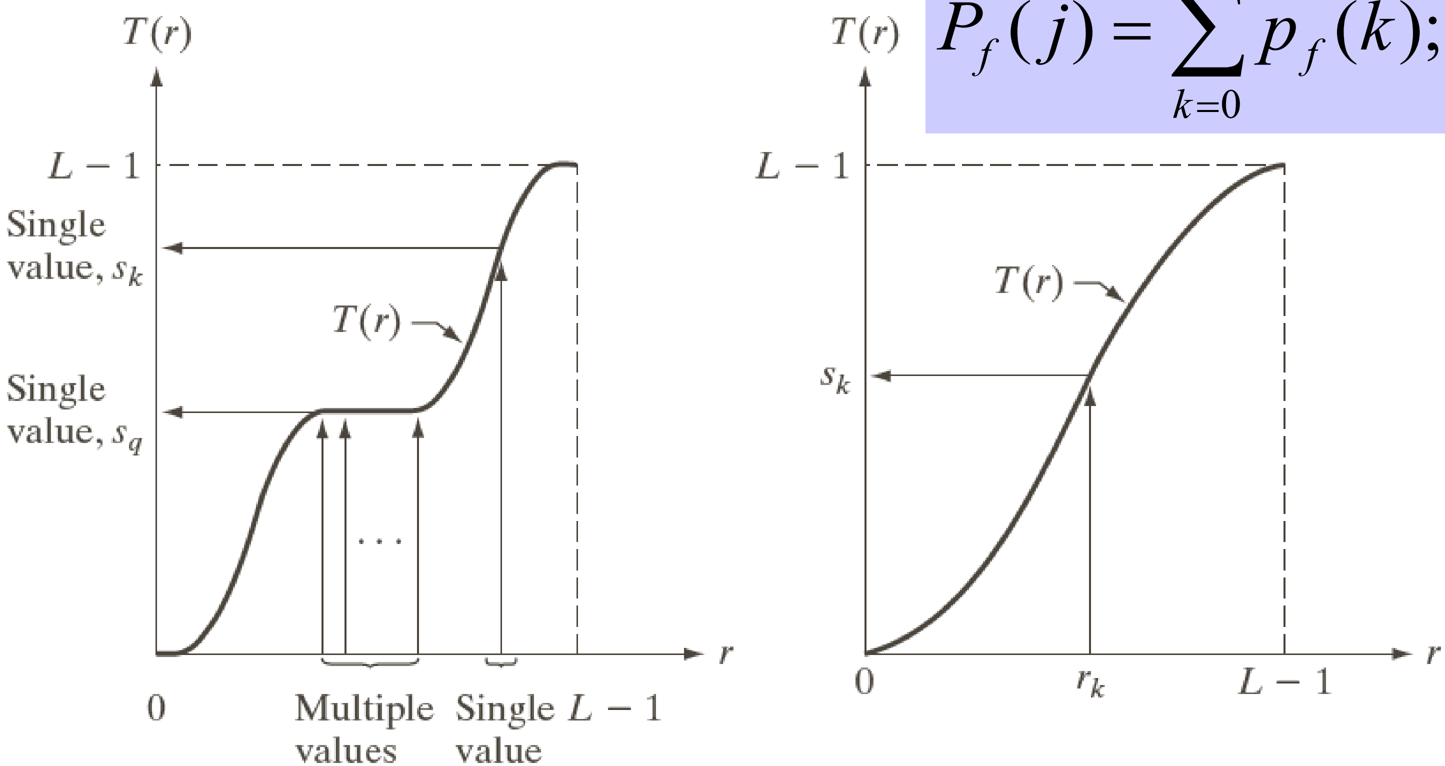
用cumulative處理影像就會讓所有顏色變平均

Histogram Equalization

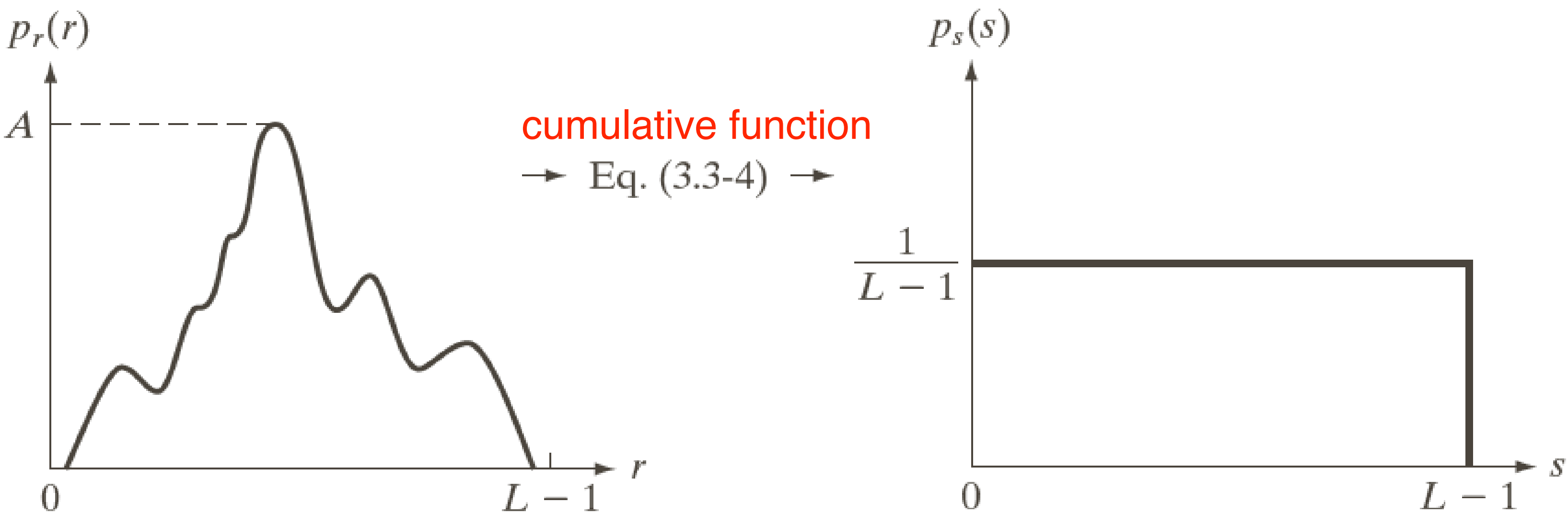
- Normalized histogram of an image f

$$g(x,y) = P_f[f(x,y)] \quad \text{or} \quad s = \mathrm{T}(r), \text{ where } \mathrm{T}(.) \text{ is } P_f(.)$$

$$P_f(j) = \sum_{k=0}^{j} p_f(k); \quad j = 1, \cdots, L$$



* consideration：有些discrete function 如果帶入cumulative function 時會有部分轉換吃敗導致不flat



a b

FIGURE 3.18 (a) An arbitrary PDF. (b) Result of applying the transformation in Eq. (3.3-4) to all intensity levels, r. The resulting intensities, s, have a uniform PDF, independently of the form of the PDF of the r's.

Algorithm for Histogram Equalization:

1. *Compute the histogram of the input image:*

   $h(k) = \#\{(x,y) | f(x,y)=k\}$, *where* $k = 0$ *to* 255.

2. *Compute the transformation function:*

$$T(k) = 255 * \sum_{j=0}^{k} \frac{h(j)}{n}$$

3. *Transform the value of each pixel by*

$$g(x,y)=T(f(x,y))$$

處理完後contrast變漂亮

MN = 64 * 64

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

TABLE 3.1 Intensity distribution and histogram values for a 3-bit, 64×64 digital image.

看起來不平，因為discrete的關係

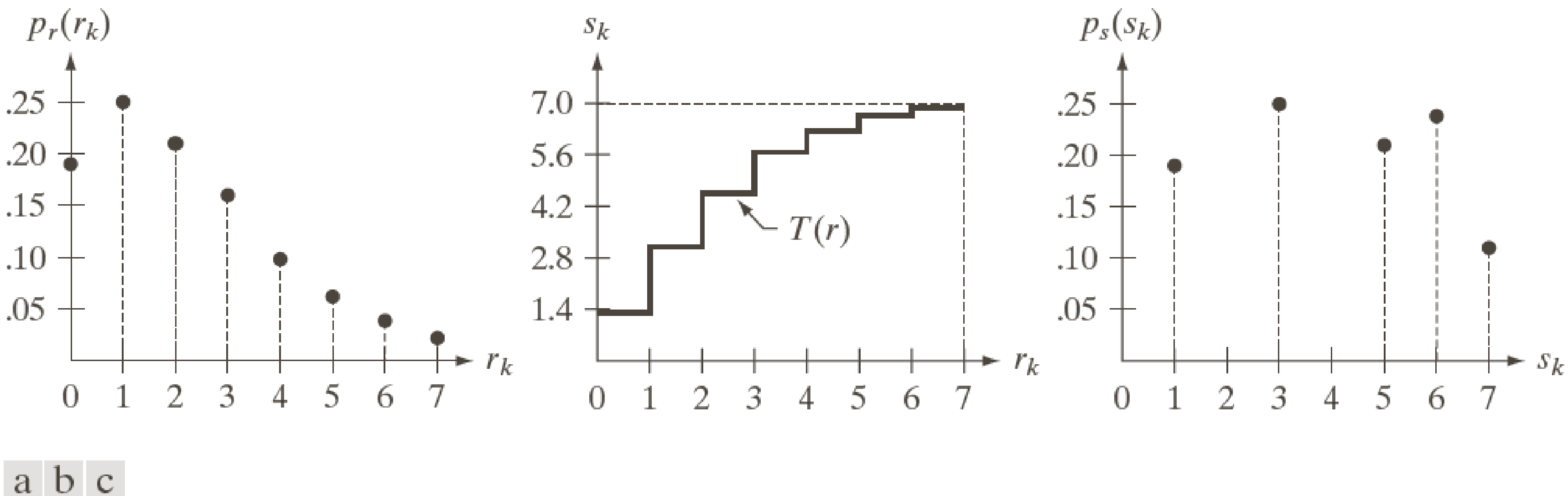
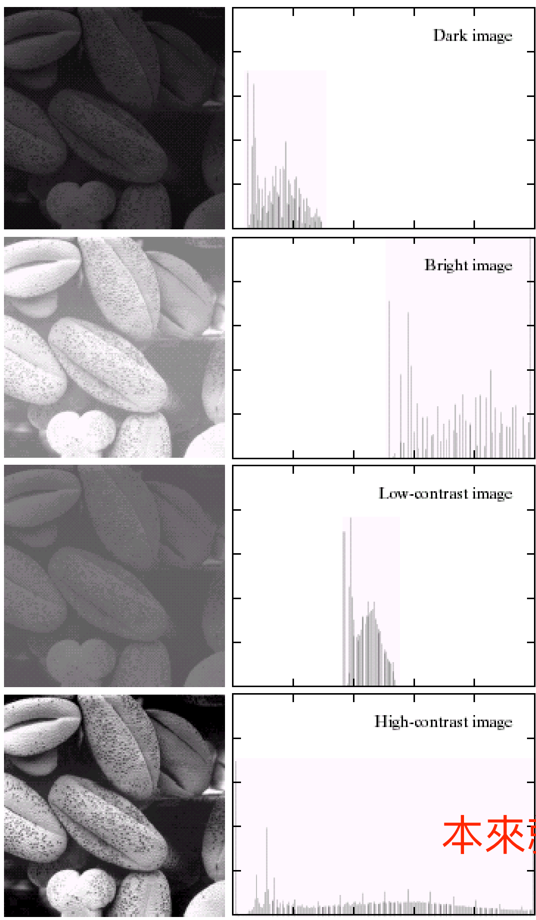
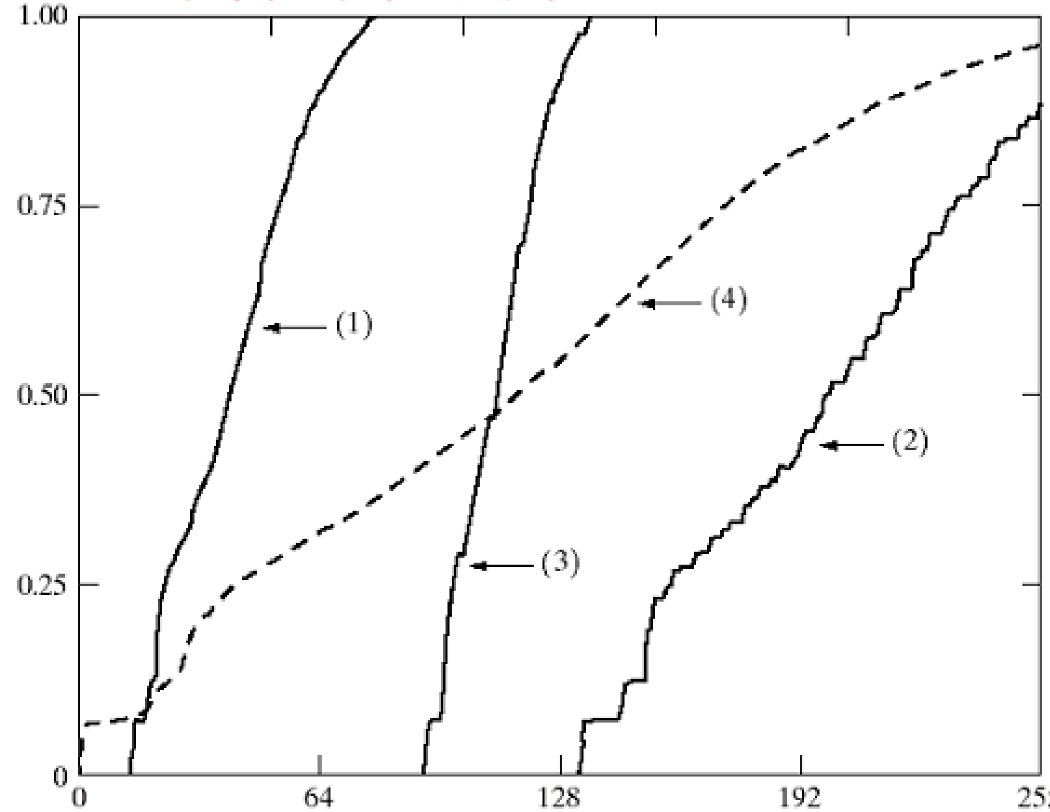


FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

Example: Histogram Equalization for the Four Images



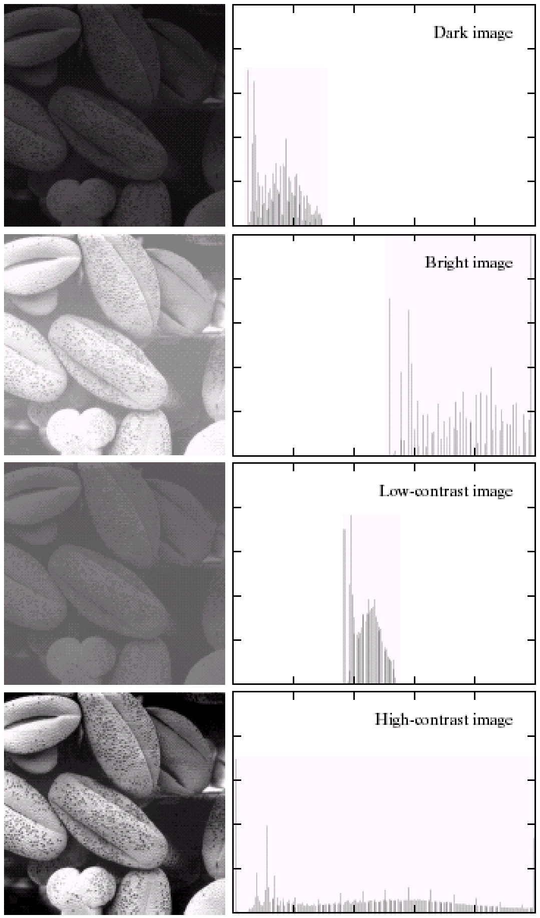
transformation function

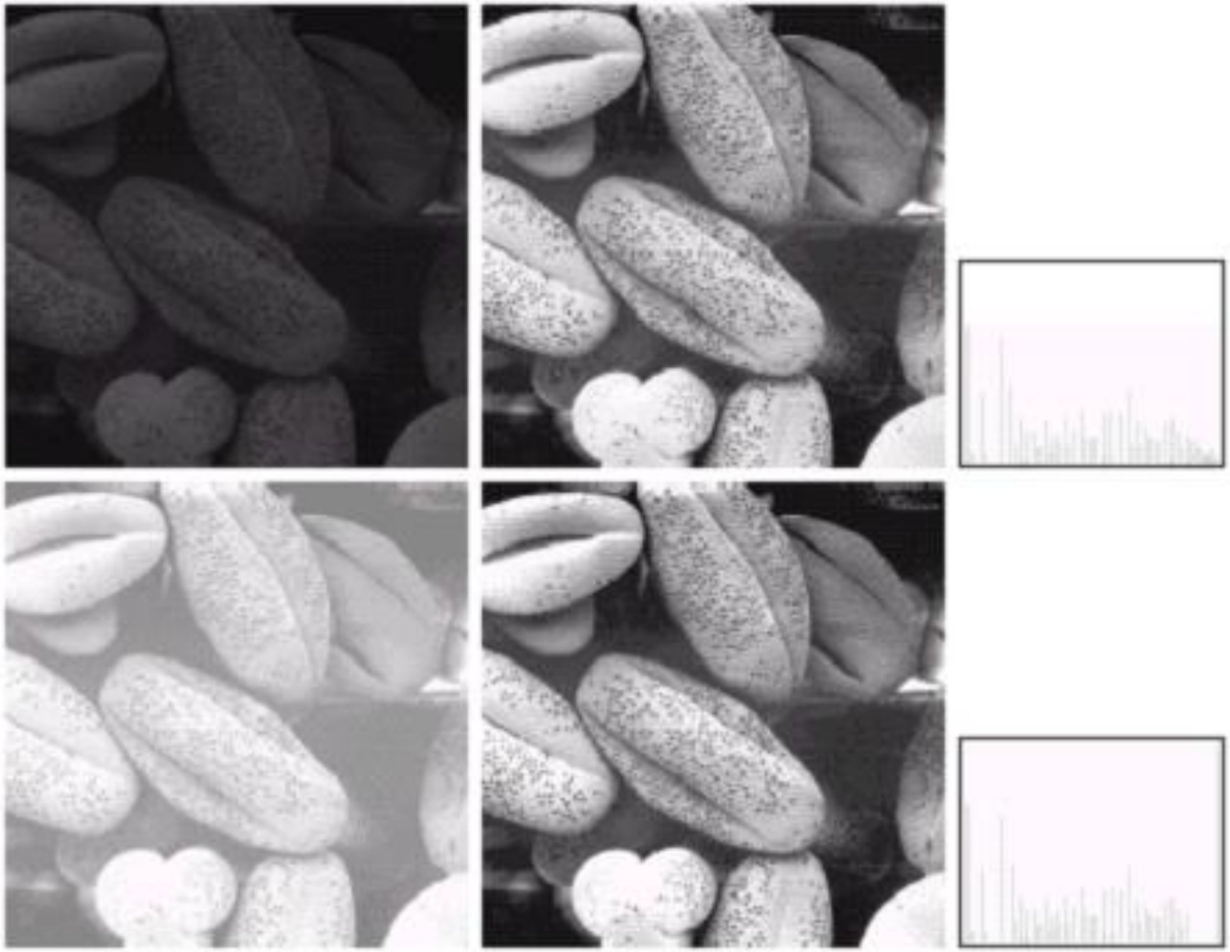


本來就很分佈，因此transformation function 非常的identity

For discrete case, histogram of g is <u>not really uniform</u>

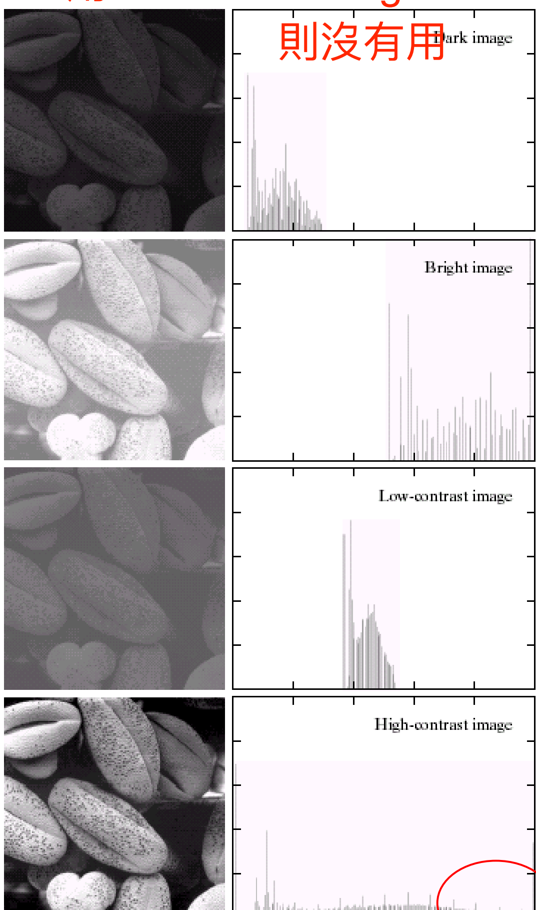
→ simply more flat than the input histogram of f

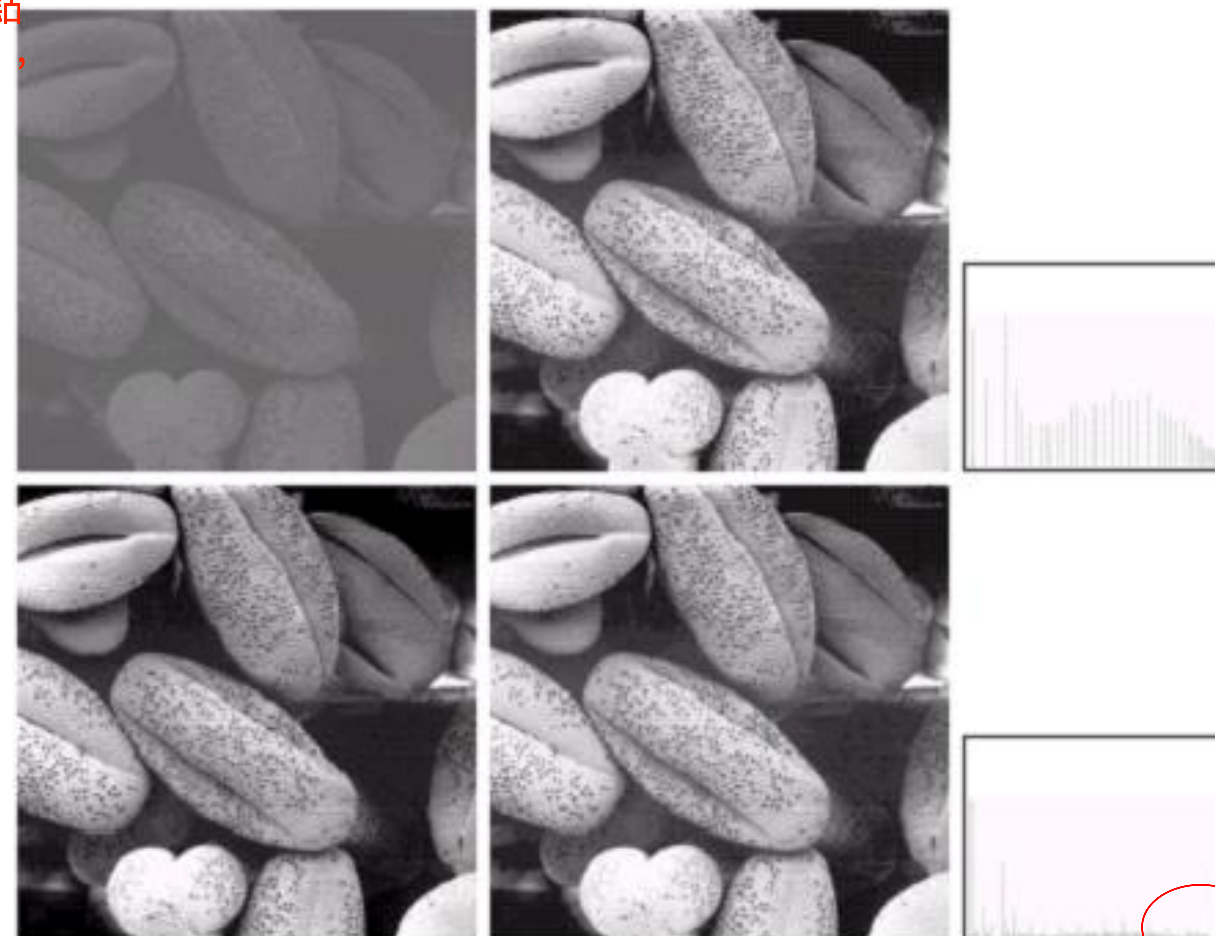




For discrete case, histogram bins are never reduced in amplitude, although they may increase if multiple gray levels map to the same value (thus destroying information)

圖一如果最亮的地方也有一點用full-scale histogram stretch，則沒有用





3.3 Histogram Processing

3.3.1 Histogram Equalization （直方圖等化、均衡化）

3.3.2 Histogram Specification (Matching)

3.3.3 Local Enhancement （局部增強）

3.3.4 Histogram Analysis for Image Thresholding

3.3.2 Histogram Specification (Matching, Shaping, Modification)

- Sometimes, "enhancement" based on a uniform histogram may not be a good approach.

因為Histogram equalization是一個color to color 的transform function

因此限制才會使用histogram specification

Histogram Equalization

雖然uniform但是不好看

- Want to transform an image into one that has a specific histogram ➔ *Histogram Specification*

First, consider the continuous case:

Given an input image $f(x,y)$ and a target histogram $h_z(k)$

Let $P_f(k)$ be the cumulative normalized histogram of $f(x,y)$.

Let $P_z(k)$ be the cumulative normalized histogram obtained from $h_z(k)$, and assume $P_z^{-1}(.)$ exists .

If we let $g(x,y) = P_z^{-1}(P_f(f(x,y))$

一個顏色只能變換成另一種顏色
ex:1/3black to gray.(x)

→ It can be shown that $g(x,y)$ has the pre-specified histogram $h_z(k)$

Proof:

We have $g(x,y) = P_z^{-1}(P_f(f(x,y))$

notation變換

inverse P

$$
\begin{aligned}
P_g(u) &= \Pr\{g \le u\} = \Pr\{P_z^{-1}[P_f(f)] \le u\} \\
&= \Pr\{P_f(f) \le P_z(u)\} = \Pr\{f \le P_f^{-1}[P_z(u)]\} \\
&= P_f[P_f^{-1}[P_z(u)]] = P_z(u)
\end{aligned}
$$

Proved!!

Next, consider the discrete case (approximation):

cumulative

$$s_k = T(r_k) = P_f(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n},$$

normalize -> histogram

$$r_k = k, \quad k = 0, \cdots, L-1$$

$$v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = \sum_{i=0}^{k} \frac{h_z(i)}{n} \approx s_{k'}$$

$$z_k = k, \quad k = 0, \cdots, L-1$$

$$z_k = G^{-1}[P_f(r_k)] = F(r_k), \quad k = 1, \cdots, L-1$$

F : 合併的轉換矩陣

Algorithm for Histogram Specification:
Given an input image f and a pre-specified histogram h_z

1. Compute $s_k = P_f(k)$, $k = 0, \ldots,$ L-1, the cumulative normalized histogram of f.

2. Compute $G(k)$, $k = 0, \ldots,$ L-1, the transformation function, from the given histogram h_z .

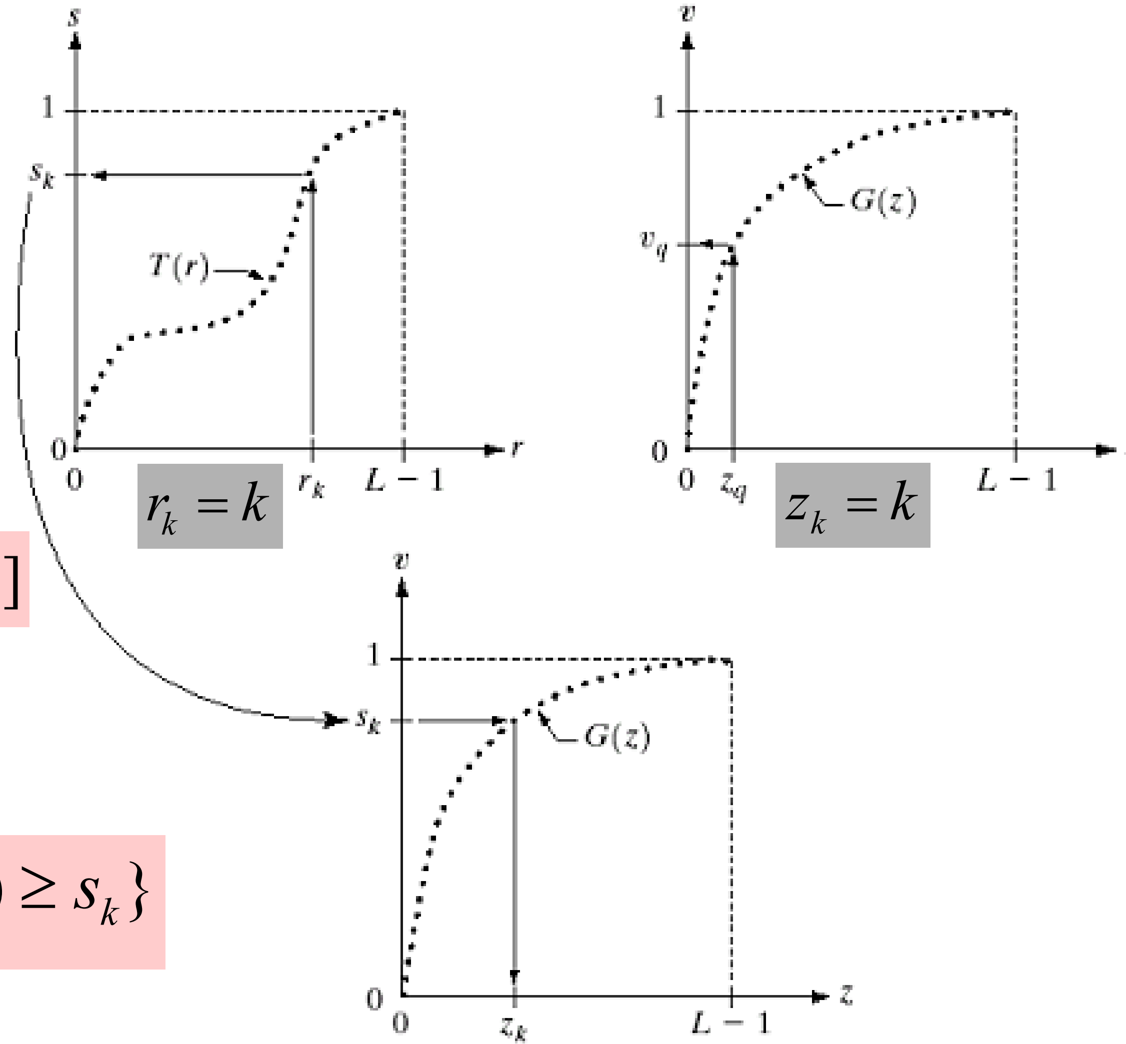
3. Compute $G^{-1}(s_k)$ for each $k = 0, \ldots,$ L-1 using an iterative method (iterate on z), or in effect, directly compute $G^{-1}(P_f(k))$

4. Transform f using $G^{-1}(P_f(k))$.

Graphical Interpretation

$$v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = \sum_{i=0}^{k} \frac{h_z(i)}{n}$$

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n}$$



$r_k = k$

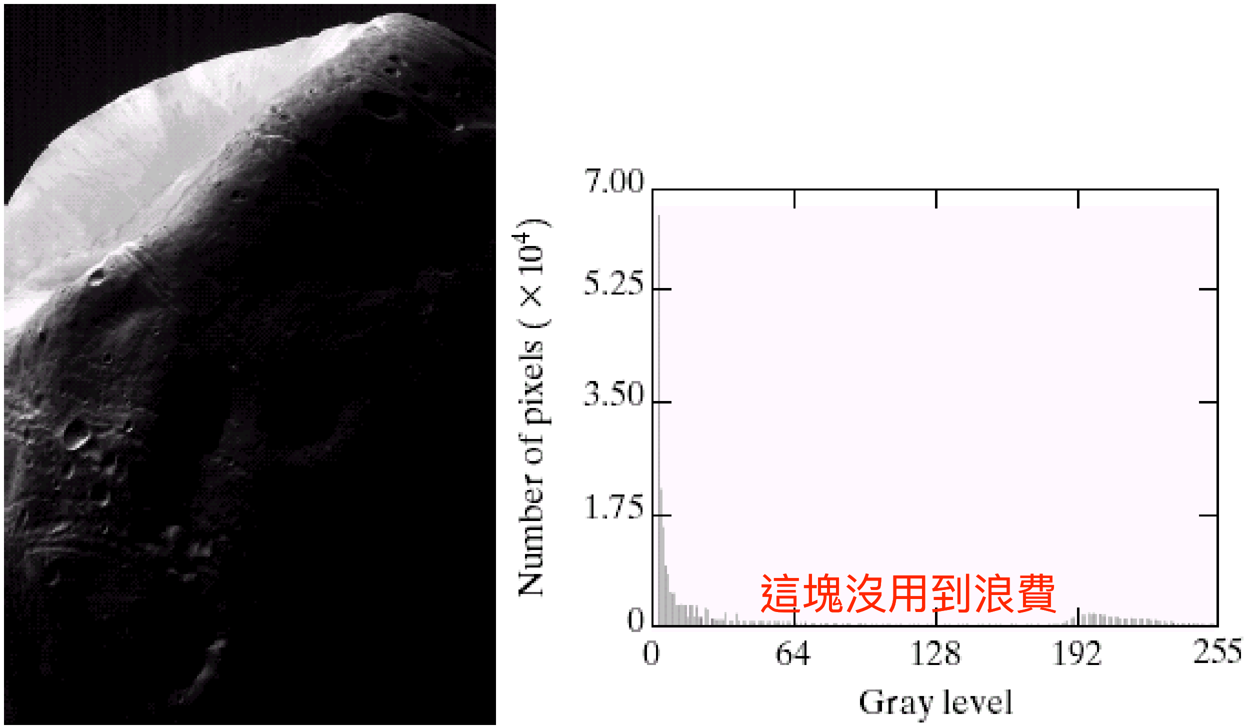
$z_k = k$

$$z_k = G^{-1}[T(r_k)] = G^{-1}[s_k]$$

程式處理方法

$$G^{-1}(s_k) = \min_{z_k}\{z_k : G(z_k) \geq s_k\}$$

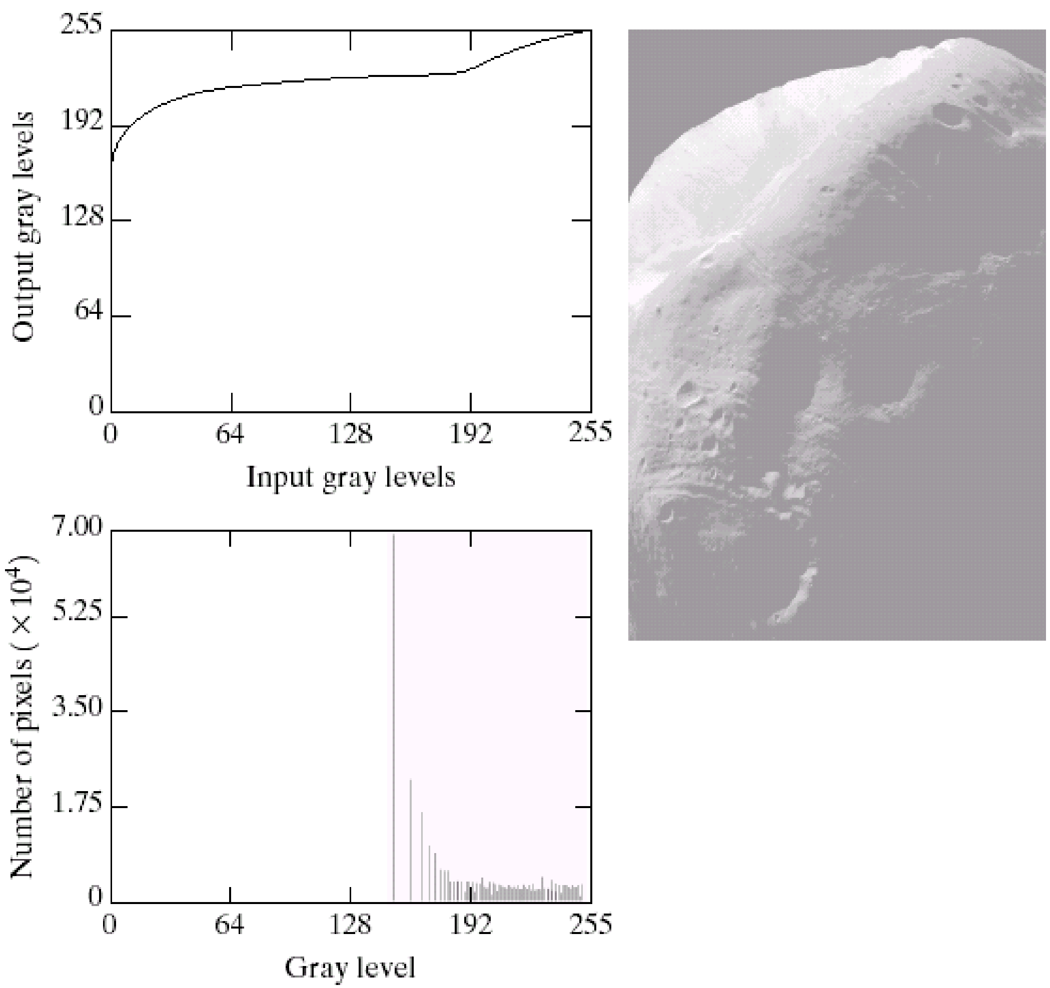
An example:



a b

FIGURE 3.20 (a) Image of the Mars moon Photos taken by NASA's *Mars Global Surveyor*. (b) Histogram. (Original image courtesy of NASA.)

An example: (continued)

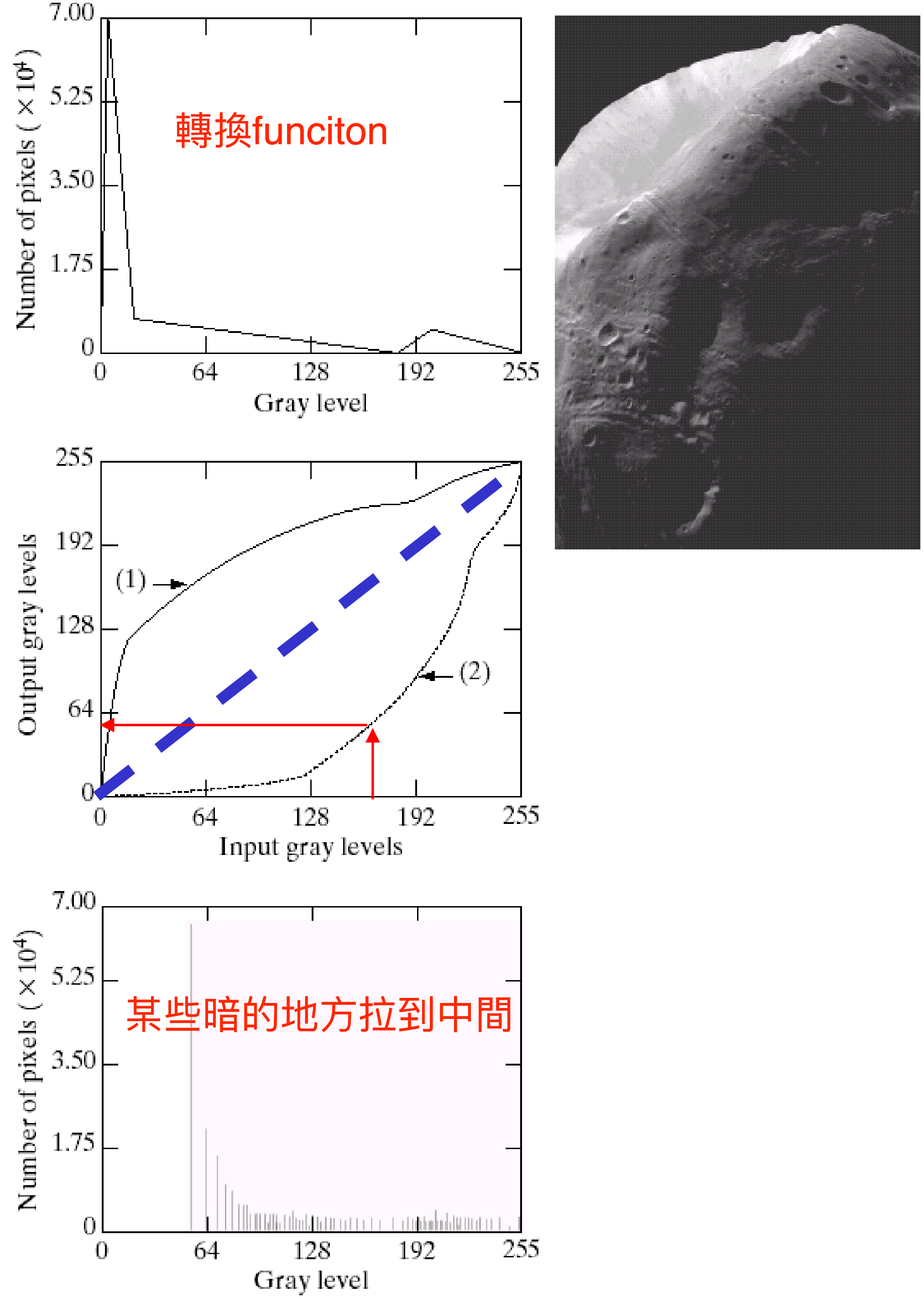


a b
c

FIGURE 3.21
(a) Transformation function for histogram equalization.
(b) Histogram-equalized image (note the washed-out appearance).
(c) Histogram of (b).

a c
b
d

FIGURE 3.22 (a) Specified histogram. (b) Curve (1) is from Eq. (3.3-14), using the histogram in (a); curve (2) was obtained using the iterative procedure in Eq. (3.3-17). (c) Enhanced image using mappings from curve (2). (d) Histogram of (c).



An example: (continued)

Some Comments on Histogram Specification

- When multiple images of the same scene, but taken under slightly different lighting conditions, are to be compared
  -- e.g., visual surveillance, image stitching, stereo, etc.
  為了做匹配，可以把其中一個histogram match另外一張圖
- Get high contrast images by using a specified V-shaped histogram. 接近二元化
- Usually, histogram specification is a trial-and-error process.

3.3 Histogram Processing

3.3.1 Histogram Equalization （直方圖等化、均衡化）

3.3.2 Histogram Specification (Matching)

用一個window找附近的pixel做histogram equalization/specification

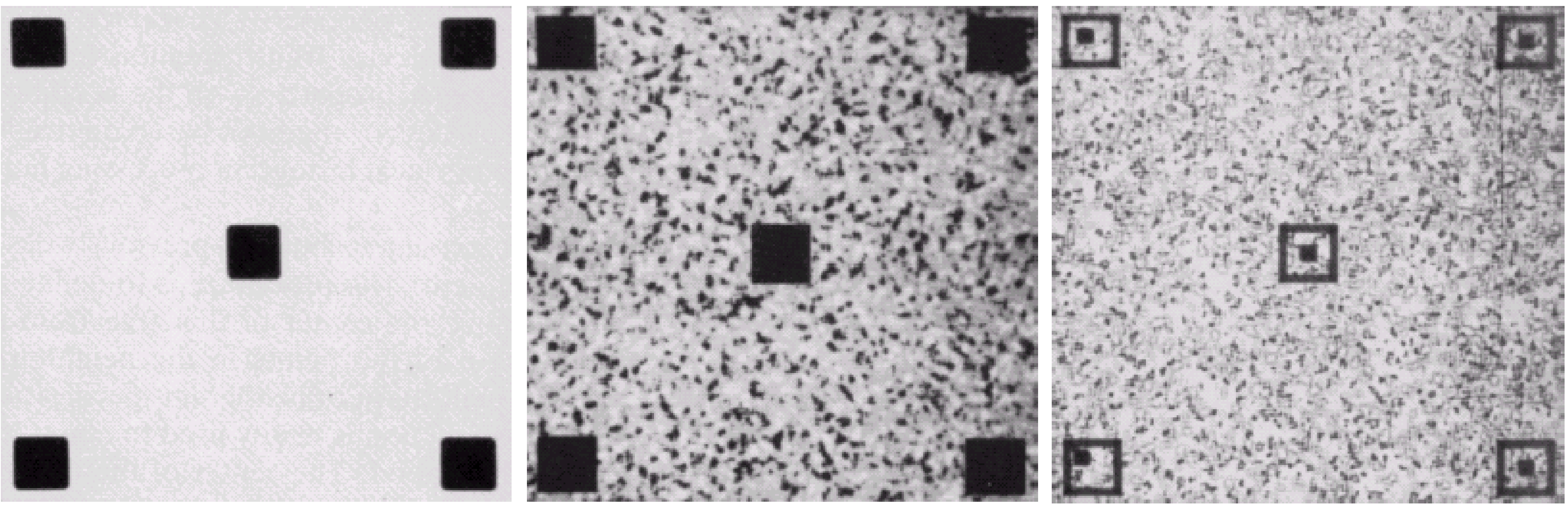
3.3.3 Local Enhancement （局部增強）

3.3.4 Histogram Analysis for Image Thresholding

3.3.3 Local Histogram Processing

- Overlapping windows – smooth, time consuming
- Non-overlapping windows – checkerboard effect, faster

細節被放大，noise也被放大



a b c

Actually, has been slightly blurred

FIGURE 3.23 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization using a 7 × 7 neighborhood about each pixel.

上述histogram processing的概念

T

s -> r

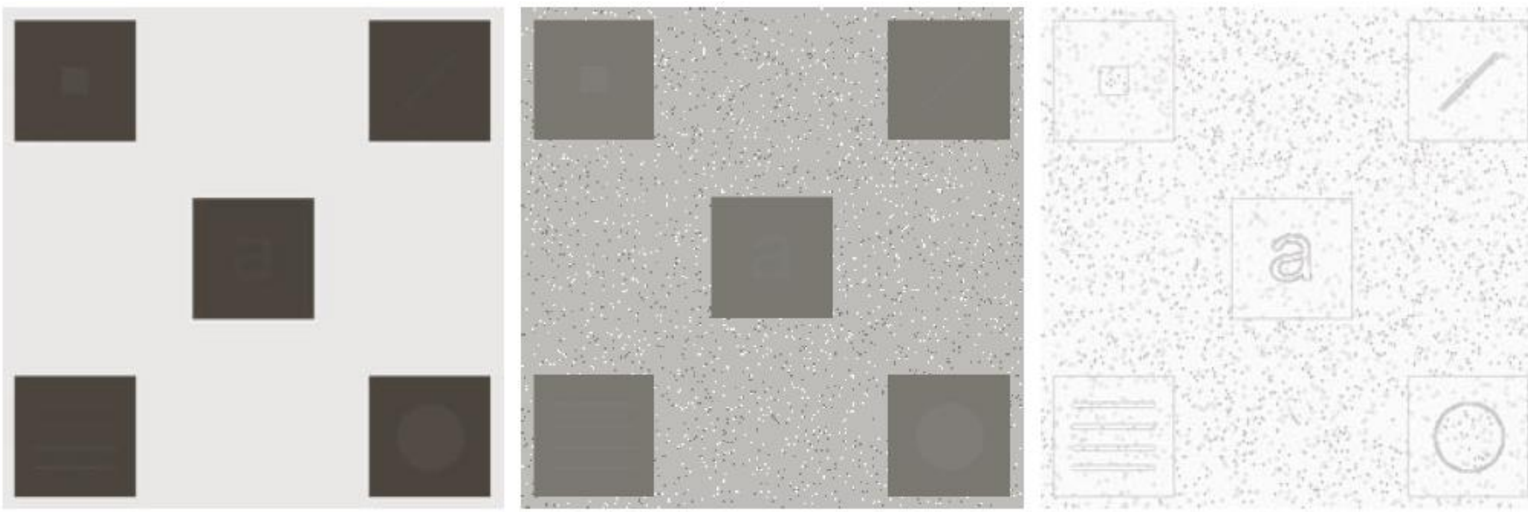


FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization applied to (a), using a neighborhood of size 3 × 3.