第五章 影像還原

- 5.1 影像衰減與復原模型
- 5.2 雜訊模型
- 5.3 雜訊影像復原 - 使用空間域濾波方法
- 5.4 週期性雜訊削減 - 使用頻域濾波方法
- 5.5 線性位置不變性衰減
- 5.6 衰減函式預測

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第五章 影像還原

- 5.7 反向濾波
- 5.8 最小均方差(Wiener)濾波
- 5.9 限制最小平方濾波
- 5.10 幾何平均濾波器
- 5.11 幾何轉換

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5.1 影像衰減與復原模型

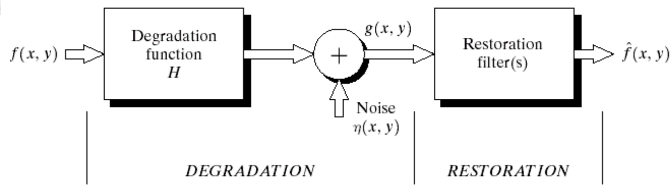

FIGURE 5.1 A model of the image degradation/ restoration process.

- 空間域衰減與復原模型

$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y) \quad (5.1\text{-}1)$$

- 頻域衰減與復原模型

$$G(u,v)=H(u,v)F(u,v)+N(u,v) \quad (5.1\text{-}2)$$

5.2 雜訊模型

- 雜訊來源
  - 影像擷取過程
    - 例如:CCD會受亮度(入光量)及溫度影響
  - 影像傳輸過程
    - 電磁干擾，大氣干擾

5.2 雜訊模型
5.2.1 雜訊的空間及頻率特性

- 白雜訊(White Noise)
  - 雜訊的傅利葉頻譜為固定數
- 僅討論與空間座標無關的雜訊

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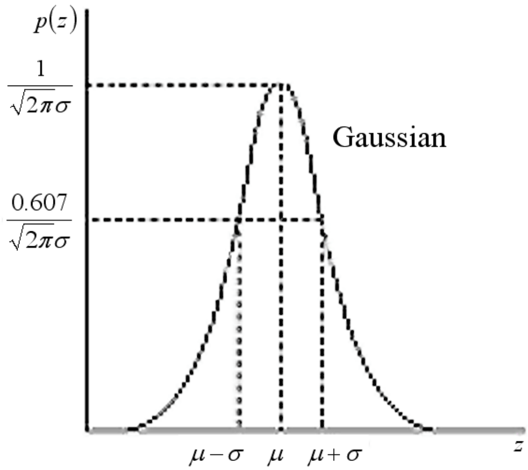
5.2 雜訊模型
5.2.2 幾個重要的雜訊機率密度函式

- Gaussian Noise
  - 機率密度函式

$$p(z)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(z-\mu)^2/2\sigma^2} \qquad (5.2\text{-}1)$$

5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式



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5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式

- ***Rayleigh Noise***
  - ***機率密度函式***

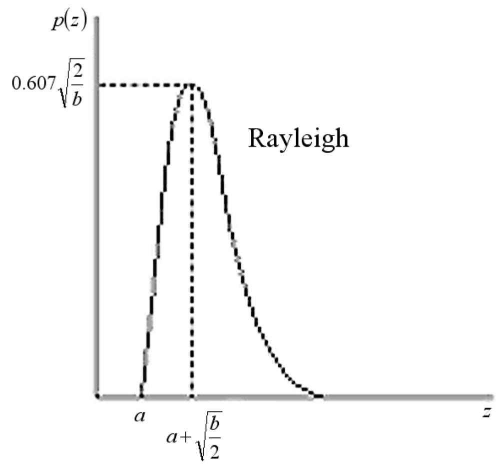
$$p(z)=\begin{cases}\frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z\geq a\\ 0 & \text{for } z<a\end{cases}\qquad (5.2\text{-}2)$$

$$\mu=a+\sqrt{\pi b/4}\qquad (5.2\text{-}3)$$

$$\sigma^2=\frac{b(4-\pi)}{4}\qquad (5.2\text{-}4)$$

5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式



5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式

- **Erlang(Gamma) Noise**
  - 機率密度函式

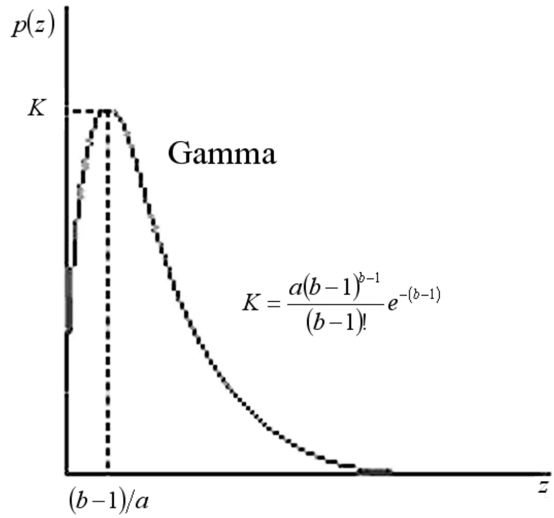
$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad (5.2\text{-}5)$$

$$\mu = \frac{b}{a} \qquad (5.2\text{-}6)$$

$$\sigma^2 = \frac{b}{a^2} \qquad (5.2\text{-}7)$$

5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式



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5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式

- **Exponential Noise**
  - 機率密度函式

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \qquad (5.2\text{-}8)$$

$$\mu = \frac{1}{a} \qquad (5.2\text{-}9)$$

$$\sigma^2 = \frac{1}{a^2} \qquad (5.2\text{-}10)$$

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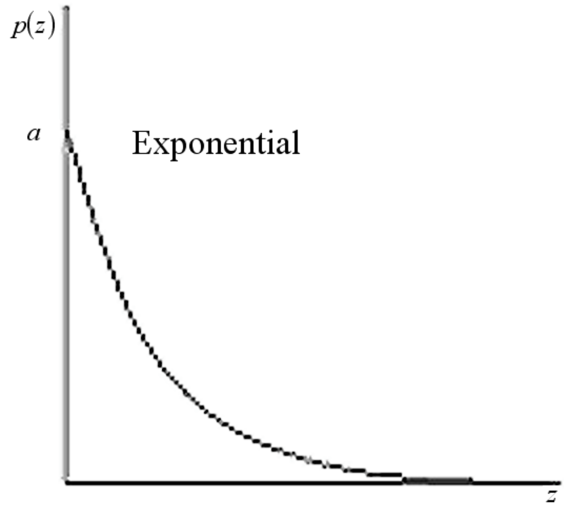
5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式



5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式

- ***Uniform Noise***
  - ***機率密度函式***

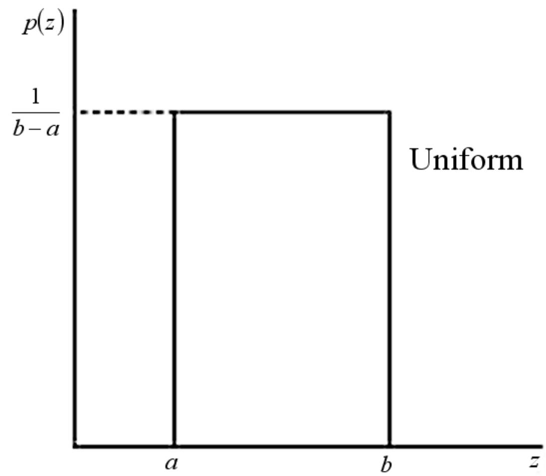
$$p(z)=\begin{cases}\dfrac{1}{b-a} & \text{if } a\le z\le b \\ 0 & \text{otherwise}\end{cases} \qquad (5.2\text{-}11)$$

$$\mu=\frac{a+b}{2} \qquad (5.2\text{-}12)$$

$$\sigma^2=\frac{(b-a)^2}{12} \qquad (5.2\text{-}13)$$

5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式



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5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式

- **Impulse (salt-and-pepper) Noise**
  - 機率密度函式

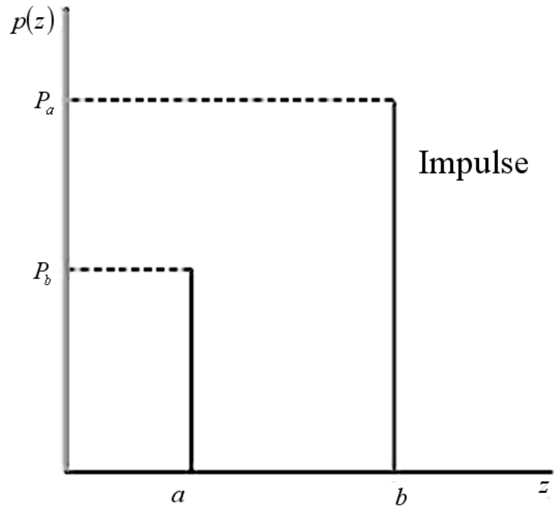
$$p(z)=\begin{cases} P_a & \text{for } z=a \\ P_b & \text{for } z=b \\ 0 & \text{otherwise} \end{cases} \qquad (5.2\text{-}14)$$

$$\mu=\frac{a+b}{2} \qquad (5.2\text{-}12)$$

$$\sigma^2=\frac{(b-a)^2}{12} \qquad (5.2\text{-}13)$$

5.2 雜訊模型

5.2.2 幾個重要的雜訊機率密度函式



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5.2 雜訊模型
5.2.2 幾個重要的雜訊機率密度函式

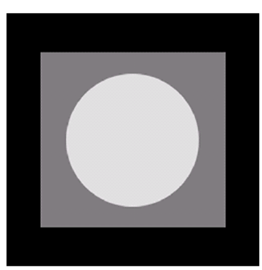

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

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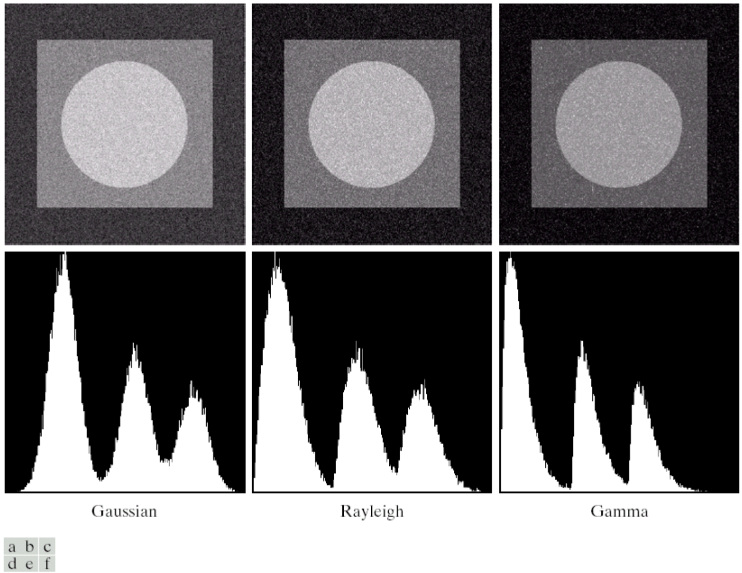

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.

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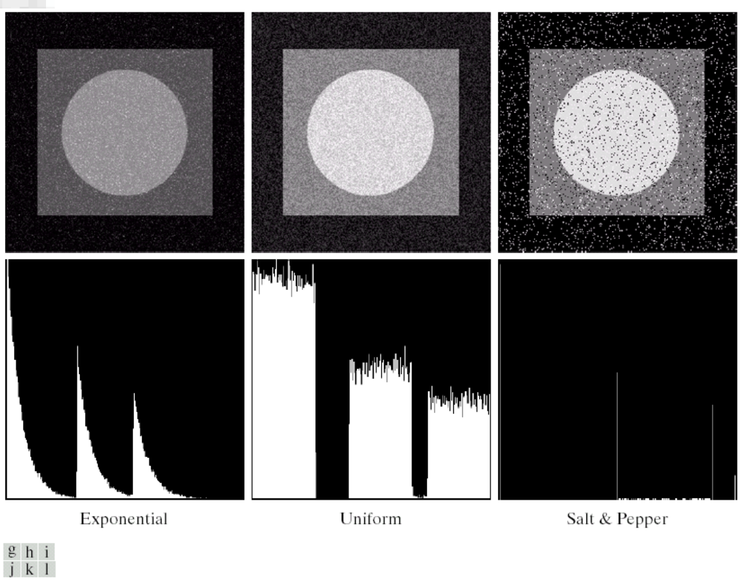

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

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5.2 雜訊模型

5.2.3 週期性雜訊

- 成因
  - 擷取影像時受電機或電磁干擾
  - 本章中唯一一個與空間相關的例子
  - 通常可經由頻域濾波方法降低

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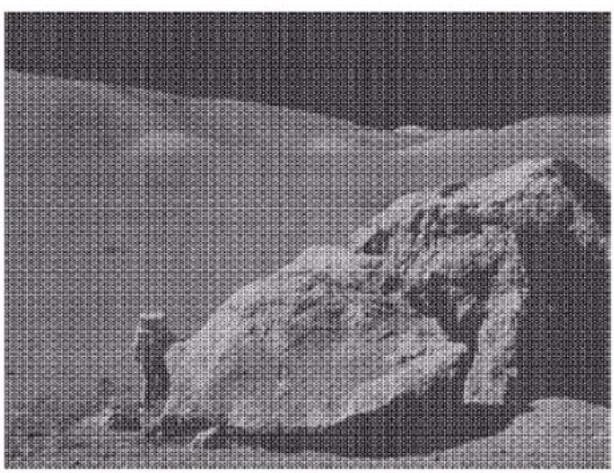
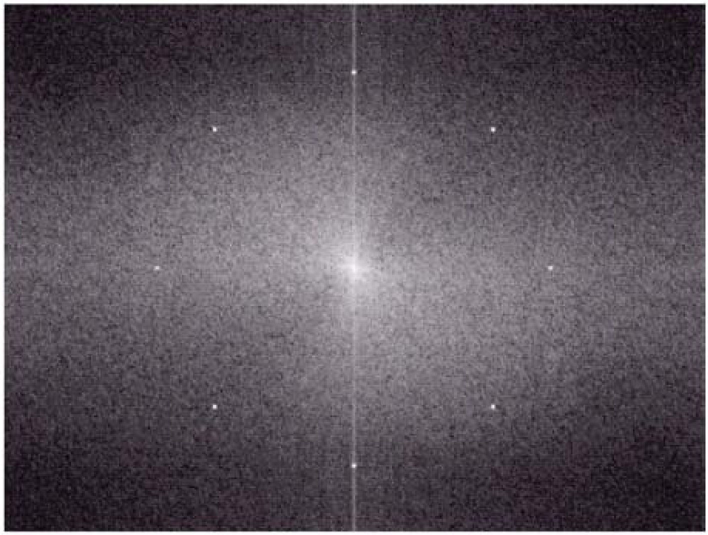
5.2 雜訊模型

5.2.3 週期性雜訊



影像受到正弦波干擾

5.2 雜訊模型
5.2.3 週期性雜訊



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5.2 雜訊模型
5.2.4 雜訊參數之估算

- 如何得知影像雜訊參數
  - 由感測器規格得知
  - 由影像像素分佈情形估算

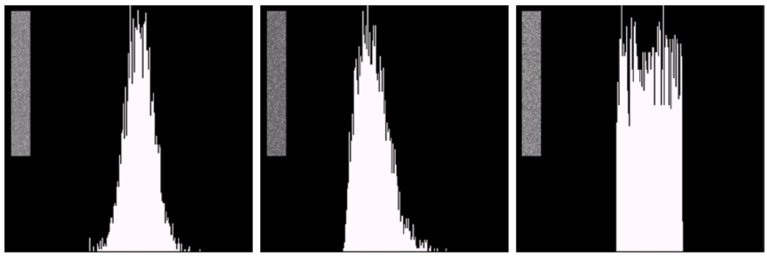
$$\mu = \sum_{z_i \in S} z_i p(z_i) \qquad (5.2\text{-}15)$$

$$\sigma^2 = \sum_{z_i \in S} (z_i - \mu)^2 p(z_i) \qquad (5.2\text{-}16)$$

z是像素值, p(z)是正規化後histogram的值

5.2 雜訊模型
5.2.4 雜訊參數之估算

- 例子
  - 圖5.4計算後的結果



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

5.3 雜訊影像復原 - 使用空間域濾波方法

- 假設造成影像衰減的唯一因素是雜訊
  - 5.1-2變成

$$g(x,y)=f(x,y)+\eta(x,y) \qquad (5.3\text{-}1)$$

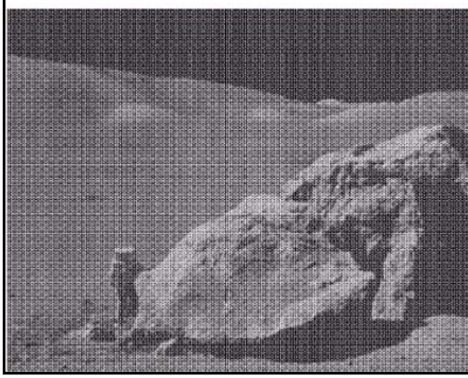
  - 且

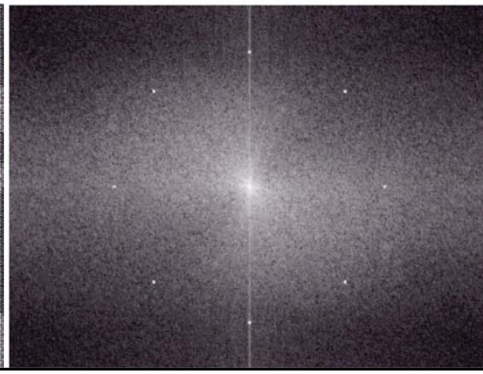
$$G(u,v)=F(u,v)+N(u,v) \qquad (5.3\text{-}2)$$

  - 不知道雜訊公式, 無法去除雜訊

5.3 雜訊影像復原 - 使用空間域濾波方法

- 一個例外的情形
  - 週期性雜訊的N(u,v)可由G(u,v)估算出來





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5.3 雜訊影像復原 - 使用空間域濾波方法

通常無法很容易估算出 N(u,v)

使用空間濾波器

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)

- 算術平均濾波器(Arithmetic mean filter)

$$\hat{f}(x,y)=\frac{1}{mn}\sum_{(s,t)\in S_{x,y}} g(s,t) \qquad (5.3\text{-}3)$$

$$\frac{1}{9}\times\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

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5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)

- 幾何平均濾波器(Geometric mean filter)

$$\hat{f}(x,y)=\left[\prod_{(s,t)\in S_{x,y}} g(s,t)\right]^{\frac{1}{mn}} \qquad (5.3\text{-}4)$$

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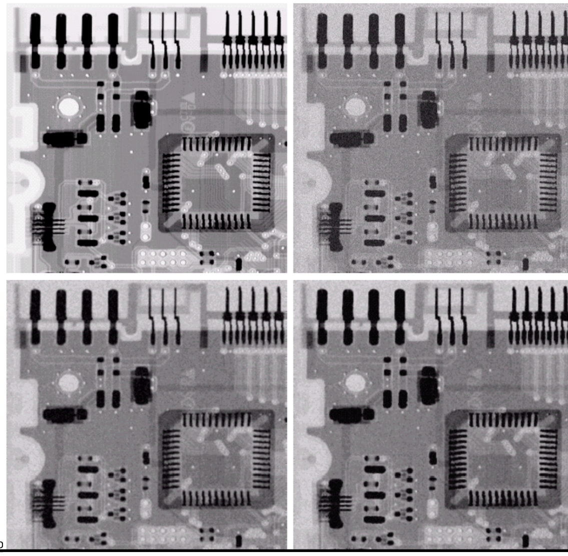
5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3 × 3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

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5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)

- 調和平均濾波器(Harmonic mean filter)

$$\hat{f}(x,y)=\frac{mn}{\sum_{(s,t)\in S_{x,y}}\frac{1}{g(s,t)}} \qquad (5.3\text{-}5)$$

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)

- 反調和平均濾波器(Contraharmonic mean filter)

$$\hat{f}(x,y)=\frac{\sum_{(s,t)\in S_{x,y}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{x,y}} g(s,t)^{Q}} \qquad (5.3\text{-}6)$$

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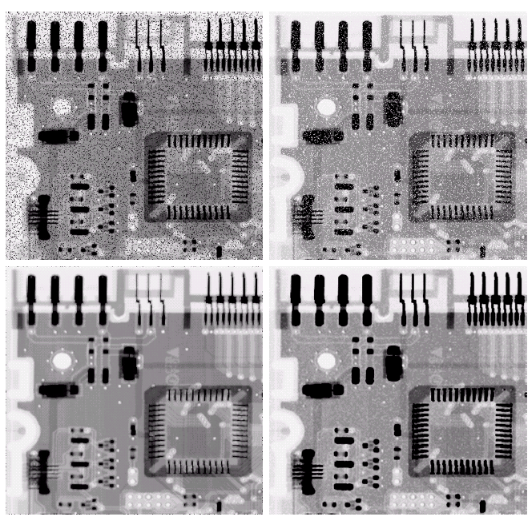
5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)

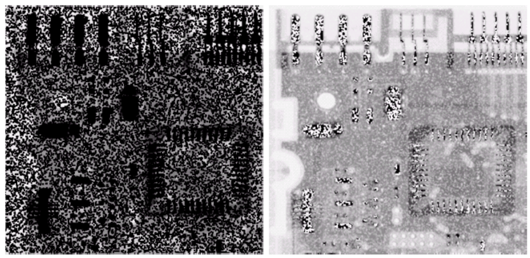
a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3 × 3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.1 平均濾波器(Mean Filters)



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

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5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 統計次序濾波器(Order-Statistics Filters)

- **中間值濾波器(Median filter)**

$$\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{median}\{g(s,t)\} \qquad (5.3\text{-}7)$$

13	28	33
15	76	32
17	30	42

={13,15,17,28,30,32,33,42,76}

median{13,15,17,28,30,32,33,42,76}=30

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 順序統計濾波器(Order-Statistics Filters)

- **最大值濾波器(Max filter)**

$$\hat{f}(x,y) = \max_{(s,t)\in S_{xy}} \{g(s,t)\} \qquad (5.3\text{-}8)$$

13	28	33
15	76	32
17	30	42

$=\{13,15,17,28,30,32,33,42,76\}$

$$\max\{13,15,17,28,30,32,33,42,76\}=76$$

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 順序統計濾波器(Order-Statistics Filters)

- **最小值濾波器(Min filter)**

$$\hat{f}(x,y) = \min_{(s,t)\in S_{xy}} \{g(s,t)\} \qquad (5.3\text{-}9)$$

13	28	33
15	76	32
17	30	42

$=\{13,15,17,28,30,32,33,42,76\}$

$$\min\{13,15,17,28,30,32,33,42,76\}=13$$

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 順序統計濾波器(Order-Statistics Filters)

- 中點濾波器(Midpoint filter)

$$\hat{f}(x,y)=\frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\}+\min_{(s,t)\in S_{xy}}\{g(s,t)\}\right] \qquad (5.3\text{-}10)$$

13	28	33
15	76	32
17	30	42

={13,15,17,28,30,32,33,42,76}

min{13,15,17,28,30,32,33,42,76}=(76+13)/2=44.5

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 順序統計濾波器(Order-Statistics Filters)

- Alpha刪減平均濾波器(Alpha-trimmed mean filter)

$$\hat{f}(x,y)=\frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t) \qquad (5.3\text{-}11)$$

砍掉最大的d個及最小的d個值,剩下的做平均

13	28	33
15	76	32
17	30	42

={13,15,17,28,30,32,33,42,76}

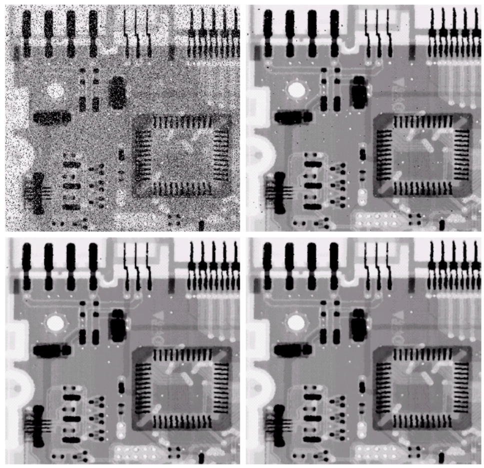
if d is 2

{13,15,17,28,30,32,33,42,76}=26

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 順序統計濾波器(Order-Statistics Filters)

Example for Median Filter



a b
c d

FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3 × 3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.

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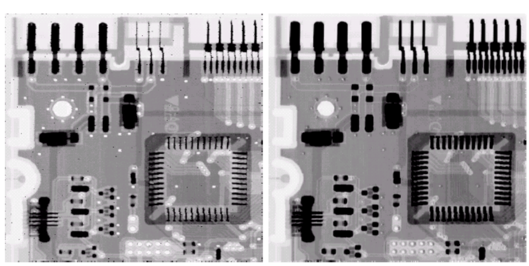
5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.2 順序統計濾波器(Order-Statistics Filters)

Example for Max and Min Filters



a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

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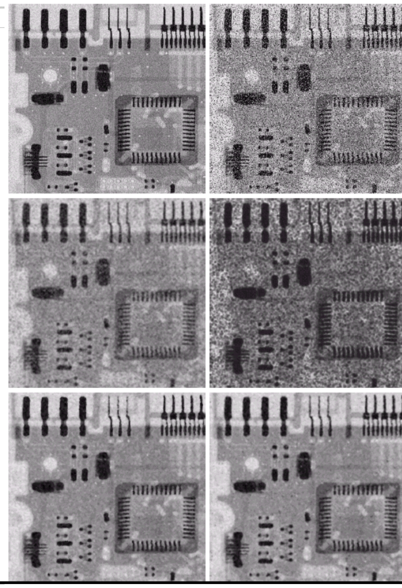
FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5×5: (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.



5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.3 適應性濾波器(Adaptive Filters)

- 適應性區域雜訊削減濾波器
  - (Adaptive, local noise reduction filter)
    - 1.假如雜訊變異數為0
      - 則 f(x,y) 等於 g(x,y)
    - 2.區域變異數遠大於雜訊變異數,
      - 則傳回接近 g(x,y) 的值
      - 區域變異數很大的影像區塊通常是邊緣
    - 3.假如區域變異數與雜訊變異數差不多
      - 則傳回算術平均值

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.3 適應性濾波器(Adaptive Filters)

$$\hat{f}(x,y)=g(x,y)-\frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x,y)-m_L\right] \qquad (5.3\text{-}12)$$

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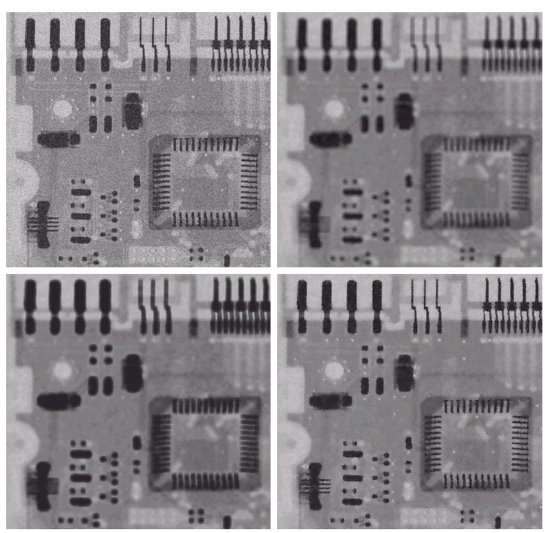
5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.3 適應性濾波器(Adaptive Filters)



a b
c d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7 × 7.

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.3 適應性濾波器(Adaptive Filters)

- **適應性中間值濾波器 (Adaptive median filter)**

Level A: $A1 = z_{med} - z_{min}$

$A2 = z_{med} - z_{max}$

不是最小及最大值 If $A1 > 0$ AND $A2 < 0$, Go to level B

Else increase the window size

If window size $\leq S_{max}$ repeat level A

Else output z_{xy}

Level B: $B1 = z_{xy} - z_{min}$

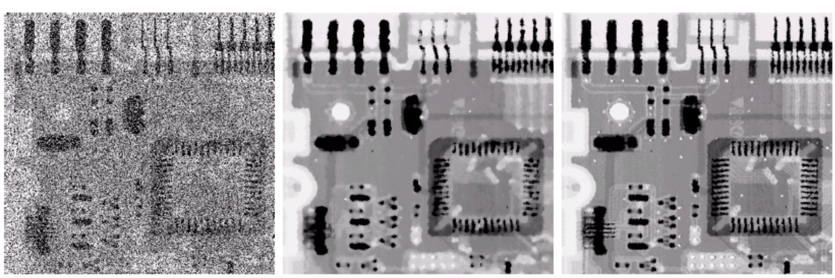
$B2 = z_{xy} - z_{max}$

不是最小及最大值 If $B1 > 0$ AND $B2 < 0$, output z_{xy}

Else output z_{med}

5.3 雜訊影像復原 - 使用空間域濾波方法

5.3.3 適應性濾波器(Adaptive Filters)



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters

- *An ideal bandreject filter*

$$H(u,v)=\begin{cases}1 & \text{if } D(u,v)<D_0-\frac{W}{2}\\ 0 & \text{if } D_0-\frac{W}{2}\le D(u,v)<D_0+\frac{W}{2}\\ 1 & \text{if } D(u,v)>D_0+\frac{W}{2}\end{cases} \qquad (5.4\text{-}1)$$

D0代表Band圓心, w代表Band寬度

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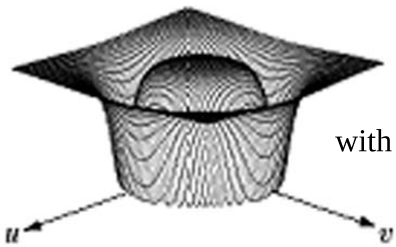
5.4 週期性雜訊削減 - 使用頻域濾波方法
5.4.1 Bandreject Filters

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters

- **Butterworth**

$$H(u,v)=\frac{1}{1+\left[\frac{D(u,v)W}{D^2(u,v)-D_0^2}\right]^{2n}} \qquad (5.4\text{-}2)$$



with Order 1

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5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters

- **Gaussian**

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D^2(u,v)-D_0^2}{D(u,v)W}\right]^2} \qquad (5.4\text{-}3)$$

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters

- *An Example*

FIGURE 5.16

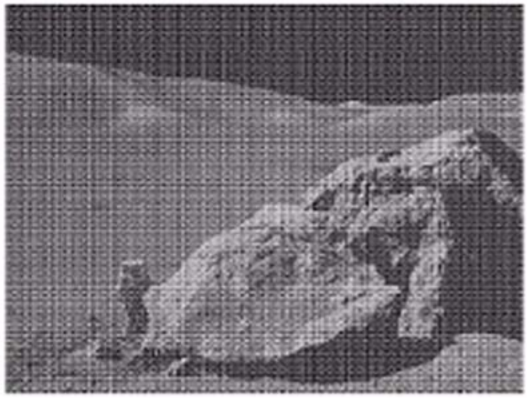


Image with sinusoidal noise

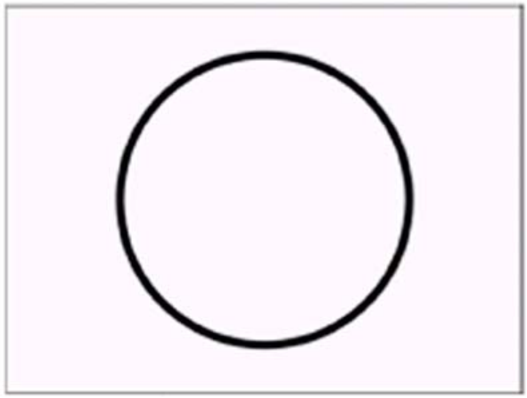
5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters

Spectrum

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters



Filter

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5.4 週期性雜訊削減 - 使用頻域濾波方法
5.4.1 Bandreject Filters

Filtered Image

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5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.1 Bandreject Filters

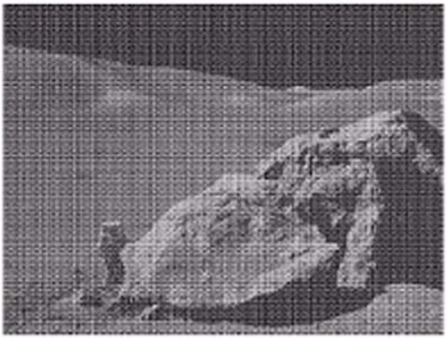


Image with sinusoidal noise

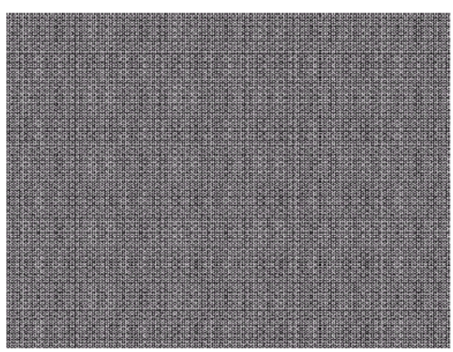
Filtered Image

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.2 Bandpass Filters

$$H_{bp}(u,v) = 1 - H_{br}(u,v) \qquad (5.4\text{-}4)$$

FIGURE 5.17 Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.



5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

- **An ideal notch filter**

$$H(u,v)=\begin{cases}0 & \text{if } D_1(u,v)\le D_0 \text{ or } D_2(u,v)\le D_0\\ 1 & \text{otherwise}\end{cases} \qquad (5.4\text{-}5)$$

$$D_1(u,v)=\left[(u-M/2-u_0)^2+(v-B/2-v_0)^2\right] \qquad (5.4\text{-}6)$$

$$D_2(u,v)=\left[(u-M/2-u_0)^2+(v-B/2-v_0)^2\right] \qquad (5.4\text{-}7)$$

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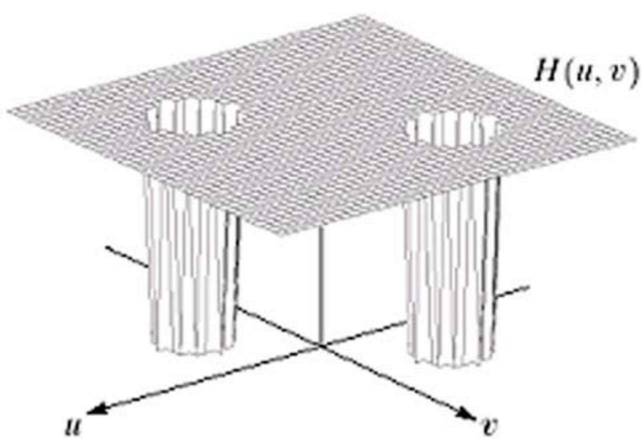
5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters



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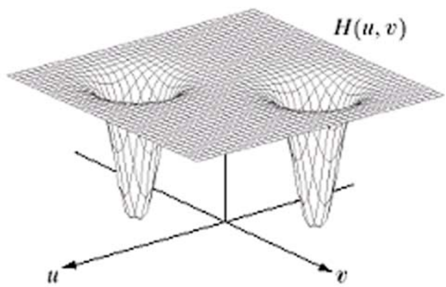
5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

- **Butterworth filter**

$$H(u,v)=\frac{1}{1+\left[\frac{D_0^2}{D_1(u,v)D_2(u,v)}\right]^n} \qquad (5.4\text{-}8)$$



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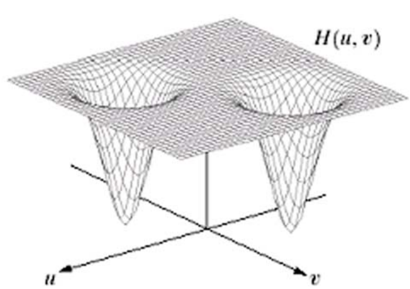
5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

- **Gaussian**

$$H(u,v)=1-e^{-\frac{1}{2}\left[\frac{D_1(u,v)D_2(u,v)}{D_0^2}\right]} \qquad (5.4\text{-}9)$$



5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

- 反向過濾器

$$H_{np}(u,v) = 1 - H_{nr}(u,v) \qquad (5.4\text{-}10)$$

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

- An Example

Image with noise

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

Spectrum

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5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

Filter

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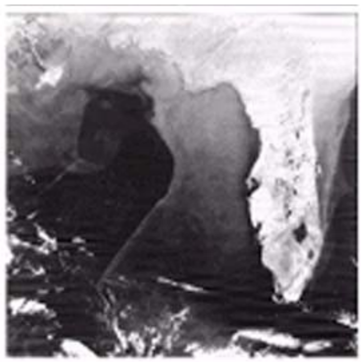
5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters



Filtered Image

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5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

Image with noise

Filtered Image

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.3 Notch Filters

Noise Image

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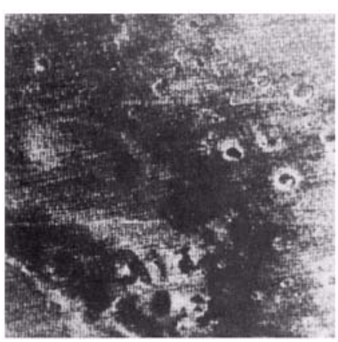
5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

- 雜訊通常含多個干擾成分



Mariner 6 太空梭拍攝

Spectrum

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

- **雜訊公式**

Frequency Domain

$$N(u,v)=H(u,v)G(u,v) \qquad (5.4\text{-}11)$$

Spacial Domain

$$\eta(x,y)=\mathfrak{I}^{-1}\{H(u,v)G(u,v)\} \qquad (5.4\text{-}12)$$

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5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

- **去除雜訊**

$$\hat{f}(x,y)=g(x,y)-\underbrace{\eta(x,y)}_{\text{雜訊}}$$

雜訊未知?

$$\hat{f}(x,y)=g(x,y)-\boxed{w(x,y)}\eta(x,y) \qquad (5.4\text{-}13)$$

加入Weighting function *w(x,y)*
雜訊處*w(x,y)*大
否則*w(x,y)*小

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

使用區域統計資訊決定w(x,y)值

變異數

$$\sigma^2(x,y)=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\left[\hat{f}(x+s,y+t)-\boxed{\bar{\hat{f}}(x,y)}\right]^2 \quad (5.4\text{-}14)$$

$$\bar{\hat{f}}(x,y)=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\hat{f}(x+s,y+t) \quad (5.4\text{-}15)$$

去除雜訊目的在將變異數最小化

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5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

$$\hat{f}(x,y)=g(x,y)-w(x,y)\eta(x,y) \quad (5.4\text{-}13)$$

$$\sigma^2(x,y)=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\left[\hat{f}(x+s,y+t)-\bar{\hat{f}}(x,y)\right]^2 \quad (5.4\text{-}14)$$

$$\begin{aligned}\sigma^2(x,y)=\frac{1}{(2a+1)(2b+1)}\sum_{s=-a}^{a}\sum_{t=-b}^{b}\{[g(x+s,y+t)\\ -w(x+s,y+t)\eta(x+s,y+t)]\\ -[\bar{g}(x,y)-\overline{w(x,y)\eta(x,y)}]\}^2\end{aligned} \quad (5.4\text{-}16)$$

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

假設w(x,y)在區域內的值都一樣

$$w(x+s, y+t) = w(x,y) \qquad (5.4\text{-}17)$$

則

$$\overline{w(x,y)\eta(x,y)} = w(x,y)\overline{\eta}(x,y) \qquad (5.4\text{-}18)$$

$$\begin{aligned}\sigma^2(x,y) = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{a}\sum_{t=-b}^{b} \{ & [g(x+s,y+t) \\ & - w(x,y)\eta(x+s,y+t)] \\ & - [\overline{g}(x,y) - w(x,y)\overline{\eta}(x,y)]\}^2\end{aligned} \qquad (5.4\text{-}19)$$

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

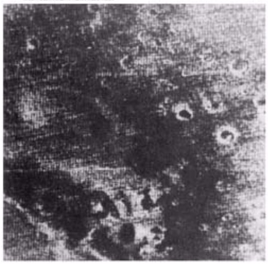
最小化變異數

$$\frac{\partial \sigma^2(x,y)}{\partial \mathrm{w}(\mathrm{x},\mathrm{y})} \qquad (5.4\text{-}20)$$

$$w(x,y) = \frac{\overline{\mathrm{g}(\mathrm{x},\mathrm{y})\eta(x,y)} - \overline{g}(x,y)\overline{\eta}(x,y)}{\overline{\eta^2}(x,y) - \overline{\eta}^2(x,y)} \qquad (5.4\text{-}21)$$

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters



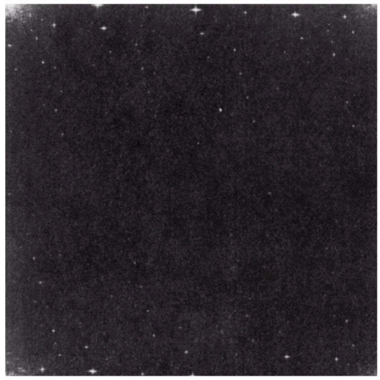
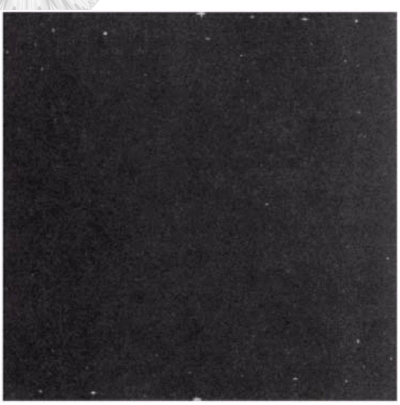


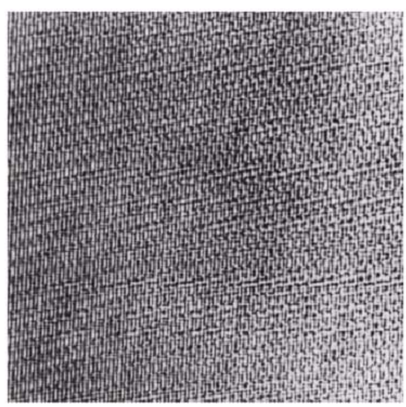
FIGURE 5.21 Fourier spectrum (without shifting) of the image shown in Fig. 5.20(a). (Courtesy of NASA.)

Spectrum

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters





N(u,v) $\eta(x, y)$

FIGURE 5.22 (a) Fourier spectrum of $N(u, v)$, and (b) corresponding noise interference pattern $\eta(x, y)$. (Courtesy of NASA.)

5.4 週期性雜訊削減 - 使用頻域濾波方法

5.4.4 Optimum Notch Filters

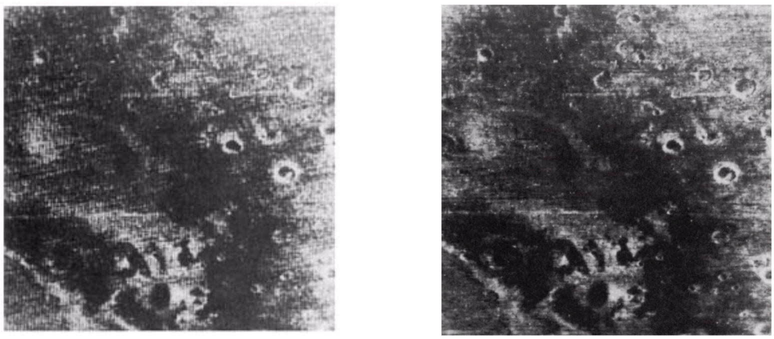


FIGURE 5.23 Processed image. (Courtesy of NASA.)

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5.5 線性位置不變性衰減

輸入影像衰減公式

$$g(\mathrm{x},\mathrm{y}) = H[f(x,y)] + \eta(\mathrm{x},\mathrm{y}) \quad (5.5\text{-}1)$$

假設雜訊為0

線性(Linear)

$$H[af_1(x,y) + bf_2(x,y)] = \mathrm{a}H[f_1(x,y)] + bH[f_2(x,y)] \quad (5.5\text{-}2)$$

相加性(Additivity)

$$H[f_1(x,y) + f_2(x,y)] = H[f_1(x,y)] + H[f_2(x,y)] \quad (5.5\text{-}3)$$

齊次性(Homogeneity)

$$H[af_1(x,y)] = aH[f_1(x,y)] \quad (5.5\text{-}4)$$

5.5 線性位置不變性衰減

位置/空間不變性(Position/Space invariant)

$$H[f_1(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta) \qquad (5.5\text{-}5)$$

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5.5 線性位置不變性衰減

連續函數之衰減模型

以脈衝函式表示f(x,y)

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\, d\alpha\, d\beta \qquad (5.5\text{-}6)$$

假設雜訊為0,則f(x,y)的衰減模型為

$$g(x,y) = H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha, y-\beta)\, d\alpha\, d\beta\right] \qquad (5.5\text{-}7)$$

假設H是線性函式

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\delta(x-\alpha, y-\beta)]\, d\alpha\, d\beta \qquad (5.5\text{-}8)$$

5.5 線性位置不變性衰減

因為 $f(\alpha,\beta)$ 與 x,y 無關

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)H[\delta(x-\alpha,y-\beta)]\,d\alpha\,d\beta \qquad (5.5\text{-}9)$$

令

$$h(x,\alpha,y,\beta)=H[\delta(x-\alpha,y-\beta)] \qquad (5.5\text{-}10)$$

$h(x,\alpha,y,\beta)$稱為脈衝響應,代表H函示對單位1之脈衝的響應,通常也稱為點分佈函數(Point Spread Function;PSF)

將5.5-10代入5.5-9

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)\,d\alpha\,d\beta \qquad (5.5\text{-}11)$$

5.5 線性位置不變性衰減

若H具位置不變性

$$H[\delta(x-\alpha,y-\beta)]=h(x-\alpha,y-\beta) \qquad (5.5\text{-}12)$$

則

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)\,d\alpha\,d\beta \qquad (5.5\text{-}11)$$

$$\downarrow$$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)\,d\alpha\,d\beta \qquad (5.5\text{-}13)$$

5.5 線性位置不變性衰減

加入雜訊

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x,\alpha,y,\beta)\ d\alpha\, d\beta+\eta(x,y) \qquad (5.5\text{-}14)$$

$$g(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)\ d\alpha\, d\beta+\eta(x,y) \qquad (5.5\text{-}15)$$

捲積表示法

$$g(x,y)=h(x,y)*f(x,y)+\eta(x,y) \qquad (5.5\text{-}16)$$

$$G(u,v)=H(u,v)F(u,v)+N(u,v) \qquad (5.5\text{-}17)$$

5.6 衰減函數之估算
5.6.1 藉由觀察影像估算

取得一張任意影像要如何估其衰減函數H

1.尋找影像中訊號較強的子影像 $g_s(x,y)$

2.找出重建方法 $\hat{f}_s(x,y)$

3.假設雜訊很小,則

$$H_s(u,v)=\frac{G_s(u,v)}{\hat{F}_s(u,v)} \qquad (5.6\text{-}1)$$

5.6 衰減函數之估算

5.6.2 藉由實驗估算

1. 建構與所要修正之影像相同的環境

2. 取得衰減影像的脈衝響應 $g(x, y)$

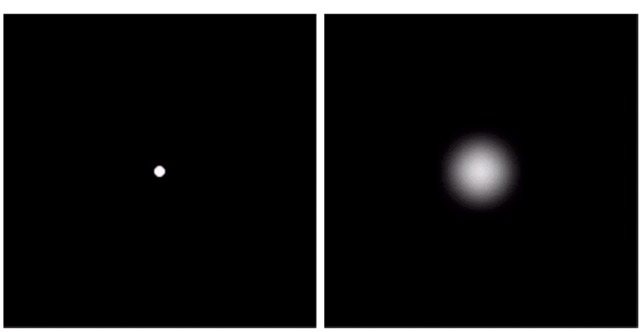
3. 取得正確影像的脈衝響應 $a(x, y)$

4. 計算衰減函數

$$H(u,v) = \frac{G(u,v)}{A} \qquad (5.6\text{-}2)$$

5.6 衰減函數之估算

5.6.2 藉由實驗估算

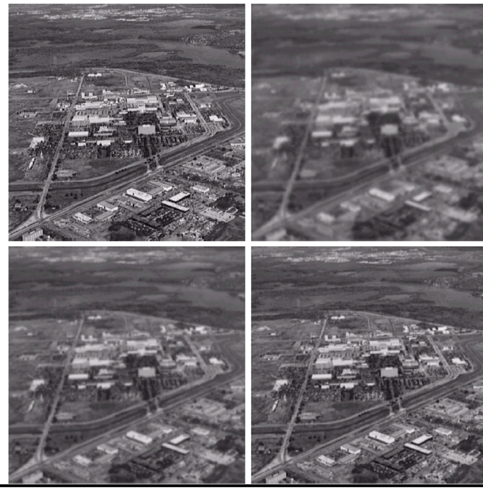


a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.

5.6 衰減函數之估算
5.6.3 藉由模型化估算

大氣干擾模型 $H(u,v)=e^{-k\left(u^2+v^2\right)^{5/6}}$ (5.6-3)



a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

5.6 衰減函數之估算
5.6.3 藉由模型化估算

隨時間均勻移動之取像模型

$$g(x,y)=\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt \qquad (5.6\text{-}4)$$

$$G(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f\left[x-x_0(t),y-y_0(t)\right]dt\right]e^{-j2\pi(ux+vy)}dxdy \qquad (5.6\text{-}5)$$

$$G(u,v)=\int_0^T\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f\left[x-x_0(t),y-y_0(t)\right]e^{-j2\pi(ux+vy)}dxdy\right]dt \qquad (5.6\text{-}6)$$

$$G(u,v)=\int_0^T F(u,v)e^{-j2\pi\left(ux_0(t)+vy_0(t)\right)}dt$$

$$=F(u,v)\int_0^T e^{-j2\pi\left(ux_0(t)+vy_0(t)\right)}dt \qquad (5.6\text{-}7)$$

$$H(u,v)=\int_0^T e^{-j2\pi\left(ux_0(t)+vy_0(t)\right)}dt \qquad (5.6\text{-}8)$$

$$G(u,v)=H(u,v)F(u,v) \qquad (5.6\text{-}9)$$

5.6 衰減函數之估算
5.6.3 藉由模型化估算

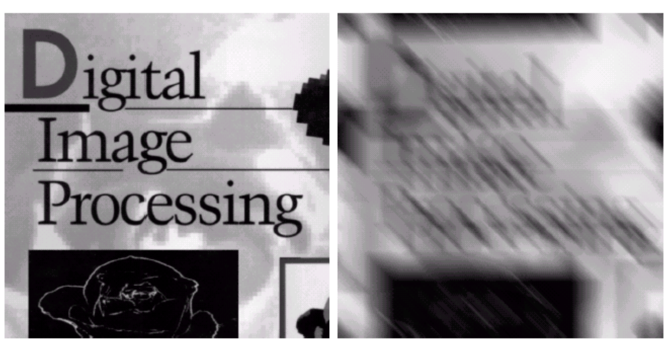
假設只有x移動,x0(t)=at/T,且t=T時x位移量為a,則

$$
\begin{aligned}
H(u,v) &= \int_0^T e^{-j2\pi u x_0(t)}\,dt \\
&= \int_0^T e^{-j2\pi uat/T}\,dt \\
&= \frac{\mathrm{T}}{\pi \mathrm{ua}} \sin(\pi \mathrm{ua}) e^{-j\pi ua} \qquad (5.6\text{-}10)
\end{aligned}
$$

假設y也移動,y0(t)=bt/T,且t=T時y位移量為b,則

$$
H(u,v) = \frac{\mathrm{T}}{\pi(ua+vb)} \sin(\pi(ua+vb)) e^{-j\pi(ua+vb)} \qquad (5.6\text{-}11)
$$

5.6 衰減函數之估算
5.6.3 藉由模型化估算



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

5.7 反向濾波

取得衰減函數H後,如何重建影像

$$\hat{F}(u,v)=\frac{G(u,v)}{H(u,v)} \qquad (5.7\text{-}1)$$

$$\hat{F}(u,v)=F(u,v)+\frac{N(u,v)}{H(u,v)} \qquad (5.7\text{-}2)$$

如果H(u,v)的值很小，則N(u,v)會導致影像變差，
H(u,v)高頻部位通常很小，所以做反向濾波時，必須適度去除
H(u,v)高頻部位，才能有效改善影像品質

5.7 反向濾波

大氣干擾的例子

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.

$$H(u,v)=e^{-k\left((u-M/2)^2+(v-N/2)^2\right)^{5/6}}$$

a.使用所有H(u,v)
b.半徑40
c.半徑70
d.半徑85

5.8 最小均方差(Wiener)濾波

$$e^2 = E\left\{ \left(f - \hat{f}\right)^2 \right\} \qquad (5.8\text{-}1)$$

$$\hat{F}(u,v) = \left[\frac{H^*(u,v) S_f(u,v)}{S_f(u,v)\left|H(u,v)\right|^2 + S_\eta(u,v)} \right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v) \qquad (5.8\text{-}2)$$

$$= \left[\frac{1}{H(u,v)} \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + S_\eta(u,v)/S_f(u,v)} \right] G(u,v)$$

針對白雜訊

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{\left|H(u,v)\right|^2}{\left|H(u,v)\right|^2 + K} \right] G(u,v) \qquad (5.8\text{-}3)$$

必須自行尋找最佳的K值

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5.8 最小均方差(Wiener)濾波

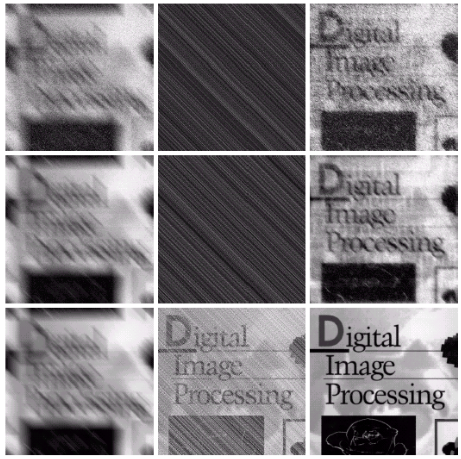
K值最好的情況

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

5.8 最小均方差(Wiener)濾波

K值最好的情況



a b c
d e f
g h i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a "curtain" of noise.

5.9 限制最小平方濾波

衰減模式之矩陣表示式

$$\mathbf{g} = \mathbf{H}\,\mathbf{f} + \eta \qquad (5.9\text{-}1)$$

求取下式之最小值

$$C = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\left[\nabla^2 f(x,y)\right]^2 \qquad (5.9\text{-}2)$$

其限制條件為：

$$\left\|\mathbf{g} - \mathbf{H}\hat{\mathbf{f}}\right\|^2 = \left\|\eta\right\|^2 \qquad (5.9\text{-}3)$$

此最佳化問題的解為：

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{\left|H(u,v)\right|^2 + \gamma\left|P(u,v)\right|^2}\right]G(u,v) \qquad (5.9\text{-}4)$$

γ為調整符合限制條件的參數，$P(u,v)$為下式之傅立葉轉換

$$p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix} \qquad (5.9\text{-}5)$$

5.9 限制最小平方濾波

a b c

FIGURE 5.30 Results of constrained least squares filtering. Compare (a), (b), and (c) with the Wiener filtering results in Figs. 5.29(c), (f), and (i), respectively.

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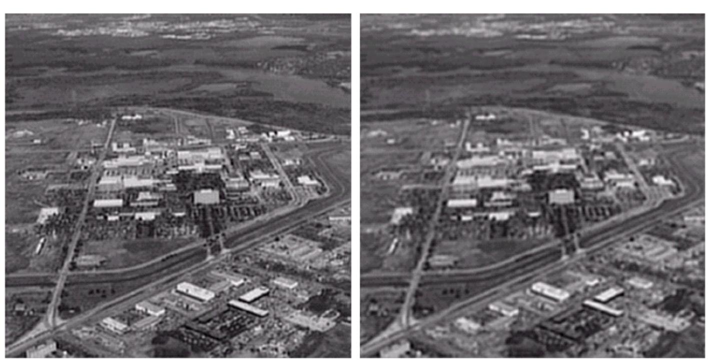
5.9 限制最小平方濾波

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.



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5.10 幾何平均濾波器

- Wiener濾波器的一般化

$$\hat{F}(u,v)=\left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha}\left[\frac{H^*(u,v)}{|H(u,v)|^2+\beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha}G(u,v) \qquad (5.10\text{-}1)$$

$\alpha=1$, Inverse Filter
$\alpha=0$, Parametric Wiener Filter
$\alpha=0$, $\beta=1$, Standard Wiener Filter
$\alpha=1/2$, $\beta=1$, Spectrum Equalization Filter

5.11 幾何轉換

5.11.1 空間轉換

幾何失真(Geometric distortion)

$$f(x,y)\Rightarrow g(x',y')$$

$$x'=r(x,y) \qquad (5.11\text{-}1)$$

$$y'=s(x,y) \qquad (5.11\text{-}2)$$

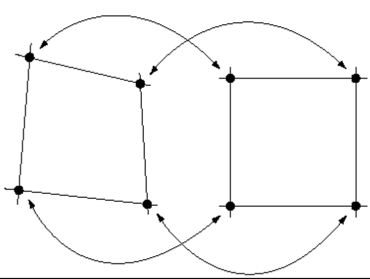

FIGURE 5.32 Corresponding tiepoints in two image segments.

5.11 幾何轉換

5.11.1 空間轉換

修復方法

使用**Tiepoints**及**bilinear equations**

$$r(x,y) = c_1 x + c_2 y + c_3 xy + c_4 \qquad (5.11\text{-}3)$$

$$s(x,y) = c_5 x + c_6 y + c_7 xy + c_8 \qquad (5.11\text{-}4)$$

$$x' = c_1 x + c_2 y + c_3 xy + c_4 \qquad (5.11\text{-}5)$$

$$y' = c_5 x + c_6 y + c_7 xy + c_8 \qquad (5.11\text{-}6)$$

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5.11 幾何轉換

5.11.2 灰階值內插法

最鄰近點(Nearest neighbor)

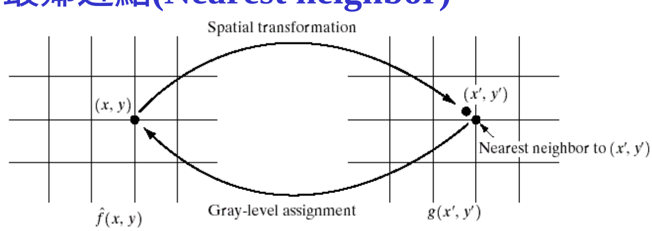


FIGURE 5.33 Gray-level interpolation based on the nearest neighbor concept.

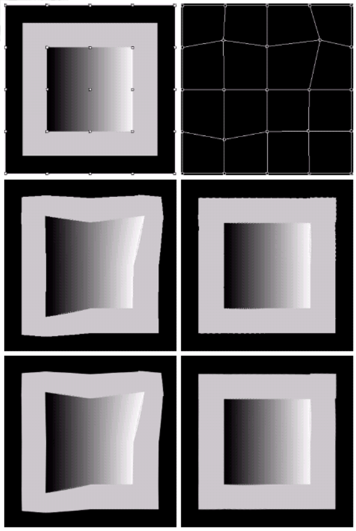
Bilinear interpolation

$$v(x', y') = ax' + by' + cx'y' + d \qquad (5.11\text{-}7)$$

Cubic Convolution

5.11 幾何轉換

5.11.2 灰階值內插法



a b
c d
e f

FIGURE 5.34 (a) Image showing tiepoints. (b) Tiepoints after geometric distortion. (c) Geometrically distorted image, using nearest neighbor interpolation. (d) Restored result. (e) Image distorted using bilinear interpolation. (f) Restored image.

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5.11 幾何轉換

5.11.2 灰階值內插法

a b
c d

FIGURE 5.35 (a) An image before geometric distortion. (b) Image geometrically distorted using the same parameters as in Fig. 5.34(e). (c) Difference between (a) and (b). (d) Geometrically restored image.