Chapter 5
Image Restoration and Reconstruction

- Image Restoration

  -- attempts to reconstruct or recover an image that has been degraded by using *a proiori* knowledge of the degradation

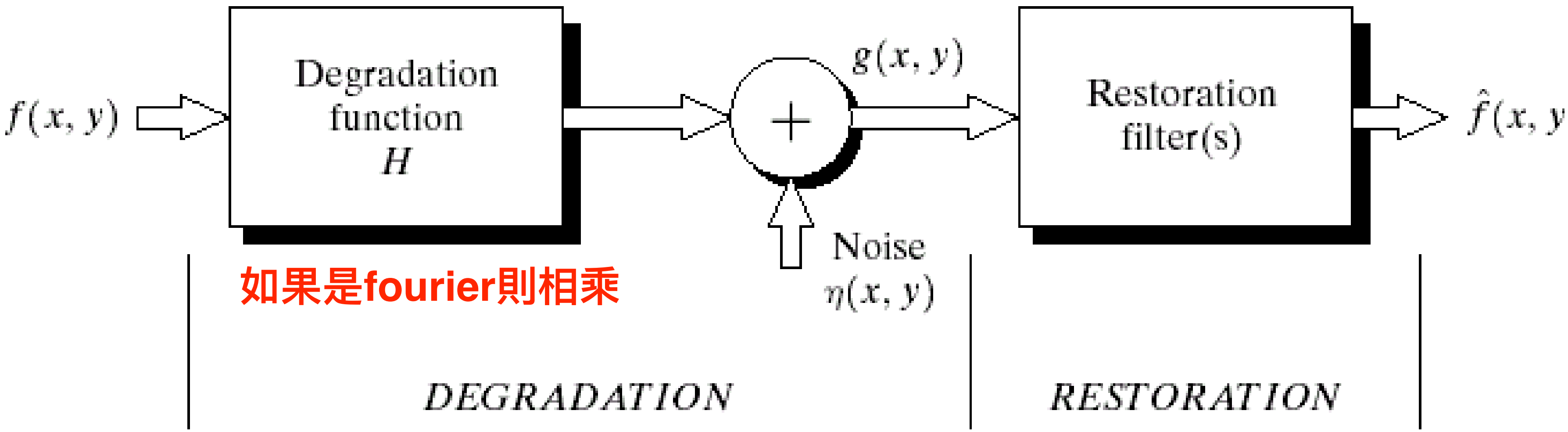
  -- model the degradation, then apply the inverse process

Image Enhancement → largely subjective
→ basically heuristic procedure

Image Restoration → more objective
→ usually formulate a criterion of goodness

A Model of the Image Degradation/Restoration Process

目的: 設計這個



如果是fourier則相乘

base:noise採用加法

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$

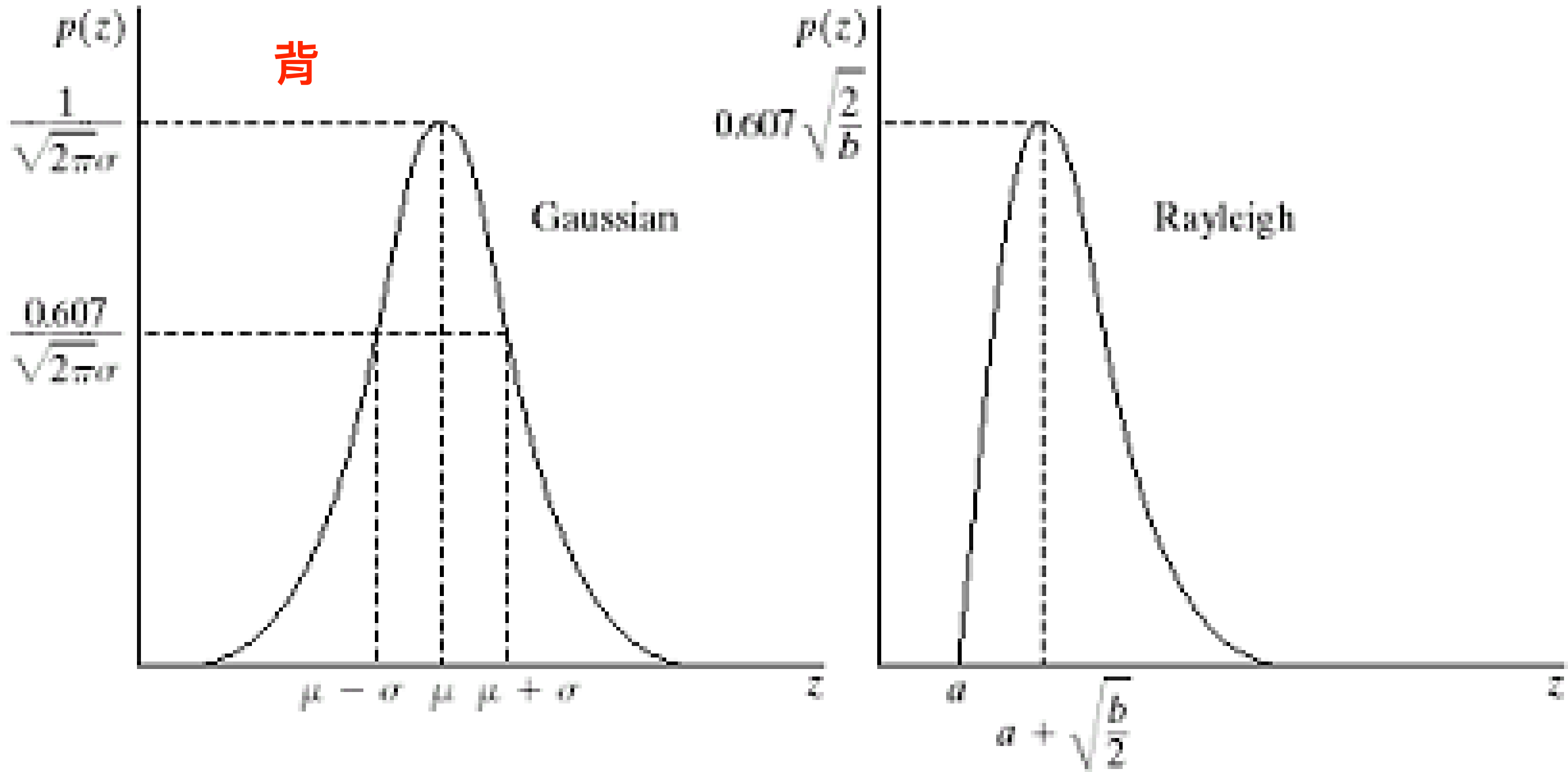
Chapter 5
Image Restoration and Reconstruction

CT: Computed Tomography (section 5.11), not covered in this course

5.1 Noise Models

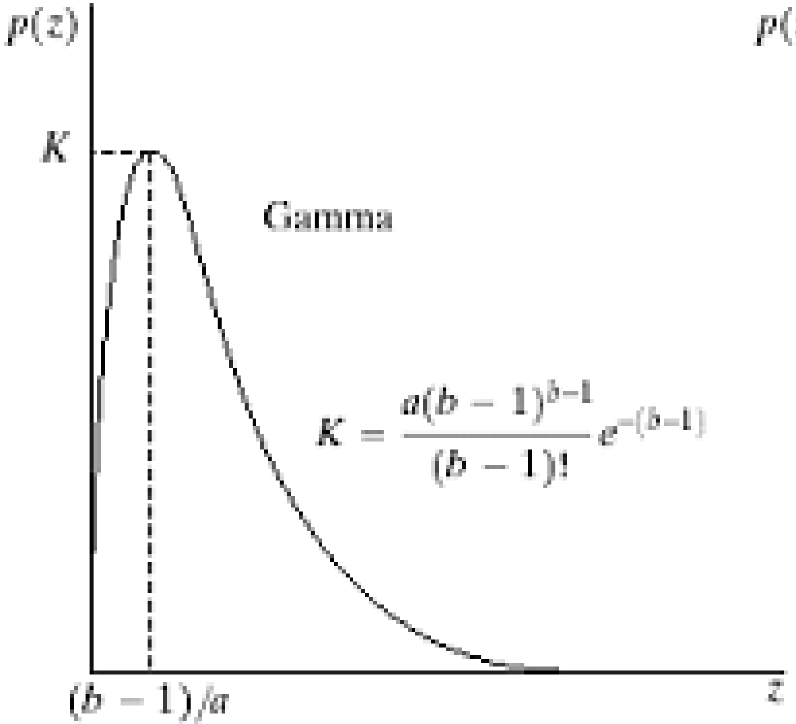
- Gaussian Noise
- Rayleigh Noise
- Gamma (Erlang) Noise
- Exponential Noise
- Uniform Noise
- Impulse (Salt-and-Pepper) Noise

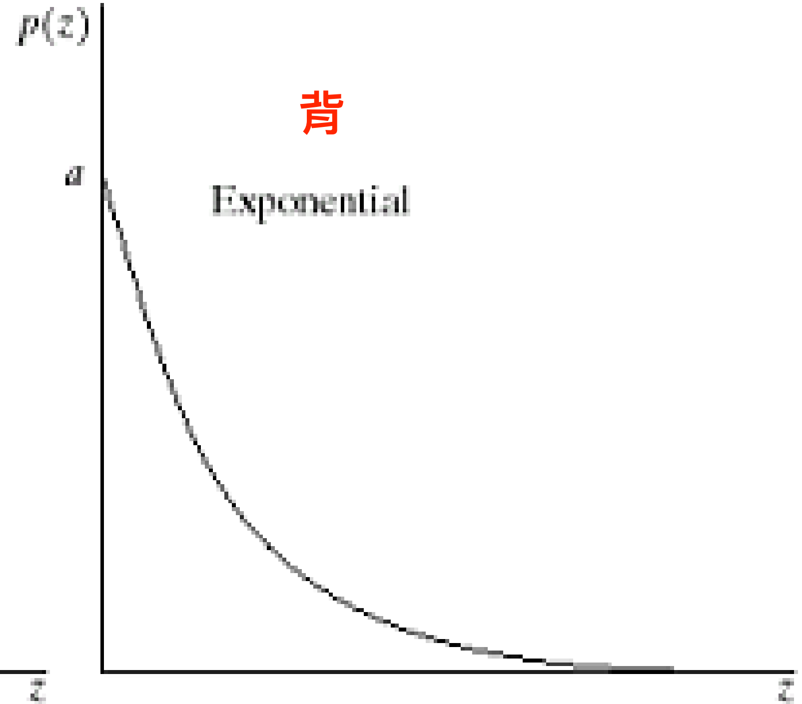
- Consider only: 假設獨立
  -- Spatially-independent (except periodic noise)
  -- Signal-uncorrelated
  相關係數=0



$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2}$$

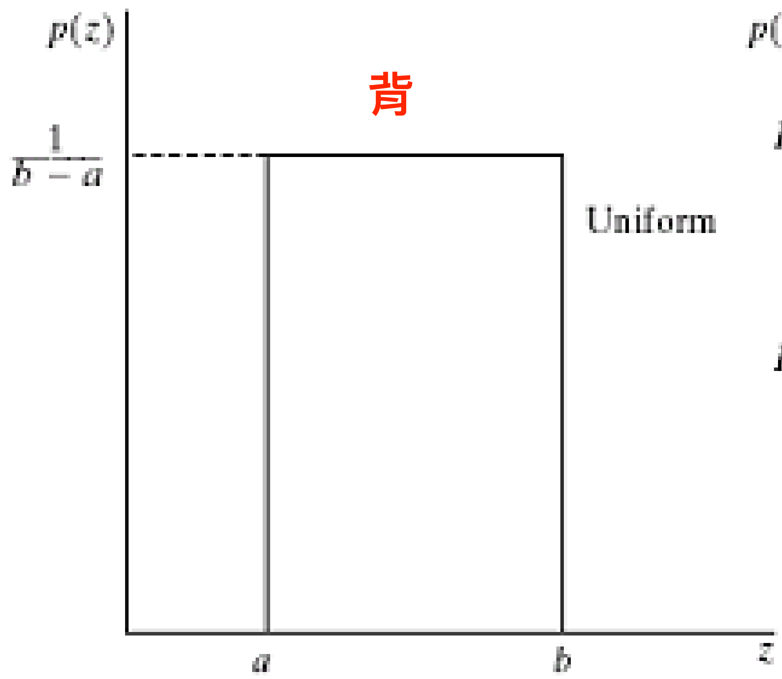
$$p(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a. \end{cases}$$

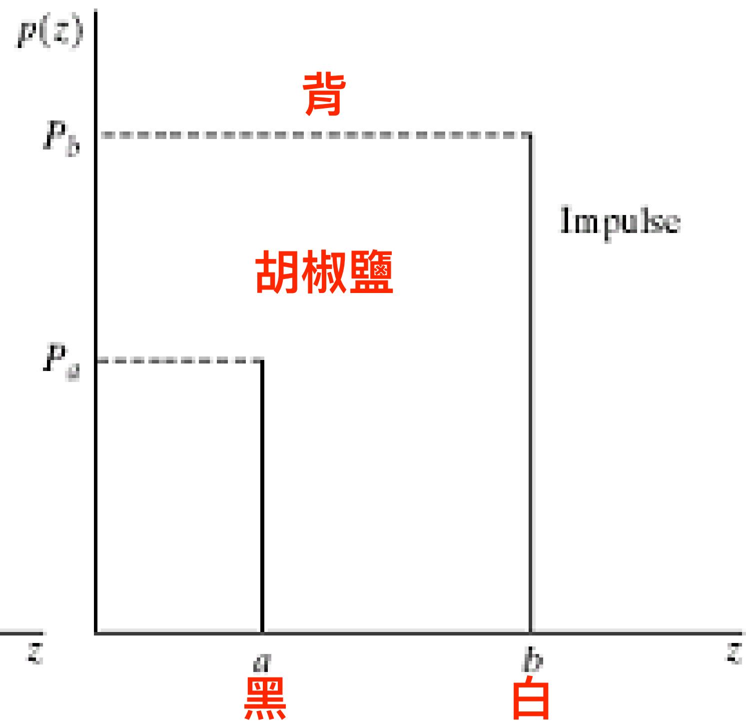




$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases}$$





$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise.} \end{cases}$$

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

Test Pattern

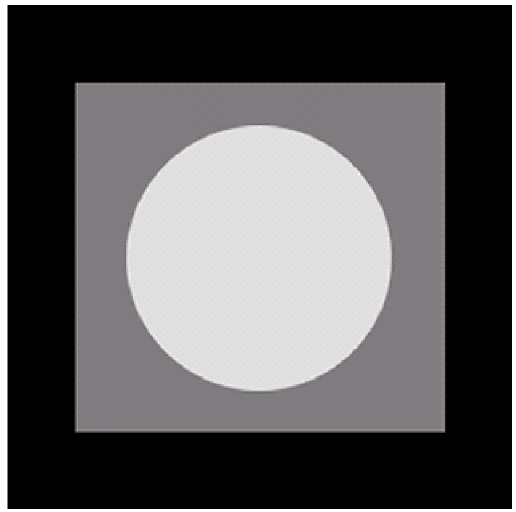
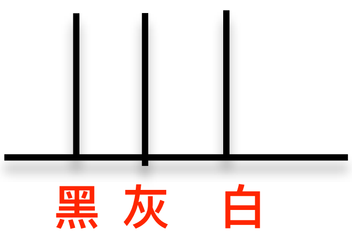
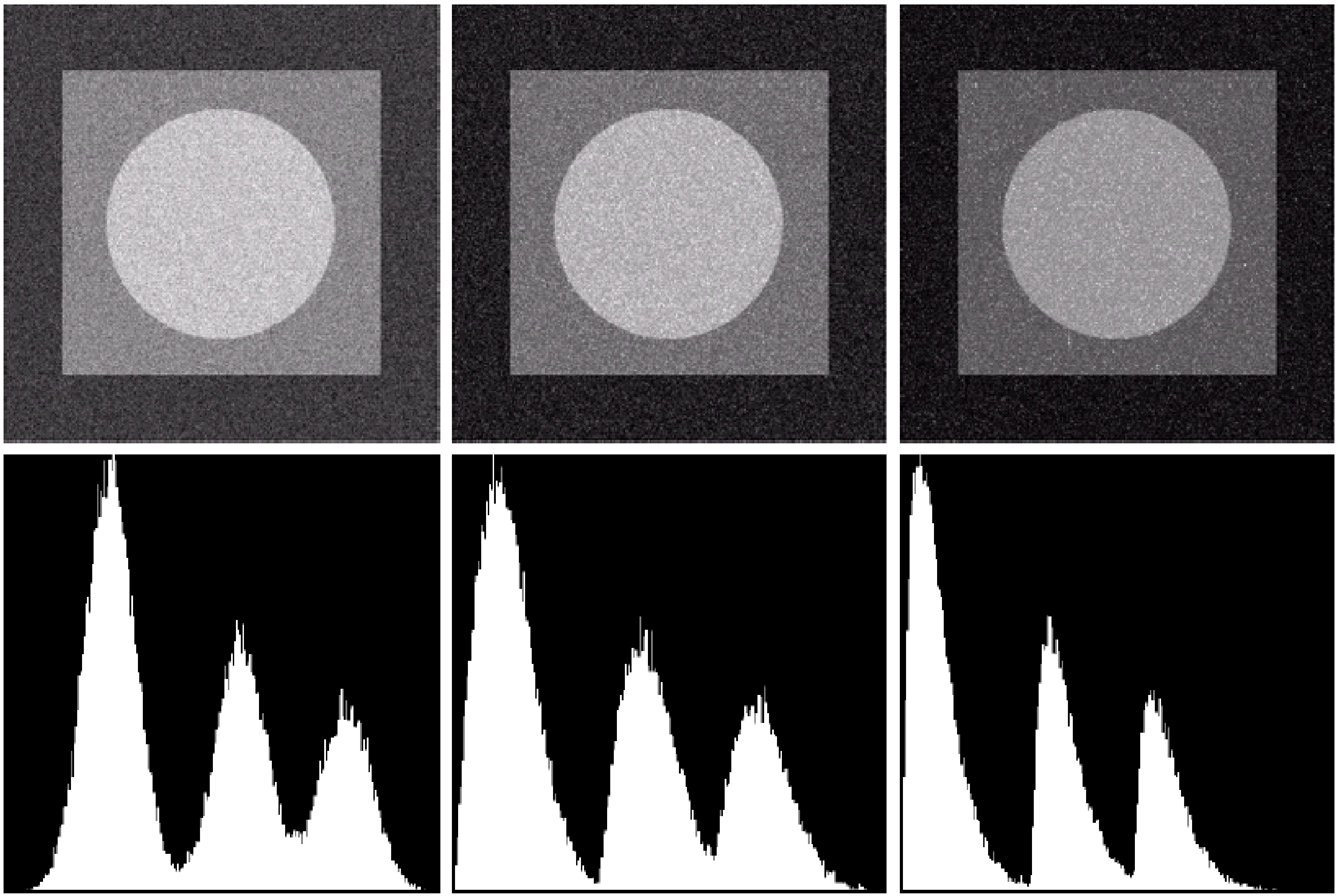


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



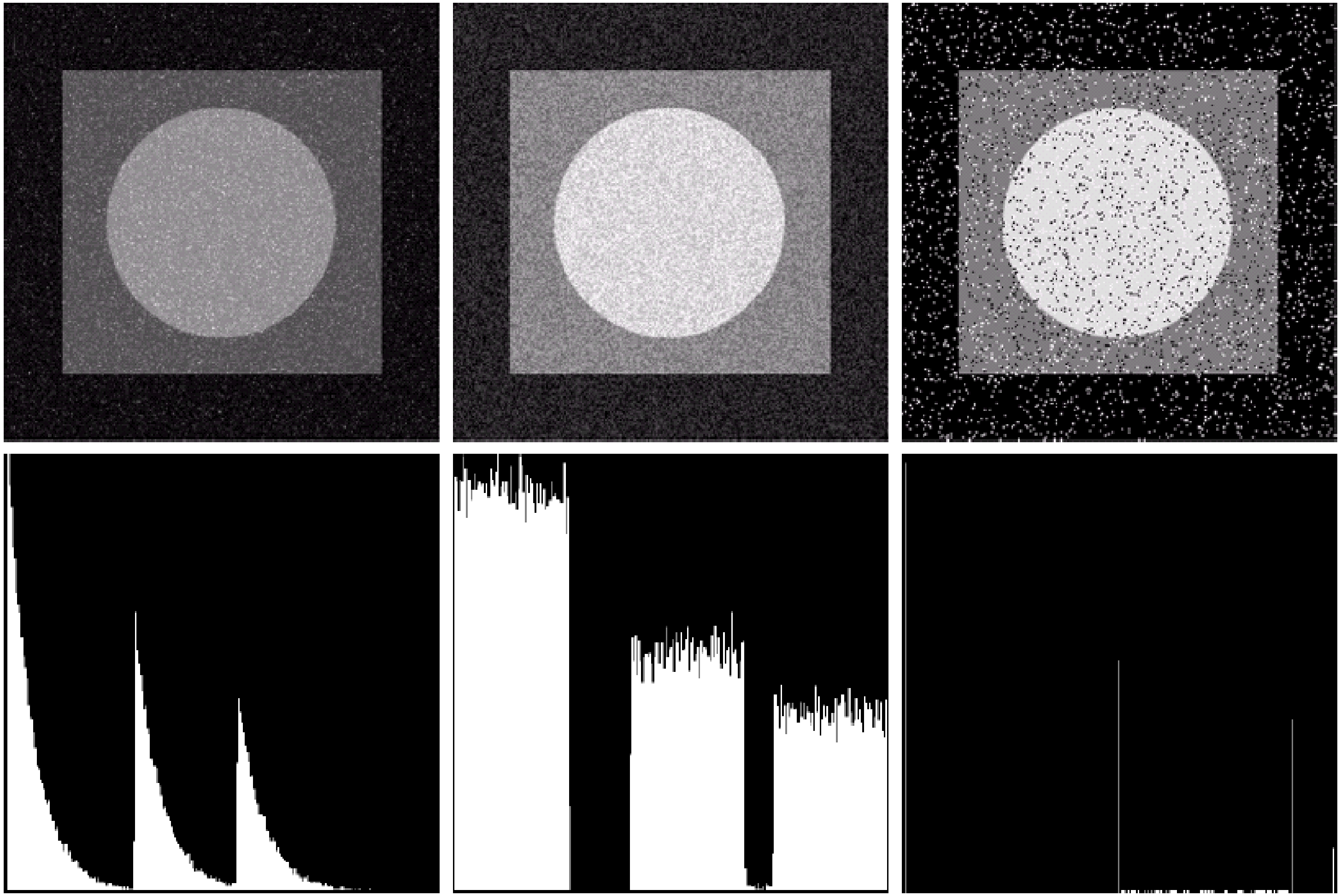
Synthetic Noise Added

histogram



Gaussian Rayleigh Gamma

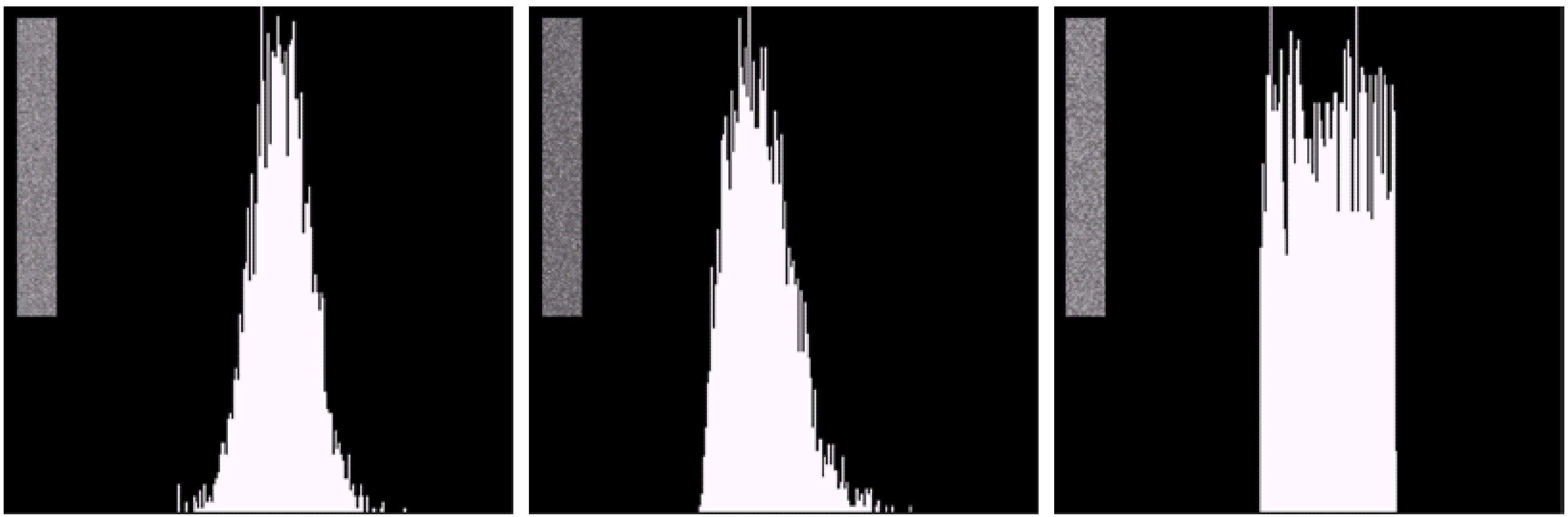
Synthetic Noise Added



Exponential Uniform Salt & Pepper

Estimation of Noise Parameters

一般estimation noise會先找一個比較uniform的地方，再去fix



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Chapter 5
Image Restoration and Reconstruction

5.1 Noise Models

5.2 Restoration in the Presence of Noise Only

– Spatial Filtering

5.3 Periodic Noise Reduction by Frequency Domain Filtering

5.4 Inverse Filtering

5.5 Minimum Mean Squared Error (Weiner) Filtering

5.2 Restoration in the Presence of Noise Only – Spatial Filtering

原來的影像　　noise

H=I $\Rightarrow$

$$g(x, y) = f(x, y) + \eta(x, y)$$

$$G(u, v) = F(u, v) + N(u, v)$$

- Mean Filters
- Order-Statistics Filters
- Adaptive Filters

Mean Filters

- Arithmetic Mean Filter

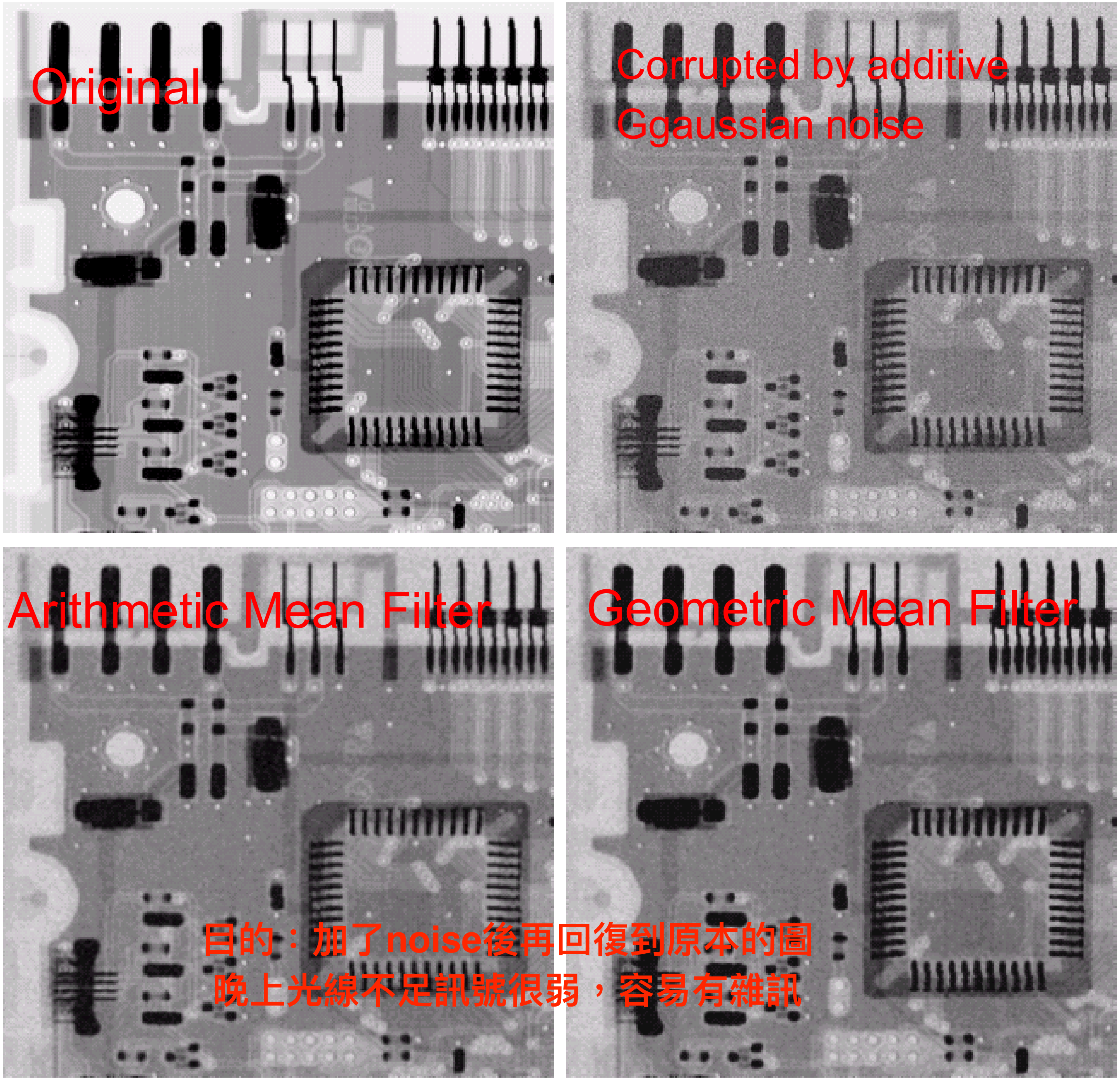
  算術平均

robust也滿常用的 $$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

- Geometric Mean Filter

  幾何平均

$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$



a b
c d

FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

- Harmonic Mean Filter
  - *good for salt noise, not good for pepper*

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

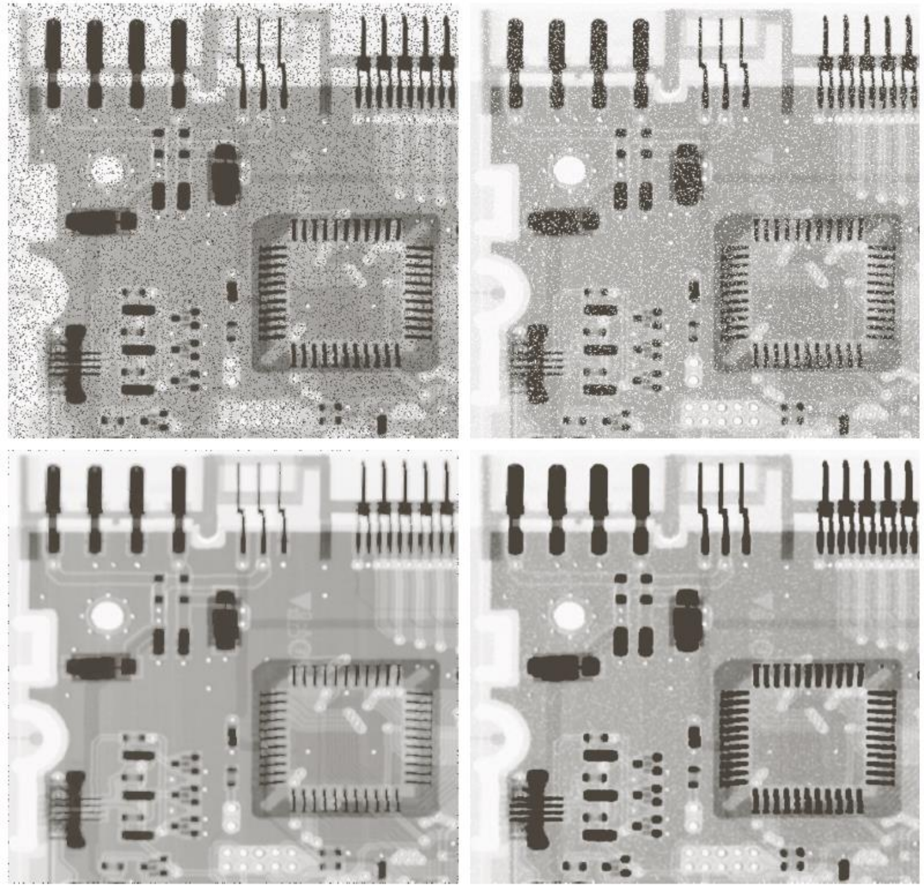
- Contraharmonic Mean Filter (see Fig 5.8)
  - Q>0 good for pepper noise; Q<0 good for salt noise

$$\hat{f}(x, y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

order

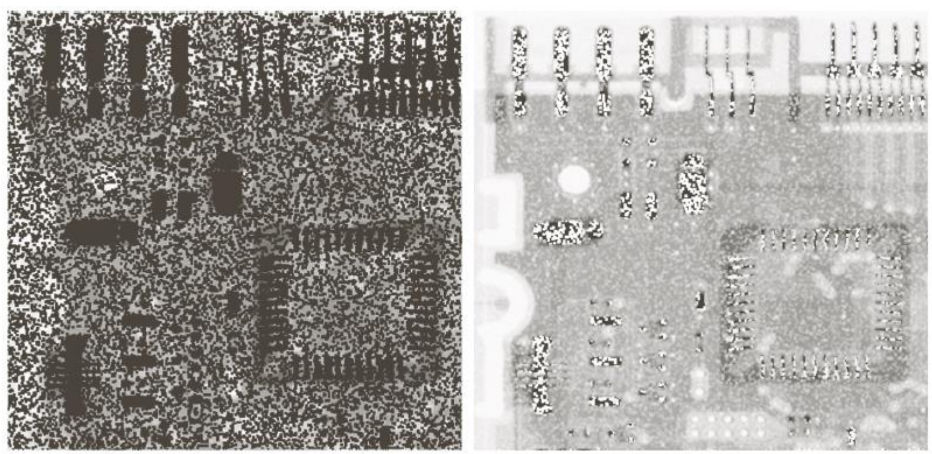
$Q = 0$?

$Q = -1$?



a b
c d

FIGURE 5.8 (a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.



a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

Order-Statistics Filters

- Median Filter

$$\hat{f}(x, y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s, t)\}$$

取中位數，對胡椒鹽雜訊好

- Max and Min Filter

$$\hat{f}(x, y) = \max_{(s,t)\in S_{xy}}\{g(s, t)\}$$

$$\hat{f}(x, y) = \min_{(s,t)\in S_{xy}}\{g(s, t)\}$$

- Midpoint Filter

mean vs median vs midpoint

$$\hat{f}(x,y) = \frac{1}{2}\left[\max_{(s,t)\in S_{xy}}\{g(s,t)\} + \min_{(s,t)\in S_{xy}}\{g(s,t)\}\right]$$

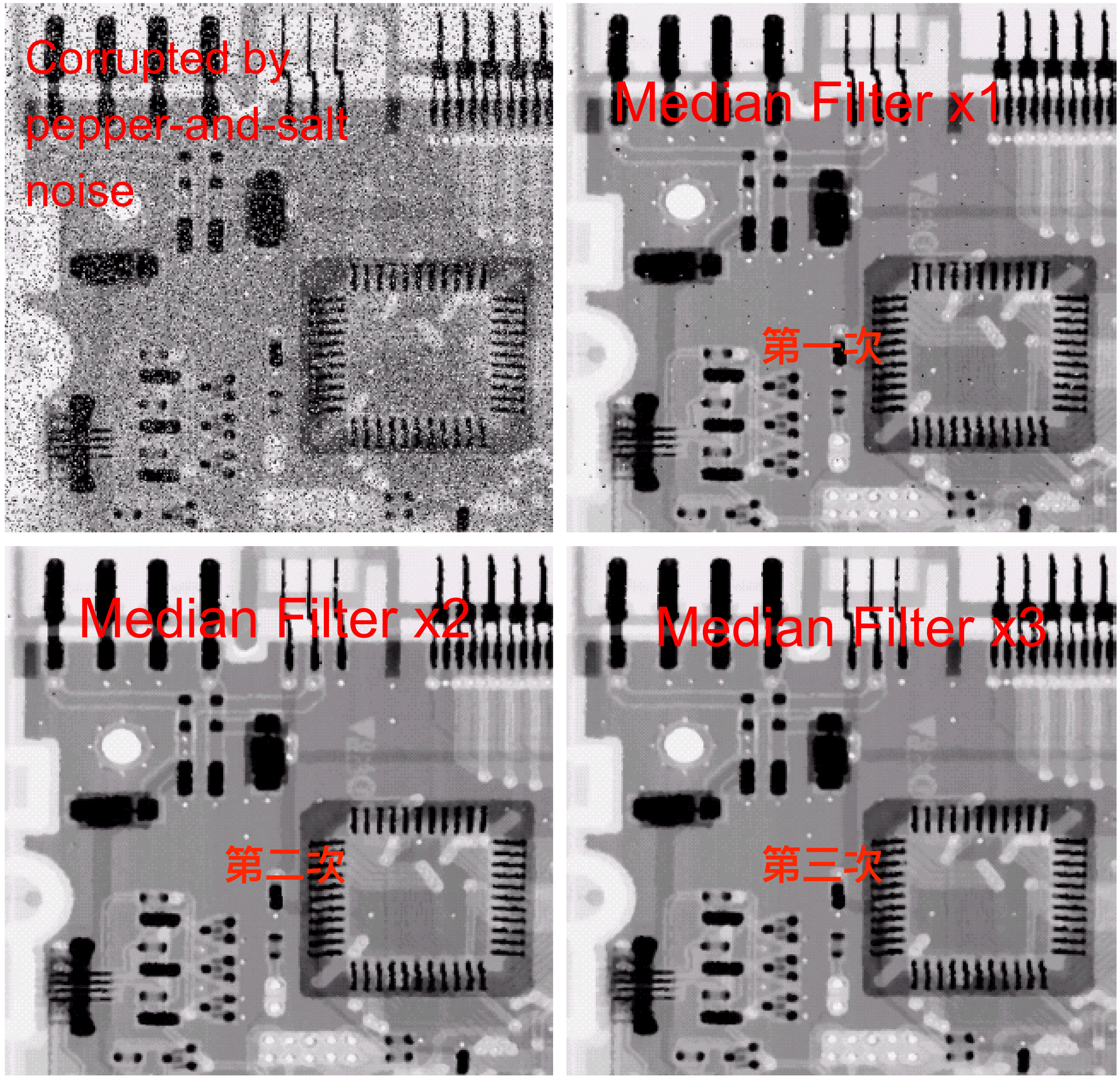
- Alpha-trimmed mean Filter

The remaining mn-d pixels

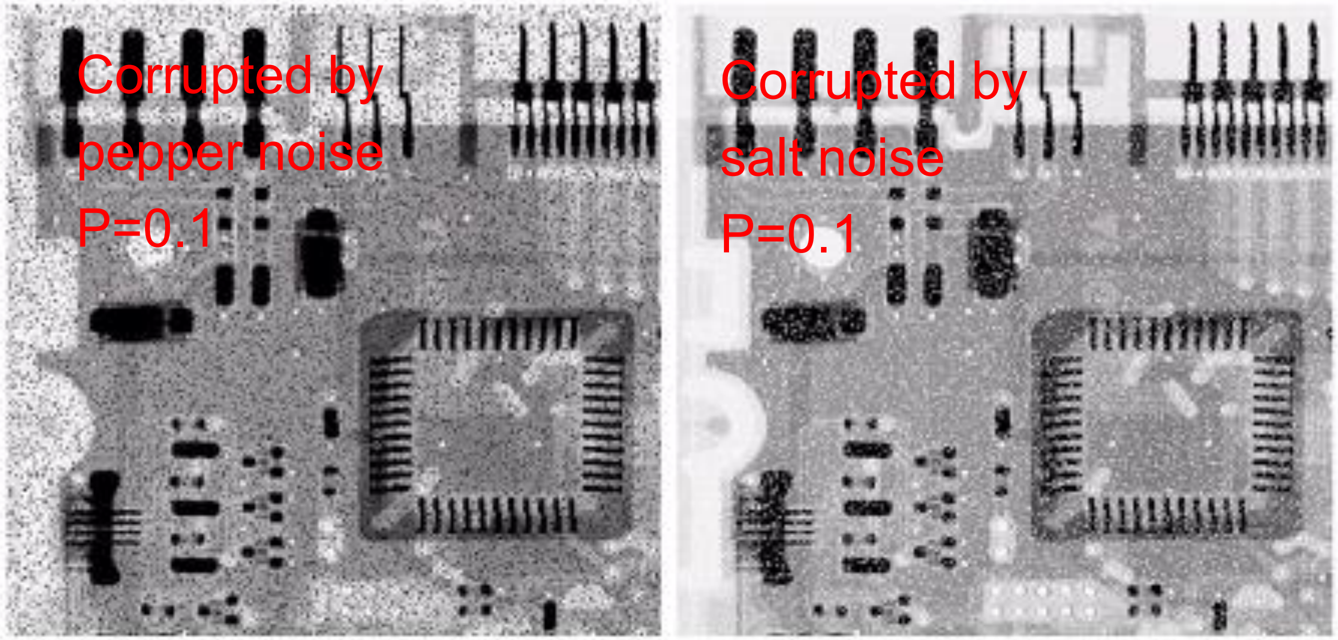
$$\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$$

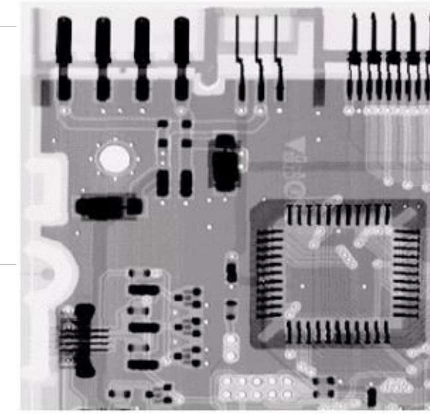
What if d → mn-1 ? **中位數**

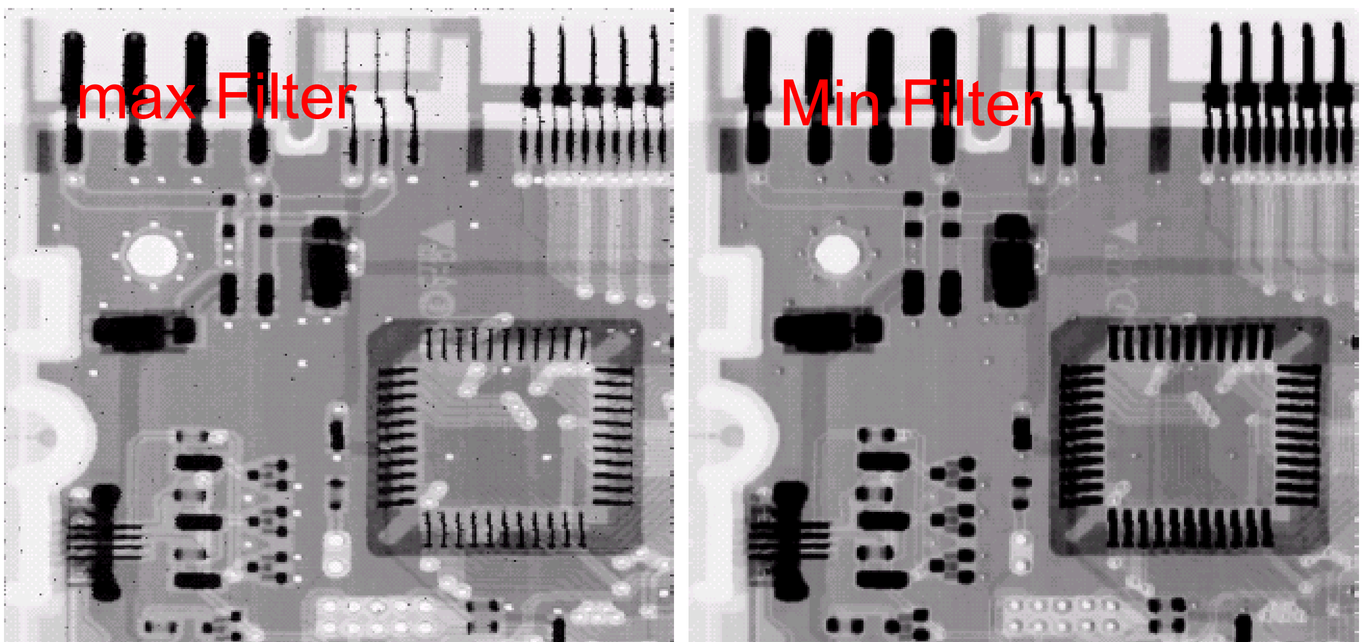
FIGURE 5.10 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$. (b) Result of one pass with a median filter of size 3×3. (c) Result of processing (b) with this filter. (d) Result of processing (c) with the same filter.



median,smooth做越多次越有差



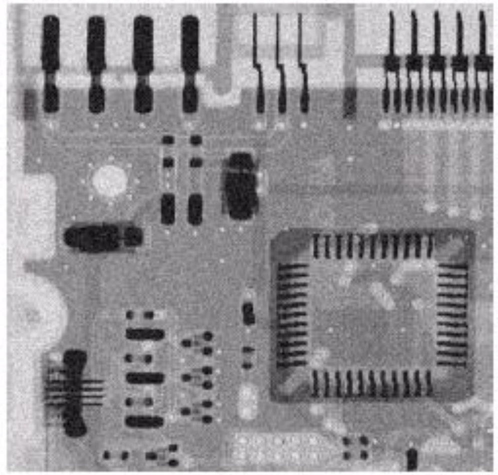




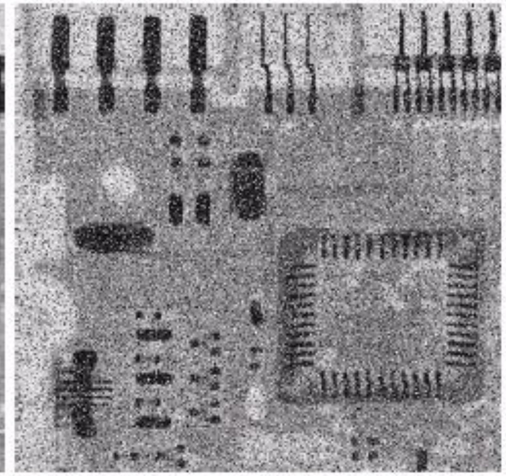
a b

FIGURE 5.11 (a) Result of filtering Fig. 5.8(a) with a max filter of size 3 × 3. (b) Result of filtering 5.8(b) with a min filter of the same size.

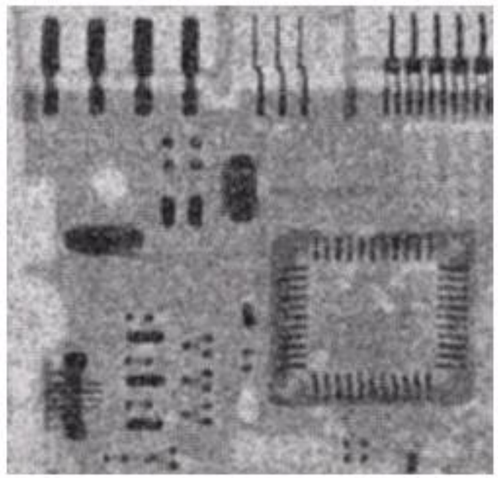
Corrupted by additive uniform noise



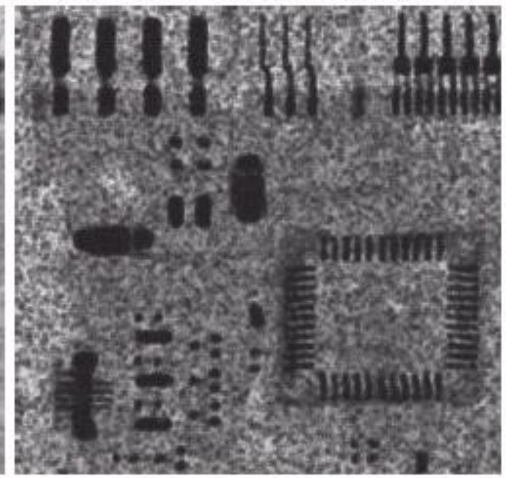
Extra salt-and-pepper noise



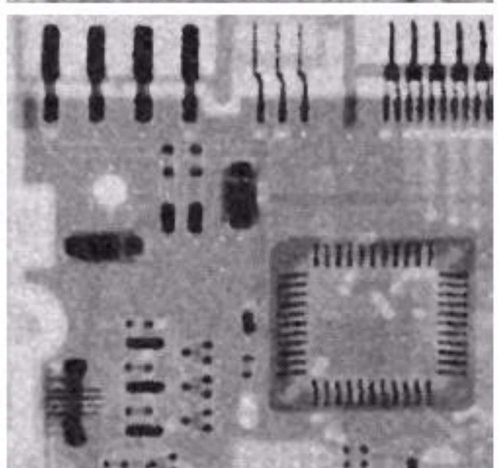
Arithmetic Mean Filter



Geometric Mean Filter

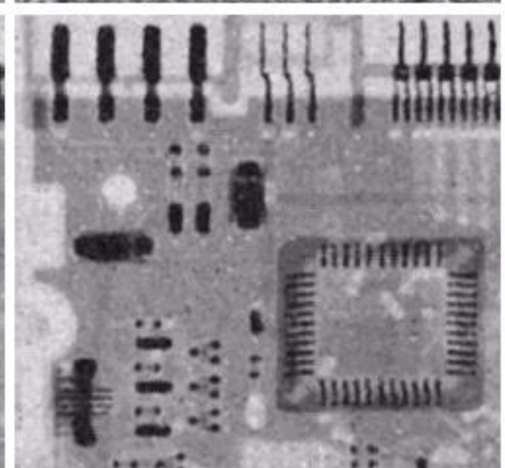


Median Filter



Alpha-trimmed Mean Filter

類似median



Adaptive Filters

類似高斯

- Adaptive, Local Noise Reduction Filter
  （Adaptive Mean Filter）
- Adaptive Median Filter

Adaptive Mean Filter

- local variance >> noise variance (eg., near edge)

  →return a value close to g(x,y)

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_L^2}\left[g(x, y) - m_L\right]$$

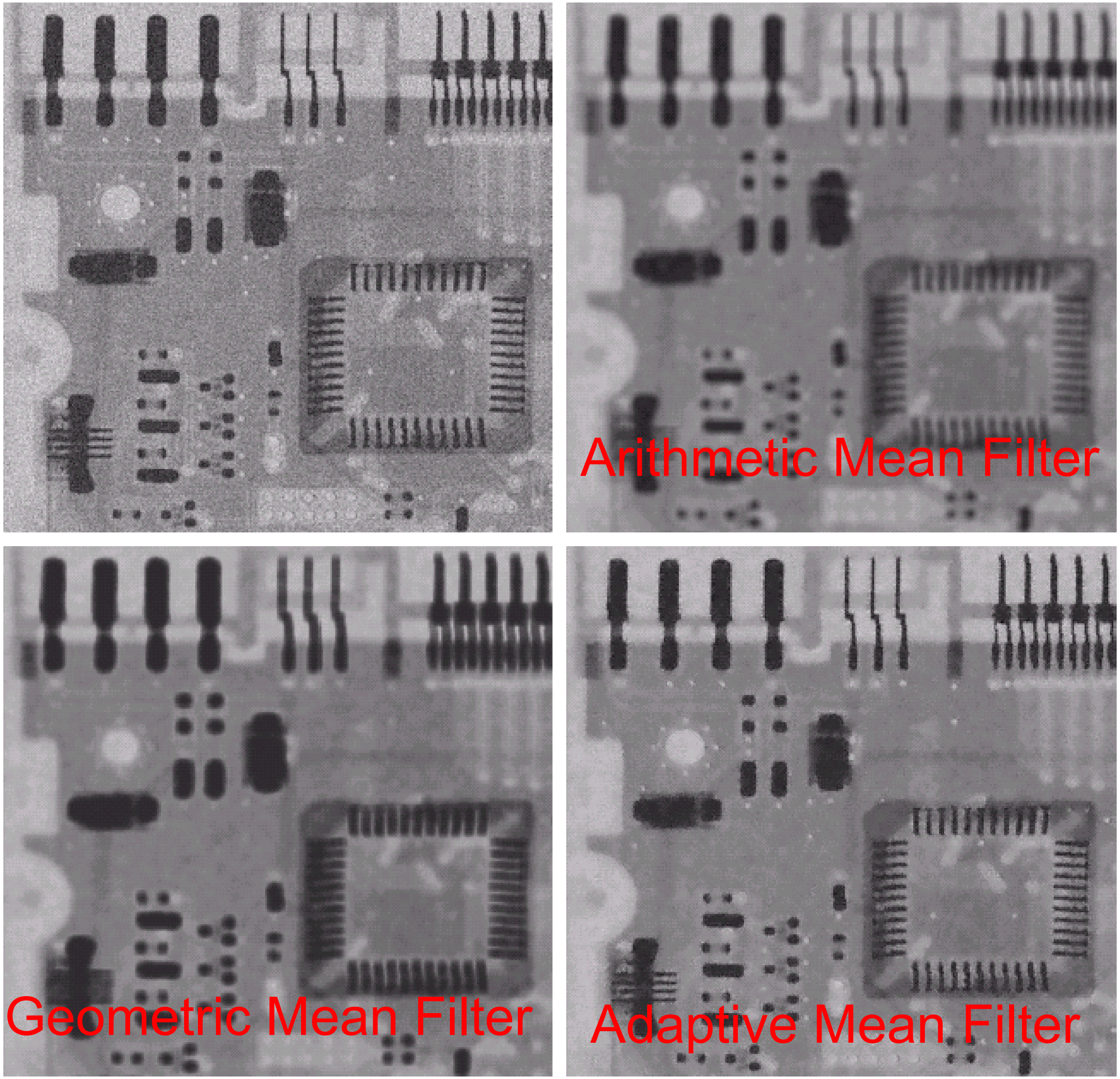
$$if \quad \sigma_L^2 > \sigma_\eta^2$$

- local variance ~ noise variance

  → return mean

a b
c d

FIGURE 5.13
(a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



Adaptive Median Filter

Level A: $A1 = z_{\text{med}} - z_{\text{min}}$

$A2 = z_{\text{med}} - z_{\text{max}}$

If $A1 > 0$ AND $A2 < 0$, Go to level B

Else increase the window size

If window size $\leq S_{\text{max}}$ repeat level A

Else output z_{xy}.

Level B: $B1 = z_{xy} - z_{\text{min}}$

$B2 = z_{xy} - z_{\text{max}}$

If $B1 > 0$ AND $B2 < 0$, output z_{xy}

Else output z_{med}.

z_{min} = minimum gray level value in S_{xy}

z_{max} = maximum gray level value in S_{xy}

z_{med} = median of gray levels in S_{xy}

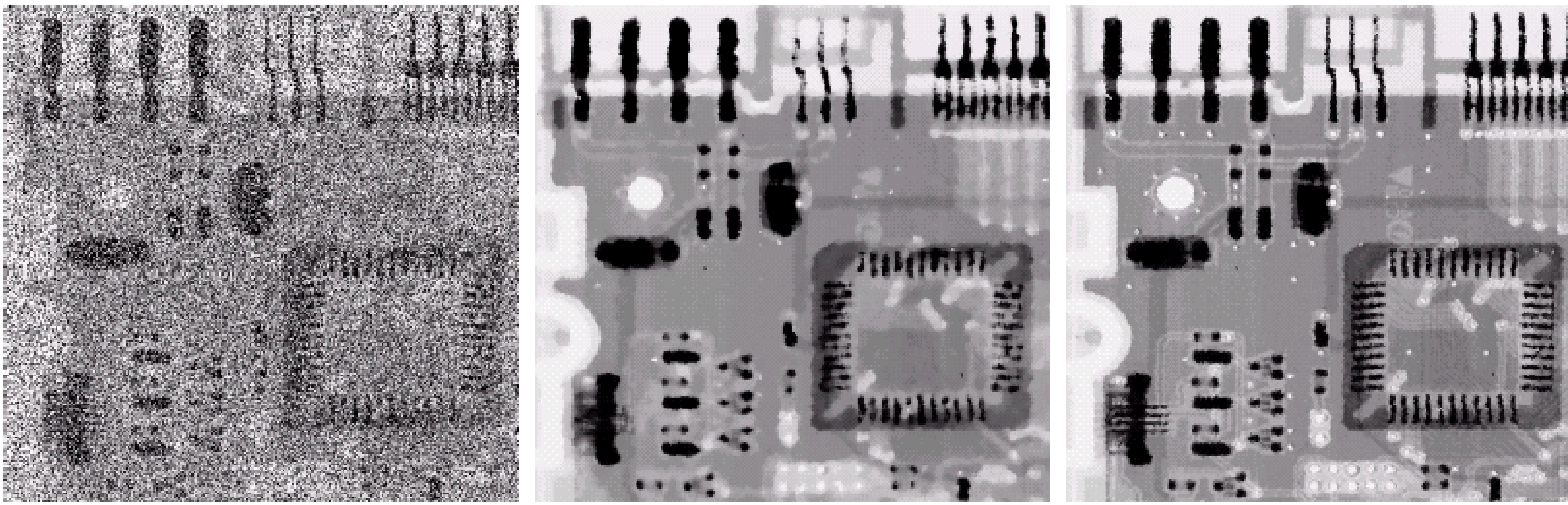
z_{xy} = gray level at coordinates (x, y)

S_{max} = maximum allowed size of S_{xy}.

Example for adaptive median filter

Median Filter

Adaptive Median Filter



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.

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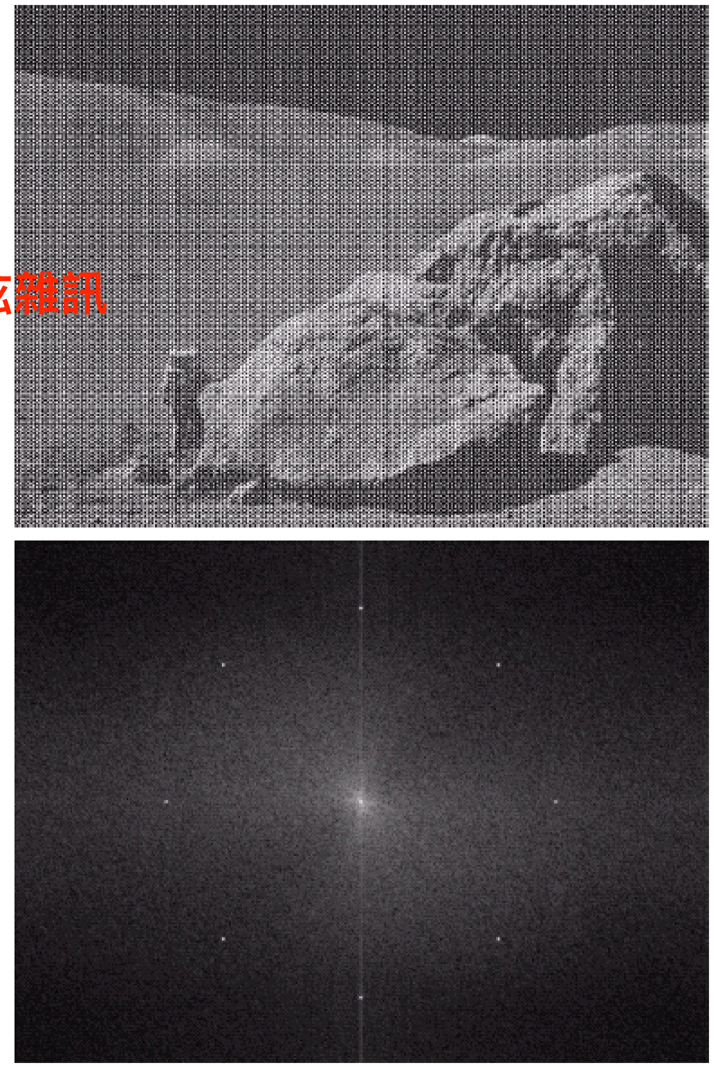
5.3 Periodic Noise Reduction by Frequency Domain Filtering

a
b

FIGURE 5.5 (a) Image corrupted by sinusoidal noise. (b) Spectrum (each pair of conjugate impulses corresponds to one sine wave). (Original image courtesy of NASA.)

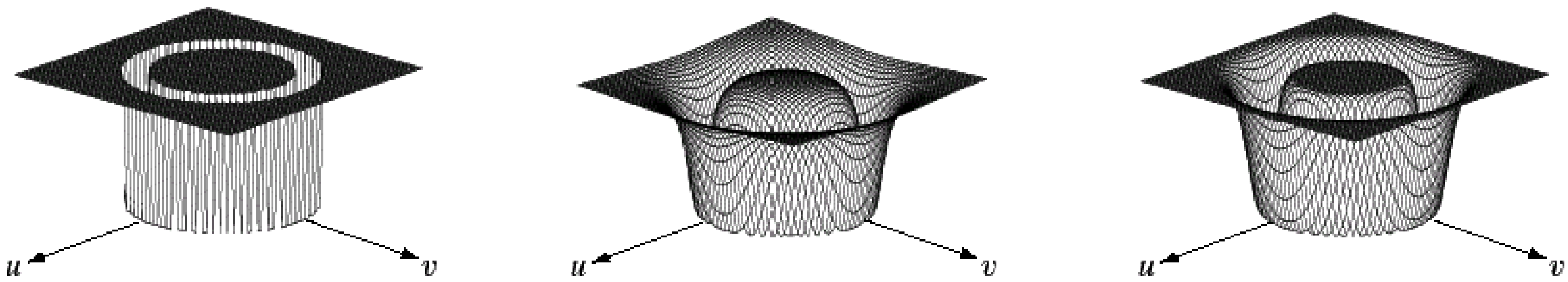
正弦雜訊



Periodic Noise

Bandreject Filters

拿掉這一圈



a b c

FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.

TABLE 4.6
Bandreject filters. W is the width of the band, D is the distance $D(u, v)$ from the center of the filter, D_0 is the cutoff frequency, and n is the order of the Butterworth filter. We show D instead of $D(u, v)$ to simplify the notation in the table.

Ideal	Butterworth	Gaussian
$H(u,v) = \begin{cases} 0 & \text{if } D_0 - \frac{W}{2} \le D \le D_0 + \frac{W}{2} \\ 1 & \text{otherwise} \end{cases}$	$H(u,v) = \dfrac{1}{1 + \left[\dfrac{DW}{D^2 - D_0^2}\right]^{2n}}$	$H(u,v) = 1 - e^{-\left[\frac{D^2 - D_0^2}{DW}\right]^2}$

Example for Butterworth bandreject filter

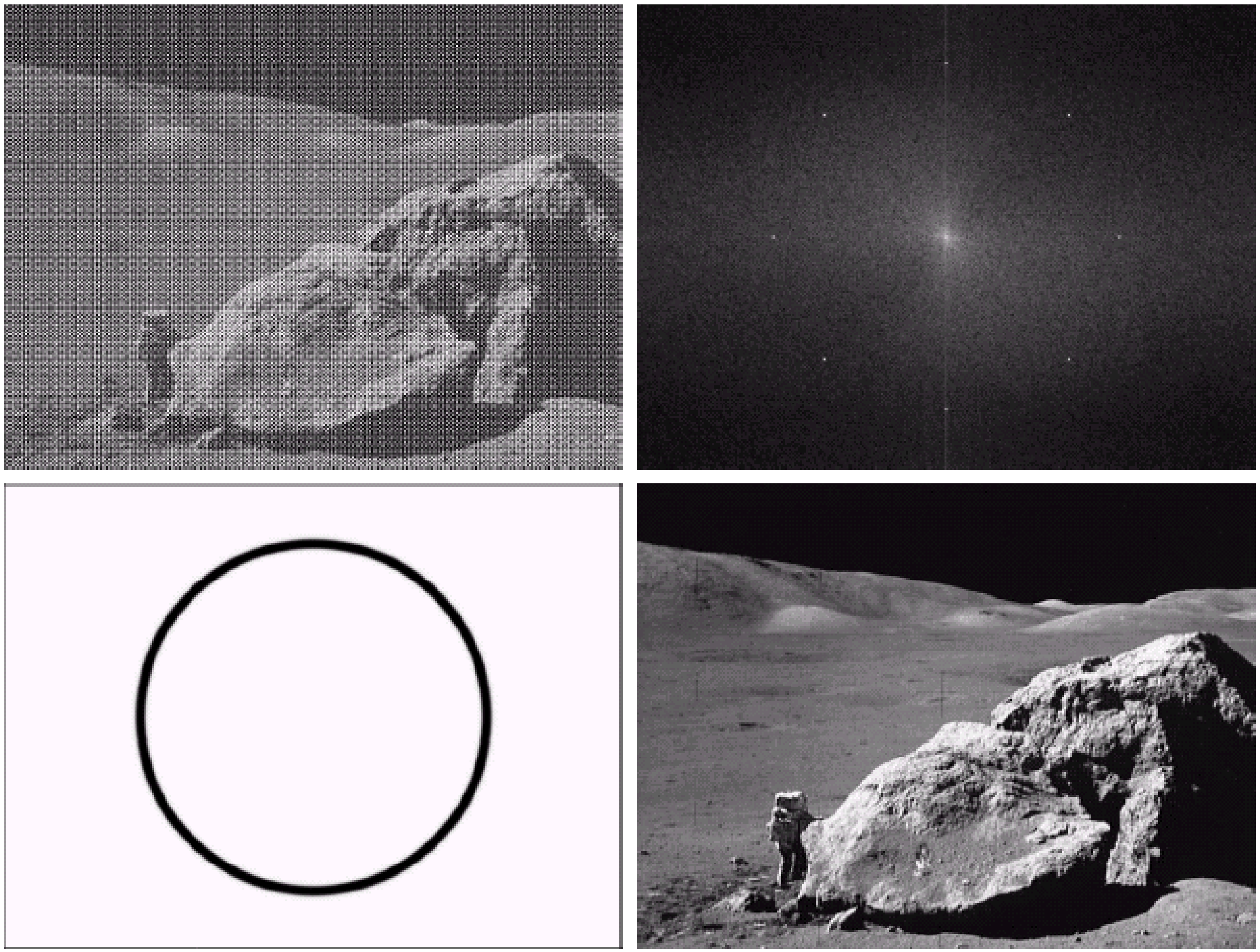
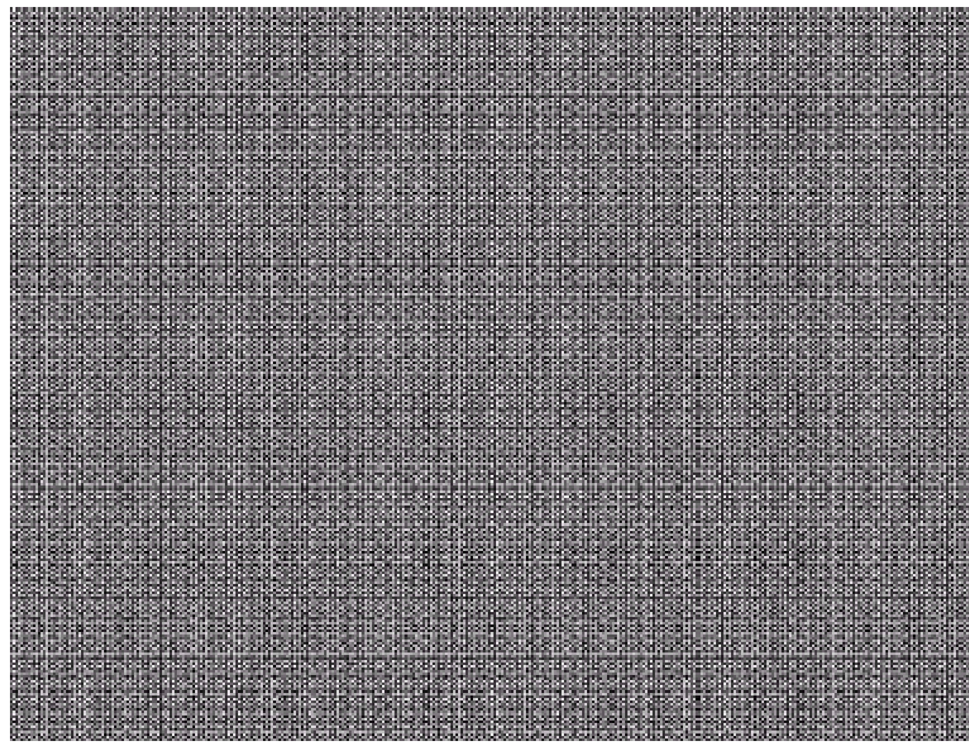


FIGURE 5.16 (a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

Noise pattern obtained by bandpass filter

FIGURE 5.17
Noise pattern of the image in Fig. 5.16(a) obtained by bandpass filtering.

See Eq 5.4-1



Notch Filters

Notch reject filter
vs
Notch pass filter

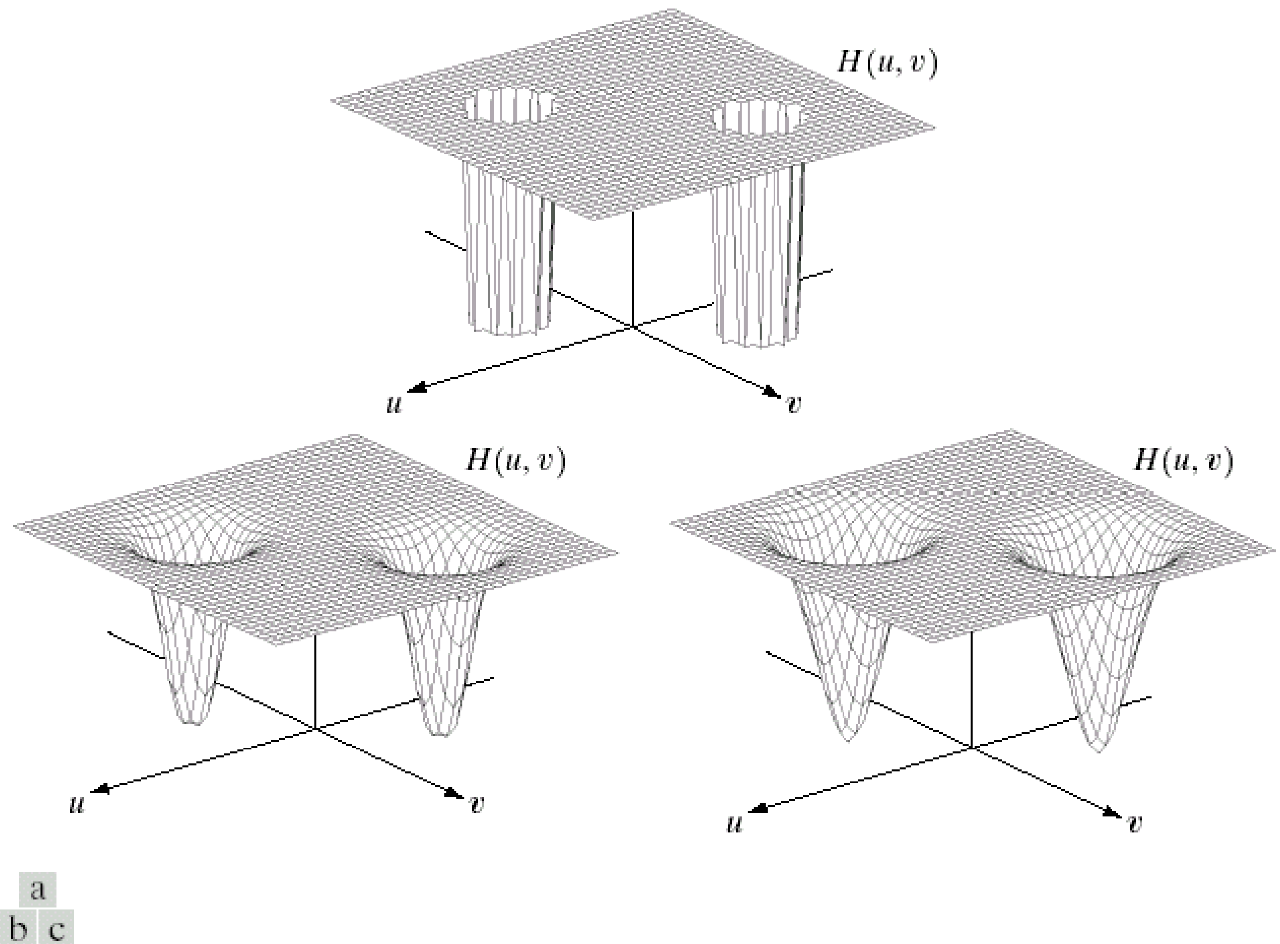
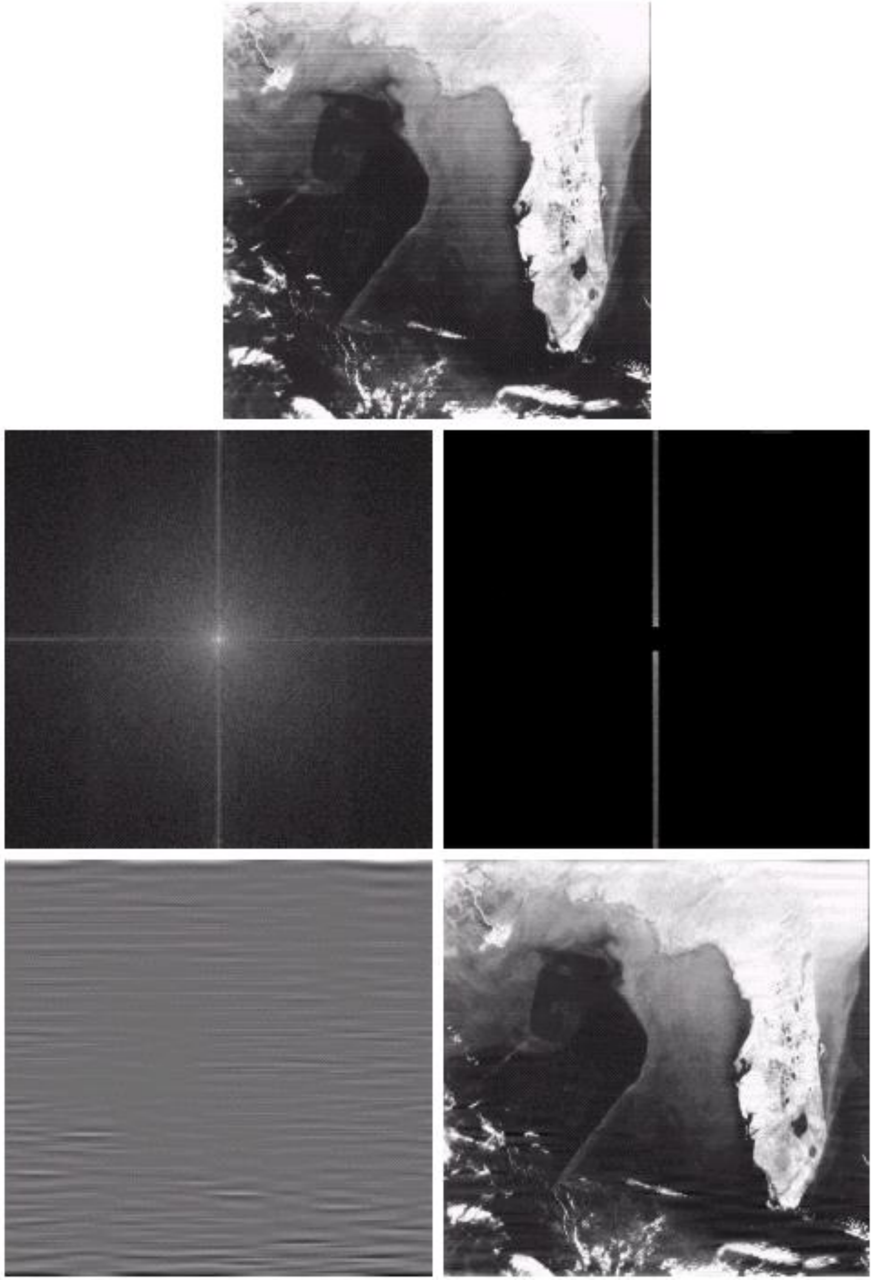


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.



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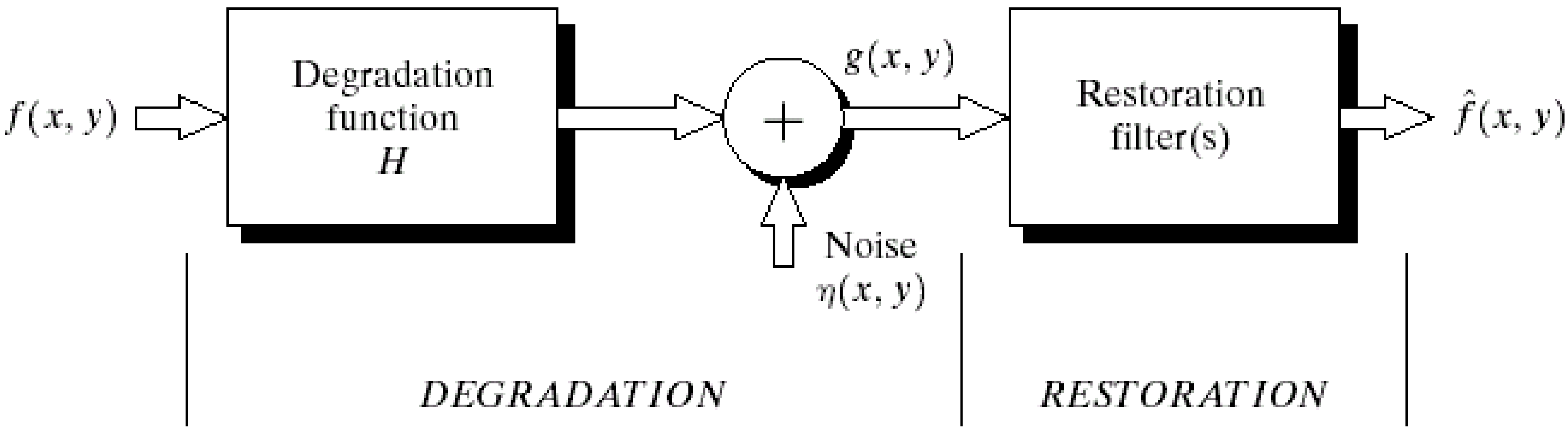
A Model of the Image Degradation/Restoration Process



$$g(x, y) = \underset{\text{degradation}}{h(x, y)} * f(x, y) + \eta(x, y)$$

$$G(u, v) = \underset{\text{degradation}}{H(u, v)} F(u, v) + N(u, v)$$

5.4 Inverse Filtering

convolution of multiple

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Let $$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$ 有G想找F

$$\Longrightarrow \quad \hat{F}(u,v) = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

- Even if H is known, can not recover F exactly due to N
- If H is 0 or very small, N/H will dominate

  -- pseudoinverse filtering

[Jain89] p.276

$$H^{-}(\omega_1,\omega_2) = \begin{cases} \dfrac{1}{H(\omega_1,\omega_2)}, & H \neq 0 \\ 0, & H = 0 \end{cases}$$

pseudo inverse

-- In practice, H^- is set to zero when |H| is too small

Example of atmospheric turbulence model

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)

找出一個適合的cut-off

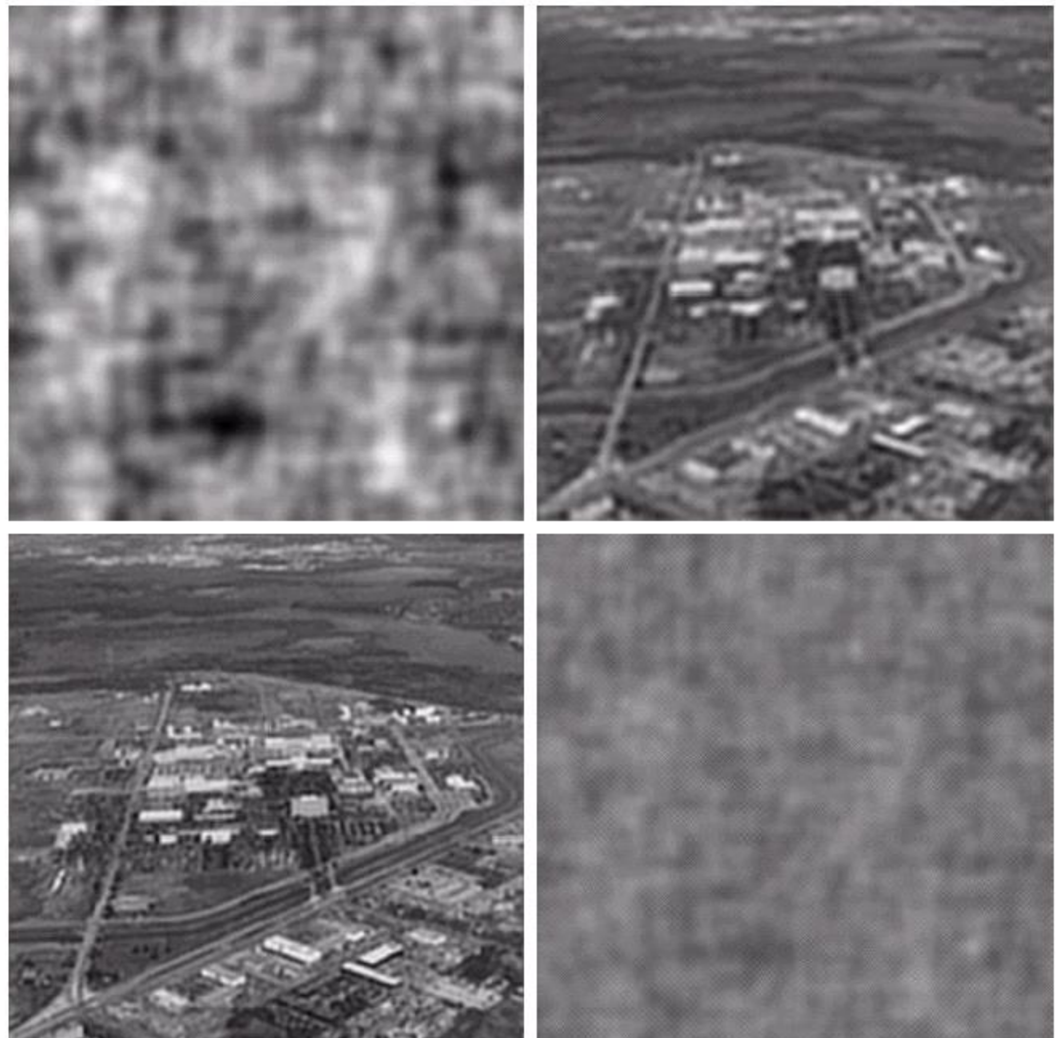
$$H(u, v) = e^{-k(u^2+v^2)^{5/6}}$$

Example of Inverse Filtering

Another Solution: apply cut-off radius (p.351 in textbook)

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



radially limited inverse filter

Butterworth LP of order 10

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

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5.5 Minimum Mean Squared Error (Weiner) Filtering

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

Given g, want to find an estimate of f such that the *mean square error* is minimum

$$\boxed{e^2 = E\{(f - \hat{f})^2\}}$$

Solution: Wiener filter !!!

Wiener Filter, MMSE filter, LSE filter (1942)

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right]G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

$$= \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right]G(u,v)$$

power spectrum of the noise

$S_\eta(u,v)/S_f(u,v)$ unknown?

Usually assume spectrally white noise

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,v)$$

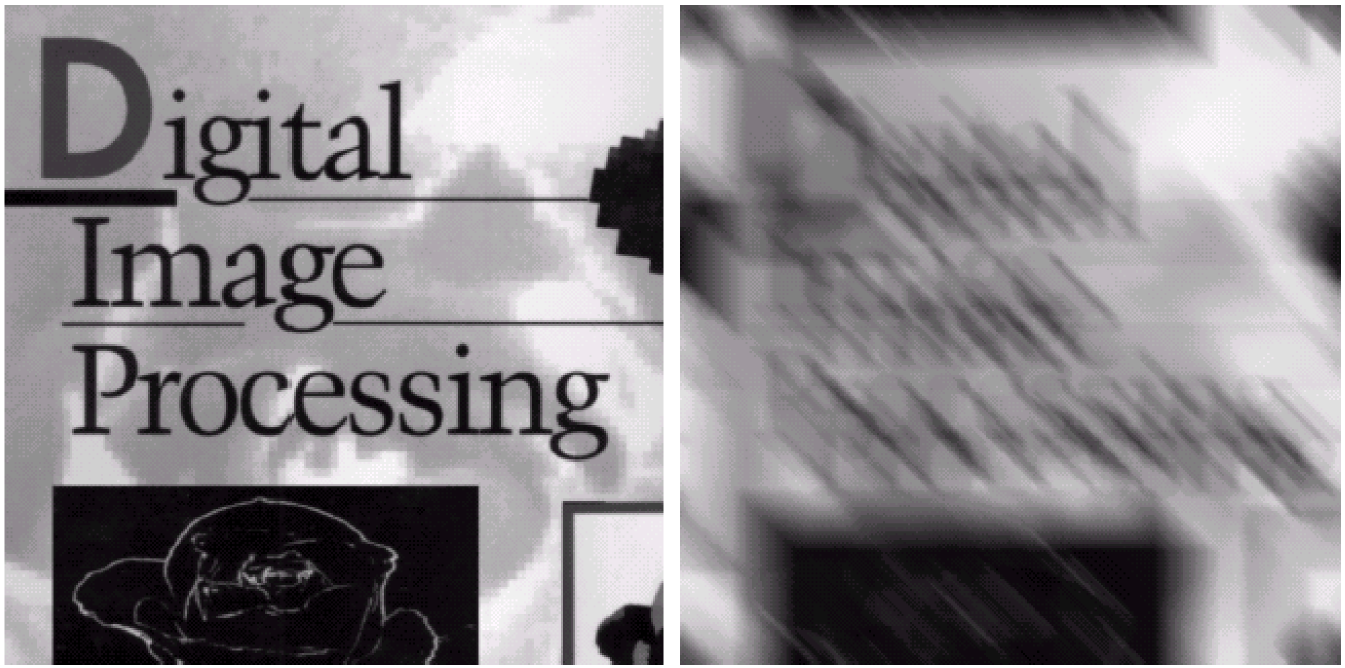
approximation

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

try interactively

Example: Image blurring by uniform linear motion

每一點都是很多點的積分



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Image blurring by uniform linear motion

$$g(x, y) = \int_0^T f[x - x_0(t), y - y_0(t)]\, dt$$

$$\begin{aligned} G(u, v) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y) e^{-j2\pi(ux+vy)}\, dx\, dy \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\int_0^T f[x - x_0(t), y - y_0(t)]\, dt\right] e^{-j2\pi(ux+vy)}\, dx\, dy \end{aligned}$$

$$G(u, v) = \int_0^T \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)] e^{-j2\pi(ux+vy)}\, dx\, dy\right] dt$$

$$G(u, v) = \int_0^T \left[\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f[x - x_0(t), y - y_0(t)] e^{-j2\pi(ux+vy)}\, dx\, dy \right] dt$$

$$\begin{aligned} G(u, v) &= \int_0^T F(u, v) e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt \\ &= F(u, v) \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt \end{aligned}$$

$$\Rightarrow \quad G(u, v) = H(u, v) F(u, v)$$

where $H(u, v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}\, dt$

$$H(u,v) = \int_0^T e^{-j2\pi u x_0(t)}\,dt$$

$x_a(t) = a\,t/T$

$$= \int_0^T e^{-j2\pi uat/T}\,dt \qquad (5.6\text{-}10)$$

$$= \frac{T}{\pi ua}\sin(\pi ua)e^{-j\pi ua}.$$

HW

$$H(u,v) = \frac{T}{\pi(ua+vb)}\sin[\pi(ua+vb)]e^{-j\pi(ua+vb)}$$

考題！

Results of inverse filtering and Wiener filtering

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Another example: inverse filtering vs Wiener filtering