Chapter 3 Intensity Transformations and Spatial Filtering

Image Enhancement in the Spatial Domain

- *No general theory* of image enhancement
- A certain amount of *trial and error* is usually required before a particular image enhancement approach is selected.

Outlines

3.4 Fundamentals of Spatial Filtering

- Filter, Mask, Kernel,
  Template, Window
- Coefficients
- Linear Filtering *vs*
  Nonlinear Filtering

  (e.g., median filtering)

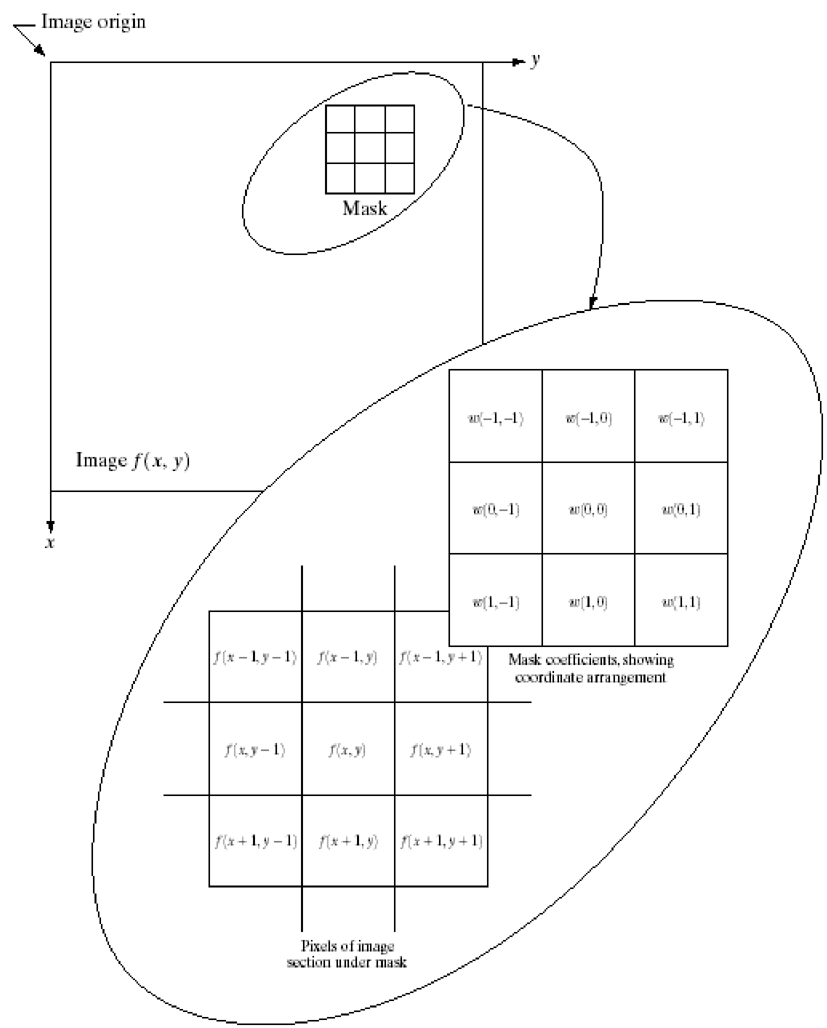
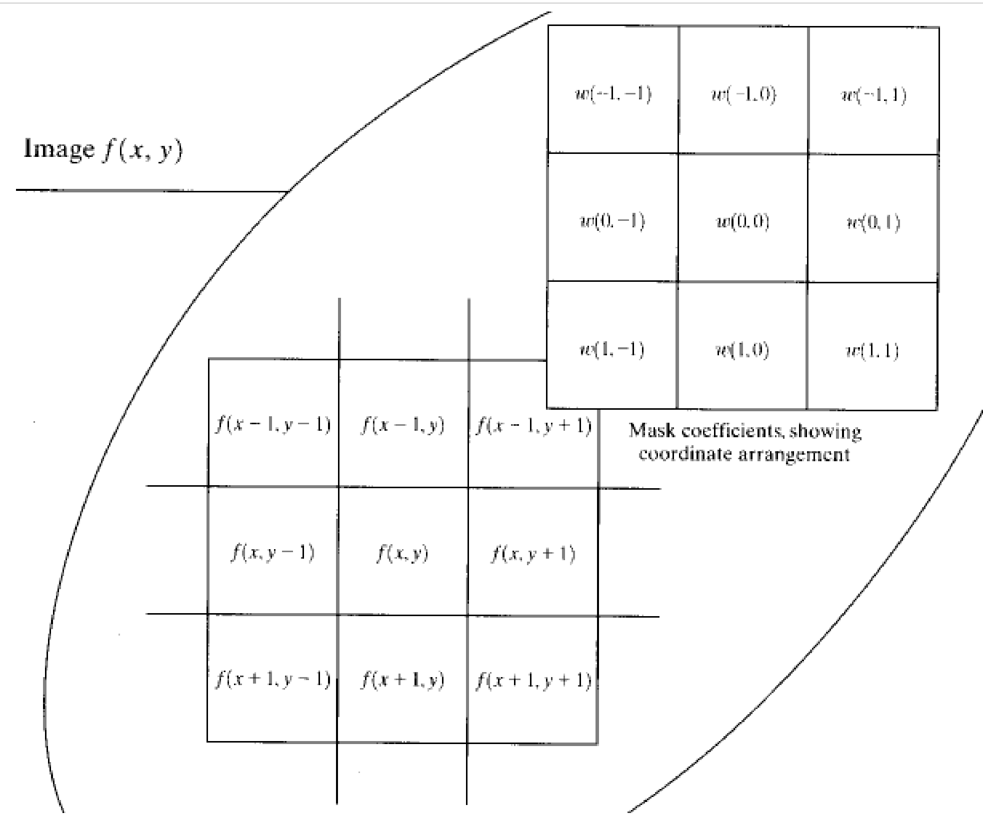


FIGURE 3.32 The mechanics of spatial filtering. The magnified drawing shows a 3 × 3 mask and the image section directly under it; the image section is shown displaced out from under the mask for ease of readability.

3 x 3 Mask, Sum of Products

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$
\begin{aligned}
R &= w_1 z_1 + w_2 z_2 + \ldots w_9 z_9 \\
&= \sum_{i=1}^{9} w_i z_i .
\end{aligned}
$$



$$
\begin{aligned}
R = {} & w(-1,-1)f(x-1, y-1) + w(-1,0)f(x-1, y) + \cdots \\
& + \underline{w(0,0)}\underline{f(x,y)} + \cdots + w(1,0)f(x+1, y) + w(1,1)f(x+1, y+1)
\end{aligned}
$$

M x N Mask

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)$$

for all pixels (x, y)

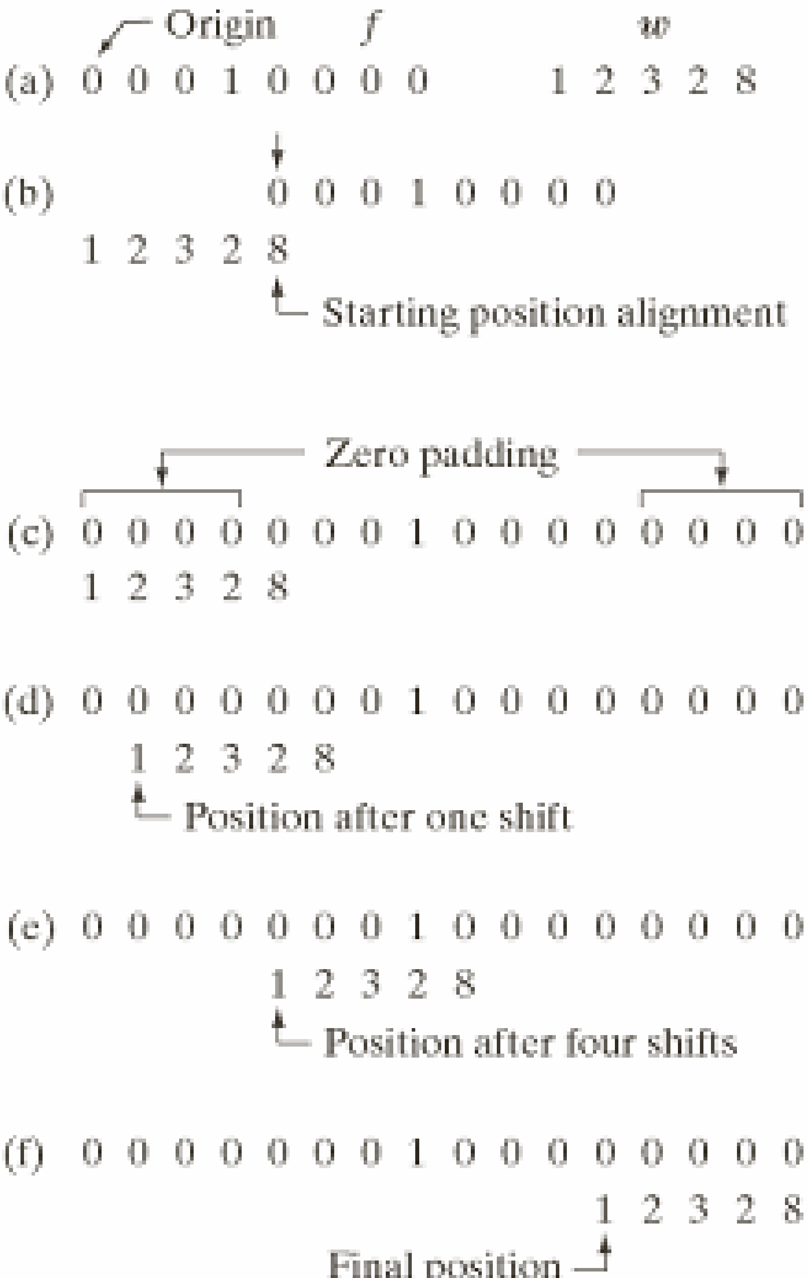
$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$
$$= \sum_{i=1}^{mn} w_i z_i$$

Spatial Correlation and Convolution

- Correlation

- Convolution

1D Correlation



1D Correlation

Origin f w

(a) 0 0 0 1 0 0 0 0 1 2 3 2 8

(b) 0 0 0 1 0 0 0 0
1 2 3 2 8

Starting position alignment

Zero padding

(c) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8

(f) 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
1 2 3 2 8

Final position

Full correlation result

(g) 0 0 0 8 2 3 2 1 0 0 0 0

Cropped correlation result

(h) 0 8 2 3 2 1 0 0

1D Correlation

1D Convolution

Origin f w

(a) 0 0 0 1 0 0 0 0 1 2 3 2 8

(b) 0 0 0 1 0 0 0 0

1 2 3 2 8

Starting position alignment

Origin f w rotated 180°

0 0 0 1 0 0 0 0 8 2 3 2 1 (i)

0 0 0 1 0 0 0 0 (j)

8 2 3 2 1

1D Convolution

Origin f w rotated 180°

0 0 0 1 0 0 0 0 8 2 3 2 1 (i)

0 0 0 1 0 0 0 0 (j)
8 2 3 2 1

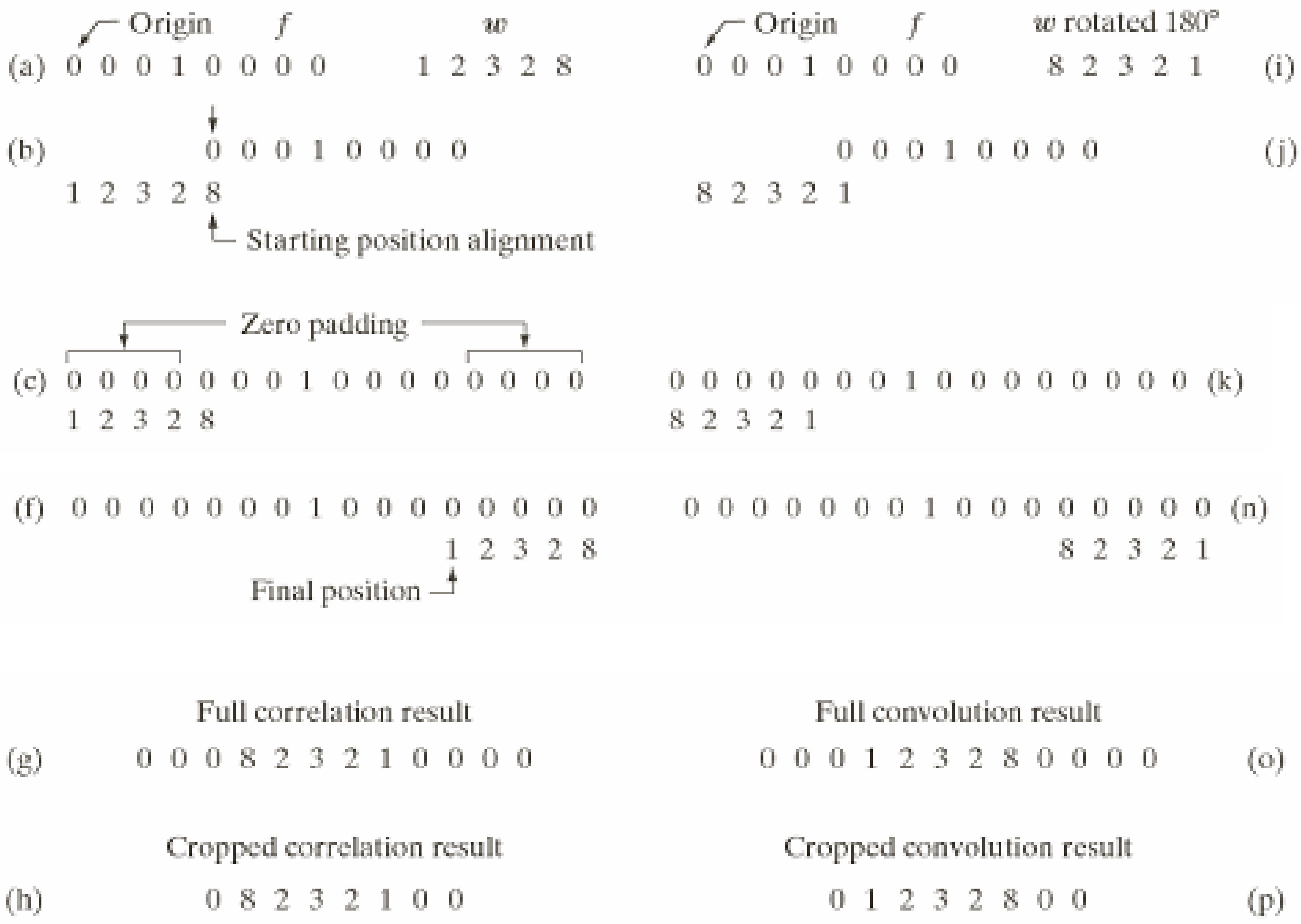
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (k)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (l)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (m)
8 2 3 2 1

0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 (n)
8 2 3 2 1

1D Correlation 1D Convolution



2D Correlation

Origin $f(x, y)$

$$\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}$$

$w(x, y)$

$$\begin{array}{ccc}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{array}$$

(a)

Padded f

$$\begin{array}{ccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

(b)

Initial position for w

$$\begin{array}{ccccccccc}
1 & 2 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 5 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 8 & 9 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

(c)

Full correlation result

$$\begin{array}{ccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 9 & 8 & 7 & 0 & 0 & 0 \\
0 & 0 & 0 & 6 & 5 & 4 & 0 & 0 & 0 \\
0 & 0 & 0 & 3 & 2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

(d)

Cropped correlation result

$$\begin{array}{ccccc}
0 & 0 & 0 & 0 & 0 \\
0 & 9 & 8 & 7 & 0 \\
0 & 6 & 5 & 4 & 0 \\
0 & 3 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}$$

(e)

2D Convolution

Origin $f(x, y)$

0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	0	0	0
0	0	0	0	0

$w(x, y)$

1	2	3
4	5	6
7	8	9

(a)

Padded f

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(b)

Rotated w

9	8	7	0	0	0	0	0	0
6	5	4	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(f)

Full convolution result

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	2	3	0	0	0
0	0	0	4	5	6	0	0	0
0	0	0	7	8	9	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(g)

Cropped convolution result

0	0	0	0	0
0	1	2	3	0
0	4	5	6	0
0	7	8	9	0
0	0	0	0	0

(h)

Correlation vs Convolution

- To perform convolution, all we do is rotate one function by 180^o and perform correlation.
  - Notice that it makes no difference which of the two function we rotate
- Using correlation or convolution to perform spatial filtering is *a matter of preference*
  - as long as the filter mask is specified correctly
- In the IP literature, "convolving a mask with an image" may refer to "correlation" operation.
- The concept of convolution is important for Chapter 4.

Operations concerning the border of the image

1. Limit the excursions of the mask
2. Padding zero
3. Padding by replicating rows or columns

Outlines

3.1 Background

3.2 Some Basic Intensity Transformation Functions

3.3 Histogram Processing

3.4 Fundamentals of Spatial Filtering

3.5 Smoothing Spatial Filters

3.6 Sharpening Spatial Filters

3.7 Combining Spatial Enhancement Methods

3.8 Using Fuzzy Techniques for Intensity Transformation and Spatial Filtering

3.5 Smoothing Spatial Filters

-- for blurring and for noise reduction

- Linear Smoothing Filters – averaging filters
- Nonlinear Smoothing Filters – median filters

3.5.1 Averaging Filters: *linear*

$$\frac{1}{9} \times \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

Box Filter

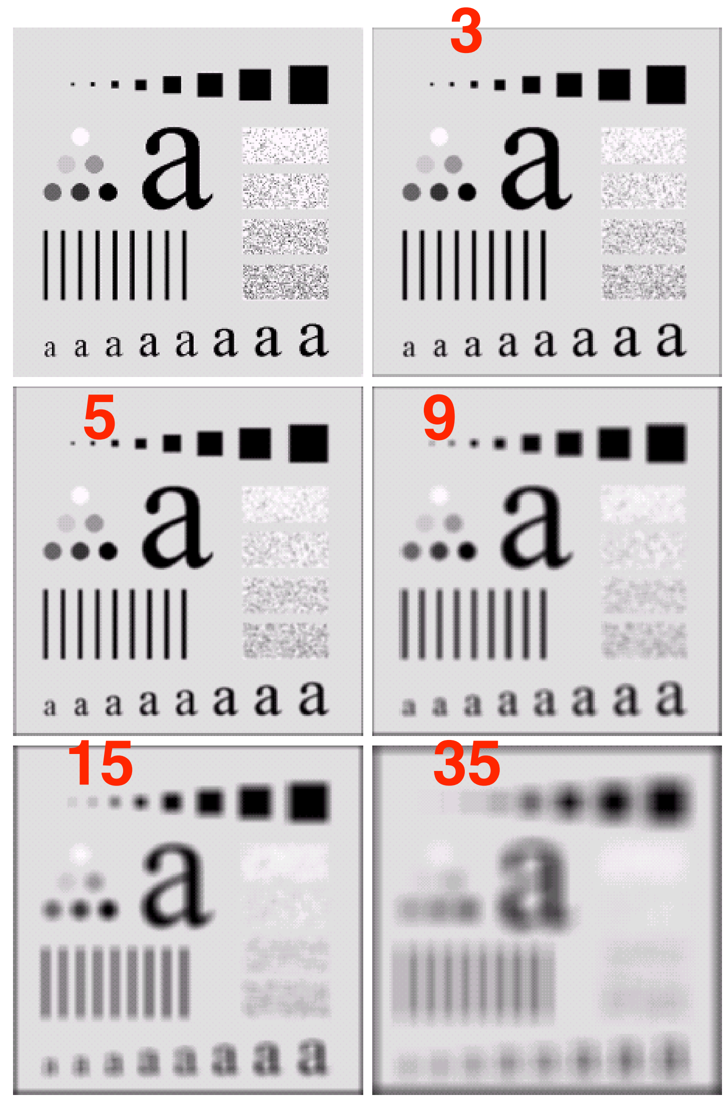
$$\frac{1}{16} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array}$$

Weighted Average

Why 16?
When to divide?

$$g(x, y) = \frac{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t)}{\sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t)}.$$

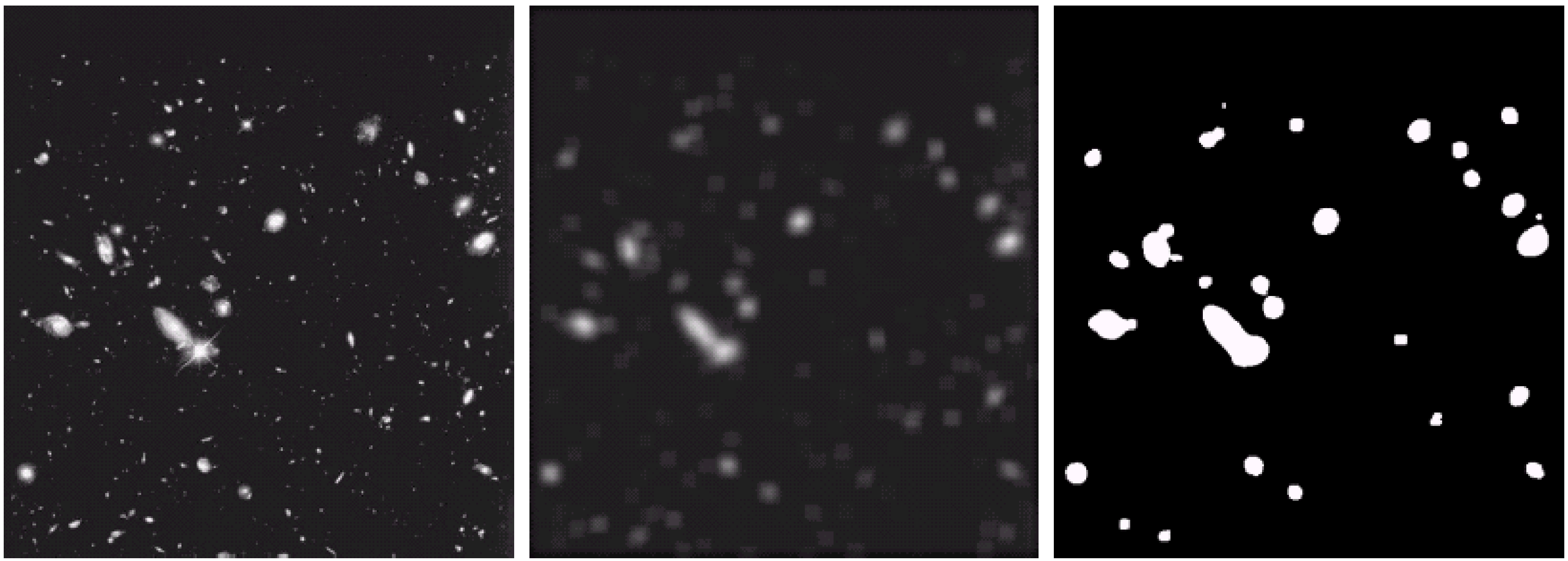
Averaging with different mask sizes



a b
c d
e f

FIGURE 3.35 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes $n = 3, 5, 9, 15$, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their gray levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

An Application of Averaging: Averaging before Thresholding



a b c

FIGURE 3.36 (a) Image from the Hubble Space Telescope. (b) Image processed by a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)

3.5.2 Order-Statistics Filters: *nonlinear*

中值濾波器藉由每一個pixel鄰近pixel灰階值排序的中間值來取代該pixel的灰階值。中值的計算是先將鄰近pixel(濾波器視窗範圍)灰階值排序，在取出排序居中的值作為濾波器中間位置影像的像素值。

Median Filter

-- the 50^{th} percentile of a ranked set of numbers

-- effective for reducing *impulse noise*, or *salt-and-pepper noise*

Max Filter

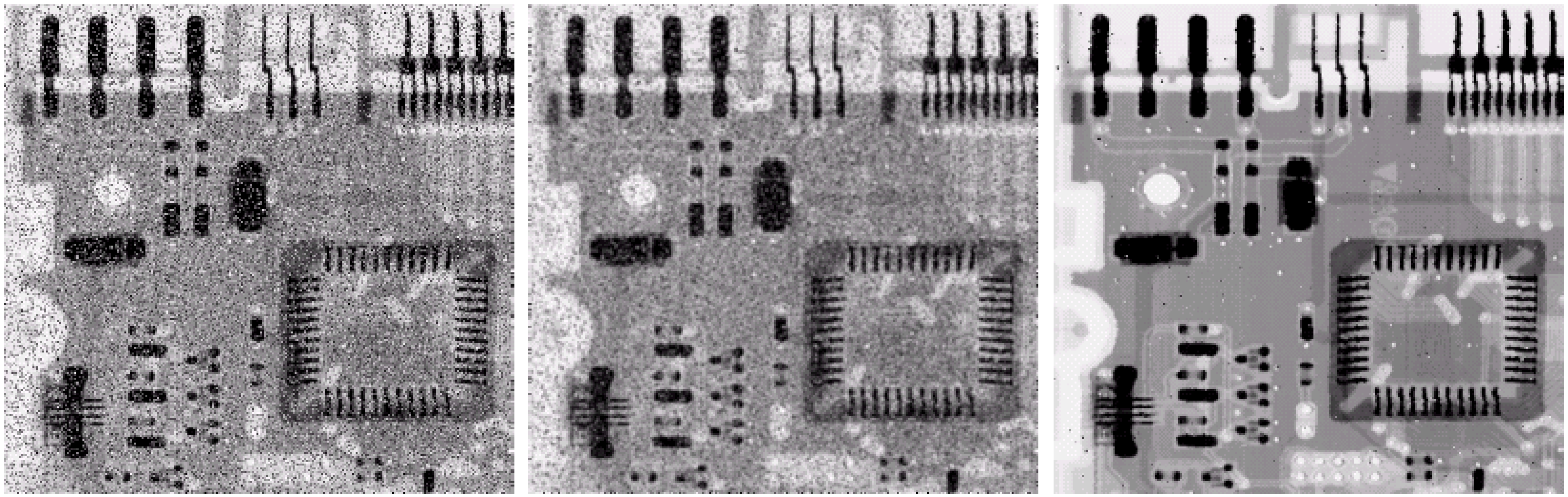
-- the 100^{th} percentile filter

More in chapter 5

Min Filter

-- the 0^{th} percentile filter

Comparison between Averaging Filter and Median Filter



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Outlines

3.6 Sharpening Spatial Filters

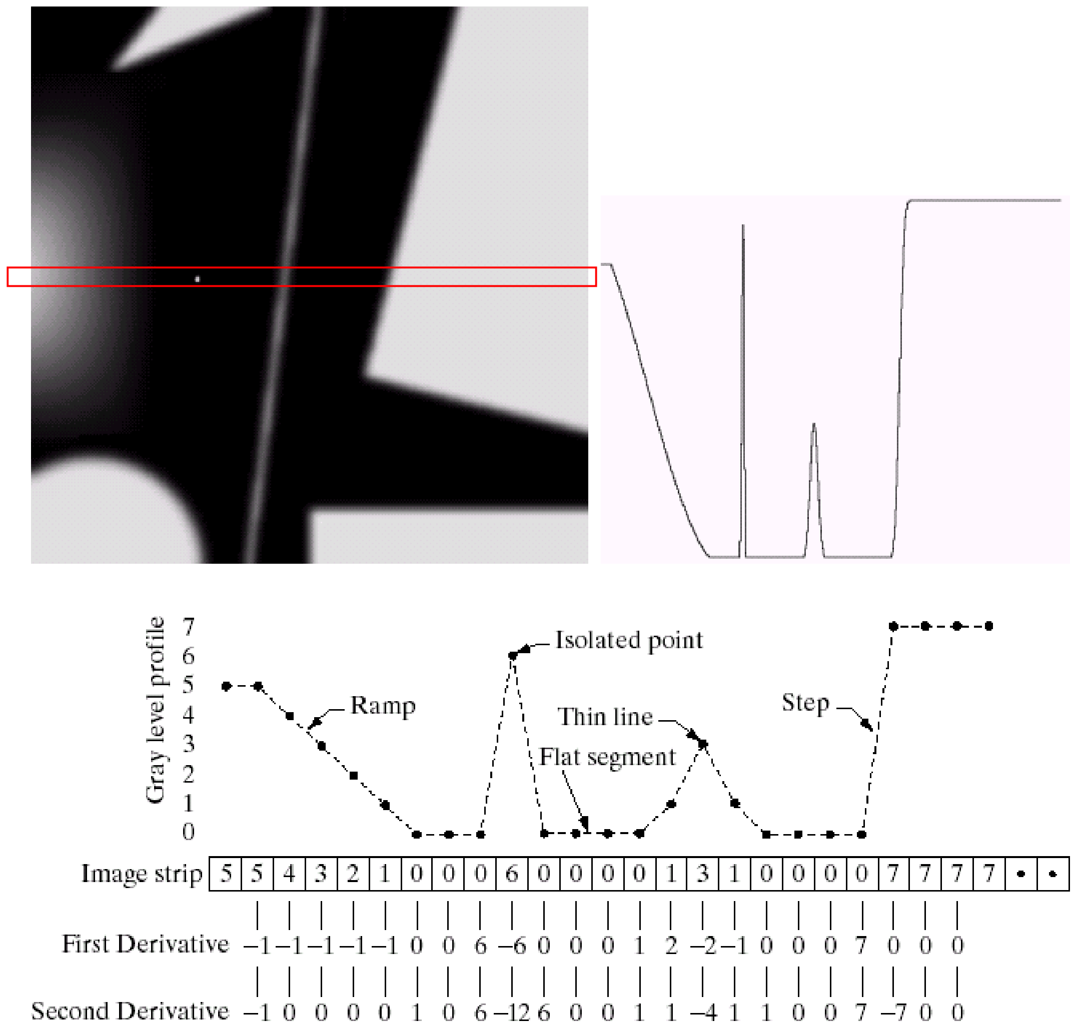
First-order derivatives

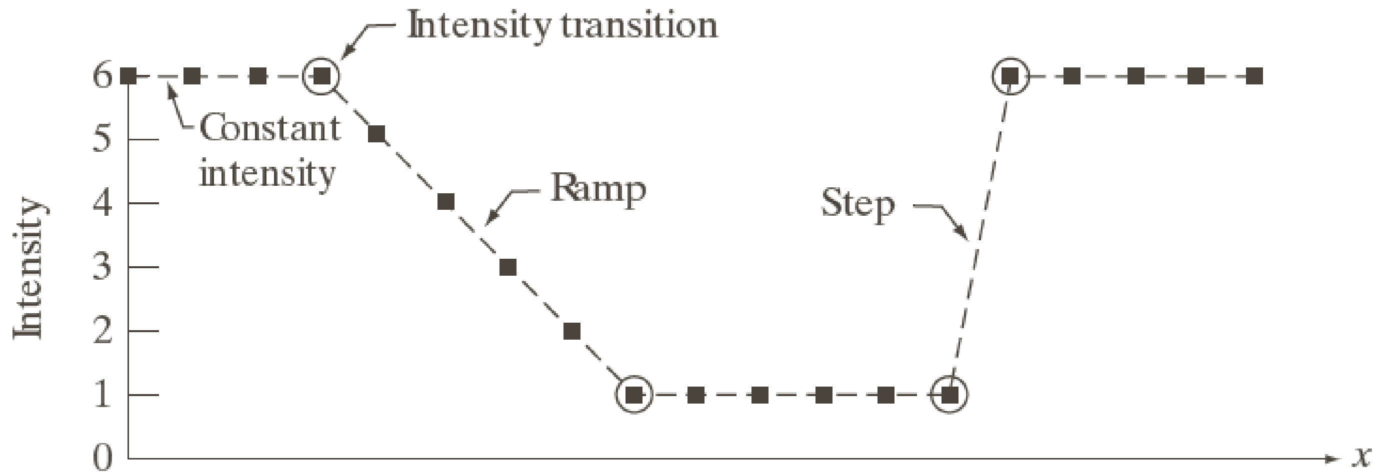
Second-order derivatives

3.6.1 Foundation -- A 1-D example

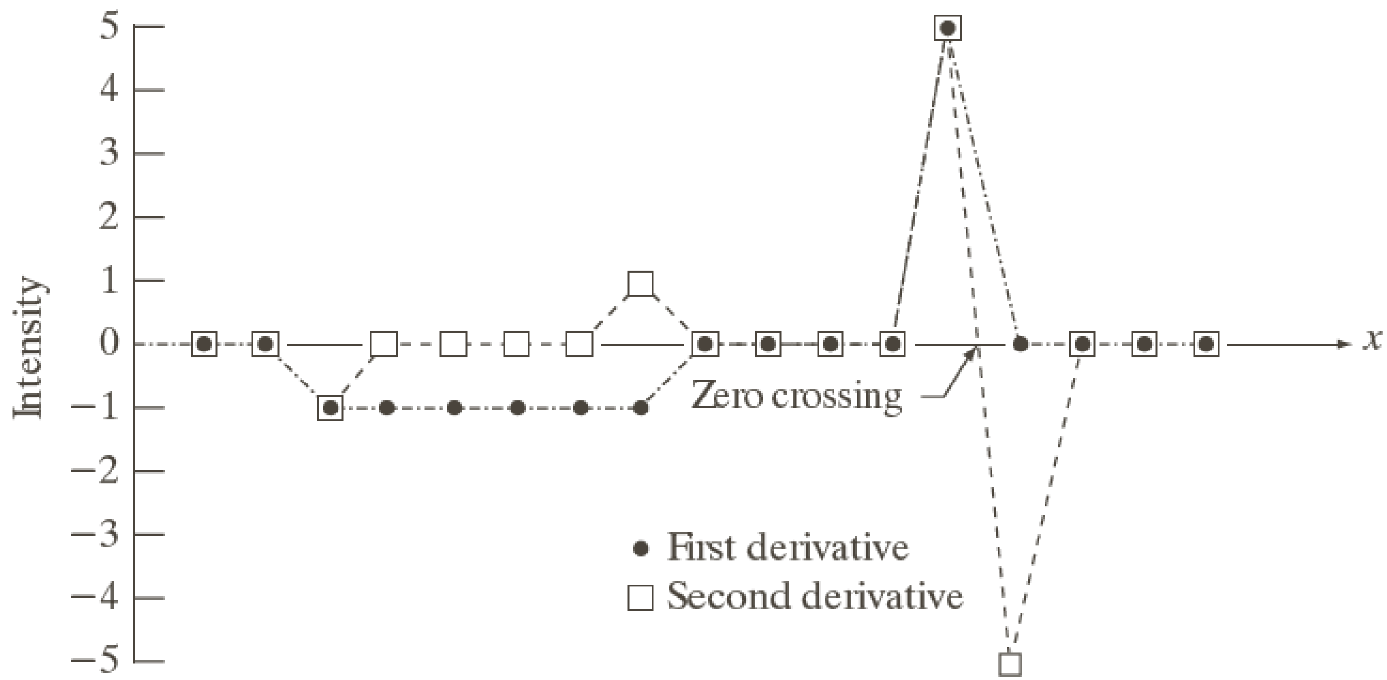
a b
c

FIGURE 3.38
(a) A simple image. (b) 1-D horizontal gray-level profile along the center of the image and including the isolated noise point. (c) Simplified profile (the points are joined by dashed lines to simplify interpretation).





Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



3.6.2 The Laplacian

-- isotropic, i.e., rotation-invariant ?

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

對x,y微兩次

Discrete form
$$\left\{ \begin{aligned} \frac{\partial^2 f}{\partial^2 x^2} &= f(x+1, y) + f(x-1, y) - 2f(x, y) \\ \frac{\partial^2 f}{\partial^2 y^2} &= f(x, y+1) + f(x, y-1) - 2f(x, y) \end{aligned} \right.$$

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y).$$

Filter Mask for Digital Laplacian

isotropic for increments of 90°

including diagonal neighbors
isotropic for increments of 45°

0	1	0
1	–4	1
0	1	0

1	1	1
1	–8	1
1	1	1

0	–1	0
–1	4	–1
0	–1	0

–1	–1	–1
–1	8	–1
–1	–1	–1

a	b
c	d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

Laplacian-Based Enhancement -- Unsharp Masking, Crispening

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

$$g(x, y) = \begin{cases} f(x, y) - \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is negative} \\ f(x, y) + \nabla^2 f(x, y) & \text{if the center coefficient of the Laplacian mask is positive.} \end{cases}$$

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

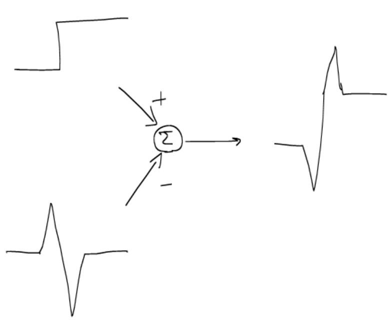
$$\begin{aligned} g(x, y) &= f(x, y) - [f(x+1, y) + f(x-1, y) \\ &\quad + f(x, y+1) + f(x, y-1)] + 4f(x, y) \\ &= 5f(x, y) - [f(x+1, y) + f(x-1, y) \\ &\quad + f(x, y+1) + f(x, y-1)]. \end{aligned}$$

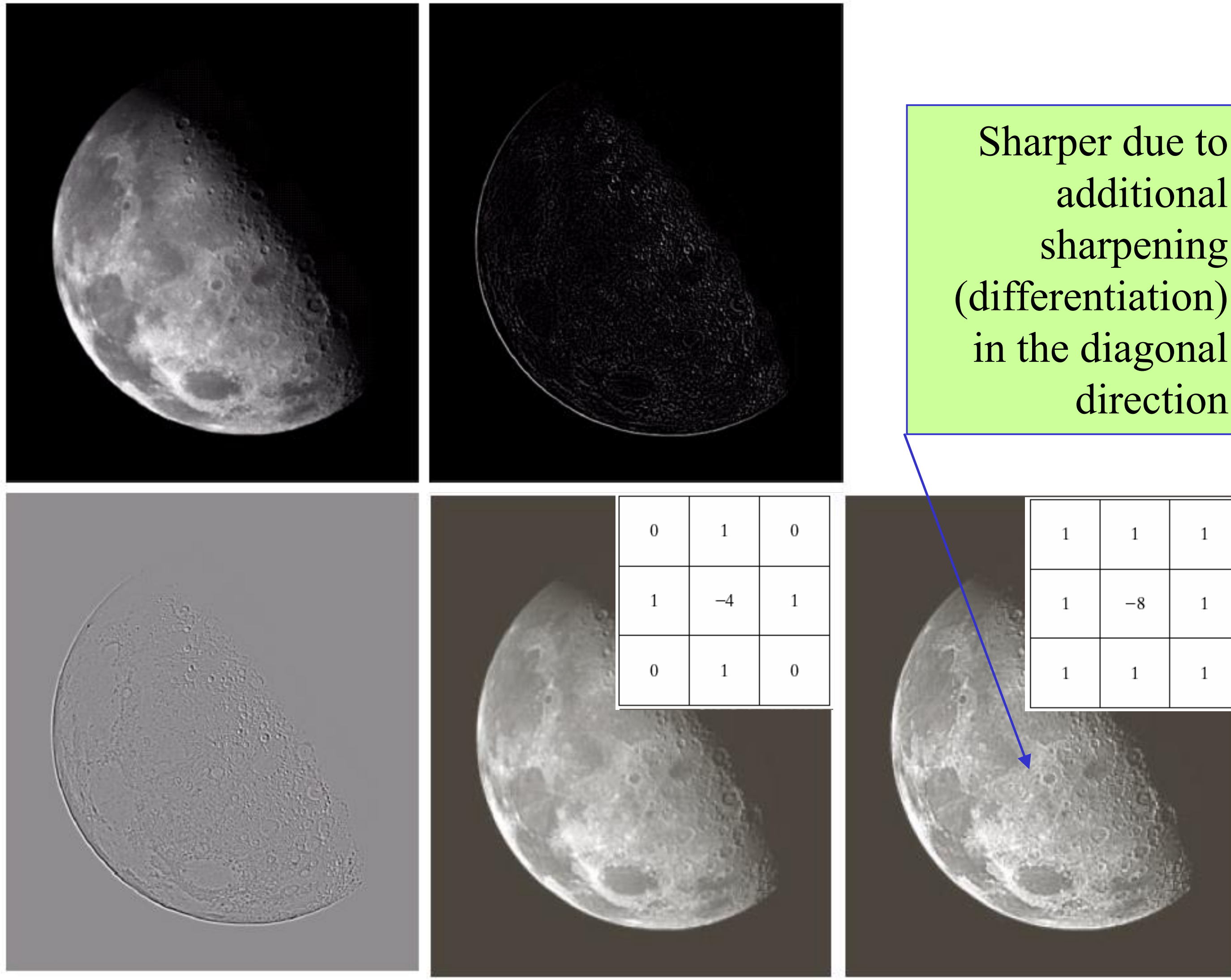
An Example of Laplacian-Based Enhancement

0	0	100	100
0	0	100	100
0	0	100	100

0	1	0
1	-4	1
0	1	0

0	100	-100	0
0	100	-100	0
0	100	-100	0





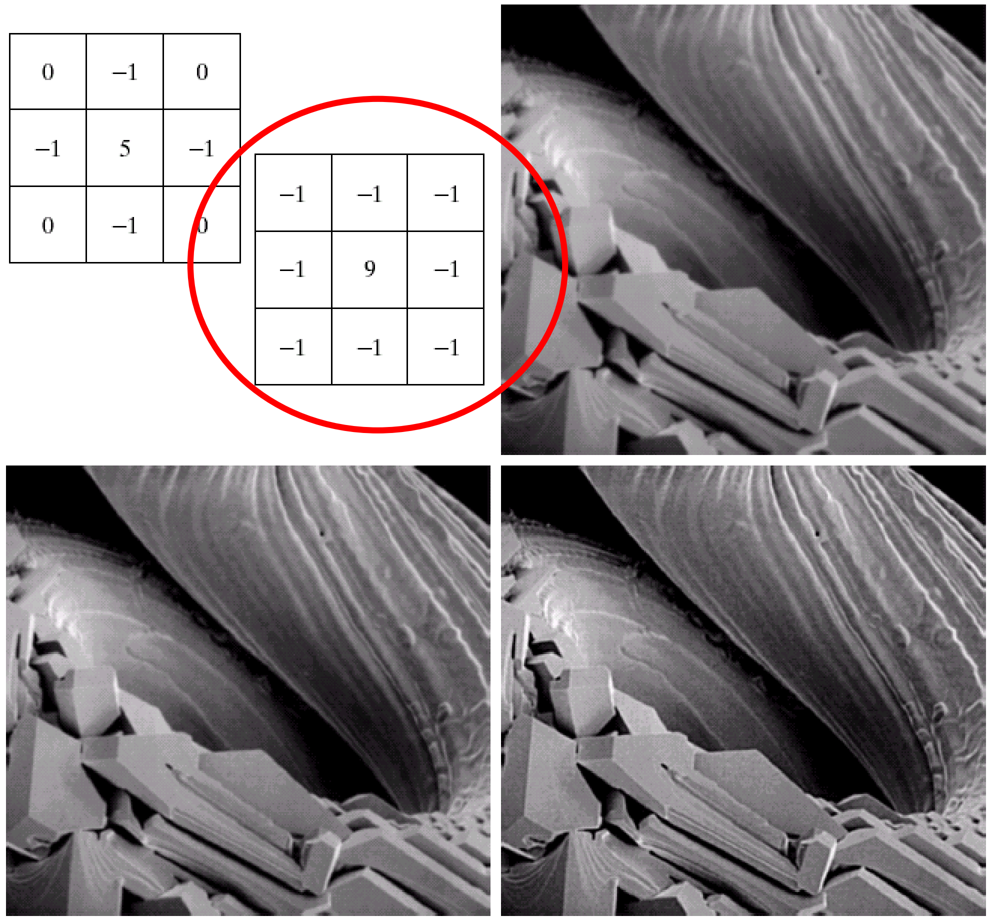
0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

Sharper due to additional sharpening (differentiation) in the diagonal direction

Laplacian-Based Enhancement
-- implemented with one pass of a single mask

$$\begin{matrix} 1 & -2 & 1 \\ -2 & 5 & -2 \\ 1 & -2 & 1 \end{matrix}$$



0	−1	0
−1	5	−1
0	−1	0

−1	−1	−1
−1	9	−1
−1	−1	−1

a b c
d e

FIGURE 3.41 (a) Composite Laplacian mask. (b) A second composite mask. (c) Scanning electron microscope image. (d) and (e) Results of filtering with the masks in (a) and (b), respectively. Note how much sharper (e) is than (d). (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene.)

Unsharp Masking (Crispening)

-- a common process in the publishing industry

- Enhance the edges
  - Edge enhancement means first isolating the edges in an image (high freq, e.g., Laplacian), amplifying them, and then adding them back into the image

$$
\begin{aligned}
f_s(x,y) &= f(x,y) + c\, f_{HF}(x,y) \\
&= f(x,y) + c\,\nabla^2 f(x,y)
\end{aligned}
$$

- Subtracts the "unsharp" (smoothed)
  - subtract a specified fraction of the smoothed ("unsharp") image from the original, then add the result back to the original

$$
\begin{aligned}
f_s(x,y) &= f(x,y) + k\, g_{mask}(x,y) \\
&= f(x,y) + k\,[f(x,y) - \bar{f}(x,y)]
\end{aligned}
$$

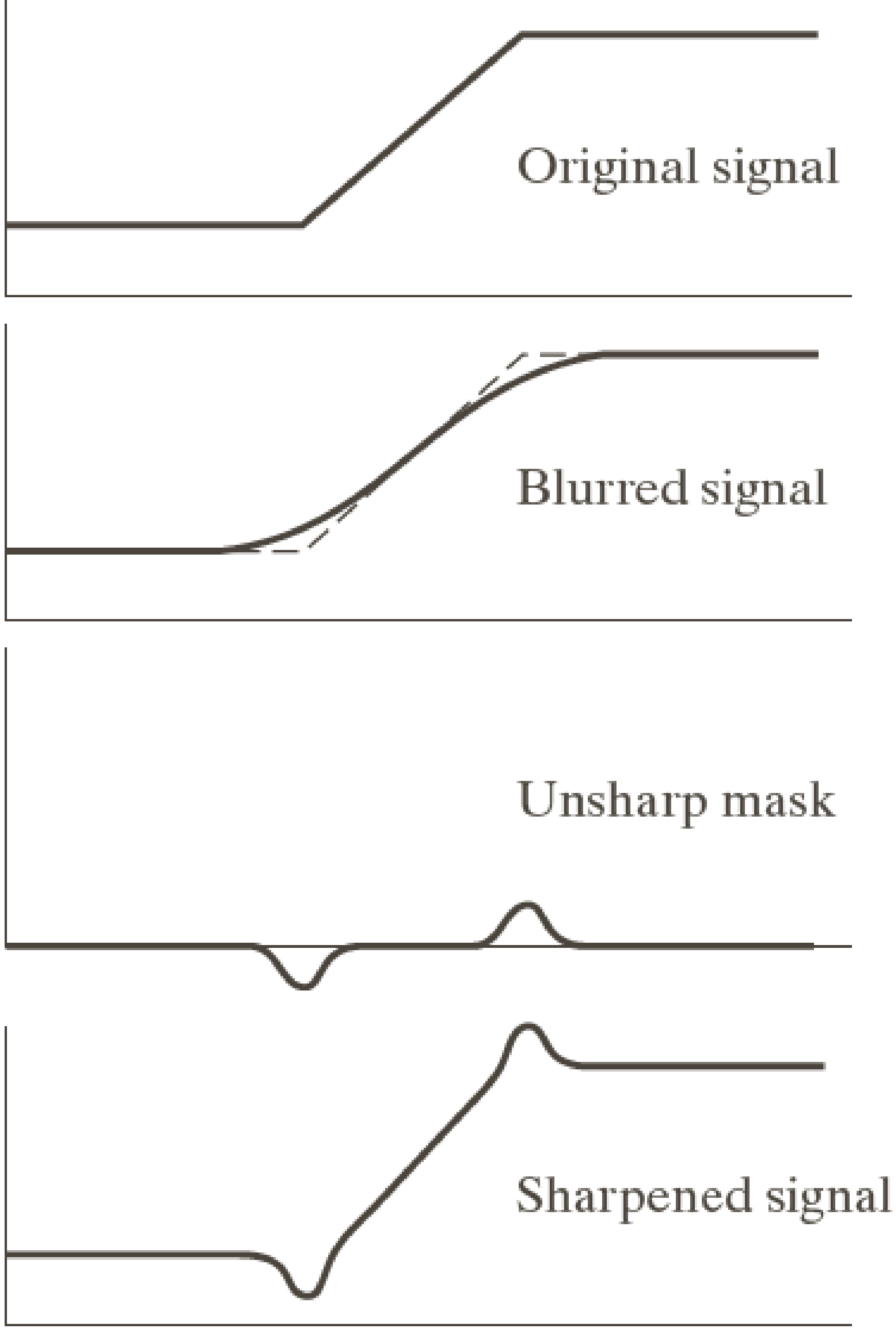
Unsharp Masking

1. Blur the original image
2. Subtract the blurred image from the original
   - -> mask
3. Add the mask to the original

$$f_s(x,y) = f(x,y) + k\ g_{mask}(x,y)$$
$$= f(x,y) + k\,[f(x,y) - \bar{f}(x,y)]$$

k=1, ➔ unsharp masking

k>1, ➔ highboost filtering



a
b
c
d

FIGURE 3.39 1-D illustration of the mechanics of unsharp masking. (a) Original signal. (b) Blurred signal with original shown dashed for reference. (c) Unsharp mask. (d) Sharpened signal, obtained by adding (c) to (a).

a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

3.6 Sharpening Spatial Filters

3.6.1 Foundation

3.6.2 The Laplacian

3.6.3 The Gradient (i.e., Gradient Magnitude)

First-order derivatives

Second-order derivatives

3.6.3 The Gradient

- Gradient Vector

$$\nabla \mathbf{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix}.$$

- Gradient Magnitude

$$\begin{aligned} |\nabla f| &= \mathrm{mag}(\nabla \mathbf{f}) \\ &= \left[G_x^2 + G_y^2\right]^{1/2} \\ &= \left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2}. \end{aligned}$$

Approximation of Gradient Magnitude

$$\nabla f \approx |G_x| + |G_y|.$$

Digital Approximation ➔ Filter Masks

- Simplest Approximation

0	0	0
0	-1	0
0	1	0

$$G_x = (z_8 - z_5)$$

0	0	0
0	-1	1
0	0	0

$$G_y = (z_6 - z_5)$$

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Common Filter Masks for Computing the Gradient

a
b c
d e

FIGURE 3.44
A 3 × 3 region of an image (the z's are gray-level values) and masks used to compute the gradient at point labeled z_5. All masks coefficients sum

derivative operator.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Roberts Cross-Gradient

−1	0
0	1

0	−1
1	0

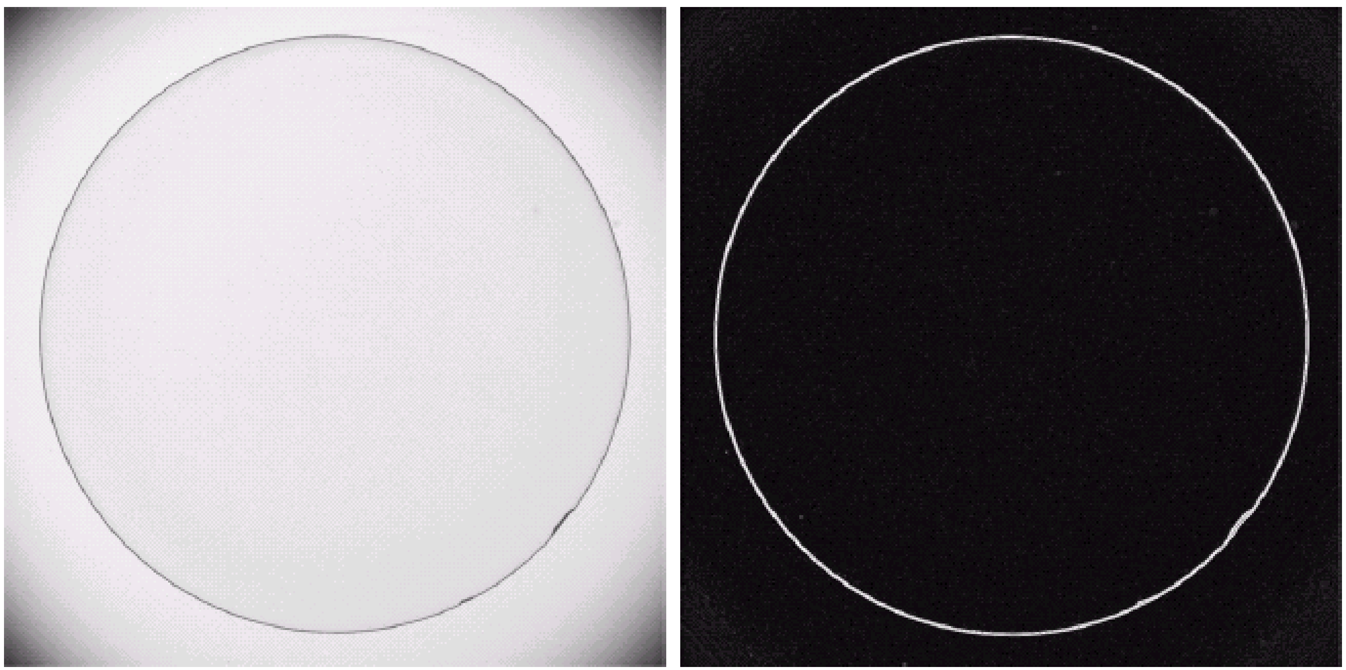
Sobel Operators

−1	−2	−1
0	0	0
1	2	1

−1	0	1
−2	0	2
−1	0	1

Gradient-based enhancement for automated inspection

-- Used as a pre-processing step for automated inspection

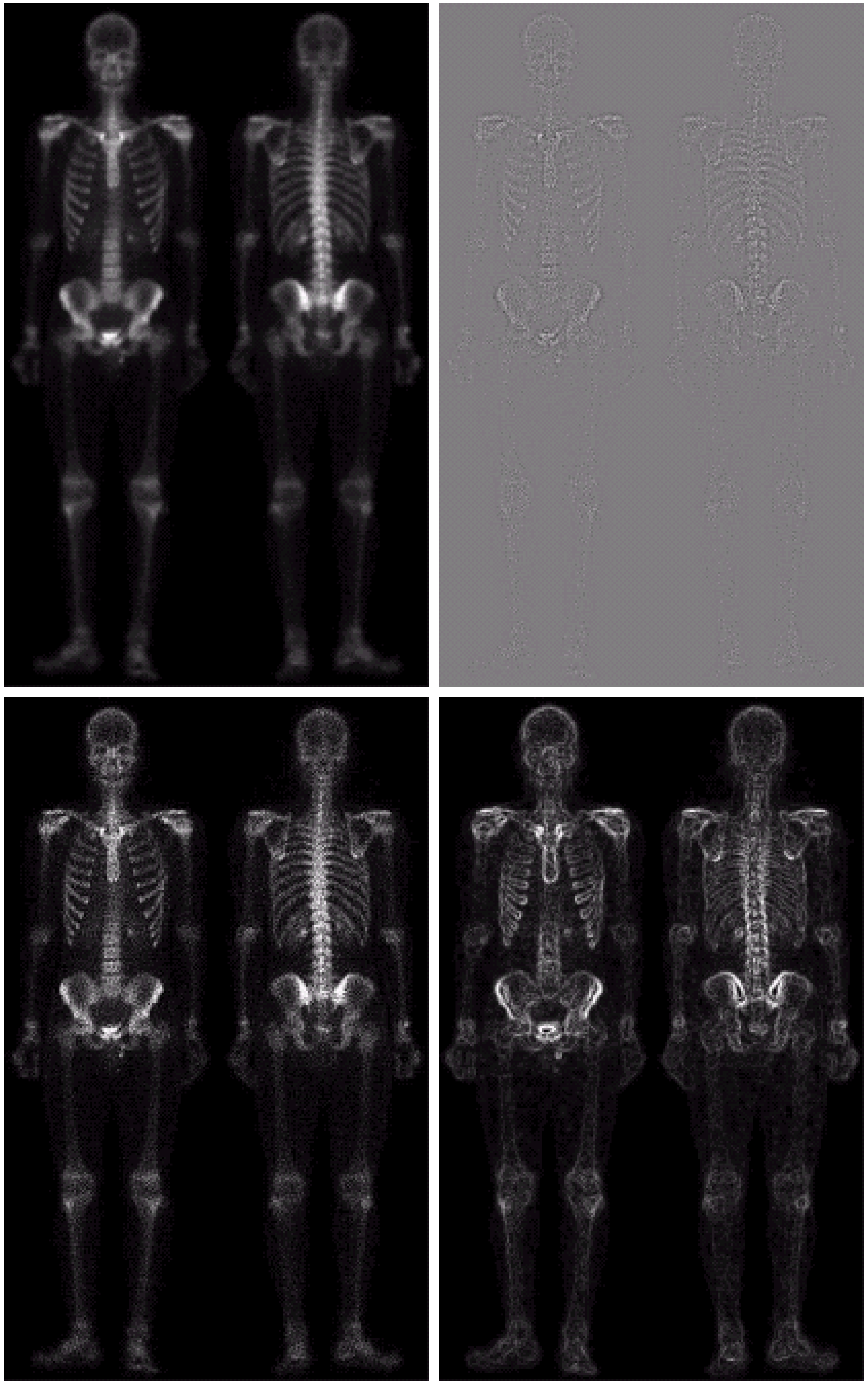


a b

FIGURE 3.45 Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Mr. Pete Sites, Perceptics Corporation.)

Outlines

3.7 Combining Spatial Enhancement Methods



a	b
c	d

FIGURE 3.46
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel of (a).

e	f
g	h

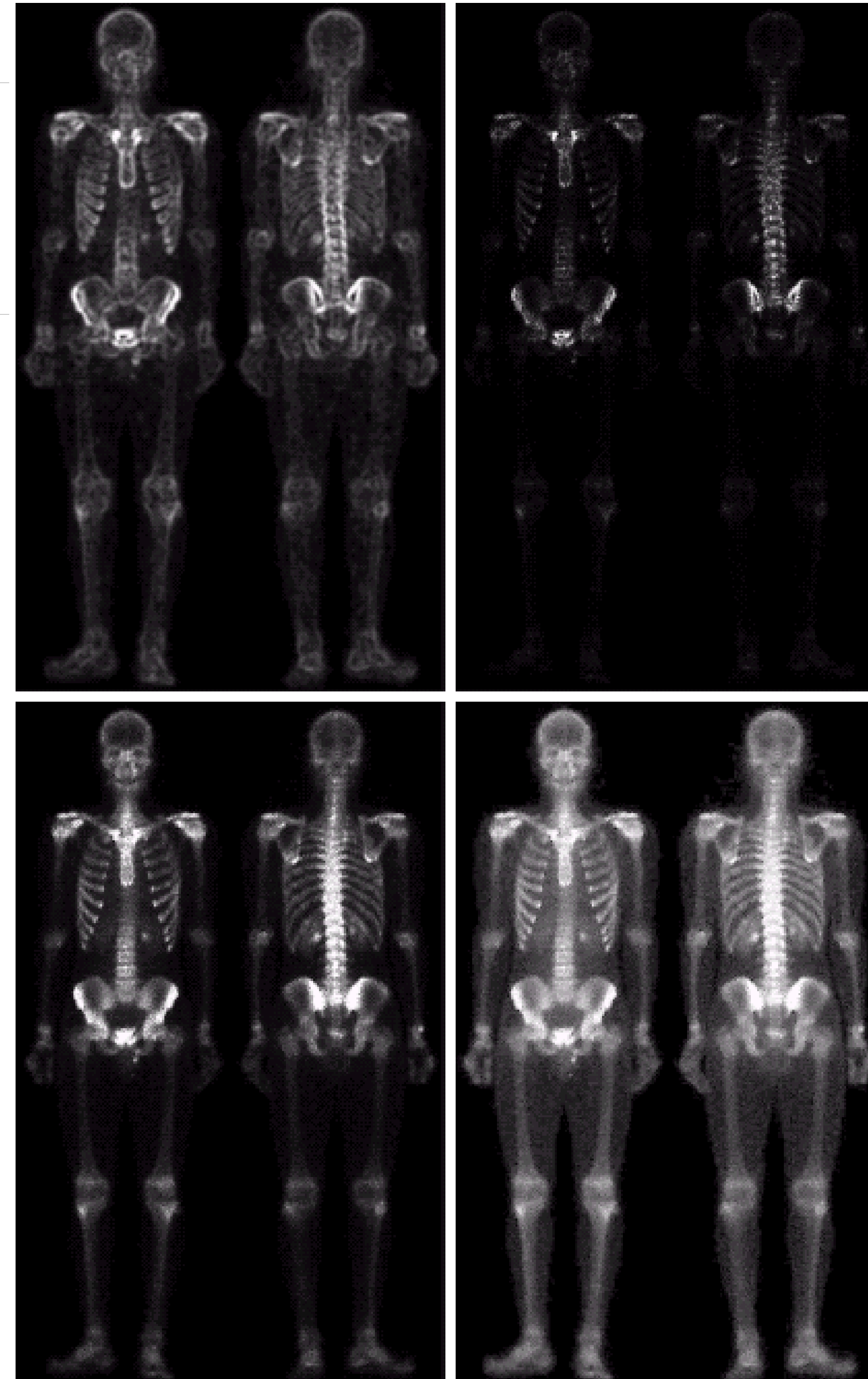


FIGURE 3.46 *(Continued)* (e) Sobel image smoothed with a 5 × 5 averaging filter. (f) Mask image formed by the product of (c) and (e). (g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)