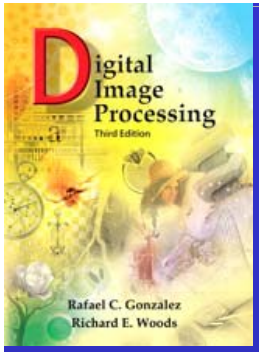
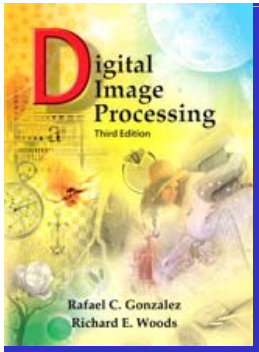


Chapter 3

強度轉換與空間濾波

空間濾波的基本原理與各式濾波器



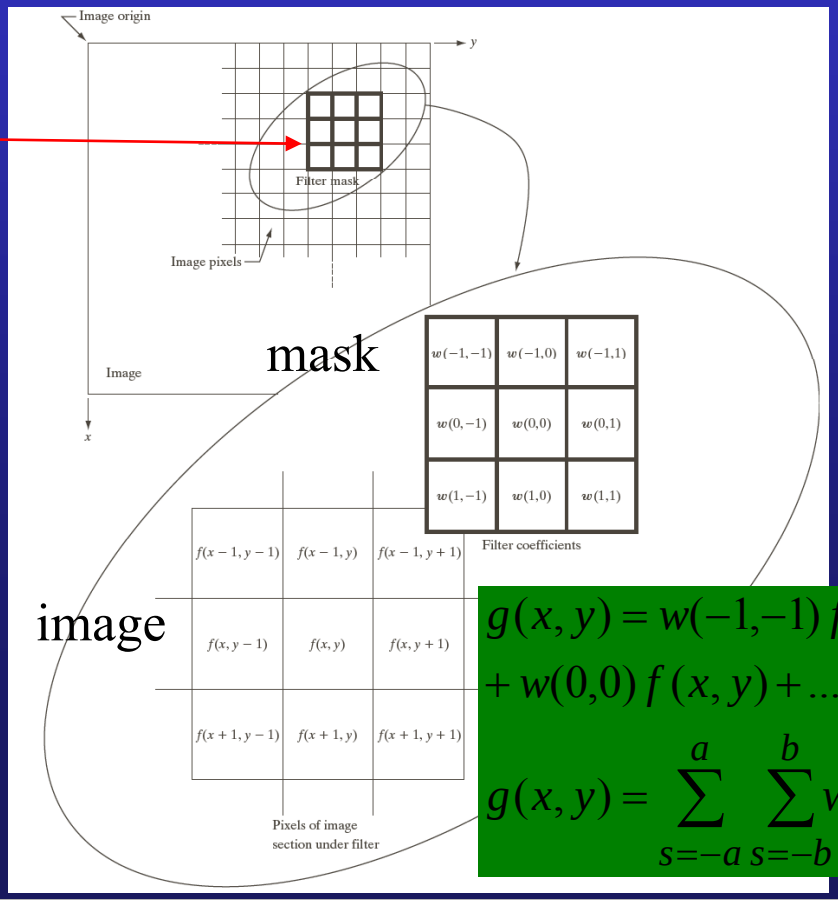
Chapter 3

強度轉換與空間濾波

空間濾波(Spatial filtering)

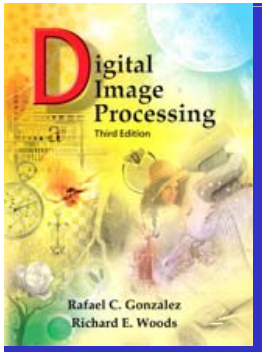
將濾波遮罩在影像中移動，在每一點(x,y)計算濾波係數及遮罩所涵蓋的影像區域的像素乘積和

遮罩mask
濾波器filter
核心 kernel
模板template
窗window



$$g(x,y) = w(-1,-1)\,f(x-1,y-1) + w(-1,0)\,f(x-1,y) + \ldots$$
$$+ w(0,0)\,f(x,y) + \ldots + w(1,1)\,f(x+1,y+1)$$

$$g(x,y) = \sum_{s=-a}^{a} \sum_{s=-b}^{b} w(s,t)\,f(x+s,y+t)$$

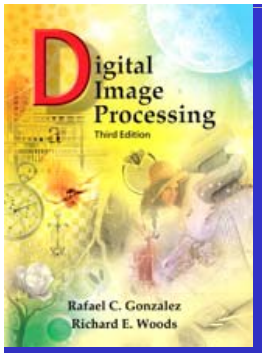


Chapter 3

強度轉換與空間濾波

當執行線性空間濾波時，有兩個緊密相關的觀念必須清楚了解，一個是相關性 (correlation)，另一個是迴旋積 (convolution)。

- 相關性是一個程序，此程序是把一濾波器遮罩在影像上移動並在每一個位置上計算乘積的和，和前一節中所解釋的完全一樣。
- 迴旋積的技術與相關性相同，只不過濾波器先旋轉180°。



Chapter 3

強度轉換與空間濾波

相關性　　迴旋積

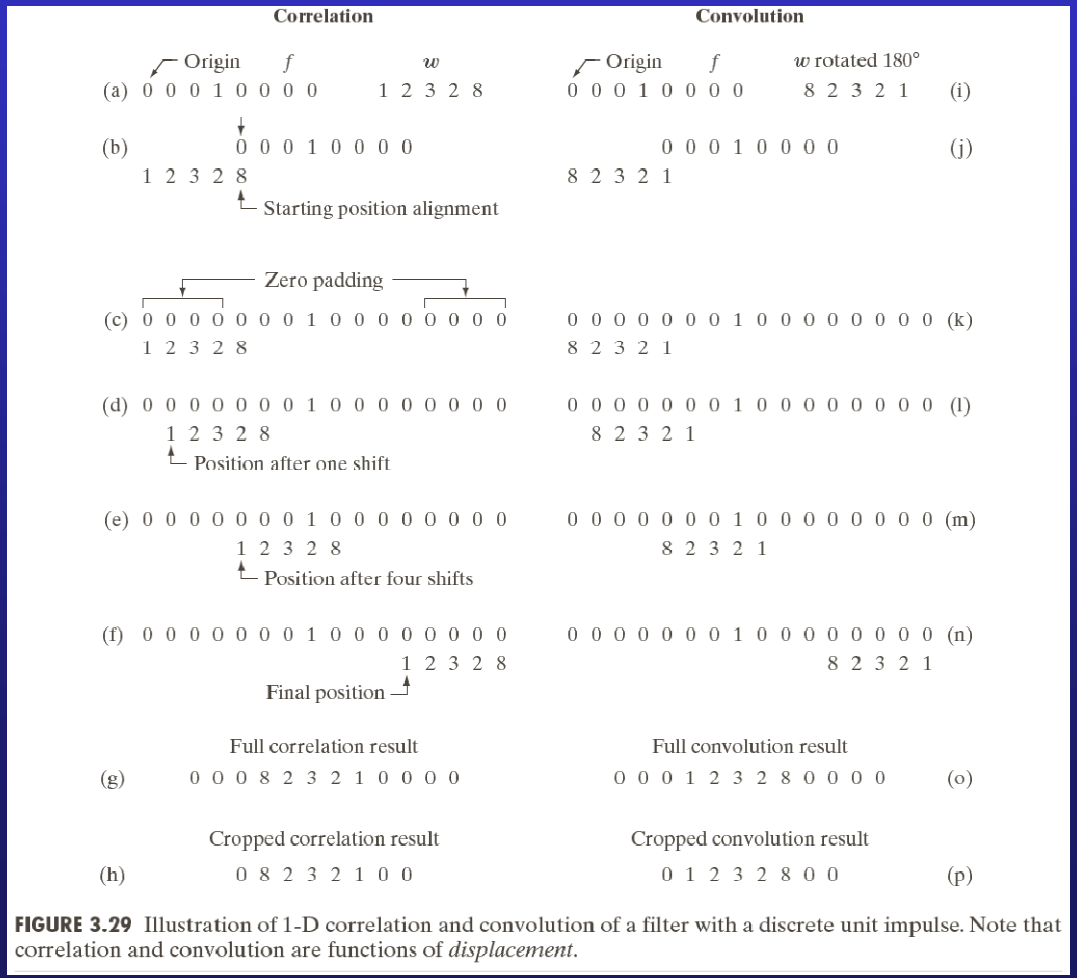


FIGURE 3.29 Illustration of 1-D correlation and convolution of a filter with a discrete unit impulse. Note that correlation and convolution are functions of *displacement*.

相關性

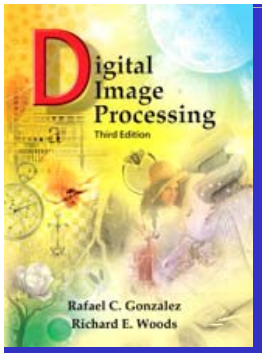
$$w(x,y) \circ f(x,y)$$

$$= \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x+s, y+t)$$

迴旋積

$$w(x,y) \bullet f(x,y)$$

$$= \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) f(x-s, y-t)$$



Chapter 3

強度轉換與空間濾波

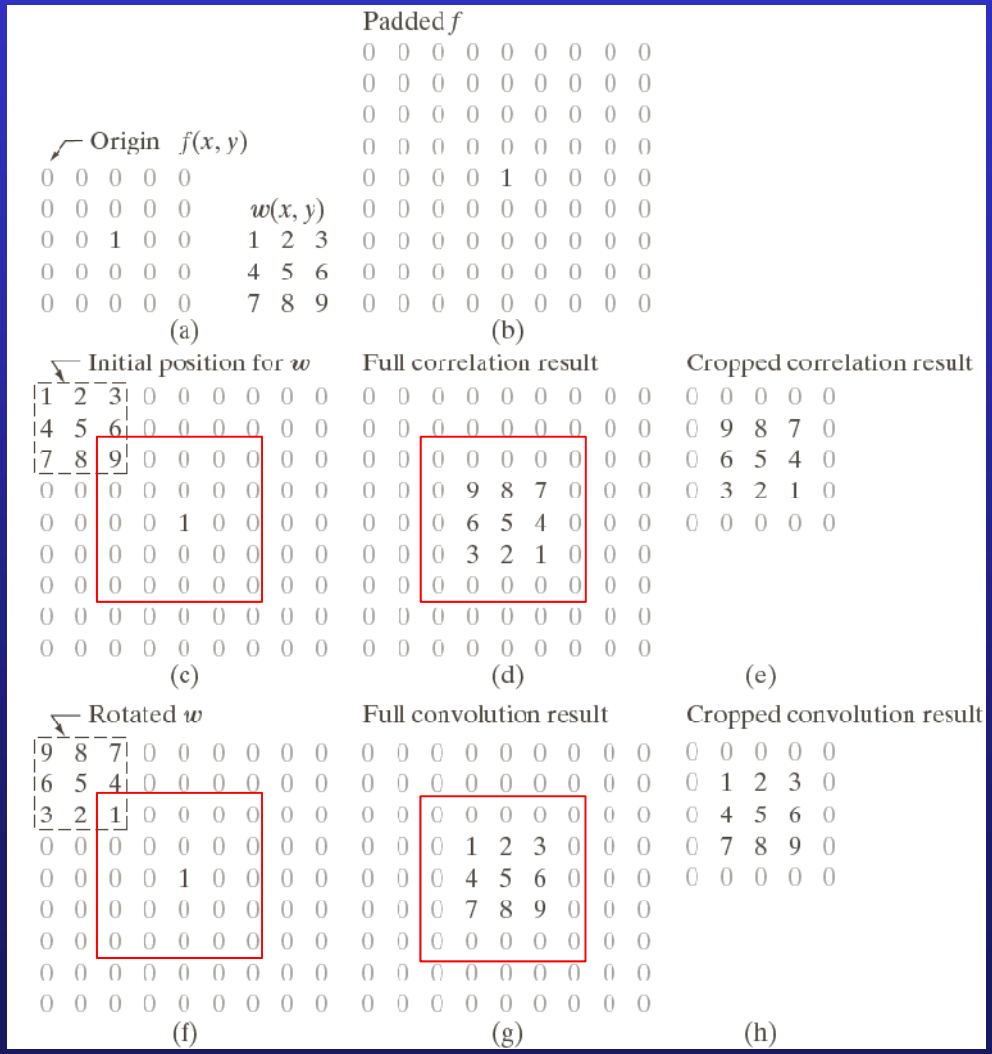
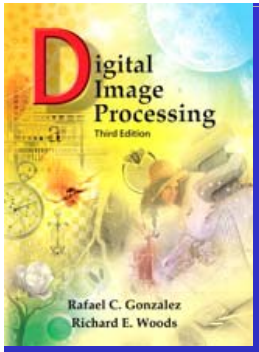


FIGURE 3.30 Correlation (middle row) and convolution (last row) of a 2-D filter with a 2-D discrete, unit impulse. The 0s are shown in gray to simplify visual analysis.



Chapter 3

強度轉換與空間濾波

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_{mn} z_{mn}$$
$$= \sum_{i=1}^{mn} w_k z_k$$
$$= w^T z$$

FIGURE 3.31 Another representation of a general 3 × 3 filter mask.

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

h(-1,-1)	*h*(-1,0)	*h*(-1,1)
h(0,-1)	*h*(0,0)	*h*(0,1)
h(1,-1)	*h*(1,0)	*h*(1,1)

對於3*3的遮罩

$$R = w_1 z_1 + w_2 z_2 + \ldots + w_9 z_9$$
$$= \sum_{k=1}^{9} w_k z_k$$
$$= w^T z$$

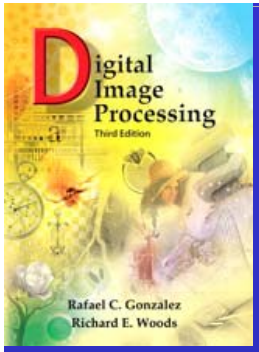
產生空間濾波器遮罩 *h*(*x*,*y*)

1.平均濾波器

$$h(x, y) = 1 \quad x = -1,0,1 \text{ and } y = -1,0,1$$

2. 高斯濾波器

$$h(x, y) = e^{\frac{x^2+y^2}{2\sigma^2}}, \quad x = -1,0,1 \text{ and } y = -1,0,1$$



Chapter 3

強度轉換與空間濾波

平滑空間濾波器
Smoothing spatial filters

- 用於模糊化跟減少雜訊
- 以濾波器所定義的平均值取代原灰階值
- 對於銳利的邊緣也有模糊的負效果

General form

$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)\,f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)}$$

Box filter
(所有係數都相等)

Weighted average
(每個像素有不同的重要性)

$$R=\frac{1}{9}\sum_{i=1}^{9}z_i$$

$\frac{1}{9}\times$

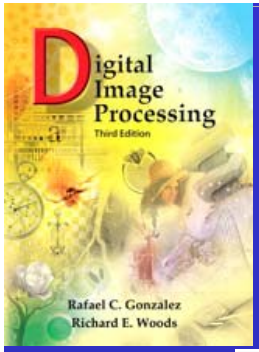
1	1	1
1	1	1
1	1	1

$\frac{1}{16}\times$

1	2	1
2	4	2
1	2	1

a b

FIGURE 3.32 Two 3×3 smoothing (averaging) filter masks. The constant multiplier in front of each mask is equal to 1 divided by the sum of the values of its coefficients, as is required to compute an average.



Chapter 3

強度轉換與空間濾波

N=3

使用方形濾波器的結果

N=5

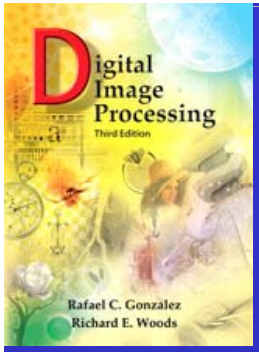
N=9

N=15

N=35

FIGURE 3.33 (a) Original image, of size 500 × 500 pixels. (b)–(f) Results of smoothing with square averaging filter masks of sizes m = 3, 5, 9, 15, and 35, respectively. The black squares at the top are of sizes 3, 5, 9, 15, 25, 35, 45, and 55 pixels, respectively; their borders are 25 pixels apart. The letters at the bottom range in size from 10 to 24 points, in increments of 2 points; the large letter at the top is 60 points. The vertical bars are 5 pixels wide and 100 pixels high; their separation is 20 pixels. The diameter of the circles is 25 pixels, and their borders are 15 pixels apart; their intensity levels range from 0% to 100% black in increments of 20%. The background of the image is 10% black. The noisy rectangles are of size 50 × 120 pixels.

a b
c d
e f



Chapter 3

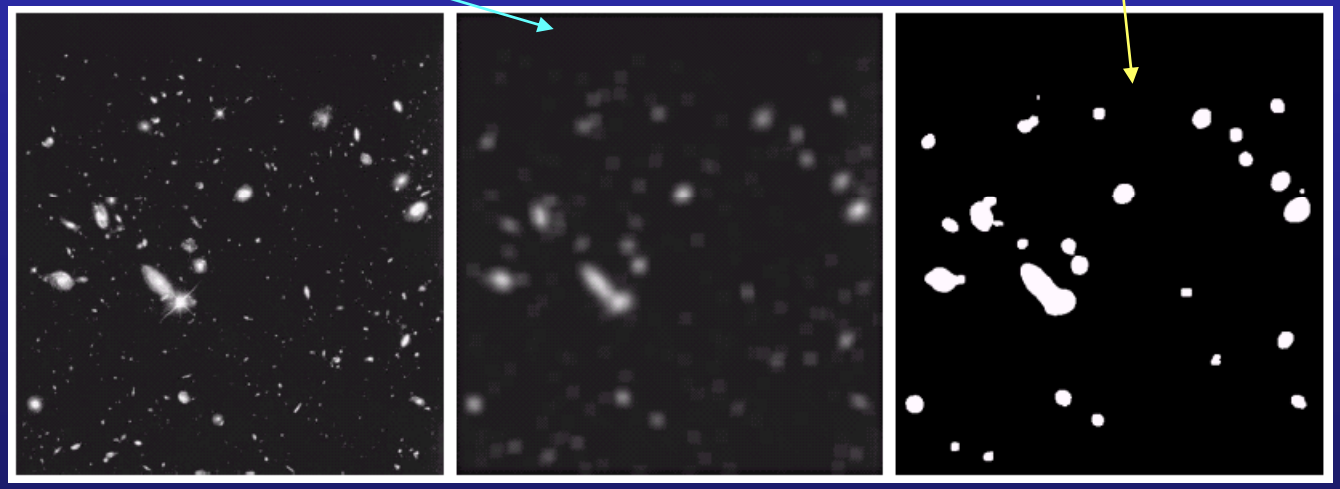
強度轉換與空間濾波

影像平均

遮罩的大小建立了會被融入背景的物體的相對大小，可用於標示較大較亮的物體

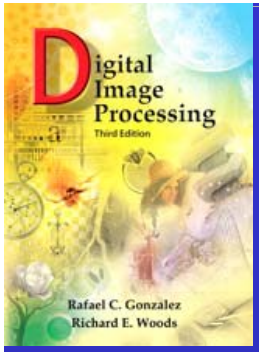
After averaging, the small objects blend with background

Threshing with a threshold value equal to 25% of the highest intensity in the blurred image, the small objects are eliminated.



a b c

FIGURE 3.34 (a) Image of size 528 × 485 pixels from the Hubble Space Telescope. (b) Image filtered with a 15 × 15 averaging mask. (c) Result of thresholding (b). (Original image courtesy of NASA.)



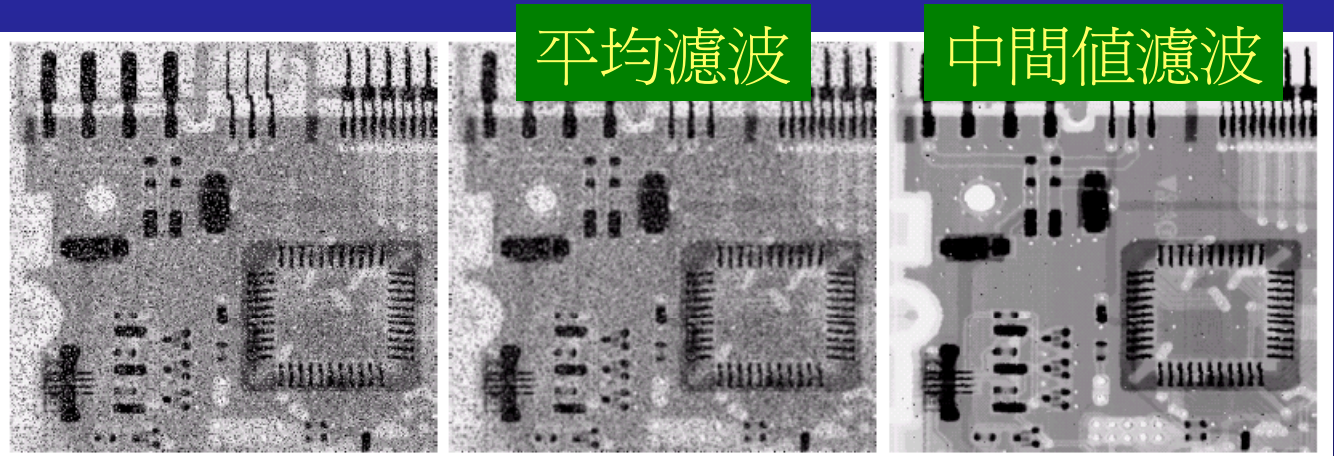
Chapter 3

強度轉換與空間濾波

排序統計濾波器 (Order-statistics filters)

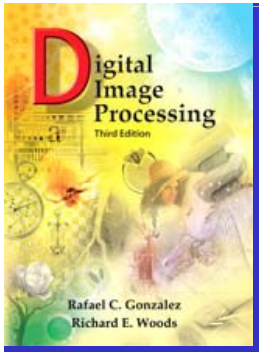
非線性的濾波器，其響應建立在由濾波器所包圍的影像區域中所含的像素順序上

- 中值濾波器 Median filter
  - 將像素值用該鄰近區域像素的 “中間值” 代替
  - 適用於胡椒鹽式雜訊(salt and pepper noise)
- Max filter, Min filter



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)



Chapter 3

強度轉換與空間濾波

- 中值濾波器

將九宮格中的灰階值重新排序

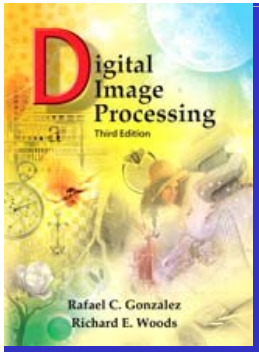
18	18	19	19	19	20	20	21	155

中值

18	19	18	20	21
19	155	20	21	21
20	19	21	20	19
19	20	21	160	18
17	19	22	20	19

胡椒鹽式雜訊

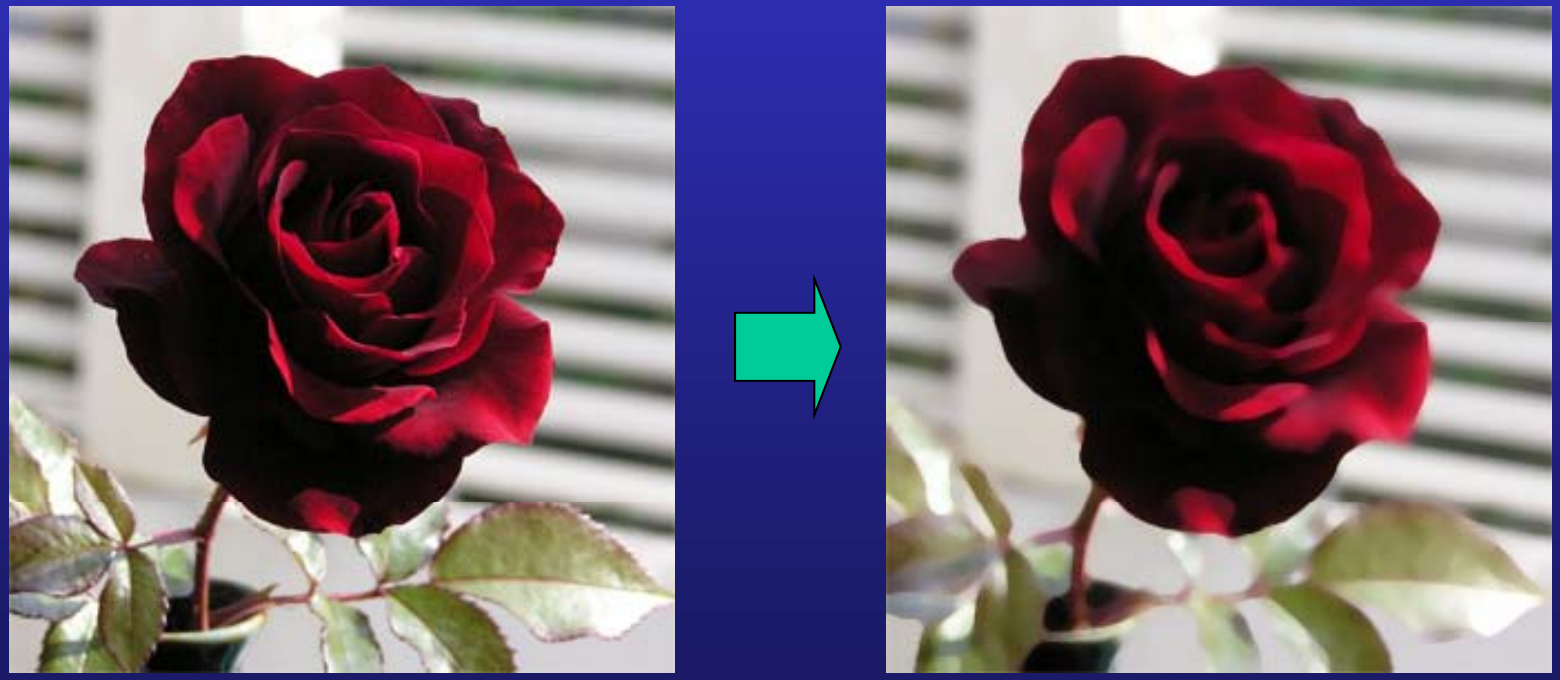
18	19	18	20	21
19	19	20	21	21
20	19	21	20	19
19	20	21	160	18
17	19	22	20	19

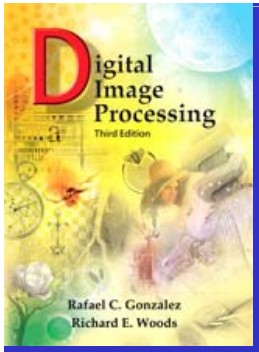


Chapter 3

強度轉換與空間濾波

- 中值濾波器—水彩畫特效





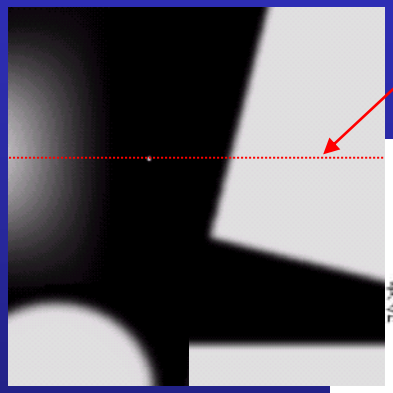
Chapter 3

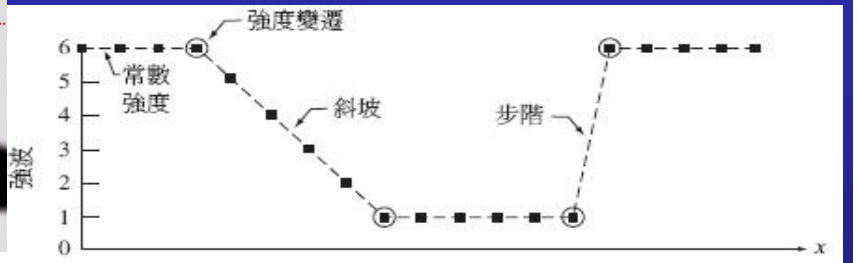
強度轉換與空間濾波

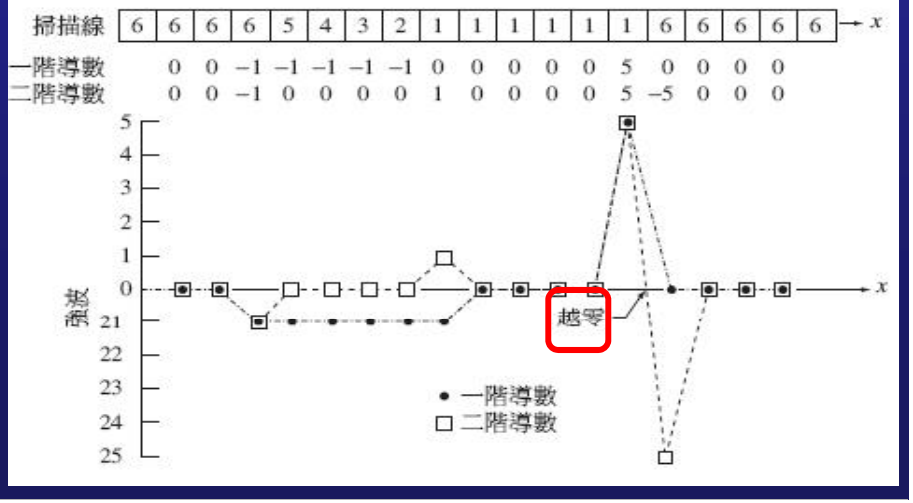
銳化空間濾波器(Sharpening spatial filters)

突顯影像中細微的部分或增強模糊的細節, 藉由微分來達成

- 微分運算子的響應強度正比於所在影像處的不連續程度
- 微分增強邊緣和不連續處





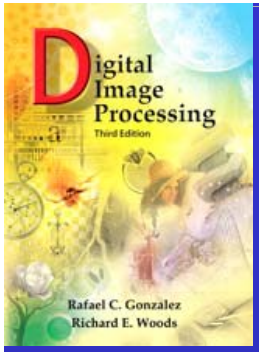


First - order derivative

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

Second - order derivative

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$



Chapter 3

強度轉換與空間濾波

銳化空間濾波器(Sharpening spatial filters)

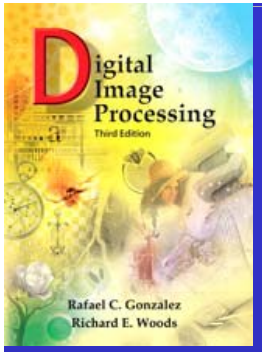
微分運算子的響應強度正比於運算子在其運用點處影像強度不連續的程度

一階導數 first derivative

- 在常數強度區域中爲零
- 在強度步階斜面起始處不爲零
- 沿著斜面不爲零

二次導數 Second-order derivatives

- 在常數強度區域中爲零
- 在強度步階或斜面起始處以及尾端不爲零
- 沿著有常數斜率的斜面爲零



Chapter 3

強度轉換與空間濾波

二次導數濾波器 Laplacian

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1,y) - 2f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

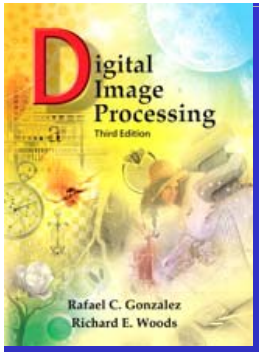
$$\nabla^2 f = [f(x+1,y) + f(x-1,y) + f(x,y+1) + f(x,y-1)] - 4f(x,y)$$

0	1	0	1	1	1
1	−4	1	1	−8	1
0	1	0	1	1	1
0	−1	0	−1	−1	−1
−1	4	−1	−1	8	−1
0	−1	0	−1	−1	−1

圖3.37
(a)用來實現 (3.6-6) 式的濾波器遮罩；

(b) 實現此式之延伸所用的遮罩，其中包括對角項；
(c)和 (d) 兩個實際上常見之拉普拉斯的其它實現。

中間值正負均可



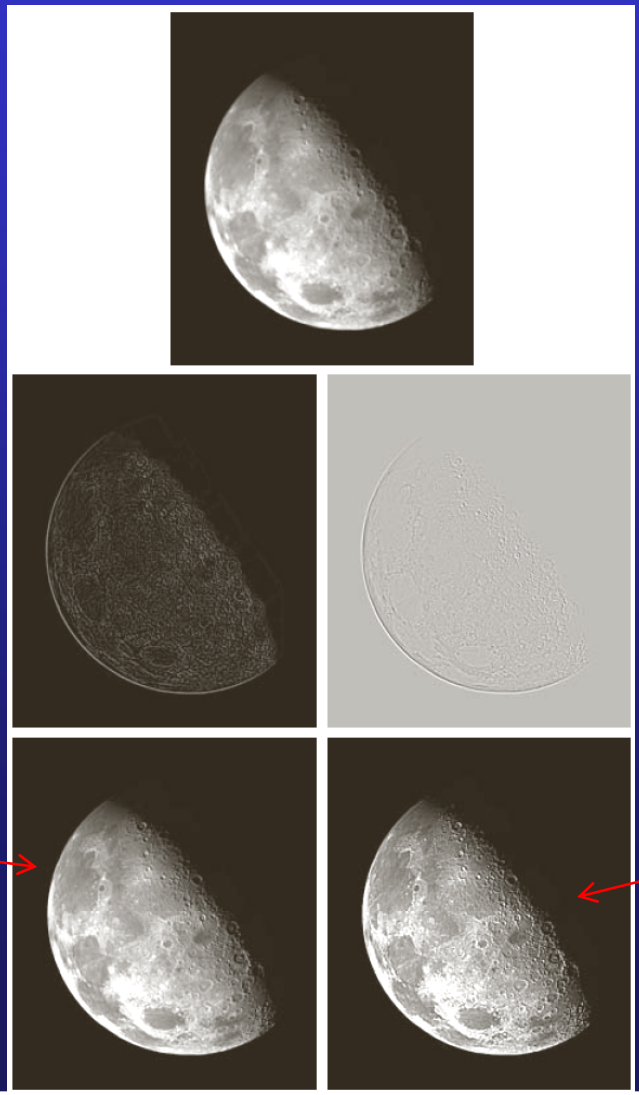
Chapter 3

強度轉換與空間濾波

$$f(x,y)+c[\nabla^2 f(x,y)]$$

- c = -1 if the center coefficient of the Laplacian mask is negative
- c = 1 if the center coefficient of the Laplacian mask is positive

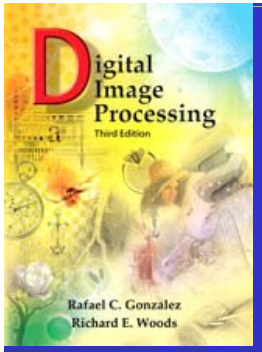
1	1	1
1	−8	1
1	1	1



0	1	0
1	−4	1
0	1	0

a
b c
d e

FIGURE 3.38
(a) Blurred image of the North Pole of the moon. (b) Laplacian without scaling. (c) Laplacian with scaling. (d) Image sharpened using the mask in Fig. 3.37(a). (e) Result of using the mask in Fig. 3.37(b). (Original image courtesy of NASA.)



Chapter 3

強度轉換與空間濾波

鈍化遮罩(unsharp masking)

$\overline{f}(x,y)$ 表示模糊的影像

$$g_{mask}(x,y) = f(x,y) - \overline{f}(x,y)$$

$$g(x,y) = f(x,y) + k * g_{mask}(x,y)$$

k=1 鈍化遮罩(unsharp masking)
k>1 高增幅濾波(highboost filtering)

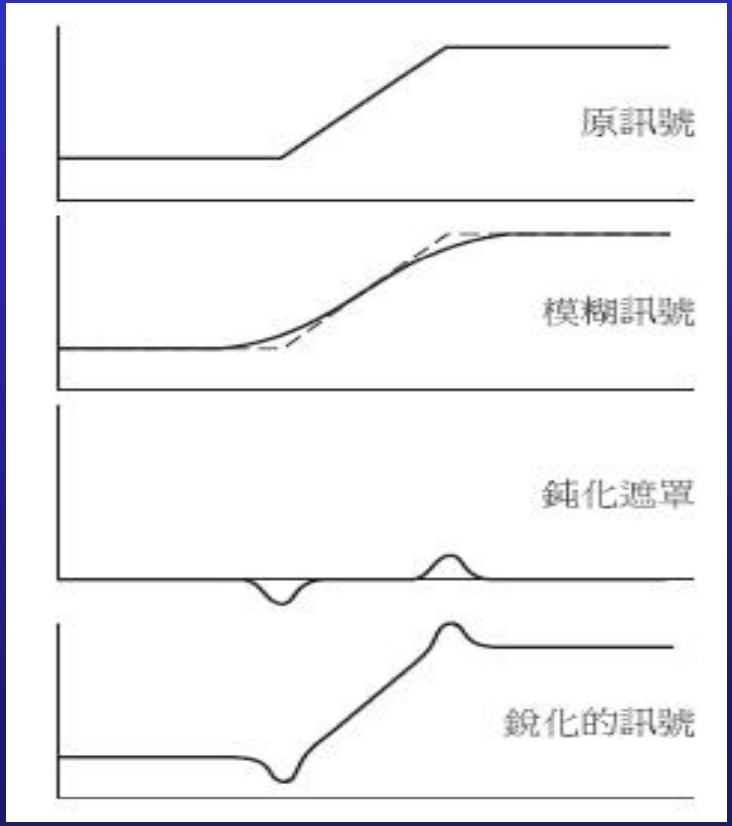
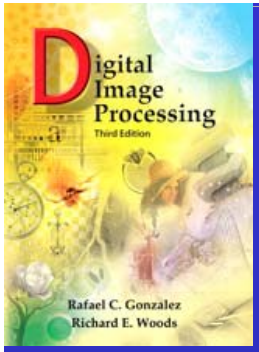


圖3.39 鈍化遮罩技術的一維展示。
(a) 原訊號；
(b) 模糊的訊號，其中原訊號顯示成虛線供參考；
(c) 鈍化遮罩；
(d) 將 (c) 加到
(a) 所得的銳化訊號。



Chapter 3

強度轉換與空間濾波

鈍化遮罩的二維範例

a
b
c
d
e

FIGURE 3.40
(a) Original image.
(b) Result of blurring with a Gaussian filter.
(c) Unsharp mask. (d) Result of using unsharp masking.
(e) Result of using highboost filtering.

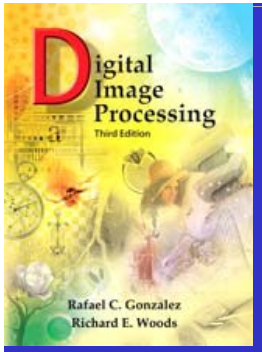
原圖

高斯濾波器模糊化（σ=3, 5＊5）

鈍化遮罩

原圖+鈍化遮罩 (k=1)

高增幅濾波(k=4.5)



Chapter 3

強度轉換與空間濾波

使用一階倒數於影像銳化-梯度 (First Derivatives – The Gradient)

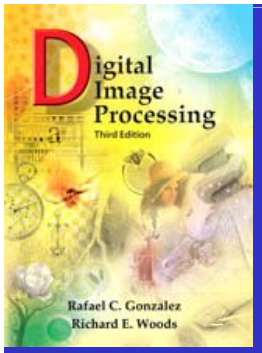
$$\nabla \mathrm{f} = \begin{bmatrix} G_x \\ G_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

The magnitude of this vector

$$M(x,y) = mag(\nabla f) = \sqrt{g_x^2 + g_y^2}$$

在實際應用上以絕對值來近似平方及平方根

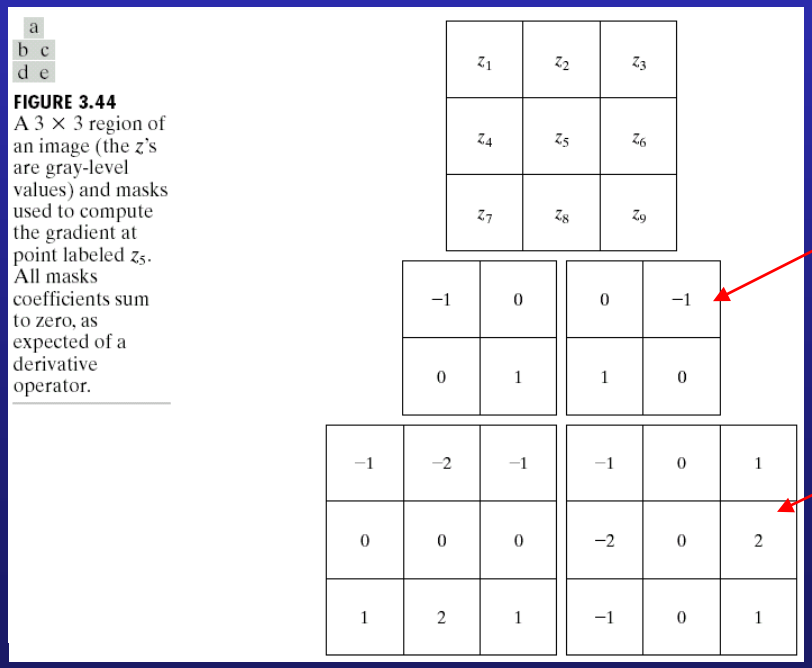
$$M(x,y) \approx |g_x| + |g_y|$$



Chapter 3

強度轉換與空間濾波

一階倒數增強-梯度
(First Derivatives – The Gradient)

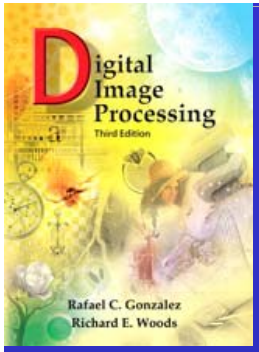


Roberts cross-gradient operators

$$g_x = (z_9 - z_5)$$
$$g_y = (z_8 - z_6)$$
$$M(x,y) = \left[(z_9 - z_5)^2 + (z_8 - z_6)^2\right]^{1/2}$$
$$M(x,y) \approx |z_9 - z_5| + |z_8 - z_6|$$

Sobel operators

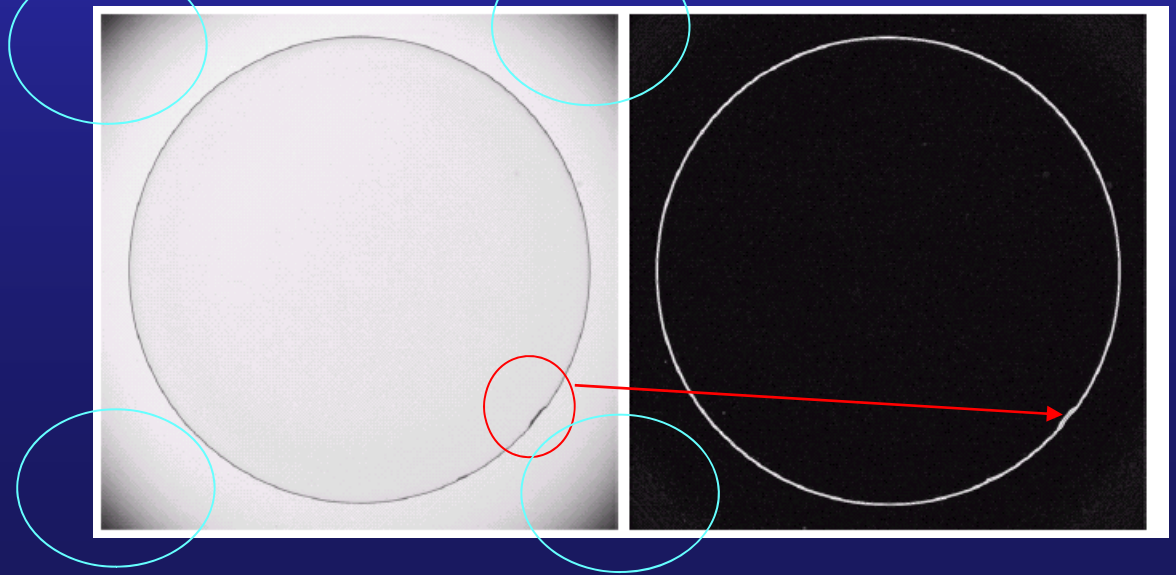
$$g_x = \frac{\partial f}{\partial x} = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)$$
$$g_y = \frac{\partial f}{\partial y} = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)$$
$$M(x,y) \approx |(z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3)| + |(z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7)|$$



Chapter 3

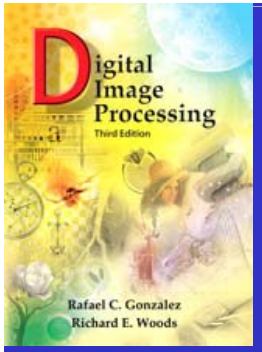
強度轉換與空間濾波

Part with Slowly varying shades of gray are eliminated in the gradient image



a b

FIGURE 3.42 (a) Optical image of contact lens (note defects on the boundary at 4 and 5 o'clock). (b) Sobel gradient. (Original image courtesy of Pete Sites, Perceptics Corporation.)



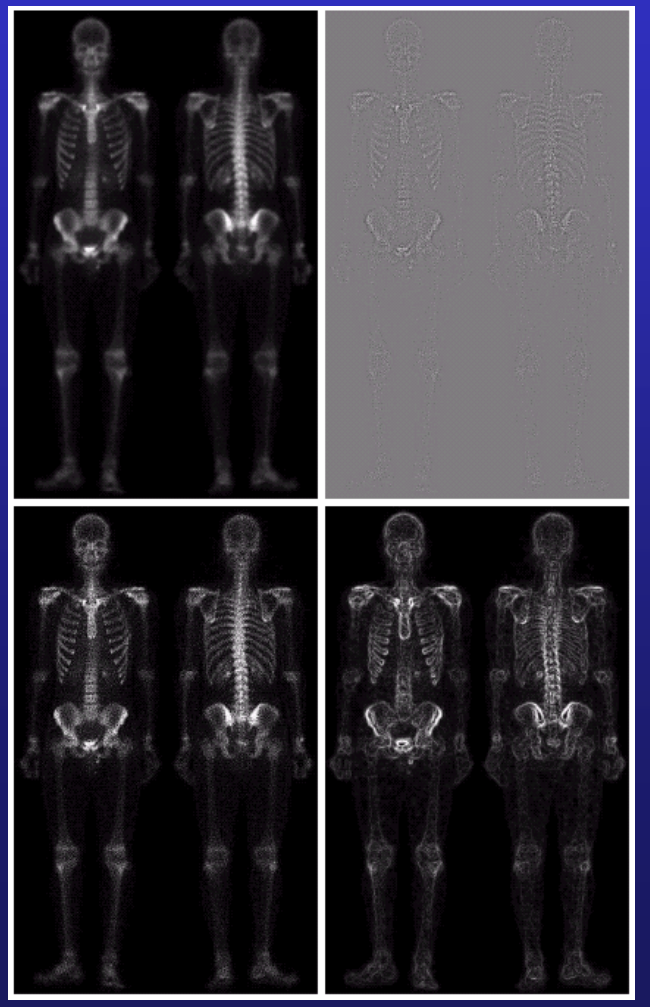
Chapter 3

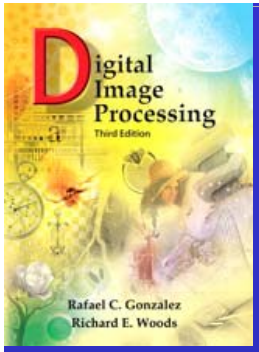
強度轉換與空間濾波

結合空間域增強的方法

全身骨骼核子掃描

- 處理目標
  - 藉由銳化及突顯更多骨骼細節來增強影像
- 影像特徵
  - 窄灰階動態範圍
  - 內容具有高雜訊
- 採取策略
  - Laplacian – 突顯細節
  - Gradient – 強化主要邊緣
  - Gray-level transformation – 增加灰階的動態範圍

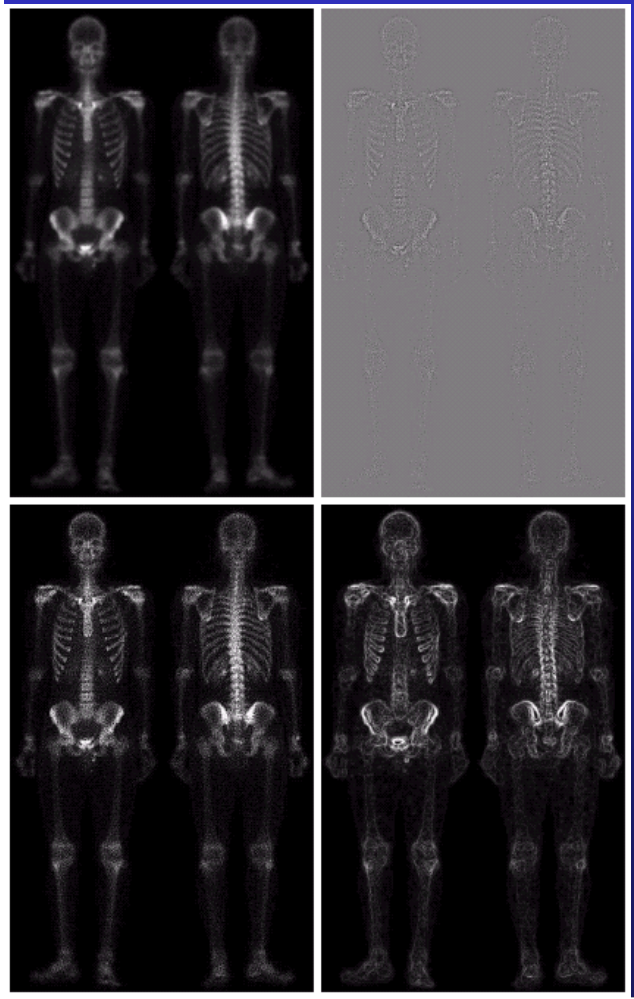




Chapter 3

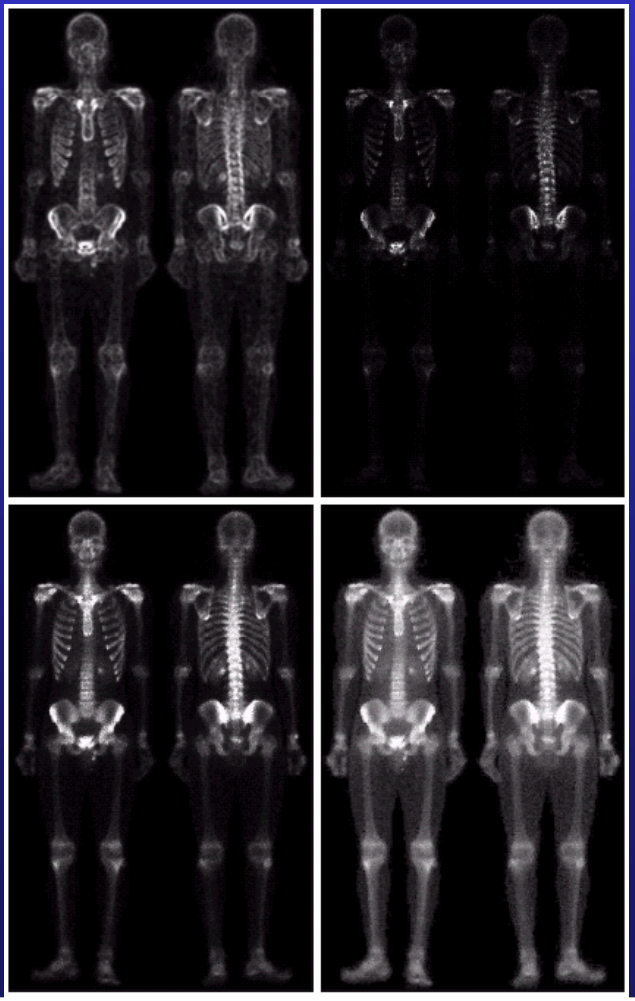
強度轉換與空間濾波

Combining spatial enhancement methods



a b
c d

FIGURE 3.43
(a) Image of whole body bone scan.
(b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).



e f
g h

FIGURE 3.43
(Continued)
(e) Sobel image smoothed with a 5×5 averaging filter. (f) Mask image formed by the product of (c) and (e).
(g) Sharpened image obtained by the sum of (a) and (f). (h) Final result obtained by applying a power-law transformation to (g). Compare (g) and (h) with (a). (Original image courtesy of G.E. Medical Systems.)