演算法：每個迴圈固定住其他feature只sample一個feature，並更新他的probability

# Gibbs Sampling

$\boldsymbol{y}^0 = \{y_1^0, y_2^0, \cdots, y_N^0\}$ ← Initialization

For t = 1 to T: ← T samples

$y_1^t \sim P(y_1 | y_2 = y_2^{t-1}, y_3 = y_3^{t-1}, y_4 = y_4^{t-1}, \cdots, y_N = y_N^{t-1}, \boldsymbol{x})$

$y_2^t \sim P(y_2 | y_1 = y_1^t, y_3 = y_3^{t-1}, y_4 = y_4^{t-1}, \cdots, y_N = y_N^{t-1}, \boldsymbol{x})$

$y_3^t \sim P(y_3 | y_1 = y_1^t, y_2 = y_2^t, y_4 = y_4^{t-1}, \cdots, y_N = y_N^{t-1}, \boldsymbol{x})$

$\vdots$

$y_N^t \sim P(y_N | y_1 = y_1^t, y_2 = y_2^t, y_3 = y_3^t, \cdots, y_{N-1} = y_{N-1}^t, \boldsymbol{x})$

Get a sample: $\boldsymbol{y}^t = \{y_1^t, y_2^t, \cdots, y_N^t\}$

↓

$\boldsymbol{y}^1, \boldsymbol{y}^2, \boldsymbol{y}^3, \ldots, \boldsymbol{y}^T$ ← As sampling from P(**y**|**x**)

# Gibbs Sampling

- Is $P(y_i | y_1, y_2, \ldots, y_{i-1}, y_{i+1}, \ldots, y_N, \mathbf{x})$ easy to be computed?

所有y同時sample

$y_i \epsilon \{+1, -1\}$ ➡ $2^N$ possible y

$$P(y|x) = \frac{e^{F(x,y)}}{\boxed{\sum_{y''} e^{F(x,y'')}}}$$ → Enumerate all possible y may not be tractable

$$P(y_i | y_1, y_2, \cdots, y_{i-1}, y_{i+1}, \cdots, y_N, \boldsymbol{x})$$

fix住所有y只留一個feature去做sample

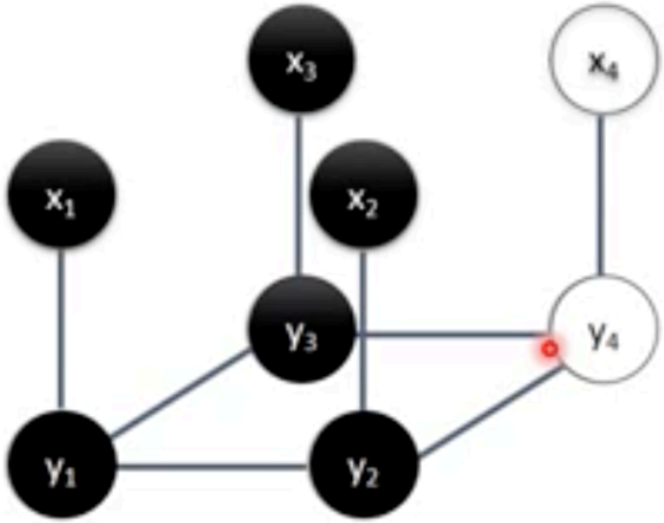
$y_i \epsilon \{+1, -1\}$ ➡ 2 possible $y_i$

$$= \frac{e^{F(x, y_{-i}, y_i)}}{\boxed{\sum_{y_i'} e^{F(x, y_{-i}, y_i')}}}$$ → Enumerate all possible $y_i$ may be tractable

這樣下來可能性變成很少因此可以窮舉，以剛剛的例子來看這邊只有兩種可能0 or 1

# 一開始先隨機初始化所有Y

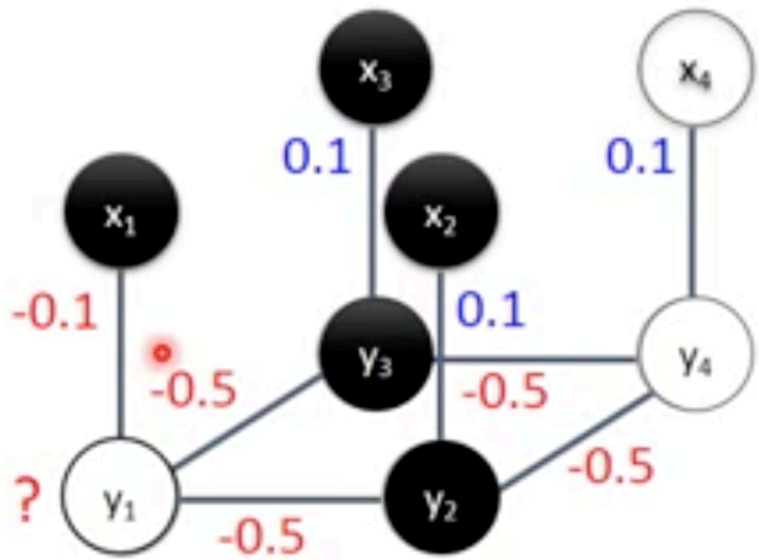
***Initialization*** $y_1, y_2, y_3, y_4 = -1, -1, -1, 1$




Sample from P(**y**|**x**)
by Gibbs sampling

Sample $y_1$ given all the other variables

$$y_1 \sim P(y_1|\boldsymbol{y}_{-1}, \boldsymbol{x}) \quad \boldsymbol{y}_{-1} = \{y_2, y_3, y_4\}$$




$$= \frac{P(x,y)}{\frac{e^{F(x,y)}}{\sum_{x',y'} e^{F(x',y')}}}$$

Compute $P(y_1 = 1|\boldsymbol{y}_{-1}, \boldsymbol{x})$ and $P(y_1 = -1|\boldsymbol{y}_{-1}, \boldsymbol{x})$

$$P(y_1 = 1|\boldsymbol{y}_{-1}, \boldsymbol{x}) = \frac{P(\boldsymbol{x}, y_1 = 1, \boldsymbol{y}_{-1})}{P(\boldsymbol{x}, y_1 = 1, \boldsymbol{y}_{-1}) + P(\boldsymbol{x}, y_1 = -1, \boldsymbol{y}_{-1})}$$

$$= \frac{e^{\boxed{F(x, y_1 = 1, \boldsymbol{y}_{-1})}} \; = -1.8}{e^{\boxed{F(x, y_1 = 1, \boldsymbol{y}_{-1})}} + e^{F(x, y_1 = -1, \boldsymbol{y}_{-1})}}$$

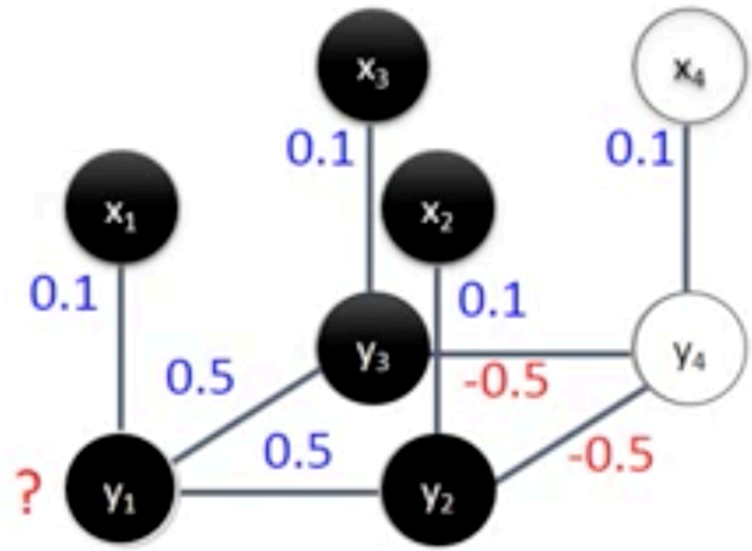
$= -1.8$



固定住所有Ｙ只留一個變動，sample所有可能，這邊是y1 =1

Sample $y_1$ given all the other variables

$$y_1 \sim P(y_1|\boldsymbol{y}_{-1}, \boldsymbol{x}) \qquad \boldsymbol{y}_{-1} = \{y_2, y_3, y_4\}$$




Compute $P(y_1 = 1|\boldsymbol{y}_{-1}, \boldsymbol{x})$ and $P(y_1 = -1|\boldsymbol{y}_{-1}, \boldsymbol{x})$

$$P(y_1 = 1|\boldsymbol{y}_{-1}, \boldsymbol{x}) = \frac{P(\boldsymbol{x}, y_1 = 1, \boldsymbol{y}_{-1})}{P(\boldsymbol{x}, y_1 = 1, \boldsymbol{y}_{-1}) + P(\boldsymbol{x}, y_1 = -1, \boldsymbol{y}_{-1})}$$

$$= \frac{e^{\boxed{F(x, y_1=1, y_{-1})}} \; = -1.8}{e^{\boxed{F(x, y_1=1, y_{-1})}} + e^{\boxed{F(x, y_1=-1, y_{-1})}}} = 0.10 \longrightarrow y_1 = -1$$

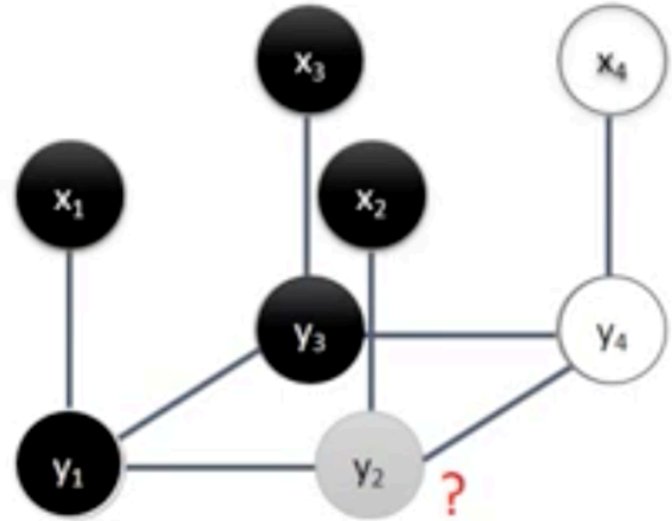
= -1.8 =0.4

y1=1的機率 Random sample sample的結果

當y1=−1的時候是0.4

Sample $y_2$ given all the other variables

$$y_2 \sim P(y_2 | \boldsymbol{y}_{-2}, \boldsymbol{x})$$




$$P(y_2 = 1 | \boldsymbol{y}_{-2}, \boldsymbol{x}) = \frac{P(\boldsymbol{x}, y_2 = 1, \boldsymbol{y}_{-2})}{P(\boldsymbol{x}, y_2 = 1, \boldsymbol{y}_{-2}) + P(\boldsymbol{x}, y_2 = -1, \boldsymbol{y}_{-2})}$$

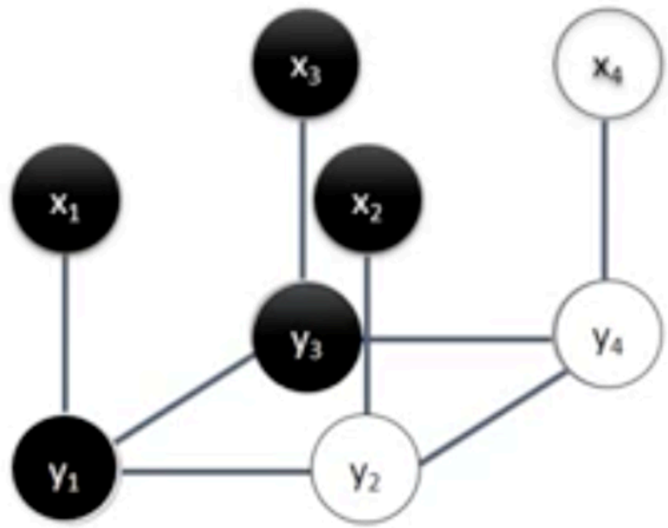
$$= \frac{e^{\boxed{F(\boldsymbol{x}, y_2=1, \boldsymbol{y}_{-2})}} \; =0.2}{e^{\boxed{F(\boldsymbol{x}, y_2=1, \boldsymbol{y}_{-2})}} + e^{\boxed{F(\boldsymbol{x}, y_2=-1, \boldsymbol{y}_{-2})}}} = 0.45 \xrightarrow[\text{Random}]{} y_2 = 1$$

y2=1的機率

sample的結果

Sample $y_3$ given all the other variables

$$y_3 \sim P(y_3 | \boldsymbol{y}_{-3}, \boldsymbol{x})$$

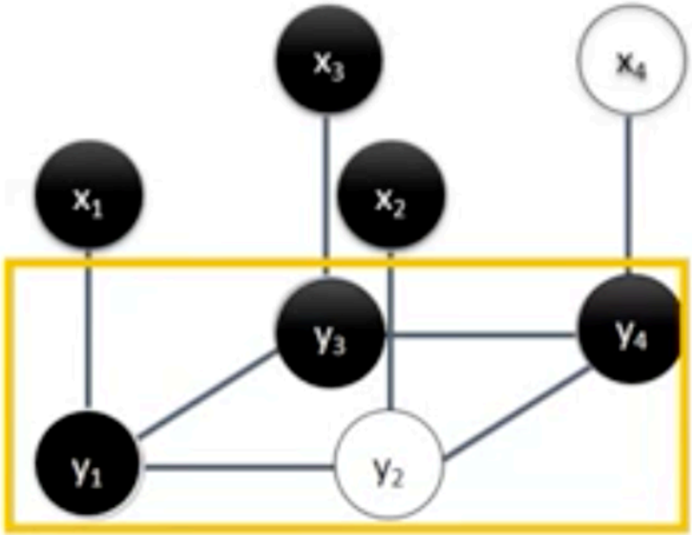



$P(y_3 = 1 | \boldsymbol{y}_{-3}, \boldsymbol{x}) = ?$ 0.45

Random sample → $y_3 = -1$

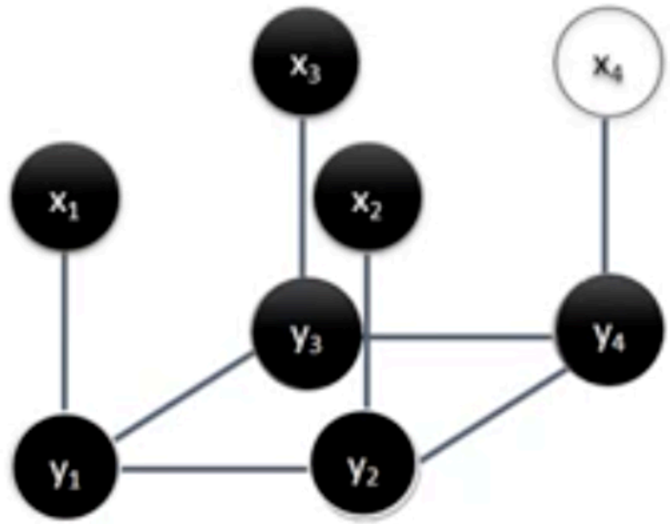
Sample $y_4$ given all the other variables

$$y_4 \sim P(y_4 | \boldsymbol{y}_{-4}, \boldsymbol{x})$$




Get ***1-st*** sample $y_1$=-1, $y_2$=1, $y_3$=-1, $y_4$=-1

得到第一筆sample的資料




Get ***1-st*** sample $y_1$=-1, $y_2$=1, $y_3$=-1, $y_4$=-1

Get ***2-nd*** sample $y_1$=-1, $y_2$=-1, $y_3$=-1, $y_4$=-1

Get ***1-st*** sample $y_1$=-1, $y_2$=1, $y_3$=-1, $y_4$=-1

Get ***2-nd*** sample $y_1$=-1, $y_2$=-1, $y_3$=-1, $y_4$=-1

Get ***3-rd*** sample $y_1$=1, $y_2$=1, $y_3$=-1, $y_4$=1

Get ***4-th*** sample $y_1$=-1, $y_2$=1, $y_3$=-1, $y_4$=1

Get ***5-th*** sample $y_1$=1, $y_2$=1, $y_3$=1, $y_4$=1

⋮

Until you want to stop