HW2 - Roots of Equations

1) Consider $f(x) = e^{-x}$ at $x_{i+1} = 1$ using a base value of $x = 0.25$ and a step size of $h = 0.75$

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x)$$

$f(x) = e^{-x}$

$f'(x) = -e^{-x}$

$f'' = e^{-x}$

$f''' = -e^{-x}$

$f^4 = e^{-x}$

$$f(x+h) = e^{-x} + h(-e^{-x}) + \frac{h^2}{2}e^{-x} + \frac{h^3}{6}(-e^{-x}) + \frac{h^4}{24}(e^{-x})$$

$$= e^{-x}\left(1 - h + \frac{h^2}{2} - \frac{h^3}{6} + \frac{h^4}{24}\right) + O(h^4)$$

b) Zero order approx $x = 0$ $h = 0.25$

$$f(0+0.25) = e^0(1) = 1$$

actual Val $e^{-0.25} = 0.7788$

$$\varepsilon_r = \left|\frac{0.7788 - 1}{0.7788}\right| = 0.2840$$

c) 1st order

$$f(0+0.25) = e^0(1-0.25) = 0.75$$

$$\text{True } \varepsilon_r = \left|\frac{0.7788 - 0.75}{0.7788}\right| = 0.0370$$

d) 2nd order

$$f(0+0.25) = e^0\left(1 - 0.25 + \frac{0.25^2}{2}\right) = 0.7813$$

$$\text{True } \varepsilon_r = \left|\frac{0.7788 - 0.7813}{0.7788}\right| = 0.0032$$

e) 3rd order

$$f(0.25) = e^0\left(1 - 0.25 + \frac{0.25^2}{2} - \frac{0.25^3}{6}\right) = 0.7786$$

$$\text{True } \varepsilon_r = \left|\frac{0.7788 - 0.7786}{0.7788}\right| = 0.00025$$

2) Determine the real root of $f(x) = (0.8 - 0.3x)/x$

a) $0 = \frac{(0.8 - 0.3x)}{x}$

$\cancel{x \neq 0}$ $\quad 0 = 0.8 - 0.3x$

$x = \frac{0.8}{0.3} \approx 2.667$

b) See Wolfram Alpha Screen shot

$x = 2.67$

c) $x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$ initial guess $(1, 3)$

① $x_r = 3 - \frac{f(3)(1-3)}{f(1) - f(3)} = 3 - \frac{(0.333)(-2)}{0.5 - (-0.333)}$

$= 2.875$

$\varepsilon_t = \left| \frac{2.667 - 2.875}{2.667} \right| \times 100 = 7.81\%$

$f(x_l) f(x_r) = f(1) f(2.875) = \frac{1}{2}(-0.02174)$

$= -0.01087$

replace upper ~~estimate~~ estimate w/ 2.875

② $x_r = 2.875 - \frac{f(2.875)(1 - 2.875)}{f(1) - f(2.875)} = 2.7969$

$\varepsilon_a = \left| \frac{x_{new} - x_{old}}{x_{new}} \right| \times 100 = 2.793\%$

$\varepsilon_t = \left| \frac{x_{true} - x_{old}}{x_{true}} \right| = 4.88\%$

$f(x_l) f(x_r) = f(1) f(2.7969) = -0.00698$ replace upper bound w/ x_r

③ $x_r = 2.7969 - \frac{f(2.7969)(1 - 2.7969)}{f(1) - f(2.7969)} = 2.74805$

$\varepsilon_a = \left| \frac{2.74805 - 2.7969}{2.74805} \right| \times 100 = 1.78\%$

$\varepsilon_t = \left| \frac{2.667 - 2.74805}{2.667} \right| \times 100 = 3.05\%$

d) ① initial guess $(1,3)$

calculate midpoint $\quad c = \frac{a+b}{2} = \frac{3+1}{2} = 2$

$f(c) = f(2) = 0.1$

$f(a) = f(1) = 0.5$

> same sign so $a = c$

② $(2,3)$ $\quad c = \frac{2+3}{2} = \frac{5}{2} = 2.5$

$f(2.5) = 0.02$

$f(2) = 0.1$

> same sign $a = c$

③ $(2.5, 3)$ $\quad c = \frac{2.5+3}{2} = \frac{5.5}{2} = \cancel{3.25}\ 2.75$

$f(2.75) = -0.00909091$

$f(2.5) = 0.02$

> diff sign so $b = c$

④ $(2.5, 2.75)$ $\quad c = \frac{2.5 + 2.75}{2} = 2.625$

Secant $x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$