HW3

$[A] = \begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix}$ $[B] = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix}$ $\{C\} = \begin{Bmatrix} 3 \\ 6 \\ 1 \end{Bmatrix}$ $[D] = \begin{bmatrix} 9 & 4 & 3 & -6 \\ 2 & -1 & 7 & 5 \end{bmatrix}$

$[E] = \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix}$ $[F] = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 7 & 3 \end{bmatrix}$ $[G] = [7 \ 6 \ 4]$

a) A → 3x2 B → 3x3 C → 3x1 D → 2x4 E → 3x3
F → 2x3 G → 1x3

b) B and E are square matracies
C is a column matrix
G is a row matrix

c)

	A	B	C	D	E	F	G
a_{12}	7	3	nan	4	5	0	2
b_{23}	nan	7	nan	7	3	3	nan
d_{32}	6	0	nan	nan	0	nan	nan
f_{12}	7	3	nan	4	5	0	2
g_{12}	7	3	nan	4	5	0	2

d) $[E]+[B] = \begin{bmatrix} 5 & 8 & 15 \\ 8 & 4 & 10 \\ 6 & 0 & 10 \end{bmatrix}$

$[A] \times [F] = \begin{bmatrix} 4 & 7 \\ 1 & 2 \\ 5 & 6 \end{bmatrix} \times \begin{bmatrix} 3 & 0 & 1 \\ 1 & 7 & 3 \end{bmatrix} = \begin{bmatrix} 17 & 26 \\ 27 & 39 \end{bmatrix}$

$[B] - [E] = \begin{bmatrix} 3 & -2 & -1 \\ -6 & 0 & 4 \\ -2 & 0 & -2 \end{bmatrix}$

$7([B]) = \begin{bmatrix} 28 & 21 & 49 \\ 7 & 14 & 49 \\ 14 & 0 & 28 \end{bmatrix}$

element wise multiplication

$$[E] \times [B] = \begin{bmatrix} 4 & 3 & 7 \\ 1 & 2 & 7 \\ 2 & 0 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 5 & 8 \\ 7 & 2 & 3 \\ 4 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 4 & 15 & 56 \\ 7 & 4 & 21 \\ 8 & 0 & 24 \end{bmatrix}$$

★ cannot apply the cross product ★
dimensions do not allow

$$\{C\}^T = \{3 \ 6 \ 1\}$$

$[B] \times [A]$ cannot multiply ~ dimensions do not match for cross product or element wise multiplication

$$[D]^T = \begin{bmatrix} 9 & 2 \\ 4 & -1 \\ 3 & 7 \\ -6 & 5 \end{bmatrix}$$

a) $-2.2x_1 + 20x_2 = 240$
$-1x_1 + 8.7x_2 = 87$

$$\rightarrow \left[\begin{array}{cc|c} -2.2 & 20 & 240 \\ -1 & 8.7 & 87 \end{array}\right] = [A]$$

or

$$\begin{bmatrix} -2.2 & 20 \\ -1 & 8.7 \end{bmatrix} = \begin{bmatrix} 240 \\ 87 \end{bmatrix}$$

$$[A^T] = \begin{bmatrix} -2.2 & -1 \\ 20 & 8.7 \\ 240 & 87 \end{bmatrix}$$

$$A^T = \begin{bmatrix} -2.2 & -1 \\ 20 & 8.7 \end{bmatrix}$$

$-2.2(8.7) - (-1)(20)$
$-19.14 + 20$
$A^T = -0.86$

The system is not singular b/c the determinant $\neq 0$

b) $x_1 \approx 404.651 \quad x_2 = 56.5116$

see plot

$-2.2x_1 + 20x_2 = 240$ ①

$-22(-x_1 + 8.7x_2 = 87)$

$2.2x_1 + -19.14x_2 = -191.4$ ②

① + ②

$0.86x_2 = 48.6$

$x_2 = 56.5116$

$x_1 = 8.7(56.5116) - 87$

$x_1 = 404.6511$

3) $5x_1 + x_2 - 0.5x_3 = 13.5$

$-6x_1 - 12x_2 + 4x_3 = -123$

$2x_1 + 2x_2 + 10x_3 = -43$

$$\left[\begin{array}{ccc|c} 5 & 1 & -.5 & 13.5 \\ -6 & -12 & 4 & -123 \\ 2 & 2 & 10 & -43 \end{array}\right]$$

$\frac{R_1}{5}$ ①

$\rightarrow R_2 + 6R_1$ ②

$\rightarrow R_3 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & -0.1 & 2.7 \\ 0 & -10.8 & 3.4 & -116.8 \\ 0 & 1.6 & 10.2 & -48.4 \end{array}\right]$$

$R_2 / -10.8$

$$\left[\begin{array}{ccc|c} 1 & 0.2 & -0.1 & 2.7 \\ 0 & 1 & -\frac{17}{54} & \frac{89}{9} \\ 0 & 0 & \frac{289}{27} & -\frac{578}{9} \end{array}\right]$$

$R_3 - 1.6R_2$ ✓

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{27} & \frac{13}{18} \\ 0 & 1 & -\frac{17}{54} & \frac{89}{9} \\ 0 & 0 & 1 & -6 \end{array}\right] \quad \begin{array}{l} \frac{1}{27}R_3 + R_1 \\ R_2 + \frac{17}{54}R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & -6 \end{array}\right]$$

$x_1 = \frac{1}{2}$

$x_2 = 8$

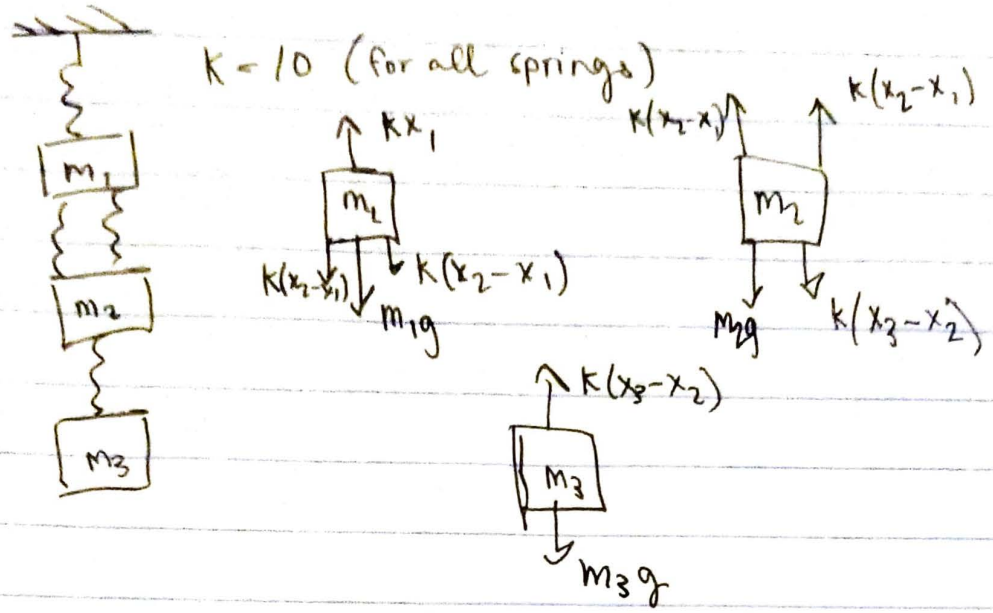
$x_3 = -6$

$5\left(\frac{1}{2}\right) + 8 - \frac{1}{2}(-6) = 2.5 + 8 + 3 = 13.5$ ✓

$-6\left(\frac{1}{2}\right) - 12(8) + 4(-6) = -3 - 96 - 24 = -123$ ✓

$2\left(\frac{1}{2}\right) + 2(8) + 10(-6) = 1 + 16 - 60 = -43$ ✓

4)



$$\Sigma F_{m_1} = kx_1 - 2(k(x_2 - x_1)) - m_1 g = 0$$

① $3kx_1 - 2kx_2 = m_1 g$

$$\Sigma F_{m_2} = 2(k(x_2 - x_1)) - m_2 g - (kx_3 - x_2) = 0$$

② $-2x_1$ ② $3kx_2 - kx_3 = m_2 g$

$$\Sigma F_{m_3} = k(x_3 - x_2) - m_3 g = 0$$

③ $kx_3 - kx_2 = m_3 g$

$$\left[\begin{array}{ccc|c} 30 & -20 & 0 & 19.62 \\ -20 & 30 & -10 & 29.43 \\ 0 & -10 & 10 & 24.525 \end{array}\right]$$

Augmented Matrix

$x_1 = 7.36$ $x_2 = 10.06$ $x_2 = 12.51$