1.3. **Topological Decoupling of QuasiPeriodic Videos.** Recall that if a function $f : \mathbb{R} \longrightarrow \mathbb{C}$ is quasiperiodic, then its sliding window point cloud $\mathbb{SW}_{d,\tau} f \subset \mathbb{C}^{d+1}$ is dense in a torus of dimension equal to the number of linearly independent frequencies in f. The number of independent oscillators. The story is exactly the same for a quasiperiodic video (such as the `dots_quasi.wmv` example from Weeek 1 - Day 4, SW1PerS II), which for a video with colors can be thought of as a quasiperiodic function $f : \mathbb{R} \longrightarrow \left(\mathbb{R}^H \times \mathbb{R}^W\right)^3$, where $f(t) = \left[f_1(t),\ f_2(t),\ f_3(t)\right]$ is so that each $f_j(t)$ is the image frame of height H and width W at time t, corresponding to the color channels red ($j = 1$), green ($j = 2$) and blue ($j = 3$).

The goal of this project is to device algorithms which, given a quasiperiodic video showing N independent oscillators, the output is a reordering of the frames in N different ways — resulting in N different videos. This is done in such a way that for the n-th video only the n-th oscillator is moving while the others appear almost stationary. Going back to the example of the `dots_quasi.wmv` video, we want to generate two videos: one where the leftmost dot is moving from left to right while the right dot appears stationary (moving very slowly), and a second video where the leftmost dot appears stationary while the rightmost dot is moving from left to right.

A place to start:

(1) Think of how you can recover the N-torus from the video data. Try your ideas on the `dots_quasi.wmv` example.
(2) Try to think of the relation between traversing this torus in different ways, and reordering the frames of the video to get the desired results.

References.

(1) (Quasi)Periodicity Quantification in Video Data, Using Topology, by Christopher J. Tralie and and Jose A. Perea, https://arxiv.org/pdf/1704.08382.pdf
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