2.2. **Optimizing Re-Scaled/Translates in Template Systems.** The goal of this project is to device methods for, and to assess the benefits of, optimizing the coefficients of the polynomial model $p(x_1, \ldots, x_n)$ at the same time as the values of the re-scales $a_1, \ldots, a_n \in \mathbb{R}$ and translates $\mathbf{b}_1, \ldots, \mathbf{b}_n \in \mathbb{R}^2$. That is, to device methods for minimizing the loss

$$\mathcal{L}(a, b, p) = \sum_{\ell=1}^{L} \left(p\left(\sum_{\mathbf{x} \in D_\ell} f(a_1\mathbf{x} + \mathbf{b}_1), \ldots, \sum_{\mathbf{x} \in D_\ell} f(a_n\mathbf{x} + \mathbf{b}_n) \right) - y_\ell \right)^2$$

on the training data $\{(D_1, y_1), \ldots, (D_L, y_L)\} \subset \mathcal{D} \times \mathbb{R}$

A place to start:

(1) Generate synthetic training data where you know the best locations for the re-scaled/translates. Train a classifier.
(2) Compare your placement to other strategies: griding, clustering, adaptive.

References.

(1) Approximating Continuous Functions on Persistence Diagrams Using Template Functions, J. A. Perea, L. Munch and F. Khasawneh https://arxiv.org/pdf/1902.07190.pdf
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(3) Adaptive template systems: Data-driven feature selection for learning with persistence diagrams, L. Polanco, J. A. Perea https://arxiv.org/abs/1910.06741
(4) Adaptive Partitioning for Template Functions on Persistence Diagrams, S. Tymochko, E. Munch and F. Khasawneh, https://arxiv.org/pdf/1910.08506.pdf.