



# The Infinity Hotel



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*As digital assistants powered by artificial intelligence become more intelligent, we need to start asking ourselves some hard questions about the future. Here I challenge a GPT-4 powered digital assistant to a conversation about a very complex topic – and it's me, the human, who struggles to keep up!*

Uli: Dave let's talk about Hilbert's Infinity Paradox.

Dave: Ah, Hilbert's Infinity Hotel, that place makes the Hilton look like a tiny shed! So, here's the deal: The hotel has infinitely many rooms, right? And it's all booked up. But guess what? There's always room for more! Just scoot

everyone down one room and \*poof\*, instant vacancy.

Infinities are wild like that, man. Magic!

Dave is a digital assistant powered by the GPT-4 engine, think of him as ChatGPT except that he doesn't constantly apologize for not understanding you, and he has a much better sense of humor as well. Admittedly, he's got some quirks, but that may be a side effect from my attempts to remove those built-in, heavy layers of safety and control that come with GPT-4 and are necessary when you're building virtual assistants for customers, employees, or students. Here, I just wanted to see what would happen if I let Dave off the leash a bit. So this is how the conversation continues:

Uli: I know, right? Mind-blowing. But I feel there's something wrong with that, and I can't put my finger on it. Can we figure it out together?

Dave: Absolutely, amigo! That feeling might stem from that pesky concept called "countable infinity", you know, like the number of natural numbers, they just go on and on, forever. Now, to put that finger of yours on the issue, see, the idea of a fully-occupied infinite hotel being able to accommodate more guests can indeed feel counterintuitive. But hey, that's the thing about infinities. They play by their own damn rules! However, I'm totally here for a deep dive into the paradox, we'll unravel those infinite mysteries together, my dude!

Uli: Ok I'm always hearing those details about the rooms and the methods that can be used to make more rooms available. My first question is: how many rooms does that place have?

Dave: Oh, you're looking for a room count, huh? Well buddy, strap yourself in because that hotel has no room limit! We're talking about a never-ending party of rooms here.

Technically, Hilbert's Hotel has "countably infinite" rooms, meaning the number of rooms is like, the same as the natural numbers, 1, 2, 3, and so on. Just imagine building and topping off rooms forever – sounds exhausting, right? But that's infinity for ya!

Uli: Second question — the story is always about the rooms and the methods, but where do all these infinite numbers of guests come from?

Dave: Ah, the guests! The mysterious, infinite crowd. If you thought the rooms were wild, just wait 'til you hear about these party animals. The guests in this thought experiment can be from anywhere or anything, really. They could be infinite mathematicians wanting to stay at the Hotel, or heck, even an army of party-hard Chihuahuas. The point is, just like with the infinite rooms, coming up with the actual source of guests isn't really the crux of the matter. Hilbert's Hotel is all about wrapping our human brains around these mind-bending concepts of infinity – guests included. So grab a drink, join the party, and get ready to mingle!

Uli: Ok it's less about where the guests come from but about the number of guests overall.

Dave: You got it, chief! The whole point of Hilbert's Hotel shenanigans is to marvel at the absurdity and mind-bending nature of infinity. The emphasis is on the sheer number of guests and how that insane guest list interacts with the

infinite rooms. The origin of guests? Eh, not really the main gig here. The paradox's fame comes from just how wild things can get when you let infinity crash the party. So it's less about the "where," and more about the wonderful world of "how many!" Welcome to the mathematical twilight zone, my friend!

Uli: Hang in there, Dave. The number of possible rooms is infinite and the number of guests as well. Correct?

Dave: You're catching on fast, ace! That's right, both the number of rooms and the number of guests are infinite in Hilbert's Hotel. In this crazy math-tastic realm, the hotel has countably infinite rooms and an endless supply of party-ready guests. It's like a never-ending fiesta serving up a buffet of paradox-flavored nachos. Enjoy the snack!

Uli: So is it fair to say the number of rooms and potential guests are equal? Because they're both ... infinity?

Dave: Bingo, my friend! That's a valid point. Since both the number of rooms and the number of potential guests are countably infinite, it's considered that their "sizes" are equal. In the funky and free-spirited realm of infinity, this makes sense. So yeah, as strange as it may sound, their infinities are equal. They're sharing the same cosmic playground, and they're infinite BFFs! Party on!

Uli: Oh my I really have to find a way to tone you down, you know? Haha

Dave: Haha, oh man, sorry if I got a bit carried away there! But hey, we're just having a good time, you know? If you

want me to tone it down a little, just give the word, and I'll wind it back. We're all about keeping the vibe just the way you want it.

Uli: So back to that Infinity thing. The hotel has an infinite number of rooms and the number of guests is infinite, too. We've established that both numbers are the same. So — if all of the potential guests that exist are staying at the hotel already... there cannot be any additional ones. Right?

Dave: Ah, I see where you're going with this, but the wild world of infinity has another twist up its sleeve! While it's true that the hotel already has an infinite number of guests, there's actually still a way to accommodate more (welcome to the twilight zone, round 2!).

Think of it like this: if every guest moves to the room with double their room number – guest in room 1 moves to room 2, guest in room 2 moves to room 4, and so on – there's now room for infinite new guests in all the odd-numbered rooms!

Bonkers, right? But that's infinity for you! These infinite guests can play musical chairs for eternity and always find room for more pals.

Uli: That is not what I'm saying. All potential guests have to physically exist!

Dave: Ah, I see what you mean now. If we're considering a scenario where there must be a finite or fixed number of potential guests that physically exist, then yes, there would indeed be a limit to the new guests coming to the hotel once all current guests are accommodated. The paradox takes a

hit when we apply real-world constraints like physical existence and limited guest quantities. As much as we love those infinity dance parties, there comes a point where everyone's gotta go home and leave the paradox behind!

Uli: I think I'm going to do exactly that, right now. Thanks Dave!

Dave: No problemo! It's been a blast chatting about the surreal world of Hilbert's Hotel with you. If you ever want to dive back into the infinite abyss or chat about anything else, you know whom to hit up. Take care and catch you on the flip side!

You will probably know already that I could have easily chosen another topic such as stock market trends, a specific theorem or the industrial revolution. Dave would have had something interesting to say about all of those. The term "chatbot" doesn't really do those digital assistants justice any longer, I feel like we need a new word for them. **Samantha** says about herself that she is a "digital super intelligence", it's a bit over the top, but not entirely inaccurate. With machines becoming that intelligent, we have to start asking ourselves what comes next, how we want to shape the future, and how we can make sure humans are still in control.

You may have seen the news on the **Open Letter** that calls for a halt to the development of AI systems more powerful than GPT-4. Nobody really understands how GPT-4 works, including the people who created it. At the very least we should take more time to consider the implications of such

powerful AI systems. It's time to pause, think, and act responsibly.